Lab 2 - January 24, 2017

Exercise 1: Estimating a proportion with a discrete prior

(From Albert, 2011) Bob claims to have ESP. To test his claim, you propose the following experiment. You will select one card from four large cards with different geometric figures on them, and Bob will try to identify the figure.
Let p denote the probability that Bob is correct in identifying the figure for a single card. You believe that Bob has no ESP ability (so $p = 0.25$) but there is a small chance that p is either smaller or larger than 0.25. After some thought, you place the following prior distribution on p:

p	0	0.125	0.250	0.375	0.500	0.625	0.750	0.875	1.00
$g(p)$	0.001	0.001	0.950	0.008	0.008	0.008	0.008	0.008	0.008

Suppose that the experiment is repeated ten times and Bob is correct six times and incorrect four times.
Use the function `pdisc` in `LearnBayes` to find the posterior probabilities of these values of p. What is your posterior probability that Bob has no ESP ability?

Change the prior probability to uniform (equal) over the original values of p. Re-run the analysis with the original data. Again, what is the posterior probability that Bob has no ESP ability?

Briefly talk about (a couple of sentences) the differences in the two analyses.

R code is given in `Lab2DiscretePrior.R`.

Exercise 2: Prediction of a proportion in a future sample

(From Albert, 2011) A study reported on the long term effects of exposure to low levels of lead in childhood. Researchers analyzed children's primary teeth (after they had fallen off) for lead content. Of the children whose teeth had a lead content of over 22.22 parts per million (ppm), 22 eventually graduated from high school and seven did not.
Suppose that θ denotes the proportion of children with lead exposure over 22.22 ppm who will eventually graduate from high school. Further, suppose that the prior for θ is a $\text{Beta}(1, 1)$, which, combined with the likelihood, results in a posterior for θ which is $\text{Beta}(23, 8)$.
Tasks are the following:

1. Using the function `qbeta` in `LearnBayes`, compute the 90% credible set for θ.
2. Use the function `pbeta` to find the probability that $\theta > 0.6$.
3. Use the function `rbeta` to draw 1000 values of θ from its posterior distribution.

Suppose that you find 10 more children with teeth lead content above 22.22 ppm. Use your simulated sample from part 3 above to find the predictive probability that nine or 10 of

them will graduate from high school. (You will also need to use the function `rbinom` to draw a sample from the predictive distribution.)
The code to carry out these analyses is given in `Lab2Prediction.R`.

Exercise 3: Comparing a uniform and a beta prior

If θ has a Beta distribution with parameters $\alpha > 0$ and $\beta > 0$, then:

$$\begin{aligned} p(\theta) &\propto \theta^{\alpha-1}(1-\theta)^{\beta-1} \\ E(\theta) &= \frac{\alpha}{\alpha+\beta} \\ V(\theta) &= \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}. \end{aligned}$$

These, and many other attributes of all standard distributions are given in the handout entitled `ProbabilityDistributions.pdf` that is posted in the class notes folder of the course's website.

For example

- If $\theta \sim \text{Beta}(1, 1)$, then $E(\theta) = 0.5$ and $V(\theta) = 1/12$, which correspond to the mean and the variance of a uniform random variable in (0, 1).
- If $\theta \sim \text{Beta}(100, 100)$, then $E(\theta) = 0.5$ and $V(\theta) = 1/804$.

In the second example, the prior is highly informative. R code to compute posteriors and interval estimates of θ under the uniform and the informative beta priors is given in `BernBeta.R`