Common Force Part II : Resitive Force

Introduction

In this unit, we will understand some forces related to other forces acting on an object or the state of motion of an object. Usually, the magnitude or direction of these forces will be affected by the current state of the object. For example, the magnitude of friction is related to the horizontal and vertical forces acting on the object at that moment.

Friction

Friction is a common resistive force in daily life. When an object experiences friction, the force exerted on the object mainly manifests as preventing the object from sliding. For example, when a block slides on a non-smooth surface, friction forms a force opposite to the direction of sliding, preventing the object from sliding. In physics, friction is divided into static friction and kinetic friction. Static friction refers to the friction experienced when the object is at rest, while kinetic friction refers to the friction experienced when the object is in motion.

Static Friction

Static friction refers to the friction experienced when an object is at rest. For example, when we push a stone on the ground, we apply an external force, but the stone does not move. This is because the static friction experienced by the object counteracts the force applied to push the stone, keeping it at rest. From the above example, we can understand that when we apply a force parallel to the ground on an object, a static friction force opposite to that force will be generated. Therefore, we can write the formula for static friction and external force as follows, where $\vec{F}_{\text{ext}}$ represents the external force acting on the object, and $\vec{f}_s$ represents the static friction force acting on the object.

$$\vec{F}_{\text{ext}} = -\vec{f}_s \qquad (1)$$

Maximum Static Friction

Static friction describes the resistance of an object on a non-smooth interface, but static friction cannot always prevent the object from moving. Imagine a large stone that cannot be pushed by a person, but if we use a truck, we can easily pull the stone. So we know that static friction has an upper limit, which we denote as $\vec{f}_{\max}$, the maximum static friction.

The maximum static friction is also related to the interaction between the object and the interface (i.e., the normal force applied to the object). The greater the interaction between the interface and the object, the harder it is to move the object, and the greater the maximum static friction of the object. We can easily imagine two stones, one large and one small. The larger stone requires more force to move, which means the maximum static friction of the larger stone is greater.

Additionally, the maximum static friction is also related to the roughness of the interface. The greater the roughness, the greater the maximum static friction of the object. Physicists write the interaction between the interfaces as the maximum static friction coefficient μ_s. The larger the value, the rougher the interface.

Therefore, we can write the maximum static friction as the following formula, where $\vec{f}_{\max}$ represents the maximum static friction, μ_s represents the friction coefficient of the interface, and $\vec{N}$ represents the normal force .

$$|\vec{f}_{\max}| = \mu_s|\vec{N}| \tag{2}$$

Kinetic Friction

Kinetic friction refers to the frictional force experienced by an object in motion. When an object slides over a surface, kinetic friction generates a force opposite to the direction of motion, which slows down the object's speed. For example, when we slide on ice, although the ice surface is relatively smooth, there is still kinetic friction, which is why we eventually stop.

Similar to the maximum static friction, kinetic friction is also related to the roughness of the contact surface and the normal force. We can express the magnitude of kinetic friction using the following formula, where $\vec{f}_k$ represents kinetic friction, μ_k represents the coefficient of kinetic friction, and $\vec{N}$ represents the normal force.

$$|\vec{f}_k| = \mu_k|\vec{N}| \tag{3}$$

According to experiments, we can know that the magnitude of kinetic friction is usually smaller than the maximum static friction. This is because when an object starts to move, the contact points between surfaces decrease, thereby reducing friction. This is also why when we push a heavy object, it feels difficult at first, but once the object starts moving, it becomes easier to push.

Sample Problem #1

If an object with mass m is placed against a vertical wall, and the maximum static friction coefficient and kinetic friction coefficient between the object and the wall are μ_{max} and μ_{k} respectively, (a) what is the minimum horizontal force required to keep the object fixed against the wall? (b) What is the horizontal force required to make the object slide down the wall at a constant velocity?

Sol.

Sample Problem #2

A block with a mass of $1\ \mathrm{kg}$ is placed on a rough surface. If we apply a horizontal force of $10\ \mathrm{N}$ to push the block from rest, and the block moves $2\ \mathrm{m}$ in two seconds after it starts moving, (a) what is the magnitude of the frictional force acting on the block? (b) What is the kinetic friction coefficient between the block and the surface?

Sol.

