Two and Three Dimensional Motion

Position and Displacement in Two or Three-Dimensional Space

In one-dimensional motion, we use x to denote the position of an object, and Δx to denote the displacement. Both are scalars, as we can simply use positive or negative numbers to represent them.

However, in two or three-dimensional motion, we need to describe the position in a more complex space. Therefore, we use x and y to denote the position of an object in two or three-dimensional space, and Δx and Δy to denote the displacement. If we combine these terms, we can form a vector $\overrightarrow{r} = (x, y, \ldots)$ (sometimes written as $\overrightarrow{x}, \mathbf{r}, \mathbf{x}$) to represent the position, and $\Delta\overrightarrow{r} = (\Delta x, \Delta y, \ldots)$ to represent the displacement in two or three-dimensional space.

Unit Vector Notation

In physics, we use unit vector notation to represent the position and displacement of an object in two or three-dimensional space. For example, we use $\hat{i}, \hat{j}, and \hat{k}$ to represent the unit vectors in the $x, y, and z$ directions, respectively. Therefore, we can represent the position of an object in multi-dimensional space as $\overrightarrow{r} = x\hat{i} + y\hat{j} + \ldots$, and the displacement as $\Delta\overrightarrow{r} = \Delta x\hat{i} + \Delta y\hat{j} + \ldots$.

Sample Problem #1

Assume a car's position in a two-dimensional space is described as $\overrightarrow{r} = \cos\left(\frac{2\pi t}{T}\right)\hat{i} + \sin\left(\frac{2\pi t}{T}\right)\hat{j}$, where T is a constant. (a) What is the car's displacement between $\frac{T}{4}$ and $\frac{T}{2}$? (b) Does the car pass the origin at any point in time? If yes, when? (c) Does the car perform periodic motion? If yes, what is the period? (Periodic motion implies that the car will follow a path after a fixed time.)

$Sol.$

Sample Problem #2 [Halliday 4.4]

Sol.

Velocity in Two or Three-Dimensional Space

Similarly, we also need to describe the velocity of an object in two or three-dimensional space. We can use the formula below to represent the velocity of an object in two or three-dimensional space:

$$\overrightarrow{v} = (v_x, v_y, ...) = v_x\hat{i} + v_y\hat{j} + ... \tag{1}$$

Average Velocity

Recall that the average velocity in one-dimensional space can be written as follows:

$$v_{avg} = \frac{\Delta x}{\Delta t} \tag{2}$$

We can use a similar formula to calculate the average velocity in two or three-dimensional space, as they are based on the same physics.

$$\overrightarrow{v_{avg}} = \frac{\Delta\overrightarrow{r}}{\Delta t} = \frac{\Delta x\hat{i}+\Delta y\hat{j}+...}{\Delta t} = \frac{\Delta x}{\Delta t}\hat{i} + \frac{\Delta y}{\Delta t}\hat{j} + ... \tag{3}$$

Compared with (1), we can easily find that the vector element in the average velocity can be described as $(v_{avg})_i = \frac{\Delta r_i}{\Delta t}$

Instantaneous Velocity

Similar to the instantaneous velocity in one-dimensional space, the instantaneous velocity in multiple-dimensional space can be written as follows:

$$\overrightarrow{v} = \lim_{\Delta t\to 0}\frac{\Delta\overrightarrow{r}}{\Delta t} = \lim_{\Delta t\to 0}\frac{\Delta x}{\Delta t}\hat{i} + \lim_{\Delta t\to 0}\frac{\Delta y}{\Delta t}\hat{j} + ... = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + ... \tag{4}$$

Compared with the instantaneous velocity in one-dimensional space $v = \frac{dx}{dt}$, we can also write the instantaneous velocity in two or three-dimensional space as follows:

$$\overrightarrow{v} = \frac{d\overrightarrow{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + ... \tag{5}$$

Compared with (1), we can easily find that the vector element in the instantaneous velocity is the derivative of the vector element in the position, that is $v_i = \frac{dr_i}{dt}$

Sample Problem #3

Assume that a car's position in a two-dimensional space is described as $\overrightarrow{r} = 3t^2\hat{i} + 2t\hat{j}$. Answer the following questions: (a) What is the average velocity between 0 and 5 seconds? What is the magnitude of the average velocity? (b) What is the function of velocity over time?

Sol.

