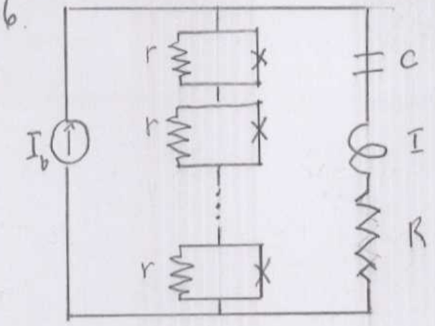
$\dot{\phi} = \Omega + a\sin\phi_K + K\sum_{j=1}^{2}\sin\phi_j$ 4.6.6.



$$\frac{\hbar}{2er}\cdot\frac{d\phi_K}{dt} + I_c\sin\phi_K + \frac{dQ}{dt} = I_b$$

$$L\frac{d^2Q}{d^2t} + R\frac{dQ}{dt} + \frac{Q}{C} = \boxed{\frac{\hbar}{2e}\sum_{j=1}^{N}\frac{d\phi_j}{dt}}$$

Chapter 5: Linear Systems

$\dot{x} = v$
$\dot{v} = -\omega^2 x$

5.1.1. a. $\frac{\dot{x}}{\dot{v}} = \frac{dx}{dv} = \frac{v}{-\omega^2 x}$; $-\omega^2 x + C = v$; $\boxed{\omega^2 x^2 + v^2 = C}$

b. Conservation of Energy: $\sum \frac{1}{2}mv^2 = E$; $\frac{1}{2}m\omega^2 x^2 + \frac{1}{2}mv^2 = C$

$\boxed{KE_{rot} + KE_{Lin} = KE_{TOT.}}$

$\dot{x} = ax$
$\dot{y} = -y$

5.1.2 $\frac{\dot{y}}{\dot{x}} = \frac{dy}{dx} = \frac{-y}{ax} = \frac{-e^{-t}}{ae^{at}} = \frac{-1}{ae^{(a+1)t}}$; $\lim_{t\to\infty}\frac{dy}{dx} = \lim_{t\to\infty}\frac{-1}{ae^{(a+1)t}} = \boxed{-\infty}$ ∥ y-axis

$\lim_{t\to-\infty}\frac{dy}{dx} = \lim_{t\to-\infty}\cdot\frac{-1}{ae^{(a+1)t}}\ \boxed{=0}$ ∥ x-axis.

$\dot{x} = -y$
$\dot{y} = -x$

5.1.3. $\begin{bmatrix}\dot{x}\\ \dot{y}\end{bmatrix} = \boxed{\begin{bmatrix}-1\\ -1\end{bmatrix}\begin{bmatrix}x & y\end{bmatrix}}$

$\dot{x} = 3x - 2y$
$\dot{y} = 2y - x$

5.1.4 $\begin{bmatrix}\dot{x}\\ \dot{y}\end{bmatrix} = \boxed{\begin{bmatrix}3 & -2\\ -1 & 2\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}}$

$\dot{x} = 0$
$\dot{y} = x + y$

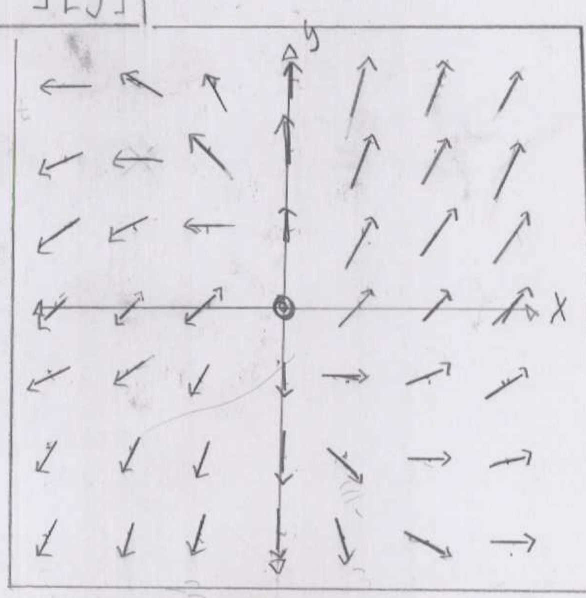
5.1.5. $\begin{bmatrix}\dot{x}\\ \dot{y}\end{bmatrix} = \boxed{\begin{bmatrix}0 & 0\\ 1 & 1\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}}$

$\dot{x} = x$
$\dot{y} = 5x + y$

5.1.6. $\begin{bmatrix}\dot{x}\\ \dot{y}\end{bmatrix} = \boxed{\begin{bmatrix}1 & 0\\ 5 & 1\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}}$

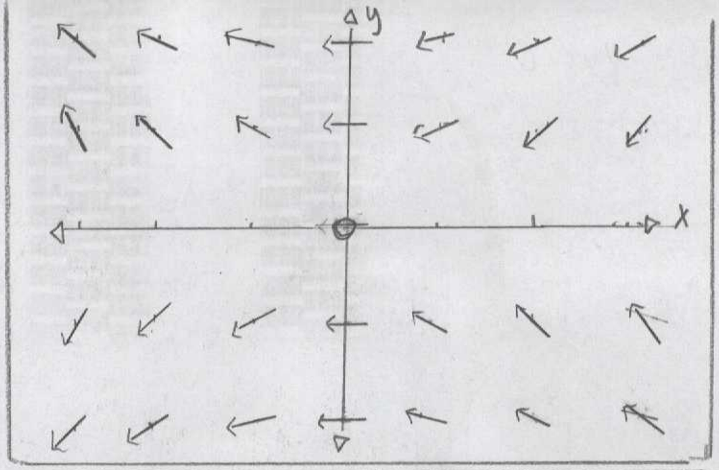
$\dot{x} = x$
$\dot{y} = x + y$

5.1.7. $\frac{dy}{dx} = \frac{x+y}{x}$



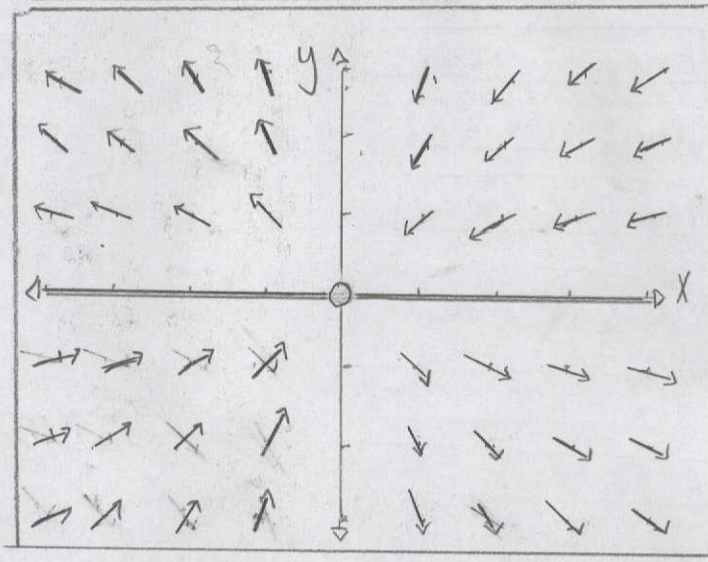
$\dot{x} = -2y$ 5.1.8. $\frac{\dot{y}}{\dot{x}} = \frac{dy}{dx} = \frac{-x}{-2y} = \frac{x}{2y}$.

