Condensed Matter Field Theory:

Chapter 1: From particles to Fields

Exercise #1: (1.19) "Faraday's Law/Gauss' Magnetic Law in Homogeneous fields"

$$\nabla \times E + \partial_t B = 0 \qquad \nabla \cdot B = 0$$

(1.20) "Electromagnetic field tensor"

$$F = \{F_{\mu\nu}\} = \begin{bmatrix} 0 & E_1 & E_2 & E_3 \\ -E_1 & 0 & -B_3 & B_2 \\ -E_2 & B_3 & 0 & -B_1 \\ -E_3 & -B_2 & B_1 & 0 \end{bmatrix}$$

(pg 16) "Field-potential relation"

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

(from exercise)

$$\partial_\lambda F_{\mu\nu} + \partial_\mu F_{\nu\lambda} + \partial_\nu F_{\lambda\mu}$$

$\boxed{\text{Left side}}$

$$= \partial_\lambda \begin{bmatrix} 0 & E_1 & E_2 & E_3 \\ -E_1 & 0 & -B_3 & B_2 \\ -E_2 & B_3 & 0 & -B_1 \\ -E_3 & -B_2 & B_1 & 0 \end{bmatrix} + \partial_\mu \begin{bmatrix} 0 & E_1 & E_2 & E_3 \\ -E_1 & 0 & -B_3 & B_2 \\ -E_2 & B_3 & 0 & -B_1 \\ -E_3 & -B_2 & B_1 & 0 \end{bmatrix} + \partial_\nu \begin{bmatrix} 0 & E_1 & E_2 & E_3 \\ -E_1 & 0 & -B_3 & B_2 \\ -E_2 & B_3 & 0 & -B_1 \\ -E_3 & -B_2 & B_1 & 0 \end{bmatrix}$$

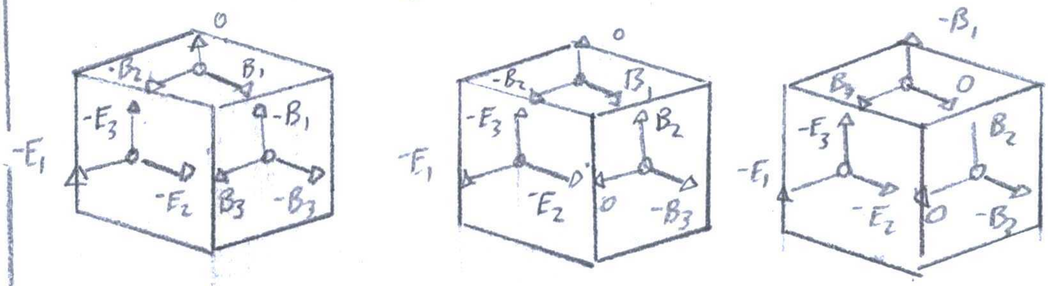
$$= \nabla \times E + \partial_t B + \nabla \cdot B \qquad \ldots \text{(pg 16)} \quad \partial_\mu = (\partial_t, \nabla)$$

$\boxed{\text{Right side}}$

$$= \partial_\lambda(\partial_\mu A_\nu - \partial_\nu A_\mu) + \partial_\mu(\partial_\nu A_\lambda - \partial_\lambda A_\nu) + \partial_\nu(\partial_\mu A_\lambda - \partial_\lambda A_\nu)$$

$$= 0$$

A random drawing about four-tensors...



$$E_1 \ldots \quad + \quad E_2 \ldots \quad + \quad E_3 \ldots$$

$$x\begin{bmatrix} -E_1 & B_3 & -B_2 \\ -E_2 & -B_3 & B_1 \\ -E_3 & -B_1 & 0 \end{bmatrix} \qquad x\begin{bmatrix} -E_1 & 0 & -B_2 \\ -E_2 & -B_3 & B_1 \\ -E_3 & B_2 & 0 \end{bmatrix} \qquad x\begin{bmatrix} -E_1 & 0 & B_3 \\ -E_2 & -B_3 & 0 \\ -E_3 & B_2 & -B_1 \end{bmatrix}$$

