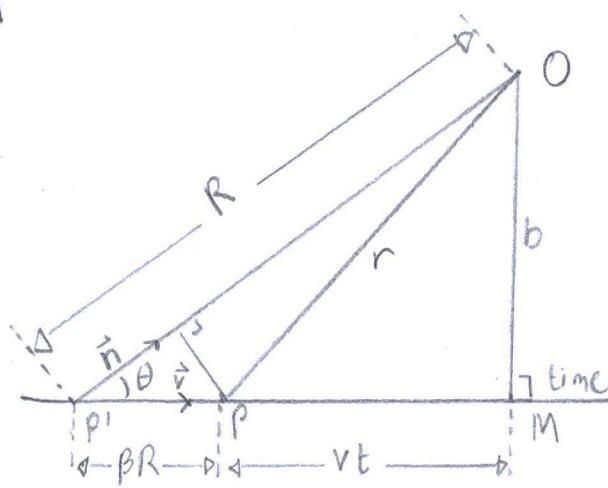


Chapter 14: Radiation by Moving Charges:

14.1



"Figure 14.2"

$$\overline{PQ} = \beta R \cos \theta \\ = \beta \cdot n \cdot R$$

$$\overline{OQ} = (1 - \beta \cos \theta) R \\ = (1 - \beta \cdot n) R$$

$$\overline{OM} = b \\ = R \sin \theta$$

$$\overline{OQ}^2 = \overline{OP}^2 - \overline{PQ}^2$$

$$= r^2 - \beta^2 R^2 \sin^2 \theta$$

$$= b^2 + v^2 t^2 - \beta^2 b^2$$

$$= \frac{1}{\gamma^2} (b^2 + \gamma^2 v^2 t^2)$$

← It's the law,
Pythagorean
Law

(11.152) "Transverse Component"

$$E = \frac{e \gamma b}{(b^2 + \gamma^2 v^2 t^2)^{3/2}}$$

$$= e \left[\frac{b}{\gamma^2 (1 - \beta \cdot n)^3 R^3} \right] \text{ when } \overline{OQ}^2 \text{ is } (1 - \beta \cdot n) R$$

(14.14) "Velocity fields"

$$E(x, t) = e \left| \frac{n - \beta}{\gamma^2 (1 - \beta \cdot n)^3 R^3} \right| + \frac{e}{c} \left[\frac{\hat{n} \times [(n - \beta) \times \hat{\beta}]}{(1 - \beta \cdot n)^3 R} \right]$$

14.2

a) "Taylor Expansion"

(e)

$$x(t') = x + v(t-t_0) + \frac{1}{2} a(t'-t_0)^2 + \dots$$

$$\approx \frac{1}{2} a(t'-t_0)^2$$

"Particle moving
with nearly
nonrelativistic
motion"

$$\beta(t) = \frac{1}{c} a(t-t_0) + \dots$$

$$\dot{\beta}(t) = \frac{1}{c} a + \dots$$

"retarded time"

$$\hat{t} = t' + |x - r(t)|/c$$

$$= t' + x/c + \dots$$

$$t' = t_0 - x/c$$

$$r = \frac{x^2}{2c^2} a \quad ; \quad \beta = \frac{x}{c^2} a \quad ; \quad \dot{\beta} = \frac{1}{c} a$$

$$\begin{aligned} R &= x - r \\ &= x - \frac{x^2}{2c^2} a \\ &= x \left(1 - \frac{x}{2c^2} a\right) \end{aligned}$$

$$\hat{n} \equiv x - R$$

$$\equiv x - \frac{x}{2c^2} [a - \dot{x}(\hat{x} \cdot a)]$$

(14.14) "Velocity Fields"

$$E(x, t) = e \left[\frac{n - \beta}{8^2 (1 - \beta \cdot n)^3 R^2} \right]_{\text{ret}} + \frac{e}{c} \left[\frac{n x [(n - \beta) \times \dot{\beta}]}{(1 - \beta \cdot n)^3 R} \right]_{\text{ret}}$$

