Bond Pricing and Capital Requirements

FM 9528 Banking Analytics

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Executive Summary

In this report, we analyzed a dataset consisting of 6262 corporate bonds traded in the US on May 31, 2024 and 10 features including price, maturity, treasury spread, offering date, principal, interest frequency, coupon rate, rating, and security level. We re-priced these bonds using the Canadian yield curve on the same day and the yield curve on January 13, 2025 to analyze the effect of the interest rate on bond prices. To get the accurate risk-free rate, we interpolated the yield curves using quadratic interpolation. For coupon payment dates, we listed the coupon payment dates between the transaction date ("today") to the maturity using the interval determined by interest frequency. We then discounted future coupon payments and the principal using the risk-free rates plus the spread at each payment date, getting the present value of bonds. Results show that 80.93% of bonds achieved higher prices, with an average price increase of $1.18, under the Canadian yield curve which was approximately 1.39% lower than the U.S. Bonds were valued even higher on January 13, 2025 when the short-term interest rate had dramatically decreased, resulting in an average price difference of $5.04.

We therefore concluded that a Canadian investor subject to Canadian interest rates would invest in U.S. bonds, because domestic interest rates were lower than those abroad and money flows to wherever there are high returns. In other words, a Canadian investor would consider U.S. bonds underpriced when they discount using the Canadian rates.

Under banking regulations, banks are required to set aside economic capital for risk management. For a portfolio of bonds, the risk is credit risk. Thus we computed the capital requirements and provision for this portfolio. We first derived risk components according to the OSFI capital adequacy guideline using the Foundation Internal Rating Based (F-IRB) approach. Specifically, the Probability of Default (PD) was simulated using the log-normal distribution, and LGD were mapped from seniority of the bonds. We then applied the capital requirement equation to each individual bond to get the capital requirements, which were then aggregated to the portfolio level. The provision was calculated in a similar fashion.

We analyzed the distribution of the capital requirement and the provision using descriptive statistics. We found that for $100 exposure to a portfolio of bonds, the bank needs to set aside $9.82 ~ $9.93 as economic capital, and $0.46 ~ $0.48 as provision. We further analyzed each rating's contribution to the total capital requirement. CC bonds contributed the most: for $1 exposure to them, $0.64 was unexpected loss and thus needed to be set aside as regulatory capital. This result is useful for capital budgeting and portfolio performance measurement.

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Part 1

Repricing Bonds Using Canadian Interest Rates

The data set is a portfolio consisting of 6262 bonds traded in the US on May 31, 2024. Figure 1 explores the variables in the dataset. It is shown that most prices were between 75% to 125% percent of the principal; most bonds were offered in recent 25 years; coupons are paid semi-annually but are not always so; coupon rate ranges between 2.5% to 7.5%.