Exercise #2:

(1.16) "Functional"

$$S[\phi] = \int_M d^m x \, \mathcal{L}(\phi^i, \partial_\mu \phi^i)$$ ← Lagrangian

(1.22) "Functional consistent to Gauge and Lorentz invariance"

$$S[\phi] = \int d^4x \left(\underbrace{c_1 F_{\mu\nu} F^{\mu\nu}}_{\text{"Lorentz invariance"}} + \underbrace{c_2 A_\mu j^\mu}_{\text{"Aj-coupling"}} \right)$$

<u>Gauge Transformation: Aj-coupling:</u>

(pg 16) "Gauge Transformation"

$$A_\mu \longrightarrow A_\mu + \partial_\mu \Gamma$$ ...where Γ = arbitrary function

$$S[\phi] = c_2 \int d^4x \, A_\mu j^\mu$$

$$= c_2 \int d^4x \, (A_\mu + \partial_\mu \Gamma) j^\mu$$

$$= C_2 \int d^4x \, A_\mu j^\mu + \int d^4x \, \partial_\mu T j^\mu$$

$$= C_2 \int d^4x \, A_\mu j^\mu + \left[\int d^4x (\partial_\mu T) j^\mu + \underbrace{\int d^4x \, T (\partial_\mu j^\mu)}_{=0 \text{ by continuity equation}} \right]$$

$$C_2 \int d^4 A_\mu j^\mu = C_2 \int d^4x (A_\mu + \partial_\mu T) j^\mu$$

∴ interpretation to physical invariance, current adds or subtracts in similar quantities within each (or new) reference frame.

<u>Gauge transformation : Lorentz invariance :</u>

$$S[\phi] = C_1 \int d^4x (F_{\mu\nu} F^{\mu\nu})$$

$$S[\phi]_{var} = C_1 \int d^4x (F_{\mu\nu} + \partial_\mu T)(F^{\mu\nu} + \partial^\mu T)$$

$$= C_1 \int d^4x (F_{\mu\nu} F^{\mu\nu} + F_{\mu\nu} \partial^\mu + F^{\mu\nu} \partial_\mu + (\partial_\mu T)(\partial^\mu T))$$

$$C_1 \int d^4x (F_{\mu\nu} F^{\mu\nu}) \neq C_1 \int d^4x (F_{\mu\nu} F^{\mu\nu} + F_{\mu\nu} \partial^\mu + F^{\mu\nu} \partial_\mu + (\partial_\mu T)(\partial^\mu T))$$

∴ not invariant by Gauge transformation.

In unhappy manner, later the book "Condensed Matter Field theory" removes Planck's constant (h) from the content. A brief memory considered a new book, "Deformation and Fracture Mechanics of Engineering materials"

Exercise #3

(1.25) "Canonical Commutation"

$$[\hat{\pi}(x), \hat{\Phi}(x)] = -i\hbar\,\delta(x - x')$$

... where $\hat{\pi}(x), \hat{\Phi}(x)$ are quantum fields

(1.31) "Ladder/Creation-Annihilation/raising-lowering/Adjoint/A-representation of the Hamiltonian"

$$\hat{H} = \omega(\hat{a}^{\dagger}\hat{a} + 1/2)$$

$$\hat{H}|n\rangle = \omega\left[\frac{\hat{p}^2}{2m} + \frac{m\omega^2}{2}\hat{x}^2\right]|n\rangle$$

.. (pg 21) Hamiltonian

$$= \omega\left[\frac{\hat{p}^2}{2m} - \frac{i}{2}(\hat{x}\hat{p} - \hat{p}\hat{x}) + \frac{m\omega^2}{2}\hat{x}^2 + \frac{1}{2}\right]|n\rangle$$