(1.1a) "Faraday's Law"

$$\nabla \times E - \frac{1}{c} \frac{\partial B}{\partial t} = 0$$

$$\frac{\partial B}{\partial t} = -c \nabla \times E$$

$$= -\frac{e}{c} \frac{\hat{r} \times a}{r^2}$$

$$B = -\frac{e}{c} \frac{\hat{r} \times (a(t-t_0))}{r^2}$$

$$= -\frac{e}{c} \frac{\hat{r} \times v(t)}{r^2}$$

$$= \frac{e}{c} \frac{v(t) \times \hat{r}}{r^2}$$

(5.5) "Bio and Savart Law"

$$B = \mu q \frac{v \times X}{|X|^3}$$

$$= \frac{e}{c} \frac{v \times X}{X^{3/2}}$$

... (pg 176) $\mu = 1/c$

empirical searches
about the constant (μ)

The problem derived a
similar answer to the
book.

Curl of Magnetic field:

$$\nabla \times E = \nabla \times \left(\frac{er}{r^3} \right) - \frac{e}{2c^2} \nabla \times \left(\frac{a}{r} + \frac{r(r \alpha)}{r^3} \right)$$

$$= -\frac{e}{2c^2} \left(\nabla \left(\frac{1}{r} \right) \times a + \frac{1}{r^2} \nabla (r \alpha) \times \hat{r} \right)$$

$$= -\frac{e}{2c^2 r^2} (-\hat{r} \times a + a \times \hat{r})$$

$$= \frac{e}{c^2 r^2} \hat{X} \times a$$

$$= e \left[\frac{x - \frac{\hat{x}}{2c^2} [a - \hat{x}(\hat{x} \cdot a)] + \frac{\hat{x}}{c^2} a}{x^2 (1 - \frac{\hat{x}}{2c^2} \cdot \hat{x} \cdot a) (1 + \frac{\hat{x}}{c^2} \hat{x} \cdot a)} \right] + \frac{e}{c} \left[\frac{x \cdot (\hat{x} \cdot \frac{1}{c} a)}{\gamma} \right]$$

$$= \frac{e \hat{x}}{x^2} + \frac{e}{2c^2 \gamma} [a - 3\hat{x}(\hat{x} \cdot a)] - \frac{e}{c} \frac{\hat{x} \cdot (\hat{x} \cdot \frac{1}{c} a)}{\gamma}$$

$$\equiv \frac{E_x}{x} + E_a$$

$$E_{\text{tot}} = \frac{e \hat{x}}{x^2} - \frac{e}{2c^2 \gamma} [a + \hat{x}(\hat{x} \cdot a)]$$

\uparrow \uparrow
 "Static" "Dynamic"
 coulomb relativistic
 term($1/x^2$) term"

b) (14.13) "Velocity field"

$$\mathbf{B} = [\mathbf{n} \times \mathbf{E}]_{\text{ret}}$$

$$= \left(x - \frac{\hat{x}}{2c^2} [a - \hat{x}(\hat{x} \cdot a)] \right) \circ \left(\frac{e \hat{x}}{x^2} - \frac{e}{2c^2 x} [a + \hat{x}(\hat{x} \cdot a)] \right)$$

$$= -\frac{e}{2c^2 x} (a \cdot x + x \cdot a)$$

$$= 0$$

The particle has no magnetic induction.

c) Zero-divergence Electric field:

$$\nabla \cdot \left(\frac{a + r(r \cdot a)}{r} \right) = \nabla \cdot \left(\frac{a}{r} \right) + \nabla \cdot \left(\frac{r(r \cdot a)}{r^3} \right)$$

$$= \nabla \left(\frac{1}{r} \right) \cdot a + \nabla \left(\frac{1}{r^3} \right) (r(r \cdot a))$$

$$+ \frac{1}{r^3} \nabla \cdot (r(r \cdot a))$$

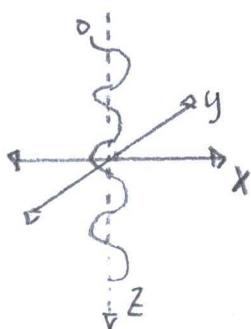
$$= -\frac{1}{r^2} \hat{r} \cdot a - \frac{3}{r^2} \hat{r} \cdot a + \frac{4}{r^2} \hat{r} \cdot a$$

