The Evolution of Trade Modularity: A Spectral Analysis of Global Trade Networks (1950–2014)

Miles Guerin

Advised by:
Professor Yannis M. Ioannides
Professor James M. Murphy

Tufts University

January 2025

Abstract

This study examines the evolution of trade modularity from 1950 to 2014. Using data from Fouquin and Hugot (2016), which includes over 1.8 million bilateral trade observations, we construct weighted networks of trade for each year. Using spectral graph theory, we provide a scale-invariant metric of modularity for each network, where modularity measures the extent to which trade networks form distinct regional blocs, with higher modularity indicating more fragmented trade clusters and lower modularity reflecting a more interconnected network. We find that trade modularity was highest in the 1950s, reflecting a more regionalized world economy, followed by a steady decline as trade networks became more interconnected. In the network of intra-European trade, the dissolution of the USSR in 1991 was followed by a significant drop in modularity. We then compare the observed modularity to simulated null models and inter-state trade within the United States. The motivation of this paper is to define a metric of modularity that can be used to compare trade networks across regions, sectors, and time.

Keywords: International Trade Network, Spectral Graph Theory, Graph Modularity

JEL: F14, F15, N70

1 Introduction

Globalization has fundamentally reshaped international trade, creating a vast, interconnected network of economic relationships among nations. This complex web of trade interactions cannot be fully understood through traditional economic analyses, which often focus on trade volumes, bilateral balances, or isolated economic indicators (Fagiolo, 2010). Such approaches miss the influence of interdependencies and the overall structure of international trade on outcomes. To address these limitations, a subset of the international trade literature has applied network theory to examine the structure of international trade networks, to explore the centrality of certain countries in trade, the formation of trade communities, and the diffusion of shocks across the trade network (Chaney, 2014; De Benedictis et al., 2014; Fagiolo, 2016; Hoang, Piccardi, & Tajoli, 2023).

This paper employs network theory to analyze two key aspects of the international trade network: centrality and modularity. Centrality metrics indicate the influence of individual countries within the trade network, reflecting how some countries function as major trade hubs, while others occupy more peripheral roles. Metrics like PageRank centrality, which accounts for both direct and indirect connections, allow us to identify countries that have a disproportionate impact on global trade flows due to their strategic position within the network (Deguchi et al., 2014; Hoang, Piccardi, & Tajoli, 2023). By examining the evolution of centrality, we can trace shifts in economic power, such as the stable centrality of the United States, the decline of certain European powers, and the rise of China as a central node in the trade network.

Modularity, on the other hand, highlights the degree to which the trade network is partitioned into distinct clusters or communities. In international trade, high modularity may indicate the presence of regional trade blocs or tightly knit groups of countries with dense internal trade and fewer external connections, while low modularity reflects a more integrated global system. We choose to analyze the dynamics of modularity to better explain how geopolitical events, such as the dissolution of the USSR, impact the overall cohesiveness and regionalization of the trade network. Metrics of modularity act as a barometer of the globalization-regionalization balance in trade networks. High modularity suggests a fragmented network dominated by regional or local clusters, which may point to a trend of regionalization or trade protectionism. Conversely, low modularity indicates a more integrated global system, where countries trade freely between blocs. Furthermore, modularity can measure the resilience of the trade network to shocks. For instance, if a modular trade network is highly dependent on a single cluster or hub, a disruption in that cluster—such as economic sanctions, a natural disaster, or political instability—could ripple through the entire global system. In contrast, a less modular network may more easily absorb the shock to a single node or collection of trade relationships.

The core objective of this paper is to map the structural evolution of international trade from 1950 to 2014 by examining changes in the centrality of specific countries and the overall modularity of the trade network. Using historical trade data from Fouquin and Hugot (2016), we construct directed weighted networks for each year of data, where each network maps out aggregate flows of trade between countries measured in British Pounds. Given this dynamic model of international trade, we attempt to measure the rise and fall of influential countries, the formation and dissolution of trade blocs, and the effects of major geopolitical events on the structure of international trade. To this end, we utilize key methodologies frequently used in the computer science literature: PageRank centrality and spectral graph theory.

PageRank centrality enables us to capture the influence of countries beyond raw trade volume. In international trade, PageRank centrality has been utilized by Deguchi et al. (2014); Kim and Yun (2022); and Hoang, Piccardi, and Tajoli, (2023) across different time periods and levels of data to analyze the dynamics of countries becoming more or less central in the network of international trade.

In contrast, spectral graph theory has been less commonly applied in economics but is widely utilized in other disciplines, including computer science for tasks like clustering and image segmentation, physics for studying complex systems, and biology for analyzing protein interaction networks and gene regulatory systems. In the study of international trade, Laenen and Sun (2020) employed spectral graph theory to cluster countries into four trading clusters, but otherwise, little work has been done on determining trade modularity through spectral graph theory. That being said, clustering and the formation of trade communities have been explored quite extensively in the international trade literature, namely: Fagiolo (2010); Chaney (2014); and Abbate et al. (2018). We chose to use spectral graph theory in this analysis because, in addition to its utility for community detection, spectral methods provide a means to determine the algebraic connectivity of a graph (Bapat, 2010). In the context of international trade, a higher algebraic connectivity indicates a well-integrated trade network where countries are strongly interconnected, making it less susceptible to fragmentation. Conversely, lower algebraic connectivity suggests that the trade network is more modular or loosely connected, potentially reflecting the presence of distinct trade blocs or regional clusters. Crucially, this metric of graph modularity is scale-invariant, so even though countries are added or removed from the network year-to-year, we can compare graph modularity across all years of data.

In sum, this paper offers a longitudinal analysis of the international trade network by focusing on the structural evolution of centrality and modularity from 1950 to 2014. We examine how shifts in global economic power, regional trade dynamics, and major geopolitical events have reshaped the structure of international trade.

2 Theoretical Background

In this section, we explore the theoretical underpinnings of international trade literature by reviewing two prominent frameworks: gravity models and network theory. These models provide complementary perspectives on the mechanisms that drive trade flows and the structural characteristics of international trade networks.

To model international trade, gravity models are commonly used to explain the volume of trade between countries. The gravity model, first introduced by Tingergen (1962), borrows from Newton's law of gravitation, stating that the trade flow between two countries is directly proportional to their economic sizes (measured by GDP) and inversely proportional to the geographic distance between them. The naive gravity model is formulated as:

$$X_{ij} = G\frac{Y_i^{\beta_1} Y_j^{\beta_2}}{D_{ij}^{\beta_3}}\epsilon_{ij}$$

Where X_{ij} is the aggregate bilateral trade from country i to country j. Y_i and Y_j are the GDP of country i and j respectively. D_{ij} is the distance between country i and j under some metric. And ϵ_{ij} is a normally distributed error term with a mean of 0. Numerous augmentations have been made to this model to account for variables that can further influence bilateral trade between countries, such as the inclusion of common language, common borders, and joint membership in trade agreements (Kabir et al., 2017). For empirical applications, the main strength of gravity models lie in their ability to econometrically estimate the effect of trade agreements, common currencies, and other factors on the bilateral flow of trade (Baier et al., 2014). In practice, these models have been quite successful in explaining the relationship between country characteristics and variations in bilateral trade flows. However, gravity models are limited in their ability to capture the network effects present in international trade, where trade flows between countries are not independent but rather influenced by the global structure of trade relationships. This is where network theory provides a valuable complement that enables us to consider the centrality of certain countries and the overall interdependence of trade.

Network theory offers an approach to analyzing international trade that emphasizes the interdependence of trade flows and the broader structure of the international trade network. In this framework, countries are represented as nodes, and yearly trade flows are represented by linkages from exporter to importer countries. Furthermore, weights may be assigned to these linkages, or edges, to represent heterogeneous flows of trade between countries. Repeating such a construction year-by-year results in a time series of networks. Formally, for each year t, the set of nodes is the set of countries, $N^t = \{1, ..., n^t\}$ and define the edge,

$a_{ij}^t \geq 0$ to be the value of exports from country i to country j in year t where $i, j \in N^t$. Note that due to the asymmetry of imports and exports, a_{ij}^t is not necessarily equal to a_{ij}^t. If we were to define A^t to be the set, or adjacency matrix, of all possible directed bilateral flows between countries, a weighted directed graph, $G^t(N^t, A^t)$ can be constructed to represent the full network of international trade in year t.

If we wanted to consider trade flows by commodity, bilateral flows could be broken down into industry-specific flows, $a_{ij}^t(h)$ where $h \in \{1, ..., H\}$ indicates a specific commodity (Fagiolo, 2016). This decomposition results in H different networks for each year t and the time series of all these networks is called a multilayer network (Kivelä et al., 2014). Alternatively, if we wanted to model flows between M industries across different countries, each node in the network would be represented as a country-industry pair. Let $\mathcal{I} = \{1, ..., M\}$ be the set of all industries, then the flow from industry $k \in \mathcal{I}$ in country i to industry $l \in \mathcal{I}$ in country j in year t is given by $a_{(i,k)\to(j,l)}^t$ or $a_{ik,jl}^t$.

Regardless of the type of network constructed, an assortment of metrics is commonly used to analyze the topological properties of trade networks and their dynamics. For example, graph connectivity measures the extent to which countries are linked within the trade network, either directly or indirectly. A highly connected network indicates that goods can flow efficiently between countries, reducing barriers and enhancing resilience against localized disruptions. In contrast, a network with low connectivity might suggest potential bottlenecks or dependencies on specific trade routes or partners, making the network more vulnerable to disruptions. Beyond the overall connectivity of the network, we may also be concerned with the centrality of individual countries within the network. Different types of centrality—such as degree centrality, betweenness centrality, and eigenvector centrality—can measure various types of influence on the overall trade graph. Degree centrality, for instance, measures the number of direct trade connections a country has, indicating how integrated it is within the network. Betweenness centrality, on the other hand, measures how often a country lies on the shortest path between pairs of countries, potentially representing a country pivotal in facilitating indirect trade. Eigenvector centrality considers not only the number of connections but also the influence of those connections, highlighting countries that are well-connected to other influential trade partners. Beyond centrality and connectivity, we are particularly interested in the clustering of certain communities in the network and the overall modularity of the trade network.

Modularity in network theory refers to the degree to which a network can be partitioned into distinct communities or clusters. In the context of international trade, these clusters may represent regional trade blocs or groups of countries with dense internal trade links but relatively sparse external connections. High modularity indicates a fragmented network with distinct trade communities, while low modularity suggests a more integrated global trade system. Despite significant trade globalization, the world trade network remains a largely modular network as trade becomes clustered around large advanced economies (Fagiolo, 2016).

2.1 Random Graphs and Null Models

Beyond the construction of trade networks from data, the mechanisms behind their generation have been explored using tools from random graph theory. First defined by Erdös and Renyi (1959), random graphs are used to study the properties of networks where the edges between nodes are determined probabilistically. This modeling approach is suited for systems where the network structure plays a critical role in shaping outcomes but cannot be fully predicted from the initial parameters. In particular, random graph theory has been used extensively by mathematicians, sociologists, computer scientists, and biologists in analyzing social and biological networks. In the domain of economics, Ioannides (2015) has provided an extensive overview of various types of random graphs and their respective usages in economics; and has provided a model for random graphs of trade in differentiated products between countries. While most models of random graphs assume the number of nodes to be static, Jackson (2010) reviews the construction of growing random networks in which nodes are added to the graph dynamically and form edges with existing nodes. Such a model of random graphs extends beyond the analysis of this paper, but provides a means to simulate the introduction of new countries or firms to the network of international trade over time. In particular, Setayesh, et al. (2022) have utilized exponential random graph models to explore the structural, economic, geographical, and political factors that influence the evolution of the trade network.

Beyond modeling, random graphs can be used to assess the statistical significance of a metric or structural feature of the actual graph (Jackson, 2008; De Clerck et al., 2024). In the context of international trade, comparing the observed trade network to a simulated distribution of random graphs enables a form of hypothesis testing. This approach allows us to determine whether a change in the trade network, such as shifts in centrality or graph modularity, can be attributed to an external factor, such as a political or economic shock, or if it is indistinguishable from random fluctuations (noise) within the network. By comparing the observed metric to its distribution in randomized versions of the network, we can determine whether the observed change is statistically significant or likely a result of random chance. In constructing random graphs, or null models, Fagiolo (2016) emphasizes the importance of carefully selecting which features of the world trade network are preserved during the randomization process. The constraints on these features should strike a balance: they must be strict enough to retain a sense of realism while remaining flexible enough to generate a meaningful distribution of the metric of interest for hypothesis testing.

3 Data

We use data from Fouquin and Hugot (2016), which compiles the British Pound value of total nominal exports between pairs of countries between 1950 and 2014. For this time period, the data is primarily sourced from IMF data and supplemented with primary sources from trading countries. The trade amounts are at the country-pair year level. We drop non-sovereign ISO-3 codes from the dataset. For each year, we have between 93 and 196 countries with recorded trade flows. Across all years, we have over 1.8 million bilateral trade observations. The total volumes of all exports for each year are shown in Figure 14. Given the dramatic rise in international trade over the previous decades, it is important to move beyond mere volumetric analysis and instead examine the network structure of trade.

One difficulty of analyzing trade data—particularly for earlier time periods—is the occurrence of missing data, as not all countries report their trade flow. Sajedianfard et al. (2021) explore the impact of the missing data on metrics of network centrality. The authors conclude that since network behavior in the trade network is motivated by leading and major participating countries that consistently report their trade flow; missing data from smaller non-reporting countries has no significant impact on the structure of the network. This means we can draw inferences from measures of network centrality and modularity, despite the occurrence of missing data.

3.1 Construction of the Trade Network

For each year between 1950 and 2014, we construct an adjacency matrix denoting the total exports from one country to another. Let $N^t = \{1, ..., n^t\}$ be the set of n^t countries that we have trading data for in year t. Now, for each year, define the weighted directed adjacency matrix, A^t, as an $n^t \times n^t$ matrix with matrix entries denoted as a^t_{ij}, where a^t_{ij} represents the total value of exports (in British Pounds) from country i to country j in year t, where $i, j \in N_t, i \neq j$. As we are considering aggregate flows between countries, and not intra-country inter-industry flows, $a^t_{ii} = 0$ for all i in N^t. We then define the graph, $G^t = (N^t, A^t)$ as the graph of directed bilateral exports between all n^t countries in year t. $a^t_{ij} = 0$ implies there is no value of recorded exports from country i to j in the given year, and $a^t_{ij} > 0$ implies some value of exports was recorded from country i to country j in year t.

The number of nodes (countries) in the trade graphs, the number of non-zero edges, and edge density (as computed by the number of non-zero edges divided by $(N^t * (N^t - 1))$ for each year between 1950 and 2014 are shown in Figure 15. These summary statistics are chosen to replicate Bhattacharya's (2007) summary of international trade networks using a different dataset. The number of countries represented by the trade graph increased from 93 in 1950 to 185 in 2014, capturing the creation of new states and more existing states reporting their export totals. The number of non-zero edges and edge density also increased since 1950, indicating both more countries reporting trade flows and countries trading with a wider array of other countries.

