PHYS 260 Electromagnetism

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Chapters 21-22

1 Electrostatics Basics

1.1 Introduction

1. As you may know, matter is made of atoms. Atoms have a nucleus, which is made up of protons (positive) and neutrons (neutral), and electrons (negative). Opposite charges are attracted to one another with a inverse square force, and like charges are similarly repelled.
2. The electrons[1] in conductors are able to move freely, while in insulators they cannot unless they come off entirely
3. Some materials tend to give off electrons (e.g. fur) and some tend to attract them (e.g. plastic)
4. Not all metals are conductors
5. A neutral object can be attracted to a charged object via inducted polarity. In a conductor, this means the electrons move through the entire object to create a dipole. In an insulator, the electrons of the atoms realign to create billions of of tiny dipoles. This makes a difference due to the inverse square law, which will be discussed further later. The former creates a strong attraction, while the latter creates a weak attraction.

1.2 Electrical Grounding

An object can be "grounded" by connecting it to a big conductor, which serves as a resevoir for charge, e.g. you can ground the wall outlet by sticking a fork into it. The gruond in this case is you body and the ground. The electricity from the wall will flow through your body and into the ground

[1] Not *all* of the electrons in a conductor can move, it's just that the ones that don't move can be ignored because they do not contribute to the net charge of a material

2 Coulomb's Law

The magnitude of the force between two point charges q_1 and q_2 C, with distance r m is equal to the following

$$F = \frac{1}{4\pi\epsilon_0}\frac{|q_1 \cdot q_2|}{r^2}$$

- where ϵ_0 is vacuum permittivity, the value of the absolute dielectric permittivity of classical vacuum, or also just "the electric constant".
- $\epsilon_0 \approx 8.854189 \cdot 10^{-12}$
- the Coulomb constant $k = \frac{1}{4\pi\epsilon_0}$
- electric forces add as vectors, which is called the "Superposition Principle"

3 Electric Fields

- Electric Fields are best represented by vector fields
- For the electric field of a point charge $\vec{F} = q\vec{E} \rightarrow E = \frac{F}{q} \rightarrow E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2} = \frac{kq}{r^2}\ \frac{N}{C}$
- Field lines do not indicate the trajectory of a test charge, but rather are lines tangent to the electric field at a given point

3.1 Electric Field Integrations

As it turns out, it is unrealistic to use Coulomb's Law just by itself to calculate the electric field at a point. This would require knowing the location and charge of every charge in an object. One way this problem can be simplified is with the use of charge density and calculus.

- Linear density: $\lambda = \frac{Q}{L}$
- Area density: $\sigma = \frac{Q}{A}$
- Volumetric density: $\rho = \frac{Q}{V}$

Basically, what you do is find the ds/dA/dV and you multiply it times the charge density to get the dQ. You then usually use the point charge electric field formula and integrate it over the relevant domain. When the differential is a point charge, you usually need to make sure you aren't summing magnitudes. However, if the differential you are given is an area, this has probably already been taken care of.

Infinite Sheet of Charge

$$|\vec{E}| = \frac{\sigma}{2\epsilon_0}$$

[2]**Near the Surface of a Conductor**

$$|\vec{E}| = \frac{\sigma}{\epsilon_0}$$

4 Gauss' Law

$$\Phi = \oint \vec{E} \cdot d\vec{A} = \oint \vec{E} \cdot \hat{n}\ dA = \frac{Q_{net}}{\epsilon_0}$$

The above equation is Gauss' Law, which states that the flux through a surface is a measure of how orthogonally an electric field passes through a surface. Positive flux means that the passing it outward, while negative flux means that it is inward (overall). Finding flux is useful because it is proportional to the net charge of within the surface of the integral. The surface used in the integral typically is not a real surface, but a contrived one, used for determining the net charge of what it encloses. It is called a Gaussian Surface and is usually a rectangular prism, cylinder, or sphere, depending on what is most convienient.

Furthermore, if the electric field is uniform and the angle between the field and the surface is constant, then the following applies.

$$\Phi = \vec{E} \cdot \vec{A} = EAcos(\theta) = \frac{Q_{net}}{\epsilon_0}$$

where θ is the angle between the electric field and the normal vector $\hat{n}$

4.1 Flux

Based on the previous simplified equation, it becomes obvious that the units of flux are $\frac{N}{C} \cdot m^2$. If you think about the electric field as the flowing of water, in a river perhaps, then flux is like a measure of that flow times an area, i.e. a rate of volume. Obviously, an electric field is, in fact, not flowing water, but this analogy is useful.