Sample Problem #4

Assume that a ball follows a path $y(x) = x^3 + x^2 - 3x - 2$. And x is a function of time, $x(t) = 4t^3 + (t-2)^2 - 5$. Answer the following questions: (a) Will the ball pass the origin at any point in time? If yes, when? (b) Find the average velocity between 0 and 1 seconds. (c) Find the function of the ball's velocity over time. (d) Will the ball stop at any time? If yes, when?

Sol.

Sample Problem #5

Suppose the motion of a small ball over time is given by $\overrightarrow{r} = 4t\hat{i} + 3t^2\hat{j}$. What is the magnitude of the ball's velocity? What is the relationship between the direction of motion and the angle with the x-axis over time?

Sol.

Acceleration in Two or Three-Dimensional Space

Similarly, we can describe the acceleration in a two or three-dimensional space. We can use the formula below to represent the acceleration of an object in a two or three-dimensional space:

Average Acceleration

$$\overrightarrow{a}_{avg} = \frac{\Delta\overrightarrow{v}}{\Delta t} = \frac{\Delta v_x}{\Delta t}\hat{i} + \frac{\Delta v_y}{\Delta t}\hat{j} + \ldots \tag{6}$$

Instantaneous Acceleration

$$\overrightarrow{a} = \frac{d\overrightarrow{v}}{dt} = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} + \ldots \tag{7}$$

Sample Problem #6

If the velocity of an object in a two-dimensional space is described as $\overrightarrow{v} = \cos(t)\hat{i} + 2t\sin(t)\hat{j}$. (a) What is the average acceleration between 0 and $\frac{\pi}{3}$ seconds? (b) Find the function of acceleration over time.

Sol.

Sample Problem #7

(a) Follow the step in formula (2) to (5), derive the formula (6) and (7).
(b) Find the derivative relation between insantaneous acceleration and position in three-dimension space.

Sol.

Integrate Relation of Position, Velocity and Acceleration

Similar to one-dimensional motion, we can integrate the derivative relation to get the relation between the position, velocity and acceleration.

Integrate Relation between Position and Velocity

In (1)(5), we can easily find that each vector element in the instantaneous velocity is the derivative of the vector element in the position, so we can integrate them respectively to get the relation between the position and velocity.

$$(1)\ (5) \Rightarrow v_i = \frac{dr_i}{dt} \Rightarrow r_i(t) = \int dr_i = \int v_i dt \tag{8}$$

If the velocity v_i is constant ,and the position at time $t = 0$ is $r_i(0) = r_{i0}$, then we can integrate the relation between the position and velocity.

$$(8) \Rightarrow r_i(t) = C_i + v_i t \Rightarrow r_i(t) = r_{i0} + v_i t \tag{9}$$

Other Relations between Position, Velocity and Acceleration

Similarly, we can also find the relations below:

$$v_i(t) = \int dv_i = \int a_i dt \tag{10}$$

$$v_i(t) = v_{i0} + a_i t \quad \text{(Under the condition that the acceleration is constant)} \tag{11}$$

$$r_i(t) = r_{i0} + v_{i0} t + \frac{1}{2} a_i t^2 \quad \text{(Under the condition that the acceleration is constant)} \tag{12}$$

Sample Problem #8

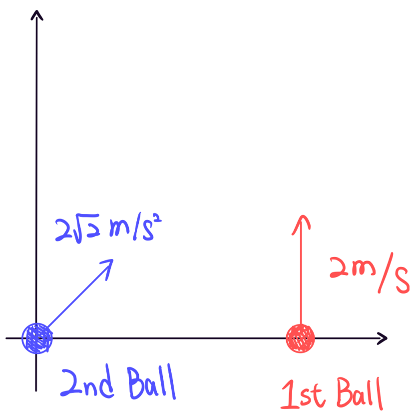
(a) Derive the formula (10) to (12). (b) If the acceleration is not constant, given by $a(t) = 4t + 3$, find the relation between position and velocity.

Sol.

Sample Problem #9

There is a game with two balls, the first ball is moving on with the velocity $v_1 = 2\hat{j}\ \mathrm{m/s}$.The player need to launch the second ball with 45 degrees and the acceleration $a_2 = 2\sqrt{2}\ \mathrm{m/s^2}$ from the origin from the rest. If the second ball hit the first ball, get the formula goal. If the first ball is launched from the position $r_{x0} = 9\hat{i}\ \mathrm{m}$, how many time the player need to wait for launching the second ball making the second ball hit the first ball? How many seconds to hit the first ball after the first ball launched?