The adjacency matrices for the years 1950 and 2014 are displayed as logarithmic heatmaps in Figure 16 and Figure 17 respectively. The rows and columns are arranged by the total sum of a country's exports (e.g. the largest exporting country will be in the first column and row). Note, since the total country i exports to country j is equivalent to the total country j imports from country i, it follows that the transpose of A^t, $(A^t)^T$ represents the weighted directed import matrices, where each entry represents the total value that country i imports from county j in a given year. The difference between country i's exports to country j and imports from country j is thus given by $a^t_{ij} - a^t_{ji}$ (Since a^t_{ji} is equal to the the ij entry of $(A^t)^T$). The matrix of this difference for all countries in 2014 can be found in Figure 18. This matrix illustrates how some countries export more in the majority of their bilateral connections, whereas others import more in the majority of their bilateral trades.

3.2 Intra-European Trade Network

Due to potential issues of missing data in the complete trade network, we also consider a smaller trade network in which all exporting and importing countries are in the European region. Like above, we construct an adjacency matrix of all exports between 1950 and 2014, and all entries represent trade that flows from one European country to another. The adjacency matrix of intra-European trade for years 1950 and 2014 are displayed as logarithmic heatmaps in Figure 19 and Figure 20 respectively. Note, for 2014, we have trade values for almost every pair-wise combination of intra-European trade. Below are visualizations of the intra-European trade network for 1950 and 2014 in Figure 1 and Figure 2 respectively.

Network of Intra-European Trade in 1950 (Two Largest Export Destinations).

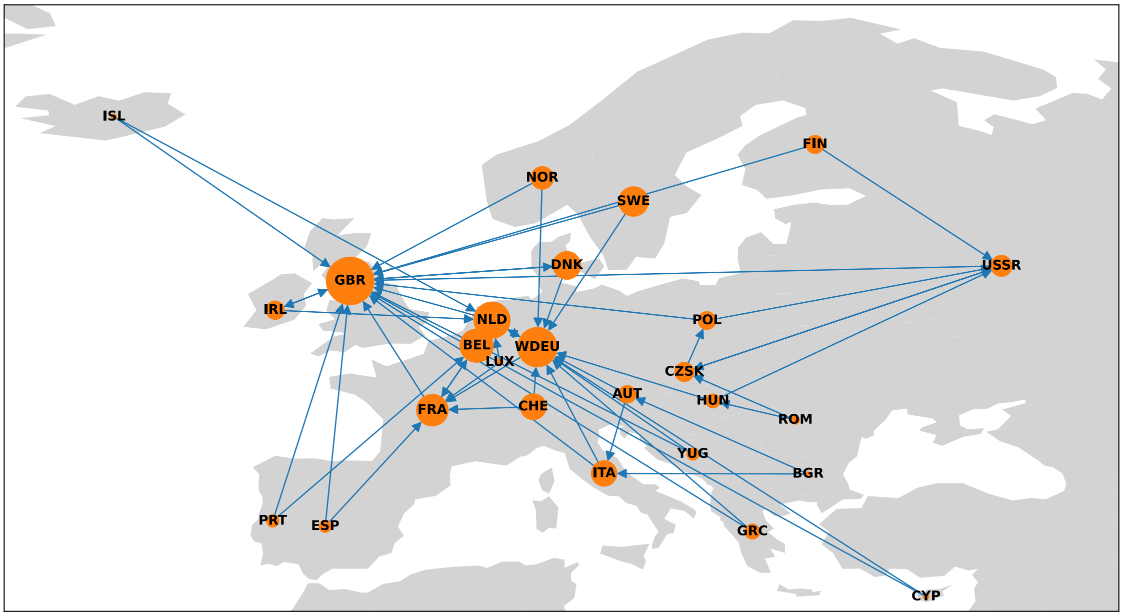

Figure 1: Using the 1950 network of intra-European trade, directed edges are drawn for each country's two largest export destinations. Country sizes are based on their in-degree, representing the value of goods they import.

Network of Intra-European Trade in 2014 (Two Largest Export Destinations).

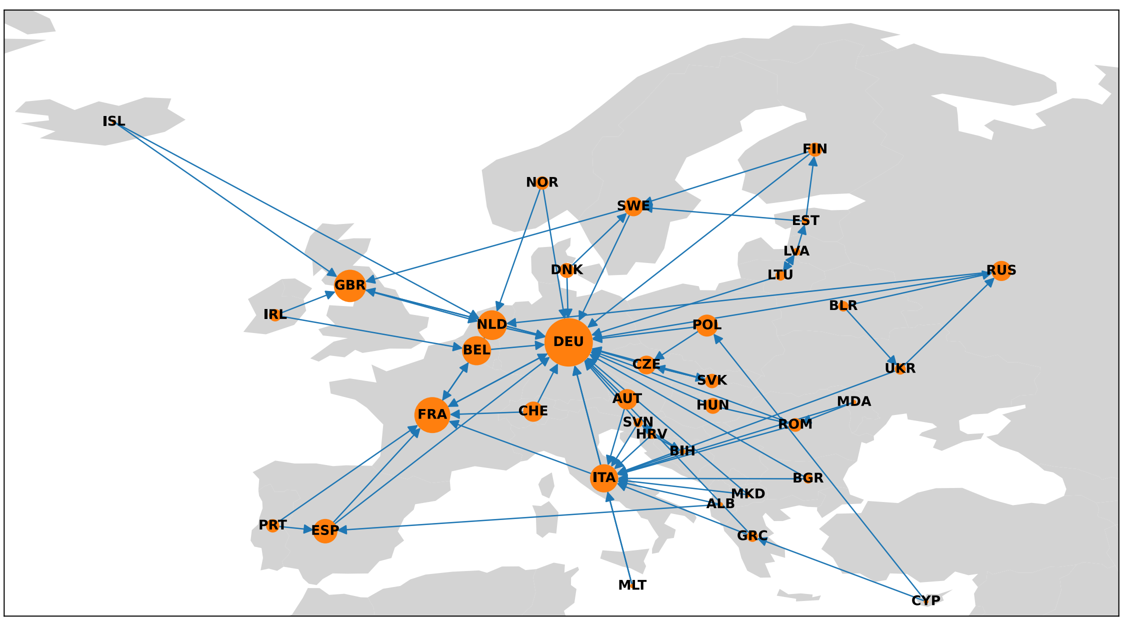

Figure 2: Using the 2014 network of intra-European trade, directed edges are drawn for each country's two largest export destinations. Country sizes are based on their in-degree, representing the value of goods they import.

4 Results

In this section, we utilize various measures of graph centrality and modularity to infer how the structure of the trade network has evolved between 1950 and 2014. To analyze centrality, we use the PageRank algorithm developed by Brin and Page (1998). In our analysis of graph modularity, we rely on spectral graph theory and the characteristics of a graph's Laplacian matrix.

4.1 Centrality Analysis

PageRank evaluates the importance of a country not only by its direct connections but also by the significance of its connections' links. This means a country is deemed central not merely by the volume of its trade, but by how central its trade partners are within the network. Zhang et al. (2021) apply weighted PageRank to World Input-Output networks, and introduce a tuning parameter to adjust for the relative importance of weights and degree in the model. Hoang, Piccardi, and Tajoli (2023) utilize weighted and unweighted PageRank to analyze bilateral trade volumes between 1996 and 2019. The authors interpret a decrease in the network's average PageRank as an indication of homogenization as large countries' PageRanks decrease relative to small economies.

To analyze the adjacency graphs we constructed above, we will utilize weighted PageRank with the damping factor set to 0.85 or 1. Given the weighted directed adjacency matrix of exports, A^t, we start by taking the transpose of the matrix such that a_{ij}^t now represents the values of the imports from country i to country j. We take the transpose of the matrix so that our computed PageRank scores will capture which countries are the most central as exporters, rather than importers. The PageRank value of country i in year t is denoted as $PR(i)$, is defined recursively as:

$$PR(i)^t = \beta \sum_{j \in N^t, j \neq i} a_{ij}^t \frac{PR(j)^t}{d_j^{\text{out}}} + \frac{1-\beta}{n_t}$$

Here,

- n^t is the total number of countries.
- $\beta \in (0, 1]$ is the damping factor. Where $\beta = 0.85$ is the standard value in PageRank analysis (Brin and Page, 1998).
- $d_j^{\text{out}} = \sum_{j \in N^t, j \neq i} a_{ij}^t$ is the weighted out-degree of country j.
- $PR(j)^t$ is the weighted PageRank of country j in year t.

The chart of each country's weighted PageRank in each year for $\alpha = 0.85$ is displayed in Figure 3. The increase in West Germany's (WDEU) in the 1950s and early 1960s reflects the recovery of the West German economy under the Marshal Plan, and the country's exports to GDP ratio more than doubled in the 1950s (Vonyó, 2018). In parallel, we observe the decline in Great Britain's (GBR) PageRank with the collapse of the British Empire.

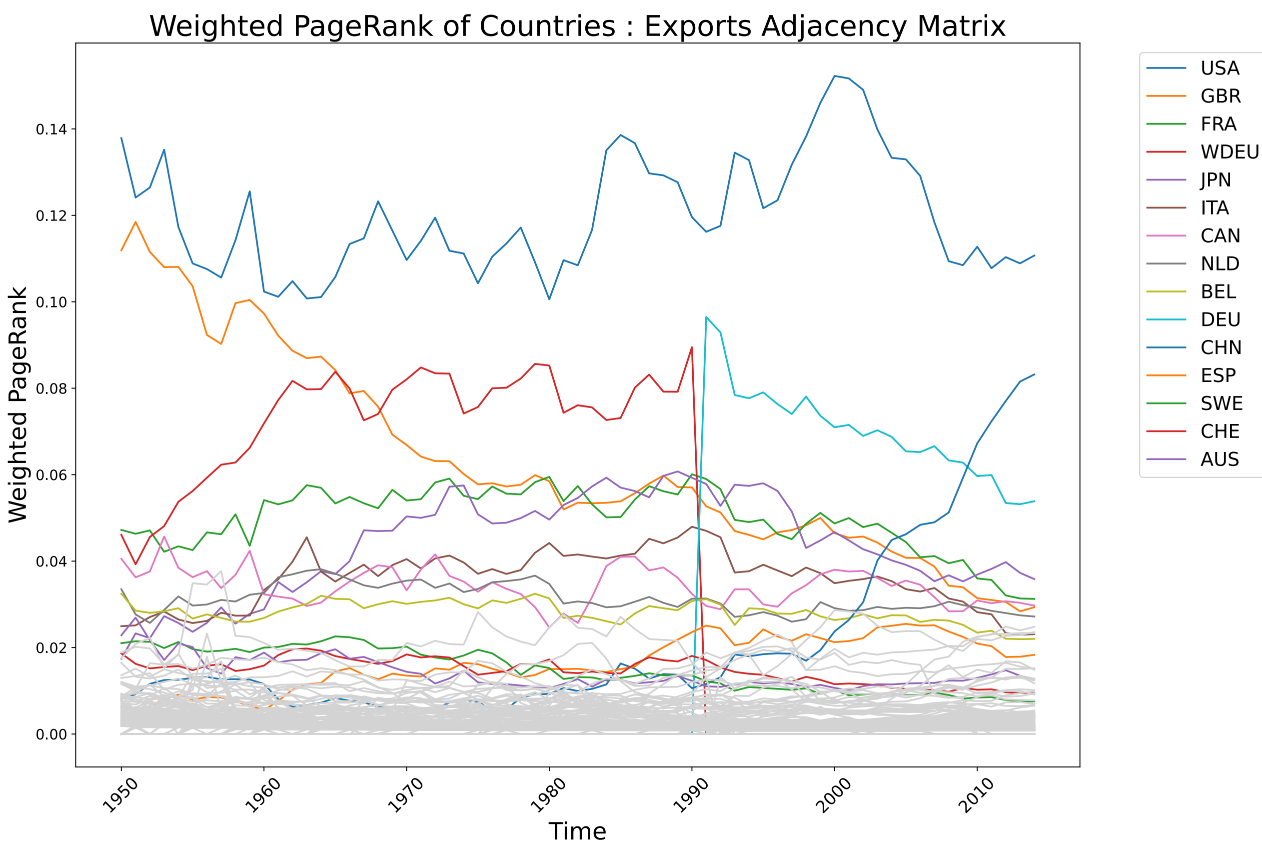

Figure 3: Weighted PageRank of Countries with $\alpha = 0.85$. Countries with the 15 highest average PageRank values are labeled.

Critically, analyzing weighted PageRank values captures the centrality of countries in the trade network beyond their raw trade volumes. While China exported more than the United States in 2014 (\$1.63 vs \$2.34 trillion USD), the United States has a higher weighted PageRank than China in that same year. By construction of the weighted PageRank, this is indicative of the United States trading with other major economies that also have high weighted PageRanks, where China's lower weighted PageRank captures exports to smaller and developing economies with lower weighted PageRanks.

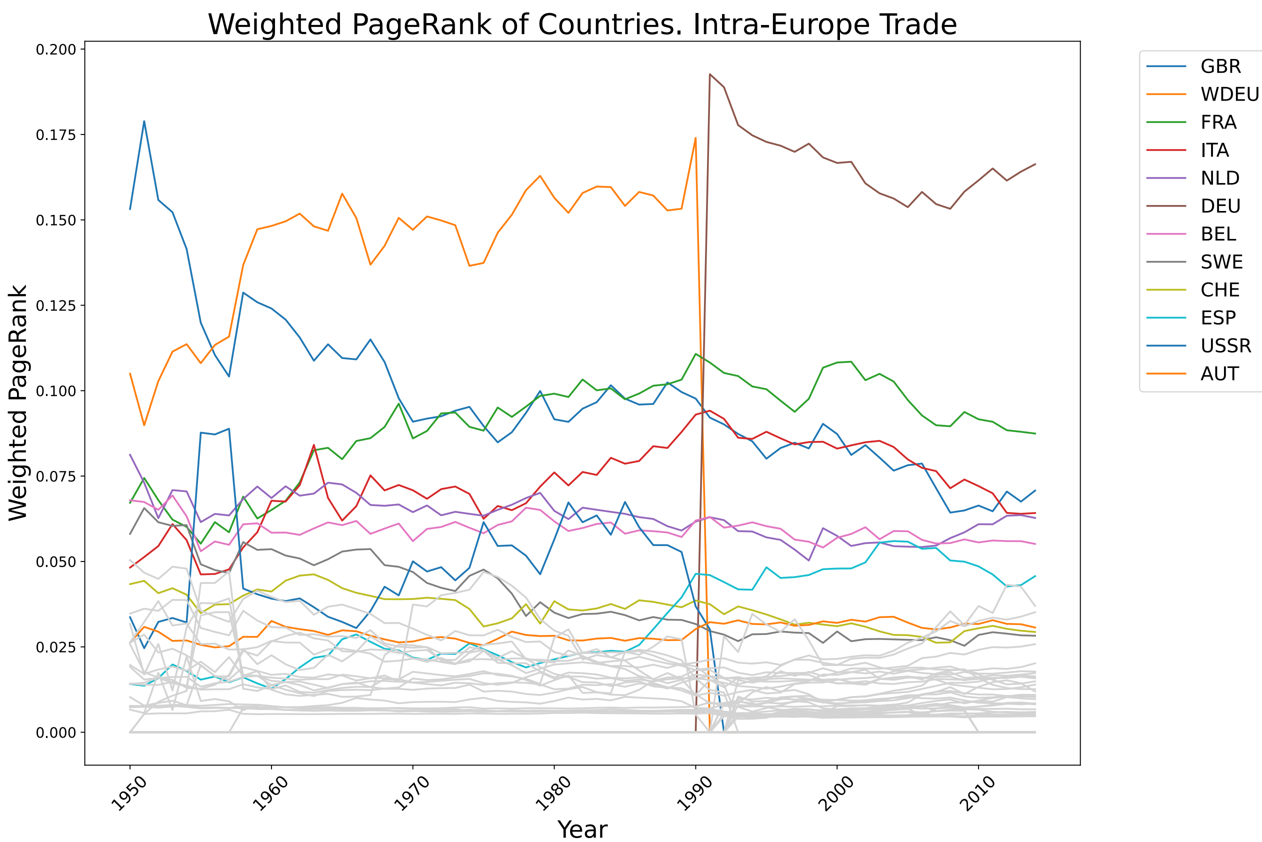

Figure 4: Weighted PageRank of Countries for Intra-European trade $\alpha = 0.85$. Countries with the 12 highest average PageRank values are labeled.

