Chapter 7: Plane Electromagnetic Waves and Wave Propagation:

7.1 a) (Equation 7.27) $s_0 = |\epsilon_1 \cdot E|^2 + |\epsilon_2 \cdot E| = a_1^2 + a_2^2$

$$s_1 = |\epsilon_1 \cdot E| - |\epsilon_2 \cdot E| = a_1^2 - a_2^2$$

$$s_2 = 2\mathrm{Re}[(\epsilon_1 \cdot E)^*(\epsilon_2 \cdot E)] = 2a_1 a_2 \cos(\delta_2 \delta_1)$$

$$s_3 = 2 \cdot \mathrm{Im}[(\epsilon_1 \cdot E)^*(\epsilon_2 \cdot E)] = 2a_1 a_2 \sin(\delta_2 \delta_1)$$

(Equation 7.26) $E_1 = a_1 e^{i\delta_1}$

$$E_2 = a_2 e^{i\delta_2}$$

s_0, $s_0 = |E_1|^2 + |E_2|^2$ ... when $a = \epsilon_1$ or ϵ_2

$$s_1 = |E_1|^2 - |E_2|^2$$

$$s_2 = 2\mathrm{Re}(E_1 * E_2)$$

$$s_3 = 2\,\mathrm{Im}(E_1 * E_2)$$

A solution to E_1 by adding:

$$s_0 + s_1 = 2|E_1|^2$$

$$|E_1| = \sqrt{\frac{s_0 + s_1}{2}}$$

A solution to E_2 by subtraction:

$$s_0 - s_1 = 2|E_2|^2$$

$$|E_2| = \sqrt{\frac{s_0 - s_1}{2}}$$

A solution to $\delta_2 - \delta_1$ by inverting cosine:

$$s_2 = 2E_1E_2\cos(\delta_2 - \delta_1)$$

$$\delta_2 - \delta_1 = \cos^{-1}\left(\frac{s_2}{(s_0+s_1)(s_0-s_1)}\right)$$

$$= \cos^{-1}\left(\frac{s_2}{\sqrt{s_0^2 - s_1^2}}\right)$$

$$E = \begin{bmatrix} |E_1| e^{i\theta_1} \\ |E_2| e^{i\theta_2} \end{bmatrix}$$

$$= e^{i\theta_1} \begin{bmatrix} |E_1| \\ |E_2| e^{i(\theta_2 - \theta_1)} \end{bmatrix}$$

$$= \frac{1}{\sqrt{2}} \begin{bmatrix} \sqrt{S_0 + S_1} \\ \dfrac{S_2 \pm i S_3}{\sqrt{S_0 + S_1}} \end{bmatrix}$$

$$= \frac{1}{\sqrt{2}} \begin{bmatrix} \sqrt{2} \\ \sqrt{2} - i\sqrt{2} \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 1 - i \end{bmatrix}$$

Circularly Polarized light:

(Equation 7.26) $E_+ = a_+ e^{i\delta_+}$

$E_- = a_- e^{i\delta_-}$

(Equation 7.28) $S_0 = |\epsilon_+^* \circ E| + |\epsilon_- \circ E|^2 = a_+^2 + a_-^2$

$S_1 = 2\,\mathrm{Re}\left[(\epsilon_+^* \circ E) * (\epsilon_-^* \circ E)\right] = 2a_+ \cdot a_- \cdot \cos(\delta_- - \delta_+)$

$S_2 = 2\,\mathrm{Im}\left[(\epsilon_+^* \circ E) * (\epsilon_-^* \circ E)\right] = 2a_+ \cdot a_- \sin(\delta_- - \delta_+)$

$S_3 = |\epsilon_+^* \circ E|^2 - |\epsilon_- \circ E|^2 = a_+^2 - a_-^2$

So, $S_0 = |E_+|^2 + |E_-|^2$

$S_1 = 2|E_+||E_-|\cos(\delta_- - \delta_+)$

$S_2 = 2|E_+||E_-|\sin(\delta_- - \delta_+)$

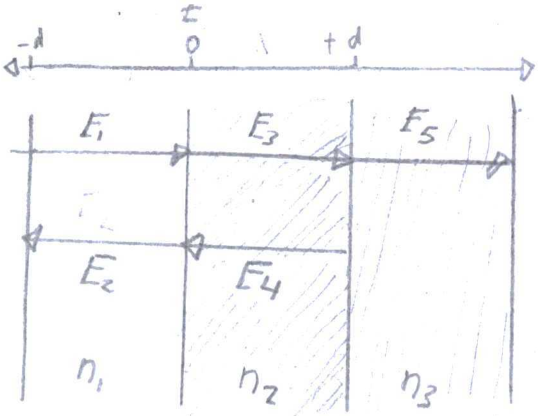
$S_3 = |E_+|^2 - |E_-|^2$

A solution to E_+ by adding:

$S_0 + S_3 = 2|E_+|^2$

$|E_+| = \sqrt{\dfrac{S_0 + S_3}{2}}$

7.2.



"A plane wave incident on a layered surface"

Interface #1:

$$E_1 + E_2 = E_3 + E_4$$

$$\sqrt{\frac{\epsilon_1}{\mu_0}}\left(E_1 e^{ik_1 z} - E_2 e^{-ik_1 z}\right) = \sqrt{\frac{\epsilon_2}{\mu_0}}\left(E_3 e^{ik_1 z} - E_4 e^{ik_1 z}\right)$$

$$\sqrt{\frac{\epsilon_1}{\mu_0}}\left(E_1 - E_2\right) = \sqrt{\frac{\epsilon_2}{\mu_0}}\left(E_3 - E_4\right) \ldots \text{ at } z=0$$

Interface #2:

$$\sqrt{\frac{\epsilon_2}{\mu_0}}\left(E_3 e^{ik_2 z} - E_4 e^{-ik_2 z}\right) = \sqrt{\frac{\epsilon_3}{\mu_0}}\left(E_5 e^{ik_3(z-d)}\right)$$

If $\alpha = \sqrt{\frac{\epsilon_3}{\epsilon_2}}$ and $\beta = \sqrt{\frac{\epsilon_2}{\epsilon_1}}$

$$E_1 + E_2 = E_3 + E_4$$

$$(E_1 - E_2) = \beta(E_3 - E_4) \quad \ldots \text{ at } z=d$$

$$E_3 e^{ik_2 d} + E_4 e^{-ik_2 d} = E_5$$

$$E_3 e^{ik_2 d} - E_4 e^{-ik_2 d} = \alpha E_5$$

$$2E_1 = (1+\beta)E_3 + (1-\beta)E_4$$

$$2E_2 = (1-\beta)E_3 + (1+\beta)E_4$$

$$2E_3 e^{ik_2 d} = (1+\alpha)E_5$$

$$2E_4 e^{ik_2 d} = (1-\alpha)E_5$$

then, $$E_1 = \left[\frac{(1+\alpha)(1+\beta)e^{-ik_2 d} + (1-\alpha)(1-\beta)e^{ik_2 d}}{4}\right] E_5$$

$$= \left[\frac{(\alpha\beta+1)\cos(k_2 d) - (\alpha+\beta)i\sin(k_2 d)}{2}\right] E_5$$

A solution to E_- by subtracting:

$$S_0 - S_3 = 2|E_-|^2$$

$$|E_-|^2 = \sqrt{\frac{S_0 - S_3}{2}}$$

A solution to phase$(\delta_- - \delta_+)$ by inverting cosine:

$$S_1 = 2|E_+||E_-|\cos(\delta_- - \delta_+)$$

$$\delta_- - \delta_+ = \cos^{-1}\left(\frac{S_1}{\sqrt{(S_0+S_1)(S_0-S_1)}}\right)$$

$$= \cos^{-1}\left(\frac{S_1}{\sqrt{S_0^2 - S_1^2}}\right)$$

$$E = \begin{bmatrix} |E_1| e^{i\delta_-} \\ |E_2| e^{i\delta_+} \end{bmatrix}$$

$$= \frac{1}{\sqrt{2}} \begin{bmatrix} \sqrt{S_0 + S_3} \\ \frac{S_1 + iS_2}{\sqrt{S_0 + S_3}} \end{bmatrix}$$

$$= \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1+2i \end{bmatrix}$$

Transmission: $T = \left(\frac{E_1}{E_5}\right)^2$

$$= \frac{(\alpha\beta+1)^2 - (1-\alpha^2)(1-\beta^2)\sin^2(n_2 d)}{4}$$

$$= \frac{4 n_1 n_2^2 n_3}{n_2^2(n_1+n_3)^2 + (n_2^2-n_3^2)(n_2^2-n_1^2)\sin^2\left(\frac{n_2 w d}{c}\right)}$$

Reflection: $R = 1 - T$

$$= 1 - \frac{4 n_1 n_2^2 n_3}{n_2^2(n_1+n_3)^2 + (n_2^2-n_3^2)(n_2^2-n_1^2)\sin^2\left(\frac{n_2 w d}{c}\right)}$$

If $n_1 = 3$, $n_2 = 2$, and $n_3 = 1$, then

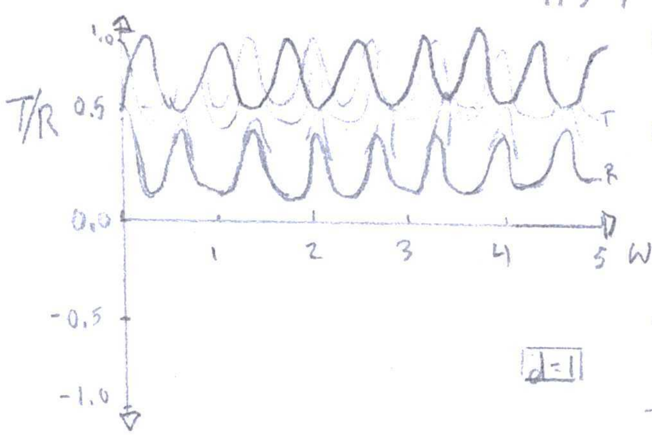
$$T = \frac{96}{113 + 15\cos(4wd/c)}$$

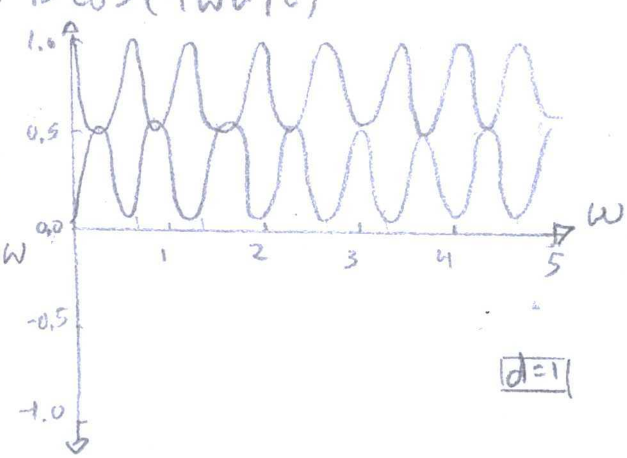
If $n_1 = 2$, $n_2 = 4$, and $n_3 = 1$, then

$$T = \frac{32}{9(4 + 5\sin^2(4wd/c))}$$

If $n_1 = 1$, $n_2 = 2$, and $n_3 = 3$, then

$$T = \frac{96}{113 + 15\cos(4wd/c)}$$





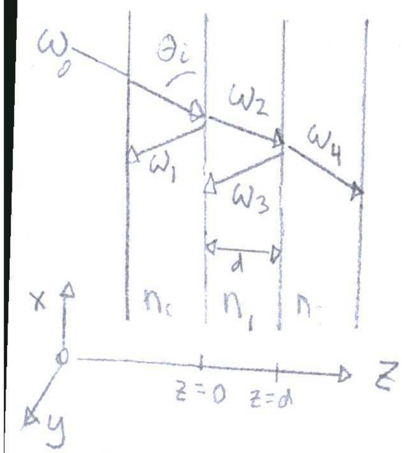
b) $n_3 = 1$, $R(\omega_0) = 1 - \dfrac{4n_1}{(1+n_1)^2 + \dfrac{(1-n_2)^2(n_1^2-n_2^2)\sin^2(n_2\omega d/c)}{n_2^2}}$