$\dot{y} = -x$



$\dot{x} = -y$ 5.1.9 a) $\frac{\dot{y}}{\dot{x}} = \frac{dy}{dx} = \frac{y}{x}$

$\dot{y} = -x$



b.) $x\dot{x} = -xy$; $y\dot{y} = -xy$; therefor, $x\dot{x} = y\dot{y}$; $x\dot{x} - y\dot{y} = 0$

and $\boxed{x\,dx - y\,dy = x^2 - y^2 = C}$

c) $\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$; $\begin{pmatrix} -\lambda & -1 \\ -1 & -\lambda \end{pmatrix} = \lambda^2 + 1 = (\lambda+1)(\lambda-1) = 0$; $\lambda_1 = 1$, $\lambda_2 = -1$

$\lambda_1 = 1$; $\begin{pmatrix} +1 & -1 \\ -1 & +1 \end{pmatrix} \begin{pmatrix} V_{11} \\ V_{12} \end{pmatrix} = 0$; Guess $V_{11} = 1$, $V_{12} = -1$; $x = -C_1 e^{t}$; $y = C_2 e^{t}$

$\lambda_2 = -1$; $\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} V_{11} \\ V_{12} \end{pmatrix} = 0$; Guess $V_{11} = 1$, $V_{12} = 1$; $x = C_1 e^{-t}$; $y = C_2 e^{-t}$

General Solution: $x(t) = C_1 e^{-t} - C_1 e^{t}$; $y(t) = C_2 e^{-t} + C_2 e^{t}$

$\lim_{t \to \infty} x(t) = -\infty$; $\lim_{t \to -\infty} x(t) = \infty$: $\boxed{\text{Unstable Manifold}}$

$\lim_{t \to \infty} y(t) = \infty$; $\lim_{t \to -\infty} y(t) = \infty$: $\boxed{\text{Stable Manifold}}$

d) $u = x + y$; $\dot{u} = \dot{x} + \dot{y} = -y - x = -u$; $\boxed{u(t) = u_0 e^{-t}}$

$v = x - y$; $\dot{v} = \dot{x} - \dot{y} = x - y = v$; $\boxed{v(t) = v_0 e^{t}}$

e) $\lim_{t \to \infty} u(t) = 0$; $\lim_{t \to -\infty} u(t) = \infty$; $\lim_{t \to \infty} v(t) = \infty$; $\lim_{t \to -\infty} v(t) = 0$;

$\boxed{\text{Stable}}$ $\boxed{\text{Arbitrary}}$ $\boxed{\text{Arbitrary}}$ $\boxed{\text{Unstable}}$

f) See part C.

5.1.10

$\dot{x}=y; \dot{y}=-4x$ a) $\begin{bmatrix}\dot{x}\\ \dot{y}\end{bmatrix} = \begin{bmatrix}0 & 1\\ -4 & 0\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}; \begin{bmatrix}-\lambda & 1\\ -4 & -\lambda\end{bmatrix} = \lambda^2+4=0; \lambda_1 = \pm 2i; \lambda_2 = \pm 2i$

$\lambda_1 = 2i; \begin{bmatrix}-2i & 1\\ -4 & -2i\end{bmatrix}\begin{bmatrix}v_{11}\\ v_{12}\end{bmatrix} = 0; \quad -2iv_{11}+v_{12}=0; \quad v_{11}=1; v_{22}=2i$

$-4v_{11}-2iv_{12}=0$

$\lambda_2 = -2i; \begin{bmatrix}2i & 1\\ -4 & 2i\end{bmatrix}\begin{bmatrix}v_{11}\\ v_{12}\end{bmatrix} = 0; \quad 2v_{11}+v_{12}=0, \quad v_{11}=1; v_{22}=2i$

$-4v_{11}+2v_{12}=0$

$\boxed{\text{Liapunov Stable}}$

Identity: $e^{\lambda t} = \cos(t)+i\sin(t)$

$e^{\lambda t}V = e^{\lambda t}\begin{bmatrix}v_{11}\\ v_{12}\end{bmatrix} = \begin{bmatrix}\cos(2t)+i\sin(2t)\\ 2i(\cos(2t)+i\sin(2t))\end{bmatrix}$

$\dot{x}=2y; \dot{y}=x$ b) $\boxed{\text{None of the Above}}$

$\dot{x}=0, \dot{y}=x$ c) $\boxed{\text{None of the Above}}$

$X_1 = \begin{bmatrix}x = C_2\cos(2t) + C_1\sin(2t)\\ y = 2C_2\cos(2t) - 2C_1\sin(2t)\end{bmatrix}$

$\dot{x}=0, \dot{y}=-y$ d) $\boxed{\text{None of the Above}}$

$\dot{x}=-x; \dot{y}=-5y$ e) $\boxed{\text{Asymptotically stable}}$

$X_i = C_1 \mathrm{Re}(e^{\lambda t}V_1) + C_2 \mathrm{Im}(e^{\lambda t}V_1)$

$\dot{x}=x; \dot{y}=y$ f) $\boxed{\text{Asymptotically stable}}$

5.1.11 a) $\|x(t) = C\cos(2t) + C\sin(2t) - x^*\| < C^2 = \epsilon$

$\|x(0) = C - x^*\| < C + \delta$

b) $\begin{bmatrix}\dot{x}\\ \dot{y}\end{bmatrix} = \begin{bmatrix}0 & 2\\ 1 & 0\end{bmatrix}\begin{bmatrix}v_{11}\\ v_{12}\end{bmatrix}; \begin{bmatrix}-\lambda & 2\\ 1 & -\lambda\end{bmatrix} = \lambda^2-2=0; \lambda_1 = +\sqrt{2}; \lambda_2 = -\sqrt{2}$

$\lambda_1 = +\sqrt{2}; \begin{bmatrix}-\sqrt{2} & 2\\ 1 & -\sqrt{2}\end{bmatrix}\begin{bmatrix}v_{11}\\ v_{12}\end{bmatrix} = 0; -\sqrt{2}v_{11}+2v_{12}=0; \quad v_{11}=1; v_{12}=\frac{1}{\sqrt{2}}$

$v_{11}-\sqrt{2}v_{12}=0;$

$\lambda_2 = -\sqrt{2}; \begin{bmatrix}\sqrt{2} & 2\\ 1 & \sqrt{2}\end{bmatrix}\begin{bmatrix}v_{11}\\ v_{12}\end{bmatrix} = 0; \quad \sqrt{2}v_{11}+2v_{12}=0; \quad v_{11}=1; v_{12}=-\frac{1}{\sqrt{2}}$

$x(t) = C_1\cosh(t) + C_2\sinh(t/\sqrt{2}); \quad y(t) = x_0\cosh(t) + C_4\sinh(-t/\sqrt{2})$

$= x_0\cosh(t) + \frac{y_0}{\sqrt{2}}\sinh(t/\sqrt{2}); \quad y(t) = x_0\cosh(t) - \frac{y_0}{\sqrt{2}}\sinh(-t/\sqrt{2})$