Considering the weighted PageRanks of countries in just the intra-European trade graph, the chart of each country's weighted PageRank for each year is shown in figure Figure 4. Unlike in the graph of all countries where most European states show a decline in their overall weighted PageRanks, when it comes to intra-European trade, most nations maintain a similar weighted PageRank between 1950 and 2014, apart from a sharp decline in the UK's score. The UK's sharp decline since the 1950s indicates that not only has its centrality in global trade decreased, but its regional importance in European trade has also decreased substantially. While Germany's weighted PageRank has decreased in the overall network of trade since 1960, its centrality in the intra-Europe network has increased, indicating increased regional importance despite a decline in its global importance as an exporter.

4.2 The Modularity of the Trade Network

Similar to social and biological networks, trade networks are characterized by communities or clusters, where countries tend to trade more extensively with others within the same community than with those outside of it. These trade communities may parallel the grouping of countries by identical consumption preferences, factor endowments, and proximity (Krugman, 1979). The detection of these network communities has generated an extensive field of study. In particular, spectral graph theory is implemented to explain the relationship between the eigenvalues of a graph's Laplacian matrix and the graph's overall connectivity (Chung, 1997). For an undirected graph, $G = (V, E)$, with an ordered vertex set $V = \{1, 2, ..., n\}$ where

$$E \subset \{\{i, j\} \mid i, j \in V\}$$

The Laplacian matrix of the graph G, L, is a $n \times n$ matrix with entries, L_{ij} given by:

$$L_{i,j} = \begin{cases} -1 & \text{if } \{i, j\} \in E, \\ d(i) & \text{if } i = j, \\ 0 & \text{otherwise.} \end{cases}$$

Equivalently, the Laplacian matrix is the matrix subtraction:

$$L = D - A$$

Where D and A are the degree and adjacency matrices of G, respectively. If the graph is weighted, replace A with the graph's weights matrix, W.

A weighted graph's normalized symmetric Laplacian matrix, L_{sym} is defined as

$$L_{sym} := D^{-1/2} L D^{-1/2} = I - D^{-1/2} W D^{-1/2}$$

Crucially, the eigenvalues of the normalized Laplacian matrix are scale-invariant. This scale invariance is important for analyzing export graphs across years since the number of countries, $|N_t|$ varies by year. Since L_{sym} is symmetric, its eigenvalues are all real and non-negative. Thus, for L_{sym}, we can find n eigenvalues (not necessarily unique), and label them $0 \leq \lambda_1 \leq \lambda_2 \leq ... \leq \lambda_n$. It can be shown that the smallest eigenvalue of L_{sym} will always be zero; $\lambda_1 = 0$ (Chung, 1997). The eigenvalues of the Laplacian matrix explain the connectivity of the graph, G. The graph G is connected if and only if $\lambda_2 > 0$. More generally, the number of zero eigenvalues is equal to the number of connected components in G, e.g. if $\lambda_i = 0$, the graph has i disjoint components (Chung, 1997). Lastly, in the case of a connected graph, the value of λ_2 indicates the algebraic connectivity of the graph (Bapat, 2010).

4.2.1 Methods to Symmetrize the Trade Network

Computing the normalized Laplacian matrix requires an undirected weight matrix for each year of trade, so computing the algebraic connectivity requires some symmetrization of the weighted directed network of trade. Fagiolo et al. (2010) discuss contributions that utilize both directed and undirected approaches to analyzing the world trade networks. They make the point that analyzing the undirected network of trade might underestimate the role of heterogeneity in bilateral trade flows. For instance, a lot of trade linkages are heavily skewed to one direction, such that intense trade linkages coexist with relatively weaker linkages. Consequently, we must consider under what conditions the network should be symmetrized, and by which method the symmetrization in computed. Fagiolo (2006) provides a procedure for deciding whether or not an observed network is sufficiently symmetric to warrant an undirected analysis, and in Fagiolo (2010), he concludes that the international network of aggregate trade flows is sufficiently symmetric to warrant an undirected analysis. Furthermore, we would argue that in assessing the connectivity of internal trade, a symmetrization of the aggregate flows network provides some proxy for the weight of an economic relationship and interdependence between states, so the algebraic connectivity may serve to capture the overall interdependence of the network.

As for the method used to symmetrize the network, we need to consider the structural properties of the trade network we hope to preserve. If the priority is to have edge weights capture some degree of the volume of trade between edges, summing or taking the maximum of the two edge weights would be a reasonable method, i.e.

$$A^t_{sym} = (A^t + (A^t)^T) \quad \text{or} \quad a^t_{sym,ij} = max\{a^t_{ij},\ a^t_{ji}\}$$

If, instead, the priority is to capture how strong the bidirectional ties of trading relationships are, or how reciprocal they are, averaging the edge weights, or a more involved approach where we take the square root of the product of the edge weights, can be used.

For this analysis, since we are most interested in the political connections associated with trade and overall network connectivity, we opted to use an averaging of the edge weights, so our symmetric matrices are of the form,

$$a^t_{sym,ij} = \frac{1}{2}(a^t_{ij} + a^t_{ji}) \quad \implies \quad A^t_{sym} = \frac{1}{2}(A^t + (A^t)^T)$$

so entry $a^t_{sym,ij}$ is equal to the average value of exports from country i to country j and exports from country j to country i.

To reiterate Fagiolo's point, we find that the less symmetric the initial trading networks are, the more algebraic connectivity varies based on the symmetrization method used. So, for intra-European trade, which is relatively more symmetric than other trade networks, the choice of symmetrization method has less of an impact on the algebraic connectivity of the network. In contrast, when we apply these methods to networks with much more asymmetric trading flows, like commodity trade networks, the normalized Laplacian varies significantly based on which symmetrization method is chosen. So in the case of those networks, methods that allow for directed networks should be used to provide a notion of network connectivity.

4.2.2 Network Cut

Using the symmetric normalized Laplacian matrix, we can analyze how partitionable the world trade network is for each year in our data. Replacing W with A^t_{sym}, we then define the symmetric normalized Laplacian matrix, L^t_{sym}, as defined above for each year. Consider the second smallest eigenvalue of L^t_{sym}, λ^t_2, which represents the algebraic connectivity of the graph in year t. In a connected trade graph, smaller values of λ^t_2 represent a more bi-partitionable graph, as in, the graph could be split into two disjoint communities through the removal of just a few edges. Assuming A^t_{sym} could be partitioned into two disjoint sets B^t and C^t—representing two autarkic trading groups—such that $B^t \cup C^t = N^t$, $B^t \cap C^t = \emptyset$. The bi-partionability of A^t can be computed as the total weight of edges that need to be removed in order to partition the graph, otherwise referred to as the *cut* (Shi and Malik, 2000).

$$cut(B^t, C^t) = \sum_{i \in B^t, j \in C^t} a^t_{sym,ij}$$

The optimal way to divide the graph is given by the choice of $B^t, C^t \subset A^t_{sym}$ such that $cut(B^t, C^t)$ is minimized. The relationship between λ^t_2 and the concept of a cut in the graph aligns with the intuition that a good partition of a graph minimizes the connections between B^t and C^t—the ones that are 'cut'—leading to what is effectively a measure of the graph's resilience to being split into disjoint clusters. Given the graph of trade connectivity, A^t_{sym} in year t, a smaller λ^t_2 implies that the network can be separated into two groups, B^t and

C^t, with a relatively small number of trading relationships being disrupted. This scenario corresponds to a network structure entirely, or almost entirely, fractured into two distinct trading communities. In contrast, a large value of λ_2^t indicates a well-connected trading network that would require the disruption of many trading links in order to partition the network.

4.2.3 On Network Conductance

A major limitation of using the normalized Laplacian and associated eigenvalues for analyzing the network of trade is the loss of directional information as a result of symmetrizing the weighted adjacency matrix, as we briefly touched on above. Ideally, we would prefer a method that allows us to maintain the directed component of the network as we analyze its connectivity and modularity. In theory, graph conductance can provide of measure of how well-connected the graph is and provide a sense of how "bottle-necked" the network of trade is. For a weighted directed graph, a common approach for computing graph conductance is to define conductance based on the flow of a random walk over the network (Spielman, 2004). To do this, we start by defining a Markov chain. Given a weighted directed graph $G = (V, W)$, for nodes i and j in the graph, define the transition probability between them as:

$$P(i,j) = \frac{w_{ij}}{\sum_{k \in V} w_{ik}}$$

Assuming the chain is irreducible and aperiodic, as it is for all years of our network data, there must exist a stationary distribution, π, for this Markov chain. Now, for any strict subset of vertices, $S \subsetneq V$, the conductance of S, $\Phi(S)$ is defined as:

$$\Phi(S) = \frac{\sum_{i \in S} \sum_{j \notin S} \pi(i) P(i,j)}{min\{\pi(S),\ 1 - \pi(S)\}}$$

The overall conductance of the directed graph, Φ_G is then given by the minimum of $\Phi(S)$ over strict subsets.

$$\Phi_G \coloneqq \min_S \Phi(S)$$

And Φ_G is in the range $0 < \Phi_G < 1$. A low conductance indicates that there exists a relatively small number of edges connecting two parts of the graph together, indicating a higher modularity, where a larger conductance represents a more connected and less modular graph. Unfortunately, computing graph conductance is an NP-hard problem since testing all subsets of vertices has time complexity $O(2^n)$.

4.3 Comparative Statics of Modularity with Varying Network Size.

Having discussed the implications of various magnitudes of the Fiedler vector, λ_2^t, on graph modularity λ_2^t, we move now to discuss how λ_2^t is expected to change as the network of trade evolves. First, we discuss how changes in the magnitude of trade across the graph varies λ_2^t. Second, we explore how the introduction or removal of countries in the trading network impacts λ_2^t through an analysis of the Laplacian submatrices.

4.3.1 Uniform Scaling of the Trade Network

We can consider an increase in trade across the network as a perturbation of the weighted adjacency matrix representing exports between countries. Consider a network of trade, $G = (N, A)$ where $A \in \mathbb{R}^{n \times n}$ is the weighted adjacency matrix with the corresponding diagonal degree matrix, D (where $D_{ii} = \sum_j A_{ij}$). Additionally, the network has a symmetric normalized Laplacian given by $L_{sym} = D^{-1/2} L D^{-1/2} = I - D^{-1/2} A D^{-1/2}$ with eigenvalues $\lambda_1 \leq \cdots \leq \lambda_n$. Now consider a new network given by a scaling of A by a constant factor, $\alpha > 0$, as $A' = \alpha A$, representing a $(\alpha 100 - 100)\%$ change in trade volume for each exporting link. Since the entries of the degree matrix are defined by summing the rows of the adjacency matrix, the diagonal entries of the new diagonal degree matrix will be,

$$D'_{ii} = \sum_j A'_{ij} = \sum_j (\alpha A)_{ij} = \alpha D_{ii} \implies D' = \alpha D$$

And the symmetric normalized Laplacian of this new graph will be

$$L'_{sym} = I - (D')^{-1/2} A' (D')^{-1/2} = I - (\alpha D)^{-1/2} (\alpha A)(\alpha D)^{-1/2}.$$

Since $(\alpha D)^{-1/2} = (\alpha^{-1/2} D^{-1/2})$. Substituting gives

$$(\alpha D)^{-1/2}(\alpha A)(\alpha D)^{-1/2} = \alpha^{-1/2} D^{-1/2} \alpha A \alpha^{-1/2} D^{-1/2} = (\alpha^{-1/2}\alpha\alpha^{-1/2})(D^{-1/2} A D^{-1/2})$$

And since $(\alpha^{-1/2}\alpha\alpha^{-1/2}) = \alpha^0 = 1$,

$$(\alpha D)^{-1/2}(\alpha A)(\alpha D)^{-1/2} = (1)(D^{-1/2} A D^{-1/2}) = D^{-1/2} A D^{-1/2}.$$

Therefore,

$$L'_{sym} = I - (D')^{-1/2}(A')(D')^{-1/2} = I - D^{-1/2} L D^{-1/2} = I - D^{-1/2} A D^{-1/2} = L_{sym}.$$

Which means any constant scaling of the trade network results in no change in the algebraic connectivity of the network. This should be expected as a result of the normalization of the adjacency matrix in the $D^{-1/2} A D^{-1/2}$ term.

4.3.2 Eigenvalue Interlacing

We move now to discuss how graph modularity is expected to change as a result of individual trade connections being disrupted or as a consequence of countries being added or removed from the trade network. Mathematically, we will be looking to define upper and lower bounds for Laplacian eigenvalues as the underlying graphs are modified. The following paragraphs rely heavily on Eigenvalue Interlacing and build upon Cauchy's Interlacing Theorem.

Cauchy's Interlacing Theorem: Let G be a graph of of n vertices with adjacency matrix A (i.e. $A \in \mathbb{R}^{n \times n}$) and let B be the adjacency matrix of an induced subgraph of A with m vertices, where $m < n$. If $\lambda_1 \leq \cdots \leq \lambda_n$ are the eigenvalues of A and $\mu_1 \leq \cdots \leq \mu_m$ are the eigenvalues of B, then we have

$$\lambda_i \leq \mu_i \leq \lambda_{i+(n-m)} \qquad \forall i \in \{1, \ldots, m\}.$$

In the case where $m = n - 1$, implying the removal of just one node, then we simply have

$$\lambda_i \leq \mu_i \leq \lambda_{i+1} \qquad \forall i \in \{1, \ldots, n-1\}.$$

Proofs of the Eigenvalue Interlacing Theorem are given by Golub et al. (1983) using the Courant-Fischer minimax theorem, and by Fisk (2005) using properties of polynomials.