$= 0$

$(n_2-1)^2 n_2^2 + (1-n_2^2)(n_1^2-n_2^2)\sin^2(n_2\omega d/c) = 0$

$n_1 n_2 = 1$ and $d = \dfrac{2\pi k c}{\omega n_2}$

7.3



"Two plane semi-infinite slabs... seperated by an air gap"

Parallel:

$$E_1 = \epsilon_0 E_0 e^{i(\frac{n}{c}\omega_0 k_\cdot x - \omega_0 t)} + \epsilon_1 E_1 e^{i(\frac{n}{c}\omega_1 k x - \omega t}$$

$$E_2 = \epsilon_2 E_2 e^{i(\frac{n}{c}\omega k - \omega_2 t)} + \epsilon_3 E_3 e^{i(\frac{n}{c}\omega_3 k - \omega_3 t)}$$

$$E_3 = \epsilon_4 E_4 e^{i(\frac{n}{c}\omega_4 k(x-d) - \omega t)}$$

Boundary Conditions:

@ $z=0,\ n_1 = 1$

$$e^{(\frac{n}{c}\omega k x - \omega t)} = e^{i(\frac{n}{c}\omega_1 k x - \omega_1 t)} = e^{i(\frac{1}{c}\omega_2 k x - \omega_2 t)} = e^{i(\frac{1}{c}\omega_3 k x - \omega t)}$$

@ $z=d,\ n_1 = 1$

$$e^{i(\frac{1}{c}\omega_2 k x - \omega_2 t)} = e^{i(\frac{1}{c}\omega_3 k x - \omega_3 t)} = e^{i(\frac{n}{c}\omega_4 k(x-d) - \omega_4 t)}$$

If $\omega = \omega_1 = \omega_2 = \omega_3 = \omega_4$

then $n\hat{k}x = n\hat{k}_1 x = k_2 x = k_3 x$

and

$k_2 x = k_3 x = n k_4 (x-d)$

Thus, $k_x = n\sin\theta \quad k_z = n\cos\theta$

By "snell's Law"

$n_1 \sin\theta_{i} = \sin\theta_r = n \sin\theta_{i}$

Snell's Law
$n\sin\theta_i = n\sin\theta_r$

and

$$\sin\theta_{g,i} = \sin\theta_{g,r} = n\sin\theta_{t,i}$$

Eventually,

$$E_0 = \epsilon_0 E_0 e^{i(\frac{n}{c}\omega(\sin\theta x + \cos\theta z) - \omega t)} + \epsilon_1 E_1 e^{i(\frac{n}{c}\omega(\sin\theta x - \cos\theta z) - \omega t)}$$

$$E_1 = \epsilon_2 E_2 e^{i(\frac{1}{c}\omega(n\sin\theta_i x + \sqrt{1-n^2\sin^2\theta}\, z) - \omega t)} + \epsilon_3 E_3 e^{i(\frac{1}{c}\omega(n\sin\theta x - \sqrt{1-n^2\sin^2\theta}\, z) - \omega t)}$$

$$E_3 = \epsilon_4 E_4 e^{i(\frac{n}{c}(\sin\theta x + \cos\theta(z-d)) - \omega t)}$$

$\therefore$ if $1 = K_x^2 + K_y^2$

and $\cos\theta_i = \sqrt{1-n^2\sin^2\theta}$

Simplification,

$$E_0 = \epsilon_0 E_0 e^{i(\frac{n}{c}\omega\cos\theta z)} + \epsilon_1 E_1 e^{-i(\frac{1}{c}\omega\cos\theta_i \cdot z)}$$

$$E_1 = \epsilon_2 E_2 e^{i(\frac{1}{c}\cos\theta_i z)} + \epsilon_3 E_3 e^{-i(\frac{1}{c}\omega\cos\theta_i z)}$$

$$E_3 = \epsilon_4 E_4 e^{i(\frac{n}{c}\omega\cos\theta_i(z-d))}$$

<u>Perpendicular:</u>

$\circ\, B = (n/c)\, K \times E$ is in the y-direction.

$$E_0 = \hat{y} E_0 e^{i(\frac{n}{c})\omega\cos\theta_i z} + \hat{y} E_1 e^{-i\frac{n}{c}\omega\cos\theta z}$$

$$E_1 = \hat{y} E_2 e^{i\frac{1}{c}\omega\cos\theta_i z} + \hat{y} E_3 e^{-i\frac{1}{c}\omega\cos\theta_i z}$$

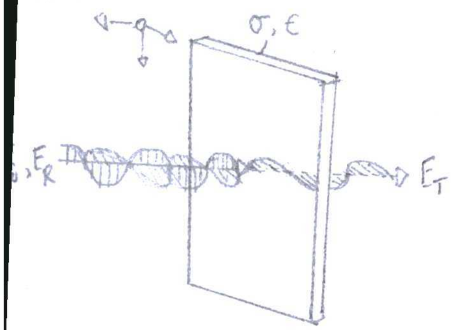
$$E_2 = \hat{y} E_4 e^{i\frac{n}{c}\omega\cos\theta(z-d)}$$

$$B_0 = \frac{n}{c}(\sin\theta z - \cos\theta x) E_0 e^{i\frac{n}{c}\cos\theta z} + \frac{n}{c}(\sin\theta z + \cos\theta x) E_1 e^{-i\frac{n}{c}\omega\cos\theta}$$

$$B_1 = \frac{1}{c}(n\sin\theta z - \cos\theta x) E_2 e^{i\frac{1}{c}\cos\theta z} + \frac{1}{c}(n\sin\theta z + \cos\theta x) E_3 e^{-i\frac{1}{c}\cos\theta t}$$

$$B_2 = \frac{n}{c}(\sin\theta z - \cos\theta x) E_4 e^{i\frac{n}{c}\omega\cos\theta(z-d)}$$

7.4.



"Plane polarized electromagnetic wave... incident normally on a flat surface"

a) Boundary conditions:

$$E_i = E_T - E_R$$

Note: Only in plane-polarized and 0° angle

$$kE = kE_R + k_T E_T$$

Incident wave:

$$E_R = \frac{k}{k_T}E - \frac{k}{k_T}E_R - E$$

$$= \frac{k - k_T}{k + k_T}E$$

$$\frac{E_R}{E} = \frac{1 - \sqrt{\frac{\epsilon}{\epsilon_0} + i \cdot \frac{\sigma}{\epsilon_0 \omega}}}{1 + \sqrt{\frac{\epsilon}{\epsilon_0} + i \cdot \frac{\sigma}{\epsilon_0 \omega}}}$$

Wavenumber:

$$k = \omega / c$$

$$= \sqrt{\epsilon(\omega)\mu_0}\,\omega$$

Dielectric:

$$\epsilon(\omega) = \epsilon + i\frac{\sigma}{\omega}$$

$$\left|\frac{E_R}{E}\right| = \sqrt{\frac{\left(1 - \sqrt{\frac{\epsilon}{\epsilon_0} - i\frac{\sigma}{\epsilon_0 \omega}}\right)\left(1 - \sqrt{\frac{\epsilon}{\epsilon_0} + i\frac{\sigma}{\epsilon_0 \omega}}\right)}{\left(1 + \sqrt{\frac{\epsilon}{\epsilon_0} - i\frac{\sigma}{\epsilon_0 \omega}}\right)\left(1 + \sqrt{\frac{\epsilon}{\epsilon_0} + i\frac{\sigma}{\epsilon_0 \omega}}\right)}}$$

"Amplitude"

$$= \sqrt{\frac{1 + \sqrt{\frac{\epsilon^2}{\epsilon_0^2} + \frac{\sigma^2}{\epsilon_0^2 \omega^2}} - 2\,\mathrm{Re}\sqrt{\frac{\epsilon}{\epsilon_0} + i\frac{\sigma}{\epsilon_0 \omega}}}{1 + \sqrt{\frac{\epsilon^2}{\epsilon_0^2} + \frac{\sigma^2}{\epsilon_0^2 \omega^2}} + 2\,\mathrm{Re}\sqrt{\frac{\epsilon}{\epsilon_0} + i\frac{\sigma}{\epsilon_0 \omega}}}}$$

$$\mathrm{Arg}\left(\frac{E_R}{E}\right) = \tan^{-1}\left(-i\,\frac{\frac{E_R}{E} - \frac{E_R}{E}^*}{\frac{E_R}{E} + \frac{E_R}{E}^*}\right)$$

Identity:

$$\mathrm{Arg}(z) = \tan^{-1}\left(-i\frac{z - z^*}{z + z^*}\right)$$

$$= \tan^{-1}\left(\frac{2\,\mathrm{Im}\sqrt{\frac{\epsilon}{\epsilon_0} - i\frac{\sigma}{\epsilon_0 \omega}}}{1 - \sqrt{\frac{\epsilon^2}{\epsilon_0^2} + \frac{\sigma^2}{\epsilon_0^2 \omega^2}}}\right)$$

"Phase"

b) Poor conductor $\longleftrightarrow$ good conductor

$\boxed{\sigma \ll \epsilon_0 \omega}$ $\boxed{\sigma \gg \epsilon_0 \omega}$

$$\left|\frac{E_R}{E}\right| = \sqrt{\frac{1+\frac{\epsilon}{\epsilon_0} - 2\sqrt{\frac{\epsilon}{\epsilon_0}}}{1+\frac{\epsilon}{\epsilon_0} + 2\sqrt{\frac{\epsilon}{\epsilon_0}}}}$$

$$= \frac{1-\sqrt{\frac{\epsilon}{\epsilon_0}}}{1+\sqrt{\frac{\epsilon}{\epsilon_0}}}$$

= "Amplitude of a poor conductor"

Identity:

$$Im(\sqrt{x^2+y^2}) = Im(x^2+y^2)e^{\frac{i}{2}\tan^{-1}(\ldots}$$

$$= (x^2+y^2)^{1/4}\sin(\tfrac{1}{2}\tan^{-1}(y/\ldots$$

Small angle identity:

$y \ll x$, $\tan^{-1}(y/x) = y/x$

$\sin(\tfrac{1}{2}y/x) = \tfrac{1}{2}y/x$

$$Arg\left(\left|\frac{E_R}{E}\right|\right) = \tan^{-1}\left(\frac{\sqrt{\frac{\epsilon}{\epsilon_0}}\,\frac{\sigma}{\epsilon\omega}}{\frac{\epsilon}{\epsilon_0}-1}\right)$$ ... when $Im(\sqrt{x+iy}) = \frac{1}{2}\sqrt{x}\,y/x$

$$\lim_{\sigma\to 0} Arg\left(\left|\frac{E_R}{E}\right|\right) = \pi + \frac{\sqrt{\frac{\epsilon}{\epsilon_0}}\,\frac{\sigma}{\epsilon\omega}}{\frac{\epsilon}{\epsilon_0}-1}$$ ... "phase of a poor conductor"

When $\sigma \gg \epsilon_0\omega$,

Amplitude

$$\left|\frac{E_R}{E}\right| = \sqrt{\frac{1+\frac{\sigma}{\epsilon_0\omega} - \sqrt{2}\sqrt{\frac{\sigma}{\epsilon_0\omega}}}{1+\frac{\sigma}{\epsilon_0\omega} + \sqrt{2}\sqrt{\frac{\sigma}{\epsilon_0\omega}}}}$$

$$= \sqrt{\frac{1-\sqrt{2}\sqrt{\frac{\epsilon_0\omega}{\sigma}}}{1-\sqrt{2}\sqrt{\frac{\epsilon_0\omega}{\sigma}}}}$$

$$= \frac{1-\frac{1}{2}\sqrt{2}\sqrt{\frac{\epsilon_0\omega}{\sigma}}}{1+\frac{1}{2}\sqrt{2}\sqrt{\frac{\epsilon_0\omega}{\sigma}}}$$

$$= \frac{1+\frac{1}{2}\frac{\epsilon_0\omega}{\sigma} - \sqrt{2}\sqrt{\frac{\epsilon_0\omega}{\sigma}}}{1-\frac{1}{2}\frac{\epsilon_0\omega}{\sigma}}$$

$$= 1-\sqrt{2}\sqrt{\frac{\epsilon_0\omega}{\sigma}}$$

Binomial Expansion:

$\sqrt{1+x} = 1 + (1/2)x + \ldots$

Phase

$$Arg\left(\frac{E_R}{E}\right) = \tan^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{\sigma}{\epsilon_0\omega}}}{\frac{\sigma}{\epsilon_0\omega}-1}\right)$$

$$= \tan^{-1}\left(\sqrt{2}\sqrt{\frac{\epsilon_0\omega}{\sigma}}\right)$$

$$= \pi + \sqrt{\frac{2\epsilon_0 \omega}{\sigma}}$$

Reflection Coefficient:

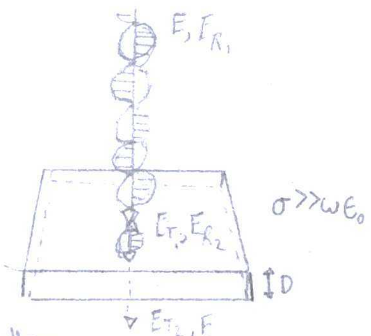
$$R = \left|\frac{E_R}{E}\right|^2 = \left(1 - \sqrt{2}\sqrt{\frac{\epsilon_0 \omega}{\sigma}}\right)^2$$

$$= 1 + 2\frac{\epsilon_0 \omega}{\sigma} - 2\sqrt{2}\sqrt{\frac{\epsilon_0 \omega}{\sigma}}$$

$$\cong 1 - 2\sqrt{2}\sqrt{\frac{\epsilon_0 \omega}{\sigma}}$$

$$\cong 1 - 2\frac{\omega}{c}\delta \quad \ldots \text{where } \delta = \sqrt{\frac{2}{\mu_0 \sigma \omega}}$$

7.5.