[2]The field due to *everything*

5 Dipoles

$\vec{p} = q\vec{d}$
$\vec{\tau} = \vec{p} \times \vec{E}$
$|\vec{\tau}| = pEsin(\theta)$
$W = \int_{\phi_1}^{\phi_2} (-pEsin(\phi))d\phi = pE(cos(\phi_2) - cos(\phi_1))$
$U(\phi) = -\vec{p} \cdot \vec{E} = -pEcos(\phi)$

Chapters 23-25

6 Electric Potential Energy

Recall: Work done by a conservative force is equal to the following

$$W_{a\to b} = -\Delta U$$

Recall: Electrostatic force

$$F_e = qE$$

Therefore, in a uniform electric field between parallel plates

$$U = (qE)y$$

$$W_{a\to b} = F_e d = (qE)\delta y$$

For a point charge

$$F = \frac{1}{4\pi\epsilon_0}\frac{qq_0}{r^2}$$

$$U = \frac{1}{4\pi\epsilon_0}\frac{qq_0}{r}$$

Superposition applies

$$U = \frac{q_0}{4\pi\epsilon_0}\Sigma_i \frac{q_i}{r_i}$$

7 Electric Potential

Potential is just potential energy per unit charge

$$V = \frac{U}{q_0}$$

$$U = q_0 V$$

The units are volts (V, $\frac{J}{C}$) The potential difference ΔV is also called voltage. It is equal to the work in joules that must be done to move a 1 couloumb charge from a to b against the electric force.

$$\frac{W_{a\to b}}{q_0} = \frac{-\Delta U}{q_0} = -\Delta V$$

Usually since a and b are arbitrary, the voltage is usually given as an absolute value
A 9V battery has a potential difference or voltage of 9V which means that the difference between the positive and negative terminals is 9V
The electric potential due to a point charge:

$$V = \frac{1}{4\pi\epsilon_0}\frac{q}{r} = \frac{U}{q_0}$$

Accordingly:

$$V = \frac{1}{4\pi\epsilon_0}\Sigma_i \frac{q_i}{r_i}$$

$$V = \frac{1}{4\pi\epsilon_0}\int \frac{dq}{r}$$

The electric potential due to a field $\vec{E}$

$$W_{a\to b} = \int_a^b \vec{F}\cdot d\vec{l} = \int_a^b q_0\vec{E}\cdot d\vec{l}$$

Divide by q_0

$$-\Delta V = \int_a^b \vec{E}\cdot d\vec{l} = \int_a^b Ecos\phi\ dl$$

ito $\vec{E}$ we get:

$$\vec{E} = -\nabla V$$

Think of just

$$-\Delta V = \int_a^b E_x dx$$

$$-\int_a^b dV = \int_a^b E_x dx$$

$$-dV = E_x dx$$

$$-\frac{dV}{dx} = E_x$$

If you do for y, z then you get negative gradient

8 Equipotential Surfaces

Equipotential surfaces for electric fields can be entirely generalized to the idea of a topographical map.

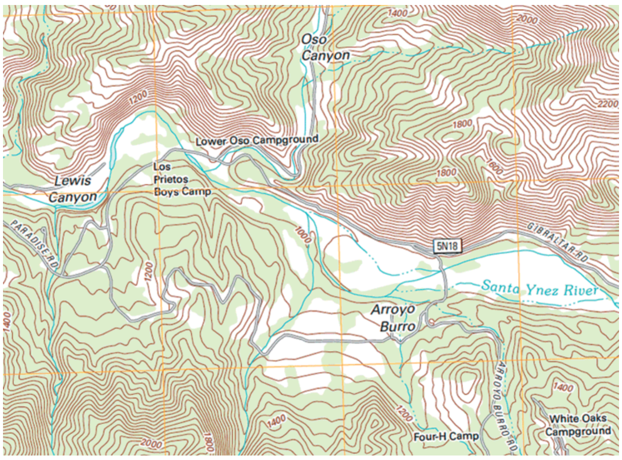


Figure 1: *Topographical map*

Because of $\vec{E} = -\nabla V$, given that the topographical lines are like potential lines, the field lines are perpindicular and in the downhill direction. These could be imagined as the lines of errosion.

9 Capacitors and Capacitance

A **capacitor** is any two conductors separated by an insulator (or vaccum[3]). It is usually used to store energy in the electric field between the plates. We basically just assume that all capacitors are parallel plate capacitors for simplicity. A parallel plate capacitor has oppositely charged plates.

$$|\vec{E}_{capacitor}| = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A}\ ; |d << L$$

Capacitance is the capability of a material object or device to store electric charge. It is based on the geometry of the capacitor. It's units are Farads (F, $\frac{C}{V}$)

$$C = \frac{Q}{\Delta V} = \frac{\epsilon_0 A}{d}$$

10 Capacitors in Series and Parallel

Capacitors in a circuit can be combined into an equivalent capacitor in order to simplify problems.