$$\begin{aligned}
& + \frac{1}{(1-\beta \cdot n)^3 R^2} \left[n_0 (n \cdot \beta)^3 - \beta^2 (n \cdot \beta) - \beta (n \cdot \beta)^2 + \beta^2 \beta + \right. \\
& \quad \left. 3n (n \cdot \beta)^2 - 2\beta (n \cdot \beta) - \beta n - 3n (n \cdot \beta)^3 \right. \\
& \quad \left. + 2\beta (n \cdot \beta)^2 + \beta^2 n (n \cdot \beta) \right] \\
& = \frac{n x (n \cdot \beta) - \beta}{c (1-\beta \cdot n)^3 R} + \frac{1}{(1-\beta \cdot n)^3 R^2} \left[-2n (n \cdot \beta)^3 + \beta (n \cdot \beta)^2 + \beta^2 \beta \right. \\
& \quad \left. + 3n (n \cdot \beta)^2 - 2\beta (n \cdot \beta) - \beta^2 n \right]
\end{aligned}$$

$$\begin{aligned}
E_{\text{ret}} &= e \left[\frac{n}{R^2} \right] + e \left[\frac{R}{c} \right] + \frac{n(n \cdot v) - v}{R} + e \frac{3n(n \cdot \beta) - \beta}{(1-\beta \cdot n) R^2} \\
&= \frac{n - \beta}{8^2 (1-\beta \cdot n)^2 R^2}
\end{aligned}$$

$$E(x, t) = e E_{\text{ret}} + \frac{e}{c} \left[\frac{n x (n \cdot \beta) \times \beta}{(1-\beta \cdot n)^3 R} \right]_{\text{ret}}$$

14.4



$$a) Z(t) = a \cos(\omega t)$$

(14.21) "time-averaged Power radiated per solid angle"

$$\begin{aligned}
\frac{dP(t)}{d\Omega} &= \frac{e^2}{4\pi c^3} |v|^2 \sin^2 \theta \\
&= \frac{e^2 a^2 \omega_0^4}{4\pi c^3} \cos^2(\omega_0 t) \sin^2 \theta
\end{aligned}$$

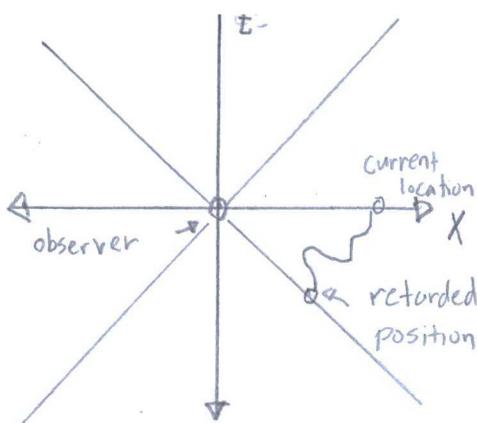
"Particle on
z-axis"

$$\langle \frac{dP(t)}{d\Omega} \rangle = \frac{e^2 a^2 \omega_0^4}{8\pi c^3} \sin^2 \theta \quad \dots \text{when } \langle \cos^2(\omega_0 t) \rangle = 1/2$$

Final Answer

Final

14.3



"retarded time"

(14.14) "Lienard-Wiechart"

$$E(x, t) = e \left[\frac{n - \beta}{\gamma^2 (1 - \beta \cdot n)^3 R^2} \right]_{ret} + \frac{e}{c} \left[\frac{n x [(n - \beta) \times \dot{\beta}]}{(1 - \beta \cdot n)^3 R} \right]_{ret}$$

(14.13) "Lienard-Wiechart field"

$$B = [n \times E]_{ret}$$

$$\text{where } E_{ret} = e \left[\frac{n}{R^2} \right]_{ret} + e \left[\frac{R}{c} \right]_{ret} + \frac{d}{dt} \left[\frac{n}{R^2} \right]_{ret} + e \frac{1}{c^2} \frac{d^2}{dt^2} \left[n \right]_{ret}$$

From (14.3),

$$\frac{dt}{dt'} = 1 - \beta \cdot n$$

3rd term: $\frac{d}{dt} \left[\frac{n}{R^2} \right] = \frac{n(n \cdot v) - v}{R}$

