# Motion Along a Straight Line: Constant Acceleration

## Constant Acceleration

If an object moves along a straight line and its acceleration is constant, we can say that the object has constant acceleration.

## Integrate Relation of Velocity and Acceleration

Recalling that acceleration is the rate of change of velocity, shown as below, where $a$ is acceleration, $v$ is velocity, and $t$ is time.

$$a = \frac{dv}{dt} \tag{1}$$

We can easily rewrite the equation (1) as Integrate relation of velocity and acceleration.

$$v = \int a dt \tag{2}$$

As acceleration is constant, we can take it out of the integral sign.

$$v = a \int dt = at + c \tag{3}$$

When $t = 0$,

$$v_0 = a \cdot 0 + c = c \tag{4}$$

Finally, we get the equation of velocity as a function of time.

$$v(t) = v_0 + at \tag{5}$$

## Sample Problem #1

Consider a car that moves along a straight line with a constant acceleration of $a = 8\ \mathrm{m/s^2}$. At $t = 2\ \mathrm{s}$, we find that the velocity of the car is $13\ \mathrm{m/s}$. Find the initial velocity of the car at $t = 0\ \mathrm{s}$.

*Sol.*

## Sample Problem #2 [Halliday 2.27]

An electron has a constant acceleration of $+3.2\ \mathrm{m/s^2}$. At a certain instant its velocity is $+9.6\ \mathrm{m/s}$. What is its velocity (a) 2.5 s earlier and (b) 2.5 s later?

*Sol.*

## Integrate Relation of Position and Velocity

Similarly, we can integrate the relation of position and velocity.

$$x = \int v dt \tag{6}$$

Putting (5) into (6), we get

$$x = \int (v_0 + at)dt = v_0 t + \frac{1}{2}at^2 + c \tag{7}$$

If we consider $x = x_0$ at $t = 0$, we can easily get

$$x_0 = 0 + 0 + c \tag{8}$$

and finally we get the equation of position as a function of time.

$$x(t) = x_0 + v_0 t + \frac{1}{2}at^2 \tag{9}$$

or

$$\Delta x = v_0 t + \frac{1}{2}at^2 \tag{9}$$

if we use (9) to rewrite (5), we get

$$(5)^2 \Rightarrow v^2 = v_0^2 + 2atv_0 + a^2t^2 = v_0^2 + 2(v_0 t + \frac{1}{2}at^2) = v_0^2 + 2a\Delta x \tag{10}$$

## Graphic Analysis

In some cases, we can use the graphical analysis to help us understand how the position ,velocity and acceleration are related. For example, if we want to know the position of an object at a certain time but the velocity and the acceleration are an complicated function, we can use the graphical analysis to help us.

## Sample Problem #3

Two people are traveling on a straight road by car and train. Both train and car have constant acceleration until they reach their greatest velocities, and they start with zero velocity. The acceleration of the car is $a = 6\ \mathrm{m/s^2}$, and the acceleration of the train is $a = 5\ \mathrm{m/s^2}$. The greatest velocity of the car is $v = 30\ \mathrm{m/s}$, and the greatest velocity of the train is $v = 40\ \mathrm{m/s}$. Answer the following questions: (a) Which vehicle is leading at $t = 3\ \mathrm{s}$? (b) Will another vehicle catch up? If so, when?

*Sol.*

## Free fall

Free fall is the process of an object falling from rest under the influence of gravity, where the acceleration due to gravity is constant and equals to $g = 9.8\ \mathrm{m/s^2}$.

### Example: Drop a Ball From a Height of $H$.

Assuming up is in the positive direction, since the acceleration due to gravity is $-g$, we can easily find the relation between height and time by using formula (9) ($v_0 = 0$ because the object is dropped from rest).

$$y(t) = H - \frac{1}{2}gt^2 \tag{11}$$

We can also find the velocity-height relation from (10).

$$(5)(10) \Rightarrow \left\{ \begin{aligned} v(t) &= -gt \\ t &= \sqrt{2(H-y)/g} \end{aligned} \right. \Rightarrow v = -\sqrt{2g(H-y)} \tag{12}$$

## Sample Problem #4

Assume that the constant acceleration of gravity is $g = 9.8\ \mathrm{m/s^2}$,if we throw a ball vertically from the ground with initial velocity $v_0 = 29.4\ \mathrm{m/s}$, answer the following questions: (a) What is the height and velocity of the ball at $t = 2\ \mathrm{s}$? (b) What is the maximum height of the ball? When does it reach that height? (c) When does the ball hit the ground?

