The Evolution of Trade Connectivity: A Spectral Analysis of Global Trade Networks

Author: Miles Guerin [1] Advised by: Yannis M. Ioannides[1] and James M. Murphy[1]

[1]Tufts University

Research Statement and Findings

In this paper, we explore the use of normalized graph Laplacians and graph conductance in measuring the evolution of interconnectedness in international trade. We construct networks using bilateral trade data from 1950 to 2014, and estimate **the connectivity** of these networks. **Higher connectivity indicates that countries are more integrated across the network**, whereas **lower connectivity means they can be more easily partitioned into separate communities.** So in the context of trade, lower network connectivity implies a trade network characterized by stronger fragmentation into regional trade blocs.

Research Questions

- Has trade become more interconnected or has it stayed regionally fragmented?
- How does computed graph connectivity compare to **graph conductance** and **null-models** of international trade?
- Have certain regions, like the intra-European trade network, become more strongly interconnected overtime?

Advantages of Using Normalized Graph Laplacian Eigenvalues

- **Scale invariance** allows us to compare graph connectivity between networks with different magnitudes of nominal flows.
- We can determine **comparative statics for connectivity** with respect to node addition or removal, and bound changes with respect to edge removals.
- The entries of the corresponding eigenvectors can be used to identify trading communities and a notion of distance between them.

Empirical Findings

- Following 1990 **intra-European trade becomes more connected** than we would expect for similar randomly constructed trade networks.
- When trade networks are sufficiently symmetric, normalized graph Laplacians provide a good estimate of trade connectivity.

Potential Extensions

- We are exploring the feasibility of these methods when analyzing **sector-level** and **commodity-level data**, as these metrics could be used to identify which supply chains have a "bottlenecked" or modular structure.

The Network Structure of Trade Data

Let each year of trade data be represented by **a weighted directed graph**, $G = (V, W)$, where the set of vertices, V, represent countries and edge weights, w_{ij} represent the value of exports from country i to country j. These weights are directional as the exports from i to j are not necessarily equal to the flows from j to i. Additionally, we set $w_{ii} = 0$. Analyzing trade as a network like this has been discussed extensively by **Thomas Chaney**, **Giorgio Fagiolo**, and many other authors. [1] [2]

For this analysis, we use data from **Fouquin and Hugot** (2016) that records more than **1.6 million bilateral trade flows from 1950-2016** [3]. We use these bilateral flows to construct weight matrices and corresponding graphs for all years. The average network size, number of edges, and edge density for each year are plotted below.

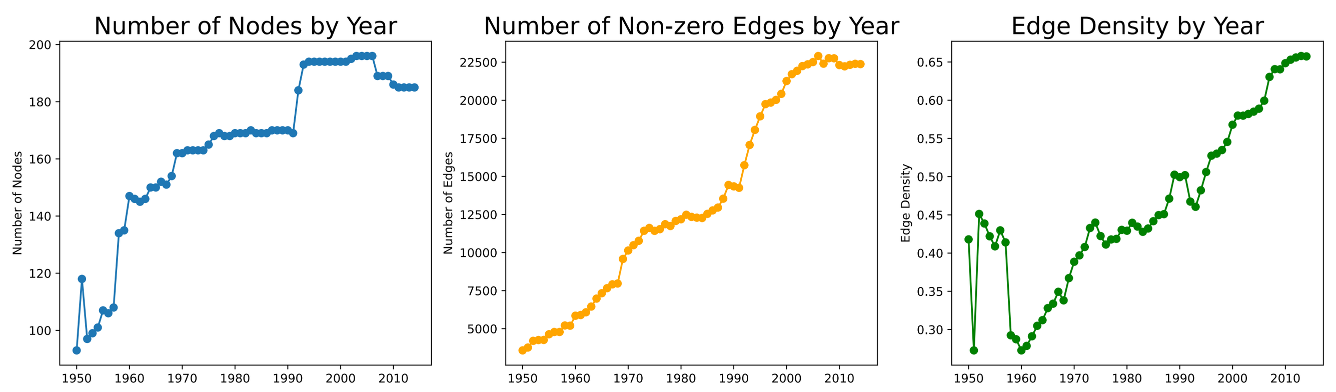


Figure 1. Global trade network summary stats 1950-2014. Edge density is defined as $\frac{edges}{N*(N-1)}$

In addition to the global trade network, we also consider the **network of intra-European trade** that only considers flows between European countries.

