Introduction

In the social sciences, few regularities attain law-like status. The idea that war's deadliness follows a power-law distribution holds this coveted spot. The power-law is a simple heavy-tailed probability distribution used to describe a range of phenomena that have many small values and only a few large ones. Lewis F. Richardson first noticed that international war fatalities follow such a distribution in the mid-twentieth century (Richardson 1948, 1960), and in the years since his initial discovery scholars attributed the power-law to various other dimensions of war (Cioffi-Revilla and Midlarsky 2010) and to several other types of violence, including terrorism and civil wars (Clauset 2020). This places human conflict in the company of wide-ranging phenomena, from earthquakes to solar flares, and fluid turbulence to city population size (Clauset, Shalizi, and Newman 2009).

While the idea that wars obey the power-law has a long tradition in IR, it received special attention with the publication of Bear Braumoeller's (2019) book *Only the Dead*, which provides a thorough, data-driven rebuttal to claims made by proponents of the so-called decline-of-war thesis (see Pinker 2011). Adherents to the view claim that for reasons ranging from Enlightenment Humanism to shifting cultural norms, human violence is in a secular decline and has been for some time.

Contrary to this optimistic view of human nature, Braumoeller (2019) finds, among many things, that power-law fits to international wars before and after 1945 (a significant pivot-point for decline-of-war theorists) are statistically indistinguishable and each consistent with a potentially infinite mean for war fatalities—a unique possibility that can arise in power-law probability distributions. Taken together, these findings simultaneously deal a blow to a key claim of the decline-of-war thesis and warn against complacency about the deadly potential of any future war.

While not questioning the totality of evidence Braumoeller (2019) levies against the decline-of-war thesis, results from this study raise questions specifically about the power-law's applicability to war intensity. Does an alternative probability distribution provide a better fit for the data? If so, does this alternative provide different inferences about whether a shift in war's deadliness occurred in the last century?

The case is made here that the power-law *does not* provide the best fit for the data. Rather, a log-logistic distribution provides a better fit and supports different inferences about a change in war intensity. Using data on the intensity of major international wars from the past 200 years, a comparison of the power-law model and a log-logistic model demonstrates the superiority of the latter. The log-logistic model of war intensity not only provides a much better fit for *all* of the data, its predictions have far greater efficiency than that of the power-law in the extreme tails of the distribution of war intensity.

This finding has important general implications for theory, statistical inference, and hypothesis testing with war intensity data (see Cederman 2003; Cederman, Warren, and Sornette 2011; and Friedman 2014). More specifically, as further replication and extension of the analysis done by Braumoeller (2019) shows, the choice between using the power-law or the log-logistic probability distribution leads to different conclusions about whether a significant shift in war's deadliness occurred after 1945. Consistent with Braumoeller's (2019) findings, when modeled with the power-law, a statistical test fails to reject the null that war intensities pre- and post-1945 come from different distributions. However, when modeled with the log-logistic distribution, a statistical test rejects the null, and the data are consistent with a substantial decline in war intensity after 1945.

The paper proceeds as follows. Motivation and background on modeling wars with the power-law is provided. A summary of the power-law model of war intensity and an alternative log-logistic specification follows. I then summarize the data used to compare the power-law and log-logistic modeling approaches. Data on war fatalities come from the Correlates of War (COW) Project. War intensity is calculated as the total battle deaths of all countries involved in a war per the combined population of the relevant countries at the time of the conflict. Next, results comparing the power-law with the log-logistic fit are presented, followed by a replication and extension of Braumoeller (2019).

The final section addresses the implications of these findings. While the power-law may apply in numerous settings, in the case of international wars it provides a poorer fit than decades of tradition would lead one to believe. In contrast to the power-law, the log-logistic model offers a better fit and does so with greater statistical precision. This finding has significant ramifications for the quantitative study of war. Scholars have predicated theoretical models of war's microfoundations, missing data imputation, and hypothesis testing on the power-law (Cederman 2003; Cederman, Warren, and Sornette 2011; Friedman 2014). A range of important research may need to be reevaluated to confirm its robustness.

Outside of its implications for quantitative research on war, the theme of this study falls in line with a broader trend across scientific fields questioning the appropriateness of the power-law where once scholars took it for granted. For example, recent studies show that more than a 1,000 real-world networks (Broido and Clauset 2019) and even solar flares (Verbeeck et al. 2019), which researchers long thought obeyed the power-law, instead may arguably follow a log-normal distribution (something very close to the log-logistic). The time, therefore, is well overdue to revisit the power-law and war.

Power-Laws and War Deaths

Regarding power-laws and war, the title of Johnson and colleagues' (2013) article in *Scientific Reports* states plainly that a "Simple Mathematical Law Benchmarks Human Confrontations." They of course mean the power-law, which enjoys a special place in the quantitative study of war. As some of the examples discussed in this section highlight, scholars rely on the power-law for theorization of war's microfoundations, prediction and missing value imputation, and hypothesis testing. With such important objectives predicated on the power-law, the choice to use it as opposed to other models has far-reaching implications.

The idea that war intensity obeys the power-law—a simple mathematical model that describes phenomena with many small magnitude events and few extreme magnitude ones—originates with the influential physicist, meteorologist, and quantitative scholar of human violence, Lewis F. Richardson (1948, 1960). Sometimes referred to as "Richardson's Law" (Clauset, Young, and Gleditsch 2007), the power-law of war holds a privileged position as one of the few regularities in the social sciences granted law-like status.

