Bond Pricing and Capital Requirements

FM 9528 Banking Analytics

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Executive Summary

In this report, we analyzed a dataset consisting of 6262 corporate bonds traded in the US and 10 features including price, maturity, treasury spread, offering date, principal, interest frequency, coupon rate, rating, and security level. We re-priced these bonds based on Canadian yield curve on May 31, 2024 and yield curve on January 13, 2025 to analyze the effect of the interest rate on bond prices. To get the accurate risk-free rate, we interpolated the yield curves using quadratic interpolation. We generated the coupon payment dates from the offering date to the maturity using the interval determined by interest frequency. We then discounted future coupon payments and the principal using the risk-free rates plus the spread at each date, getting the present value of bonds. Results show that 80.4% of bonds had higher prices, with an average price increase of $3.75 under the Canadian yield curve, which was approximately 1.39% lower than the U.S. Bonds were valued even higher on January 13, 2025, before which the short-term interest rate had dramatically decreased, resulting in a further average price difference of $1.18.

We therefore concluded that a Canadian investor subject to Canadian interest rates would invest in U.S. bonds, because domestic interest rates were lower than those abroad and money flows to wherever there are high returns. In other words, a Canadian investor would consider U.S. bonds underpriced when they discount using the Canadian rates.

Under banking regulations, banks are required to set aside regulatory capital for risk management. For a portfolio of bonds, the risk is credit risk. Thus we computed the capital requirements and provision for this portfolio. We first derived risk components according to the OSFI CAR guideline using the Foundation Internal Rating Based (F-IRB) approach. Specifically, the Probability of Default (PD) was simulated using the log-normal distribution in each rating, and LGD were mapped from seniority of the bonds. We then applied the capital requirement equation to each individual bond to get the capital requirements, which were then aggregated to the portfolio level. The provision was calculated in a similar fashion.

We analyzed the distribution of the capital requirement and the provision using descriptive statistics. We found that for $100 exposure to a portfolio of bonds, the bank needs to set aside $3.21 ~ $3.26 as capital, and $0.123 ~ $0.129 as provision. We further analyzed each rating's contribution to the capital requirement and provision. CC bonds contributed the most: for $1 exposure to them, $0.18 was unexpected loss and $0.013 expected loss and thus needed to be set aside as regulatory capital and provision. This shows that although CC bonds have a high yield, they are likely to have low risk-adjusted return on capital (RAROC).

Contents

Part 1

Repricing Bonds Using Canadian Interest Rates

The data set is a portfolio of 6262 bonds traded in the US on May 31, 2024. Figure 1 explores the variables in the dataset. It is shown that most prices were between 75% to 125% percent of the principal; most bonds were offered in recent 25 years; coupons are paid semi-annually but are not always so; and the annual coupon rate ranges between 2.5% to 7.5%.