$\|x(t) - x^*\| = \|x_0\cosh(t) + \frac{y_0}{\sqrt{2}}\sinh(t/\sqrt{2}) - 0\| \nless \epsilon$

$\|x(0) - x^*\| = \|x_0\| < \delta$ $\boxed{\text{None of the above}}$

c) $\begin{bmatrix}\dot{x}\\ \dot{y}\end{bmatrix} \ldots \lambda_1 = +2i; \lambda_2 = -2i$

$\lambda_1 = 2i \ldots e^{\lambda t}V = e^{\lambda t}\begin{bmatrix}v_{11}\\ v_{12}\end{bmatrix} = \ldots$

$\lambda_2 = -2i \ldots v_{11} \ldots \frac{1}{2}; \quad X = \begin{bmatrix}x = C_1\cos(t) + C_2\sin(t/2)\\ \ldots\cos(t) + C_2\sin(t/2)\end{bmatrix}$

c $\dot{x}=0$; $x=1+C$; $\dot{y}=x$; $\boxed{\text{None of the Above}}$

$= \text{"}C\text{"}$; $y=x_0t+C$

$=x_0$; $=x_0t+y_0$

d. $\dot{x}=0$; $x=1+C$; $\dot{y}=x$; $\boxed{\text{None of the above}}$

$=C$; $=x_0t+C$

$=x_0$; $=x_0t+y_0$

e. $\dot{x}=-x$; $\dot{y}=-5y$; $\boxed{\text{Asymptotically Stable}}$

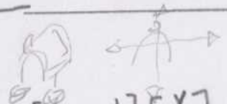
$x=x_0e^{-t}$; $\dot{y}=y_0e^{-5t}$

f. $\dot{x}=x$; $\dot{y}=y$; $\boxed{\text{Asymptotically Stable}}$.

$x=e^{t}$; $y=e^{+t}$

$\dot{x}=v$; $\dot{v}=-x$ 5.1.12 v-axis @ $(0,-v_0)$; x-axis @ $(x,0)$; $\boxed{\dot{v}(0)=-v=v_0 ;\ \dot{x}(x)=0}$

5.1.13 The "saddle point" is a category of bifurcation that is parabolic beyond a coordinate. A connection to real saddles is the "curved" shape where the rider sits



$\dot{x}=4x-y$

$\dot{y}=2x+y$

5.2.1 a. $\dot{x}=A\vec{x}$; $\begin{bmatrix}\dot{x}\\ \dot{y}\end{bmatrix}=\begin{bmatrix}4 & -1\\ 2 & 1\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}$; $\begin{bmatrix}4-\lambda & -1\\ 2 & 1-\lambda\end{bmatrix}=(4-\lambda)(1-\lambda)+2=\lambda^2-5\lambda+6=0$

$\lambda_1=2$; $\lambda_2=3$

$\lambda_1=2$; $\begin{bmatrix}2 & -1\\ 2 & -1\end{bmatrix}\begin{bmatrix}v_{11}\\ v_{12}\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$; $2v_{11}-v_{12}=0$; $v_{11}=1$; $v_{12}=2$; $\vec{v}_1=\begin{bmatrix}1\\ 2\end{bmatrix}$

$\lambda_2=3$; $\begin{bmatrix}1 & -1\\ 2 & -2\end{bmatrix}\begin{bmatrix}v_{21}\\ v_{22}\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$; $v_{21}-v_{22}=0$; $v_{21}=1$; $v_{22}=1$; $\vec{v}_2=\begin{bmatrix}1\\ 1\end{bmatrix}$

b) General Solution: $X(t)=\begin{bmatrix}x(t)\\ y(t)\end{bmatrix}=C_1e^{\lambda_1t}V_1+C_2e^{\lambda_2t}V_2=C_1e^{2t}\begin{bmatrix}1\\ 2\end{bmatrix}+C_2e^{3t}\begin{bmatrix}1\\ 1\end{bmatrix}$

$\boxed{x(t)=C_1e^{2t}+C_2e^{3t}}$

$\boxed{y(t)=2C_1e^{2t}+C_2e^{3t}}$

c) $\boxed{\text{Stable}}$

d) $(x_0,y_0)=(3,4)$; $3=C_1+C_2$; $4=2C_1+C_2$

$C_1=3-C_2$; $4=2(3-C_2)+C_2=6-2C_2+C_2$

$=6-C_2$; $C_2=2$; $C_1=1$

$\boxed{x(t)=e^{2t}+2e^{3t}}$

$\boxed{y(t)=2e^{2t}+2e^{3t}}$

$\dot{x} = x - y$
$\dot{y} = x + y$

5.2.2. a) $X = Ax$; $\begin{bmatrix}\dot{x}\\ \dot{y}\end{bmatrix} = \begin{bmatrix}1 & -1\\ 1 & 1\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}$; $\begin{vmatrix}1-\lambda & -1\\ 1 & 1-\lambda\end{vmatrix} = (1-\lambda)^2 + 1 = \lambda^2 - 2\lambda + 2$

$= \lambda_1 = 1 - i$; $\lambda_2 = 1 + i$

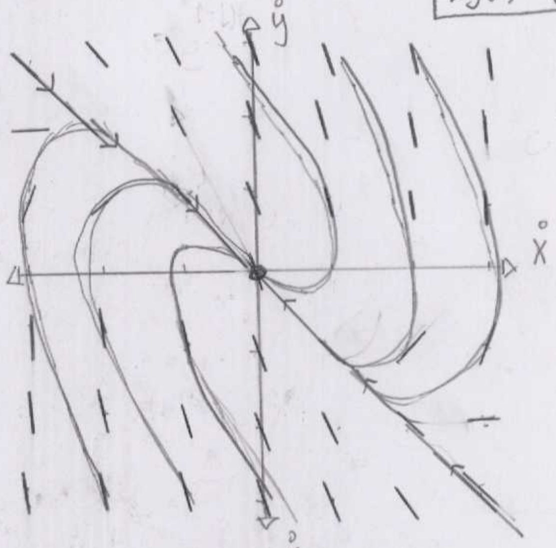
$\lambda_1 = 1 - i$; $\begin{bmatrix}1-(1-i) & -1\\ 1 & 1-(1-i)\end{bmatrix}\begin{bmatrix}V_{11}\\ V_{12}\end{bmatrix} = \begin{bmatrix}0\\ 0\end{bmatrix}$; $iV_{11} - V_{12} = 0$; $\boxed{\vec{V}_1 = \begin{bmatrix}1\\ i\end{bmatrix}}$

$\lambda_2 = 1 + i$; $\begin{bmatrix}1-(1+i) & -1\\ 1 & 1-(1+i)\end{bmatrix}\begin{bmatrix}V_{21}\\ V_{22}\end{bmatrix} = \begin{bmatrix}0\\ 0\end{bmatrix}$; $-iV_{21} - V_{22} = 0$; $\boxed{\vec{V}_2 = \begin{bmatrix}1\\ -i\end{bmatrix}}$

b. General Solution: $\vec{X}(t) = V_1 e^{\lambda_1 t} + V_2 e^{\lambda_2 t} = \boxed{\begin{bmatrix}x(t) = e^t \circ 2\cos(t)\\ y(t) = e^t \circ 2i\sin(t)\end{bmatrix}}$