Tradition and convenience lie behind the inertia that propelled the power-law to ubiquity. Power-law distributions have the unique (and convenient) characteristic of linearity when plotted in log-log space. The slope of the probability density distribution of a power-law over the range of possible intensity levels of some variable in log-log space lets one seamlessly map possible intensities to a specific probability of occurrence. In addition to making projections of intensity exceptionally easy, the method of estimating the relevant slope parameter has a simple closed-form solution (Clauset, Shalizi, and Newman 2009).

Power-laws also have a rhetorically useful feature. Power-laws have only a single parameter, and if this parameter takes a value at or less than 2 the mean of the variable in question can possibly be infinite. An infinite mean implies that there is no upper limit on intensity (at least none that is mathematically identifiable), making random variation in war fatalities all the more alarming. Analyses such as Braumoeller's (Braumoeller 2019) have identified power-law slopes that are consistent with this posibility. This gives shockingly ferocious and deadly international wars the qualities of other "black swan" events, namely, a high degree of unpredictability coupled with massive impact (Taleb 2010).

With few exceptions (see Zwetsloot 2018), the power-law abides as the framework of choice for studying war. With respect to theory generation, Cederman (2003) predicated an agent-based model of war on the stylized fact that the most intense wars seem to conform to the power-law. His study establishes microfoundations of human conflict that generate power-law distributed war intensities.

Scholars have extended the power-law to other dimensions of conflict, too. Cioffi-Revilla and Midlarsky (2010) conducted the first systematic replication of Richardson's (1948, 1960) analysis, using the most up-to-date COW data on international conflicts and extending the analysis to civil wars as well. They also fit war onset and duration to the power-law, expanding the range of war's characteristics that plausibly conform to the power-law distribution. They conclude with a call for more rigorous theorization of the data-generating processes that give rise to these unique characteristics of conflict. Other studies have used the power-law to study terrorism and all manner of human violence, including attacks by lone-wolf actors (Clauset, Young, and Gleditsch 2007; and Johnson et al. 2013).

The power-law also serves as a framework for missing data imputation. In one prominent example, Friedman (2014) leveraged the power-law to make inferences about unobserved fatalities in Native American and US conflicts from 1776 to 1890. Friedman (2014) lauds the potential of the power-law to assist in missing value imputation for a range of important conflicts for which data either are unavailable or untrustworthy.

Lastly, the power-law is used in hypothesis testing. Cederman, Warren, and Sornette (2011) collected data on the fatalities in major power wars since 1495 and fit the data to the power-law using different successive splits in the sample to identify a significant shift in war deadliness following the Napoleonic Wars. Braumoeller (2019), as already discussed, used the power-law to test a claim from the decline-of-war thesis that wars became less severe after 1945.

In sum, a lot rides on the appropriateness of the fit between observed war intensity and the power-law; not just inferences important for the decline-of-war thesis, but theorization, hypothesis testing, and prediction more generally. A substantial mismatch between the data and the power-law therefore would raise questions about the reliability of analyses that use the approach. This would call for a reexamination of several important studies (including many of those cited here).

Possible Problems with the Power-Law

The power-law reigns supreme, in part, because of a preoccupation with modeling the extreme ends of the distribution of war intensity. This preoccupation comes as no surprise. However, the focus on the thick tail of the distribution of wars has led to the neglect of the other end of the distribution.

Methodology may bear some of the blame. Clauset, Shalizi, and Newman (2009) established the state-of-the-art in fitting power-laws to empirical data. They provide a helpful and more consistent closed-form maximum likelihood estimator for parameter estimation in lieu of the more typical ordinary least squares approach. They also recommend and describe a procedure for fitting a power-law model to data that involves identifying a lower bound beyond which the data conform well to the power-law, but below which the data do not. This particular recommendation makes it practically feasible to ignore smaller magnitude events in favor of events in the extreme tail of the distribution.

Identifying a lower-bound has a practical purpose. Rarely do observed data perfectly conform to our theoretical models of their generation, and the need to identify a lower bound reflects this reality. But at what point does this necessary concession to make the model fit data slip into an effort to make the data fit the model? As Clauset, Shalizi, and Newman (2009) warn, their method will "tell us only the best fit to the power-law form, not whether the power law is in fact a good model for the data" (667).

The idea that there might be some inconsistencies between the power-law and forms of human conflict has precedent. The most recent and comprehensive critique to this author's knowledge appears in a working paper by Zwetsloot (2018). Zwetsloot (2018) engages in a mammoth data collection effort, assessing the fit of the power-law to over 685,000 conflict events across 16 datasets. His data is restricted to civil conflict events for which data are publicly available, limiting our ability to draw inferences from his analysis to major international wars. Nonetheless, his analysis reveals a less-than-consistent fit between the power-law distribution and a large number of civil conflict events. Zwetsloot finds that the power-law provides an excellent fit for only a third of the data and a probable fit for only half.

Recent studies raise questions about the appropriateness of the power-law for a host of other phenomena from the physical and social world as well. Connections within networks of various kinds have been long thought to obey the power-law, but recent research that examines more than 1,000 real-world networks from social, biological, technological, and informational domains finds otherwise (Broido and Clauset 2019). Solar flares have also been argued to obey the power-law, but a recent study suggests poor data bear the blame for this inference. Verbeeck et al. (2019) find that after correcting for background noise, the log-normal distribution better describes both peak flux and fluence of solar flares, overturning decades of precedent.

In sum, the idea that the power-law provides the best description of war intensity, while ubiquitous, deserves reexamination. The following section discusses the technical details of the power-law model and contrasts it with a proposed alternative: a log-logistic model of war intensity. The latter model has the potential to better fit not only the observations in the extreme tail of the distribution but also the less extreme (and more likely) war intensities on the left side of the distribution that fitting the power-law to the data requires we ignore.