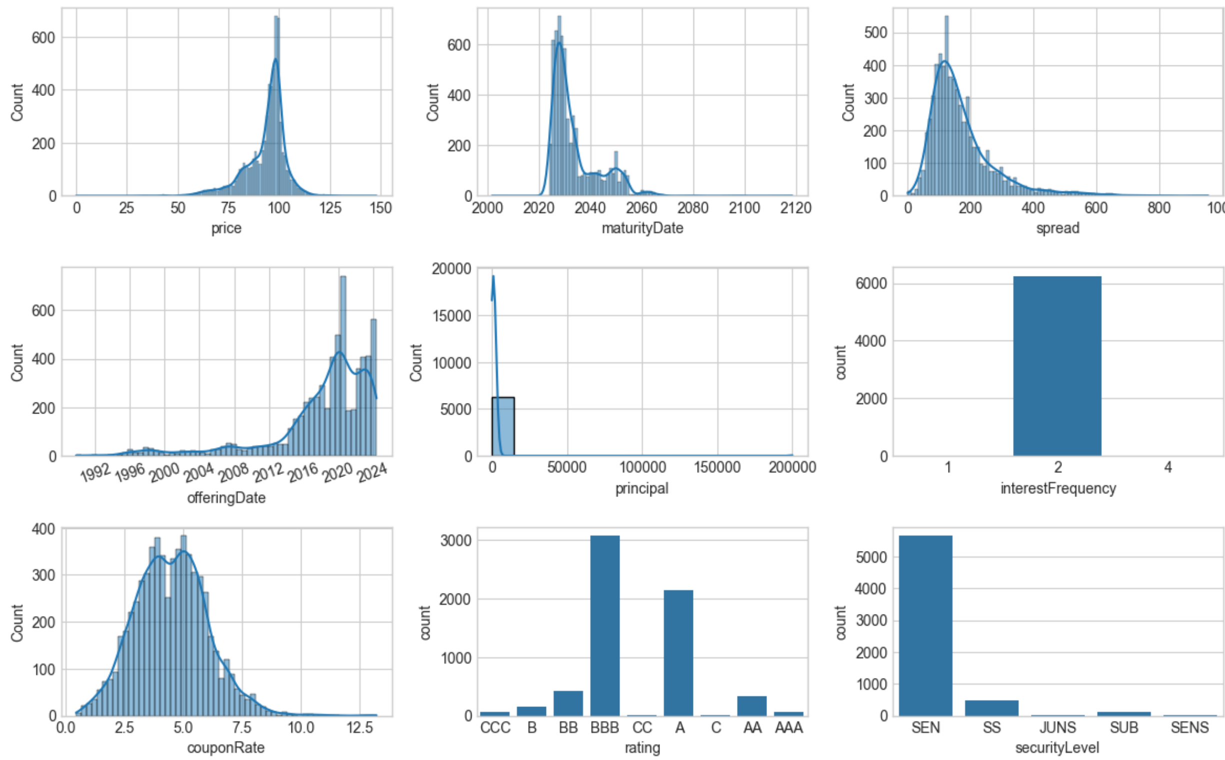

Figure 1. Exploratory Data Analysis

As can be seen in Figure 2, on May 31 2024, Canadian interest rates were lower than the US. To analyze the impact of bonds being traded in Canada and thus being subject to Canadian rates, we repriced the bonds by calculating the present value of these bonds' cash flows with the time-varying discount rates, which is the sum of risk-free rates represented by the Canadian yield curve on May 31, 2024, and the credit spread. Risk free rates were interpolated using quadratic interpolation. Figure 3 is an example of interpolated risk-free rates for the first bond in the dataset. They follow the yield curve closely.

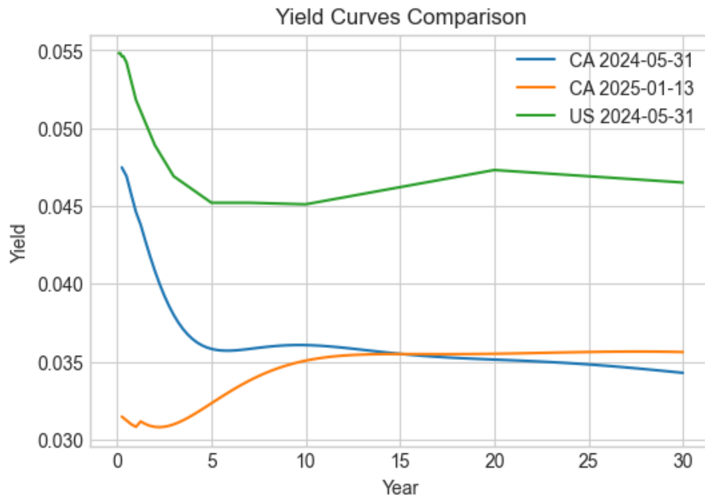

Figure 2. Yield Curves Comparison

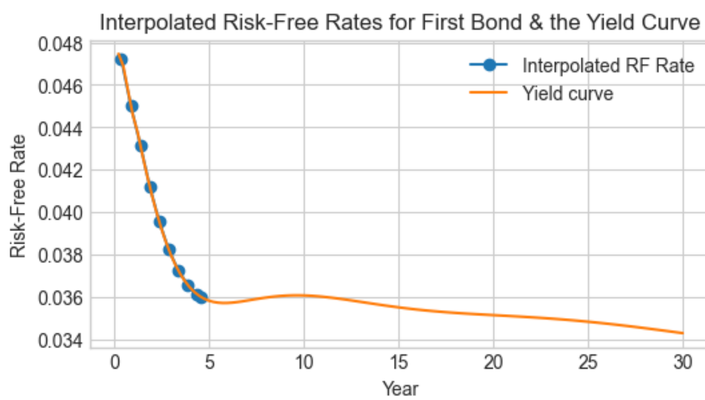

Figure 3. Yield Curve Interpolation

Comparison and Analysis

Having discounted the future cash flows and obtained the new prices, we observed that the majority (80.4%) of bonds had a price increase under the Canadian interest rate, in line with the expectation that lower interest rates should result in higher bond prices. The average price difference was 3.75% of the principal. Nevertheless not all bonds were priced higher. We think market microstructure may be the reason: the influences of supply and demand, liquidity, and market players may have influenced the actual traded price of the bonds; while the Canadian price is the theoretical.