"Plane-polarized light... incident a flat sheet"

a) $E = E_0 e^{i(kx-\omega t)}$

Boundary Conditions:

$$E + E_{R_1} = E_T + E_{R_2}$$

$$(E - E_{R_1}) = \beta(E_{T_1} - E_{R_2}) \quad \ldots \text{where } \beta = \sqrt{\frac{\epsilon}{\epsilon_0}} = \sqrt{1 - i\frac{\sigma}{\epsilon_0 \omega}} = \alpha^{-1}$$

$$E_{T_2} = E_T e^{ik_2 D} + E_{R_2} e^{-ik_2 D}$$

$$\alpha E_{T_2} = E_T e^{ik_2 D} - E_{R_2} e^{-ik_2 D}$$

Incident Wave:

$$\left(\frac{E_T}{E}\right)^{-1} = \cos(k_2 D) - (\alpha + \beta)\, i \sin(k_2 D)$$

If $1 \ll \sigma/\omega_0 \epsilon_0$, then $\beta = \sqrt{i\frac{\sigma}{\epsilon_0 \omega}}$

$$= (1+i)\sqrt{\frac{\sigma}{2\epsilon_0 \omega}}$$

$$= \frac{2}{\gamma} \quad \ldots \text{where } \gamma = \sqrt{\frac{2\epsilon_0 \omega}{\sigma}}(1-i)$$

If $\lambda = \frac{(1-i) \cdot D}{\delta}$, then $k_2 D = \frac{2\omega D}{\gamma c} \approx i\lambda \qquad \delta = \sqrt{\frac{2}{\mu_0 \sigma \omega}}$

$$\left(\frac{E}{E_T}\right) = \cos(i\lambda) - \frac{\beta}{2} i \sin(i\lambda)$$

$$= \frac{2\gamma e^{\lambda}}{\gamma(1+e^{-2\lambda})+2(1-e^{-2\lambda})}$$

$$E_{R_1} = \frac{(1-\alpha\beta)\cos(k_2 D)-(\alpha-\beta)i\sin(k_2 D)}{(1+\alpha\beta)\cos(k_2 D)-(\alpha+\beta)i\sin(k_2 D)} E$$

$$\frac{E_{R_1}}{E} = \frac{\frac{2}{\gamma} i \sin(i\lambda)}{2(E/E_T)}$$

$$= -\frac{E_T}{2\gamma E}(e^{\lambda}-e^{-\lambda})$$

$$= -\frac{1}{2\gamma}\frac{E_T}{E}\frac{1-e^{-2\lambda}}{e^{-\lambda}}$$

$$= \frac{-(1-e^{-2\lambda})}{(1-e^{-2\lambda})+\gamma(1+e^{-2\lambda})}$$

b) Zero thickness: $\lim_{D\to 0} = \lim_{\lambda\to 0}$

$$\lim_{D\to 0}\frac{E_{R_1}}{E} = \lim_{\lambda\to 0}\frac{E_{R_1}}{E}$$

$$= 0$$ "Zero reflection"

$$\lim_{D\to 0}\frac{E_T}{E} = \lim_{\lambda\to 0}\frac{E_T}{E}$$

$$= 1$$ "100% transmission at zero thickness"

Infinite Thickness:

$$R = \lim_{D\to\infty}\left(\left|\frac{E_{R_1}}{E}\right|^2\right)$$

$$= \lim_{\lambda\to\infty}\left(\left|\frac{E_{R_1}}{E}\right|^2\right)$$

$$= \left(\frac{-1}{1+\gamma}\right)$$

$$T = \lim_{D\to\infty}\left(\left|\frac{E_T}{E}\right|^2\right)$$

$$= \lim_{\lambda \to \infty} \left(\left| \frac{E_T}{E} \right|^2 \right)$$

$$= 0$$

c) $$\lim_{\gamma \to 0} T = \lim_{\gamma \to 0} \left(\left| \frac{E_T}{E} \right| \right)$$

$$= \lim_{\gamma \to 0} \left(\frac{2\gamma e^{-\lambda}}{\gamma(1+e^{-2\lambda}) + 2(1-e^{-2\lambda})} \right)^2$$

$$\cong \left(\frac{2\gamma e^{-\lambda}}{(1-e^{-2\lambda})} \right)^2$$

$$\cong \frac{4|\gamma|^2 e^{-2D/\delta}}{1+e^{-4D/\delta} - 2Re(e^{-2\lambda})}$$

$$\cong \frac{8Re(\gamma)^2 \cdot e^{-2D/\delta}}{1 - 2\cos(2D/\delta) e^{-2D/\delta} + e^{-4D/\delta}}$$

4 very small thickness,

$$Re(\lambda) = Re[(1-i)D/\delta]$$

$$\lambda > D/\delta$$

$$> D/\sqrt{2/\omega\mu\sigma}$$

$$D < \frac{2\lambda}{\omega\mu\sigma}$$

7.6.

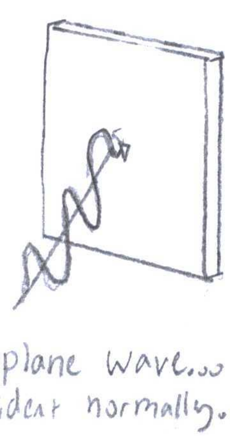
$$n(\omega) = [n^2(\omega) = \epsilon(\omega)/\epsilon_0]$$

Plane, linearly polarized, monochromatic electromagnetic

wave equation:

$$E = \epsilon_0 E_0 e^{i(kx - \omega t + \theta)}$$

$$= \epsilon_0 E_0 e^{i((\beta + i\alpha/2) - \omega t + \theta)}$$



"A plane wave... incident normally... from vacuum on a semi-infinite slab"

@ $z=0$, $t=\Theta/\omega$

(1.1a) "Faradays Law"

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

$$B = -\int (\nabla \times E)\, dt$$

$$= \frac{E_0}{i\omega}(k \times \epsilon_1)\, e^{-\alpha \hat{k} \circ x/2} \circ e^{i\beta k \cdot x - i\omega t + i\theta}$$

$$= -\frac{i}{c} E_0 (k \times \epsilon_1)\, e^{-\alpha k \cdot x/2} \circ e^{i\beta k \cdot x - i\omega t + i\theta}$$

(7.47) "Poynting Vector"

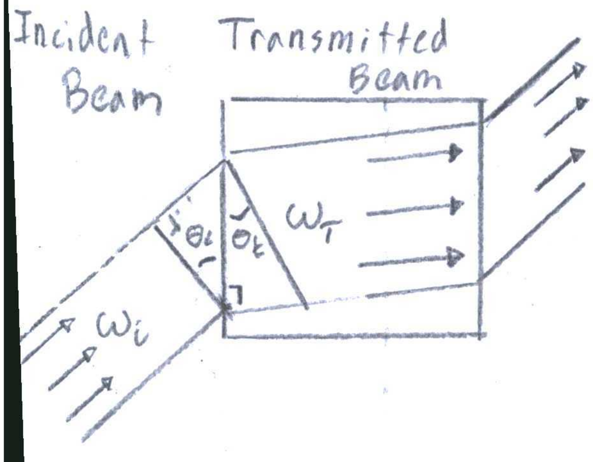
$$k \circ S = \frac{1}{2\mu} k \cdot E \times B^*$$

$$= \frac{|E_0|^2}{2\mu c} e^{-\alpha \hat{k} \cdot x}$$

% Transmitted

$$\text{Incident Power} = \frac{\text{Incident Power Transmitted}}{\text{Total Incident power}}$$

$$= \frac{\mathrm{Real}(\hat{k} \circ S_T(x=0))\, A_t}{\mathrm{Real}(\hat{k} \circ S_i(x=0))\, A_i}$$



$$\frac{A_t}{A_i} = \frac{l w_T}{l w_i}$$

$$= \frac{l w \cos \theta_T}{l w \cos \theta_i}$$

$$= \frac{\cos \theta_T}{\cos \theta_i}$$

$$= \frac{\sqrt{1 - (n/n')^2 \sin^2 \theta_i}}{\cos \theta_i}$$

"Snell's law"

Perpendicular:

$$T = \frac{Re(k \cdot S(x=0))}{Re(k \cdot S(x=0))} \frac{\sqrt{1-(n_i/n_T)^2 \sin^2\theta_i}}{\cos\theta_i}$$

$$= \frac{Re(n)}{Re(n)} \frac{\mu_i \sqrt{1-(n_i/n_t)^2 \sin^2\theta_i}}{\mu_T \cos\theta_i} \left| \frac{E_T}{E_i} \right|^2$$

$$= \frac{\mu_i \sqrt{1-(n/n')^2 \sin^2\theta_i}}{\mu_T \cos\theta_i} \left| \frac{\frac{n_i}{\mu_i}\cos\theta_i + \frac{n_i}{\mu_i}\cos\theta_i}{\frac{n_i}{\mu_i}\cos\theta_i + \frac{n_t}{\mu_t}\cos\theta_t} \right|$$

$$= \frac{\mu_i \sqrt{1-(n/n')^2 \sin^2\theta}}{\mu_t \cos\theta_i} \left| \frac{2\frac{n_i}{\mu_i}\cos\theta_i}{\frac{n_i}{\mu_i}\cos\theta_i + \frac{n_t}{\mu_t}\cos\theta_t} \right|$$

$$R = \left| \frac{E_R}{E_i} \right|^2$$

$$= \left| \frac{\frac{n_i}{\mu_i}\cos\theta_i - \frac{n_t}{\mu_t}\cos\theta_t}{\frac{n_i}{\mu_i}\cos\theta_i + \frac{n_t}{\mu_t}\cos\theta_t} \right|$$

Parallel:

$$T = \frac{Re(n)}{Re(n)} \frac{\mu_i \sqrt{1-(n_i/n_t)^2 \sin^2\theta}}{\mu_t \cos\theta_i} \left| \frac{\frac{n_i}{\mu_i}\cos\theta_i + \frac{n_i}{\mu_i}\cos\theta_i}{\frac{n_t}{\mu_t}\cos\theta_i + \frac{n_t}{\mu_i}\cos\theta_t} \right|^2$$