The Power-Law versus the Log-Logistic Distribution

Many probability distributions can characterize variables with many small events and a few larger ones. These include exponential, log-normal, or log-logistic distributions in addition to the power-law. To narrow the scope of this study, in this section and those that follow the power-law is contrasted with the log-logistic distribution. The log-logistic model represents a promising alternative to the power-law for characterizing war intensity for two reasons: (1) it should provide a better fit for smaller wars while not interfering with the fit to more extreme conflicts and (2) it has thicker tails than distributions such as the log-normal which provides some additional advantages in dealing with the thicker-tailed empirical distribution that various forms of conflict tend to follow.

First, consider the power-law model of war deaths. The power-law specifies that the probability of observing a war deadlier than some value $x > 0$ is

$$\Pr(X > x) = \frac{\exp(\alpha)}{x^{\beta}} \quad \forall \quad x \geqslant x_{min} \tag{1}$$

where x_{min} is a lower bound such that x follows a power-law distribution to the degree $\beta > 0$ and α is a constant. $\Pr(X > x)$ is one minus the empirical cumulative density of observed war intensities. Expressed in log-log space, this is

$$\log[\Pr(X > 0)] = \alpha - \beta \log(x) \quad \forall \quad x \geqslant x_{min} \tag{2}$$

Given this convenient form, many studies have used ordinary least squares to fit the model parameters. However, Clauset, Shalizi, and Newman (2009) show that this approach can be inconsistent and recommend instead using maximum likelihood. The maximum likelihood estimator for the slope β has the closed form solution:

$$\hat{\beta} = 1 + n / \sum_{i=1}^{n} \log(x_i / x_{min}), \tag{3}$$

where $n > 0$ is the number of observations such that $x \geqslant x_{min}$ where i denotes the first element in this vector.

While the solution for β is simple enough, its appropriateness depends on identifying the best x_{min}. Calculating x_{min} requires a more involved process. Clauset, Shalizi, and Newman (2009) recommend choosing different values of x to serve as x_{min} and estimating a new β per each iteration. Then, for each iteration, a Kolmogorov-Smirnov test should be used to assess the fit between the model and the data. The x_{min} that provides the best fit is selected.

As mentioned earlier, the need to identify a lower-bound is a necessary and even justifiable concession in the face of real-world data. However, if the lower-bound leads to a more than trivial truncation of observations, the lack of perfect linearity between all x and $\Pr(X > x)$ in log-log space may actually signal poor specification. In this case, it is worth considering whether other functional forms make for a better fit for *all* the data—not just those beyond a lower bound.

Unlike equation (1), the log-logistic model is specified as

$$\text{Odds}(X > x) = \frac{\Pr(X > x)}{1 - \Pr(X > x)} = \frac{\exp(\alpha)}{x^{\beta}}. \tag{4}$$

This can be expressed linearly as

$$\log[\text{Odds}(X > x)] = \alpha - \beta \log(x). \tag{5}$$

The idea with this specification is that the left-hand side of the equation is better modeled as the log of the *odds* of observing a war deadlier than x than as the log of the *probability*. With this form, the solutions for α and β can be identified using the logit maximum likelihood estimator.

Following Braumoeller (2019), a bootstrapping procedure is used for statistical inference for both the power-law and log-logistic distributions.

In the sections that follow, the data used to compare the power-law and log-logistic fits is presented. A pooled analysis that pits these two probability distributions head to head follows. After this, I turn to a replication and extension of Braumoeller (2019) to see whether the log-logistic fit provides different inferences relevant to the decline-of-war thesis to those provided by the power-law.

Data

To compare the log-logistic and power-law specifications for war intensity I use a measure of total battle related deaths per capita of major international wars from 1816 to 2007. I accessed the data using the newly available `{peacesciencer}` R package which provides users access to a wide range of datasets relevant to the study of conflict (Miller 2022). Data on war deaths originated from the Correlates of War Project's 4.0 data (Sarkees and Wayman 2010). To ensure that the measure reflects the relative intensity of wars, total battle deaths were divided by the summed population of the countries involved in fighting a given war. Estimates of country population data originated from Anders, Fariss, and Markowitz (2020).

The raw data was originally at the level of directed dyad-years. Before analysis, the data were collapsed to the level of individual wars so that each unit of observation was a unique war and the variable of interest the total battle deaths per the populations of the countries involved in the war. After aggregation, there were a total of 95 unique observations. Total battle related deaths ranged from 2,000 to more than 134 million, with the median number of fatalities clocking in at just over 20 thousand.

Results

This section summarizes the results from a pooled comparison of the power-law and log-logistic fits for war intensity. Figure 1 shows a scatter plot of the probability of observing a war as deadly as that observed in both log-log space and in logit-log space. Both the left and right panels show the log of per capital battle deaths along the x-axis. In the left panel the y-axis shows the log of the empirical *probability* of a war deadlier than the one observed, and in the right panel the y-axis shows the log of the empirical *odds* of observing a war deadlier than the one observed. Each panel includes the estimated model fit for the power-law and logit specifications, respectively. In the case of the power-law fit, the

Table 1. Power-Law and Log-Logistic Parameter Estimates

	log[Pr(X > x)]	**log[Odds(X > x)]**
x_{min} (% War Deaths)	0.025 [0.002; 0.156]	
Power-law Slope (β_{pl})	1.657 [1.408; 2.097]	
Log-logistic Intercept (α_{ll})		-6.452 [-7.571; -5.566]
Log-logistic Slope (β_{ll})		-0.739 [-0.850; -0.636]
N	47	95

Inference done with 200 bootstrapped samples. Bootstrapped 95% confidence intervals shown in [brackets]. Estimates used to draw regression lines in Figure 1.

regression line has slope $\beta_{pl} = 1.66$ for observations past the lower bound x_{min}. For the log-logistic fit, the regression slope is $\beta_{lg} = -0.74$.[1] Table 1 provides a summary of the estimates along with their bootstrapped credible intervals.