4.3.3 Disruption of Individual Trading Relationships

An equally important consideration is the impact of a single trade relationship's disruption or removal on the modularity of the entire trade network. While in practice, the impact of the trade relationship's disruption will be a function of the magnitude of trade disrupted and the structure of the network, we can provide lower and upper bounds for the new graph modularity ex-post. The theorem for the following inequalities are given by Chen et al. (2004) and Butler (2007). Consider a network of trade, $G = (N, A)$ where $A \in \mathbb{R}^{n \times n}$ is symmetric with normalized Laplacian, $L_{sym}(G)$. Then let $G - e$ be a subgraph of G where one edge, e, is removed. Conceptually, this could be thought of as a bilateral, or unilateral, application of sanctions between two trading countries in the network. Let $\lambda_1 \leq \cdots \leq \lambda_n$ and $\mu_1 \leq \cdots \leq \mu_n$ denote the eigenvalues of the normalized Laplacian of G and $(G - e)$ respectively. Then

$$\lambda_{i-1} \leq \mu_i \leq \lambda_{i+1} \qquad \forall i \in \{1, \ldots, n\}.$$

There may exist a further restriction of the above inequality that allows us to compare the algebraic connectivity of G and $(G - e)$ as such inequality between the algebraic connectivity of graphs and subgraphs with edges removed exists for non-normalized Laplacians (Chen et al., 2004). Proving such an inequality would assist with analyzing simulated trade networks. While not proven, intuitively, as individual trade relationships are disrupted, we would expect an overall more modular network.

4.3.4 Removal or Addition of a Country in the Network

To interpret the impact of a decrease in the number of countries on the modularity of the trade network, we can consider a contraction of two countries into one trading partner.
To represent this contraction as a network, we will use a formalization given by Chen et al. (2004). Consider a network of trade, $G = (N, A)$ where A has been symmetrized. Then, for any two vertices u and v of G, denote $G/\{u, v\}$ as the graph obtained by contracting u and v into a single vertex, where the neighborhood of this new vertex is the union of u and v. i.e. export edge weights of this new node are equal to $a_{ui} + a_{vi}$ and import edge weights are equal to $a_{iu} + a_{iv}$ for all $i \neq u, v$. Now, if we consider u and v to be in the same equivalence class, while treating all other nodes as singletons, we can use a result from Butler (2007) and Aliniaeifard et al. (2021) to compare the eigenvalues of the normalized Laplacians of G and $G/\{u, v\}$. If the eigenvalues of normalized Laplacian of G and $G/\{u, v\}$ are $\lambda_1 \leq \cdots \leq \lambda_n$ and $\nu_1 \leq \cdots \leq \nu_{n-1}$, then we have the following

$$\lambda_1 \leq \nu_1 \leq \lambda_2 \leq \ldots \lambda_{n-1} \leq \nu_{n-1} \leq \lambda_n.$$

This implies that in the event of a removal of a country from a network of trade, assuming some reallocation of the trading being imported and exported by said country, we would expect a decrease in the modularity of the overall network–assuming such a removal results in no other changes across the network. Similarly, we can restate the above relationship to consider the impact of adding a country on the modularity of the network. If we were to consider $G/\{u, v\}$ to be the current network of trade, and we were to add a country by splitting some existing node into u and v where the existing export and import values are distributed between u and v, then we would have graph G as described above. Thus, we can reuse the same inequalities to state that in lieu of any other changes across the network, if we were to add an additional country to the network, we would expect a decrease in overall modularity.

While the assumption that the structure of the rest of the network is not endogenous to changes in individual trade relationships or the addition or removal of countries is unrealistic, exploring the theoretical comparative statistics provides us with a direction of bias for a variety of observed networks. For instance, if new countries are introduced into the network without significantly altering overall trade volumes, we would expect an increase in network modularity. However, if modularity instead decreases, this suggests that other structural changes in the network are at play. In this case, the addition of the new country alone would have biased the network towards greater modularity, meaning the observed decrease must be a result of other shifts in trade relationships. For example, if countries are added to the network without a major change in trade volumes across the network, we would expect an in increase in network modularity. If, instead, we observed a decreased modularity of the new network, we can conclude that this change in modularity must in some part be a function of some other shift in the network, as the addition of the node would be biasing the network towards being more modular.

4.4 Observed Modularity of World and European Trade Networks

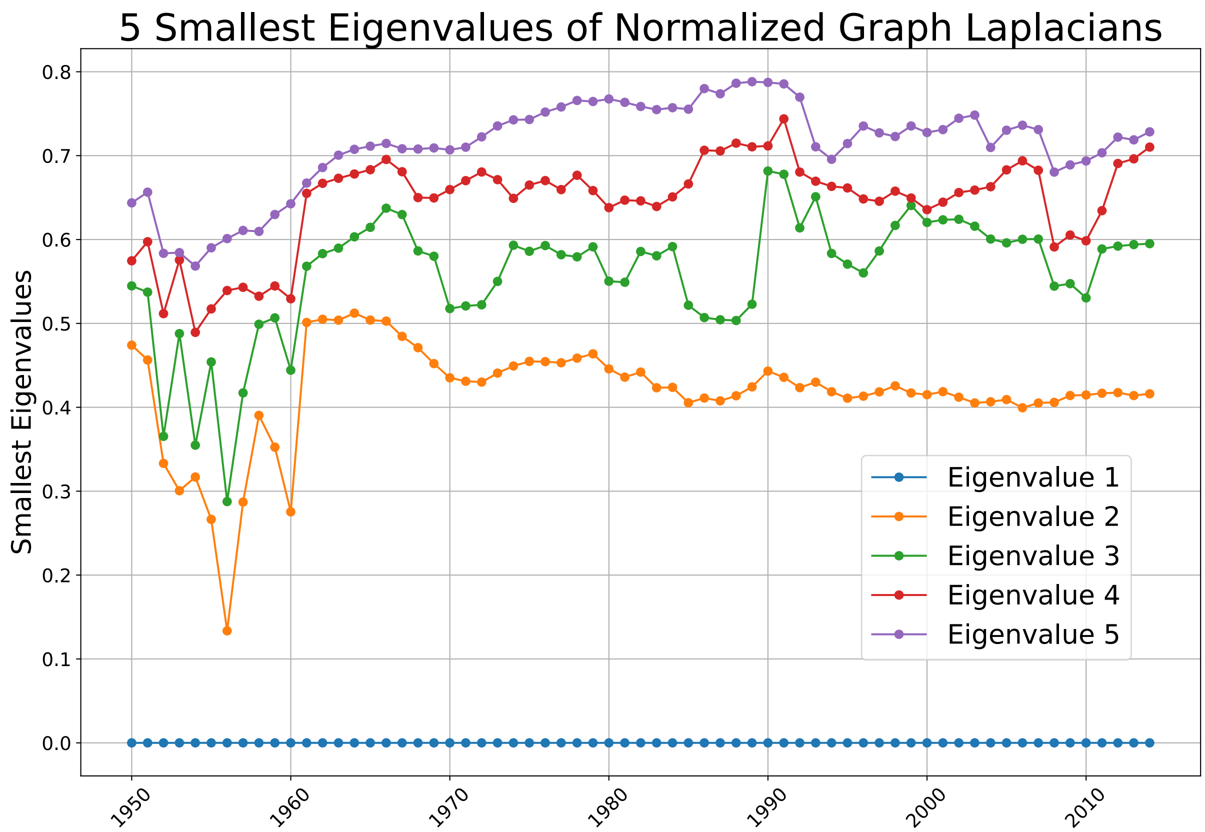

Figure 5: Five smallest eigenvalues of L^t_{sym}. λ^t_2 in orange.

The five smallest eigenvalues of L^t_{sym} for each year are graphed in Figure 5. The decrease in the value of λ^t_2 during the 1950s means the trade graph, A^t_{sym} can be partitioned into two distinct communities by removing fewer edges (as measured by their weights) compared to periods when λ^t_2 is larger. Furthermore, λ^t_3 is also smaller during the same period, meaning that the *cut* cost of dividing the graph into three disjoint sub-graphs is also lower than in later years. Interpreting eigenvalues larger than λ^t_2 is important since while two graphs may have the same optimal cut cost, one of the graphs could have more connectivity in the resulting sub-graphs. The value of λ^t_3 represents the separability of A^t_{sym} into three disjoint sub-graphs. For instance, in the early 1990s, λ^t_2 stays relatively constant, but λ^t_3 increases quite dramatically. This may indicate that while the world trade network maintained the same modularity, connectivity increased within established trading blocs during that time.

In looking at just intra-European trade, the five smallest eigenvalues of L^t_{sym} for each year are shown in Figure 6. The value of λ^t_2 fluctuates substantially between 1950 and 1990 before spiking between 1991 and 1992, coinciding with the dissolution of the USSR. Most strikingly, fluctuations in the value of λ^t_2 in the intra-European network mirror fluctuations in the value of λ^t_3 in the overall network of trade. This suggests that a three-way partition of international trade would involve some bi-partition of European trade.

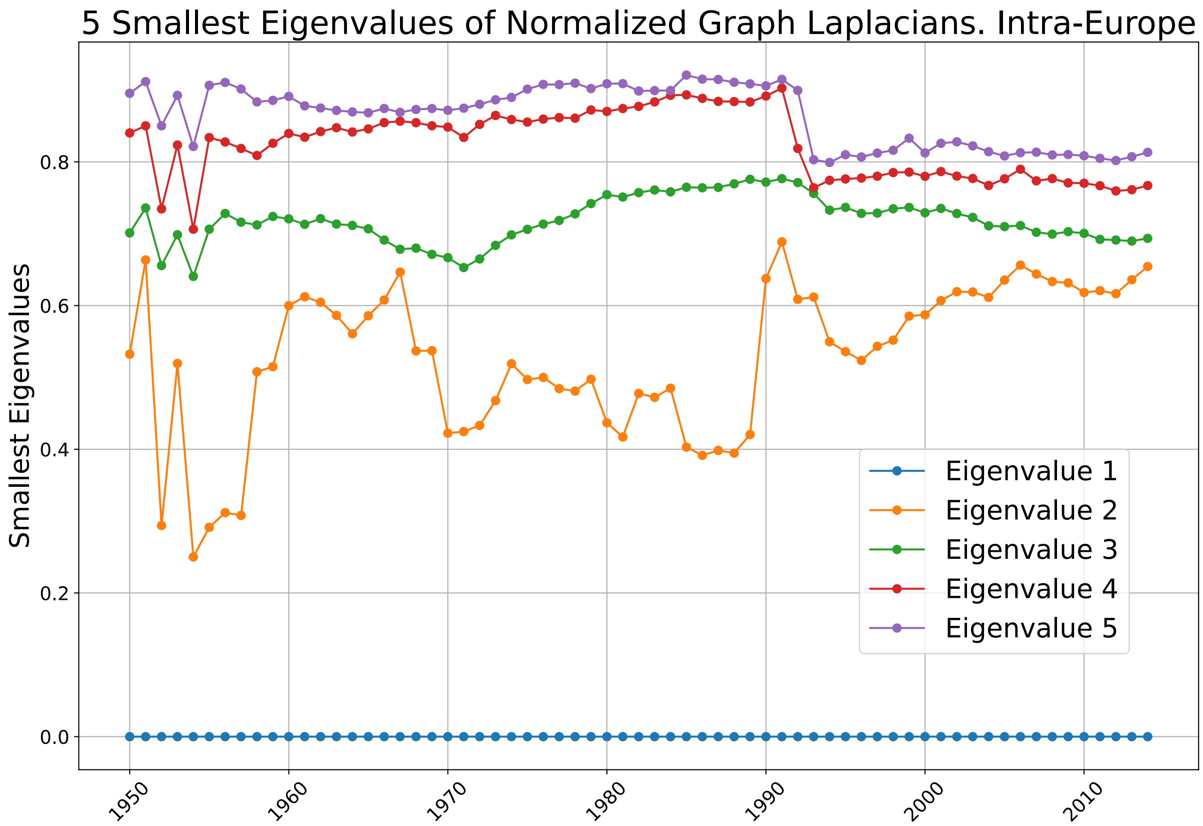

Figure 6: Five smallest eigenvalues of L^t_{sym} for intra-European trade. λ^t_2 in orange.

4.5 Comparison with Null Models

In considering the significance of the above eigenvalues, we have to ask whether these eigenvalues are unique to the network of international trade and if they respond to real-world economic and political shifts, or if we would expect similar eigenvalues for any randomly generated graph with similar characteristics. Conceptually, given each country's total ability to export, and their propensity to trade larger amounts with certain countries, does the modularity of the international trade graph statistically differ from the modularity of a random graph with similar characteristics? These random graphs with similar characteristics are referred to as "null" statistical models, where constraints are added to maintain various characteristics of the observed graph (Squartini et al., 2014; Fagiolo, 2016). To compare how our observed graphs differ from the null models, we compare the observed eigenvalues of the trade networks to the distribution of eigenvalues that result from running a Monte Carlo simulation with 1000 replicates for a random graph that has similar characteristics to the actual trade network we observe for each year.

To construct these random graphs, we fix every country's weighted out-degree to preserve nodes' relative importance, and we fix the distribution of weighted out-going edges, and then randomly assign these weights to all the node's edges using a surjective assignment, allowing new edges from the old distribution of weights. Denote the random graph for time period t as, $\mathcal{G}^t(N^t, \{d_i\}, \{W_i\}, \mathcal{R})$. Where N^t is the set of all nodes, or countries, in the actual network of trade for year t. $\{d_i\}$ is the set of out-degrees for each node, where d_i represents the number of outgoing edges for node i in the observed network of trade. And where $\{W_i\}$ defines a set of weight vectors for each node i of the $|N^t|$ nodes in the graph. Such that for each i,

$$W_i = \{w_{i1}, w_{i2} ... w_{id_i}\}$$

where each w_{ij} denotes the weight of the out going edge from node i to node j in the actual trade network, A_t (which excludes self-loops from country i to itself). Lastly, we define $\mathcal{R}$ as a random permutation or matching function that ensures a one-to-one random assignment of the d_i outgoing edges of each node i to d_i distinct nodes in N_t. The randomness of $\mathcal{R}$ can be described as $\mathcal{R} \sim \mathcal{U}(\mathcal{P})$, where $\mathcal{P}$ is the set of all possible permutations of edge weights to edges, excluding the edges between countries and themselves.

For both the world trade network and the network of intra-European trade, we simulate 1000 random graphs for each year between 1950 and 2014 using the manner described above.

For each simulated random graph, we compute the normalized symmetric Laplacian and the corresponding 5 smallest eigenvalues. The mean value of each eigenvalue as the associated standard deviation across 1000 replications for each year is plotted for the world trade network in Figure 21 and the intra-European network in Figure 22. Additionally, we plot the distribution of the second smallest eigenvalues across the replications of the random graph against the actual values of the world trade network's second smallest eigenvalue in Figure 7 and for the intra-European network in Figure 8. Observe that for the world trade network, the second smallest eigenvalue, λ_2^t is more than two standard deviations away from the mean of the distribution of eigenvalues for the random graph from 1961 onwards. This implies that the structure of the trade network we observed in those years is significantly more interconnected than we would expect from a random graph generated with similar characteristics. For the graph of intra-European trade, λ_2^t varies significantly prior to 1990, and in most years before 1990 the observed λ_2^t is within one or two standard deviations of the mean eigenvalue for the random graph. However, beginning in 1990 with the introduction of additional countries into the network, we would expect to see a decrease in the second smallest eigenvalue as observed in the Monte Carlo simulation, but we instead observed a significant increase in λ_2^t for 1990 and all subsequent years. This suggests that despite the introduction of numerous more nodes into the graph, intra-European trade became more interconnected and less modular following the dissolution of the USSR, and that the structure of trade we observe is distinct from a random graph constructed with similar weights.