Sol.



Exercises

Exercise #1 [halliday 4.3]

Sol.

Exercise #2 [halliday 4.86]

Sol.

Exercise #3 [halliday 4.7]

Sol.

Exercise #4 [halliday 4.9]

Sol.

Exercise #5 [halliday 4.10]

Sol.

Exercise #6 [halliday 4.12]

Sol.

Exercise #7 [halliday 4.11]

Sol.

Exercise #8 [halliday 4.17]

Sol.

Exercise #9 [halliday 4.19]

Sol.

Exercise #10 [halliday 4.20]

Sol.

Exercise #11 [halliday 4.95]

Sol.

Solutions

Sample Problem #1

(a) What is the car's displacement between $\frac{T}{4}$ and $\frac{T}{2}$

The displacement is the change in position over the change in time. We can find the position at $t = \frac{T}{4}$ and $t = \frac{T}{2}$, subtract them, and then divide by the change in time.

$$\Delta\overrightarrow{r} = \overrightarrow{r}(\tfrac{T}{2}) - \overrightarrow{r}(\tfrac{T}{4}) = (-1, -1) \tag{1}$$

(b) Does the car pass the origin at any point in time? If yes, when?

when $\cos\left(\frac{2\pi t}{T}\right) = 0 \Rightarrow \sin\left(\frac{2\pi t}{T}\right) = \pm 1$

The car will never pass the origin.

(c) Does the car perform periodic motion? If yes, what is the period?

If the car have a periodic motion, the car will following the formula below,where T_p is the period.

$$\overrightarrow{r(t)} = \overrightarrow{r(t+T_p)} \tag{2}$$

In this case, We can rewrite the equation as follows.

$$\begin{aligned} \overrightarrow{r(t+T_p)} &= \cos\left(\frac{2\pi(t+T_p)}{T}\right)\hat{i} + \sin\left(\frac{2\pi(t+T_p)}{T}\right)\hat{j} \\ &= \cos\left(\frac{2\pi t}{T} + \frac{2\pi T_p}{T}\right)\hat{i} + \sin\left(\frac{2\pi t}{T} + \frac{2\pi T_p}{T}\right)\hat{j} \end{aligned} \tag{3}$$

If we take T_p equal to T, then we get

$$\begin{aligned} \overrightarrow{r(t+T)} &= \cos\left(\frac{2\pi t}{T} + 2\pi\right)\hat{i} + \sin\left(\frac{2\pi t}{T} + 2\pi\right)\hat{j} \\ &= \cos\left(\frac{2\pi t}{T}\right)\hat{i} + \sin\left(\frac{2\pi t}{T}\right)\hat{j} = \overrightarrow{r(t)} \end{aligned} \tag{4}$$

Sample Problem #2 [Halliday 4.4]

(a)

The minute hand moves from the 3 o'clock position to the 6 o'clock position. The magnitude of the displacement vector is the straight-line distance between these two points.

$$|\vec{r}| = \sqrt{(10\,\mathrm{cm})^2 + (10\,\mathrm{cm})^2} = \sqrt{200}\,\mathrm{cm} = 10\sqrt{2}\,\mathrm{cm} \approx 14.14\,\mathrm{cm} \tag{5}$$

(b)

The angle is the change in the angle of the minute hand. From 3 o'clock to 6 o'clock, the minute hand moves 90 degrees.

$$\theta = 90^\circ \tag{6}$$

(c)

The minute hand moves from the 6 o'clock position to the 12 o'clock position. The magnitude of the displacement vector is the straight-line distance between these two points.

$$|\vec{r}| = 20 \text{ cm} \tag{7}$$

(d)

The angle is the change in the angle of the minute hand. From 6 o'clock to 12 o'clock, the minute hand moves 180 degrees.

$$\theta = 180^\circ \tag{8}$$

(e)

The minute hand moves from the 12 o'clock position back to the 12 o'clock position. The magnitude of the displacement vector is zero because the starting and ending points are the same.

$$|\vec{r}| = 0\,\mathrm{cm} \tag{9}$$

(f)

The angle is the change in the angle of the minute hand. From 12 o'clock to 12 o'clock, the minute hand moves 360 degrees.

$$\theta = 360^\circ \tag{10}$$

Sample Problem #3

(a) What is the average velocity between 0 and 5 seconds? What is the magnitude of the average velocity?