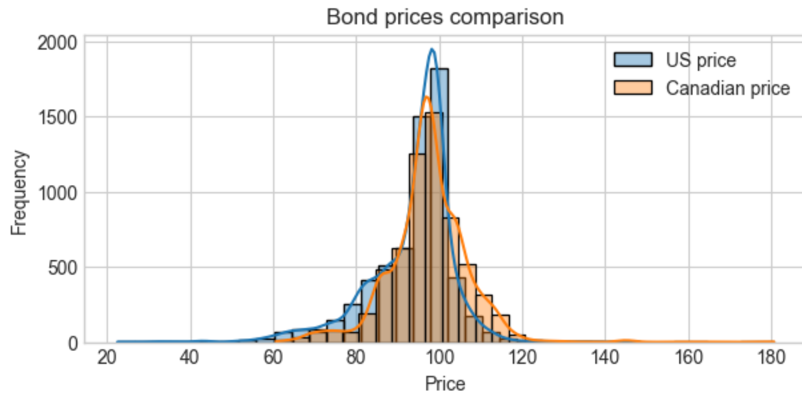

Figure 4. Bond Prices Comparison

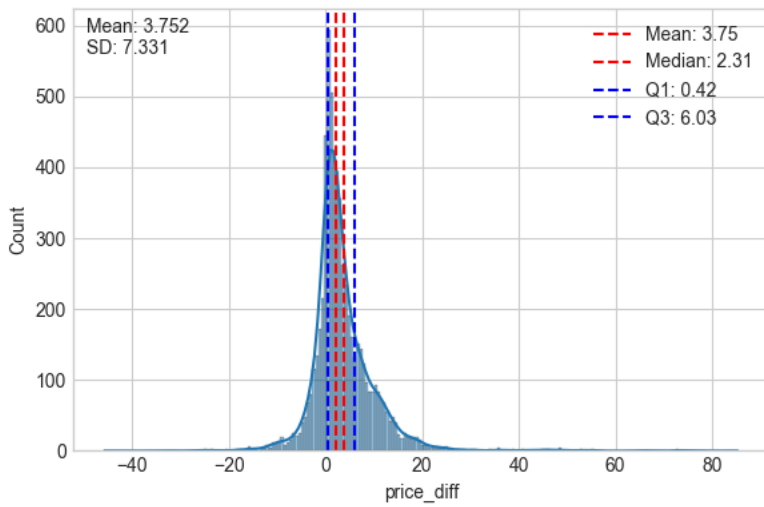

Figure 5. The Distribution of the Bond Price Difference

The difference in bond prices between the two markets shows the impact of different interest rate environments. Given the same future cash flows, lower domestic interest rates lead to foreign assets being valued higher.

The U.S. market was more attractive, as when a Canadian investor discounts a bond using the Canadian rates, he would find U.S. bonds underpriced (assuming the Canadian dollar was on par with the US dollar) so he would go to the US bonds market to purchase these bonds instead of investing domestically.

Part 2

We now reprice the bonds again, but with the Canadian yield curve on Jan 13, 2025.

Comparison of Bond Price Distributions

Using the same method as above, we got the bond prices under the Canadian yield curve on Jan 13, 2025. The price distributions are shown in Figure 7.