4th term: $\frac{1}{c^2} \frac{dt'}{dt} \frac{dn}{dt} = \frac{1}{c^2} \left[\frac{1}{1 - \beta \cdot n} \right] \left[\frac{d}{dt'} \frac{1}{1 - \beta \cdot n} \frac{d}{dt'} \hat{n} \right]$

$$= \frac{1}{c^2} \left[\frac{1}{1 - \beta \cdot n} \cdot (\dot{\beta} \cdot n + \beta \cdot \dot{n}) \frac{d}{dt} n + \frac{1}{1 - \beta \cdot n} \frac{d^2}{dt^2} n \right]$$

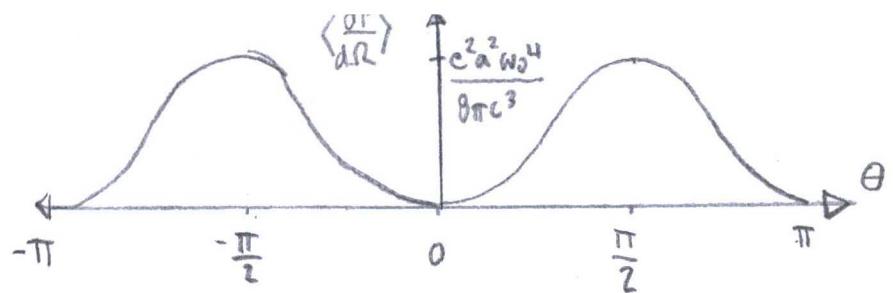
$$= \frac{1}{c^2} \left[\frac{1}{(1 - \beta \cdot n)^3} \left(n \cdot \ddot{\beta} + \frac{(n(n \cdot v) - v)}{R} \beta \right) \frac{n(n \cdot v) - v}{R} \right]$$

$$+ \frac{1}{(1 - \beta \cdot n)^2} \left[\frac{n x (n \times v)}{R} + \frac{3n(n \cdot v)^2 - 2v(n \cdot v)v^2}{R^2} \right]$$

$$= \frac{1}{cR} \left[\frac{n \cdot \ddot{\beta}}{(1 - \beta \cdot n)^3} [n(n \cdot \beta) - \beta] + \frac{n x (n \times \dot{\beta})}{(1 - \beta \cdot n)^2} \right]$$

$$+ \frac{1}{R^2} \frac{[(n(n \cdot \beta) - \beta)\beta][n(n \cdot \beta) - \beta]}{(1 - \beta \cdot n)^3} + \frac{3n(n \cdot \beta)^2 - 2\beta(n \cdot \beta)\beta^2 n}{(1 - \beta \cdot n)^2}$$

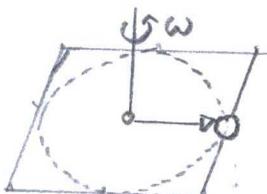
$$= \frac{1}{c(1 - \beta \cdot n)^3 R} [n x (n \times \dot{\beta}) - n x (\beta \times \dot{\beta})]$$



$$\text{b. } \chi(t) = R \langle \cos(\omega_0 t), \sin(\omega_0 t) \rangle$$

$$\dot{\vec{v}}(t) = -\omega_0^2 R \langle \cos(\omega_0 t), \sin(\omega_0 t) \rangle$$

"particle in plane"



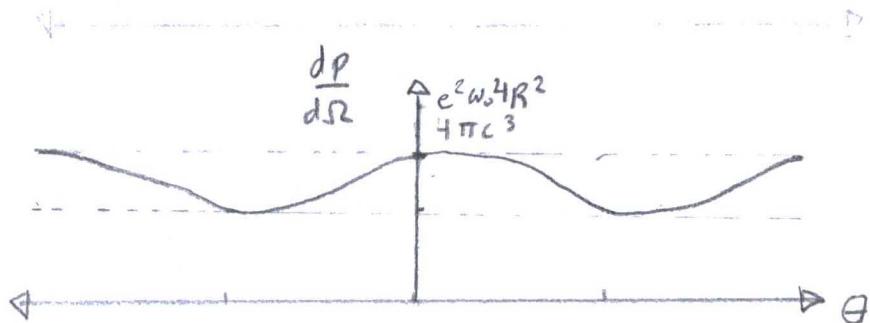
$$\frac{dP}{d\Omega} = \frac{e^2}{4\pi c^3} |\dot{v}|^2 \sin^2 \theta$$

$$= \frac{e^2 w_0^4 R^2}{4\pi c^3} (1 - \sin^2 \theta \cdot \cos^2(\phi - \omega t))$$