Sample Problem #3

An object with mass m is placed on an inclined plane with an angle of θ. The kinetic friction coefficient between the inclined plane and the object is μ. Find the acceleration of the object sliding down the inclined plane.

Sol.

Air Resistance

Air resistance describes the resistance that an object encounters when moving through the air. When an object moves through the air, air resistance generates a force opposite to the direction of the object's motion, slowing down its speed. For example, when we ride a bicycle, air resistance slows us down.

Typically, air resistance is related to the following factors: it is proportional to the square of the object's speed, proportional to the object's surface area, and proportional to the air density. Therefore, we can describe air resistance using the following formula, where $\vec{f}_a$ represents air resistance, v represents the object's speed, A represents the object's surface area, ρ represents air density, and C_d represents the drag coefficient.

$$|\vec{f}_a| = \frac{1}{2} C_d \rho A v^2 \tag{4}$$

For example, when we ride a bicycle, we can clearly feel the effect of air resistance. The faster we ride, the stronger the headwind we feel, which is due to the increased air resistance with speed. Additionally, if we wear loose clothing, the increased frontal area will result in more resistance; whereas if we wear tight-fitting sportswear, the resistance will be reduced. Similarly, in high-altitude areas, due to lower air density, the resistance encountered while riding will also be relatively smaller.

Terminal Velocity

It is foreseeable that if we have an external force acting on a moving object, the speed will increase with the acceleration of the external force, and thus the air resistance will also increase. When the air resistance becomes equal to the external force, the forces reach equilibrium, resulting in constant speed motion, and this speed is called terminal velocity. Therefore, we can derive the terminal velocity of an object as follows:

First, we know that at terminal velocity, the external force on the object is equal to the air resistance, so we can write the following equation:

$$\vec{F}_{\text{ext}} = -\vec{f}_a = \frac{1}{2} C_d \rho v_{\text{term}}^2 \tag{5}$$

Then we can calculate the terminal velocity as follows:

$$v_{\text{term}} = \sqrt{\frac{2\vec{F}_{\text{ext}}}{C_d \rho A}} \tag{6}$$

From this, we can also know that the terminal velocity of an object in free fall can be expressed as:

$$v_{\text{term}} = \sqrt{\frac{2mg}{C_d \rho A}} \tag{7}$$

Sample Problem #4

An object with a mass of $10\ \mathrm{kg}$ experiences air resistance described by $F_{\text{air}} = -kv$ at low speeds. Here, k is the drag coefficient and v is the velocity. What is the terminal velocity of the object when it falls freely at low speeds?

Sol.

Sample Problem #5

In a science competition, participants are required to design rockets, and the competition task is to achieve the highest terminal velocity. Suppose there are two rockets participating. Rocket A has a powerful engine with a thrust of $500,000\ \mathrm{N}$, and its shape is look like a bullet, with a drag coefficient of 0.295. The frontal area is $10\ \mathrm{m}^2$, and the mass of the rocket is $10,000\ \mathrm{kg}$. Rocket B has a weaker engine with a thrust of $150,000\ \mathrm{N}$, but its shape is airfoil, with a drag coefficient of 0.045. The frontal area is $8\ \mathrm{m}^2$, and the mass of the rocket is also $10,000\ \mathrm{kg}$.

Assume both rockets are launched under the same conditions, and the air density is $1.225\ \mathrm{kg/m}^3$. Please answer the following questions: First, calculate the terminal velocity of Rocket A. Second, calculate the terminal velocity of Rocket B. Finally, compare the terminal velocities of the two rockets and explain which rocket has the higher terminal velocity and why.

Sol.

Exercises

Exercise #1 [Halliday 6.12]

Sol.

Exercise #2 [Halliday 6.19]

Sol.

Exercise #3 [Halliday 6.20]

Sol.

Exercise #4 [Halliday 6.34]

Sol.

Exercise #5 [Halliday 6.25]

Sol.

Exercise #6 [Halliday 6.27]

Sol.

Exercise #7 [Halliday 6.88]

Sol.

Exercise #8 [Halliday 6.66]

Sol.

Exercise #9 [Halliday 6.36]

Sol.

Exercise #10 [Halliday 6.39]

Sol.

Solutions

Sample Problem #1

If an object with mass m is placed against a vertical wall, and the maximum static friction coefficient and kinetic friction coefficient between the object and the wall are $\mu_{\max}$ and μ_{k} respectively, (a) what is the minimum horizontal force required to keep the object fixed against the wall? (b) What is the horizontal force required to make the object slide down the wall at a constant velocity?