The average velocity is the change in position divided by the change in time. We can find the position at $t = 0$ and $t = 5$, subtract them, and then divide by the change in time.

$$v_{avg} = \frac{\Delta r}{\Delta t} = \frac{r(5) - r(0)}{5 - 0} = \begin{pmatrix} 15 \\ 2 \end{pmatrix} \tag{11}$$

The magnitude of the average velocity is the square root of the sum of the squares of the components of the average velocity.

$$||v_{avg}|| = \sqrt{(v_{avg})^2} = \sqrt{15^2 + 2^2} = 15.13 \tag{12}$$

(b) What is the function of velocity over time?

The velocity is the derivative of the position. We can find the derivative of each component of the position to get the velocity.

$$v = \frac{dr}{dt} = \begin{pmatrix} 6t \\ 2 \end{pmatrix} \tag{13}$$

Sample Problem #4

(a) Will the ball pass the origin at any point in time? If yes, when?

If we put $x = 0$ into $y(x) \Rightarrow y(0) = -2$. Therefore, the ball will not pass the origin at any point in time.

(b) Find the average velocity between 0 and 1 seconds.

The average velocity is the change in position divided by the change in time. We can find the position at $t = 0$ and $t = 1$, subtract them, and then divide by the change in time.

$$r(0) = \begin{pmatrix} x(0) \\ y(x(0)) \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}, r(1) = \begin{pmatrix} x(1) \\ y(x(1)) \end{pmatrix} = \begin{pmatrix} 0 \\ -2 \end{pmatrix} \Rightarrow \\ \Delta r = \begin{pmatrix} 1 \\ -3 \end{pmatrix} \Rightarrow v_{avg} = \frac{\Delta r}{\Delta t} = \begin{pmatrix} 1 \\ -3 \end{pmatrix} \tag{14}$$

(c) Find the function of the ball's velocity over time.

The velocity is the derivative of the position. We can find the derivative of each component of the position to get the velocity.

$$v_x = \frac{dx}{dt} = 12t^2 + 2(t - 2) = 12t^2 + 2t - 4 \tag{15}$$

$$v_y = \frac{dy}{dt} = \frac{dx}{dt}\frac{dy}{dx} = (3x^2 + 2x - 3)(12t^2 + 2t - 4) \tag{16}$$

(d) Will the ball stop at any time? If yes, when?

Stop means that the velocity is 0. When we see the v_x we can find that it is 0 when $t = \frac{1}{2} \lor t = -\frac{2}{3}$. If we check the v_y we can find that it is 0 when $v_x = 0$. So the ball will stop at $t = \frac{1}{2} \lor t = -\frac{2}{3}$.

Sample Problem #5

Suppose the motion of a small ball over time is given by $\overrightarrow{r} = 4t\hat{i} + 3t^2\hat{j}$. What is the magnitude of the ball's velocity? What is the relationship between the direction of motion and the angle with the x-axis over time?

Sol.

(a) What is the magnitude of the ball's velocity?

The velocity vector is the derivative of the position vector with respect to time.

$$\overrightarrow{v} = \frac{d\overrightarrow{r}}{dt} = \frac{d}{dt}(4t\hat{i} + 3t^2\hat{j}) = 4\hat{i} + 6t\hat{j} \tag{17}$$

The magnitude of the velocity vector is given by:

$$||\overrightarrow{v}|| = \sqrt{(4)^2 + (6t)^2} = \sqrt{16 + 36t^2} = 2\sqrt{4 + 9t^2} \tag{18}$$

(b) What is the relationship between the direction of motion and the angle with the x-axis over time?

The direction of motion is given by the angle θ between the velocity vector and the x-axis. This angle can be found using the tangent function:

$$\tan(\theta) = \frac{v_y}{v_x} = \frac{6t}{4} = \frac{3t}{2} \tag{19}$$

Therefore, the angle θ is:

$$\theta = \tan^{-1}\left(\frac{3t}{2}\right) \tag{20}$$

This shows that the angle θ increases with time, indicating that the direction of motion changes as the ball moves.

Sample Problem #6

(a) What is the average acceleration between 0 and $\frac{\pi}{3}$ seconds?