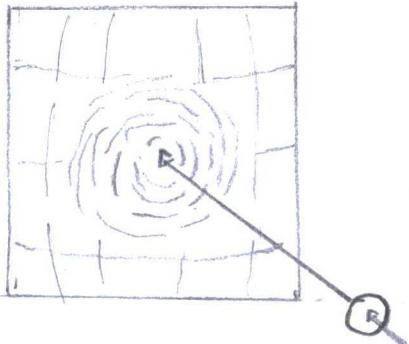
$$\langle \frac{dP}{d\Omega} \rangle = \frac{e^2 w_0^4 R^2}{4\pi c^3} \left(1 - \frac{1}{2} \sin^2 \theta\right) \quad \text{when } \cos^2(\phi - \omega t) = \frac{1}{2}$$

Total Power:

$$P = \int \frac{e^2 w_0^4 R^2}{4\pi c^3} \left(1 - \frac{1}{2} \sin^2 \theta\right) d\Omega$$



14.5.



"Nonrelativistic particle
head on collision
with a force field"

a) (14.22) "Total Instantaneous Power radiated"

$$P dt = \frac{2}{3} \frac{z^2 e^2}{c^3} |V|^2 dt$$

$$\mathbf{F} = m\mathbf{a}$$

$$= m \ddot{\mathbf{r}}$$

$$= -\nabla V$$

$$= -\frac{dV}{dr} \hat{\mathbf{e}}_r$$

$$E = \frac{1}{2} m v^2 + V(r)$$

$$= V(r_{min})$$

$$v = \sqrt{\frac{2}{m} (V(r_{min}) - V(r))}^{1/2}$$

$$P dt = \frac{2}{3} \frac{z^2 e^2}{m^2 c^3} \sqrt{\frac{m}{2}} \left| \frac{dv}{dr} \right|^2 \frac{dr}{\sqrt{V(r_{min}) - V(r)}}$$

(14.51 & 14.53) "Total energy radiated"-modified"

$$\Delta W = 0.00$$

$$= \frac{4}{3} \frac{z^2 e^2}{m^2 c^3} \sqrt{\frac{m}{2}} \int_{r_{min}}^{\infty} \left| \frac{dv}{dr} \right|^2 \frac{dr}{\sqrt{V(r_{min}) - V(r)}}$$

b) If $V(r) = z^2 e^2 / r$

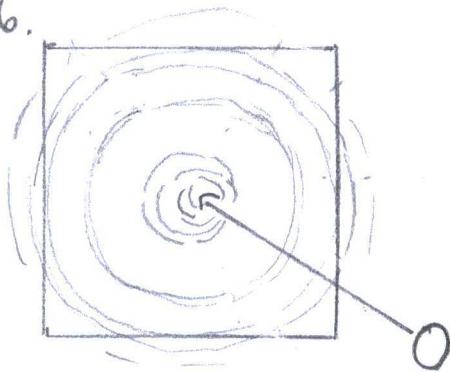
$$\Delta W = \frac{4}{3} \frac{z^2 e^2}{m^2 c^3} \sqrt{\frac{m}{2}} (z^2 e^2)^{3/2} \int_{r_{min}}^{\infty} \frac{1}{r^4} \frac{dr}{\sqrt{\frac{1}{r_{min}} - \frac{1}{r}}}$$

$$= \frac{4}{3} \frac{Z^2 e^2}{m^2 c^3} \sqrt{\frac{m}{2}} (Z^2 e^2)^{3/2} r_{\min} \int_{r_{\min}}^{\infty} \frac{dr}{\sqrt{\frac{1}{r_{\min}} - \frac{1}{r}}}$$

$$\begin{aligned}
 &= \frac{4}{3} \frac{Z^2 e^2}{m^2 c^3} \sqrt{\frac{m}{2}} (Z^2 e^2)^{3/2} r_{\min} \frac{16}{15 r_{\min}^3} \\
 &= \frac{64}{45} \sqrt{\frac{m}{2}} \frac{(Z^2 e^2)^{3/2}}{m^2 c^3} \left(\frac{Z^2 e^2}{m v_0^2} \right)^{-5/2} \\
 &= \frac{8}{45} \frac{Z m v_0^5}{Z c^3}
 \end{aligned}$$