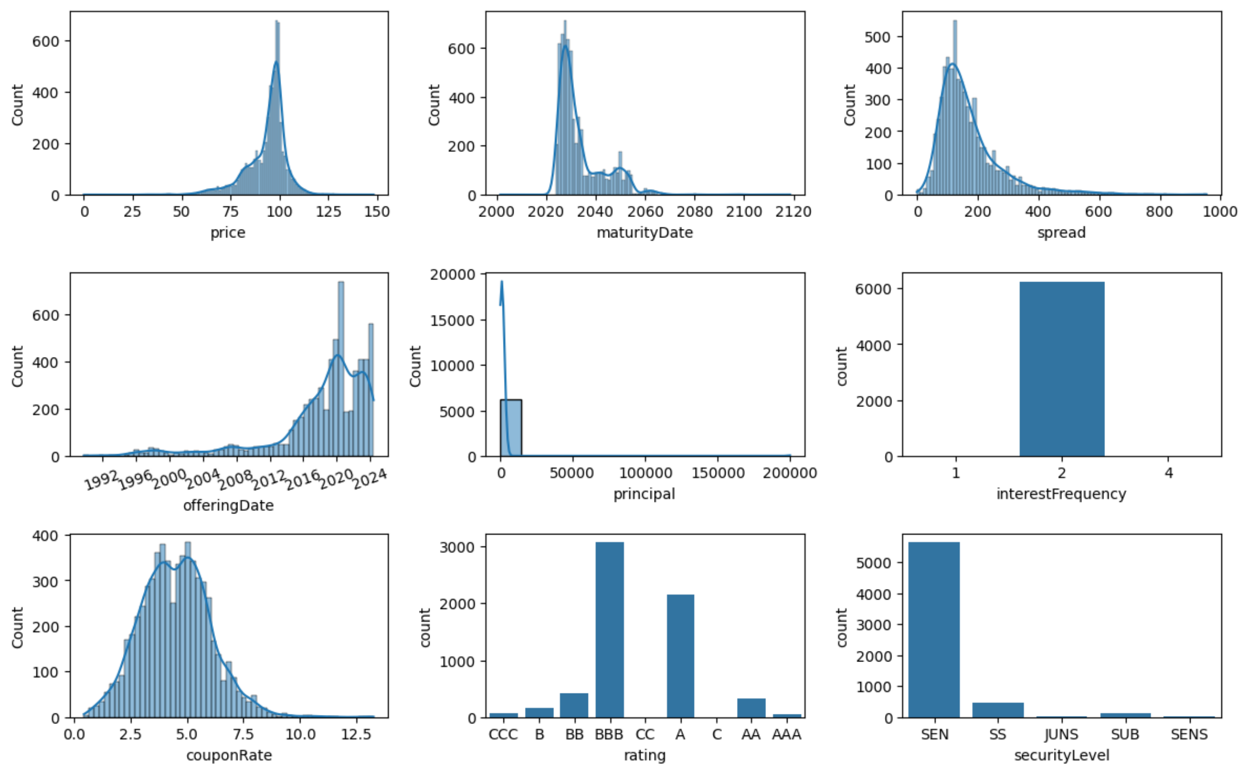

Figure 1. Exploratory Data Analysis

As can be seen in Figure 2, on May 31 2024, Canadian interest rates were lower than the US. To analyze the impact of bonds being traded in Canada and thus being subject to Canadian rates, we repriced the bonds by calculating the present values of these bonds' cash flows with the time-varying discount rates, which is the sum of risk-free rates represented by the Canadian yield curve on May 31, 2024, and the credit spread. Risk free rates were interpolated using quadratic interpolation. Figure 3 is an example of interpolated risk-free rates for the first bond in the dataset. They follow the yield curve closely.