At face value, the power-law specification provides a good fit for the data past the lower bound. But, the loss of war deaths data with the power-law fit is quite substantial with only less than 50% of all wars providing the information necessary to model the probability of ever deadlier conflicts. As evident from Figure 1, the power-law model is not equipped to explain or predict the likelihood of war deaths in the the remaining lower half of the sample.

While the power-law model poorly fits half the sample, the log-logistic model provides an excellent fit for the *entire* sample. Across the range of observed war deaths, the odds of observing a war deadlier than the one observed appears to be well approximated by a linear function of the observed magnitude of war deaths in log-log space. In fact, most researchers would be fortunate to observe a linear fit a fraction as tight as the one observed in Figure 1.

The limitations of the power-law model become all the more evident with Figure 2. The x-axis shows the log of war deaths per 100 million, and the y-axis shows the percent error in model fit relative to the data. The solid line shows the error with the power-law

[1]The power-law parameter is positive and the logit parameter negative simply due to the method of estimation. The linear relationships they correspond with in log-log space are both negative since the likelihood of ever deadlier conflicts is decreasing in war intensity.

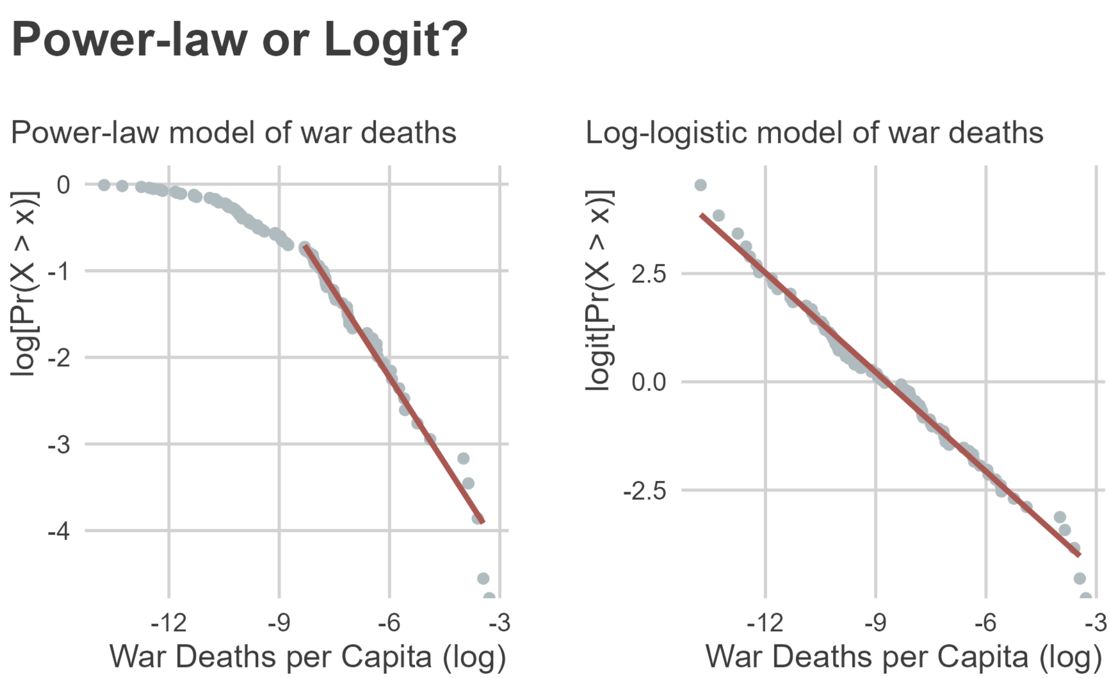

Figure 1. Empirical probability of wars as deadly as those observed in log-log space (left) and logit-log space (right). The latter provides a better fit for all the data while the former requires ignoring observations below a certain threshold. Table 1 estimates were used to draw the regression lines.

model, and the dashed line shows the error with the logit model. Positive values mean that the model *overestimates* the probability of a war as deadly as the one observed, while negative values mean that the model *underestimates* the probability of a war as deadly as the one observed. Predictions are based on the fitted parameters in Table 1, and the percent error is calculated as the difference between the fit and the empirical CDF per the value of the empirical CDF.

The difference in model fit for smaller wars shines through. The power-law model poorly predicts the probability of war deaths around and below 100 thousand per 100 million in population size. These are quite deadly conflicts, making the poor fit of the power-law model for these observations all-the-more consequential. Compare these poor predictions with the far superior ones of the log-logistic model. While both models make for nearly identical (and good) fits for the data for wars that kill beyond 100 thousand per 100 million in population size, the log-logistic model provides an exceptionally better fit for smaller wars as well.[2]

The limitations of the power-law fit go beyond prediction error for smaller conflicts. While the log-logistic and power-law models yield similar predictions for war deaths beyond the fitted power-law's lower bound, the power-law model does so with far less precision.

Figure 3 shows the point-estimate of the power-law and logit model predictions of the likelihood of observing a war that kills more than 10% of the belligerents' populations—a truly deadly conflict. The x-axis shows the probability in percentages and the y-axis denotes the respective model that generated the prediction. The 2.5 and 97.5 percentiles of the bootstrapped empirical sampling distribution of predictions are included.