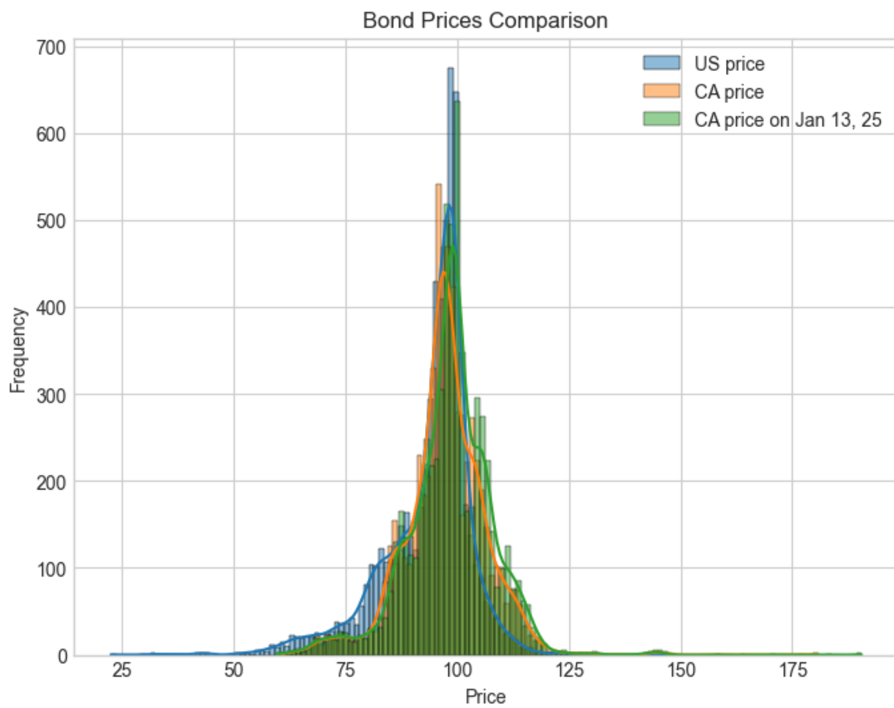

Figure 7. Bond Prices Comparison

There was a shift of the distribution to the right due to even lower interest rates. Approximately 88.4% of the bonds experienced a price increase. The average increase in the Canadian price was 1.18% of the principal.

Events, Rate Cuts, and Their Impact on the Fixed-Income Market

Between May 31, 2024 and January 13, 2025, Bank of Canada (BoC) decreased the policy interest rate five times, from 5.0% to 3.25% (Bank of Canada, 2025) since inflation was under control and the BoC had shifted its focus towards growth (Dewan, 2024). Therefore, we observed the yield curve as shown in Figure 2. Thus, bonds prices increased due to lower interest rates.

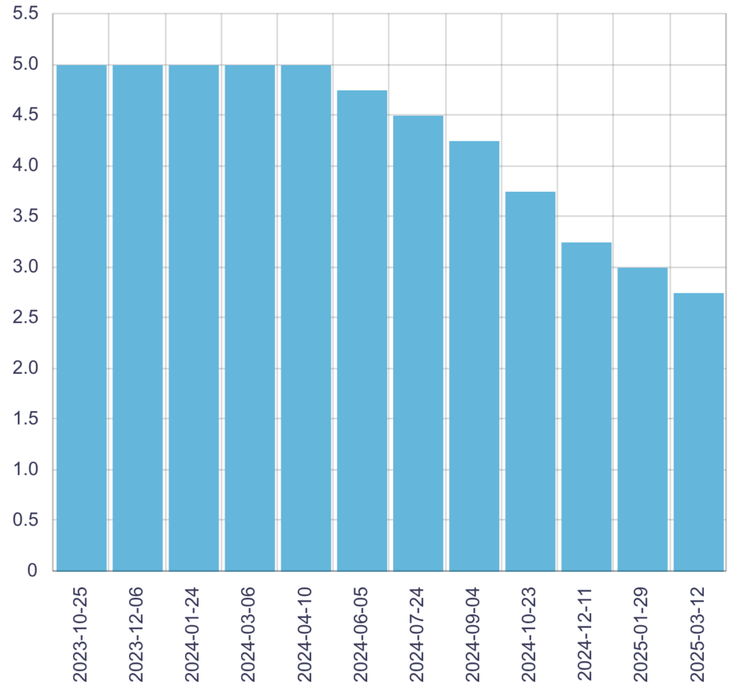

Figure 6. BoC Policy Rate Changes (Bank of Canada, 2025)

Part 3

To obtain the required regulatory capital and provision for a portfolio of bonds, we first calculated the capital requirement for individual bonds. To do so, we first derived risk components (including PD, LGD, and EAD) in the capital requirement equation using the Foundation Internal-Ratings Based (F-IRB) approach according to the Capital Adequacy Requirements Guideline (OSFI, 2023), which is based on Basel framework. The capital requirement equation was then applied to each bond.