Commutator Table:

Name:	Symbol:	Operator:
Position	$\hat{x}$	x
Momentum	$\hat{p}$	$-i\hbar\frac{\partial}{\partial x}$

$$= \omega\left(\left[\left(-\sqrt{\frac{m\omega}{2}}\left(\hat{x} - \frac{i}{m\omega}\hat{p}\right)\right)\left(-\sqrt{\frac{m\omega}{2}}\left(\hat{x} + \frac{i}{m\omega}\hat{p}\right)\right)\right] + \frac{1}{2}\right)|n\rangle$$

$$= \omega(\hat{a}^{\dagger}\cdot\hat{a} + 1/2)|n\rangle$$

$$= \omega(n + 1/2)|n\rangle$$

.. when $\hat{a}^{\dagger}\hat{a} = n$

... again, book denotes speed of light equal to one ($c=1$) and Plancks constant to one ($\hbar=1$).

Exercise #5: (pg 24) "Lagrangian density of the electromagnetic field"

$$\mathcal{L}(A_\mu, \partial_\nu A_\mu) = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + A_\mu j^\mu$$

(pg 34) "Energy-Momentum tensor"

$$T_\mu{}^\nu(x) = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi^i)} \partial_\nu \phi^i - \delta_{\mu\nu} \mathcal{L}$$

$$= \frac{\partial}{\partial(\partial_\mu \phi^i)} \left(-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + A_\mu j^\mu\right) \partial_\nu \phi^i$$

$$- \delta_{\mu\nu}\left(-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + A_\mu j^\mu\right)$$

with (pg 33) "Noether's conserved local current"

$$\partial_\mu j_\mu{}^\alpha = 0$$

(pg 16) "Field-potential relation"

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$T_\mu{}^\nu(x) = \frac{\partial}{\partial(\partial_\mu A_\lambda)}\left(-\frac{1}{4} F_{\mu\nu} F^{\mu\nu}\right) \partial_\nu \Phi^i - \delta_{\mu\nu}\left(-\frac{1}{4} F_{\mu\nu} F^{\mu\nu}\right)$$

$$= \frac{\partial}{\partial(\partial_\mu A_\lambda)}\left(-\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial_\mu A_\nu - \partial_\nu A_\mu)\right) \partial_\nu A_\lambda$$

$$- \delta_{\mu\nu}\left(-\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial_\mu A_\nu - \partial_\nu A_\mu)\right)$$

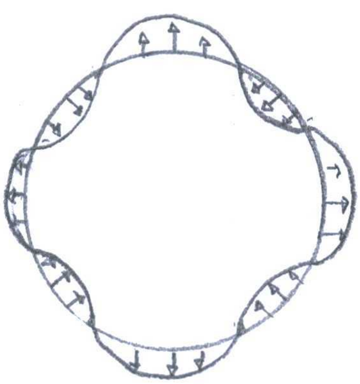
$$= \frac{\partial}{\partial(\partial_\mu A_\lambda)}\left(-\frac{1}{2}(\partial^\mu A^\nu \partial_\mu A_\nu - \partial^\mu A^\mu \partial_\nu A_\mu)\right) \partial_\nu A_\lambda$$

$$- \delta_{\mu\nu}\left(-\frac{1}{2}(\partial^\mu A^\nu \partial_\mu A_\nu - \partial^\mu A^\mu \partial_\nu A_\mu)\right)$$

Normalization:

$$\langle n|n\rangle = \langle \hat{a}^{\dagger}\hat{a}|\hat{a}\hat{a}^{\dagger}\rangle$$
$$= [\hat{a}^{\dagger}\hat{a} - \hat{a}\hat{a}^{\dagger}]$$
$$= \frac{m\omega}{2}\left[\left(\hat{x}+i\frac{\hat{p}}{m\omega}\right)\left(\hat{x}-i\frac{\hat{p}}{m\omega}\right)-\left(\hat{x}-i\frac{\hat{p}}{m\omega}\right)\left(\hat{x}+i\frac{\hat{p}}{m\omega}\right)\right]$$
$$= \frac{m\omega}{2}\left[\frac{-2i}{m\omega}(\hat{p}\hat{x}-\hat{x}\hat{p})\right]$$
$$= \frac{m\omega}{2}\left[\frac{2i}{m\omega}(-i)\right]$$
$$= 1$$