*Sol.*

# Exercises

## Exercise #1 [halliday 2.23]

*Sol.*

## Exercise #2 [halliday 2.34]

*Sol.*

## Exercise #3 [halliday 2.36]

*Sol.*

## Exercise #4[halliday 2.53]

*Sol.*

## Exercise #5 [halliday 2.56]

*Sol.*

## Exercise #6 [halliday 2.67]

*Sol.*

## Exercise #7 [halliday 2.99]

*Sol.*

## Exercise #8 [halliday 2.113]

*Sol.*

## Exercise #9 [halliday 2.61]

*Sol.*

# Solutions

## Sample Problem #1

$$v(t) = v_0 + at, v(2) = v_0 + 8 \cdot 2 = 13 \Rightarrow v_0 = 13 - 16 = -3 \text{ (m/s)}$$

It means the car is moving backward at $t = 0$.

## Sample Problem #2 [Halliday 2.27]

Answer in Hallidays book.

## Sample Problem #3

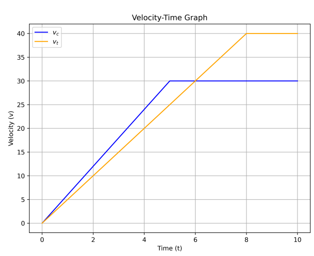
(a) At $t = 3$, the person with car is leading, because at the first 3 seconds, the car has a greater velocity than the train.

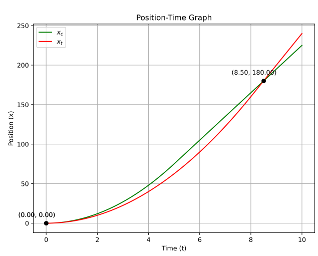
(b) In this case, we write down the path of the train and the path of the car.(we use $x_c$ and $x_t$ to denote the x-coordinate of the car and the train respectively, and $v_c$ and $v_t$ to denote the velocity of the car and the train respectively.)

$$v_c = \begin{cases} 6t & t \leq 5 \\ 30 & t \geq 5 \end{cases} \Rightarrow x_c = \begin{cases} 3t^2 & t \leq 5 \\ 75 + 30(t-5) & t \geq 5 \end{cases}$$

$$v_t = \begin{cases} 5t & t \leq 8 \\ 40 & t \geq 8 \end{cases} \Rightarrow x_t = \begin{cases} 2.5t^2 & t \leq 8 \\ 160 + 40(t-8) & t \geq 8 \end{cases}$$

If we solve the equation $x_c = x_t$, we can get $t = 0 \text{ or } 8.5$,(see the figure below, the figures show the velocity-time and position-time of the car and the train), so the person with train will catch up to the person with car at $t = 8.5 \text{ s}$.



## Sample Problem #4

(a) Using the equations of motion:

$$\begin{cases} v(t) = v_0 - gt \\ y(t) = v_0 t - \frac{1}{2}gt^2 \end{cases} \Rightarrow \begin{cases} v(2) = 9.8 \\ y(2) = 39.2 \end{cases}$$

(b) The maximum height is reached when the velocity becomes zero:

$$v(t_{\max}) = v_0 - gt_{\max} = 0 \Rightarrow t_{\max} = \frac{v_0}{g} = \frac{29.4}{9.8} = 3 \text{ s}$$

$$y_{\max} = v_0 t_{\max} - \frac{1}{2}g{t_{\max}}^2 = 29.4 \times 3 - \frac{1}{2} \times 9.8 \times 3^2 = 44.1 \text{ m}$$

(c) The ball hits the ground when $y(t) = 0$. Using the equation:

$$y(t) = v_0 t - \frac{1}{2}gt^2 = 29.4t - \frac{1}{2} \times 9.8t^2 = 0 \Rightarrow t = 0 \vee 6$$

Therefore, the ball hits the ground at $t = 6$ s.

## Exercise #1 [Halliday 2.23]

$1.62 \times 10^{15}\ \text{m/s}^2$

## Exercise #2 [Halliday 2.34]

(a) $-13.9$ m/s; (b) $-2\ \text{m/s}^2$ (minus sign means the direction is opposite to the positive direction.)

## Exercise #3 [Halliday 2.36]

(a) 56.6 s; (b) 31.8 m/s

## Exercise #4 [Halliday 2.53]

4m/s

## Exercise #5 [Halliday 2.56]

3.0 m/s

## Exercise #6 [Halliday 2.67]

0.56 m/s

## Exercise #7 [Halliday 2.99]

22 m/s

## Exercise #8 [Halliday 2.113]

(a) 5.44s; (b) 53.3m/s; (c) 5.80m

## Exercise #9 [Halliday 2.61]

20.4m