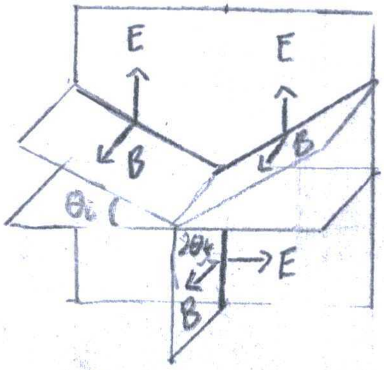
$$= \frac{\mu_i \sqrt{1-(n_i/n_t)^2 \sin^2\theta}}{\mu_t \cos\theta_i} \left| \frac{2\frac{n_i}{\mu_i}\cos\theta_i}{\frac{n_t}{\mu_t}\cos\theta_i + \frac{n_t}{\mu_i}\cos\theta_t} \right|^2$$

$$R = \left| \frac{E_R}{E_i} \right|^2$$

$$= \left| \frac{\frac{n_t}{\mu_t}\cos\theta_i - \frac{n_i}{\mu_i}\cos\theta_t}{\frac{n_t}{\mu_t}\cos\theta_i + \frac{n_t}{\mu_i}\cos\theta_t} \right|^2$$

Note: Other methods about derivation derive from two images:

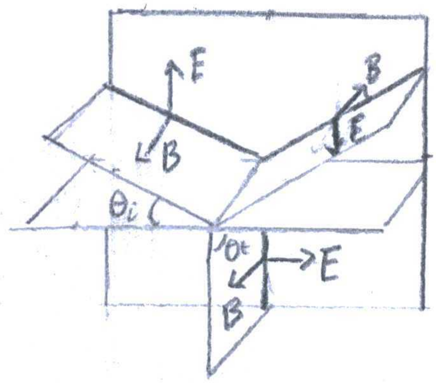
<u>Perpendicular</u>



$$R_\perp = \frac{n_i \cos\theta_i - n_t \cos\theta_t}{n_i \cos\theta_i + n_t \cos\theta_t}$$

S-polarized

<u>Parallel:</u>



$$R_\parallel = \frac{n_i \cos\theta_t - n_t \cos\theta_i}{n_i \cos\theta_t + n_t \cos\theta_i}$$

p-polarized

Citation: "Brown, Fresnel"

If $\theta_i = 0^\circ$, $\mu_T = \mu_i$, $n_i = 1$, and $n_t = n(\omega)$

"Normal Incidence" "Non-magnetic" "Atmosphere" "Index of refraction at a Frequency"

Perpendicular: $R = \left| \frac{1 - n(\omega)}{1 + n(\omega)} \right|^2$

$$T = \frac{4R(n(\omega))}{|1 + n(\omega)|^2}$$

Parallel: $R = \left| \frac{1 - n(\omega)}{1 + n(\omega)} \right|^2$

$$T = \frac{4R(n(\omega))}{|1 + n(\omega)|^2}$$

$$-i\omega m v_x = q(E_x + v_y B_0) \qquad \ldots \text{if } B_0 = (0,0,1)$$

$$-i\omega m v_y = q(E_y - v_x B_0)$$

$$-i\omega m v_x = q E_z$$

$$v_x = \frac{q}{m}\left(\frac{i\omega E_x - \omega_b E_y}{\omega^2 - \omega_b^2}\right)$$

$$v_y = \frac{q}{m}\left(\frac{\omega_b E_x + i\omega E_y}{\omega^2 - \omega_b^2}\right)$$

$$V_z = \frac{q}{m}\frac{i}{\omega}E_z \qquad \ldots \text{when } \omega_b = \frac{qB_0}{m}$$

(5.159) "Ohms Law"

$$J = \sigma E$$

$$\sigma = \begin{bmatrix} \frac{q_j^2 n_j}{m_i}\frac{i\omega}{\omega^2-\omega_b^2} & -\frac{q^2 n}{m}\frac{\omega_b}{\omega^2-\omega_b^2} & 0 \\ \frac{q^2 n}{m_j}\frac{\omega_b}{\omega^2-\omega_b^2} & \frac{q^2 n}{m}\frac{i\omega}{\omega^2-\omega_b^2} & 0 \\ 0 & 0 & \frac{q^2 n}{m}\frac{i}{\omega} \end{bmatrix}$$

$$\chi = \frac{1}{-i\omega\epsilon_0}\sigma$$

$$= \frac{1}{-i\omega\epsilon_0}(\sigma_{zz} + \sigma_{xy})$$

$$= \frac{1}{-i\omega\epsilon_0}\frac{q_j^2 n_j}{m_j}\left(\frac{i(\delta_{ij} - b_i b_k)}{\omega} + \frac{\omega_b b_\ell \epsilon_{ok\ell}}{\omega^2 - \omega_b^2}\right)$$

$$= -\frac{\omega_p^2}{\omega^2(\omega - \omega_b^2)}\left(\omega^2 - \omega_b^2 - i\omega\omega_b\right)$$

b) $$\epsilon_j = \epsilon_0\begin{pmatrix} \delta_{11} + \chi_{11} & 0 & 0 \\ 0 & \delta_{22} + \chi_{22} & 0 \\ 0 & 0 & \delta_{33} + \chi_{33} \end{pmatrix}$$

$$= \delta_{ij} + \frac{\omega_p^2}{\omega_0^2 - \omega^2}$$

Matrix Func.
$A = I + u \cdot v^T$

$$u = \frac{\omega_p}{\sqrt{\omega_0^2 - \omega^2}} \qquad v = \frac{\omega_p}{\sqrt{\omega_0^2 - \omega^2}}$$

$$\epsilon_j = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 + \frac{\omega_p^2}{\omega_0^2 - \omega^2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\epsilon_j - \lambda = \begin{pmatrix} 1-\lambda_1 & 0 & 0 \\ 0 & 1 + \frac{\omega_p^2}{\omega_0^2 - \omega^2} - \lambda_2 & 0 \\ 0 & 0 & 1-\lambda_3 \end{pmatrix}$$

$$\lambda_1 = 1 , \quad \lambda_2 = 1 + \frac{\omega_p^2}{\omega_0^2 - \omega^2} , \quad \lambda_3 = 1$$

c) $(1-\xi)E_j + \xi n(n \cdot E) + \sum_k \chi_{jk} \cdot E_k = 0 \qquad j = 1, 2, 3$

$$(1-\xi)E_j + \xi n(n \cdot E) + \sum_k \frac{\omega_p^2}{\omega^2(\omega^2 - \omega_B^2)}\left[\omega^2 \delta_{ij} - \omega_B^2 b_j b_k - i\omega\omega_B \epsilon_{jkl} \cdot b_l\right] \cdot E = 0$$

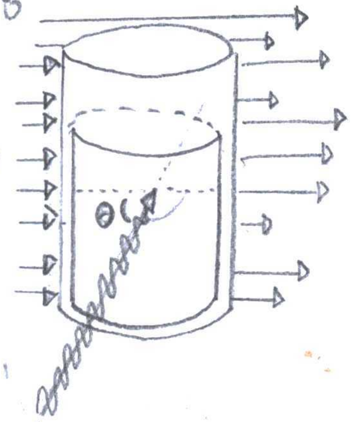
$$E_1 = +1 , \quad E_2 = 1 + \frac{\omega_p^2}{\omega_0^2 - \omega^2} , \quad E_3 = 1$$

$$\epsilon_z / \epsilon_0 = \delta_{ik} + \chi_{jk}$$

$$= 1 - \frac{\omega_p^2(\omega^2 - \omega_B^2)}{\omega^2(\omega^2 - \omega_B^2)} \mp \frac{\omega_p^2 \omega_B}{\omega^3} \qquad \cdots \quad \omega \gg \omega_B$$

$$= 1 - \frac{\omega_p^2}{\omega^2} \mp \frac{\omega_p^2 \omega_B}{\omega^2}$$

7.18



"Compressible, nonviscous, perfectly conducting fluid in a uniform static magnetic induction"

a) (1.1a) "Amperes Law"

$$\nabla \times H = J \quad \ldots \text{"Static field"}, J=0$$

(1.1a) "Gauss' law in Magnetic field"

$$\nabla \cdot B = 0 \quad \ldots \text{"no new magnetic angles"}$$

(1.1a) "Faraday's law"

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

$$\nabla \times E + \frac{\partial B}{\partial t} = 0$$

(5.159) "Ohm's Law"

$$J = \sigma E$$

$$B = \nabla \times A$$

$$\nabla \times B = \mu J$$

$$= \mu \sigma E$$

(5.160) $\nabla^2 A = \mu\sigma \frac{\partial A}{\partial t}$ "Diffusion Equation" "Wave Equation"

$$\nabla^2 B = \mu\sigma \frac{\partial B}{\partial t}$$

$$\frac{\nabla^2 B}{\mu\sigma} - \frac{\partial B}{\partial t} = 0$$

$$J = \sigma(E + v \times B)$$

$$\frac{J}{\sigma} = \nabla \times E$$

$$= \nabla \times (E + v \times B)$$

$$= \nabla \times (v \times B)$$

$$= 0$$

(7.68) $\frac{1}{\mu\sigma}\nabla^2 B - \frac{\partial B}{\partial t} = \nabla \times (v \times B)$

$$\frac{\partial B}{\partial t} = \nabla \times (v \times B) + \frac{1}{\mu\sigma}\nabla^2 B$$

$$\cong \nabla \times (v \times B)$$

(5.161) $\tau = O(\mu\sigma L^2)$ "Decay"

Notes: Decay is milliseconds to years in a uniform metal plasma, such as an iron molten core or copper sphere e.g. frequency shifts at a rate in a star, from the plasma frequency, in a millisecond to year duration, (problem 7.12).

(7.69) "Hydrodynamic Equation"

$$\frac{\partial \rho}{\partial t} + \nabla \circ (\rho v) = 0 \quad ""$$

$$\rho \frac{\partial v}{\partial t} + \rho (v \circ \nabla) v = -\nabla p - \frac{1}{\mu} B \times (\nabla \times B)$$

Note: Newton's equation of force with mechanical pressure, force density, and the magnetic force density.

$\circ\circ\circ$ When $-\frac{1}{\mu} B \times (\nabla \times B) = -\nabla\left(\frac{1}{2\mu} B^2\right) + \frac{1}{2}(B \cdot \nabla) B$

"Magnetic pressure" "additional tension"

If $B = B_0 + B_1(x,t)$, then (7.160) becomes

$$\rho = \rho_0 + \rho_1(x,t)$$

$$v = v_1(x,t)$$

$$s^2 = \gamma p_0/\rho_0$$

$$\frac{\partial \rho_1}{\partial t} + \rho_0 \nabla \cdot v_1 = 0$$ ∴ in constant magnetic induction.

$$\rho \frac{\partial v_1}{\partial t} + s^2 \nabla \rho_1 + \frac{B_0}{\mu} \times (\nabla \times B_1) = 0$$

$$\frac{\partial B_1}{\partial t} - \nabla \times (v_1 \times B_0) = 0$$

(7.72)

$$\frac{\partial^2 v_1}{\partial t^2} - s^2 \nabla(\nabla \cdot v_1) + v_A \times \nabla \times [\nabla \times (v_1 \times v_A)] = 0$$

(7.74)

$$v_1(x,t) = v_1 e^{ikx - i\omega t}$$

into becomes this

(7.75)

$$-\omega^2 v_1 + (s^2 + v_A^2)(k \cdot v_1)k + v_A \cdot k[(v_A \cdot k)v_1 - (v_A \cdot v_1)k - (k \cdot v_1)v_A] = 0$$

$$-\omega^4 v_1 + (s^2 + v_A^2)k^2\omega^2 v_1 + \omega^2 v_A \cdot k\,[(v_A \cdot k)v_1 - (v_A \cdot v_1)k - (k \cdot v_1)v_A] = 0$$

$$\omega^4 - k^2(s^2 + v_A^2)\omega^2 - k^2 v_A^3 \cos^2\theta\,[v_A \cdot k - v_A k - k v_A] = 0$$ ∴ if $\omega = k v_A \cos$

$$\omega^4 - k^2(s^2 + v_A^2)\omega^2 + k^4 v_A \cos^2\theta = 0$$ ∴ if $v_A = \sqrt{s}$

b) $$v_j^2 = \begin{pmatrix} (v_A \cos\theta^2) & 0 & 0 \\ 0 & \frac{1}{2}(s^2+v_A^2) \pm ([s^2+v_A^2]^2 - 4s^2 v_A^2 \cos^2\theta)^{1/2} & 0 \\ 0 & 0 & \frac{1}{2}(s^2+v_A^2) \pm ([s^2+v_A^2]^2 - 4s^2 v_A^2 \cos^2\theta) \end{pmatrix}$$

$$\lambda_1 = v_A \cos\theta \qquad \lambda_{2,3} = \frac{1}{2}(s^2 + v_A^2) \pm ([s^2+v_A^2]^2 - 4s^2 v_A^2 \cos^2\theta)^{1/2}$$

If $\theta = 90°$, then v_1 = Alven wave

Longitudinal: $k_\perp V_A$, V_1 is longitudinal

$k_{\parallel} V_A$, V_1 is longitudinal

Transverse $V_1 \circ V_A = 0$, V_1 is longitudinal

c) If $V_A >> S$, $V_1 = V_A \cos\theta$

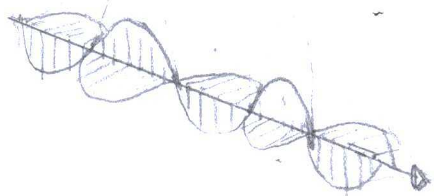
$$V_{2,3} = \frac{1}{2} V_A^2 \pm \frac{1}{2}\left[(V_A^2)^2\right]^{1/2}$$

$$= \frac{1}{2} V_A^2 \pm \frac{1}{2} V_A^2$$

$$= V_A^2 \text{ or } \emptyset$$

$$= \frac{B_0^2}{\mu \rho_0}$$ .. when parallel to field.