The average acceleration is the change in velocity divided by the change in time. We can find the velocity at $t = 0$ and $t = \frac{\pi}{3}$, subtract them, and then divide by the change in time.

$$a_{avg} = \frac{\Delta v}{\Delta t} = \frac{v(\frac{\pi}{3}) - v(0)}{\frac{\pi}{3} - 0} = -\frac{3}{2\pi}\hat{i} + \frac{\sqrt{3}}{2}\hat{j} \tag{21}$$

(b) What is the function of acceleration over time?

The acceleration is the derivative of the velocity. We can find the derivative of each component of the velocity to get the acceleration.

$$a = \frac{dv}{dt} = -\sin(t)\hat{i} + (2t\cos(t) + 2\sin(t))\,\hat{j} \tag{22}$$

Sample Problem #7

(a) Follow the step in formula (2) to (5), derive the formula (6) and (7).

The average acceleration in one-dimensional space can be written as follows:

$$a_{avg} = \frac{\Delta v}{\Delta t} \tag{23}$$

In two or three-dimensional space, the formula for average acceleration becomes:

$$\overrightarrow{a_{avg}} = \frac{\Delta \overrightarrow{v}}{\Delta t} = \frac{\Delta v_x\hat{i} + \Delta v_y\hat{j} + \ldots}{\Delta t} = \frac{\Delta v_x}{\Delta t}\hat{i} + \frac{\Delta v_y}{\Delta t}\hat{j} + \ldots \tag{24}$$

Similar to velocity, we can find that the vector element in the average acceleration can be described as:

$$\left(a_{avg}\right)_i = \frac{\Delta v_i}{\Delta t} \tag{25}$$

The instantaneous acceleration in one-dimensional space can be written as follows:

$$a = \frac{dv}{dt} \tag{26}$$

In two or three-dimensional space, the formula for instantaneous acceleration becomes:

$$\overrightarrow{a} = \frac{d\overrightarrow{v}}{dt} = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} + \ldots \tag{27}$$

Compared with the instantaneous acceleration in one-dimensional space, we can also write the instantaneous acceleration in two or three-dimensional space as follows:

$$\overrightarrow{a} = \frac{d\overrightarrow{v}}{dt} = \frac{dv_x}{dt}\hat{\imath} + \frac{dv_y}{dt}\hat{\jmath} + \ldots \tag{28}$$

Finally , we can easily find that the vector element in the instantaneous acceleration is the derivative of the vector element in the velocity, that is:

$$a_i = \frac{dv_i}{dt} \tag{29}$$

(b) Find the derivative relation between insantaneous acceleration and position in three-dimension space.

First, let's recall the definitions:

$$\overrightarrow{v} = \frac{d\overrightarrow{r}}{dt} = \frac{dx}{dt}\hat{\imath} + \frac{dy}{dt}\hat{\jmath} + \frac{dz}{dt}\hat{k} \tag{30}$$

$$\overrightarrow{a} = \frac{d\overrightarrow{v}}{dt} = \frac{d^2x}{dt^2}\hat{\imath} + \frac{d^2y}{dt^2}\hat{\jmath} + \frac{d^2z}{dt^2}\hat{k} \tag{31}$$

From these definitions, we can see that acceleration is the second derivative of position with respect to time. Therefore, the derivative relationship between instantaneous acceleration and position in three-dimensional space is:

$$\overrightarrow{a} = \frac{d^2\overrightarrow{r}}{dt^2} = \frac{d^2x}{dt^2}\hat{\imath} + \frac{d^2y}{dt^2}\hat{\jmath} + \frac{d^2z}{dt^2}\hat{k} \tag{32}$$

Sample Problem #8

(a) Derive the formula (10) to (12).

We can easily find that each vector element in the instantaneous acceleration is the derivative of the vector element in the velocity, so we can integrate them respectively to get the relation between the velocity and acceleration.

$$a_i = \frac{dv_i}{dt} \Rightarrow v_i(t) = \int dv_i = \int a_i dt \tag{33}$$

If the acceleration is constant, the velocity at time $t = 0$ is $v_i(0) = v_{i0}$, then we can integrate the relation between the position and velocity.

$$v_i(t) = C_i + a_i t \Rightarrow v_i(t) = v_{i0} + a_i t \tag{34}$$

The relation between the position and constant acceleration can be obtained by integrating derivative relation between the position and velocity.

$$r_i(t) = \int v_i dt = \int (v_{i0} + a_i t)dt = r_{i0} + v_{i0}t + \frac{1}{2}a_i t^2 \tag{35}$$

(b) If the acceleration is not constant, given by $a(t) = 4t + 3$, find the relation between position and velocity.

