Apply Newton's Laws in One-Dimensional Systems

Way to Solve

1. Analyze the problem & draw a force diagram

Identify the question being asked and draw the given conditions on a diagram. Finally, assume and draw the unknown variables on the diagram. Forces should be marked on the object, and velocity and acceleration should be marked next to the object.

2. List the formulas

Based on the force diagram, find the net force on the object and use Newton's second law $F_{\text{net}} = ma$ to write the equation of motion for the object.

3. Find the common variables

In mechanics problems, there may be common variables, such as objects connected by a rope that may have the same speed and acceleration, or two objects in contact that may exert equal and opposite forces on each other.

4. Calculate

After following the above steps, you can calculate the value of the unknown variables.

Sample Problem #1

The cable's "strength" is commonly referred to as the tension at which it snaps. To stop a moving micro unmanned submarine weighing $931\ \mathrm{N}$ ($g = 9.8\ \mathrm{m/s^2}$) within $16\ \mathrm{m}$, what is the minimum strength required for the cable if the submarine initially travels at $24\ \mathrm{m/s}$? Assume that the submarine is under the situation of constant acceleration.

Sol.

Sample Problem #2

Assume a $72\ \mathrm{kg}$ astronaut and a $12\ \mathrm{kg}$ toolbox are floating in the zero-gravity environment of outer space, $28\ \mathrm{m}$ apart but connected by a rope of negligible mass. The astronaut exerts a horizontal $F = 6\ \mathrm{N}$ force on the rope. What are the acceleration magnitudes of (a) the toolbox and (b) the astronaut? (c) How far from the astronaut's initial position do they meet? (d) How many seconds does it take the astronaut to meet the toolbox?

Sol.

Sample Problem #3

A crane is lifting a spring scale, which has a steel plate with a mass of $20\ \mathrm{kg}$ hanging from it. The steel plate is being lifted from the ground to a platform at a height of $245\ \mathrm{m}$. Assuming that $g = 9.8\ \mathrm{m/s^2}$, answer the following questions: (a) If the crane lifts the steel plate at a constant speed, what is the weight displayed on the scale? (b) If the crane lifts the steel plate with constant acceleration and reaches a height of $122.5\ \mathrm{m}$ in the fifth second, what is the weight displayed on the scale during the acceleration period? (c) If the crane starts decelerating in the fifth second and comes to a stop just as it reaches the platform, what is the weight displayed on the scale during this process?

Sol.

Sample Problem #4 [Halliday 5.55]

Sol.

Sample Problem #5

A person and a weight are hanging on opposite sides of a pulley. If the weight's mass is three times that of the person, what is the person's acceleration? If the mass of the person is m, What is the tension T in the rope?

Sol.

Exercises

Exercise #1 [Halliday 5.41]

Sol.

Exercise #2 [Halliday 5.29]

Sol.

Exercise #3 [Halliday 5.48]

Sol.

Exercise #4 [Halliday 5.87]

Sol.

Exercise #5 [Halliday 5.54]

Sol.

Exercise #6 [Halliday 5.43]

Sol.

Exercise #7 [Halliday 5.69]

Sol.

Exercise #8 [Halliday 5.50]

Sol.

Exercise #9 [Halliday 5.51]

Sol.

Exercise #10 [Halliday 5.59]

Sol.

Solutions

Sample Problem #1

Solution:

To find the minimum strength required for the cable, we need to determine the tension force needed to stop the submarine. We start by calculating the deceleration required to bring the submarine to a stop.

$$v^2 = u^2 + 2as \tag{1}$$

Here, $v = 0 \text{ m/s}$ (final velocity), $u = 24 \text{ m/s}$ (initial velocity), and $s = 16 \text{ m}$ (distance).

$$0 = (24)^2 + 2a(16) \tag{2}$$

Solving for a:

$$a = -\frac{(24)^2}{2 \cdot 16} = -18 \text{ m/s}^2 \tag{3}$$

The negative sign indicates deceleration. Next, we calculate the force required to produce this deceleration using Newton's second law:

$$F = ma \tag{4}$$

The mass m of the submarine is:

$$m = \frac{931 \text{ N}}{9.8 \text{ m/s}^2} = 95 \text{ kg} \tag{5}$$

Therefore, the tension force F is:

$$F = 95 \text{ kg} \times 18 \text{ m/s}^2 = 1710 \text{ N} \tag{6}$$

The minimum strength required for the cable is 1710 N.