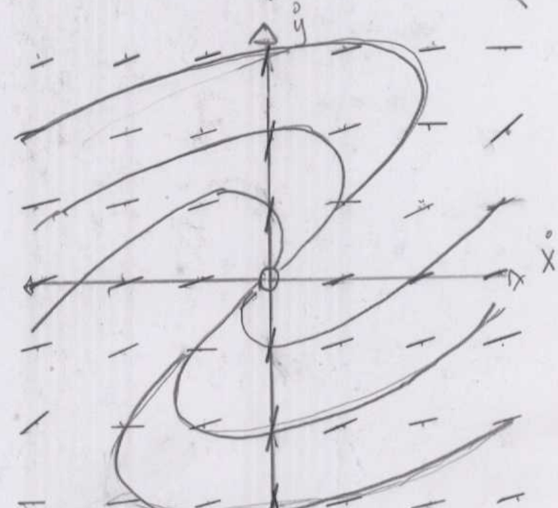
$\dot{x} = y$
$\dot{y} = -2x - 3y$

5.2.3. $\frac{\dot{y}}{\dot{x}} = \frac{dy}{dx} = -2 \cdot \frac{x}{y} - 3$



$\dot{x} = 5x + 10y$
$\dot{y} = -x - y$

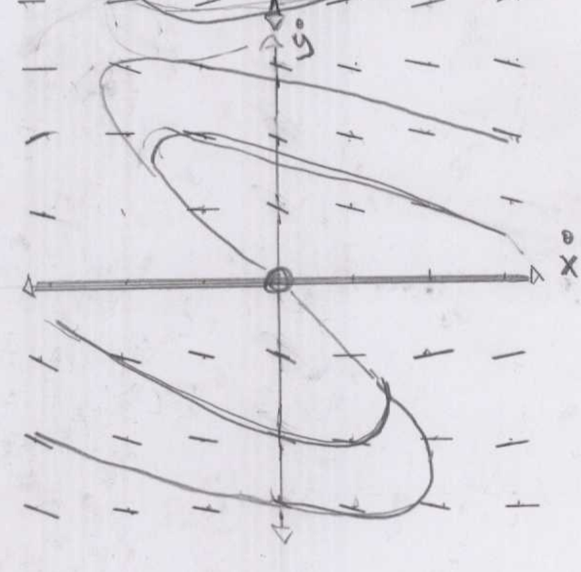
5.2.4 $\frac{\dot{y}}{\dot{x}} = \frac{dy}{dx} = \frac{-x-y}{5x+10y}$