We can easily find that each vector element in the instantaneous acceleration is the derivative of the vector element in the velocity, so we can integrate them respectively to get the relation between the velocity and acceleration.

$$a(t) = \frac{dv(t)}{dt} \Rightarrow v(t) = \int dv = \int a(t)dt = \int (4t+3)dt = 2t^2 + 3t + C_1 \tag{36}$$

If the acceleration is not constant, the velocity at time $t = 0$ is $v(0) = v_0$, then we can integrate the relation between the position and velocity.

$$v(t) = C_1 + \int a(t)dt \Rightarrow v(t) = v_0 + 2t^2 + 3t \tag{37}$$

The relation between the position and non-constant acceleration can be obtained by integrating derivative relation between the position and velocity.

$$x(t) = \int v(t)dt = \int (v_0 + 2t^2 + 3t)dt = x_0 + v_0 t + \frac{2}{3}t^3 + \frac{3}{2}t^2 \tag{38}$$

Sample Problem #9

let t is the time after the first ball launched, and $t_{\text{wait}}$is the time the player need to wait for launching the second ball.

First Ball:

The first ball is moving vertically upwards with a velocity of 2 m/s from the position 9 m along the x-axis. So, its position at any time t can be described as $r_1(t) = 9\hat{i} + 2t\hat{j}$.

Second Ball:

The second ball is launched from rest at a 45-degree angle with an acceleration of $2\sqrt{2}\ \mathrm{m/s^2}$.We can easily get the $a_2 = 2\hat{i} + 2\hat{j}\ \mathrm{m/s^2}$. The position of the second ball at any time t (where t $\geq t_{\text{wait}}$) is given by:

Along x-axis: $x_2(t) = \frac{1}{2} \cdot 2(t - t_{\text{wait}})^2 = (t - t_{\text{wait}})^2$

Along y-axis: $y_2(t) = \frac{1}{2} \cdot 2(t - t_{\text{wait}})^2 = (t - t_{\text{wait}})^2$

Collision Condition:

$9 = (t - t_{\text{wait}})^2, 2t = (t - t_{\text{wait}})^2$

Therefore, the time to hit the first ball is $t = 4.5\mathrm{s}$, $t_{\text{wait}} = 1.5\mathrm{s}$

Exercise #1 [halliday 4.3]

$(-2.0)\hat{i} + (6.0)\hat{j} - (10)\hat{k} \qquad \text{(unit: meter)}$

Exercise #2 [halliday 4.86]

(a) $\vec{r}_1 = 276\hat{i} + 231\hat{j}\ (\mathrm{m}), \vec{\mathrm{r}}_2 = -755\hat{\mathrm{i}} - 231\hat{\mathrm{j}}\ (\mathrm{m}), \vec{\Delta}\mathrm{r} = -1031\hat{\mathrm{i}}\ (\mathrm{m})$; (b) westward;

Exercise #3 [halliday 4.7]

$-0.70\hat{i} + 1.4\hat{j} - 0.40\hat{k}\ (\mathrm{m/s})$

Exercise #4 [halliday 4.9]

(a) 0.83 cm/s; (b) 0°; (c) 0.11 m/s;

Exercise #5 [halliday 4.10]

(a) $3.5\ (\mathrm{m/s})$; (b) $-0.125\ (\mathrm{m/s^2})$;

Exercise #6 [halliday 4.12]

(a) $56.6\ (\mathrm{m})$; (b) 135°; (c) $1.89(\mathrm{m/s})$; (d) 135°; (e) $0.471\ (\mathrm{m/s^2})$; (f) 45°;

Exercise #7 [halliday 4.11]

(a) $6.00\hat{i} + 106\hat{j}\ (\mathrm{m})$; (b) $19.0\hat{i} - 224\hat{j}\ (\mathrm{m/s})$; (c) $24.0\hat{i} - 336\hat{j}\ (\mathrm{m/s^2})$; (d) -85.2°;

Exercise #8 [halliday 4.17]

$32\,\hat{i} \qquad \text{(unit: m/s)}$

Exercise #9 [halliday 4.19]

$(a)(72.0\text{ m})\hat{\mathrm{i}} + (90.7\text{ m})\hat{\mathrm{j}}; (b)49.5^\circ$

Exercise #10 [halliday 4.20]

60°

Exercise #11 [halliday 4.95]

(a) 1.5; (b) (36 m, 54 m)