LGD

Loss Given Default (LGD) is the ratio of the loss on an exposure to the amount outstanding at default. We referred to Chapter 5.4.1 (ii) for details about mapping the seniority level to LGD under the foundation approach; the idea is as follows:

- Senior unsecured claims on non-financial corporates: LGD = 40%
- All subordinated, unsecured claims on corporates: LGD = 75%
- Effective LGD applicable to a collateralized transaction = weighted average of the LGD applicable to the unsecured part of an exposure and the LGD applicable to the collateralized part of an exposure.

Thus senior bonds in the dataset are assigned an LGD of 40% as they are assumed to be non-financial corporate bonds. (Senior) secured bonds were assumed to have an LGD of 20% due to lack of information about the type of collateral associated with each bond. Junior bonds are assigned to an LGD of 75%, as they are often subordinated to senior bonds.

Table 1. Seniority-LGD Mapping

Seniority level	LGD (%)
Junior	0.75
Junior Subordinate	0.75
Senior	0.40
Senior Subordinate	0.75
Senior Secured	0.20
Subordinate	0.75

EAD

Exposure at Default (EAD) is defined as the gross exposure on the moment of default. Since these bonds have no collateral, EAD is not reduced through it. And as these bonds are not off-balance sheet items, we do not need to consider additional drawings prior to default. Due to conservatism, no downward adjustment for amortization or expected prepayment are allowed for the EAD of on-balance sheet items, so EAD is the outstanding amount at the time of capital requirement calculation, that is, the price (which is expressed as a percentage of the principal) times the face value. We derived EAD this way.

PD

Probability of Default (PD) is the likelihood of a default over a particular time horizon. For corporate exposure, the PD is the one-year PD associated with the internal borrower grade to which that exposure is assigned (OSFI, 2023). As the objective of this project does not include PD prediction, we adopted a simulation approach where we treated PD as a random variable (except for AAA rating bonds) that follows a log-normal distribution and simulated 10,000 realizations. Specifically, in each rating grade, PD is generated as follows

$$Z \sim N(0,1)$$
$$PD = e^{\mu+\sigma Z}$$
$$ln(PD) \sim N(\mu,\sigma^2) \text{ } or \text{ } PD \sim Lognormal(\mu,\sigma^2)$$

Thus the mean and the standard deviation of PD in each rating are

$$E(PD) = e^{\mu+1/2\sigma^2}, Var(PD) = (e^{\sigma^2} - 1) \times e^{2\mu+\sigma^2}$$

$E(PD)$ and $Var(PD)$ should align with Table 2. In order to produce them we backed out μ and σ where

$$\mu = ln \frac{E(PD)^2}{\sqrt{E(PD)^2 + Var(PD)^2}}$$
$$\sigma = ln(1 + \frac{Var(PD)^2}{E(PD)^2})$$

With $E(PD)$ and $Var(PD)$ provided in the dataset and shown in Table 2.

Table 2. Rating-PD Mapping

	AAA	AA	A	BBB	BB	B	CCC/C
Weighted long-term average	0	0.02	0.05	0.14	0.57	2.98	25.98
Standard deviation	0	0.06	0.1	0.25	0.96	3.23	11.73

Figure 8 shows the distributions of the simulated PD values for each grade.