7.19.



"A monochromatic plane wave"

a) $f(x) = Ne^{-\alpha|x|/2}$

$$= Ne^{ik_0x - \alpha|x|/2}$$

(7.91) "Wave number spectrum"

$$A(k) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} u(x,0)e^{-ikx}\,dx$$

$$= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-ikx + ik_0x - \alpha|x|/2}\,dx$$

$$= \frac{1}{\sqrt{2\pi}}\cdot 2N\left[\int_{-\infty}^{\infty}\cos((k-k_0)x)e^{-\alpha|x|/2}\,dx\right]$$

$$= \frac{1}{\sqrt{2\pi}}\cdot 2N\left[e^{-\alpha|x|/2}\cdot\frac{-\frac{\alpha}{2}\cos((k-k_0)x) + (k-k_0)\sin((k-k_0)x)}{\alpha^2/4 + (k-k_0)^2}\right]_0^{\infty}$$

Identities:

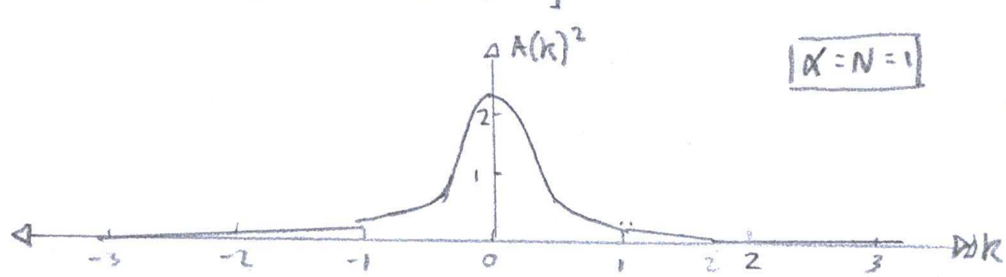
$\cos x\cos y = \frac{1}{2}[\cos(x-y) + \cos(x+y)]$

$\sin x\sin y = \frac{1}{2}[\sin(x+y) + \sin(x-y)]$

$$= \frac{1}{\sqrt{2\pi}}\left[\frac{N\alpha}{\alpha^2/4 + (k-k_0)^2}\right]$$

$$A(k)^2 = \frac{1}{2\pi}\left[\frac{N^2}{\alpha^2}\,\frac{1}{\frac{1}{4} + \left(\frac{k}{\alpha} - \frac{k_0}{\alpha}\right)^2}\right]^2$$

$\boxed{\alpha = N = 1}$



Spatial spread:

(7.102) "Variance"

$$\Delta X = \sqrt{(\Delta X_0)^2 + \left(\frac{\omega'' t}{\Delta X_0}\right)^2}$$

$$E\langle \Delta X \rangle = \sqrt{\langle \Delta X^2 \rangle - \langle \Delta X \rangle^2}$$

$$= \sqrt{\frac{\int_{-\infty}^{\infty} x^2 f(x)\,dx - \left[\int_{-\infty}^{\infty} x f(x)\,dx\right]^2}{\int_{-\infty}^{\infty} f(x)\,dx}}$$

$$= \sqrt{\frac{\int_{-\infty}^{\infty} x^2 e^{-\alpha x}\,dx}{\int_{-\infty}^{\infty} e^{-\alpha |x|}\,dx}}$$

$$= \sqrt{2}/\alpha$$

Wavevector spread:

$$E\langle \Delta k \rangle = \sqrt{\frac{\int_{-\infty}^{\infty} k^2 \left[\frac{1}{\alpha^2/4 + k^2}\right] dk}{\int_{-\infty}^{\infty} \left[\frac{1}{\alpha^2/4 + k^2}\right]^2 dk}}$$

(7.82) "Uncertainty Principle"

$$\Delta X \Delta k \geq 1/2$$

$$= \sqrt{2}/2$$

b) $f(x) = Ne^{-\alpha^2 x^2/4}$

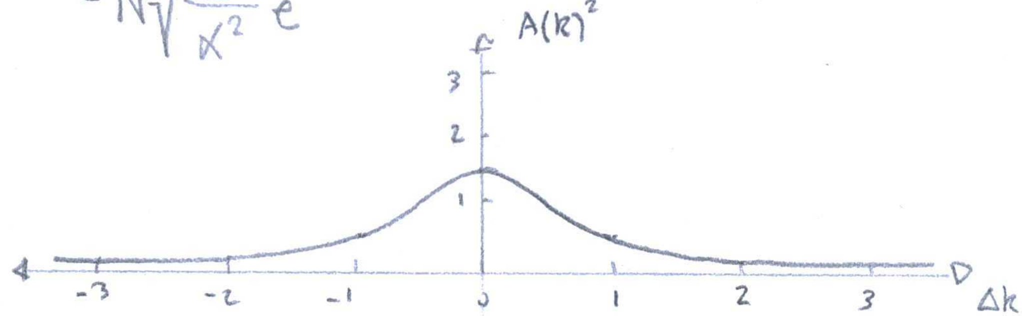
$u(x,0) = Ne^{ik_0 x - \alpha^2 x^2/4}$

Wavenumber spectrum:

$$A(k)^2 = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} u(x,t)\, e^{-ikx}\, dx$$

$$= \frac{2}{\sqrt{2\pi}} N \int_{0}^{\infty} \cos((k-k_0)x)\, e^{-\alpha^2 x^2/4}\, dx$$

$$= N\sqrt{\frac{2}{\alpha^2}}\, e^{-(k-k_0)^2/\alpha^2}$$



<u>Wavepacket spread:</u>

$$E\langle\Delta x\rangle = \sqrt{\frac{\int_{-\infty}^{\infty} x^2 e^{-\alpha^2 x^2/2}\, dx}{\int_{-\infty}^{\infty} e^{-\alpha^2 x^2/2}\, dx}}$$

$$= 1/\alpha$$

<u>Wavenumber spread:</u>

$$E\langle\Delta k\rangle = \sqrt{\frac{\int_{-\infty}^{\infty} k^2 e^{-2k^2/\alpha^2}\, dk}{\int_{-\infty}^{\infty} e^{-2k^2/\alpha^2}\, dk}}$$

$$= \alpha/2$$

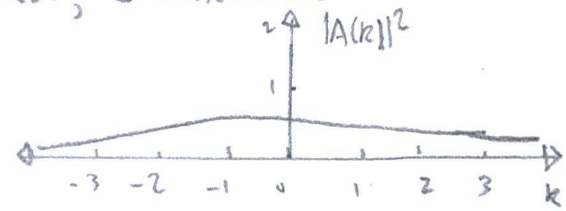
Uncertainty: $\Delta x \Delta k \geq 1/2$

$$\geq 1/2$$

c) $$f(x) = \begin{cases} N(1-\alpha|x|) & \text{for } \alpha|x|<1 \\ 0 & \text{for } \alpha|x|>1 \end{cases}$$

$u(x,0) = Ne^{ik_0 x}$ for $|x|<a$, 0 otherwise

$$|A(k)|^2 = N^2 a^2 \frac{2}{\pi} \frac{\sin^2(\Delta k/a)}{(\Delta k/a)^2}$$



<u>Wavepacket spread:</u>

$$\frac{|u(x,0)|^2}{N^2} = 1 \quad \text{for} \quad |x/a|<1, \text{ 0 otherwise}$$

<u>Wavenumber spread:</u>

$$E\langle \Delta x \rangle = \sqrt{\frac{\int_{-a}^{a} x^2 dx}{\int_{-a}^{a} dx}}$$

<u>Wavenumber spread:</u>

$$E\langle \Delta k \rangle = \sqrt{\frac{\int_{-\infty}^{\infty} \sin^2(ka) dk}{\int_{-\infty}^{\infty} \frac{\sin^2(ka)}{k^2} dk}}$$

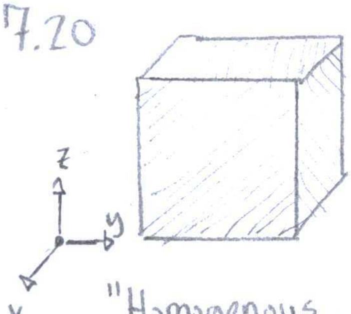
$$= 0$$

Uncertainty: $\Delta x \Delta k = \infty$

$\geq 1/2$

d) ow i skipped.

7.20



$\epsilon_x = \epsilon_y = \epsilon_z$

$n_x(\omega) = n_y(\omega) = n_z(\omega)$

"Homogenous, isotropic, nonpermeable dielectric"

a) (7.3) "Helmholtz wave equation"

$$(\nabla^2 + \mu\epsilon\omega^2)\begin{Bmatrix} E \\ B \end{Bmatrix} = 0$$

$$(7.6)\ u(x,t) = ae^{ikx - i\omega t} + be^{-ikx - i\omega t}$$

$$= e^{-i\omega t}\left[ae^{ikx} + be^{-ikx}\right]$$

$$= e^{-i\omega t}\left[ae^{in(\omega)/c \cdot \omega x} + be^{-in(\omega)/c \cdot \omega x}\right]$$

Fourier Transform:

$$u(x,t)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}e^{-i\omega t}\left[a e^{i(\omega/c)n(\omega)x}+b e^{-i(\omega/c)n(\omega)x}\right]d\omega$$

b) (7.126) $$n^*(\omega)=\frac{ck(\omega)}{\omega}+i\frac{ck(\omega)}{\omega}$$

$$=\frac{ck(\omega)}{\omega}-i\frac{ck(\omega)}{\omega}$$

$$=\frac{-ck(-\omega)}{\omega}+\frac{ick(\omega)}{\omega}$$

$$=n(-\omega)$$

c) @ $x=0$, $$u(x=0,t)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\left[A(\omega)+B(\omega)\right]e^{-i\omega t}\,d\omega$$

$$\frac{du(x=0,t)}{dx}=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\left[A(\omega)-B(\omega)\right]\frac{i\omega k}{c}e^{-i\omega t}\,d\omega$$

$$A(\omega)+B(\omega)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}u(0,t)\,e^{i\omega t}\,dt$$

$$A(\omega)-B(\omega)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}-\frac{ic}{\omega n}\frac{\partial u(x=0,t)}{\partial x}e^{i\omega t}\,dt$$

$$A(\omega)=\frac{1}{2\sqrt{2\pi}}\int_{-\infty}^{\infty}\left[u(0,t)-\frac{ic}{\omega n}\frac{\partial u(x=0,t)}{\partial x}\right]e^{i\omega t}\,dt$$

$$B(\omega)=\frac{1}{2\sqrt{2\pi}}\int_{-\infty}^{\infty}\left[u(0,t)+\frac{ic}{\omega n}\frac{\partial u(x=0,t)}{\partial x}\right]e^{i\omega t}\,dt$$

$$\begin{Bmatrix}A(\omega)\\B(\omega)\end{Bmatrix}=\frac{1}{2\sqrt{2\pi}}\int_{-\infty}^{\infty}\left[u(0,t)\mp\frac{ic}{\omega n}\frac{\partial u(0,t)}{\partial x}\right]e^{i\omega t}\,dt$$