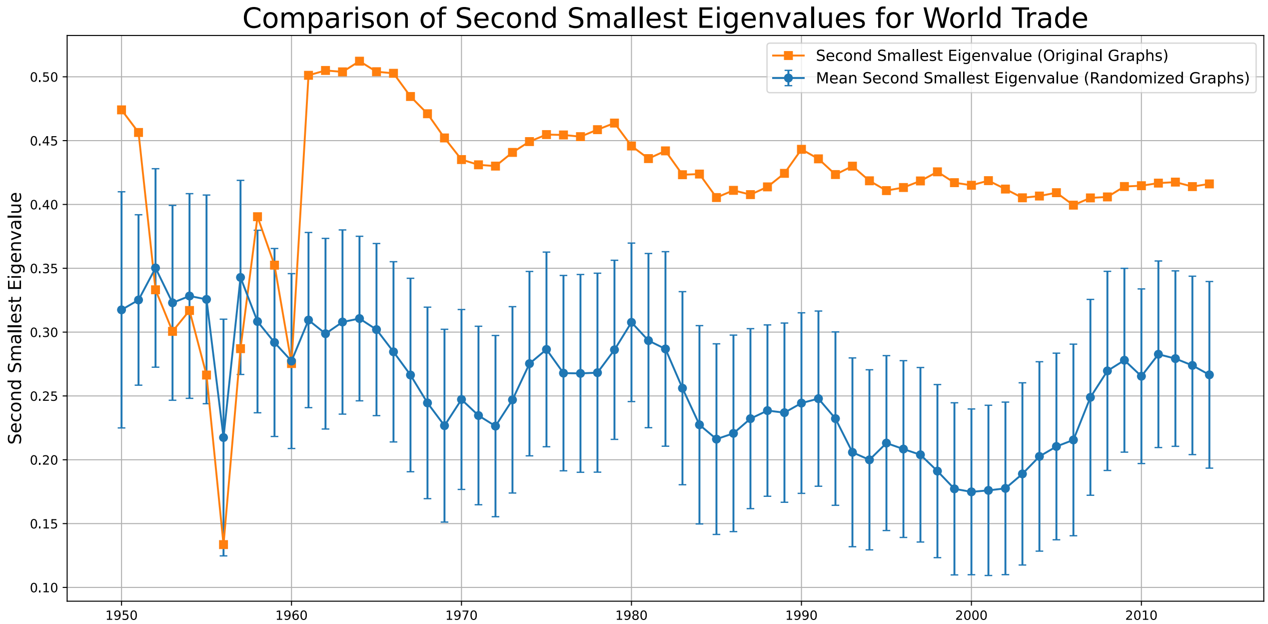

Figure 7: World Trade Network: Second smallest eigenvalue of the symmetric normalized Laplacian (λ_2^t) of the actual trade network in each year plotted against the distribution of the second smallest eigenvalue for 1000 replications of the random graph $\mathcal{G}^t$ with parameter values from the full trade network.

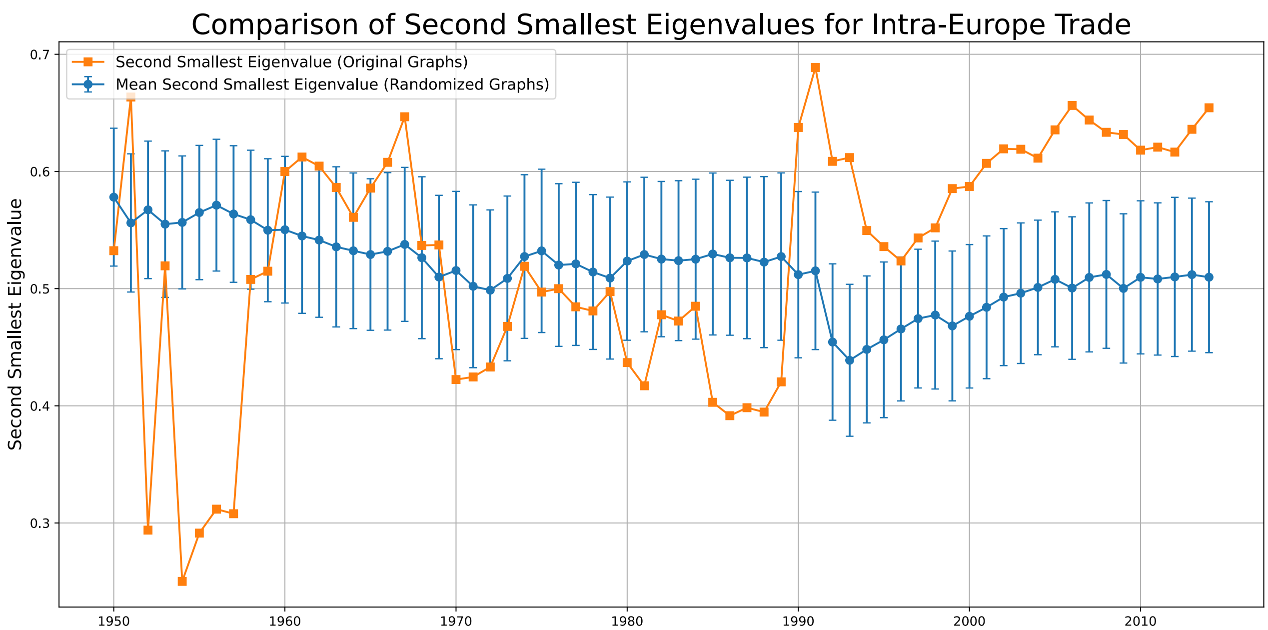

Figure 8: Intra-European Trade Network: Second smallest eigenvalue of the symmetric normalized Laplacian (λ_2^t) of the actual trade network in each year plotted against the distribution of the second smallest eigenvalue for 1000 replications of the random graph $\mathcal{G}^t$ with parameter values from the intra-European trade network.

4.6 Comparison with Inter-state Trade Within the US

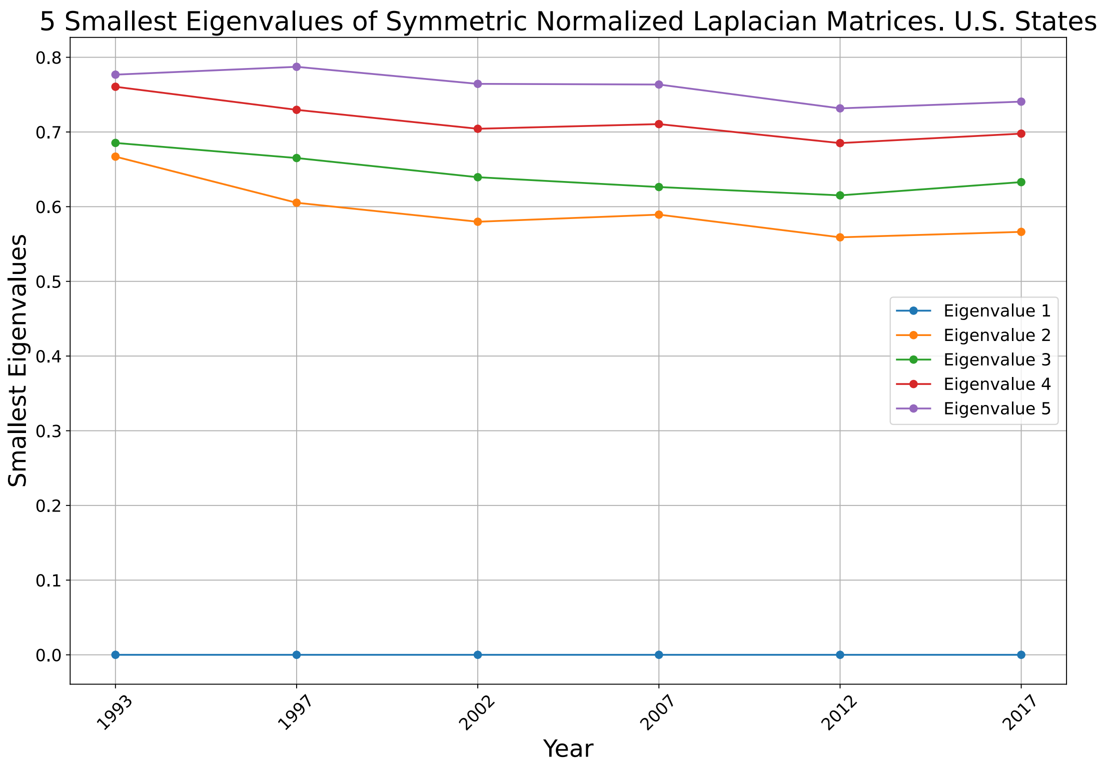

Figure 9: Five smallest eigenvalues of the symmetric normalized Laplacian of inter-state US trade. λ_2^t in orange.

In addition to a comparison with null models, we can also consider the symmetric normalized Laplacian eigenvalues we observe in already integrated trading communities, such as trade within the United States. Using data from the United States Department of Transportation. Bureau of Transportation Statistics Commodity Flow Survey (CFS) (1993, 1997, 2002, 2007, 2012, and 2017) we construct a network of state-to-state trade within the United States for six years across a 15 year range (1993-2017). While the CFS data is far from complete in terms of capturing all trade between states, since we are considering the normalized Laplacian of networks, as long as the trade values reported by CFS are proportional to the actual values of trade, then as we discussed in section 4.3.1, the resulting eigenvalues of the normalized Laplacian will be the same for the CFS data as they are the actual data. While we doubt, the networks constructed by the CFS will be exactly proportional to the actual network, the scale invariance of the normalized Laplacian will hopefully mitigate the impact of under-reported trade values on modularity. The five smallest eigenvalues of the symmetric normalized Laplacian of inter-state US trade for each year are shown in Figure 9. In this chart, we observe that the algebraic connectivity of inter-state US trade varies in a similar range as the algebraic connectivity of intra-European trade starting in 1992. While we do not claim that intra-European trade and inter-state U.S. trade are directly comparable in all respects, this analysis provides valuable context for understanding how trade modularity evolves in integrated economies. The fact that algebraic connectivity levels converge to a similar range following the year 1992 suggests that, at least in terms of network structure, Europe's trade integration process brought it closer to resembling an internal US trade network rather than a set of distinct, self-contained regional economies.

4.7 Partitioning Trade Networks and Community Detection

Beyond expressing a degree of modularity in the network, we can also use the normalized Laplacian to explore the formation of and evolution of trade blocs in the trade network. In particular, we proceed now to explore the evolution of trade blocs before and after the dissolution of the USSR in the intra-European network of trade. Given the symmetric normalized Laplacian matrix for each year, L^t_{sym}, we can use the eigenvector corresponding to the second-smallest eigenvalue, λ^t_2, of L^t_{sym} to cluster countries in the trade network for each year. This eigenvector is commonly referred to as the Fiedler vector, and we will denote it as v^t (Fiedler, 1973; Wyss-Gallifent, 2019). Each entry of v^t corresponds to a country in A^t. If we assign every country to a trade community based on the sign of its corresponding entry in the Fiedler vector, this would result in a partition of countries that attempts to remove as few edges as possible–as measured by their weights–while simultaneously trying to remain comparable size between the trade communities (Wyss-Gallifent, 2019). Alternatively, running a clustering algorithm such as k-means or hierarchical clustering can be used to construct a similar partition of the graph (von Luxburg, 2007).

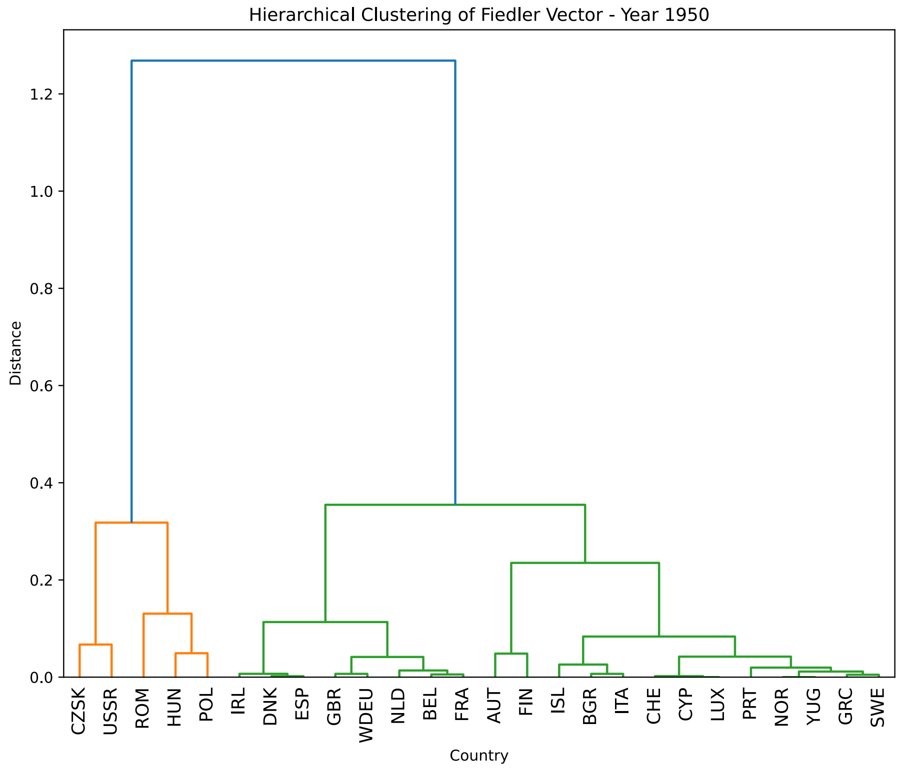

Figure 10: Hierarchical Clustering using L_{sym} of the intra-European trade graph. 1950.