Sample Problem #2

Solution:

(a) What are the acceleration magnitudes of the toolbox?

The acceleration of the toolbox can be found using Newton's second law:

$$F = ma \tag{7}$$

Given $F = 6\ \mathrm{N}$ and $m = 12\ \mathrm{kg}$, we solve for a:

$$a = \frac{F}{m} = \frac{6\ \mathrm{N}}{12\ \mathrm{kg}} = \frac{1}{2}\ \mathrm{m/s^2} \tag{8}$$

(b) What are the acceleration magnitudes of the astronaut?

Similarly, the acceleration of the astronaut is:

$$a = \frac{F}{m} = \frac{6\ \mathrm{N}}{72\ \mathrm{kg}} = \frac{1}{12}\ \mathrm{m/s^2} \tag{9}$$

(c) How far from the astronaut's initial position do they meet?

Let x be the distance the astronaut travels. The toolbox travels $28\ \mathrm{m - x}$. Since they meet at the same time, we can set up the equation:

$$\frac{x}{\frac{1}{12}} = \frac{28-x}{\frac{1}{2}} \tag{10}$$

Solving for x:

$$12x = 2(28 - x) \tag{11}$$

$$x = 4 \tag{12}$$

Therefore, they meet $4\ \mathrm{m}$ from the astronaut's initial position.

(d) How many seconds does it take the astronaut to meet the toolbox?

Using the acceleration of the astronaut and the distance traveled:

$$x = \frac{1}{2}at^2 \Rightarrow t = \sqrt{\frac{2x}{a}} = \sqrt{\frac{2\times 4}{\frac{1}{12}}} = 9.80\ \mathrm{s} \tag{13}$$

It takes $9.80\ \mathrm{s}$ for the astronaut to meet the toolbox.

Sample Problem #3

Solution:

(a) If the crane lifts the steel plate at a constant speed, what is the weight displayed on the scale?

When the crane lifts the steel plate at a constant speed, the only force acting on the plate is gravity. The weight displayed on the scale is the gravitational force:

$$W = mg \tag{14}$$

Given $m = 20\ \mathrm{kg}$ and $g = 9.8\ \mathrm{m/s^2}$:

$$W = 20\ \mathrm{kg} \times 9.8\ \mathrm{m/s^2} = 196\ \mathrm{N} \tag{15}$$

Therefore, the weight displayed on the scale is $196\ \mathrm{N}$.

(b) If the crane lifts the steel plate with constant acceleration and reaches a height of $122.5\ \mathrm{m}$ in the fifth second, what is the weight displayed on the scale during the acceleration period?

First, we need to determine the acceleration. Using the kinematic equation:

$$s = ut + \frac{1}{2}at^2 \tag{16}$$

Given $s = 122.5\ \mathrm{m}$, $u = 0\ \mathrm{m/s}$ (initial velocity), and $t = 5\ \mathrm{s}$:

$$122.5 = 0 + \frac{1}{2}a(5)^2 \tag{17}$$

Solving for a:

$$a = \frac{2 \times 122.5}{25} = 9.8\ \mathrm{m/s^2} \tag{18}$$

The total force acting on the steel plate is the sum of the gravitational force and the force due to the crane's acceleration:

$$F_{total} = m(g + a) \tag{19}$$

Given $m = 20\ \mathrm{kg}$, $g = 9.8\ \mathrm{m/s^2}$, and $a = 9.8\ \mathrm{m/s^2}$:

$$F_{total} = 20\ \mathrm{kg} \times (9.8\ \mathrm{m/s^2} + 9.8\ \mathrm{m/s^2}) = 20\ \mathrm{kg} \times 19.6\ \mathrm{m/s^2} = 392\ \mathrm{N} \tag{20}$$

Therefore, the weight displayed on the scale during the acceleration period is $392\ \mathrm{N}$.

(c) If the crane starts decelerating in the fifth second and comes to a stop just as it reaches the platform, what is the weight displayed on the scale during this process?

