NewtonsMethod

February 1, 2024

1 Newton's Method Exercise

1.1 Writing function

[13]:
```
def newton_method(num, guess, max_iter, err):
    # Make sure input is valid
    if num < 0:
        print("Newton's method failed due to negative input")
        return

    # Define f and f' according to input value
    f = lambda x: x**2 - num
    f_prime = lambda x: 2*x

    x_old = guess
    data = [guess]

    for k in range(1, max_iter):
        # Make sure code won't cause a divide by zero error
        if x_old == 0:
            print("Newton's method failed due to 'divide by zero' error")
            return

        # Calculate new guess
        x_new = x_old - f(x_old) / f_prime(x_old)
        data.append(x_new)

        # Check if max iterations exceeded
        if len(data) > max_iter:
            print(f"Newton's method failed to converge in {max_iter} iterations.
↪")
            return

        # Check if method has reached desired precision
        if abs(x_old - x_new) < err:
            break

        x_old = x_new
```

```
    # Newton's method successful! Print guesses to terminal
    for guess in data:
        print(guess)
```

1.2 Demonstrating cases

(i): $\sqrt{11}$ to an accuracy of 10^{-5}, initial seed $x_0 = 100$

[14]:
```
newton_method(11,100,25,10**-5)
```

```
100
50.055
25.13737913295375
12.787487237105545
6.8238515747148405
4.217922216174798
3.4129206687707887
3.317983288999192
3.316625068463878
3.316624790355412
```

(ii): $\sqrt{11}$ to an accuracy of 10^{-10}, initial seed $x_0 = 100$

[15]:
```
newton_method(11,100,25,10**-10)
```

```
100
50.055
25.13737913295375
12.787487237105545
6.8238515747148405
4.217922216174798
3.4129206687707887
3.317983288999192
3.316625068463878
3.316624790355412
3.3166247903554
```

(iii): $\sqrt{11}$ to an accuracy of 10^{-20}, initial seed $x_0 = 100$

[16]:
```
newton_method(11,100,25,10**-20)
```

```
100
50.055
25.13737913295375
12.787487237105545
6.8238515747148405
4.217922216174798
3.4129206687707887
```

```
3.317983288999192
3.316625068463878
3.316624790355412
3.3166247903554
3.3166247903554
```

(iv): $\sqrt{-1}$

[17]:
```
newton_method(-1,100,25,10**-10)
```

```
Newton's method failed due to negative input
```