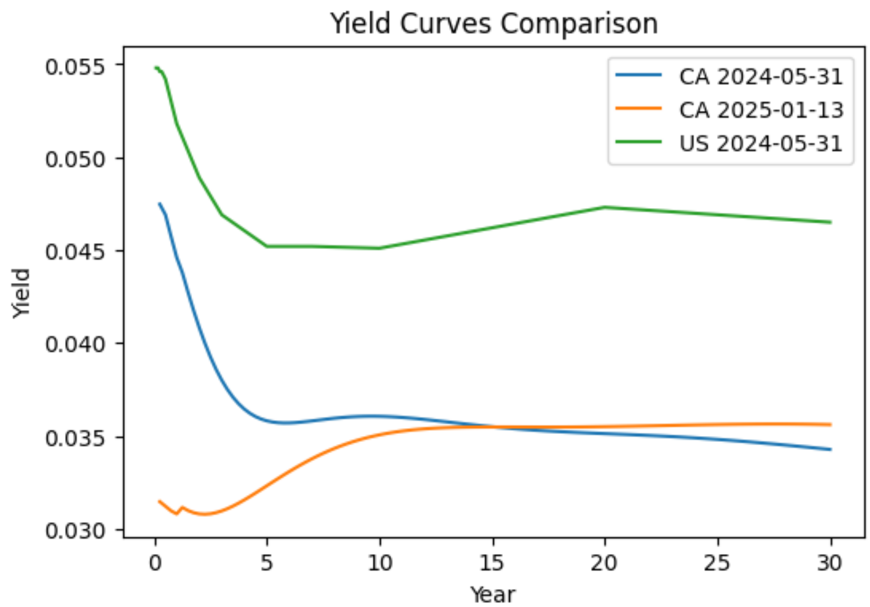

Figure 2. Yield Curves Comparison

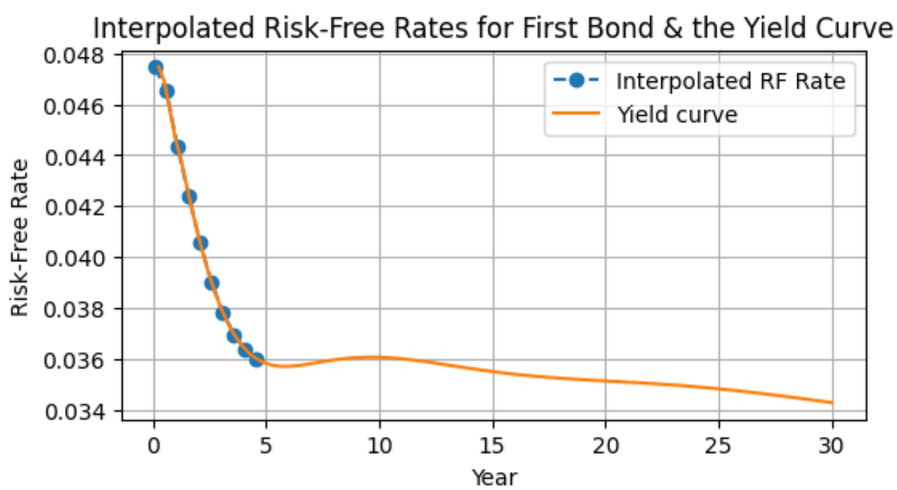

Figure 3. Yield Curve Interpolation

Comparison and Analysis

Having discounted the future cash flows and obtained the new prices, we observed that the majority (80.93%) of bonds had a price increase under the Canadian interest rate, in line with the expectation that lower interest rates should result in higher bond prices. The average price difference was US$3.86.