Switched

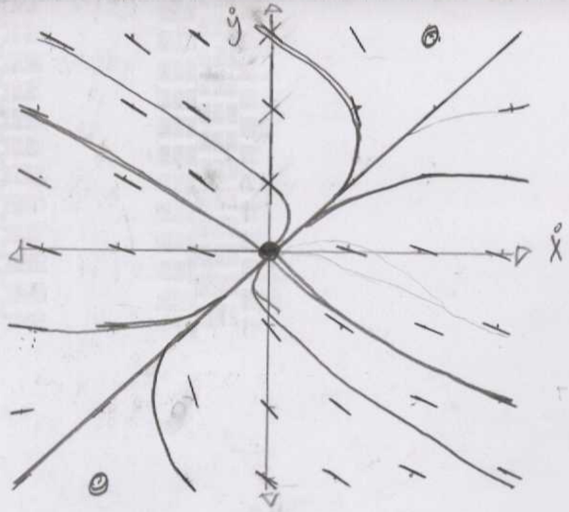
$\dot{x} = 3x - 4y$
$\dot{y} = x - y$

5.2.5. $\frac{\dot{y}}{\dot{x}} = \frac{dy}{dx} = \frac{x-y}{3x-4y}$



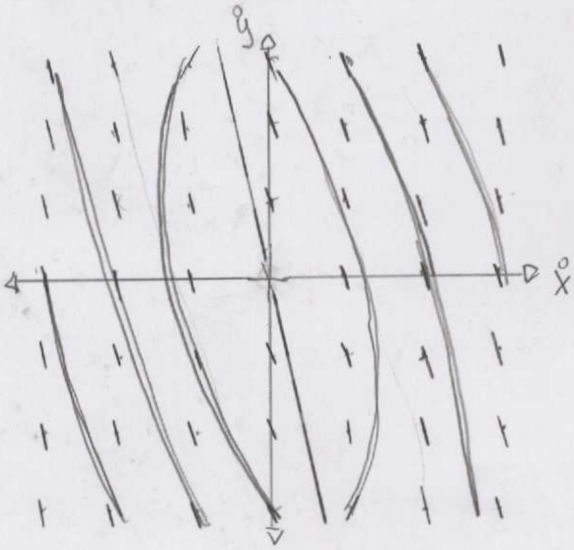
$\dot{x} = -3x+2y$
$\dot{y} = x-2y$

5.2.6. $\dfrac{\dot{y}}{\dot{x}} = \dfrac{dy}{dx} = \dfrac{x-2y}{-3x+2y}$



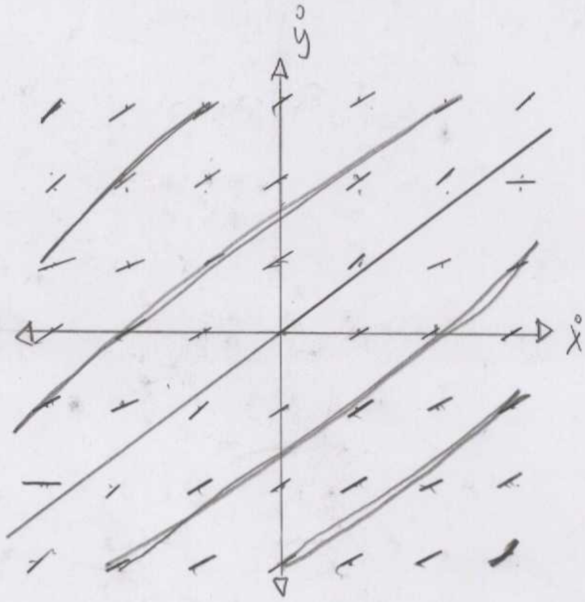
$\dot{x} = 5x+2y$
$\dot{y} = -17x-5y$

5.2.7. $\dfrac{\dot{y}}{\dot{x}} = \dfrac{dy}{dx} = \dfrac{-17x-5y}{5x+2y}$



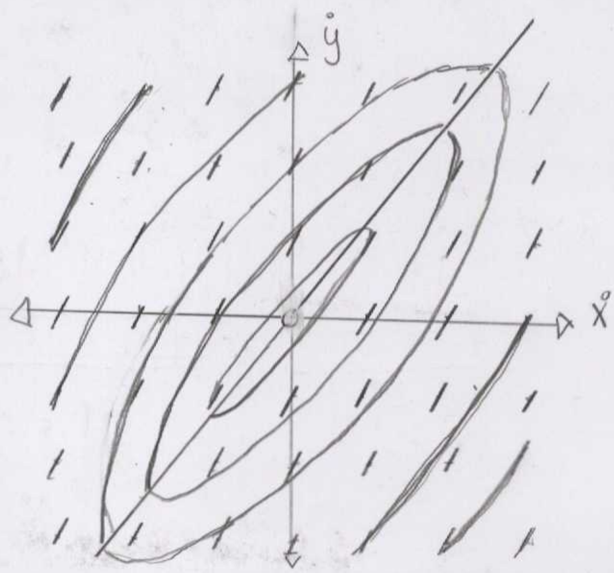
$\dot{x} = -3x+4y$
$\dot{y} = -2x+3y$

5.2.8. $\dfrac{\dot{y}}{\dot{x}} = \dfrac{dy}{dx} = \dfrac{-2x+3y}{-3x+4y}$



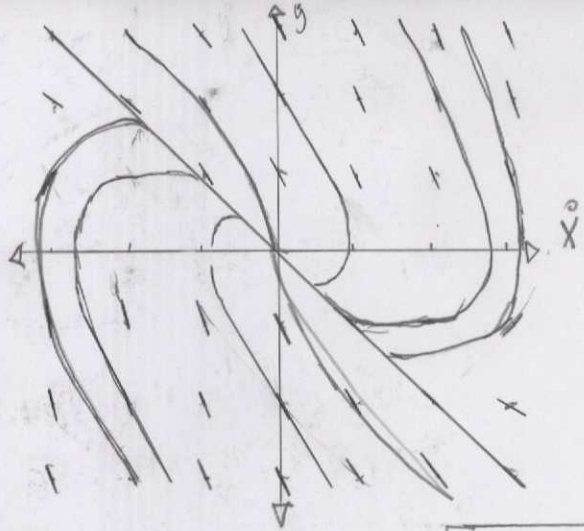
$\dot{x} = 4x-3y$
$\dot{y} = 8x-6y$

5.2.9. $\dfrac{\dot{y}}{\dot{x}} = \dfrac{dy}{dx} = \dfrac{8x-6y}{4x-3y}$



$\dot{x} = y$
$\dot{y} = -x - 2y$

5.2.10. $\frac{\dot{y}}{\dot{x}} = \frac{dy}{dx} = \frac{-x-2y}{y}$

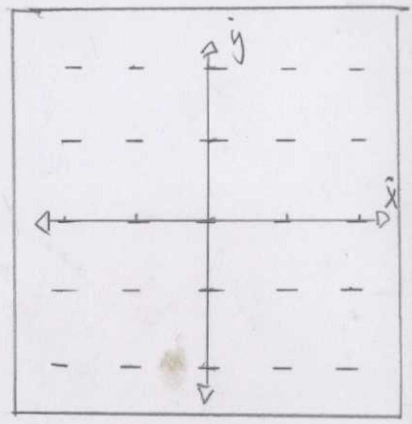


$A = \begin{pmatrix} \lambda & b \\ 0 & \lambda \end{pmatrix}; \lambda = 0$

5.2.11 $A - \begin{pmatrix} \lambda & b \\ 0 & \lambda \end{pmatrix} = \lambda^2 = 0; \lambda = 0$

$\dot{x} = Ax; \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} \lambda & b \\ 0 & \lambda \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}; \dot{x} = by; \dot{y} = 0; \frac{\dot{y}}{\dot{x}} = 0$

The book shows a typeset to the correct solution.



$L\ddot{I} + R\dot{I} + \frac{I}{C} = 0$

5.2.12

a) $x = I \quad y = \dot{I}$

$\dot{x} = \dot{I} \quad \dot{y} = \ddot{I} = -R\dot{I} - \frac{I}{C} = -Ry - \frac{x}{C}$; $\boxed{\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{1}{C} & -R \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}}$

b) $R = 0$; $\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1/C & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$; $\begin{bmatrix} -\lambda & 1 \\ -1/C & -\lambda \end{bmatrix} = \lambda^2 + \frac{1}{C} = 0$; $\lambda_{1,2} = \frac{i}{\sqrt{C}}$

$\lambda_{1,2} = \frac{i}{\sqrt{C}}$; $\begin{bmatrix} -i/C & 1 \\ -1/C & -i/\sqrt{C} \end{bmatrix} \begin{bmatrix} V_{11} \\ V_{12} \end{bmatrix} = 0$; $\frac{V_{11}}{C} + \frac{iV_{12}}{\sqrt{C}} = 0$; $V_{11} = -i\sqrt{C}$; $V_{12} = 1$

General Solution: $e^{\lambda_{1,2} t} = \cos\left(\frac{t}{\sqrt{C}}\right) + i\sin\left(\frac{t}{\sqrt{C}}\right)$

$$e^{\lambda_{1,2} t} \cdot \vec{V} = \begin{bmatrix} \sqrt{C}\sin(t/\sqrt{C}) - i\sqrt{C}\cos(t/\sqrt{C}) \\ \cos(t/\sqrt{C}) + i\sin(t/\sqrt{C}) \end{bmatrix}$$

$$X = \begin{bmatrix} x = C_1\sin(t/\sqrt{C}) - C_2\cos(t/\sqrt{C}) \\ y = C_1\cos(t/\sqrt{C}) + C_2\sin(t/\sqrt{C}) \end{bmatrix}$$

$\boxed{\text{Neutrally Stable}}$

$R > 0$; $\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{1}{C} & -R \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$; $\begin{bmatrix} -\lambda & 1 \\ -1/C & -R-\lambda \end{bmatrix} = \lambda(R+\lambda) + \frac{1}{C} = 0$; $\lambda_{1,2} = \frac{-R \pm \sqrt{R^2 - 4(1)(1/C)}}{2(1)}$

$\lambda_1 = \frac{-R + \sqrt{R^2 - 4(1)(1/C)}}{2}$; $\begin{bmatrix} \frac{R - \sqrt{R^2 - 4/C}}{2} & 1 \\ -1/C & \frac{-R - \sqrt{R^2 - 4/C}}{2} \end{bmatrix} \begin{bmatrix} V_{11} \\ V_{12} \end{bmatrix} = 0$

$$\frac{R-\sqrt{R^2-4/C}}{2}\cdot V_{11} + V_{12} = 0\ ;\quad V_{11} = 1\ ;\ V_{12} = -\frac{R+\sqrt{R^2-4/C}}{2}$$

$$\lambda_2 = \frac{-R-\sqrt{R^2-4/C}}{2}\ ;\quad \begin{bmatrix} \frac{R+\sqrt{R^2-4/C}}{2} & 1 \\ -1/C & \frac{-R+\sqrt{R^2-4/C}}{2} \end{bmatrix}\begin{bmatrix} V_{11} \\ V_{12} \end{bmatrix} = 0$$

$$\frac{R+\sqrt{R^2-4/C}}{2}\cdot V_{11} + V_{12} = 0\ ;\quad V_{11} = 1\ ;\ V_{12} = \frac{+R+\sqrt{R^2-4/C}}{2}$$