7.21 $D(x,t) = \epsilon_0 \{ E(x,t) + \int d\tau\, G(\tau) E(x, t-\tau) \}$ ... pg 332

$$\frac{\epsilon(\omega)}{\epsilon_0} = 1 + \omega_p^2 (\omega_0^2 - \omega^2 - i\gamma\omega)^{-1}$$

a) Taylor expansion:

$$D(x,t) = \epsilon_0 \left\{ \frac{\partial}{\partial t} E(x,t=0) \cdot t + \frac{\partial^2}{\partial^2 t} \frac{E(x,t=0)}{2!} t^2 + \int d\tau \cdot G(\tau) \left[\frac{\partial}{\partial t} E(x,t) + \frac{\partial^2}{\partial^2 t} \frac{E(x,t)}{2} \right.\right.$$

$$= \epsilon_0 \left\{ \frac{\partial}{\partial t} E(x,t=0) \cdot t + \frac{\partial^2}{\partial^2 t} \frac{E(x,t=0)}{2!} t^2 + \int_0^\infty d\tau\, G(\tau) e^{i\omega\tau} \right.$$

$$= \epsilon_0 \left\{ \frac{\partial}{\partial t} E(x,t=0) \cdot t + \frac{\partial^2}{\partial^2 t} \frac{E(x,t=0)}{2!} - \frac{iG(0)}{\omega} - \frac{iG'(0)}{\omega^2} \right\}$$

b) $D(x,\omega) = \epsilon(\omega) E(x,\omega)$

$$= \epsilon\left(i \frac{\partial}{\partial t}\right) E(x,t)$$

7.22. (7.120) $\mathrm{Re}[\epsilon(\omega)/\epsilon_0] = 1 + \frac{2}{\pi} P \int_0^\infty \frac{\omega' \,\mathrm{Im}[\epsilon(\omega)/\epsilon_0]}{\omega'^2 - \omega^2} d\omega'$

$$\mathrm{Im}[\epsilon(\omega)/\epsilon_0] = -\frac{2\omega}{\pi} P \int_0^\infty \frac{\mathrm{Re}[\epsilon(\omega)/\epsilon_0 - 1]}{\omega'^2 - \omega^2} d\omega$$

a) $\mathrm{Im}[\epsilon(\omega)/\epsilon_0] = \lambda[\Theta(\omega-\omega_1) - \Theta(\omega-\omega_2)]$ $\quad \omega_2 > \omega_1 > 0$

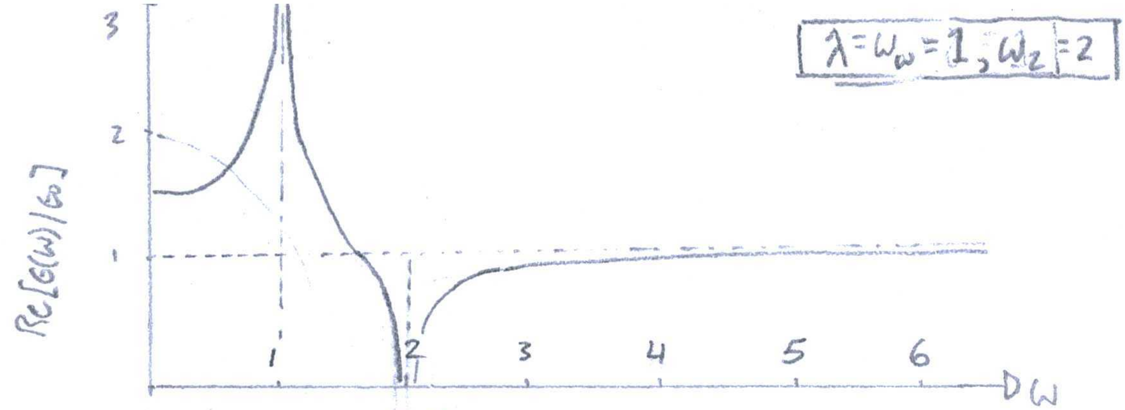
$$\mathrm{Re}[\epsilon(\omega)/\epsilon_0] = 1 + \frac{2}{\pi} P \int_0^\infty \frac{\omega' \,\mathrm{Im}[\epsilon(\omega')/\epsilon_0]}{\omega'^2 - \omega^2} d\omega$$

$$= 1 + \frac{2}{\pi} P \int_0^\infty \frac{\lambda[\Theta(\omega-\omega_1) - \Theta(\omega-\omega_2)]}{\omega'^2 - \omega^2} d\omega'$$

$$= 1 + \frac{2\lambda}{\pi} P \int_{\omega_1}^\infty \frac{\omega'}{\omega'^2 - \omega^2} d\omega' - \frac{2\lambda}{\pi} P \int_{\omega_2}^\infty \frac{\omega'}{\omega'^2 - \omega^2} d\omega'$$

$$= 1 + \frac{2\lambda}{\pi} \left[\frac{1}{2} \ln(|\omega'^2 - \omega^2|) \right]_\omega^\infty - \frac{2\lambda}{\pi} \left[\frac{1}{2} \ln(|\omega'^2 - \omega^2|) \right]_{\omega_2}^\infty$$

$$= 1 + \frac{\lambda}{\pi} \left[\ln(|\omega_2^2 - \omega^2|) - \ln(|\omega_1^2 - \omega^2|) \right]$$



b) $\mathrm{Im}(\epsilon/\epsilon_0) = \dfrac{\lambda \gamma \omega}{(\omega_0^2-\omega^2)+\gamma^2\omega^2}$

$$\mathrm{Re}(\epsilon(\omega)/\epsilon_0) = 1 + \frac{2\lambda\gamma}{\pi} P\int_0^\infty \frac{\omega'^2}{((\omega_0^2-\omega'^2)^2+\gamma^2\omega'^2)(\omega'^2-\omega^2)}\,d\omega$$

$$= 1 + \frac{2\lambda\gamma}{\pi} P\left[\int_0^\infty \frac{A}{((\omega_0^2-\omega'^2)^2+\gamma^2\omega'^2)}\,d\omega + \int_0^\infty \frac{B}{(\omega'^2-\omega^2)}\,d\omega\right]$$

Partial Fractions (Complex):

$$\frac{\omega'^2}{((\omega_0^2-\omega'^2)^2+\gamma^2\omega'^2)(\omega'^2-\omega^2)} = \frac{A}{(\omega_0^2-\omega'^2+i\gamma\omega')} + \frac{B}{(\omega_0^2-\omega'^2-i\gamma\omega')} + \frac{\omega'^2}{(\omega'^2-\omega^2)}$$

$$= \left[\frac{\omega_0^2}{2} - \frac{\omega'(\omega_0^2-\omega^2)}{2i\gamma}\right]\frac{1}{(\omega_0^2-\omega'^2+i\gamma\omega)}$$

$$+ \left[\frac{\omega_0^2}{2} + \frac{\omega'(\omega_0^2-\omega^2)}{2i\gamma}\right]\frac{1}{(\omega_0^2-\omega'^2-i\gamma\omega)}$$

$$+ \frac{\omega'^2}{(\omega'^2-\omega^2)}$$

Note: Partial fractions extend into third-, fourth-, and high-order, also, complex arithmetic.

$$\mathrm{Re}(\epsilon(\omega)/\epsilon_0) = 1 + \frac{2\lambda\gamma}{\pi}\frac{1}{((\omega_0^2-\omega^2)+\gamma^2\omega^2)}\left[-\frac{\omega_0^2}{2} P\int_0^\infty \left[\left(\frac{\omega_0^2}{2} - \frac{\omega'(\omega_0^2-\omega^2)}{2i\gamma}\right)\frac{1}{\omega_0^2-\omega'^2+i\gamma\omega}\right.\right.$$

$$+ \left(\frac{\omega_0^2}{2} + \frac{\omega'(\omega_0^2-\omega^2)}{2i\gamma}\right)\frac{1}{\omega_0^2-\omega'^2+i\gamma\omega}$$

$$\left.+ \frac{\omega'^2}{(\omega'^2-\omega^2)}\right]d\omega'$$

$$= 1 + \frac{2\lambda\gamma}{\pi} \frac{1}{((\omega_0^2-\omega^2)+\gamma^2\omega^2)} \left[-\frac{\omega_0^2}{2} \left[\frac{1}{2\sqrt{\omega_0^2+i\gamma\omega'}} \ln\left(\frac{\omega' - \sqrt{\omega_0^2+i\gamma\omega}}{\omega' + \sqrt{\omega_0^2+i\gamma\omega'}} \right) \right. \right.$$

$$\left. + \frac{1}{2\sqrt{\omega_0^2-i\gamma\omega'}} \ln\left(\frac{\omega' - \sqrt{\omega_0^2-i\gamma\omega}}{\omega' + \sqrt{\omega_0^2-i\gamma\omega}} \right) \right]_0^\infty$$

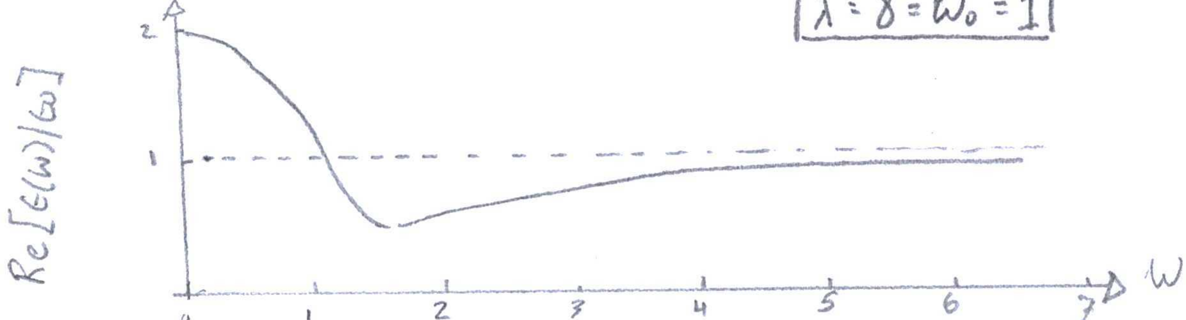
$$+ \frac{(\omega_0^2-\omega^2)}{2i\gamma} \frac{1}{2} \Big[\ln(\omega'^2-\omega_0^2-i\gamma\omega) - \ln(\omega'^2-\omega_0^2+i\gamma\omega') \Big]_0^\infty$$

Identity:
$\ln(z) = \ln(|z|) + i\,\mathrm{Arg}(z)$

$$\mathrm{Re}(\epsilon(\omega)/\epsilon_0) = 1 + \frac{2\lambda\gamma}{\pi} \frac{1}{((\omega_0^2-\omega^2)^2+\gamma^2\omega^2)} \frac{(\omega_0^2-\omega^2)}{2i\gamma} \frac{1}{2} \left[i\left(\pi + \tan^{-1}\left(-\frac{\gamma\omega'}{\omega'^2-\omega_0^2}\right)\right) - i\left(-\pi + \tan^{-1}\left(\frac{\gamma\omega'}{\omega'^2-\omega_0^2}\right)\right) \right]_0^\infty$$

$$= 1 + \lambda \frac{(\omega_0^2-\omega^2)}{((\omega_0^2-\omega^2)^2+\gamma^2\omega^2)}$$

$\boxed{\lambda = \gamma = \omega_0 = 1}$



7.23 (7.120) $$\mathrm{Re}[\epsilon(\omega)/\epsilon_0] = 1 + \frac{2}{\pi} P \int_0^\infty \frac{\omega' \mathrm{Im}[\epsilon(\omega)/\epsilon_0]}{\omega_1^2 - \omega^2} d\omega'$$

$$\mathrm{Im}[\epsilon(\omega)/\epsilon_0] = -\frac{2\omega}{\pi} P \int_0^\infty \frac{\mathrm{Re}[\epsilon(\omega)/\epsilon_0 - 1]}{\omega'^2-\omega^2} d\omega'$$

