Draft version May 13, 2021
Typeset using LaTeX **RNAAS** style in AASTeX631

Yeehaw: A Roundup of Probabilistic Transit Modeling Packages

Miles Lucas[1]

[1] *Institute for Astronomy, University of Hawai'i, USA*

ABSTRACT

In this research note we report on the statistical outputs of popular transit modeling packages: `exoplanet`, `BATMAN`, `PyTransit`, `Juliet`, and `EXOFASTv2`. We use a circular Keplerian orbit with a quadratic limb-darkening law to generate data with additive white noise and then perform statistical inference using the packages' implementations of transit curves and statistical models. Our results show that none of the packages we tested significantly biases the posterior results away from the true value, although `EXOFASTv2` had slightly different posteriors due to different prior parameterizations.

INTRODUCTION

The method of transit photometry has proven indispensable in the detection and characterization of thousands of exoplanets over the past few decades (Charbonneau et al. 2007; Winn 2009, 2010; Haswell 2010). These quantities all have to be inferred using a computed model of the planetary transit. Model accuracy directly affects the inference of these parameters and a recent study by Agol et al. (2020) has improved numerical accuracy of the popular quadratic limb-darkening law (Mandel & Agol 2002) to within $\mathcal{O}(10^{-15})$ along with analytical derivatives. The rapid growth of the exoplanet community means not every user who uses a given package will understand, in full, the statistical implications of their choices, and it is important to make sure the transit modeling ecosystem is not imparting bias into exoplanet studies. We seek to improve upon the work by Agol et al. (2020) and study how different packages affect the statistical inference of astrophysical parameters.

The following packages provide limb-darkened transit curves: `exoplanet` (Foreman-Mackey 2019), `BATMAN` (Kreidberg 2015), `PyTransit` (Parviainen 2015), and `EXOFASTv2` (Eastman et al. 2019). From these packages we use the quadratic limb-darkening law from Mandel & Agol (2002). In addition, `exoplanet` and `EXOFASTv2` have entire statistical modeling frameworks built into or on top of the transit curves. We also test `Juliet` (Espinoza et al. 2019), which adds a statistical modeling framework to the `BATMAN` transit models.

MODELING

To begin, we used the highly accurate Agol et al. (2020) transit models to simulate a light curve. The ground truth parameters were chosen to roughly mock the Kepler-101b transit (Bonomo et al. 2014) and are shown in Table 1. We built a hierarchical model using `PyMC3` (Salvatier et al. 2016) and `exoplanet` with the following parameters: the semi-major axis (aR_*), orbital period (P), time of inferior conjunction (t_0), ratio of planet to stellar radii (R_P/R_*), limb-darkening coefficients (u_1, u_2), and out-of-transit noise (σ). This parameterization is supported (at least indirectly) by all packages tested and has the benefit of no correlations in the orbital parameters, which can degrade accuracy and performance of inference methods. We built this model in a generic way that could substitute different limb-darkening laws between `exoplanet`, `BATMAN`, and `PyTransit`.

The prior parameterization is chosen to be slightly uninformative, but some tuning was done to ensure consistent outputs. For the period and semi-major axis we use log-Normal priors, for the time of inferior conjunction we use a Normal prior, for the limb-darkening coefficients we use uninformative triangular sampling (Kipping 2013) provided by `exoplanet`, and finally for the noise term we use a half-Cauchy distribution. The simulated data is compared to the models using a Gaussian likelihood without any additional noise model. For performing statistical inference

Corresponding author: Miles Lucas
mdlucas@hawaii.edu

Table 1. Posterior outputs from each statistical model experiment. The uncertainty bounds are given by the 68% highest-posterior density interval.