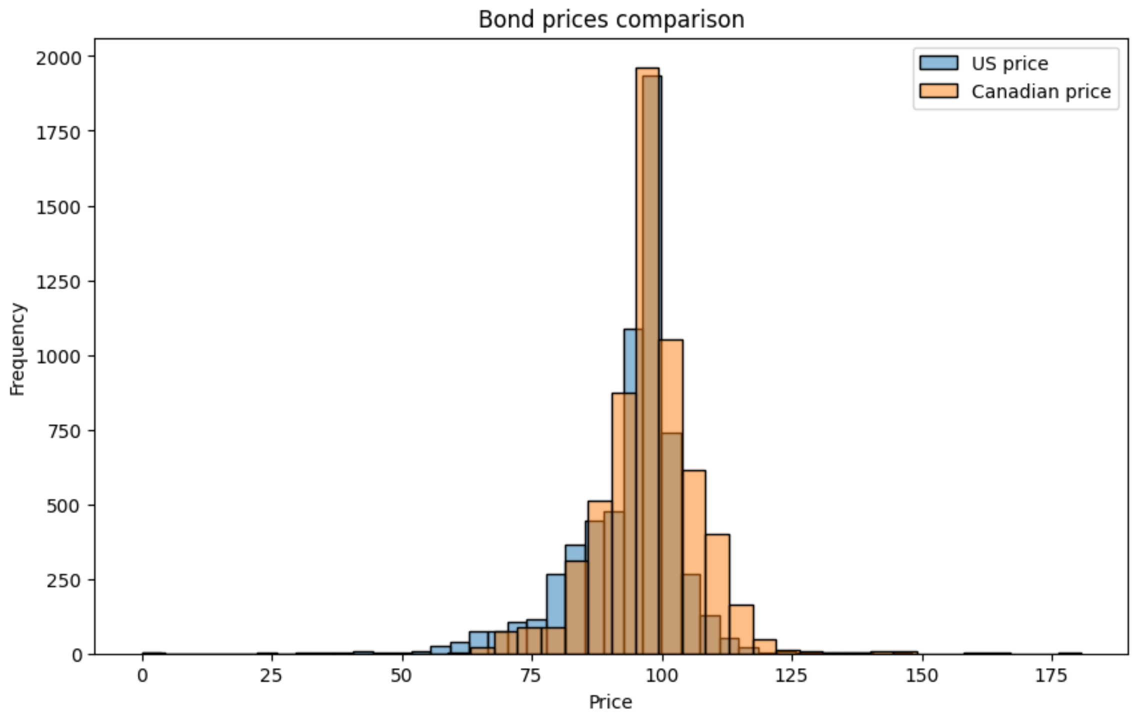

Figure 4. Bond Prices Comparison

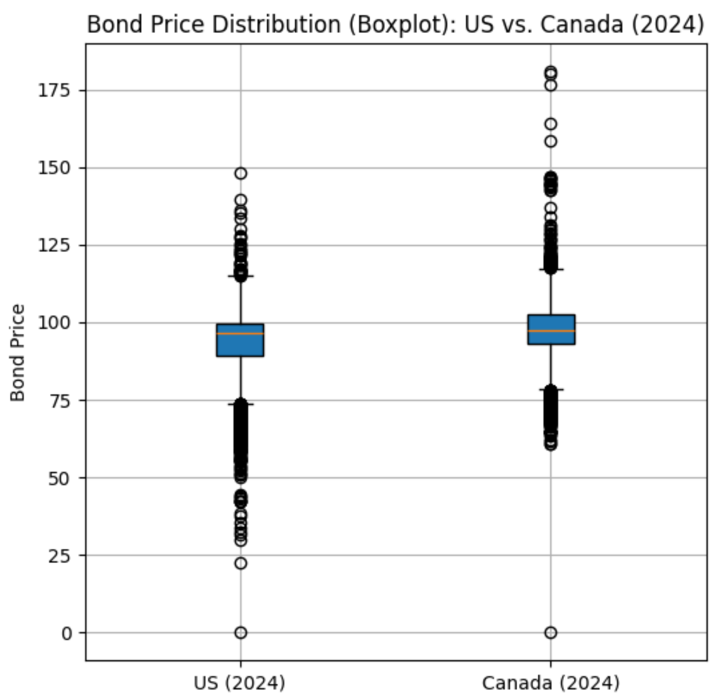

Figure 5. Bond Prices Distribution Comparison

The difference in bond prices between the two markets highlights the impact of different interest rate environments. Given the same future cash flows, lower domestic (assume Canada is 'domestic') interest rates lead to foreign assets being valued higher.

The U.S. market was more attractive, because when a Canadian investor discounts a bond using the Canadian rates, he would find U.S. bonds underpriced (assuming the Canadian dollar was on par with the US dollar) so he would go to the US bonds market to purchase these bonds instead of investing domestically.

Part 2

We now reprice the bonds again, but with the Canadian yield curve on Jan 13, 2025.

Comparison of Bond Price Distributions

Using the same method as above, we got the bond prices subject to Canadian yield curve on Jan 13 2025. The price distributions are shown in Figure 7.

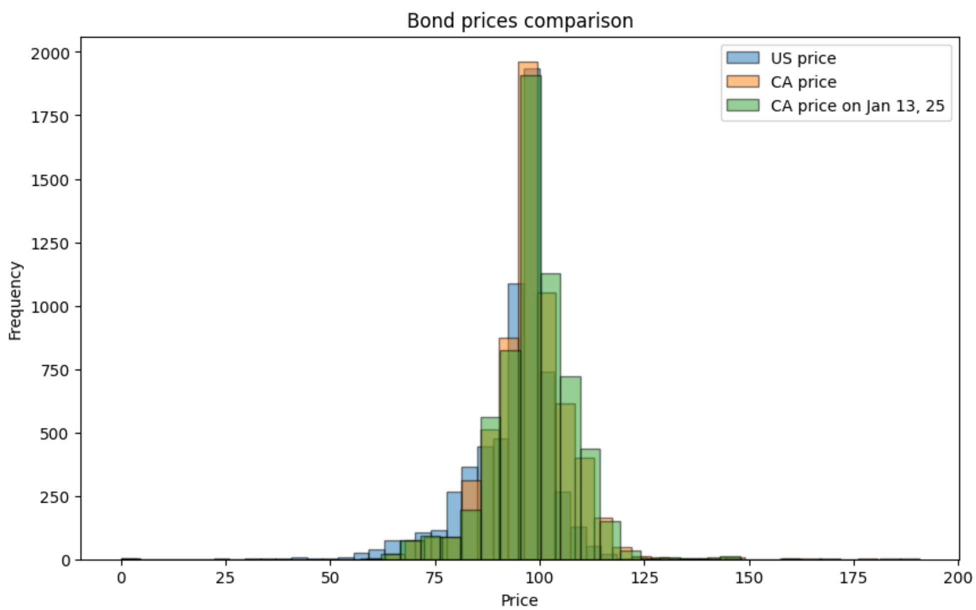

Figure 7. Bond Prices Comparison

There was a shift of the distribution to the right due to even lower interest rates. Approximately 88.41% of the bonds experienced a price increase. The average increase was US$1.18.