Exercise #4:



"Elastic chain"

(from problem) "Continuity equation"

$$\int_{x_1}^{x_2} dx\,\rho(x,t) = \partial_t\left(\phi(x_2,t)-\phi(x_1,t)\right)$$

$$\frac{d}{dt}\int_{x_1}^{x_2} dx\,\rho(x,t) = \frac{\partial}{\partial t}\left(\phi(x_2,t)-\phi(x_1,t)\right)$$
$$= v(x_2,t)\rho(x,t) - v(x,t)\rho(x,t)$$
$$= -\frac{d}{dx}\int_{t_1}^{t_2} v(x,t)\rho(x,t)\,dt$$
$$= -\frac{d}{dx}\int_{t_1}^{t_2}\phi(x,t)\,dt$$

$$\rho(x,t) = \frac{d}{dx}\phi(x,t)$$

$$\int dx\,(\text{particle density})\times(\text{velocity}) = \int dx\,\frac{d}{dt}\phi(x,t)$$

$$= -F^{\mu\nu} \circ \partial^\nu A_\lambda + \frac{1}{4} \partial^{\mu\nu} F^{\mu\nu} \circ F_{\mu\nu}$$

$$= F^{\mu\nu} \circ F_{\mu\nu} + \frac{1}{4} \partial^{\mu\nu} F^{\mu\nu} F_{\mu\nu}$$

(pg 34) "Energy - Momentum vector"

$$P^\nu \equiv \int d^d x \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi^i} \partial_\nu \phi^i - \delta_{\mu\nu} \mathcal{L} \right)$$

$$\equiv \int d^d x \left(F^{\mu\nu} \circ F_{\mu\nu} + \frac{1}{4} \partial^{\mu\nu} \circ F^{\mu\nu} \circ F_{\mu\nu} \right)$$

(1.20) "EM field tensor"

$$F = \{F_{\mu\nu}\} = \begin{bmatrix} 0 & -E_1 & -E_2 & -E_3 \\ -E_1 & 0 & -B_3 & B_2 \\ -E_2 & B_3 & 0 & -B_1 \\ -E_3 & -B_2 & B_1 & 0 \end{bmatrix}$$

... not in the book

"Faradays tensor / Hodge Dual"

$$G^{\alpha\beta} = \frac{1}{2} \epsilon_{ijk} F^{ij} = \begin{bmatrix} 0 & -B_1 & -B_2 & -B_3 \\ B_1 & 0 & -E_3 & E_2 \\ B_2 & E_3 & 0 & E_1 \\ B_3 & -E_2 & E_1 & 0 \end{bmatrix}$$

... where $\epsilon_{ijk} \epsilon_{ijk} = 2$

"Levi-Civita tensor"

$$P^\nu \equiv \int d^3 x \left(F^{0i} F_{0i} + \frac{1}{4} \partial^{0i} \circ F^{0i} F_{0i} \right)$$

$$\equiv \int d^3 x \left(\underbrace{F^{0i} F_{0i}}_{\text{"EM field tensor"}} + \frac{1}{4} \left(2 F^{0i} F_{0i} + \underbrace{F^{ij} F_{ij}}_{\text{"Faraday's tensor"}} \right) \right)$$

$$\equiv \int d^3x \ \frac{1}{2}\left(|E_1^2+E_2^2+E_3^2| + |B_1^2+B_2^2+B_3|^2\right)$$

$$\equiv \int d^3x \ \frac{1}{2}\left(|\vec{E}|^2 + |\vec{B}|^2\right)$$

Problem #1: (from problem) "Lorentz Gauge"

$$\partial t \phi = \nabla \circ A$$

"Charge density"

$$(\partial_t^2 - \nabla^2)\phi = \rho$$

"current"

$$(\partial_t^2 - \nabla^2) A = j$$

"Lagrangian Action"

$$S[A] = -\int d^4x \left(\frac{1}{4} F_{\mu\nu}F^{\mu\nu} + A_\mu j^\mu\right)$$