If $E(x,t) = E_0 e^{ikx-i\omega t}$

$B(x,t) = B_0 e^{ikx-i\omega t}$

(5.159) "Ohms Law"

$$J(x,t) = \sigma \cdot E(x,t)$$

(1.1a) "Amperes Law"

$$\nabla \times H = J + \frac{\partial D}{\partial t}$$

$$= \sigma E - (-i\omega)\epsilon E$$

$$= (-i\omega)(\epsilon + i\frac{\sigma}{\omega})E$$

$$\mathrm{Im}\left(\frac{\epsilon'(\omega)}{\epsilon_0}\right) = -\frac{2\omega}{\pi} P\int_0^\infty \frac{\mathrm{Re}\left(\frac{\epsilon'(\omega)}{\epsilon_0} - 1\right)}{\omega'^2 - \omega^2} d\omega'$$

$$= \mathrm{Im}\left(\epsilon(\omega) - \frac{i\sigma}{\omega}\right.$$

$$= -\frac{2\omega}{\pi} P\int_0^\infty \frac{\mathrm{Re}(\epsilon(\omega') - \epsilon_0)}{\omega'^2 - \omega^2} d\omega$$

$$= \frac{\sigma}{\omega} - \frac{2\omega}{\pi} P\int_0^\infty \frac{\mathrm{Re}(\epsilon(\omega') - \epsilon_0)}{\omega'^2 - \omega^2} d\omega'$$

$$\mathrm{Re}\left(\frac{\epsilon'(\omega)}{\epsilon_0}\right) = 1 + \frac{2}{\pi} P\left[\int_0^\infty \frac{\omega'\left(\frac{\epsilon(\omega)}{\epsilon_0}\right)}{\omega'^2 - \omega^2} d\omega' - \frac{1}{\epsilon_0}\int_0^\infty \frac{\frac{\omega'\sigma}{\omega}}{\omega'^2 - \omega^2} d\omega'\right]$$

$$= 1 + \frac{2}{\pi} P\left[\int_0^\infty \frac{\omega' \mathrm{Im}[\epsilon(\omega)/\epsilon_0]}{\omega_1'^2 - \omega^2} d\omega'\right]$$

7.24. a) (7.113) $\epsilon(-\omega)/\epsilon_0 = \epsilon^*(\omega^*)/\epsilon_0$

(7.114) $\mathrm{Re}[\epsilon(\omega)/\epsilon_0 - 1] = O(1/\omega^2)$, $\mathrm{Im}[\epsilon(\omega)/\epsilon_0] = O(1/\omega^3)$

(7.112) $\epsilon(\omega)/\epsilon_0 = 1 + \int_0^\infty G(\tau) e^{i\omega\tau} d\tau$

$$\cong 1 + \frac{iG(0)}{\omega} - \frac{G'(0)}{\omega^2} + \cdots$$

$$\lim_{\omega \to i\infty} \epsilon(\omega)/\epsilon_0 = \lim_{\omega \to i\infty} 1 + \frac{iG(0)}{\omega} - \frac{G'(0)}{\omega^2} + \cdots$$

$$\text{Im}\left[\lim_{\omega \to i\infty} \frac{\epsilon(\omega)}{\epsilon_0}\right] = \text{Im}\left[\lim_{\omega \to i\infty}\left(1 + \frac{iG(0)}{\omega} - \frac{G'(0)}{\omega^2}\right)\right]$$

$$= \lim_{\omega \to i\infty} \frac{iG(0)}{\omega}$$

$$= 0 \quad \text{"Real"}$$

b) $$\frac{\epsilon(\omega)}{\epsilon_0} = 1 + \frac{iG(0)}{\omega} - \frac{G'(0)}{\omega^2} + \cdots$$

$$\epsilon(\omega) = \epsilon_0\left[1 + \frac{iG(0)}{\omega} - \frac{G'(0)}{\omega^2} + \cdots\right]$$

$$= \epsilon_0$$

$$\geq 0$$

c) (7.122) $$\omega_p^2 = \frac{2}{\pi}\int_0^\infty \omega \, \text{Im}\, \epsilon(\omega)/\epsilon_0 \, d\omega$$

(7.114) $$\text{Re}[\epsilon(\omega)/\epsilon_0 - 1] = O(1/\omega^2) \quad , \quad \text{Im}[\epsilon(\omega)/\epsilon_0] = O(1/\omega^3)$$

$$\text{Im}(\epsilon(\omega)/\epsilon_0) = \text{Im}\left[\lim_{\omega \to i\infty} \frac{\epsilon(\omega)}{\epsilon_0}\right]$$

$$= 0 \quad \text{"from part a"}$$

d) (7.107) $$\epsilon(\omega)/\epsilon_0 = \omega_p^2(\omega_0^2 - \omega^2 - i\gamma\omega)$$

$$\epsilon(\omega)/\epsilon_0 \cong 1 - \frac{\omega_p^2}{\omega^2}$$

$$\omega_p^2 = \omega^2(1 - \epsilon(\omega)/\epsilon_0)$$

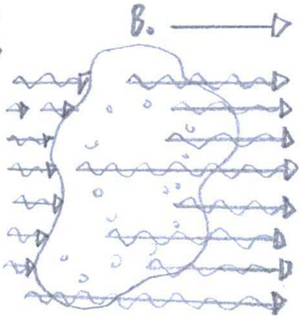
$$= \frac{2}{\pi}\int_0^\infty \omega \, \text{Im}\, \epsilon(\omega)/\epsilon_0 \, d\omega$$

$$\lim_{\omega \to i\infty} \omega_p^2 = \lim_{\omega \to i\infty} \frac{2}{\pi}\int_0^\infty \omega \, \text{Im}[\epsilon(\omega)/\epsilon_0] \, d\omega$$

$$= \frac{2}{\pi}\int_0^\infty \omega \, \text{Im}\left[\lim_{\omega \to i\infty} \epsilon(\omega)/\epsilon_0\right] d\omega$$

$= 0$ "... from part a"

7.25.



"Waves propagating along field lines through plasma in a uniform external magnetic field."

(7.67) $\frac{\epsilon}{\epsilon_0} = 1 - \frac{\omega_p^2}{\omega(\omega \mp \omega_B)}$

a) $k = \omega n(\omega)/c$ ... when $\frac{\omega_p}{\omega_B} \geq 1$

$$c\frac{dk}{d\omega} = n(\omega) + \omega\frac{dn}{d\omega}$$

$= t/t_0$ ... equation 7.12ö

$\cong 1 + \frac{\omega_p^2}{2\omega^2}$ .... equation 7.12ö.5

Positive Helicity: $\frac{\epsilon}{\epsilon_0} = 1 - \frac{\omega_p^2}{\omega(\omega - \omega_B)}$

(7.5) $n = \sqrt{\frac{\mu}{\mu_0}\frac{\epsilon}{\epsilon_0}}$

$$c\frac{dk}{d\omega} = n(\omega) + \omega\frac{dn}{d\omega}$$

$$= n(\omega) + \frac{d}{d\omega}\sqrt{\frac{\mu}{\mu_0}\left(1 - \frac{\omega_p^2}{\omega(\omega - \omega_B)}\right)}$$

$$= n(\omega) + \omega\left(\sqrt{\frac{\mu}{\mu_0}} - \frac{\omega_p^2(\omega_B - 2\omega')}{2\omega^2(\omega_B - \omega)^2\sqrt{\frac{\omega_B\omega + \omega_B^2 - \omega^2}{\omega(\omega_B - \omega)}}}\right)$$

(7.5) $v = \frac{\omega}{k}$

$= \frac{1}{\sqrt{\mu\epsilon}}$

$= \frac{c}{n}$

$$k^2 = \frac{\omega^2 n^2}{c^2}$$

$$= \frac{\omega^2}{c^2} \cdot \epsilon(\omega) \quad \ldots \text{when } \mu/\mu_0 = \epsilon_0 = 1$$

$$= \frac{\omega^2}{c^2}\left(\epsilon_0 - \frac{\omega_p^2 \epsilon_0}{\omega^2 - \omega\omega_B}\right)$$

$$= \frac{\omega^2}{c^2}\left(\epsilon_0 + \frac{\omega_p^2 \epsilon_0}{\omega\omega_B - \omega^2}\right)$$

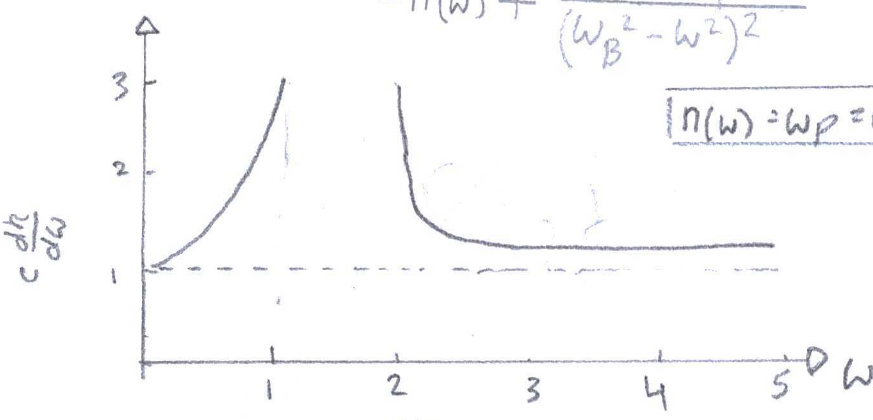
(7.128) $$c\frac{dk}{d\omega} = \frac{c\, d(n(\omega)\cdot\omega)}{c\cdot d\omega}$$

$$= n(\omega) + \omega\frac{dn(\omega)}{d\omega}$$

$$= n(\omega) + \omega\frac{d}{d\omega}\left(\epsilon_0 + \frac{\omega_p^2 \epsilon_0}{\omega_B^2 - \omega^2}\right)$$

$$= n(\omega) + \frac{2\epsilon_0 \omega^2 \omega_p^2}{(\omega_B^2 - \omega^2)^2}$$

$$\boxed{n(\omega) = \omega_p = \omega_B = \epsilon_0 = 1}$$



b) @ $\omega = 0$, $c\frac{dk}{d\omega} = n(0)$

$$\cong 1$$

c) pg 338, when $\phi(\omega) = k(\omega)x - \omega t$

$$\frac{d\phi(\omega)}{d\omega} = \frac{d}{d\omega}\left(k(\omega)x - \omega t\right)$$

$$= \frac{d}{d\omega}\left(\frac{\omega}{c} n(\omega) x - \omega t\right)$$

$$= \frac{n(w)X}{c} - t$$

$$= 0$$

$$n(w) = t/t_0$$

into $c\frac{dk}{dw} = n(w) + w\frac{dn}{dw}$

$$= \frac{t}{t_0}$$

d) Brillioun Precursor: $t = \frac{n(0)X}{c}$

$$> t_0$$

$\boxed{\begin{array}{l}\underline{\text{Airy Integral}} \\ y(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{ipx} \cdot e^{ip^3/3}\,dp \\ = Ai(x)\end{array}}$

$$\phi(w) = \phi_1(w) + \frac{1}{2}\phi''(w-w_3)^2 + \frac{1}{3!}\phi'''(w-w_3)^2$$

$$= w(t-t_0) + \frac{X}{6}\left(\frac{d^3k}{dw^3}\right)w^3$$

when $\frac{d^3k}{dw^3} = \frac{3w_p^2}{cn(0)\cdot w_3^4}$

(2.44) "Fourier Transform"

$$F(X,t) = \int e^{i\phi(w)} F(w)\,dw$$

where $F(w) = \frac{1}{\tau}\frac{1}{w^2 - (2\pi/\tau)^2}$

and

$$\phi(w) = k(w) - wt$$

When $F(X,t) \cong F(0)\int_{-\infty}^{\infty} e^{i\phi(w)}\,dw$

$$\cong F(0)\int_{-\infty}^{\infty} e^{i[w(t-t_0) + i\frac{X}{6}k_0''' w^3]}\,dw$$

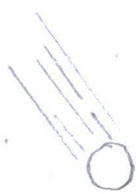
$$\cong \frac{\tau}{\sqrt{2\pi}}\sqrt{\frac{|t-t_0|}{X k_0'''}}\int_0^\infty \cos\left[\frac{3}{2} z\left(\frac{v^3}{3} \pm v\right)\right] dv$$ "Airy Integral"