Events, Rate Cuts, and Their Impact on the Fixed-Income Market

Between May 31, 2024 and January 13, 2025, Bank of Canada (BoC) decreased the policy interest rate five times, from 5.0% to 3.25% (Bank of Canada, 2025) since inflation was under control and the BoC had shifted its focus towards growth (Dewan, 2024). Rates cuts can generally only influence short-term rates, and have little influence on long-term rates (Garretson, 2024). Therefore, we observed the yield curve changing from the downward trend to an upward trend as shown in Figure 2. Thus, bonds prices increased due to lower interest rates.

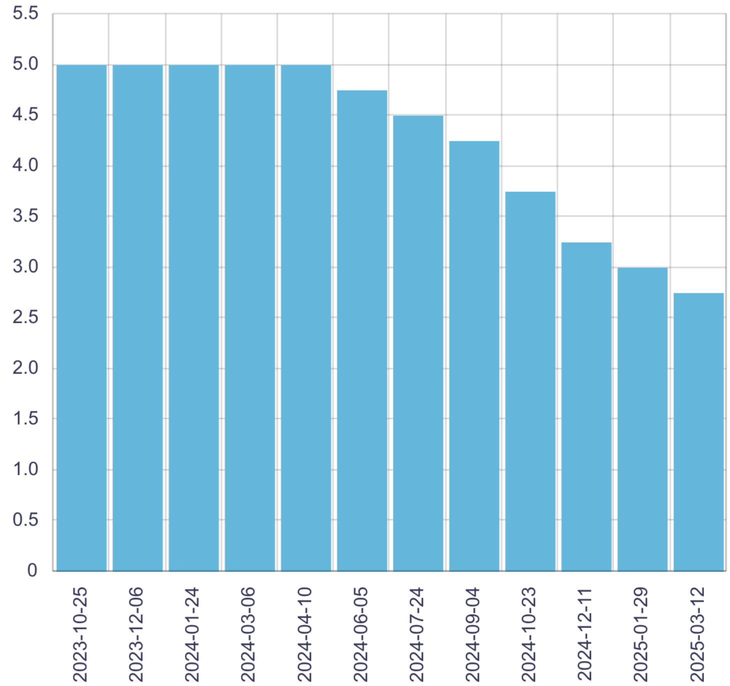

Figure 6. BoC policy Rate Changes (Bank of Canada, 2025)

Part 3

To obtain the required regulatory capital and provision of a portfolio of bonds, we first calculated the capital requirement for each individual bond. To do so, we first derived risk components in the capital requirement equation using the Foundation-Internal Ratings Based (F-IRB) approach according to the capital adequacy requirements guideline (OSFI, 2023) provided by the Superintendent of Financial Institutions (OSFI), which is based on Basel framework. The capital requirement equation was then applied to each bond.

LGD

Loss Given Default (LGD) is the proportion of an exposure that is lost if a borrower defaults (Wikipedia, 2024). We referred to Chapter 5.4.1 (ii) for specific details about mapping the seniority level to LGD. It should be noted that we assumed Junior bonds had an LGD of 75%, as there was no information in the guideline; senior bonds have an LGD of 40% as the bonds in the dataset are assumed to be corporate bonds; (senior) secured bonds were assumed to have an LGD of 20% due to lack of information in the dataset about the type of collateral associated with each bond.

EAD

Exposure at Default (EAD) is defined as the gross exposure under a facility upon default of an obligor (Wikipedia, 2024). This parameter was derived by multiplying the price (which is expressed as a percentage of the principal) and the principal. In our calculation, EAD is effectively the present value of an asset.