	P [d]	t_0 [d]	R_p/R_*	aR_*	u_1	u_2	σ
ground truth	3.5	1.3	0.03	10	0.5	0.2	$1e-4$
exoplanet+NUTS	$3.5^{+5.66e-5}_{-5.81e-5}$	$1.3^{+0.000\,186}_{-0.000\,181}$	$0.0302^{+0.000\,119}_{-0.000\,104}$	$10.0^{+0.0301}_{-0.0299}$	$0.552^{+0.0567}_{-0.067}$	$0.0479^{+0.116}_{-0.101}$	$9.97e-5^{+6.15e-7}_{-5.52e-7}$
exoplanet+MH	$3.5^{+6.22e-5}_{-5.56e-5}$	$1.3^{+0.000\,182}_{-0.000\,193}$	$0.0302^{+0.000\,111}_{-0.000\,11}$	$10.0^{+0.0306}_{-0.0296}$	$0.552^{+0.0647}_{-0.0562}$	$0.048^{+0.103}_{-0.11}$	$9.97e-5^{+5.93e-7}_{-5.59e-7}$
BATMAN+MH	$3.5^{+6.32e-5}_{-5.28e-5}$	$1.3^{+0.000\,171}_{-0.000\,198}$	$0.0302^{+0.000\,106}_{-0.000\,118}$	$10.0^{+0.0337}_{-0.0275}$	$0.551^{+0.0568}_{-0.0694}$	$0.0523^{+0.118}_{-0.106}$	$9.97e-5^{+5.69e-7}_{-5.83e-7}$
PyTransit+MH	$3.5^{+5.95e-5}_{-5.45e-5}$	$1.3^{+0.000\,194}_{-0.000\,168}$	$0.0302^{+0.000\,115}_{-0.000\,109}$	$10.0^{+0.0336}_{-0.027}$	$0.555^{+0.0687}_{-0.0599}$	$0.0422^{+0.107}_{-0.116}$	$9.97e-5^{+5.95e-7}_{-5.64e-7}$
Juliet+NS	$3.5^{+5.83e-5}_{-5.67e-5}$	$1.3^{+0.000\,175}_{-0.000\,198}$	$0.0302^{+0.000\,11}_{-0.000\,11}$	$10.0^{+0.0273}_{-0.0325}$	$0.552^{+0.0632}_{-0.0572}$	$0.0476^{+0.104}_{-0.108}$	
EXOFASTv2+DEMH	$3.5^{+5.97e-5}_{-5.8e-5}$	$1.3^{+0.000\,173}_{-0.000\,196}$	$0.0302^{+0.000\,107}_{-9.79e-5}$	$10.0^{+0.0227}_{-0.0249}$	$0.405^{+0.0274}_{-0.0268}$	$0.28^{+0.0411}_{-0.0403}$	$9.98e-5^{+6.07e-7}_{-5.58e-7}$

with `PyMC3`, we use both the No-U-Turn Sampler (NUTS; Hoffman & Gelman 2011) and Metropolis-Hastings (MH). Analytical derivatives are required for NUTS, and therefore we only tested it with the `exoplanet` light curve models.

For `Juliet` we needed to change the parameterization slightly– in the previous models the error is completely modeled by the noise term inside the Gaussian likelihood, but we had to skip modeling this term for `Juliet` because it fails to evaluate a finite likelihood if the error in its data model is 0. We followed the `Juliet` documentation for building our model– the main differences are that period and semi-major axis are sampled using a Normal distribution rather than a log-Normal. For inference, `Juliet` uses `DYNESTY` (Speagle 2020) to perform nested sampling (NS; Skilling 2004) providing both posterior samples and an estimate of the Bayesian evidence for our model.

For `EXOFASTv2` we also had to alter the parameterization slightly; rather than fitting the semi-major axis directly, we had to fit the stellar mass and fix the planetary mass. Similar to our `Juliet` setup, we use a Normal prior for period and time of inferior conjunction rather than log-Normal priors, and we also use a Uniform prior for the noise *variance*. Importantly, the limb-darkening coefficients use an informative prior from Claret & Bloemen (2011). For inference `EXOFASTv2` uses a differential evolution Metropolis-Hastings algorithm (DEMH; Ter Braak 2006) which uses an ensemble of walkers which query each other between steps to temper their proposal scales.