Where $v = \sqrt{\frac{X k_0'''}{2|t-t_0|}}\,\omega$

and

$$z = \frac{2\sqrt{2}\,|t-t_0|^{3/2}}{3\sqrt{X k_0'''}}$$

7.26



"A charged particle moves... through a medium"

$$\sigma(q,\omega) = i\omega[\epsilon_0 - \epsilon(q,\omega)]$$ "conductivity function"

$$J(q,\omega) = \sigma(q,\omega)\, E(q,\omega)$$ "Ohms Law"

a) (7.80) "Fourier transform, 1-dimensional"

$$u(x,t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} A(k)\, e^{i(kx-\omega t)}\, dk$$

(7.81)

$$A(k) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} u(x,t)\, e^{-ikx}\, dx$$

...extended to 3-dimensions

$$\rho(x,t) = \frac{1}{(\sqrt{2\pi})^4}\int_{-\infty}^{\infty} \rho(k,\omega)\, e^{i(kx-\omega t)}\, d^3k\, d\omega$$

where $$\rho(k,\omega) = \frac{1}{(\sqrt{2\pi})^4}\int_{-\infty}^{\infty} \delta(x-(x_0-vt))\, e^{-i(kx+\omega t)}\, d^3x\, dt$$

$$= \frac{Ze}{(2\pi)^4}\int_{-\infty}^{\infty} e^{-i(q(x_0-vt)+i\omega t)}\, d^3x\, dt$$

$$= \frac{Ze}{(2\pi)^4}\int^{\infty} e^{-i(qx-(qv-\omega)t)}\, dt$$

$$= \frac{Ze}{(2\pi)^4} 2\pi e^{iqx_0} \delta(\omega - qv)$$

$$= \frac{Ze}{(2\pi)^3} \delta(\omega - qv) \quad @ x_0 = 0$$

b) (1.1a) "Ampere's Law"

$$\nabla \times H = J + \frac{\partial}{\partial t} D$$

$$\nabla \cdot (\nabla \times H) = \nabla \cdot J + \frac{\partial}{\partial t} \nabla \cdot D$$

$$= 0$$

(1.1b) "Gauss' law"

$$\nabla \cdot E = \frac{\rho}{\epsilon_0}$$

$$\nabla \cdot J = -\frac{\partial}{\partial t} \rho(q,\omega)$$

$$\nabla \cdot \frac{D(q,\omega)}{\epsilon} = -i\omega \rho(q,\omega) \quad \ldots \text{ when } \rho(q,\omega) = e^{i(kx - \omega t)}$$

$$[i\omega \rho(q,\omega) - iq D(q,\omega)] = 0$$

c) $$\frac{dW}{dt} = \int J \cdot E \, d^3x$$

$$\sigma(q,\omega) = i\omega[\epsilon_0 - \epsilon(q,\omega)]$$

( ) $$\frac{dW}{dt} = \int J \cdot E \, d^3x$$

$$= \int \sigma(x,t) E(x,t) \, d^3x$$

$$= \int \sigma(x,t) E^2(x,t) \, d^3x$$

$$= (2\pi)^2 \int \sigma(x,t)\, E^2(x,t)\, d^3q$$

$$= (2\pi)^2 \int i\omega[\epsilon_0 - \epsilon(q,\omega)][-iq\Phi(q,\omega)]^2 d^3q\, dt$$

$$-\frac{dW}{dt} = (2\pi)^2 \iint (i\omega[\epsilon_0 - \epsilon(q,t)] q^2 \left(\frac{\rho(q,\omega)}{q^2\epsilon(q,\omega)}\right)^2 d^3q\, d\omega$$

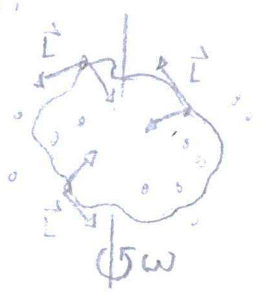
$$= (2\pi)^2 \iint (i\omega[\epsilon_0 - \epsilon(q,t)]\, q^2 \frac{1}{q^4\epsilon^2(q,\omega)} \left[\frac{Ze}{(2\pi)^3}\delta(\omega - qv)\right]^2 d^3q\, d\omega$$

$$= \frac{Z^2e^2}{(2\pi)^3} \int \frac{d^3q}{q^2} \int_{-\infty}^{\infty} d\omega\, i\omega \frac{[\epsilon_0 - \epsilon(q,t)]}{\epsilon^2(q,\omega)} \delta(\omega - qv)^2$$

$$= \frac{Z^2e^2}{8\pi^3} \int \frac{d^3q}{q^2} \int_{-\infty}^{\infty} d\omega\, \omega\, \mathrm{Im}\left\{\frac{1}{\epsilon(q,v)}\right\} \delta(\omega - qv)$$

$$= \frac{Z^2e^2}{4\pi^3} \int \frac{d^3q}{q^2} \int_{-0}^{\infty} d\omega\, \omega\, \mathrm{Im}\left\{\frac{1}{\epsilon(q,v)}\right\} \delta(\omega - qv)$$

7.27.



"a distribution of electromagnetic fields in a vacuum"

a) Note: A finite field in time means a finite region in space

$$L = \frac{1}{\mu_0 c^2}\int d^3x\, \vec{X} \times (E \times B)$$

$$= \frac{1}{\mu_0 c^2}\int d^3X\, \vec{X} \times (E \times (\nabla \times A))$$

$$= \frac{1}{\mu_0 c^2}\int d^3X\, \vec{X} \times (E \cdot \nabla A - (E \cdot \nabla) A)$$

$$= \frac{1}{\mu_0 c^2}\int d^3X (E \cdot (\vec{X} \times \nabla) A - (\vec{X} \times (E \cdot \nabla) A)$$

Higher-dimensional analogue of integration by parts:

$$\iiint (\nabla \circ f) \circ F \, dV = \iint f \circ F \, dS - \iiint f \cdot \nabla \circ F \, dV$$

$$= \frac{1}{\mu_0 c^2} \int d^3X \left(E \circ (X \times \nabla)) A + E \times A \right)$$

"orbital angular momentum" = "Spin of a photon" + "unknown"

b) $A(X,t) = \sum_\lambda \int \frac{d^3k}{(2\pi)^3} \left[\epsilon_\lambda(k) a_\lambda(k) e^{ikx - i\omega t} + c.c \right]$

where $\epsilon_\pm = \left(\frac{1}{\sqrt{2}} \right) (\epsilon_1 \pm i \epsilon_2)$

(1.1a) "Faraday's Law"

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

$$= -\frac{d}{dt} (\nabla \times A(x,t))$$

$$= i\omega A(x,t)$$

$$L_{spin} = \frac{1}{\mu_0 c^2} \int d^3k \, \frac{1}{2} E \times A$$

$$= \frac{1}{\mu_0 c^2} \int_{-\infty}^{\infty} d^3k \, \frac{1}{2} (i\omega A(x,t) \times A(x,t))$$

$$= \frac{2}{\mu_0 c^2} \int_0^\infty \frac{d^3k}{(2\pi)^3} i\omega |a_\lambda(k)|^2 (-i \lambda k \delta)$$

$$= \frac{2}{\mu_0 c^2} \int_0^\infty \frac{d^3k}{(2\pi)^3} \omega \lambda k |a_\lambda(k)|^2$$

$$= \frac{2}{\mu_0 c^2} \int \frac{d^3k}{(2\pi)^3} k \left(|a_+(k)|^2 - |a_-(k)|^2 \right)$$

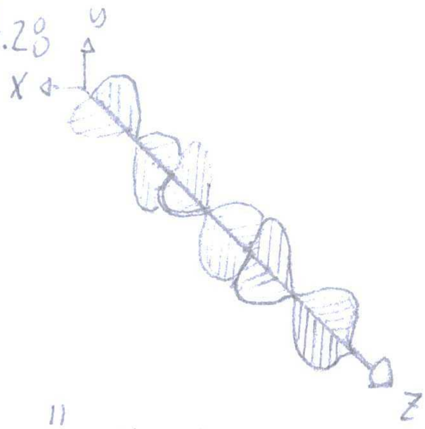
$$a_\lambda(\vec{k}) = \sqrt{\frac{\hbar}{2\epsilon_0 \omega_k}}\, b_\lambda(k)$$

$$\overset{+}{A}(x,t) = \sum \int \frac{d^3k}{(2\pi)^3} \sqrt{\frac{\hbar}{2\epsilon_0 \omega_k}}\, b_\lambda(k)\, \epsilon_\lambda(k)\, e^{i(kx-\omega t)}$$

$$\overset{+}{E}(x,t) = i \sum \int \frac{d^3k}{(2\pi)^3} \sqrt{\frac{\hbar \omega_k}{2\epsilon_0}}\, b_\lambda(k)\, \epsilon_\lambda(k)\, e^{i(kx-\omega t)}$$

$$E(x,t) = \overset{+}{E}(x,t) + \overset{-}{E}(x,t)$$

7.28



"a circularly polarized plane wave has finite extent in the x and y directions"

$$(7.27)\quad E_x(x,t) = E_0 \cos(kz - \omega t)$$

$$E_y(x,t) = \mp E_0 \sin(kz - \omega t)$$

$$(7.20)\quad E(x,t) = E_0 (\epsilon_1 \pm i\epsilon_2)\, e^{ikx - i\omega t}$$

"circularly polarized light"

where $\epsilon_\pm = \frac{1}{\sqrt{2}}(\epsilon_1 \pm i\epsilon_2)$

$$E(x,y,z,t) = \left[E_0(x,y)(\epsilon_1 + i\epsilon_2) + L(x,y)\right] e^{i(kz-\omega t)}$$

(1.1b) "Gauss' Law"

$$\nabla \cdot E = \frac{\rho}{\epsilon_0}$$

$$= 0 \quad \ldots \text{"no charge density"}$$

$$\nabla E(x,y,z,t) = \left[\frac{\partial E_0(x,y)}{\partial x} \pm i \frac{\partial E_0(x,y)}{\partial y} + L(x,y)\, ik\right] e^{i(kz-\omega t)}$$

$$-ikL(x,y) = \frac{\partial E_0(x,y)}{\partial x} \pm i \frac{\partial E_0(x,y)}{\partial y}$$

$$L(x,y) = \frac{i}{k}\left[\frac{\partial E_0}{\partial x} \pm i \frac{\partial E_0}{\partial y}\right]$$

$$E(x,y,z,t) = \left[E_0(x,y)(\epsilon_1 \pm i\epsilon_2) + \frac{i}{k}\left(\frac{\partial E_0(x,y)}{\partial x} \pm i \frac{\partial E_0(x,y)}{\partial y}\right)\epsilon_3\right] e^{i(kz-\omega t)}$$

(   ) "Faraday's Law"

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

$$-\frac{\partial B}{\partial t} \simeq \nabla \times \left[E_0(x,y)(\epsilon_1 \pm i\epsilon_2) e^{i(kz-\omega t)}\right]$$

$$\text{If } \epsilon_1 = \epsilon_1 E_1 e^{ikz-i\omega t} \quad \epsilon_2$$

$$= \epsilon_2 E_2 e^{ikz-i\omega t}$$

$$B \simeq \frac{-i}{\omega}\left[-\frac{\partial E}{\partial z}\epsilon_1 + \frac{\partial E_1}{\partial z}\epsilon_2 + \left(\frac{\partial E_z}{\partial x} - \frac{\partial E_1}{\partial y}\right)\epsilon_3\right]$$

$$\simeq \frac{-i}{\omega}\left[\mp i E_0(x,y)(ik)\epsilon_1 + E_0(x,y)(ik)\epsilon_2 + \left(\pm i\frac{\partial E}{\partial x} - \frac{\partial E}{\partial y}\right)\epsilon_3\right]$$

$$\simeq \frac{-i}{\omega}(\pm k)\left[E_0(x,y)(\epsilon_1 \pm i\epsilon_2) + \frac{i}{k}\left(\frac{\partial E_0}{\partial x} \pm \frac{\partial E_0}{\partial y}\epsilon_3\right]\right.$$

$$\simeq \mp \frac{ik}{\omega} E(x,y,z,t)$$

$$\simeq \mp \frac{n}{c} E(x,y,z,t)$$

$$\simeq \mp i\sqrt{\mu\epsilon}\, E(x,y,z,t) \quad \ldots \text{when } n=1$$