Table 1. Seniority-LGD Mapping

Seniority level	LGD (%)
Junior	0.75
Junior Subordinate	0.75
Senior	0.40
Senior Subordinate	0.75
Senior Secured	0.20
Subordinate	0.75

PD

Probability of Default (PD) is the likelihood of a default over a particular time horizon (Wikipedia, 2024). For corporate exposure, the PD is the one-year PD and is associated with the borrower grade (OSFI, 2023). We treated PD as a random variable (except for AAA rating bonds) that follows a log-normal distribution and simulated 10,000 realizations. For each bond rating, the mean and the standard deviation of PD are different, see Table 2. This information is provided in the dataset.

Table 2. Rating-PD Mapping

	AAA	AA	A	BBB	BB	B	CCC/C
Weighted long-term average	0	0.02	0.05	0.14	0.57	2.98	25.98
Standard deviation	0	0.06	0.1	0.25	0.96	3.23	11.73

Figure 8 shows the distributions of the simulated PD values for each grade.

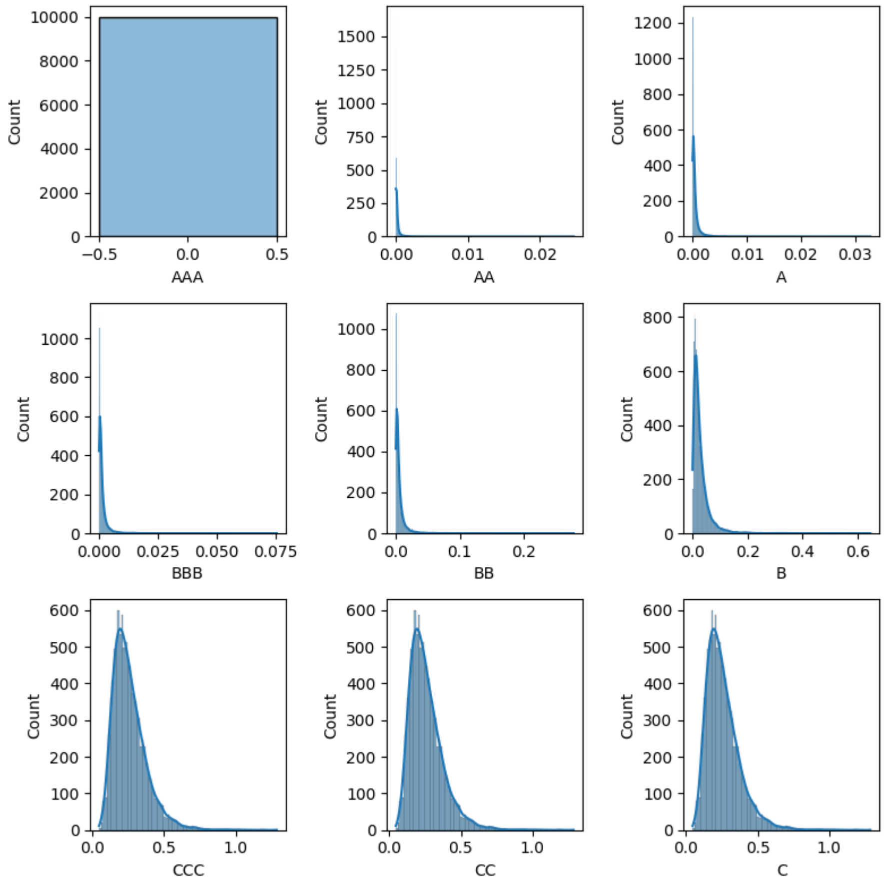

Figure 8. PD Simulation

We trimmed simulated log-normal values that are greater than 1 to be 1 to satisfy the probability notion of PD. In addition, we implemented the PD floor in Basel III.

Effective maturity, Correlation, and Maturity Adjustment

We computed the effective maturity (M) for the bonds according to 5.4.1 (iv) 133 in the OSFI guideline. We adjusted M upward to a floor of one year and a cap of five years.

We then derived Correlation (R) and Maturity Adjustment (b) according to formulas specified in 5.3.1 (i) 66 of the OSFI guideline.