RESULTS

We performed inference on all of our models, starting with a numerical optimization to decrease the time taken to converge on a solution during the MCMC inference. For the NUTS sampler we used 5.000 tuning steps and 5.000 samples. For the MH sampler we used 5.000 burn-in steps and 10.000 samples. In all cases this was enough samples to produce a Gelman-Rubin statistic close to 1.0, which implies that the chains are *well-mixed*, Nested sampling samples until convergence (see Speagle 2020, §2.4), which produced 29.000 samples which were equally resampled using their statistical weights. `EXOFASTv2` was set to sample until the Gelman-Rubin statistic for the DEMH walkers was below 1.01, which generated 50.500 samples.

The posteriors from all of our inferences are tabulated in Table 1. The table shows the median value of all posterior samples along with the 68% highest-posterior-density interval (HDI). This is similar to a "1-sigma confidence interval" but in a Bayesian context. Every model and inference combination *except* for `EXOFASTv2` was consistent with each other and fits the data very well. A key parameter, the relative radius (R_P/R_*) is recovered and almost identically distributed for all our models. We believe the differences in posteriors from `EXOFASTv2` are from the different limb-darkening prior used.

REFERENCES

Agol, E., Luger, R., & Foreman-Mackey, D. 2020, AJ, 159, 123, doi: 10.3847/1538-3881/ab4fee

Bonomo, A. S., Sozzetti, A., Lovis, C., et al. 2014, A&A, 572, A2, doi: 10.1051/0004-6361/201424617

Charbonneau, D., Brown, T. M., Burrows, A., & Laughlin, G. 2007, in Protostars and Planets V, ed. B. Reipurth, D. Jewitt, & K. Keil, 701. https://arxiv.org/abs/astro-ph/0603376

Claret, A., & Bloemen, S. 2011, A&A, 529, A75, doi: 10.1051/0004-6361/201116451

Eastman, J. D., Rodriguez, J. E., Agol, E., et al. 2019, arXiv e-prints, arXiv:1907.09480. https://arxiv.org/abs/1907.09480

Espinoza, N., Kossakowski, D., & Brahm, R. 2019, MNRAS, 490, 2262, doi: 10.1093/mnras/stz2688

Foreman-Mackey, D. 2019, exoplanet: Probabilistic modeling of transit or radial velocity observations of exoplanets. http://ascl.net/1910.005

Haswell, C. A. 2010, Transiting Exoplanets

Hoffman, M. D., & Gelman, A. 2011, arXiv e-prints, arXiv:1111.4246. https://arxiv.org/abs/1111.4246

Kipping, D. M. 2013, MNRAS, 435, 2152, doi: 10.1093/mnras/stt1435

Kreidberg, L. 2015, PASP, 127, 1161, doi: 10.1086/683602

Mandel, K., & Agol, E. 2002, ApJL, 580, L171, doi: 10.1086/345520

Parviainen, H. 2015, MNRAS, 450, 3233, doi: 10.1093/mnras/stv894

Salvatier, J., Wieckiâ, T. V., & Fonnesbeck, C. 2016, PyMC3: Python probabilistic programming framework. http://ascl.net/1610.016

Skilling, J. 2004, in American Institute of Physics Conference Series, Vol. 735, Bayesian Inference and Maximum Entropy Methods in Science and Engineering: 24th International Workshop on Bayesian Inference and Maximum Entropy Methods in Science and Engineering, ed. R. Fischer, R. Preuss, & U. V. Toussaint, 395–405, doi: 10.1063/1.1835238

Speagle, J. S. 2020, MNRAS, 493, 3132, doi: 10.1093/mnras/staa278

Ter Braak, C. J. F. 2006, Statistics and Computing, 16, 239, doi: 10.1007/s11222-006-8769-1

Winn, J. N. 2009, in Transiting Planets, ed. F. Pont, D. Sasselov, & M. J. Holman, Vol. 253, 99–109, doi: 10.1017/S174392130802629X

Winn, J. N. 2010, Exoplanet Transits and Occultations, ed. S. Seager, 55–77