The point predictions generated by each model are nearly identical. Each says that the likelihood of a war that kills more than 10% of the populations of the countries fighting a war is just less than 1%. However, while nearly identical, the logit model's predictions

[2]The logistic fit is computed as $\hat{\Pr}(X > x) = F(\hat{\alpha}_{ll} + \hat{\beta}_{ll} \log[x])$, where $F(\cdot)$ is the logistic function. The power-law fit is computed as $\hat{\Pr}(X > x) = p \times (x/\hat{x}_{min})^{-\hat{\alpha}+1}$, where p is the observed $\Pr(X > \hat{x}_{min})$.

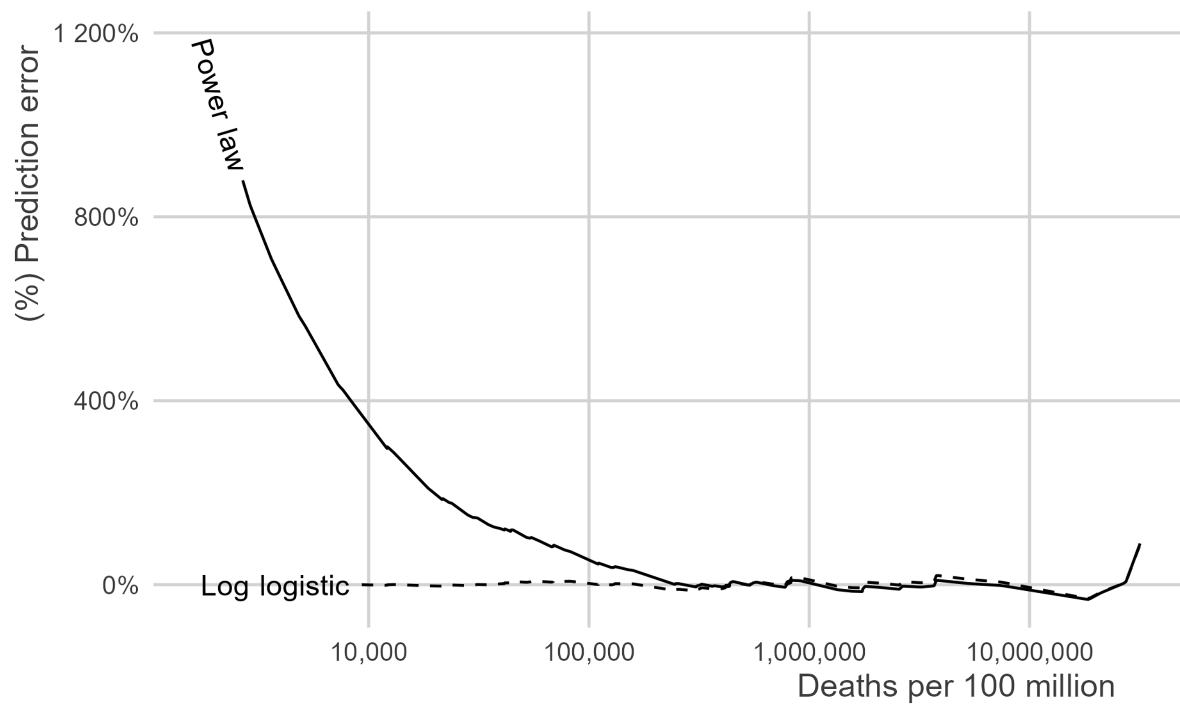

Figure 2. The percent error in model fit relative to the data. Model fit was obtained using the estimated parameters in Table 1. The percent error per a given conflict size was calculated as the difference in model fit relative to empirical CDF per the empirical CDF.

provide more than double the precision than the power-law's. While the upper bound on the bootstrapped 95% confidence interval for the power-law prediction is just below 5%, the upper bound for the logit prediction is well under 2%. These point estimates were obtained using the estimated parameters in Table 1.