To analyze both these networks using the normalized graph Laplacian, we symmetrize these graphs by taking the average of w_{ij} and w_{ji}. As symmetrization can cause information to be lost, we also consider other symmetrization methods in the paper.

The Normalized Graph Laplacian

Consider the undirected weighted network $G = (V, W)$. Where the set of countries is $V = \{1, \ldots, n\}$, and $W \in \mathbb{R}^{n \times n}$ with w_{ij} being the average trade flow between countries i and j. Define the degree matrix, D to be an $n \times n$ diagonal matrix whose i-th diagonal entry is $d_i = \sum_{j \in V} w_{ij}$.

The **normalized graph Laplacian** is defined as:

$$L = I - D^{-1/2} W D^{-1/2}$$

L has eigenvalues:

$$0 = \lambda_1 \le \lambda_2 \le \cdots \le \lambda_n \le 2$$

Properties of Normalized Graph Laplacian Eigenvalues

- The multiplicity of the eigenvalue 0 of L equals the number of connected components in G. i.e. $\lambda_k = 0$ iff G has at least k connected components.
- If the graph is weakly connected, **larger values of λ_2 indicate a more connected network. A low λ_2 indicates high modularity.**
- The entries of the eigenvector corresponding to λ_2 (the Fiedler Vector) can be used to partition the network.

Comparative Statics of Graph Connectivity

Linear scaling of the trade network:

Let $\tilde{W} = \alpha W$ for some $\alpha \in \mathbb{R}^{+}$. This also scales the degree matrix since:

$$\tilde{d}_i = \sum_{j \in V} \tilde{w}_{ij} = \alpha \sum_{j \in V} w_{ij} = \alpha d_i \implies \tilde{D} = \alpha D$$

So the normalized graph Laplacian of this network with scaled edges will be,

$$\tilde{L} = I - (\tilde{D})^{-1/2} \tilde{W} (\tilde{D})^{-1/2} = I - (\alpha D)^{-1/2} (\alpha W) (\alpha D)^{-1/2}.$$

Through some algebra, we can show $L = \tilde{L}$. Meaning network **connectivity does not change** under a linear scaling of the network.

Addition or removal of a trading country:

Consider the graph $G/\{u, v\}$ given by a contraction of nodes u and v into a single trading country. The normalized graph Laplacian of this new graph, $\tilde{L}$ will be $(n-1) \times (n-1)$ and has eigenvalues

$$0 = \mu_1 \le \mu_2 \le \cdots \le \mu_{n-1} \le 2$$

Using results from [4] and [5], we have the following inequality between the eigenvalues of $\tilde{L}$ and those of the initial graph's Normalized Laplacian,

$$\lambda_1 \le \mu_1 \le \lambda_2 \le \ldots \lambda_{n-1} \le \mu_{n-1} \le \lambda_n.$$

The implication of this inequality is that, **as new countries are added** to the network and take on existing trade flows, **network connectivity stays the same or decreases.**

Graph Conductance

To preserve directionality in the graph when computing connectivity, we also approximate the graph conductance of the trade network. For each network of trade, $G = (V, W)$ we define a **Markov chain** with transition probabilities,

$$P(i, j) = \frac{w_{ij}}{\sum_{k \in V} w_{ik}}$$

When this chain is ergodic, it has a stationary distribution π. Now, for any subset of vertices, $S \subsetneq V$, we define the conductance of this subset as:

$$\Phi(S) = \frac{\sum_{i \in S} \sum_{j \notin S} \pi(i) P(i, j)}{min\{\pi(S),\ 1 - \pi(S)\}}$$

We then define the **global conductance** of this chain as the minimum of $\Phi(S)$.