General Solution: $X_i = C_i e^{\lambda_i t} V_1$

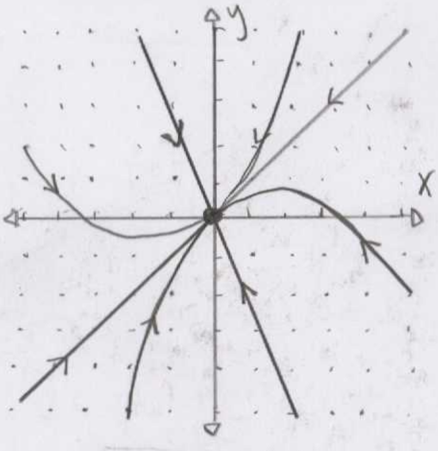
$$X_1 = C_1 e^{\frac{-R+\sqrt{R^2-4/C}}{2}t}\begin{bmatrix} 1 \\ \frac{-R+\sqrt{R^2-4/C}}{2} \end{bmatrix}$$

$$X_2 = C_2\cdot e^{\frac{-R-\sqrt{R^2-4/C}}{2}t}\begin{bmatrix} 1 \\ \frac{R+\sqrt{R^2-4/C}}{2} \end{bmatrix}$$

$$\bar{X} = X_1 + X_2 = \begin{bmatrix} x(t) = C_1 e^{\frac{-R+\sqrt{R^2-4/C}}{2}t} + C_2 e^{\frac{-R-\sqrt{R^2-4/C}}{2}t} \\ y(t) = C_1 e^{\frac{-R+\sqrt{R^2-4/C}}{2}t}\cdot\left(\frac{-R+\sqrt{R^2-4/C}}{2}\right) + C_2 e^{\frac{-R-\sqrt{R^2-4/C}}{2}t}\cdot\left(\frac{R+\sqrt{R^2-4/C}}{2}\right) \end{bmatrix}$$

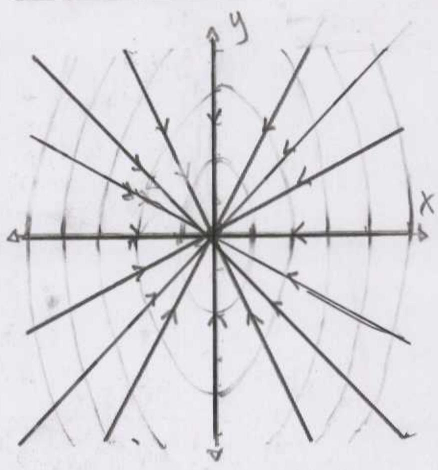
$\boxed{\text{Asymptotically Stable}}$

C. $R^2C-4L>0$



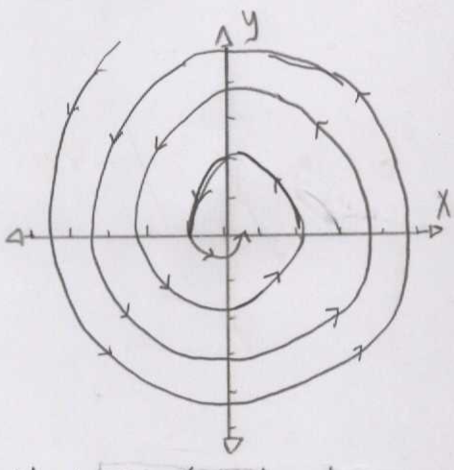
Unstable Node

$R^2C-4L=0$



Star, Degenerate Node

$R^2C-4L<0$



Unstable Spiral

$R^2C-4L>0$

$R^2C>4L$

$C>\frac{4L}{R}$

$R^2C>4L$

$m\ddot{x}+b\dot{x}+kx=0$ 5.2.13:

a. $i = x \quad j = \dot{x}$

$\dot{i} = \dot{x} \quad \dot{j} = \ddot{x} = -\frac{b\dot{x}}{m} - \frac{kx}{m} = -\frac{b}{m}j - \frac{k}{m}i$; $\boxed{\begin{bmatrix} \dot{i} \\ \dot{j} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{b}{m} & -\frac{k}{m} \end{bmatrix} \begin{bmatrix} i \\ j \end{bmatrix}}$

b. $\vec{\dot{I}} = A\vec{i}$; $\begin{bmatrix} -\lambda & 1 \\ -\frac{b}{m} & -\frac{k}{m}-\lambda \end{bmatrix} = \lambda^2 + \frac{k}{m}\lambda + \frac{b}{m} = 0$; $\lambda_{1,2} = \frac{-\frac{k}{m} \pm \sqrt{(\frac{k}{m})^2 - 4(1)(b/m)}}{2(1)}$

$\lambda_1 = \frac{-\frac{k}{m} + \sqrt{(\frac{k}{m})^2 - 4(b/m)}}{2}$; $\begin{bmatrix} \frac{\frac{k}{m} - \sqrt{(\frac{k}{m})^2 - 4(b/m)}}{2} & 1 \\ -\frac{b}{m} & \frac{-\frac{k}{m} - \sqrt{(k/m)^2 - 4(b/m)}}{2} \end{bmatrix} \begin{bmatrix} v_{11} \\ v_{12} \end{bmatrix} = 0$

$\left(\frac{k}{m} - \sqrt{(\frac{k}{m})^2 - 4(b/m)}\right) v_{11} + v_{12} = 0$

$v_{11} = 1$; $v_{12} = -\frac{k}{m} + \sqrt{(\frac{k}{m})^2 - 4(b/m)}$

$\lambda_2 = \frac{-\frac{k}{m} - \sqrt{(\frac{k}{m})^2 - 4(b/m)}}{2}$; $\begin{bmatrix} \frac{\frac{k}{m} + \sqrt{(\frac{k}{m})^2 - 4(b/m)}}{2} & 1 \\ -\frac{b}{m} & \frac{-\frac{k}{m} + \sqrt{(k/m)^2 - 4(b/m)}}{2} \end{bmatrix} \begin{bmatrix} v_{21} \\ v_{22} \end{bmatrix} = 0$

$\frac{(\frac{k}{m}) + \sqrt{(k/m)^2 - 4(b/m)}}{2} v_{21} + v_{22} = 0$

$v_{21} = 1$; $v_{22} = \frac{-(\frac{k}{m}) - \sqrt{(k/m)^2 - 4(b/m)}}{2}$

$x(t) = c_1 e^{\frac{-(\frac{k}{m}) + \sqrt{(\frac{k}{m})^2 - 4(b/m)}}{2} t} + c_2 e^{\frac{-(\frac{k}{m}) - \sqrt{(k/m)^2 - 4(b/m)}}{2} t}$

$y(t) = c_1 \cdot \frac{(-\frac{k}{m}) + \sqrt{(\frac{k}{m})^2 - 4(b/m)}}{2} \cdot e^{\frac{-(\frac{k}{m}) + \sqrt{(k/m)^2 - 4(b/m)}}{2} t} + c_2 \frac{(-\frac{k}{m}) - \sqrt{(k/m)^2 - 4(b/m)}}{2} \cdot e^{\frac{-(\frac{k}{m}) - \sqrt{(\frac{k}{m})^2 - 4(b/m)}}{2} t}$

Unstable Spiral : $\sqrt{(\frac{k}{m})^2 - 4(b/m)} < 0$

Star, Degenerate Node : $\sqrt{(\frac{k}{m})^2 - 4(b/m)} = 0$

Unstable Node : $\sqrt{(\frac{k}{m})^2 - 4(b/m)} > 0$

c. Star, Degenerate Node is critically damped. An unstable spiral is underdamped. While an unstable node is an unstable node.