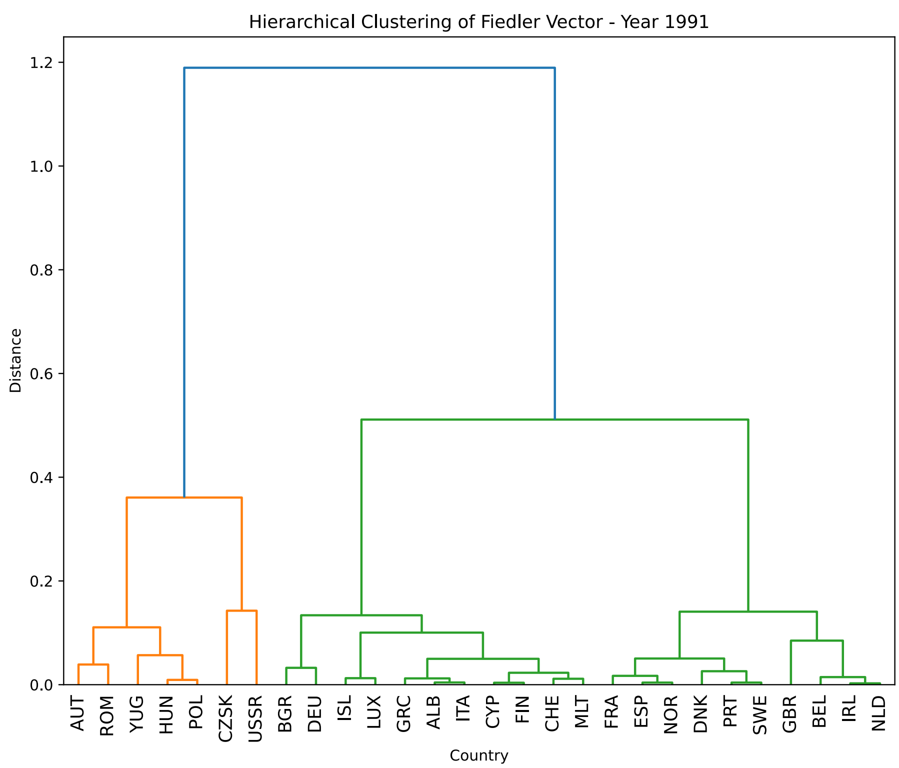

Figure 11: Hierarchical Clustering using L_{sym} of the intra-European trade graph. 1991.

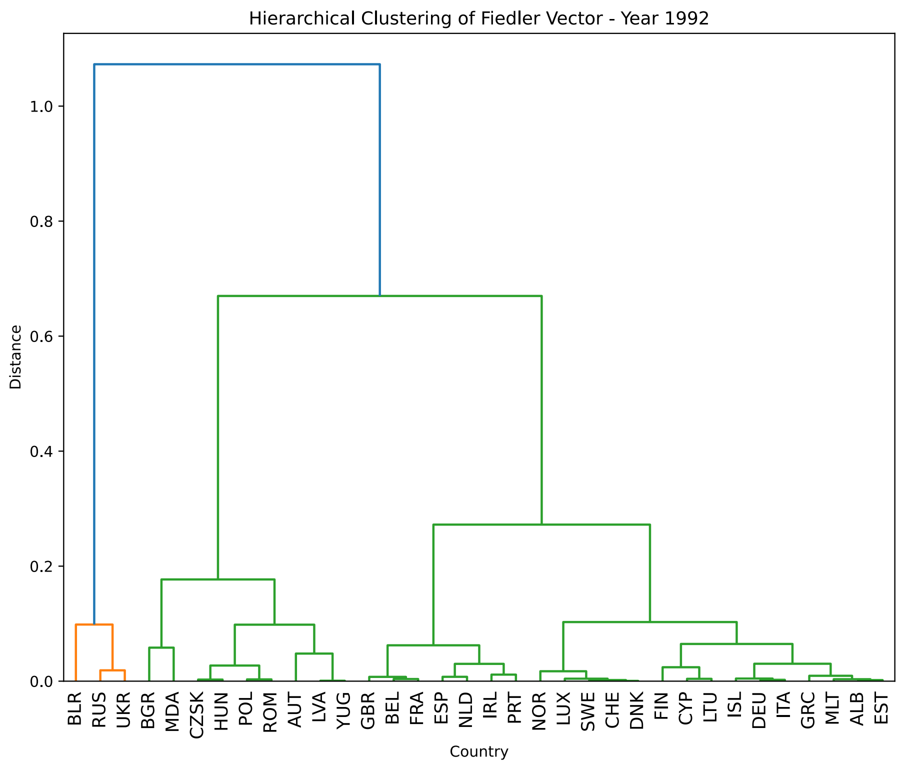

Figure 12: Hierarchical Clustering using L_{sym} of the intra-European trade graph. 1992.

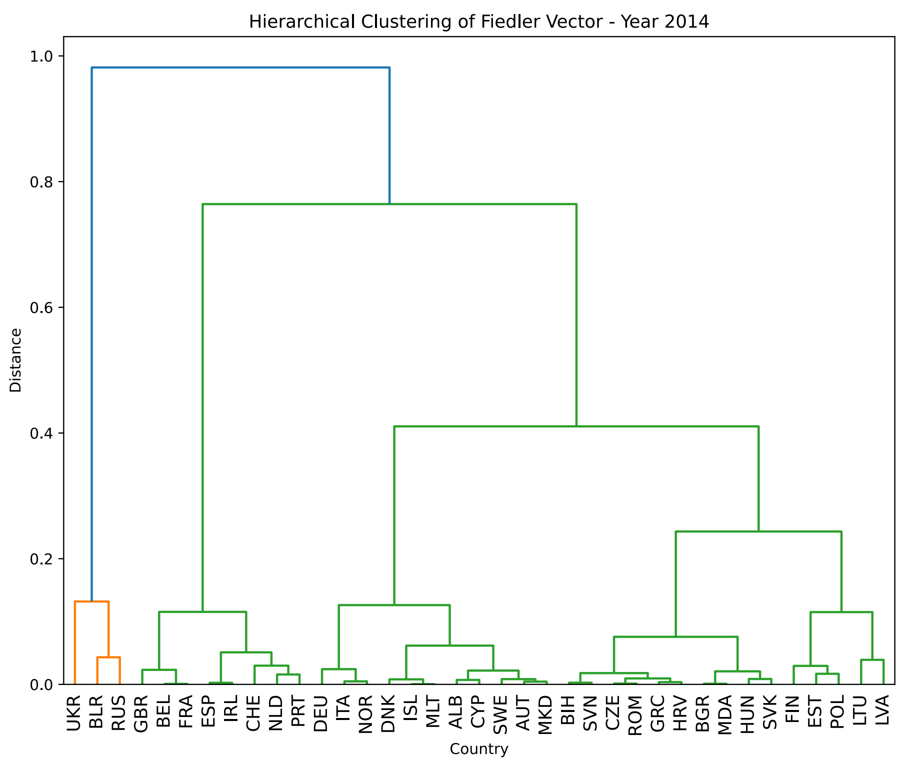

Figure 13: Hierarchical Clustering using L_{sym} of the intra-European trade graph. 2014.

Hierarchical clustering constructs a tree-like hierarchy of data points by iteratively merging or splitting clusters based on a chosen distance metric. We run hierarchical clustering on v^t using Ward's method to compute the distance between clusters (Ward, 1963). The results of hiearchical clustering for the Intra-European trade graph are shown for the years 1950, 1991, 1992, and 2014 in Figure 10, Figure 11, Figure 12, and Figure 13 respectively. In interpreting these partitions, the main metric is the distance between the final linkage (where the two partitions connect at the top of the graph) and the last sub-linkage within one of the two partitions. The greater this distance, the greater the separation between the partitions. Maps based on the partitions are displayed for 1991 in Figure 23 and 1992 in Figure 24.

Looking at the hierarchical clustering for 1950, we see the Eastern Bloc countries of Czechoslovakia, the USSR, Romania, Hungary, and Poland split off into their own sub-community separate from the rest of European trade. This same Eastern Bloc cluster still exists in 1991, and is joined by Yugoslavia and Austria, both of which were on the other side of the partition in 1950. This may be a result of Austria's successful trade relationship with the USSR and satellite states, following Austria's independence from occupation in 1955 (Sorokin, 2022). While the partition between 1950 and 1991 remained similar, the distance between the last sub-linkage and the final linkage was shorter than in 1950, indicating that the two trade communities were more connected in 1991 than 1992.

With the dissolution of the USSR in December 1991, the resulting partition of intra-European trade is quite different for 1992. The breakup of the USSR adds Belarus, Moldova, Russia, Ukraine, and the Baltic states to the network. The resulting partition isolates Belarus, Russia, and Ukraine into a community separate from the rest of Europe. However, the distance between the final linkage and the last sub-linkage is much smaller than in 1991, indicating that overall, following the collapse of the Eastern Block, the trade network became less modular. It is interesting to note that in 1992, for all the countries previously in the Eastern Bloc, apart from the aforementioned three, these countries formed a new sub-community on the other side of the partition, as indicated by the distance between the penultimate linkage and the one below it on the graph. This is further evidenced by no significant change in the third smallest eigenvalue of L_{sym}^t, λ_3^t between 1991 and 1992, which suggests that the ability to divide the graph into three clusters remains similar despite the dissolution of the USSR. In 2014, Belarus, Russia, and Ukraine are still in their own community. However, the distance before the final linkage is again smaller than it was in 1992, indicating a further decrease in the network's modularity. The sub-community of the other Eastern Bloc countries that was visible in 1992 is no longer present in 2014. In further work, the temporal evolution of sub-communities like these could be analyzed using a higher frequency of Graph Laplacian's eigenvalues.

5 Concluding Remarks

This study provides an analysis of the structural evolution of the international trade network from 1950 to 2014, focusing on centrality and modularity as key metrics to understand the network's dynamics. Shifts in each country's weighted PageRank reveal a re-centering of trade centrality as many European powers saw a declining centrality which coincided with a rapid rise in China's centrality in the network. This redistribution of influence reflects broader economic and geopolitical trends, including industrialization in emerging markets and changing trade alliances.

Having constructed a metric for graph modularity using spectral graph theory, we illustrated the persistent clustering of trade relationships into regional blocs, even as globalization deepened. However, in the case of the intra-European trade network, the dissolution of the USSR marked a significant decrease in network modularity as trade became far more interconnected across Europe. This outcome is contrary to what our constructed null models suggested—where the "fracturing" of the USSR node would have led to further graph modularity across most randomizations of the network. Such a result indicates the conscious development of trade relations between Western Europe and the former Soviet Bloc resulted in an increasingly interconnected Europe; as the new topology of the intra-European trade network did not guarantee such an outcome.

A methodological contribution of this paper is the introduction of a normalized symmetric Laplacian-based framework to measure and compare trade network modularity across different network sizes. Furthermore, by analyzing the full spectrum of eigenvalues, the connectivity within identified trade communities, and the extent to which trade blocs maintain internal cohesion and external independence can be measured.

Looking forward, this research provides a foundation for several avenues of future study. A natural extension would involve applying these methodologies to more granular data, such as industry- or commodity-specific trade flows. For instance, analyzing modularity in the automobile or semiconductor industries could measure how specific trade agreements, such as NAFTA or the US-China trade disputes, impact the structure and cohesion of production and trade networks in those sectors. Additionally, incorporating temporal or multilayer network models could allow for a deeper exploration of how global shocks—such as economic crises, technological innovations, or environmental policies—affect the trade network's evolution.

Using a combination of approaches from network science and spectral graph theory, this analysis maps the evolution of international trade since the end of World War II, where we observe a dynamic interplay between regionalization and globalization. Furthermore, the methodologies discussed in this analysis provide a framework for exploring future questions surrounding trade modularity, those that might be of particular relevance for policymakers attempting to understand the impact of bilateral policy on the modularity and resilience of the entire trade network.

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6 Appendix

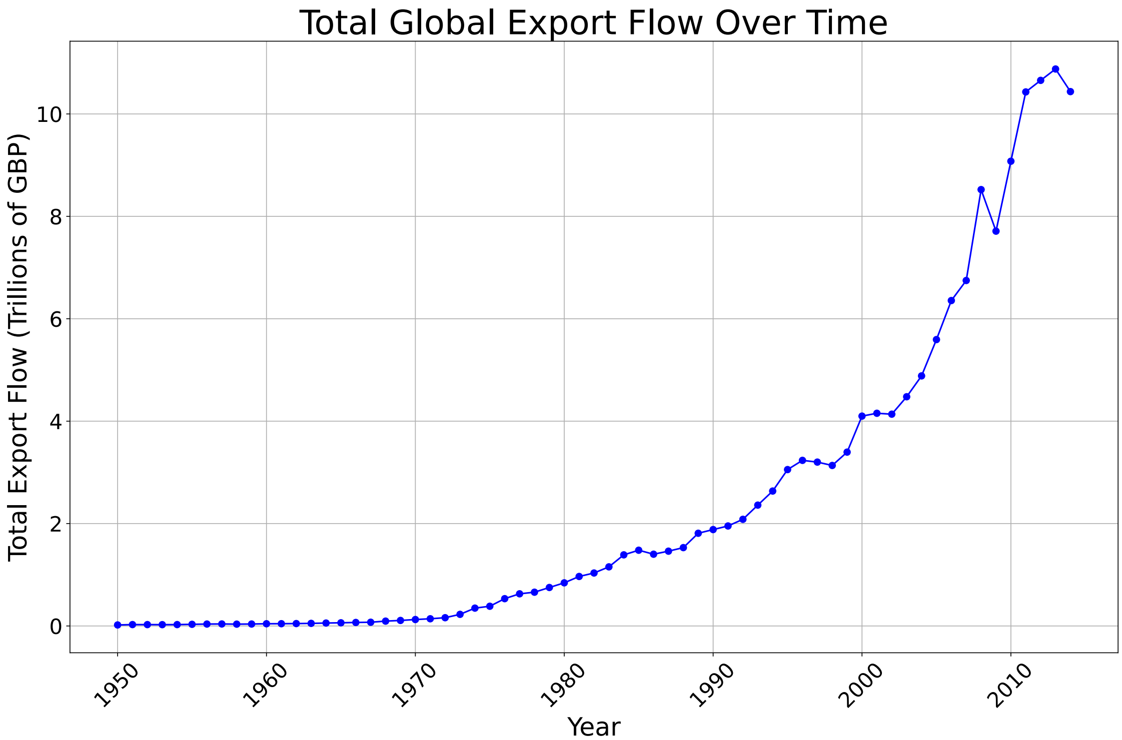

Figure 14: Total Export Flow by Year in Trillions of GBP

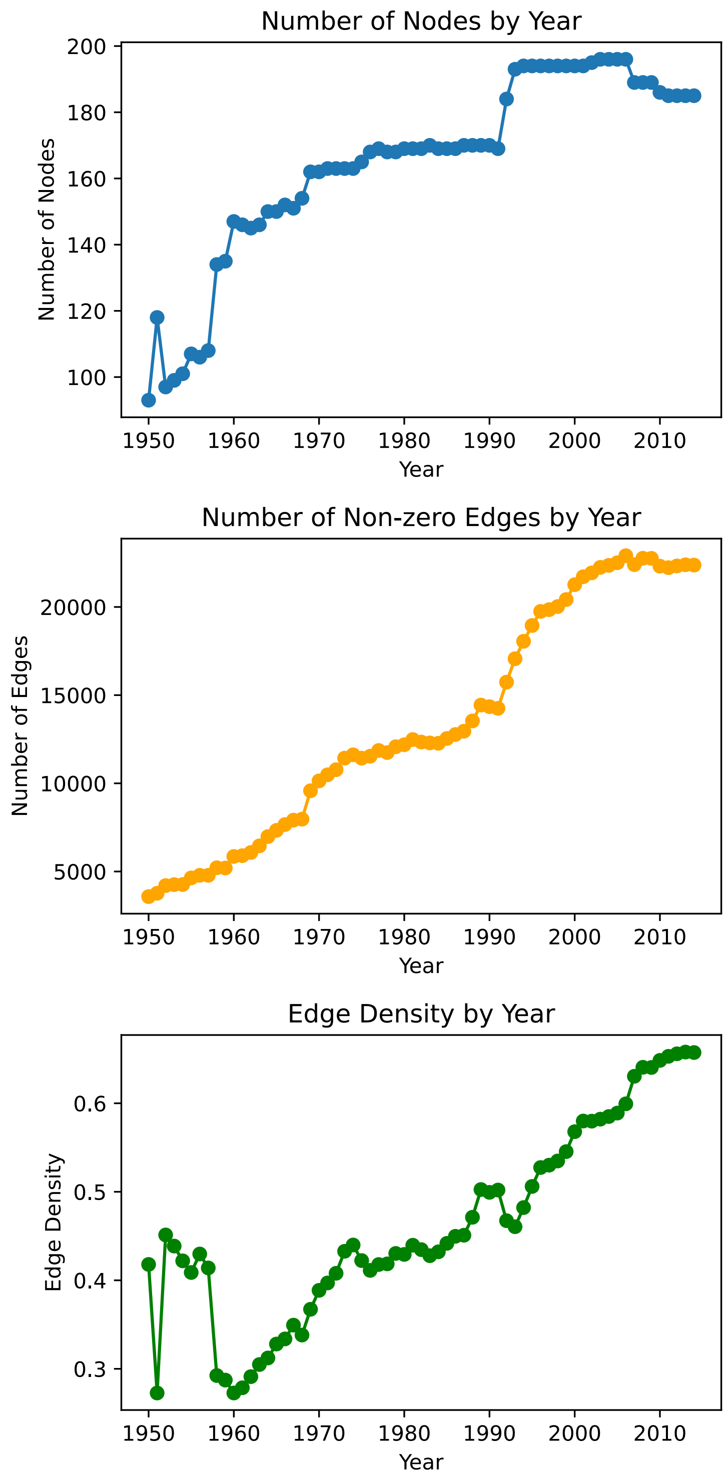

Figure 15: Summary Stats of A^t for years 1950-2014. Edge density is defined as $\frac{edges}{N*(N-1)}$. (Bhattacharya, 2007) used to select relevant summary statistics.