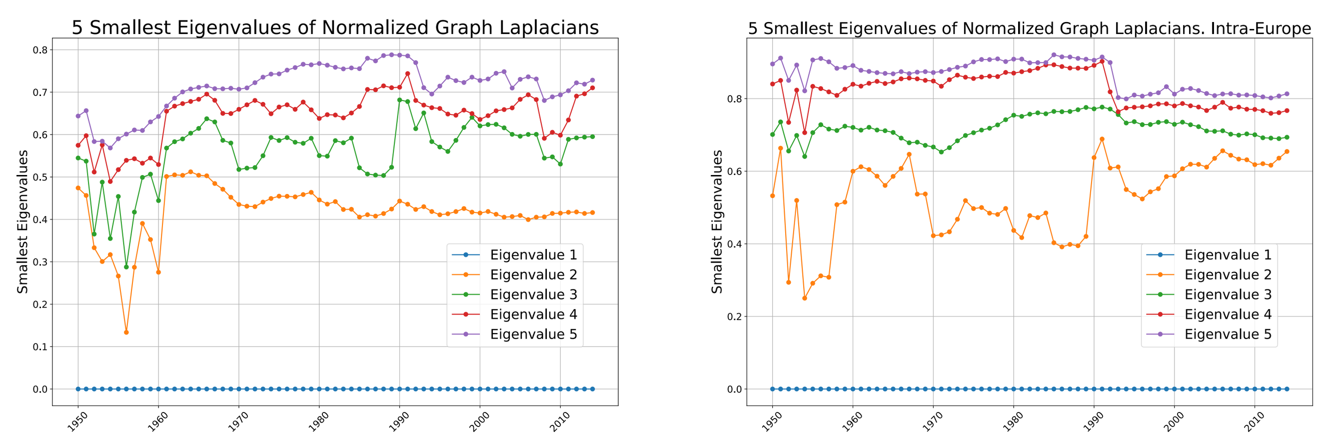
$$\Phi_G = \min_S \Phi(S)$$

Φ_G is in the range $0 < \Phi_G < 1$. Where **a higher value of Φ_G represents a higher connectivity and lower modularity of the network**

Computing Φ_G is NP-hard, so we approximate it by taking the SVD of $P = U \Sigma V^T$, and sorting the vertices based on the values of the second largest right singular vector, $\mathbf{v_2}$. Given this ordering of vertices $(v_1, \ldots, v_n)$ by $\mathbf{v_2}$, we consider the subsets $S_k = \{v_1, \ldots, v_k\}$ for $k = 1, \ldots, n-1$ and compute $\Phi(S_k)$ of each. The minimum of the conductance of these subsets is our approximated global conductance.

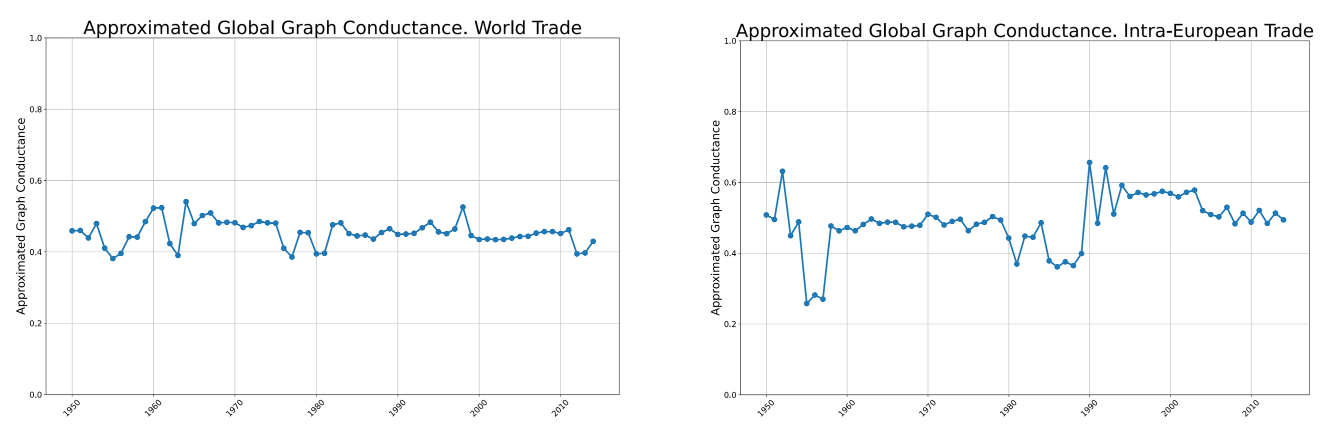
Connectivity of World and Intra-European Trade

Below we compute the normalized graph Laplacian for years 1950-2014 of the world trade network and of the intra-European trade networks. We then plot the five smallest eigenvalues, $(\lambda_1, \ldots, \lambda_5)$ of L over time. Additionally, we plot the approximated global graph conductance, Φ_G, of these networks before symmetrization.



(a) World Trade (b) Intra-European Trade

Figure 2. Five smallest eigenvalues of the normalized Laplacian for each year of world and intra-European trade. Second smallest eigenvalue, λ_2, in orange.



(a) World Trade (b) Intra-European Trade

Figure 3. Approximated global graph conductance, Φ_G for world and intra-European trade.