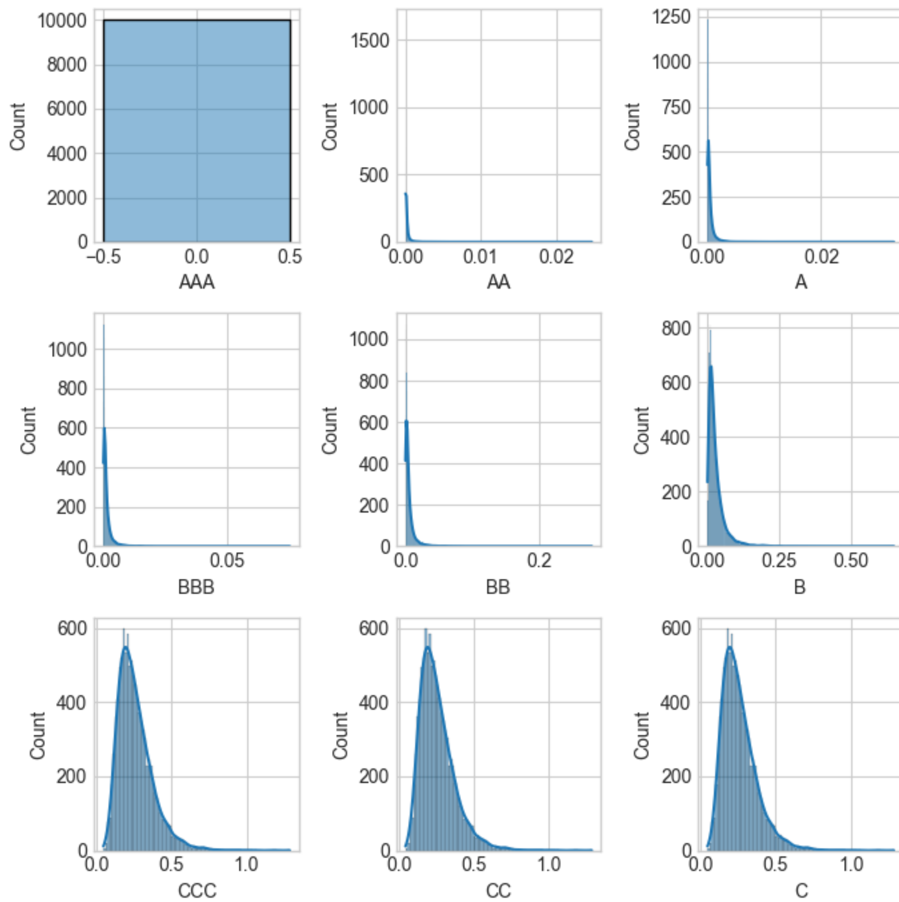

Figure 8. PD Simulation

We trimmed simulated log-normal values that are greater than 1 to be 1 to satisfy the probability notion of PD. In addition, we implemented the PD floor of 0.05% for corporate exposure.

Effective maturity, Correlation, and Maturity Adjustment

We computed the effective maturity (M) for the bonds according to 5.4.1 (iv) 133 in the OSFI guideline. We adjusted M using a floor of one year and a cap of five years.

We then derived Correlation (R) and Maturity Adjustment (b) according to formulas specified in 5.3.1 (i) 66 of the OSFI guideline.

Capital Requirement Calculation and Analysis

$$K = LGD \cdot \left\{ N\left(\sqrt{\frac{1}{1-R}} \cdot N^{-1}(PD) + \sqrt{\frac{R}{1-R}} \cdot N^{-1}(0.999) \right) - PD \right\} \left(\frac{1 + (M - 2.5)b}{1 - 1.5b} \right)$$

$$UL = K \cdot EAD$$

We plugged in these parameters to the capital requirement equation above to derive the regulatory capital required for each bond, which were then summed to obtain portfolio-level regulatory capital. Figure 9 shows the distribution of the regulatory capital of the portfolio. Table 3 displays the descriptive statistics of the regulatory capital.

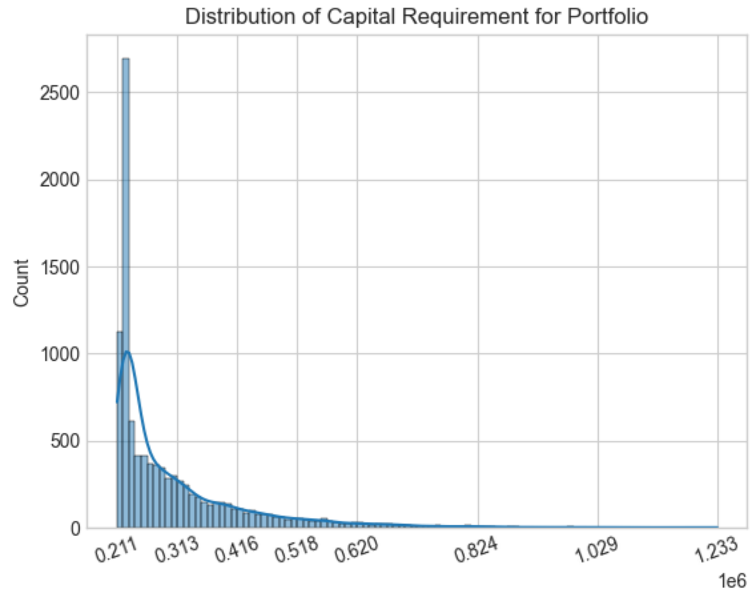

Figure 9. Distribution of Capital Requirement for the Portfolio

Table 3. Descriptive statistics for Capital Requirement

Statistic	Value
Count	10,000.00
Mean	304,803.23
Std Dev	117,154.99
Min	211,222.32
25%	227,222.94
50% (Median)	255,996.24
75%	338,371.70
Max	1,233,046.34
Skewness	2.22
Kurtosis	6.21
Lower Bound (95%)	302,506.99
Upper Bound (95%)	307,099.47

From the table, we can see that skewness is 2.22 which is greater than 0, i.e., positively skewed. This means most values are concentrated on the left with a few larger values on the right. Kurtosis is greater than 3, which means the distribution has a heavier tail than a normal distribution.