(a) What is the minimum horizontal force required to keep the object fixed against the wall?

To keep the object fixed against the wall, the static friction force must balance the weight of the object. The static friction force is given by \(f_{\text{static}} = \mu_{\text{max}} F_{\text{horizontal}} \).

$$f_{\text{static}} = mg \tag{1}$$

Therefore, the minimum horizontal force required is:

$$F_{\text{horizontal}} = \frac{mg}{\mu_{\max}} \tag{2}$$

(b) What is the horizontal force required to make the object slide down the wall at a constant velocity?

To make the object slide down the wall at a constant velocity, the kinetic friction force must balance the weight of the object. The kinetic friction force is given by \(f_{\text{kinetic}} = \mu_{\text{k}} F_{\text{horizontal}} \).

$$f_{\text{kinetic}} = mg \tag{3}$$

Therefore, the horizontal force required is:

$$F_{\text{horizontal}} = \frac{mg}{\mu_{\mathrm{k}}} \tag{4}$$

Sample Problem #2

A block with a mass of $1\ \mathrm{kg}$ is placed on a rough surface. If we apply a horizontal force of $10\ \mathrm{N}$ to push the block from rest, and the block moves $2\ \mathrm{m}$ in two seconds after it starts moving, (a) what is the magnitude of the frictional force acting on the block? (b) What is the kinetic friction coefficient between the block and the surface?

(a) What is the magnitude of the frictional force acting on the block?

First, we need to find the acceleration of the block. Using the equation of motion: $s = ut + \frac{1}{2}at^2$, where $s = 2\ \mathrm{m}$, $u = 0\ \mathrm{m/s}$, and $t = 2\ \mathrm{s}$.

$$2 = 0 \cdot 2 + \frac{1}{2}a \cdot (2)^2 \implies a = 1\ \mathrm{m/s^2} \tag{5}$$

The net force acting on the block is given by Newton's second law: $F_{net} = ma$.

$$F_{net} = 1\ \mathrm{kg} \cdot 1\ \mathrm{m/s^2} = 1\ \mathrm{N} \tag{6}$$

The applied force is $10\ \mathrm{N}$, so the frictional force f can be found using: $F_{net} = F_{applied} - f$.

$$1\ \mathrm{N} = 10\ \mathrm{N} - \mathrm{f} \implies \mathrm{f} = 9\ \mathrm{N} \tag{7}$$

(b) What is the kinetic friction coefficient between the block and the surface?

The kinetic friction force is given by: $f = \mu_k N$, where N is the normal force. For a horizontal surface, $N = mg$.

$$N = 1\ \mathrm{kg} \cdot 9.8\ \mathrm{m/s^2} = 9.8\ \mathrm{N} \tag{8}$$

Using the frictional force calculated in part (a):

$$9\ \mathrm{N} = \mu_\mathrm{k} \cdot 9.8\ \mathrm{N} \implies \mu_\mathrm{k} = \frac{9}{9.8} \approx 0.92 \tag{9}$$

Sample Problem #3

An object with mass m is placed on an inclined plane with an angle of θ. The kinetic friction coefficient between the inclined plane and the object is μ. Find the acceleration of the object sliding down the inclined plane.

Solution:

To find the acceleration of the object, we need to consider the forces acting on it. The forces are:

- The gravitational force mg acting vertically downward.
- The normal force N acting perpendicular to the inclined plane.
- The frictional force f_k acting opposite to the direction of motion.

The component of the gravitational force parallel to the inclined plane is $mg\sin\theta$, and the component perpendicular to the inclined plane is $mg\cos\theta$.

$$f_k = \mu N = \mu(mg\cos\theta) \tag{10}$$

The net force acting on the object along the inclined plane is: $F_{net} = mg\sin\theta - f_k$.

$$F_{net} = mg\sin\theta - \mu(mg\cos\theta) \tag{11}$$

Using Newton's second law, $F_{net} = ma$, we can solve for the acceleration a:

$$ma = mg\sin\theta - \mu(mg\cos\theta) \tag{12}$$

Dividing both sides by m:

$$a = g(\sin\theta - \mu\cos\theta) \tag{13}$$

Therefore, the acceleration of the object sliding down the inclined plane is:

$$a = g(\sin\theta - \mu\cos\theta) \tag{14}$$

Sample Problem #4

An object with a mass of 10 kg experiences air resistance described by $F_{\text{air}} = -kv$ at low speeds. Here, k is the drag coefficient and v is the velocity. What is the terminal velocity of the object when it falls freely at low speeds?