Capital Requirement Calculation and Analysis

We plugged in these parameters to the capital requirement equation to derive the regulatory capital required for each bond, which were then summed to obtain portfolio-level regulatory capital. Figure 9 shows the distribution of the regulatory capital of the portfolio. Table 3 displays the descriptive statistics of the regulatory capital.

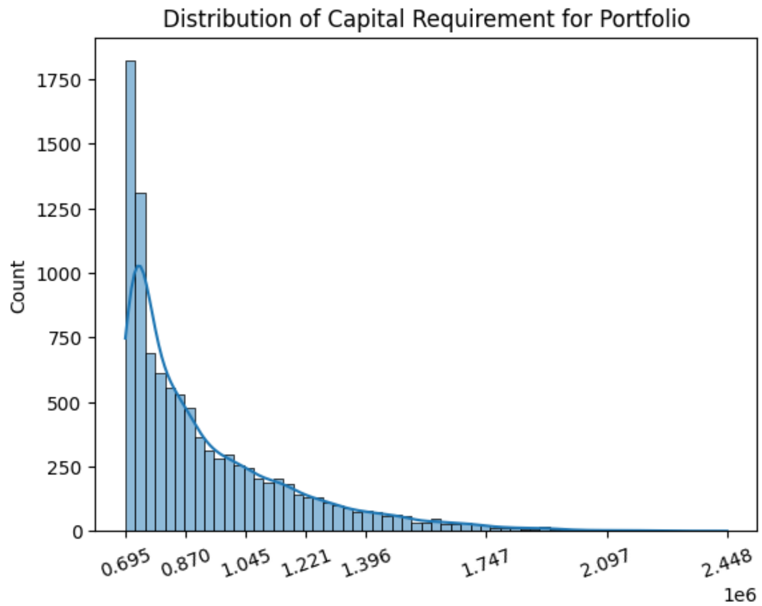

Figure 9. Distribution of Capital Requirement for the Portfolio

Table 3. Descriptive statistics for Capital Requirement

Statistic	Value
Count	10,000.00
Mean	930,241.12
Std Dev	255,258.59
Min	694,822.05
25%	728,987.78
50% (Median)	839,057.16
75%	1,040,498.99
Max	2,448,006.24
Skewness	1.66
Kurtosis	2.95
Lower Bound (95%)	925,238.05
Upper Bound (95%)	935,244.19

From the table, we can see that skewness is 1.66 which is greater than 0, i.e., positively skewed. This means most values are concentrated on the left with a few larger values on the right. Kurtosis is slightly less than 3. This means extreme values are less likely than in a normal distribution; the tail is not heavy.

Out of 10,000 simulations, on average, regulatory capital required was US$930,241.12, accounting for 9.88% of the total exposure of the portfolio (EAD was US$9,417,961.51). The 95% confidence interval for the regulatory capital was [925,238.05, 935,244.19]. Dividing these two quantities by EAD and multiplying by 100, we got (9.82, 9.93): for $100 exposure to a portfolio of bonds, the bank needs to set aside $9.82 ~ $9.93 as economic capital.

Provision Calculation and Analysis

Provision was calculated in a similar fashion. Below shows the distribution and the descriptive statistics of the provision.