Out of 10,000 simulations, on average, regulatory capital required was US$304,803.23, accounting for 3.24% of the total exposure of the portfolio (EAD was US$9,417,961.51). The 95% confidence interval for the regulatory capital was [302,506.99, 307,099.47]. Dividing these two quantities by EAD and multiplying by 100, we got (3.21, 3.26): for \$100 exposure to a portfolio of bonds, the bank needs to set aside \$3.21 ~ \$3.26 as regulatory capital.

Provision Calculation and Analysis

$$K = LGD \cdot \left\{ N\left(\sqrt{\frac{1}{1-R}} \cdot N^{-1}(PD) + \sqrt{\frac{R}{1-R}} \cdot N^{-1}(0.999) \right) - PD \right\}$$

$$EL = K \cdot EAD$$

Provision was calculated using the formulas above. Note K does not include full maturity adjustment. Below shows the distribution and the descriptive statistics of the provision.

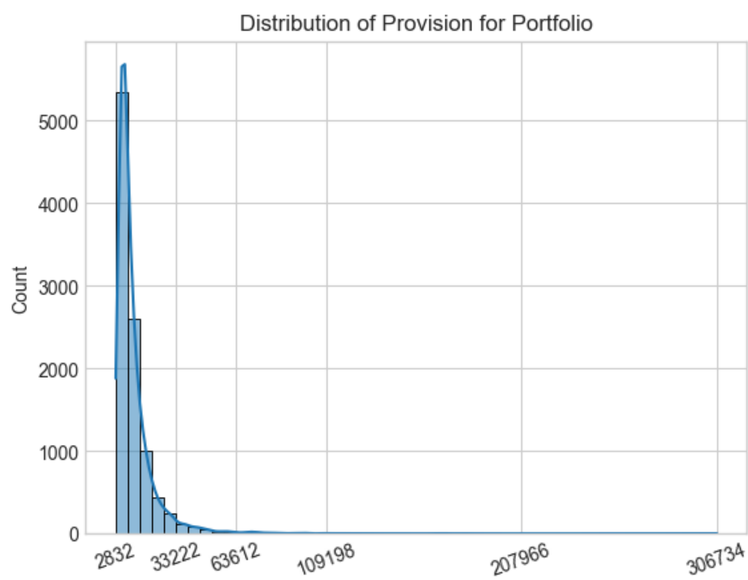

Figure 10. Distribution of Provision for the Portfolio

Table 4. Descriptive statistics for Provision

Statistic	Value
Count	10,000.00
Mean	11,896.39
Std Dev	11,984.69
Min	2,831.82
25% Quartile	6,152.84
50% (Median)	8,446.06
75% Quartile	13,345.85
Max	306,734.34
Skewness	7.36
Kurtosis	103.78

Statistic	Value
Lower Bound (95%)	11,661.49
Upper Bound (95%)	12,131.29

The provision is also positively skewed; most values are concentrated on the left with a few larger values on the right. Kurtosis is much greater than 3. This means extreme values in the tails are more likely than in a normal distribution.

Out of 10,000 simulations, on average, provision required was US$11,896.39, accounting for 0.13% of total exposure. The 95% confidence interval for the provision was [11,661.49, 12,131.29]. Dividing these two quantities by EAD and multiplying by 100, we got (0.1238, 0.1288). This means for $100 exposure to a portfolio of bonds, the bank needs to set aside $0.123 ~ $0.129 as provision.

Discussion and Conclusion

The provision and regulatory capital are mainly determined by PD, LGD, and EAD, with PD mapped from rating and LGD mapped from seniority. We now analyze the impact of rating. We grouped and aggregated the provision, the capital requirement, and EAD of the bonds by rating, leading to the result below.