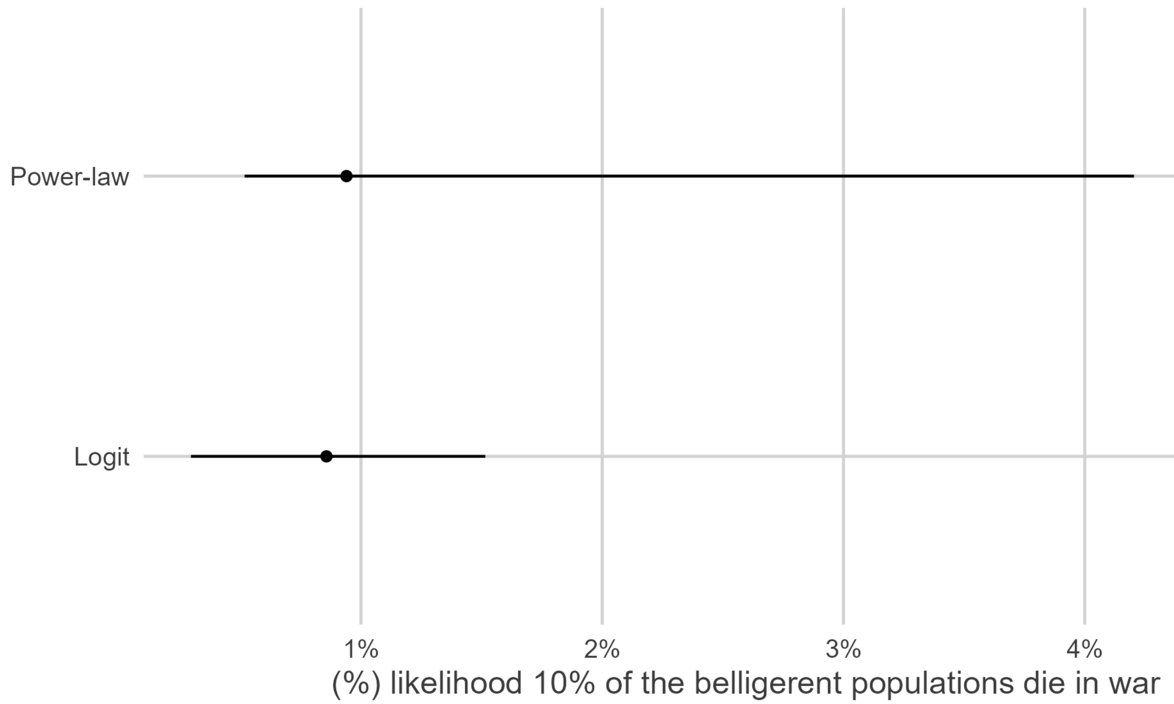

Figure 3. Comparison of power-law and logit model predictions of the probability that wars kill more than 10 percent of belligerent country populations. Predictions made using estimated parameters in Table 1.

Implications: The Decline-of-War Revisited

The above analysis shows that the power-law provides a fairly poor fit for war fatality data while the log-logistic distribution provides a far superior fit. Does modeling war intensity via a log-logistic distribution rather than the power-law yield different conclusions than those arrived at in previous studies on war's deadliness?

As noted earlier, this question has special relevance in light of recent debates about the so-called decline-of-war thesis. In response to proponents of the theory, Pinker (2011) being the most prominent target, Braumoeller (2019) makes short shrift of the decline-of-war thesis using, among many statistical tests, the power-law to estimate changes in war intensity over time. After devising a novel bootstrapping procedure for testing whether two samples originate from different power-law distributions, Braumoeller (2019) applied the method to test whether a detectable decline in war's deadliness occurred post-1945. Many adherents to the decline-of-war thesis point to this date as a turning point in war's severity, and a rejection of the null would support one of their central claims, namely, that wars became less deadly in the second half of the twentieth century. However, Braumoeller (2019) failed to identify a significant difference in war intensity. Even more, the power-law estimates he identified suggest that war's intensity could even have an infinite mean in both periods.

To test whether this finding holds up to an alternative model of war intensity, this section summarizes the results from a replication of Braumoeller's analysis followed by an extension that uses the log-logistic fit for war intensity instead. The sample consisted of the same data used in the previous analysis, save that it was divided into two sub-samples denoting pre- and post-1945 conflicts. The first sample consisted of 57 international conflicts, and the second of 38.

Using the same procedures for estimating the power-law and log-logistic parameters outlined previously, it is possible to recover estimates of the expected value of war fatalities for the different samples. For the power-law distribution, the mean is identified as $x_{min} \times (\beta - 1)/(\beta - 2)$ where, if it is not already plain to see, the mean is infinity if $\beta = 2$.[3] Further, if $\beta < 2$ the expected value of war deaths can also be negative which is out of bounds for valid war fatalities. This means that only β values less than 1 or greater than 2 are consistent with a definite and non-negative mean for war deaths.

For the log-logistic distribution, the mean can simply be recovered from the estimated slope and intercept parameters fit with the logit maximum likelihood estimator. The

[3] See Clauset, Shalizi, and Newman (2009).

Table 2. Sample sizes pre- and post-1945

	Total N	Effective N for Power-law
post-1945	38	13
pre-1945	57	34

identities of these parameters are in fact defined in terms of the structural parameters μ and σ, where the first is the mean of the log-logistic distribution and the second is its standard deviation. The parameters μ and σ can be easily backed-out of the logit estimates for α and β with the identities of each given as:

$$\alpha := \mu/\sigma \quad \text{and} \quad \beta := -1/\sigma \tag{6}$$

That implies that the relevant structural parameters can be calculated as follows after estimation:

$$\hat{\sigma} := -1/\hat{\beta} \quad \text{and} \quad \hat{\mu} = \hat{\alpha}/\hat{\beta}. \tag{7}$$

Table 2 provides a summary of the sample used in estimation. Power-law and log-logistic models were fit to the data both pre- and post-1945. The N column denotes the total sample size for each period ($N = 57$ pre and $N = 38$ post). The column next to it indicates the effective sample used to fit the power-law after identifying the best x_{min}. The fact that the power-law leaves a good deal of the variation in war intensity unexplained is made apparent by looking at the difference between the sample sizes in the two columns. Only 34 out of 57 (60%) wars provide a good fit for the power-law pre-1945, and only 13 out of 38 (34%) provide a good fit for the power-law post-1945.

Figure 4 has two panels, each of which shows a scatter plot of the empirical $\Pr(X > x)$ for war intensity on the y-axis and the observed battle deaths per capita among the belligerents in a given war on the x-axis. Values are shown in log-log space and battle deaths have been converted to a rate per 100 million to provide a better sense of scale.

Table 3. Paremeter estimates for log-logistic and power-law models

	post-1945	*pre-1945*
Log-logistic		
$\exp(\mu)$	6,349 [2,897; 11,343]	27,469 [14,572; 46,412]
σ	1.31 [1.02; 1.64]	1.27 [1.04; 1.54]
Power-law		
x_{min}	2,5471 [855; 45,044]	24,680 [2,960; 210,229]
β	1.81 [1.35; 2.52]	1.62 [1.36; 2.54]

Inference done with 200 bootstrapped samples.