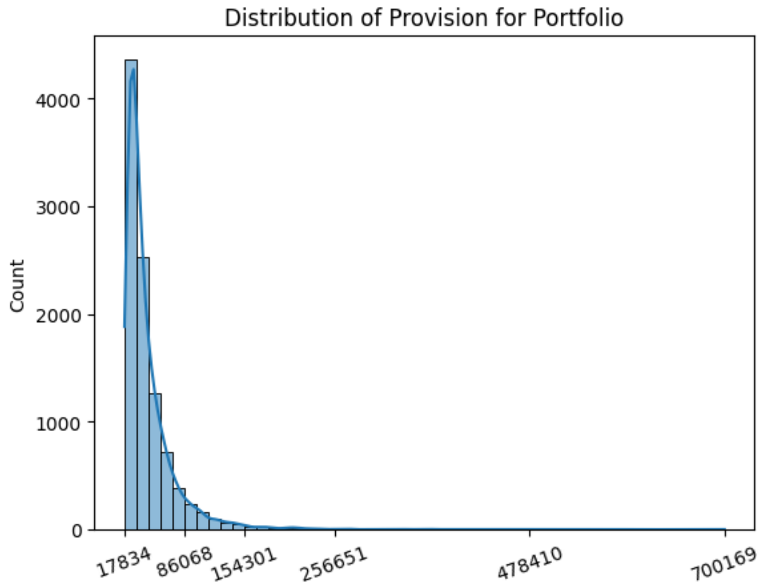

Figure 10. Distribution of Provision for the Portfolio

Table 4. Descriptive statistics for Provision

Statistic	Value
Count	10,000.00
Mean	44,350.56
Std Dev	33,323.97
Min	17,834.45
25% Quartile	25,471.38
50% (Median)	34,070.18
75% Quartile	50,647.08
Max	700,168.71
Skewness	5.23
Kurtosis	53.43
Lower Bound (95%)	43,697.41
Upper Bound (95%)	45,003.71

The provision is also positively skewed; most values are concentrated on the left with a few larger values on the right. Kurtosis is much greater than 3. This means extreme values in the tails are more likely than in a normal distribution.

Out of 10,000 simulations, on average, provision required was US$44,350.56, accounting for 0.47% of total exposure. The 95% confidence interval for the provision was [43,697.41, 45,003.71]. Dividing these two quantities by EAD and multiplying by 100, we got (0.46, 0.48). This means for $100 exposure to a portfolio of bonds, the bank needs to set aside $0.46 ~ $0.48 as provision.

Discussion and Conclusion

We now analyze which rating required the most regulatory capital. We grouped and aggregated the capital requirement and EAD of each bond by rating, leading to the following result.

Table 5. Groupwise Analysis

Rating	Regulatory Capital	Number of Bonds	EAD	Proportion
A	260,974.67	2,152	3.20E+06	0.0816
AA	22,446.31	330	3.01E+05	0.0747
AAA	3,982.34	57	5.21E+04	0.0765
B	27,529.60	153	1.41E+05	0.1951
BB	58,632.25	426	4.06E+05	0.1445
BBB	540,709.07	3,069	5.26E+06	0.1027
C	427.93	3	1.40E+03	0.3053
CC	99.57	2	3.00E+02	0.3319
CCC	15,439.37	70	5.36E+04	0.2878

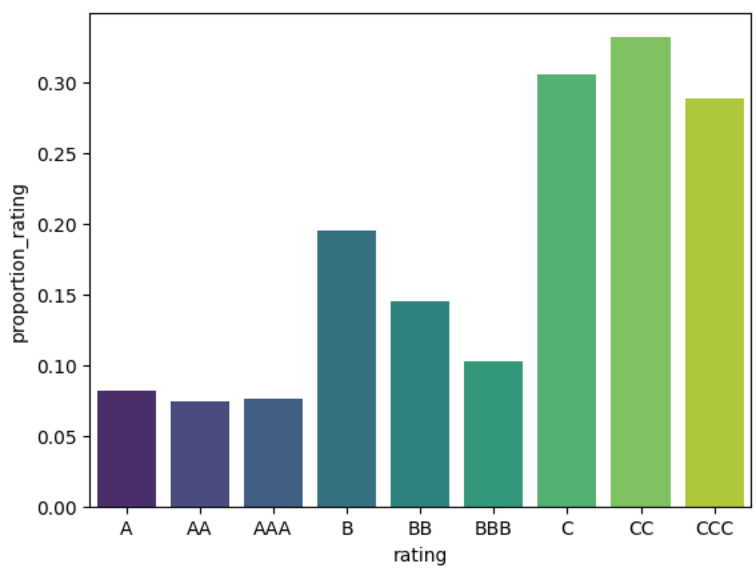

Figure 11. Contribution to the Total Capital of Each Rating

Figure 11 shows that bonds with CC rating contributes the most to the total regulatory capital. For $1 exposure to CC bonds, $0.64 will be unexpected loss and thus needs to be set aside as regulatory capital. This result arises from the multiplicative effect of PD (which is mapped from ratings) and LGD (mapped from seniority of the bonds).

Due to banking regulations, banks need to set aside economic capital for risk management. Understanding which component in the bond portfolio contributes the most to it is helpful for capital budgeting and portfolio performance measurement.

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