(pg 16) "Field Strength tensor"

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\mu A_\mu$$

$$S[A] = -\int d^4x \Big(-\frac{1}{2} A_\nu [\partial_\mu A_\nu - \partial_\nu A_\mu] \circ [\partial^\mu A^\nu - \partial^\nu A^\mu]$$
$$+ A_\mu j^\mu\Big)$$

$$= -\int d^4x \left(-\frac{1}{2} A_\mu [\partial_\mu \partial^\mu A^\mu - \partial_\mu \partial^\mu A^\mu] + A_\mu j^\mu\right)$$

$$= -\int d^4x \left(\frac{1}{2} \partial_\mu A_\nu \partial^\mu A_\nu + A_\mu j^\mu\right)$$

Problem #2: <u>Total Energy:</u>

(from problem) "Hamiltons field"

$$H = \int d^3x \, H(x) \qquad H(x) = E^2(x) + B^2(x)$$

(1.24) "Lagrangian density of the electromagnetic field"

$$\mathcal{L}(A_\mu, \partial_\nu A_\mu) = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \partial_\mu j^\mu$$

$$= -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

Noether said, "no current when without matter"

$$\partial_\mu j^\mu = 0$$

(pg 16) "Field-potential relation"

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$\mathcal{L}(A_\mu, \partial_\nu A_\mu) = -\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu A^\nu - \partial^\nu A^\mu)$$

$$= \frac{1}{2}(|E|^2 + |B|^2)$$

(1.10) "Hamiltons Density"

$$H(\phi', \partial_x \phi, \pi) = (\pi \dot{\phi} - \mathcal{L}(\phi, \partial_x \phi, \dot{\phi}))\Big|_{\dot{\phi} = \dot{\phi}(\phi, \pi)}$$

$$= \pi \partial_0 A_\mu - \mathcal{L}$$

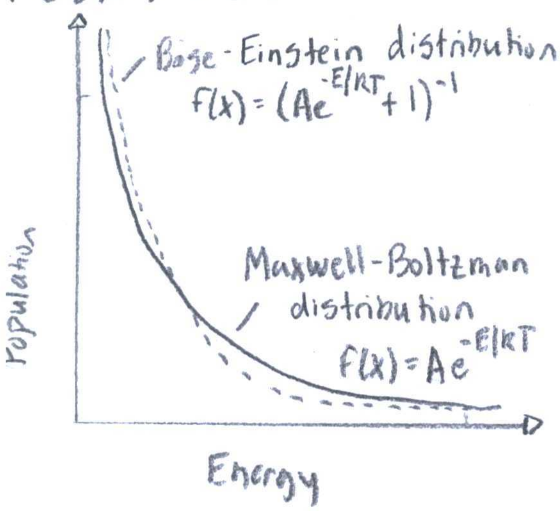
$$= \frac{1}{2}\left(2E \cdot \partial_0 A - E^2 + B^2\right)$$

$$= \frac{1}{2}\left(2\nabla\cdot(E\phi) + E^2 + B^2\right) \quad \ldots \text{When } \partial_0 A \cdot (E + \nabla\phi$$

$$H = \frac{1}{2}\int d^3x \Big(\underbrace{\nabla\cdot(E\phi)}_{\nabla\cdot E = 0,\ \text{no charge}} + E^2 + B^2 \Big)$$

$$= \frac{1}{2}\int d^3x \left(E^2 + B^2\right)$$

Problem #3:



"Bose-Einstein distribution helped low temperature ideal gas models"

Thermal Representation of Energy Density:

(pg 21) "Wavenumber"

$$k = 2\pi m / c$$

(pg 22) "Energy levels"

$$G = \omega(n + 1/2)$$

(from problem) "dispersion"

$$\omega_k = v|k|$$

$$Z = tr(e^{-\beta \hat{H}})$$

$$= \sum_{n=0}^{\infty} e^{-\beta E}$$

$$= \prod_{m=1} \sum_{n=0}^{\infty} e^{-\beta \omega_m (n_m + 1/2)}$$

$$= \prod_{m=1,2..} \frac{e^{-\beta \omega_m / 2}}{1 - e^{-\beta \omega_m}}$$

Series Identity:

$$\sum_{x=0}^{\infty} e^{-x} = \frac{e^x}{e^x - 1} = \frac{1}{1 - e^{-x}}$$

$$\ln(Z) = \ln\left[\prod_{m=1}^{\infty} \frac{e^{-\beta w_m/2}}{1-e^{-\beta w_m}}\right]$$

$$= \sum_{m=1}^{\infty}\left[-\beta w_m/2 + \ln\left(1-e^{-\beta w_m}\right)\right]$$

$$u = -(1/L)\,\partial_\beta \ln(Z)$$

$$= -(1/L)\,\partial_\beta \sum_{m=1}^{\infty}\left[-\beta w_m/2 + \ln\left(1-e^{-\beta w_m}\right)\right]$$

$$= \frac{1}{L} \circ \sum_{m=1}^{\infty}\left(\frac{w_m}{2} + \frac{\beta_m}{e^{\beta_m}-1}\right)$$

$$= \frac{1}{L} \circ \sum_{m=1}^{\infty} \frac{w_m}{2} + w_m n_\beta(w)$$

$\circ\circ$ where $n_\beta(w) = (e^{\beta\epsilon}-1)^{-1}$, energy density

Approximate sum over k by an integral:

$$u = \frac{1}{L} \circ \sum_{m=1}^{\infty} \frac{w_m}{2} + w_m n_\beta(w)$$

$$= \frac{1}{L}\int_0^{\infty}\left(\frac{w_m}{2} + w_m n_\beta(w)\right) dm$$

$$= \frac{1}{2\pi}\int_0^{\infty}\left(\frac{w_m}{2} + w_m n_\beta(w)\right) dk$$

u-substitution

$$dk = \frac{2\pi\, dm}{L}$$

$$= \frac{1}{2\pi}\int_0^{\infty}\left(\frac{v|k|}{2} + \frac{v|k|}{e^{v|k|}-1}\right) dk$$

$$= A + \frac{B}{\beta^2}$$

Specific heat:

$C = \partial_t u$

$= \frac{\partial}{\partial t}\left[A + \frac{B}{\beta^2}\right]$

$\propto \frac{\partial}{\partial t}\left[A + B \cdot kT^2\right]$

$\propto T$

Ideal gases:

$C_v = \left(\frac{\partial U}{\partial T}\right)_v$

Specific heat at classical Limits:

$$u = \frac{1}{2\pi}\int\left(\frac{v|k|}{2} + \frac{v|k|}{e^{\beta v|k|}-1}\right) dk$$

$$= \frac{1}{2\pi}\int\left(\frac{v|k|}{2} + \frac{v|k|}{\beta \cdot v|k|}\right) dk$$

Exponential Series:

$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} \ldots$

$= A + \frac{B}{\beta}$

$C = \partial_t u$

$= \frac{\partial}{\partial t}\left[A + \frac{B}{\beta}\right]$

$= \frac{\partial}{\partial t}\left[A + BkT\right]$

$= k$ "Boltzmann constant"

Isotropic solid of volume L^d:

(pg 20) "Fourier representation classical Hamiltonian"

$$\hat{H}(\hat{\phi},\hat{\pi}) = \frac{1}{2m}\hat{\pi}^2 + \frac{k_s a^2}{2}(\partial_x \hat{\phi})^2$$

(pg 20) "wavenumber"

$k = 2\pi m / L$

d-dimensional specific heat:

$$\hat{H} = \sum_{k=0}^{\infty}\left(\frac{ka^2}{2}\right)(\nabla\phi(x))^2$$

$$= \int\left(\frac{ka^2}{2}\right)(\nabla\phi(x))^2 dk$$

$$= \left(\frac{ka^2}{2}\right)^{d+1}(\nabla\phi(x))^{d+1}$$

$$u = \left(\frac{ka^2}{2}\right)^{d+1}(\nabla\phi(x))^{d+1}$$

Schrödingers equation:

$H\Psi = U\Psi$

$$C_v = \frac{\partial u}{\partial t}$$

$$= \frac{\partial}{\partial t}\left[\left(\frac{ka^2}{2}\right)^{d+1}(\nabla\phi(x))^{d+1}\right]$$

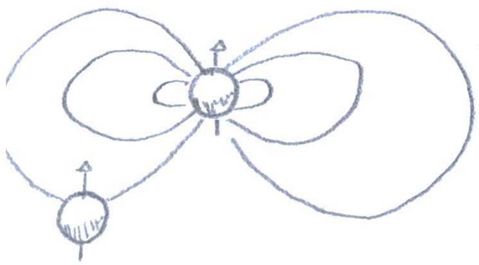
$$\propto \frac{\partial}{\partial t}\left[\left(\frac{\beta ka^2}{2}\right)^{d+1}(\nabla\phi(x))^{d+1}\right]$$

$$\propto (d+1)\left(\frac{\beta ka^2}{2}\right)^{d}(\nabla\phi(x))^{d+1}$$

$$\propto (d+1)\left(\frac{a^2}{2T}\right)^{d}(\nabla\phi(x))^{d+1}$$

$$\propto T^{-d}$$

Problem #4:



"Van der Waals force"

(from problem) "Dipole-coupled Hamiltonian"

$$\hat{H} = \underbrace{\frac{\hat{p}_1^2}{2m}}_{\substack{\text{Kinetic} \\ \text{energy} \\ \#1}} + \underbrace{\frac{\hat{p}_2^2}{2m}}_{\substack{\text{Kinetic} \\ \text{energy} \\ \#2}} + \underbrace{\frac{m\omega_0^2}{2}(\hat{x}_1^2 + \hat{x}_2^2)}_{\substack{\text{Potential} \\ \text{energy} \\ \#1 \,\&\, \#2}} + \underbrace{mK\hat{x}_1\hat{x}_2}_{\substack{\text{Dipole-dipole} \\ \text{interaction}}}$$

Spectrum (Natural Frequency):

$$U = x^T A x$$

$$= x^T \frac{m}{2}\begin{pmatrix} \omega^2 & K \\ K & \omega^2 \end{pmatrix} x$$

$$U = x^T (A - \lambda)\, x$$

$$= 0$$

$$(A-\lambda) = \frac{m}{2}\begin{pmatrix} \omega_0^2 - \lambda & K \\ K & \omega_0^2 - \lambda \end{pmatrix}$$

$$= 0$$

$$\lambda = \omega_0^2 \pm K$$

$$\omega = \sqrt{\lambda}$$

$$= \sqrt{\omega_0^2 \pm K}$$

= Natural frequency

= Characteristic frequency

Energy in a single particle oscillator:

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{m\omega^2}{2}\hat{x}^2$$

$$\hat{H}\psi = U\psi$$

$$U = KE - P.E$$

$$= \hbar\omega - V(r)$$

$$= \hbar\omega - K^2/8\omega_0^2 \quad \text{.. where } V(r) = K^2/8\omega_0^2$$

$\boxed{\text{total time} \sim 11 \text{ hours}}$