10.1 Capacitors in Series

$$C_{equivalent} = C_1 + C_2 + ...$$

10.2 Capacitors in Parallel

$$\frac{1}{C_{equivalent}} = \frac{1}{C_1} + \frac{1}{C_2} + ...$$

11 Energy in a Capacitor

Energy in Capacitor

$$U_{capacitor} = \frac{1}{2}Q\Delta V = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2$$

Energy Density in Capacitor

$$u = \frac{U_{capacitor}}{Volume} = \frac{\epsilon_0 E^2}{2}$$

12 Dielectrics

When you put an insulator between the plates of a parallel plate capacitor.

The capacitance for a capacitor with a dielectric with dielectric constant k:

$$C = \frac{k\epsilon_0 A}{d}$$

Having a dielectric lowers the field within the capacitor and it increases the capacitance.

[3] i.e. not a conductor

13 Current

Current is the rate at which charge passes through a given cross-sectional area. It "flows" in the opposite direction of the actual flow of electrons in the circuit.

It's units are Amperes (A, $\frac{C}{s}$)

$$I = \frac{dQ}{dt}$$

13.1 Current Density

Current density is the current per unit area. In terms of I, it is:

$$I = \int_{Area} \vec{J} \cdot d\vec{A}$$

If J is constant and parallel to $d\vec{A}$, then:

$$I = JA$$

14 Drift Velocity

When electrons flow through a wire, they don't go directly parallel to the direction of the wire. Instead, they randomly bounce around within the wire. If a circuit is completed, there will be a field created and the electrons will start to overall drift at a certain velocity. Even though their actual velocity is very high (avg 10^6 m/s), the drift velocity is very slow (avg 10^{-4} m/s).

$$\vec{v}_{drift} = \frac{\vec{J}}{nq}$$

if uniform J, then

$$\vec{v}_{drift} = \frac{I}{nqA}$$

15 Resistivity

$\vec{J}$ always depends on $\vec{E}$ and the properties of the material. However, in certain materials at particular tempuratures, the relationship is simply proportional. This is called **Ohm's Law**. It is an idealized model like Hooke's Law.

Resisitivity is the ratio of the electric field to the current density. It's units are $\frac{\frac{V}{m}}{\frac{A}{m^2}} = \Omega \cdot m$

$$\rho = \frac{E}{J}$$

Furthermore, the reciprocal of resistivity is conductivity $(\Omega \cdot m)^{-1}$. So you can think of resistivity as the opposite of conductivity.

	Substance	ρ ($\Omega \cdot$ m)	Substance	ρ ($\Omega \cdot$ m)
Conductors			**Semiconductors**	
Metals	Silver	1.47×10^{-8}	Pure carbon (graphite)	3.5×10^{-5}
	Copper	1.72×10^{-8}	Pure germanium	0.60
	Gold	2.44×10^{-8}	Pure silicon	2300
	Aluminum	2.75×10^{-8}	**Insulators**	
	Tungsten	5.25×10^{-8}	Amber	5×10^{14}
	Steel	20×10^{-8}	Glass	10^{10}–10^{14}
	Lead	22×10^{-8}	Lucite	$>10^{13}$
	Mercury	95×10^{-8}	Mica	10^{11}–10^{15}
Alloys	Manganin (Cu 84%, Mn 12%, Ni 4%)	44×10^{-8}	Quartz (fused)	75×10^{16}
	Constantan (Cu 60%, Ni 40%)	49×10^{-8}	Sulfur	10^{15}
	Nichrome	100×10^{-8}	Teflon	$>10^{13}$
			Wood	10^{8}–10^{11}

Figure 2: *Resistivities at 20° C*

16 Resistors

$$\vec{E} = \rho\vec{J}$$

$$R = \frac{\rho L}{A_X}$$

$$\Delta V = IR$$

17 Circuits and EMF

The symbol for EMF is $\mathcal{E}$

$$\Delta V_{Terminal} = \mathcal{E} - Ir_{int}$$

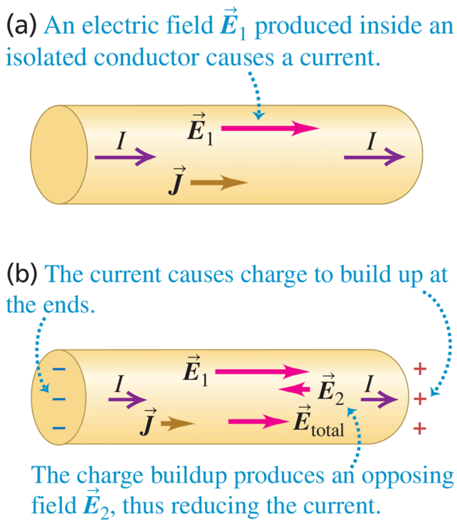
Accordingly, where R_{eq} is the equivalent resistance of all of the resistors in the ciruit besides r_{int}:

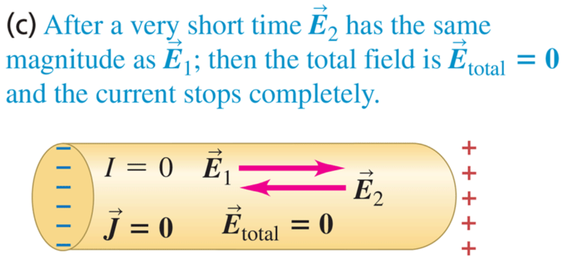
$$\Delta V_{Terminal} - IR_{eq} = 0$$

$$\mathcal{E} - Ir_{int} - IR_{eq} = 0$$

$$I = \frac{\mathcal{E}}{(r_{int} + R_{eq})}$$

Explanation for why $\Sigma\vec{E} = \vec{0}$ for a conductor with an open loop, but when the loop is closed, then it can have a net electric field:





18 Energy & Power in Circuits

Power dissipated by a resistor:

$$P_{res} = I_{res}\Delta V_{res} = I_{res}^2 R_{res} = \frac{\Delta V_{res}^2}{R_{res}}$$

19 Resistors in Series and Parallel

Many resistors in a circuit can be represented by one equivalent resistor.

19.1 Resistors in Series

$$R_{eq} = R_0 + R_1 + R_2 + ...$$

19.2 Resistors in Parallel

$$\frac{1}{R_{eq}} = \frac{1}{R_0} + \frac{1}{R_1} + \frac{1}{R_2} + ...$$

So basically:

$$R_{eq} = \frac{1}{\frac{1}{R_0} + \frac{1}{R_1} + \frac{1}{R_2} + ...}$$

20 Kirchhoff's Rules

20.1 Junction Rule

$$\Sigma I = 0$$

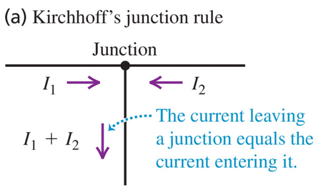


Figure 3: *Current that goes into a junction must leave the junction*

20.2 Loop Rule

$$\Sigma\Delta V = 0$$

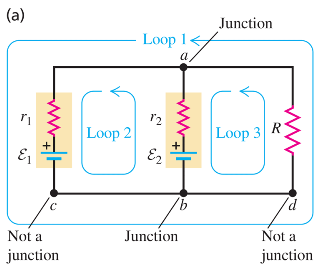


Figure 4: *The sum of the charge in voltage across a circuit loop will always be zero. So for loop 2 (let's change the rotation direction) we get this equation:* $\Sigma\Delta V = 0 = \mathcal{E}_1 - Ir_1 - Ir_2 - \mathcal{E}_2$

21 Electrical Measuring Instruments

21.1 Voltmeter

Voltmeter reads the difference in potential (voltage) between its probes. It has a really high resistance, so you usually assume that no current can flow through it. Accordingly, you put it in parallel with circuit elements.

21.2 Ammeter

Ammeter reads current and usually has a neglibly small resistance. Accordingly, you put it in series with circuit elements.

22 R-C Circuits

An RC circuit is a circuit like the one below.

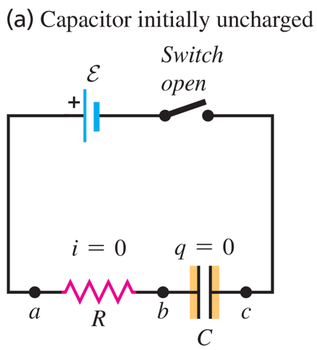


Figure 5: *RC circuit with the circuit open*

22.1 Time Constant

In the following equations, the time constant is denoted by τ and is equal to RC. The time constant determines how quickly the capacitor charges or discharges and how quickly the current decreases

$$\tau = RC$$

22.2 Charging

We use lowercase letters for changing variables.

$$q = C\mathcal{E}(1 - e^{\frac{-t}{RC}})$$

Where $C\mathcal{E}$ is the final charge when $t \to \infty$

$$i = \frac{dq}{dt} = \frac{\mathcal{E}}{R} e^{\frac{-t}{RC}}$$

Where $\frac{\mathcal{E}}{R}$ is the initial charge when $t = 0$

22.3 Discharging

$$q = Q_0 e^{\frac{-t}{RC}}$$
$$i = \frac{dq}{dt} = \frac{-Q_0}{RC} e^{\frac{-t}{RC}}$$

Where $\frac{-Q_0}{RC}$ is the initial charge when $t = 0$

23 Unit 2 Conceptual Questions

Question 1
Show Answer