$\dot{x} = Ax$ 5.2.14 $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$;

$\boxed{\begin{array}{l} \lambda_2 < \lambda_1 < 0 : \text{Stable Node} \\ \lambda_{1,2} = \kappa \pm i\omega < 0 : \text{Stable Spiral} \\ \lambda_1 = \lambda_2 = \lambda : \text{Star Node, Degenerate Node} \\ \lambda_{1,2} = \kappa \pm i\omega > 0 : \text{Unstable Spiral} \\ \lambda_2 > \lambda_1 > 0 \ : \text{Unstable Node} \end{array}}$

$$A = \begin{pmatrix} a-\lambda & b \\ c & d-\lambda \end{pmatrix} = (a-\lambda)(d-\lambda) - bc = \lambda^2 - (a+d)\lambda + (ad-bc) = 0$$

$$\lambda_{1,2} = \frac{(a+d) \pm \sqrt{(a+d)^2 - 4(ad-bc)}}{2}$$

Pseudo-code

If $(a+d)^2 - 4(ad-bc) < 0$, then λ_1 & λ_2 are imaginary.

If $(a+d) > 0$, then λ_1, λ_2 are an unstable spiral.

else, $\lambda_1 ; \lambda_2$ are a stable spiral.

Else

Doub $\lambda_1 = \dfrac{(a+d) + \sqrt{(a+d)^2 - 4(ad-bc)}}{2}$

Doub $\lambda_2 = \dfrac{(a+d) - \sqrt{(a+d)^2 - 4(ad-bc)}}{2}$

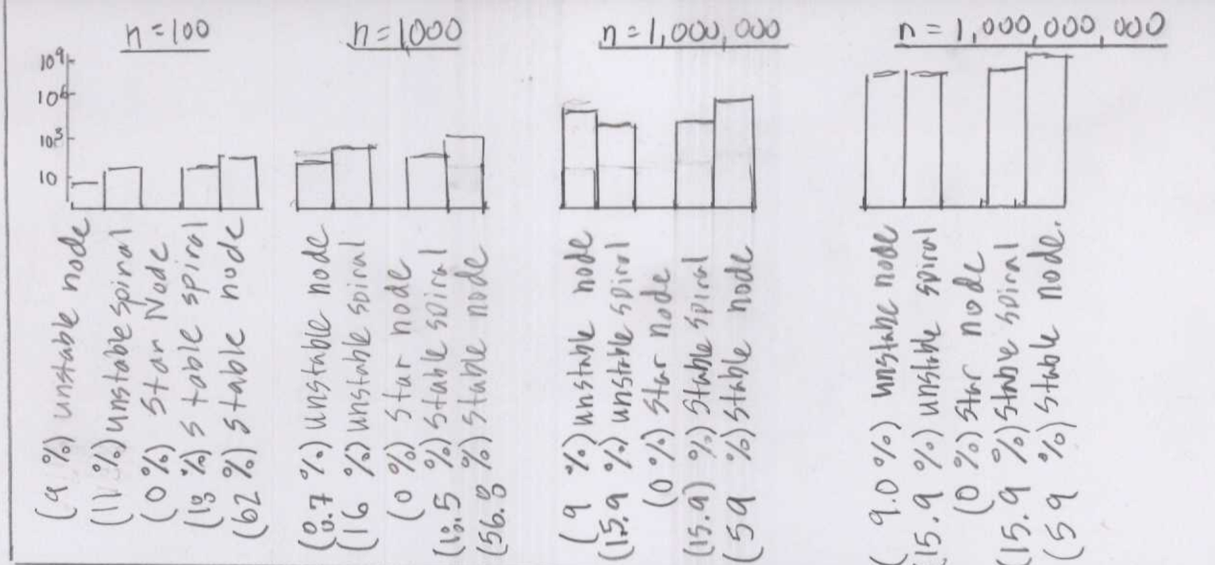
If $(\lambda_1 = \lambda_2)$, then Star Node, Degenerate Node

Else if(λ_1 & $\lambda_2 > 0$), then unstable Node

else, Stable node.

Real-code

```
#include <iostream>
#include <random>
using namespace std;
int main() {
  int i, s_node=0, u_node=0, star=0, s_spiral=0, u_spiral=0, trials=100;
  double l1, l2, a, b, c, d;
  std:default_random_engine_generator;
  std:uniform_real_distribution (-1,1);
  if(((a+d)-4(ad-bc))<0){
     if((a+d)>0) u_spiral+=1;
     else s_spiral+=1;
  }else{
    l1=((a+d)+sqrt(pow((a+d),2)-4(ad-bc)))/2;
    l2=((a+d)-sqrt(pow((a+d),2)-4(ad-bc)))/2;
    if(l1=l2) star+=1
    else if(l1&&l2>0) u_node+=1
    else s_node+=1;
  }
} }
```



An unstable spiral approaches the limit of 9%; while, stable spirals the most common, at 59%.
A normal distribution produced greater proportions of the stable phase plots (stable node [83%], stable spiral [14.8%], ...). than the uniform distribution modelled.

$\dot{R} = aR + bJ$

5.3.1. R = Romeo's love/hate; J = Juliet's love/hate; a = b = romantic style.

$\dot{R} = J$

$\dot{J} = -R + J$

5.3.2.

b \ a	(+)	(−)
(+)	Star crossed romance	cautious romance
(−)	Indifferent romance.	Sensitive romance

a. $\begin{bmatrix}\dot{R}\\ \dot{J}\end{bmatrix} = \begin{bmatrix}0 & 1\\ -1 & 1\end{bmatrix}\begin{bmatrix}R\\ J\end{bmatrix}$ "cautious romance"

b. $(R, J) = (0,0) =$ Neverending love/hate
= Stable Node

c. $R(0) = 1$; $J(0) = 0$;

$$\begin{bmatrix}0-\lambda & 1\\ -1 & 1-\lambda\end{bmatrix} = -\lambda(1-\lambda) + 1 = \lambda^2 - \lambda + 1 = 0$$

$$\lambda_{1,2} = \frac{+1 \pm \sqrt{1-4(1)(1)}}{2} = \frac{1 \pm \sqrt{3}i}{2}$$

$$\lambda_{1,2} = \frac{1+\sqrt{3}i}{2};\ \begin{bmatrix}\frac{1-\sqrt{3}i}{2} & 1\\ -1 & \frac{-1-\sqrt{3}i}{2}\end{bmatrix}\begin{bmatrix}V_{11}\\ V_{12}\end{bmatrix} = \begin{bmatrix}0\\ 0\end{bmatrix};\ \frac{-1+\sqrt{3}i}{2}V_{11} + V_{12} = 0;\ \vec{V}_{1,2} = \begin{bmatrix}1\\ \frac{+1\pm\sqrt{3}i}{2}\end{bmatrix}$$