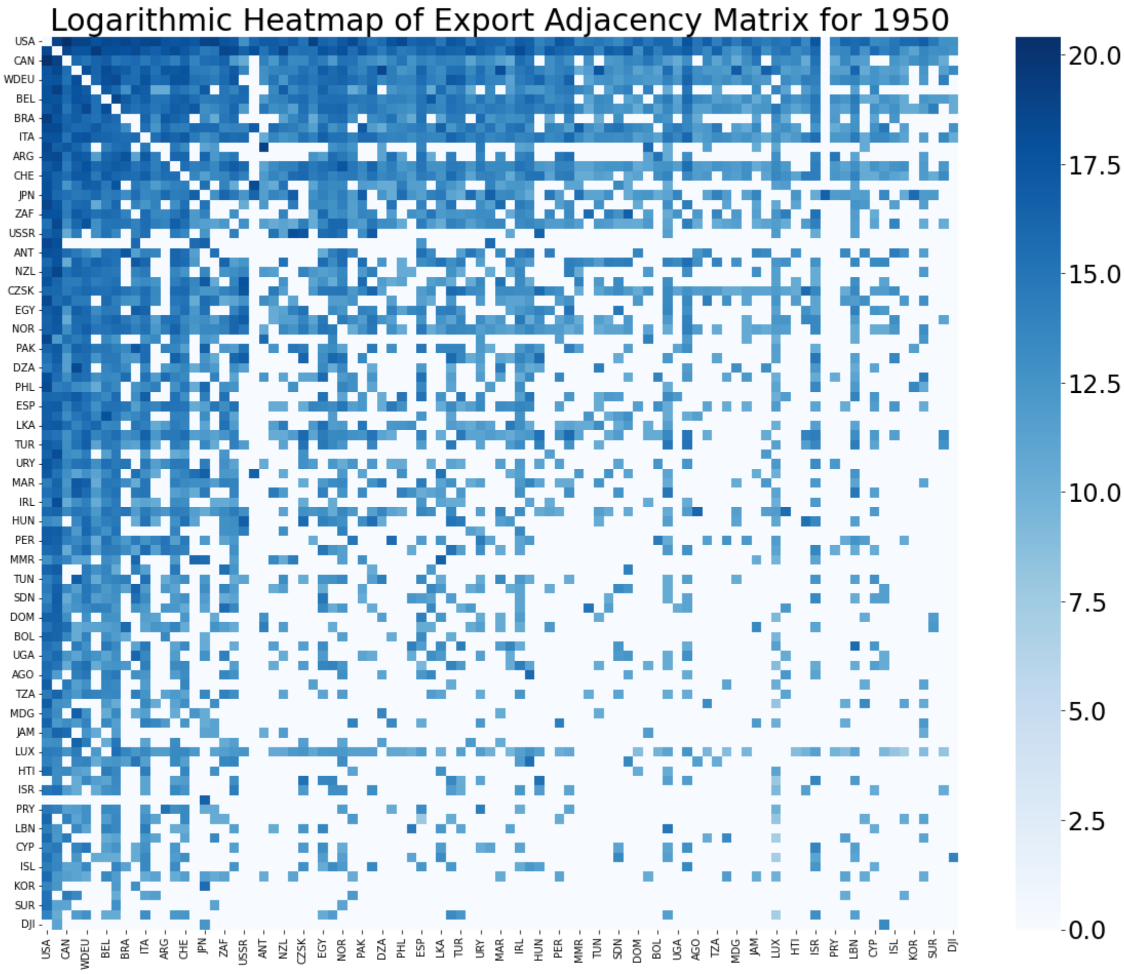

Figure 16: Logarithmic heat-map of A^t for year 1950. Rows and columns are ordered by the sum of a country's total exports to all countries.

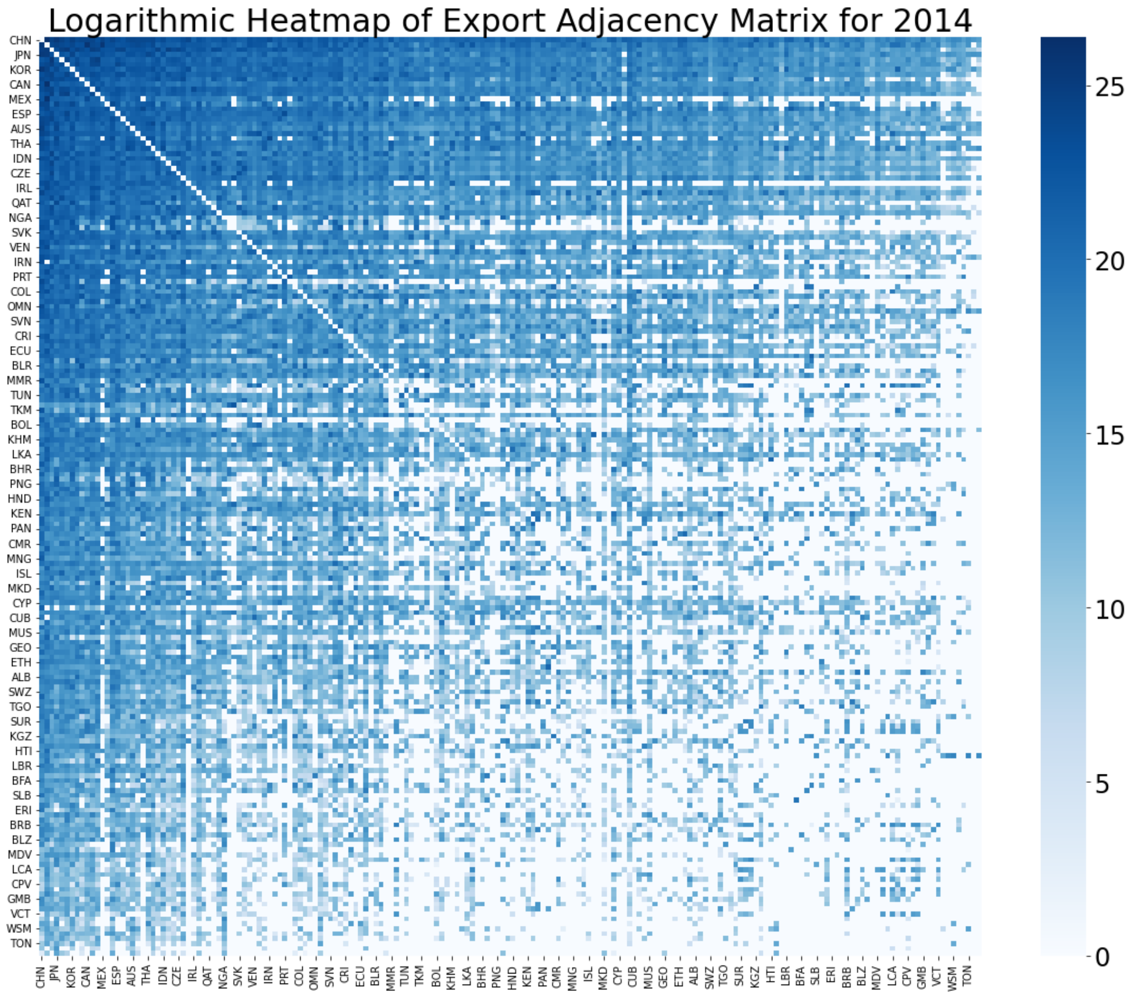

Figure 17: Logarithmic heat-map of A^t for year 2014. Rows and columns are ordered by the sum of a country's total exports to all countries.

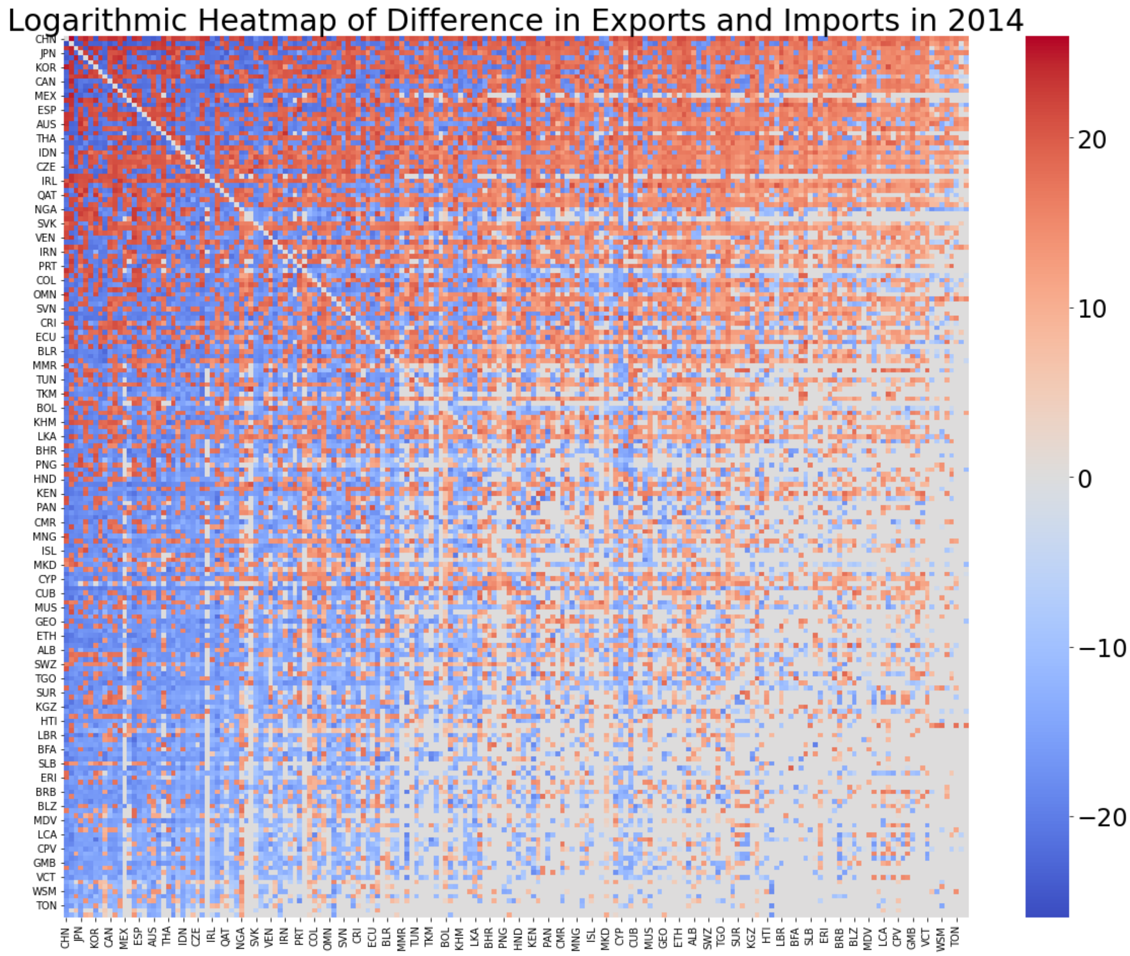

Figure 18: Logarithmic difference of exports and imports for the year 2014. Positive values represent trade surplus from country i to country j.

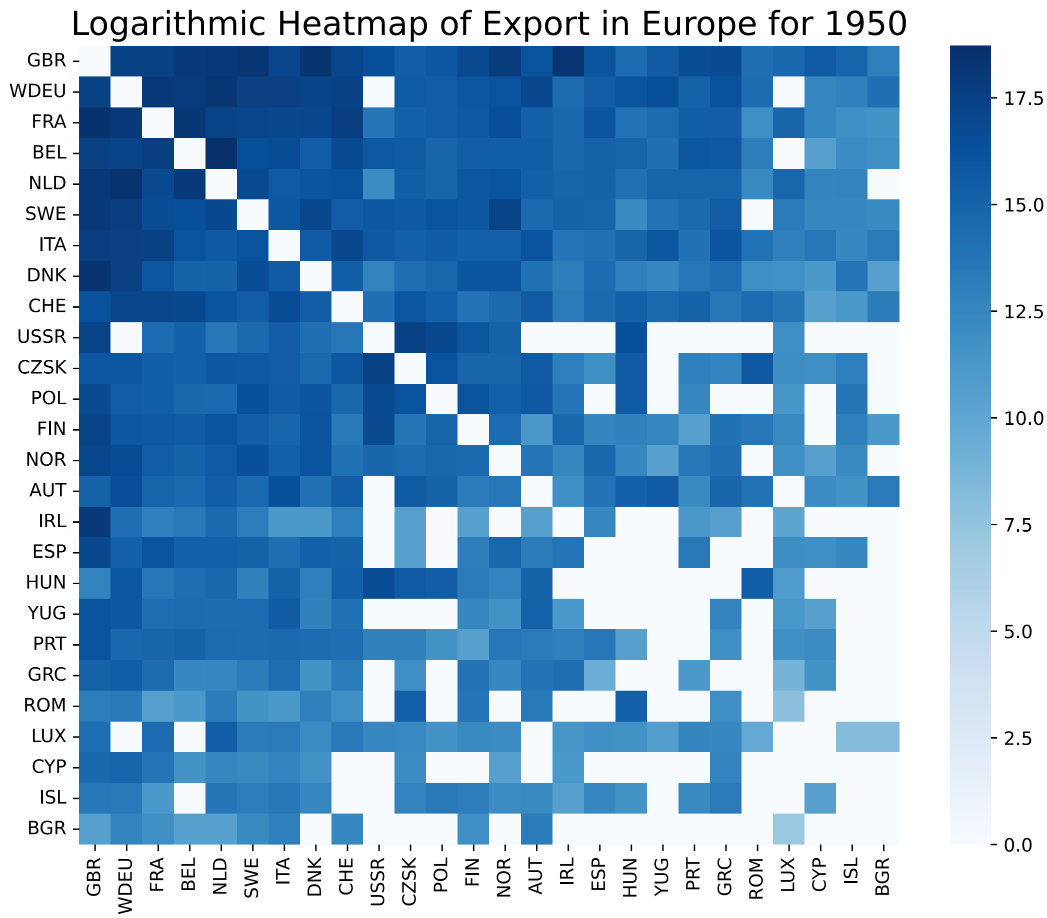

Figure 19: Logarithmic heat-map of intra-Europe trade for year 1950. Rows and columns are ordered by the sum of a country's total exports to all European countries.