Table 5. Groupwise Analysis

Rating	Provision	Reg. Cap.	# Bonds	EAD	Provision/EAD	Reg. Cap./EAD
A	903.95	78040.72	2,152	3199972.07	0.00028	0.02
AA	69.01	6774.51	330	300605.31	0.00023	0.02
AAA	10.31	1171.55	57	52052.24	0.00019	0.02
B	1455.82	12421.55	153	141092.29	0.01031	0.09
BB	890.91	21788.90	426	405755.40	0.00219	0.05
BBB	3282.11	175438.46	3,069	5263139.04	0.00062	0.03
C	133.28	234.96	3	1401.56	0.09509	0.17
CC	31.01	54.67	2	300.00	0.10337	0.18
CCC	5119.93	8877.90	70	53643.60	0.09544	0.17

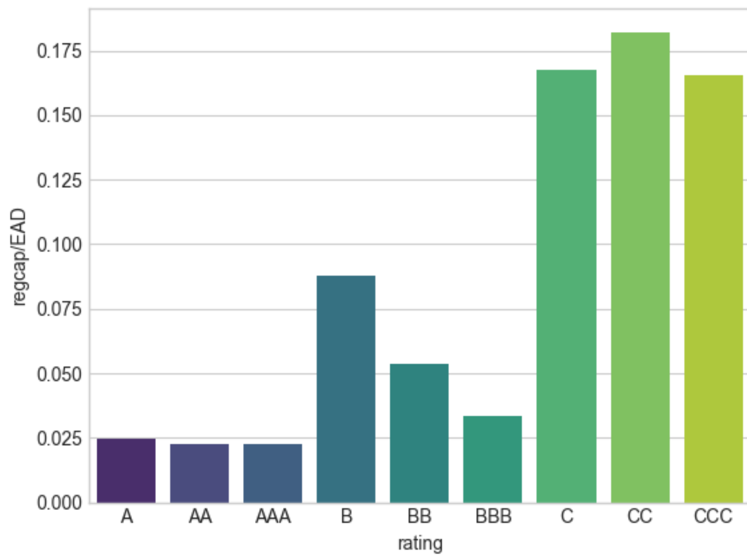

Figure 11. Contribution to the Total Capital by Rating

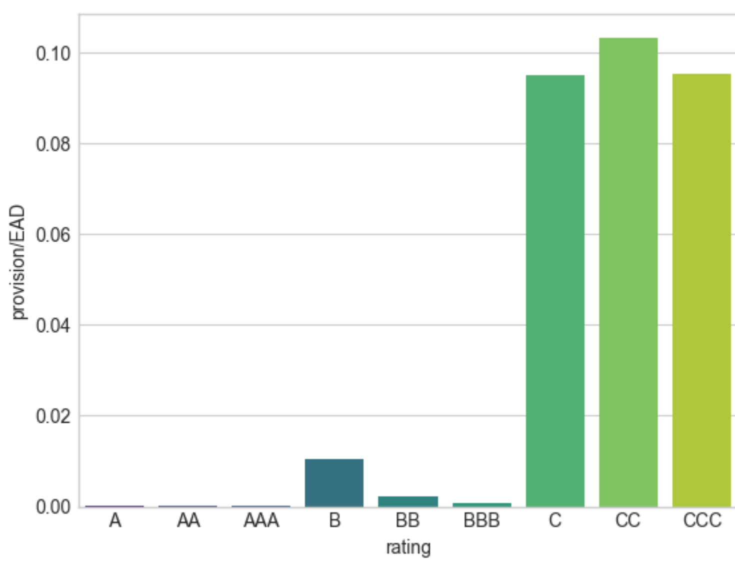

Figure 12. Contribution to the Provision by Rating

Figures 11 and 12 show that bonds of CC rating contribute the most to the total regulatory capital and provision. For \$1 exposure to CC bonds, \$0.18 and \$0.103 will be unexpected loss and expected loss, respectively, and thus need to be set aside as regulatory capital and provision. Thus although CC bonds have a high yield, they are likely to have low risk-adjusted return of capital (RAROC), which is

$$RAROC = {}^{net\ income - expected\ loss}\big/_{economic\ capital}$$

$$= {}^{coupon\ income - funding\ costs - operating\ costs - expected\ loss}\big/_{regulatory\ capital}$$

The denominator is substituted because both, the Basel IRB capital and the economic capital, aim to cover unexpected loss at a high confidence level.

Due to banking regulations, banks need to set aside economic capital for risk management. Understanding which component in the bond portfolio contributes the most to it is helpful for capital budgeting and portfolio performance measurement.

References

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