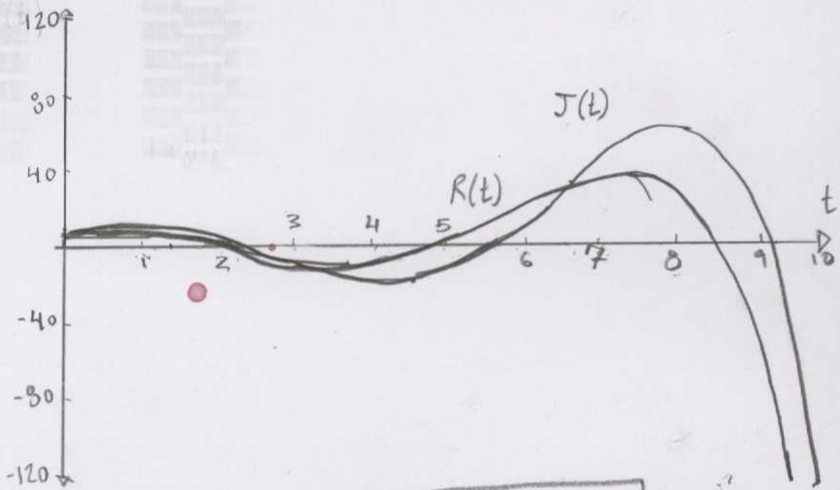
General Solution: $\vec{X} = \begin{bmatrix} R(t) = e^{\frac{t}{2}} \cdot 2\left[C_1\cos\left(\frac{\sqrt{3}}{2}t\right) + C_2\sin\left(\frac{\sqrt{3}}{2}t\right)\right] \\ J(t) = e^{\frac{t}{2}} \cdot (1+\sqrt{3}i) \cdot \left[C_1\cos\left(\frac{\sqrt{3}}{2}t\right) + C_2\sin\left(\frac{\sqrt{3}}{2}t\right)\right] \end{bmatrix}$

$$\vec{X} = \begin{bmatrix} R(t) = 2e^{\frac{t}{2}} \cdot \left[C_1\cos\left(\frac{\sqrt{3}}{2}t\right) + C_2\sin\left(\frac{\sqrt{3}}{2}t\right)\right] \\ J(t) = (1+\sqrt{3})e^{\frac{t}{2}} \cdot C_1\cos\left(\frac{\sqrt{3}}{2}t\right) + (1-\sqrt{3})e^{\frac{t}{2}}C_2\sin\left(\frac{\sqrt{3}}{2}t\right) \end{bmatrix}$$

$R(0) = 1$; $C_1 = \frac{1}{2}$

$J(0) = 0$; $0 = C_1 + \sqrt{3}C_2$; $C_2 = \frac{-1}{2\sqrt{3}}$

Final Solution: $\vec{X} = \begin{bmatrix} R(t) = e^{t/2}\left[\cos\left(\frac{\sqrt{3}}{2}t\right) - \frac{1}{\sqrt{3}}\sin\left(\frac{\sqrt{3}}{2}t\right)\right] \\ J(t) = e^{t/2}\left[\frac{(1+\sqrt{3})}{2}\cos\left(\frac{\sqrt{3}}{2}t\right) - \frac{(1-\sqrt{3})}{2\sqrt{3}}\sin\left(\frac{\sqrt{3}}{2}t\right)\right] \end{bmatrix}$



$\dot{R} = aJ$ 5.3.3.
$\dot{J} = bR$

b \ a	(+)	(−)
(+)	$a^2>b^2$: Stable $a^2<b^2$: Unstable $a=b$: Mutual	Stable center of neverending love & hate
(−)	Stable center of neverending love & hate	$a^2>b^2$: Stable $a^2<b^2$: Unstable $a=b$=Mutual

Romeo and Juliet live happily when Juliet's love is of greater amounts.

$\dot{R} = aR + bJ$ 5.3.4.
$\dot{J} = -bR - aJ$

b \ a	(+)	(−)
(+)	$a^2>b^2$: Unstable $a^2<b^2$: Stable $a=b$: Star, degen node	$a^2>b^2$: Unstable $a^2<b^2$: Stable $a=b$: Star, degen node
(−)	$a^2>b^2$: Unstable $a^2<b^2$: Stable $a=b$: Star, degen node	$a^2>b^2$: Unstable $a^2<b^2$: Stable $a=b$: Star, degen node

Yes, opposites attract when the proportion of Romeo's love is larger.

$\dot{R} = aR + bJ$ 5.3.5.
$\dot{J} = bR + aJ$

b \ a	(+)	(−)
(+)	$a^2>b^2$: Unstable Node $a^2\leq b^2$: Unstable Saddle	$a^2>b^2$: Unstable Node $a^2\leq b^2$: Unstable Saddle
(−)	$a^2>b^2$: Unstable Node $a^2\leq b^2$: Unstable Saddle	$a^2>b^2$: Unstable Node $a^2\leq b^2$: Unstable Saddle

The marraige of Romeo and Juliet of exact clone demonstrate an unstable relationship for all time.

$\dot{R} = 0$

$\dot{J} = aR + bJ$

5.3.6.

b \ a	(+)	(−)
(+)	Unstable and Fixed Relationship	$a^2 > b^2$: Stable $a^2 < b^2$: Unstable $a = b$: Isolated
(−)	$a^2 > b^2$: Unstable $a^2 < b^2$: Stable $a = b$: Isolated.	Stable and Fixed Relationship

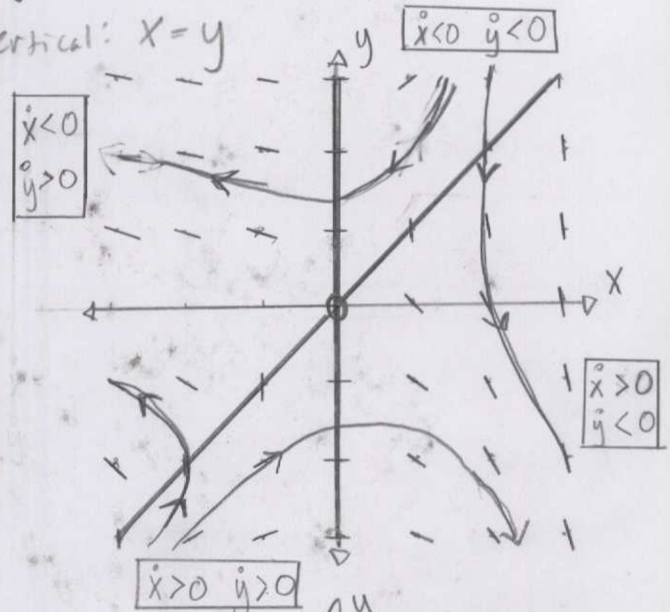
Chapter 6: Phase Plane:

$\dot{x} = x - y$

$\dot{y} = 1 - e^x$

6.1.1: $\boxed{\text{Fixed Points}}$: $\dot{x} = 0 = x - y$; $x = y$; $\dot{y} = 0 = 1 - e^x$; $x = 0$; $(x^*, y^*) = (0,0)$

$\boxed{\text{Nullclines}}$: Horizontal: $x = 0$; Vertical: $x = y$



$\dot{x} = x - x^3$

$\dot{y} = -y$

6.1.2: $\boxed{\text{Fixed Points}}$: $\dot{x} = 0 = x - x^3$

$x^* = 1, 0, -1$

$\dot{y} = 0 = -y$

$y^* = 0$

$(x^*, y^*) = (1,0), (0,0), (-1,0)$

$\boxed{\text{Nullclines}}$: Horizontal: $y = 0$

Vertical: $x = 0, 1, -1$