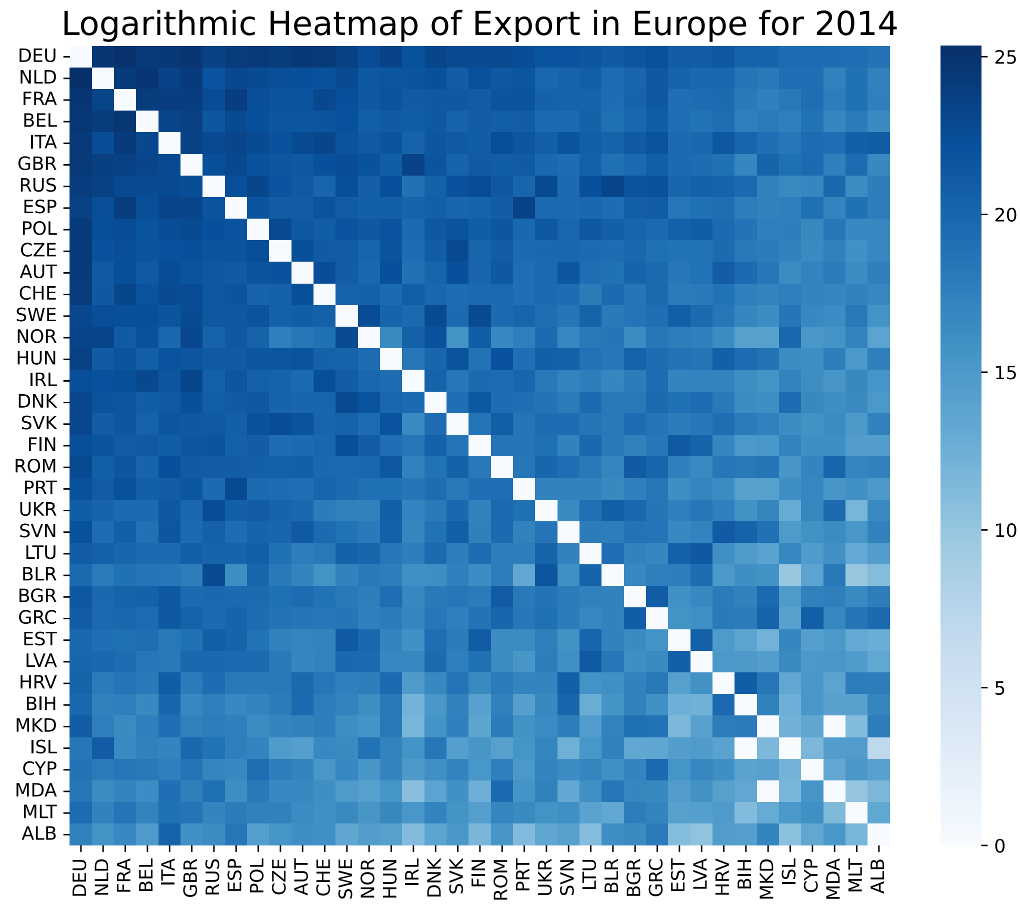

Figure 20: Logarithmic heat-map of intra-Europe trade for year 2014. Rows and columns are ordered by the sum of a country's total exports to all European countries.

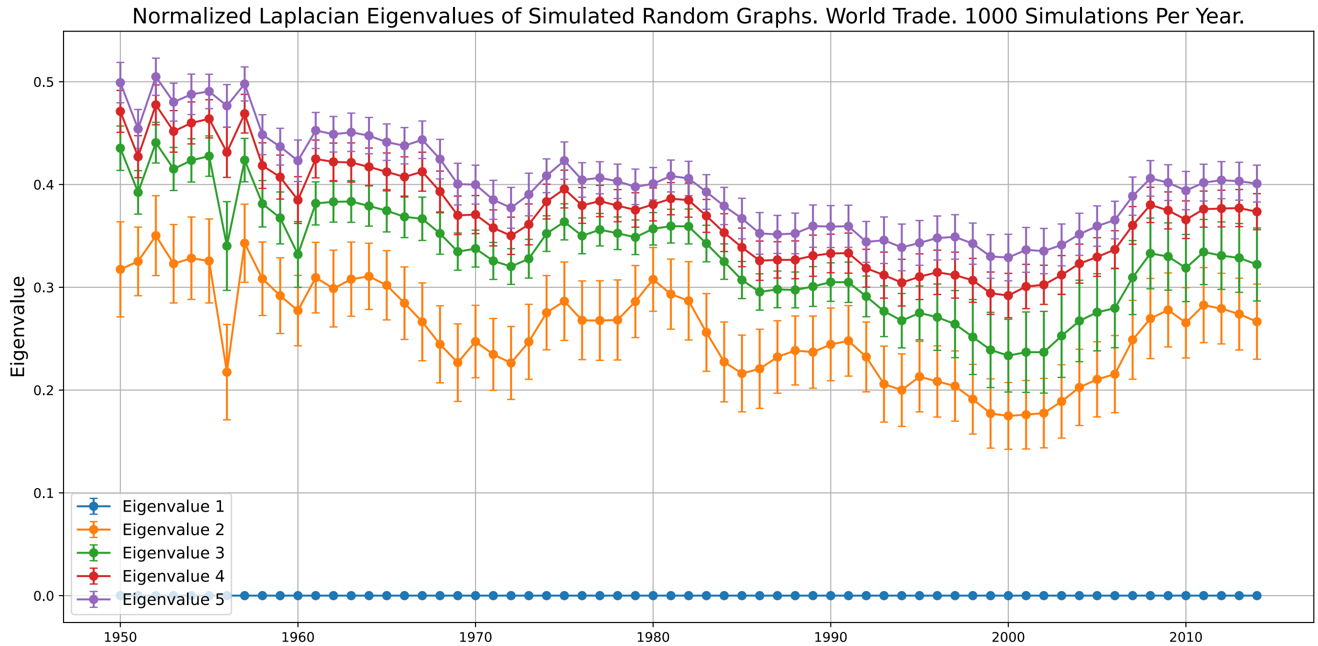

Figure 21: World Trade Network: Mean value of five smallest eigenvalues of the symmetric normalized Laplacian for 1000 replications of the random graph $\mathcal{G}$ with parameter values from the full trade network.

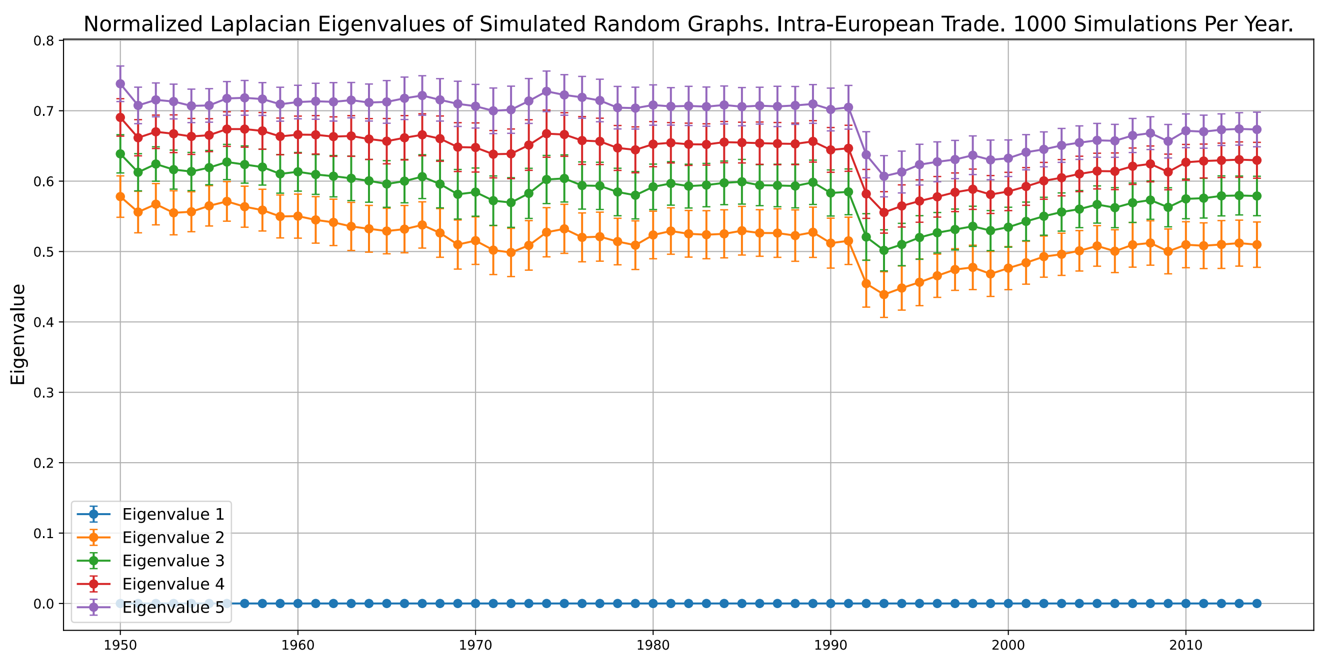

Figure 22: Intra-European Trade Network: Mean value of five smallest eigenvalues of the symmetric normalized Laplacian for 1000 replications of the random graph $\mathcal{G}$ with parameter values from the intra-European trade network.

Clustered Network of Intra-European Trade in 1991 (Two Largest Export Destinations).

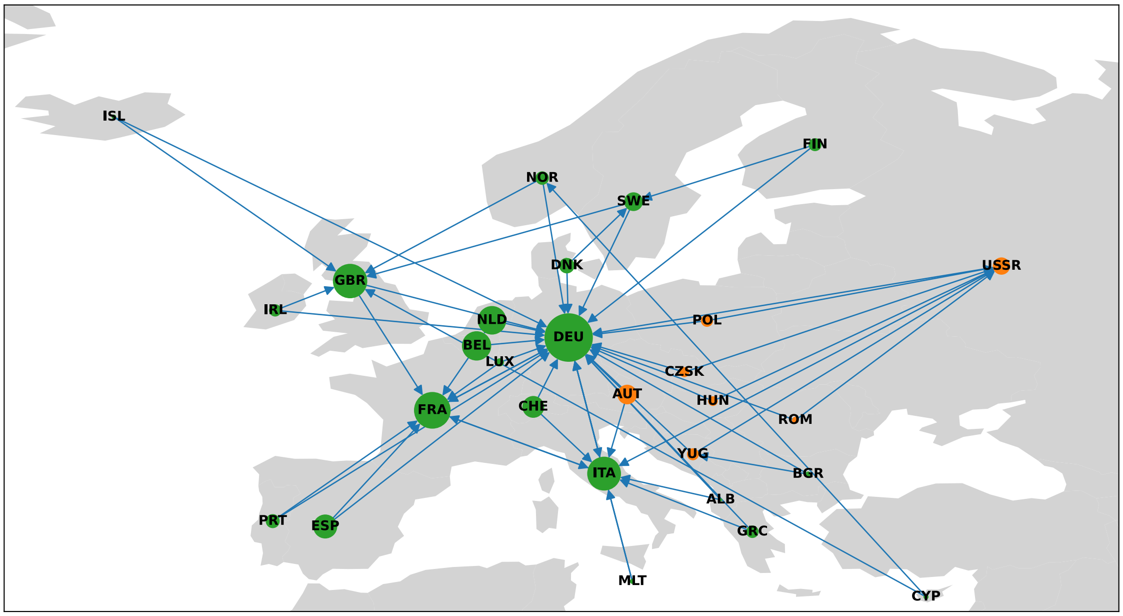

Figure 23: Directed edges are drawn for each country's two largest export destinations. Country sizes are based on their in-degree, colors based on the first hierarchical clustering partition.

Clustered Network of Intra-European Trade in 1992 (Two Largest Export Destinations).

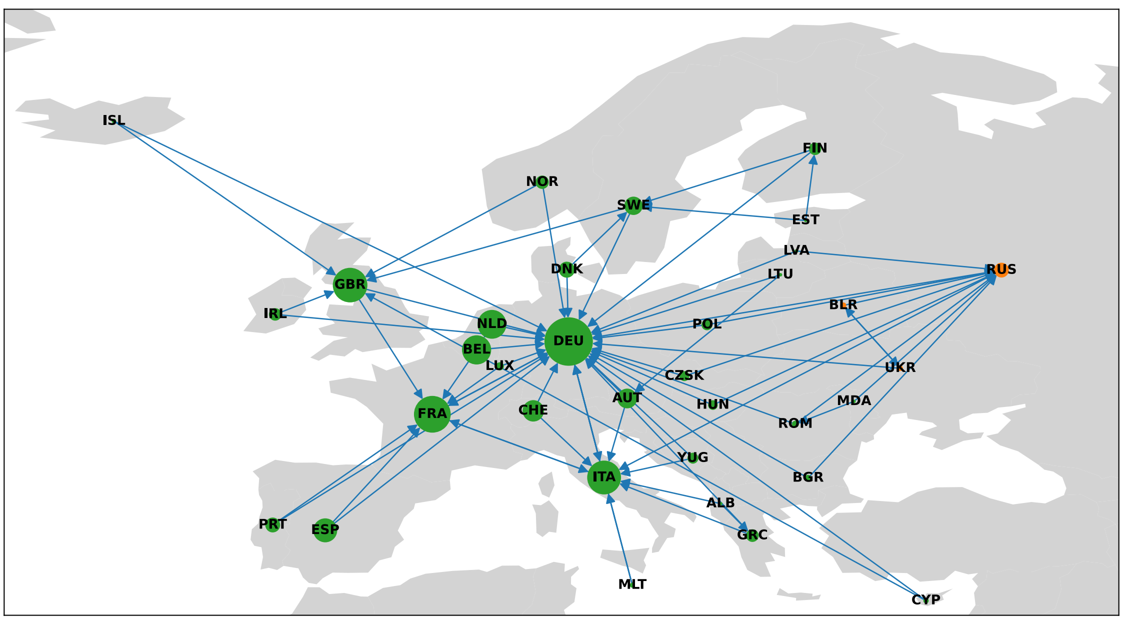

Figure 24: Directed edges are drawn for each country's two largest export destinations. Country sizes are based on their in-degree, colors based on the first hierarchical clustering partition.