Bayes Filter for Gesture Recognition

Definitions

- Hidden State Space
  - $\mathcal{X}$: the set of m objects in the scene that can be referenced and a state for no item being referenced
  - An example would be: $x =$ Object 1 is being referenced
  - Each object in the scene has an associated set of three dimensional points (x_p) and set of descriptor keywords (x_d)
- Observations
  - $\mathcal{Z}$: the set of four-tuple $\{l, r, h, s\}$
  - l represents the observed origin (l_o) and vector (l_v) for the left arm
  - r represents the observed origin (r_o) and vector (r_v) for the right arm
  - h represents the observed origin (h_o) and vector (h_v) for head
  - s represents the observed speech from the user, consisting of a list of words
- Transition Function
  - $\mathcal{T}$: a function such that $\mathcal{T}(x_a, x_b)$ is equivalent to the probability that x_a transitions to x_b
  - **ST: Model with Poisson?**
- $\mathcal{N}$: a function that returns the probability of a sample under a Gaussian distribution given a mean and variance
  - Applied as $\mathcal{N}(\mu, \sigma, \text{sample})$
- Φ: a function that, given an origin and two points, returns the angle between the two points
  - Applied as $\Phi(\text{origin}, p_1, p_2)$
- $\mathcal{I}$: an indicator function applied as $\mathcal{I}(\text{word}, \text{corpus}$ that returns 1 if the word is in the corpus and 0 otherwise

Equations

- Time Update
  - An equation used to determine the probability that $\mathcal{X}_t = x$ given only previous belief states
  - $P(\mathcal{X}_t = x|\mathcal{X}_{t-1}....\mathcal{X}_0) = \sum_{x' \in \mathcal{X}} \mathcal{T}(x, x') * bel(\mathcal{X}_{t-1} = x')$
- Measurement Update
  - An equation used to determine the belief that $\mathcal{X}_t = x$ given observation $\mathcal{Z}_{t-1}$
  - $P(\mathcal{X}_t = x|\mathcal{Z}_{t-1}) = P(X_t = x|l_{t-1}) * P(X_t = x|r_{t-1}) * P(X_t = x|h_{t-1}) * P(X_t = x|s_{t-1})$
  - $P(\mathcal{X}_t = x|l_{t-1}) = \prod_{p \in x_p} \mathcal{N}(\mu_l = 0, \sigma_l, \Phi(l_o, l_v, p))$
  - $P(\mathcal{X}_t = x|r_{t-1}) = \prod_{p \in x_p} \mathcal{N}(\mu_r = 0, \sigma_r, \Phi(r_o, r_v, p))$
  - $P(\mathcal{X}_t = x|h_{t-1}) = \prod_{p \in x_p} \mathcal{N}(\mu_h = 0, \sigma_h, \Phi(h_o, h_v, p))$
  - $P(\mathcal{X}_t = x|s_{t-1}) = \dfrac{\sum_{w \in s_{t-1}} \mathcal{I}(w, x_d)}{\sum_{x \in \mathcal{X}} \sum_{w \in s_{t-1}} \mathcal{I}(w, x'_d)}$
- Belief Update
  - An equation that produces the probability that $\mathcal{X}_t = x$ given all past observations and belief states, namely the product of the measurement and time updates
  - $bel(X_t = x) = P(\mathcal{X}_t = x|\mathcal{Z}_{t-1}) * P(\mathcal{X}_t = x|\mathcal{X}_{t-1}....\mathcal{X}_0)$
  - $bel(\mathcal{X}_0 = x) = \frac{1}{m+1}$ (uniform initialization of belief)