For intra-European trade, we observe a relatively sharp increase in connectivity in terms of both λ_2 and Φ_G at the start of the 1990s, coinciding with the reunification of Germany and further European integration. However, the combination of these methods fails to identify a shift in global trade collectivity.

Comparison with Null Models

Why compare to null models?

- To check if the observed eigenvalues reflect genuine changes in economic/political structure rather than random fluctuations.
- To see if observed network connectivity falls within the range we would expect for randomly generated graphs with similar characteristics (number of nodes, out-degree distribution, etc.).

Construction of null models

- For a given year's trade graph, $G^t(V^t, W^t)$. Fix V^t and the values of W^t.
- For each node, take its outgoing edges in G^t and randomly permute them across all other nodes in the graph.
- Compute the eigenvalues of this new graph's normalized Laplacian.

Comparison of second smallest eigenvalue in simulated and observed networks

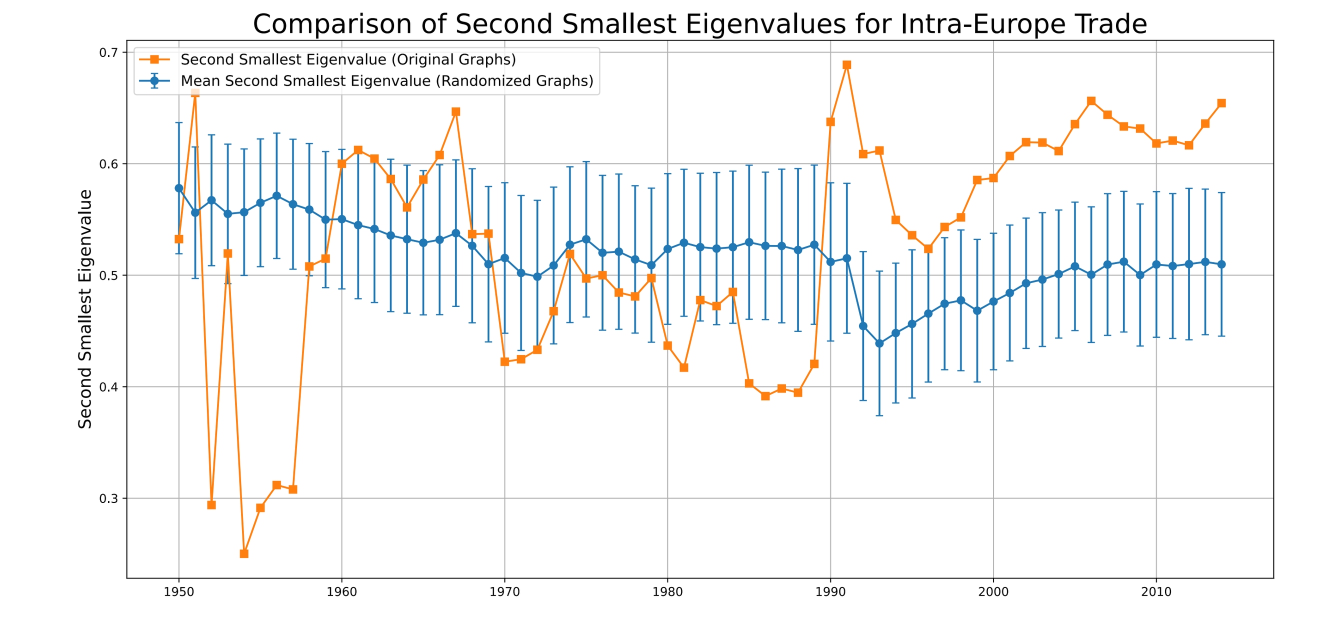
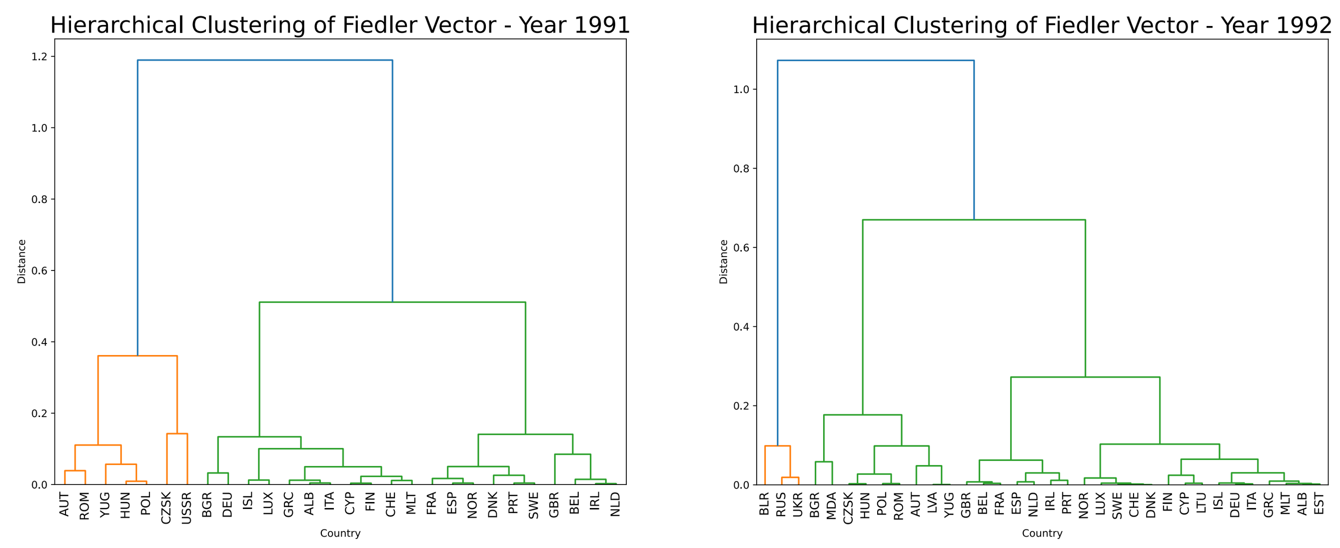