Color is used to differentiate pre-1945 (yellow) and post-1945 (blue) conflicts. The left panel includes the line-of-best fit according to the power-law model of war intensity. The right panel includes the line-of-best fit according to the log-logistic model of war intensity. Table 3 reports the parameter estimates, along with their bootstrapped 95% confidence intervals, used to plot the fits in the figure. On their face, the results presented in Figure 4 demonstrate that the log-logistic distribution makes for a much better fit for the data yet again. While the power-law only applies to a truncated set of the sample, the log-logistic model fits the entire sample (both pre- and post-1945).

Not only does the log-logistic distribution make for a better fit to the data, statistical inference supports different conclusions about the deadliness of wars after 1945. An examination of the parameters in Table 3 makes this clear. For the more visually inclined, Figures 5 and 6 plot these estimates with their 95% confidence intervals.

Figure 5 shows the expected war deaths per 100 million both pre- and post-1945 as suggested by the log-logistic distribution. The difference is stark. According to the log-logistic fit for the data, the central tendency for war intensity pre-1945 is more than 27,000 battle deaths per 100 million. Conversely, the central tendency for war intensity post-1945 is 6,394 per 100 million. This suggests wars after 1945 tend to be orders of magnitude less deadly than those before 1945.

Contrast these conclusions with those implied by the slopes for the power-law fit for the data. Figure 6 shows β estimates for the pre- and post-1945 samples. The slope

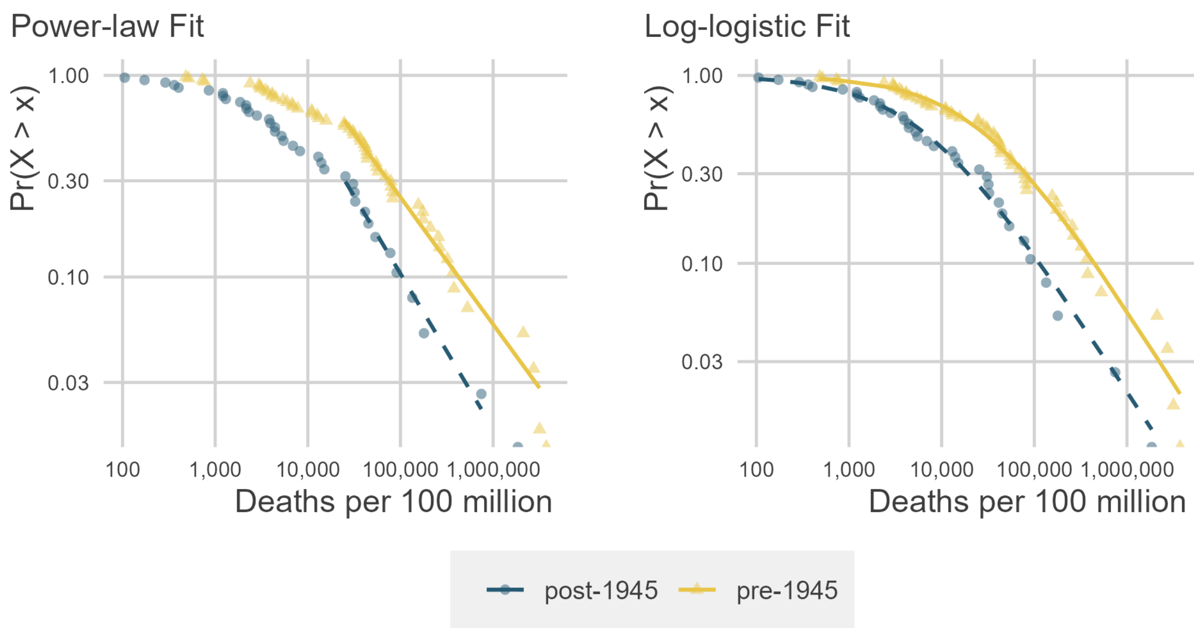

Figure 4. A comparison of the power-law and log-logistic model fits for the data. The power-law fit is on the left and the log-logistic on the right. The data are shown in log-log space. War fatalities have been rescaled to battle deaths per 100 million.

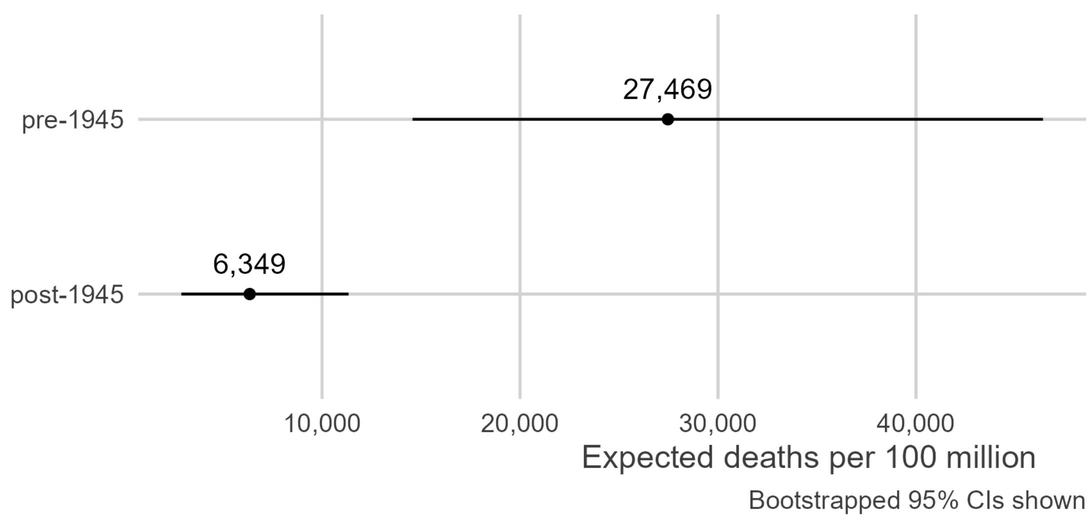

Figure 5. The expected level of war deadliness before and after 1945. Estimates obtained from a log-logistic specification of war intensity. See Tables 1 and 2 for sample and parameter estimates.

estimates are shown rather than the power-law mean because the range of possible estimates contain both negative values and infinity. The estimated slopes for both periods are between 1 and 2, implying a non-positive mean for war intensity, and the confidence intervals for both periods overlap with 2, implying a possibly infinite mean. While the estimated slope for the second period is slightly larger (suggesting wars are *less* intense after 1945), the slopes do not appear to be statistically distinguishable.