Solution:

At terminal velocity, the net force acting on the object is zero. This means the gravitational force is balanced by the air resistance force.

$$mg = kv_t \tag{15}$$

Here, v_t is the terminal velocity. Solving for v_t:

$$v_t = \frac{mg}{k} \tag{16}$$

Given that the mass $m = 10 \text{ kg}$ and the acceleration due to gravity $g = 9.8 \text{ m/s}^2$, the terminal velocity is:

$$v_t = \frac{10 \text{ kg} \cdot 9.8 \text{ m/s}^2}{k} = \frac{98 \text{ N}}{k} \tag{17}$$

Therefore, the terminal velocity of the object is $v_t = \frac{98}{k} \text{ m/s}$.

Sample Problem #5

In a science competition, participants are required to design rockets, and the competition task is to achieve the highest terminal velocity. Suppose there are two rockets participating. Rocket A has a powerful engine with a thrust of $500000\ \mathrm{N}$, and its shape is look like a bullet, with a drag coefficient of 0.295. The frontal area is $10\ \mathrm{m}^2$, and the mass of the rocket is $10,000\ \mathrm{kg}$. Rocket B has a weaker engine with a thrust of $150000\ \mathrm{N}$, but its shape is airfoil, with a drag coefficient of 0.045. The frontal area is $8\ \mathrm{m}^2$, and the mass of the rocket is also $10000\ \mathrm{kg}$.

Assume both rockets are launched under the same conditions, and the air density is $1.225\ \mathrm{kg/m^3}$. Please answer the following questions: First, calculate the terminal velocity of Rocket A. Second, calculate the terminal velocity of Rocket B. Finally, compare the terminal velocities of the two rockets and explain which rocket has the higher terminal velocity and why.

(a) Calculate the terminal velocity of Rocket A.

The terminal velocity can be found using the formula:

$$v_t = \sqrt{\frac{2 \cdot F_{thrust}}{\rho \cdot A \cdot C_d}} \tag{18}$$

For Rocket A:

$$v_{t_A} = \sqrt{\frac{2 \cdot (500000 - 10000 \cdot 9.8)}{1.225 \cdot 10 \cdot 0.295}} = 471.68\ \mathrm{m/s} \tag{19}$$

(b) Calculate the terminal velocity of Rocket B.

For Rocket B:

$$v_{t_B} = \sqrt{\frac{2 \cdot (150000 - 10000 \cdot 9.8)}{1.225 \cdot 8 \cdot 0.045}} = 485.62\ \mathrm{m/s} \tag{20}$$

(c) Compare the terminal velocities of the two rockets.

Rocket B has a higher terminal velocity of $680.14\ \mathrm{m/s}$ compared to Rocket A's terminal velocity of $471.68\ \mathrm{m/s}$. This is because Rocket B has a much lower drag coefficient and a smaller frontal area, which reduces the air resistance significantly, allowing it to achieve a higher terminal velocity despite having a weaker engine.

Exercise #1 [halliday 6.12]

$2.8 \times 10^2\ \mathrm{N}$;

Exercise #2 [halliday 6.19]

(a) No; (b) $-12\hat{i} + 5\hat{j}$ N;

Exercise #3 [halliday 6.20]

8.5 N;

Exercise #4 [halliday 6.34]

(a) $-6.1\ \mathrm{m/s^2}$; (b) $-0.98\ \mathrm{m/s^2}$;

Exercise #5 [halliday 6.25]

$1.0/times10^2$ kg;

Exercise #6 [halliday 6.27]

(a) 41 N; (b) $-3.9\hat{i}\ \mathrm{m/s^2}$; (c) $-1.0\hat{i}\ \mathrm{m/s^2}$;

Exercise #7 [halliday 6.88]

9.4 N;

Exercise #8 [halliday 6.66]

8.8 N;

Exercise #9 [halliday 6.36]

3.75

Exercise #10 [halliday 6.39]

2.3