Student: Miles Kent **Date:** 11/19/24	**Instructor:** Salvador Barone **Course:** Math 1554	**Assignment:** 7.3: Constrained Optimization

1. Find the change of variable $\mathbf{x} = P\mathbf{y}$ that transforms the quadratic form $\mathbf{x}^T A\mathbf{x}$ into $\mathbf{y}^T D\mathbf{y}$ as shown.

$$20x_1^2 + 21x_2^2 + 22x_3^2 + 24x_1x_2 - 24x_2x_3 = 38y_1^2 + 21y_2^2 + 4y_3^2$$

$$P = \begin{bmatrix} -\frac{8}{17} & \frac{12}{17} & -\frac{9}{17} \\ \frac{-12}{17} & \frac{1}{17} & \frac{12}{17} \\ \frac{9}{17} & \frac{12}{17} & \frac{8}{17} \end{bmatrix}$$

(Type an exact answer, using radicals as needed.)

2. Find (a) the maximum value of $Q(\mathbf{x})$ subject to the constraint $\mathbf{x}^T\mathbf{x} = 1$, (b) a unit vector $\mathbf{u}$ where this maximum is attained, and (c) the maximum of $Q(\mathbf{x})$ subject to the constraints $\mathbf{x}^T\mathbf{x} = 1$ and $\mathbf{x}^T\mathbf{u} = 0$.

$$Q(\mathbf{x}) = 9x_1^2 + 4x_2^2 + 4x_3^2 + 2x_1x_2 + 2x_1x_3 + 8x_2x_3$$

(a) The maximum value of $Q(\mathbf{x})$ subject to the constraint $\mathbf{x}^T\mathbf{x} = 1$ is 10.

(b) A unit vector $\mathbf{u}$ where this maximum is attained is $\mathbf{u} = \begin{bmatrix} \frac{\sqrt{6}}{3} \\ \frac{\sqrt{6}}{6} \\ \frac{\sqrt{6}}{6} \end{bmatrix}$.

(Type an exact answer, using radicals as needed.)

(c) The maximum of $Q(\mathbf{x})$ subject to the constraints $\mathbf{x}^T\mathbf{x} = 1$ and $\mathbf{x}^T\mathbf{u} = 0$ is 7.

3. Use $Q(\mathbf{x}) = 8x_1^2 + 8x_2^2 - 6x_1x_2$ to answer parts (**a**) through (**c**).

(**a**) Find the maximum value of $Q(\mathbf{x})$ subject to the constraint $\mathbf{x}^T\mathbf{x} = 1$.

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(**b**) Find a unit vector $\mathbf{u}$ where this maximum is attained.

○ **A.** $\begin{bmatrix} \frac{1}{\sqrt{3}} \\ -\frac{\sqrt{6}}{3} \end{bmatrix}$

◉ **B.** $\begin{bmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{bmatrix}$

○ **C.** $\begin{bmatrix} \frac{1}{4} \\ -\frac{\sqrt{15}}{4} \end{bmatrix}$

○ **D.** $\begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{2} \end{bmatrix}$

(**c**) Find the maximum of $Q(\mathbf{x})$ subject to constraints $\mathbf{x}^T\mathbf{x} = 1$ and $\mathbf{x}^T\mathbf{u} = 0$.

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4. Find the maximum value of the equation below subject to the constraint $x_1^2 + x_2^2 = 1$.

$$Q(\mathbf{x}) = 8x_1^2 + 5x_2^2 - 2x_1x_2$$

The constrained maximum value is $\boxed{\dfrac{\sqrt{13} + 13}{2}}$.

(Simplify your answer. Type an exact answer, using radicals as needed. Use integers or fractions for any numbers in the expression.)