Figure 4. Second smallest eigenvalue of the symmetric normalized Laplacian (λ_2) of the intra-European trade network in each year (orange) plotted against the distribution of the second smallest eigenvalue across 1000 replications of the random graph using parameter values from the corresponding intra-European trade network. Error bars represent two standard deviations.

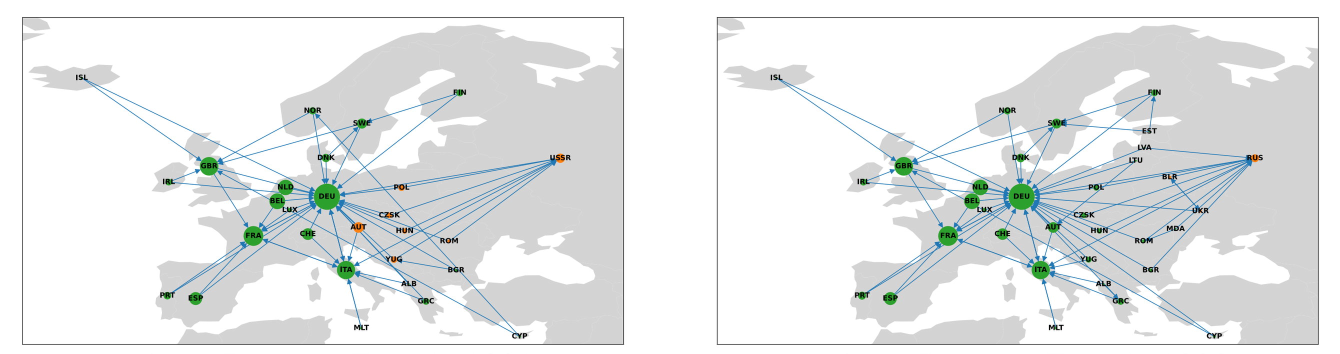
Clustering Using the Normalized Laplacian

Since each entry of the eigenvector corresponding to λ_2 (**the Fiedler vector**) corresponds to a given country, we run **hierarchical clustering** on the entries of this vector to cluster the countries into communities. Below are the visualizations of the clustering of intra-European trade in 1991 and 1992 during the dissolution of the USSR.



(a) Hierarchical Clustering for 1991 (b) Hierarchical Clustering for 1992

Figure 5. Hierarchical Clustering using the eigenvector corresponding to λ_2 of the intra-European trade graph for 1991 and 1992. Distance between clusters calculated using the Ward variance minimization algorithm.



(a) Intra-European Trade in 1991 (b) Intra-European Trade in 1992

Figure 6. Network of Intra-European Trade in 1991 and 1992. Directed edges are drawn for each country's two largest export destinations. Country sizes are based on their in-degree, colors based on the first hierarchical clustering partition.

Clustering of the Fiedler vector, we observe that from 1991 to 1992, most of the Eastern European countries–apart from Belarus, Russia, and Ukraine–are now clustered with Western Europe. And the distance between the final two clusters has decreased.

References

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