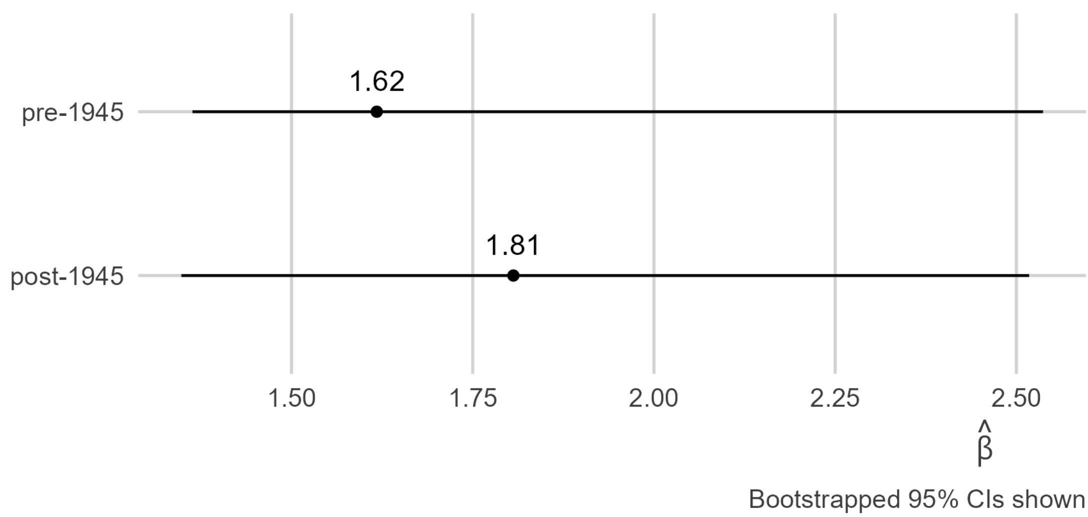

Figure 6. Power-law slopes for war deadliness before and after 1945. Estimates obtained from a power-law specification of war intensity. See Tables 1 and 2 for sample and parameter estimates.

Table 4 reports the results for bootstrapped tests of whether the samples are drawn from different distributions according to the respective models fit to the data. If we fit the power-law to the data, there is insufficient evidence to reject the null that pre- and post-1945 conflicts are drawn from different power-law distributions ($p = 0.86$). But while the null cannot be rejected in the case of the power-law fits for the data, the null can be rejected that pre- and post-1945 conflicts are drawn from the same log-logistic distribution ($p < 0.01$).

Table 4. Difference in parameter estimates post- vs. pre-1945.

	Estimate	95% Lower-Upper Bound	p-value
$\Delta\mu_{ll}$	-1.465	[-2.527; -0.716]	0.00
$\Delta\beta_{pl}$	0.188	[-0.810; 0.964]	0.94

Discussion and Conclusion

These findings have important theoretical and practical implications for the quantitative study of conflict. The idea that war deaths follow a power-law distribution is the foundation for a range of work, from theory generation to data imputation, and prediction to hypothesis testing (Braumoeller 2019; Cederman 2003; Cederman, Warren, and Sornette 2011; Cioffi-Revilla and Midlarsky 2010; Friedman 2014). Mispecification therefore carries serious repercussions for the accumulation of scientific knowledge of international conflict.

A replication and extension of recent analysis done by Braumoeller (2019) makes this point clear. Braumoeller (2019) set out to refute a claim from the decline-of-war thesis that war's deadliness has declined over time, particularly after 1945. To test this claim, he fit the power-law to pre- and post-1945 data on international war intensities and devised a statistical method to test the null that two samples come from the same power-law distribution. Using this procedure, Braumoeller (2019) failed to reject the null using the 1945 cutoff—not evidence that this particular claim of the the decline-of-war thesis is wrong, but an indictment of the idea that the data provide strong support for it all the same.

A replication of his procedure yields the same conclusion. However, when extended to using the log-logistic fit for the data rather than the power-law, the results change. A log-logistic fit suggests that post-1945 conflicts have intensities significantly and orders of magnitude less than pre-1945 conflicts.

The results from this study neither reflect a wholesale rebuttal to Braumoeller (2019), nor arrant support for the decline-of-war thesis. Many factors other than those cited by decline-of-war proponents—like Enlightenment Humanism or pacifistic norms—could explain the reduction in conflict intensity after 1945. In fact, some clues may lie in Braumoeller's (Braumoeller 2019) discussion later in his book of the role of international order in organizing patterns of international conflict—a set of ideas that share some linkages with Lake's characterization of international hierarchy (Lake 2007, 2009). Nonetheless, these results do raise questions about the reliability of quantitative conflict research that relies on the power-law to the exclusion of alternative probability distributions. Researchers may find it worthwhile to revisit prior work, and in the future, scholars should at minimum check the robustness of their inferences to alternative methods. As far as the power-law and war go, perhaps the time has come to lay this unqualified tradition to rest.

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