During deceleration, the crane's acceleration is directed downward, reducing the effective acceleration. The deceleration is equal in magnitude to the previous acceleration:

$$a_{deceleration} = -9.8\ \mathrm{m/s^2} \tag{21}$$

The effective acceleration is:

$$a_{effective} = g + a_{deceleration} = 9.8 - 9.8 = 0\ \mathrm{m/s^2} \tag{22}$$

Therefore, the weight displayed on the scale during deceleration is:

$$W = m \times a_{effective} = 20\ \mathrm{kg} \times 0\ \mathrm{m/s^2} = 0\ \mathrm{N} \tag{23}$$

The weight displayed on the scale during this process is $0\ \mathrm{N}$.

Sample Problem #4 [Halliday 5.55]

(a) Find the magnitude of the force between the two blocks.

The total force applied to the system is equal to the total mass times the acceleration (Newton's second law). So, the acceleration of the system is:

$$a = \frac{F}{m_1+m_2} = \frac{3.2}{2.3+1.2} = 0.91 \text{ m/s}^2 \tag{24}$$

The force on the smaller block (F_2) is its mass times the acceleration:

$$F_2 = m_2 a = 1.2 \times 0.91 = 1.09 \text{ N} \tag{25}$$

(b) Show that if a force of the same magnitude F is applied to the smaller block but in the opposite direction, the magnitude of the force between the blocks is 2.1 N.

If the same force is applied to the smaller block in the opposite direction, the net force on the system is now -F. So, the acceleration of the system is:

$$a = \frac{-F}{m_1+m_2} = \frac{-3.2}{2.3+1.2} = -0.91 \text{ m/s}^2 \tag{26}$$

The force on the larger block (F_1) is its mass times the acceleration:

$$F_1 = m_1 a = 2.3 \times -0.91 = -2.10 \text{ N} \tag{27}$$

The force between the blocks is the difference between the forces on each block, which is 2.1 N.

(c) Explain the difference.

The difference arises because the force is applied to different blocks. When the force is applied to the larger block, more of the force is transferred to it. When the force is applied to the smaller block, more of the force is transferred to the larger block, resulting in a larger force between the blocks.

Sample Problem #5

Let's denote the mass of the person as m and the mass of the weight as $3m$. The net force acting on the person is the tension in the rope minus the weight of the person.

$$F_{net,person} = T - mg \tag{28}$$

For the weight, the net force is the weight of the mass minus the tension in the rope.

$$F_{net,weight} = 3mg - T \tag{29}$$

Since the system is connected by a rope, the acceleration of both the person and the weight will be the same. Using Newton's second law, we can write the equations for both masses:

$$\begin{cases} T - mg = ma \\ 3mg - T = 3ma \end{cases} \Rightarrow \begin{cases} a = 0.5g \\ T = 1.5mg \end{cases} \tag{30}$$

(a) What is the acceleration of the person?

The acceleration of the person is $a = \frac{g}{2}$.

(b) What is the tension in the rope?

The tension in the rope is $T = \frac{3mg}{2}$.

Exercise #1 [halliday 5.41]

(a)$1.4\ \mathrm{m/s^2}$; (b)$4.1\ \mathrm{m/s}$;

Exercise #2 [halliday 5.29]

(a) $494\ \mathrm{N}$; (b) upward; (c) $494\ \mathrm{N}$; (d) downward;

Exercise #3 [halliday 5.48]

$176\ \mathrm{N}$;

Exercise #4 [halliday 5.87]

(a) $65\ \mathrm{N}$; (b) $49\ \mathrm{N}$;

Exercise #5 [halliday 5.54]

$23\ \mathrm{kg}$;

Exercise #6 [halliday 5.43]

(a) $1.23\ \mathrm{N}$; (b) $2.46\ \mathrm{N}$; (c) $3.69\ \mathrm{N}$; (d) $4.92\ \mathrm{N}$; (e) $6.15\ \mathrm{N}$; (f) $0.250\ \mathrm{N}$;

Exercise #7 [halliday 5.69]

$2.4\ \mathrm{N}$;

Exercise #8 [halliday 5.50]

(a) $36.8\ \mathrm{N}$; (b) $0.191\ \mathrm{m}$;

Exercise #9 [halliday 5.51]

(a) $3.6\ \mathrm{m/s^2}$; (b) $17\ \mathrm{N}$;

Exercise #10 [halliday 5.59]

(a) $4.9\ \mathrm{m/s^2}$; (b) $2.0\ \mathrm{m/s^2}$; (c) upward; (d) $120\ \mathrm{N}$;