Identity:
 $\int_a^{\infty} \frac{x}{\sqrt{ax-x^2}} dx = \frac{16}{15a^3}$

4.6.



a) (Problem 14.6)

$$E = V - L^2 / 2mr^2$$

where $L = mbv_0$

$$E = mv_0^2 / 2$$

"Nonrelativistic
particle... head
on collision with
a... force field
total energy radiated"

(Problem 14.5)

$$E = \frac{1}{2} mr^{\dot{r}^2} + \frac{1}{2} mr^2 \dot{\theta}^2 + V(r)$$

$$= \frac{1}{2} mr^{\dot{r}^2} + \frac{L^2}{2mr^2} + V(r)$$

$$\dot{r} = \sqrt{\frac{2}{m} \left[E - V(r) - \frac{L^2}{2mr^2} \right]^{1/2}}$$

(14.51 & 14.53) "Total Power radiated - modified"

$\Delta W =$

$$= \frac{4Ze^2}{3m^2 c^3} \sqrt{\frac{m}{2}} \int_{r_{\min}}^{\infty} \left| \frac{dV}{dr} \right|^2 \left[E - V(r) - \frac{L^2}{2mr^2} \right]^{-1/2} dr$$

b) (Problem 14.b)

$$E = V - \frac{L^2}{2mr^2}$$

$$\text{where } V = z^2 e^2 / r$$

$$L = m b v_0$$

$$E = \frac{1}{2} m v_0^2$$

$$\begin{aligned}
 \Delta W &= \frac{4z^2 e^2}{3m^2 c^3} \sqrt{\frac{m}{2}} \int_{r_{\min}}^{\infty} \left(\frac{dv}{dr} \right)^2 \left(E - V(r) - \frac{L^2}{2mr^2} \right)^{-1/2} dr \\
 &= \frac{4z^4 e^2 e^6}{3m^2 c^3 v_0} \int_{r_{\min}}^{\infty} r^{-4} \left(1 - 2 \frac{z^2 e^2}{mv_0^2 r} - \frac{b^2}{r^2} \right)^{-1/2} dr \\
 &= \frac{4z m v_0^5}{3z c^3 t^3} \int_{r_{\min}}^{\infty} \left(\frac{b}{r} \right)^2 \left(1 - 2 \frac{b}{tr} - \frac{b^2}{r^2} \right)^{-1/2} \frac{b dr}{r^2} \quad \text{if } t = bm v_0^2 / z^2 e^2 \\
 &= \frac{4z m v_0^5}{3z c^3 t^3} \int_0^{x_{\max}} \frac{x^2}{\sqrt{1 - 2(x/t) - x^2}} dx \quad \text{if } x = \frac{b}{r} \\
 &= \frac{4z m v_0^5}{3z c^3 t^3} \int_0^{\frac{1}{t} + \sqrt{\frac{1}{t^2} + 1}} \frac{x^2}{\sqrt{(x - (-\frac{1}{t} - \sqrt{\frac{1}{t^2} + 1}))((-\frac{1}{t} + \sqrt{\frac{1}{t^2} + 1}) - x)}} dx \\
 &= \frac{4z m v_0^5}{3z c^3 t^3} \left[-\frac{3}{2t} + \left(\frac{3}{t^2} + 1 \right) \tan^{-1} \left(-\frac{1}{t} + \sqrt{\frac{1}{t^2} + 1} \right) \right] \\
 &= \frac{4z m v_0^5}{3z c^3 t^3} \left[-\frac{3}{2t} + \frac{1}{2} \left(\frac{3}{t^2} + 1 \right) \tan^{-1} t \right] \quad \text{if } t \gg 1 \\
 &= \frac{2z m v_0^5}{z c^3} \left(-\frac{1}{t^4} + \frac{1}{t^5} \left(1 + \frac{t^2}{3} \right) \tan^{-1} t \right)
 \end{aligned}$$

c) With $t = \cot \theta / 2$; $\theta = 2 \cdot \cot^{-1}(t)$

$$\approx \frac{2z m v_0^5}{z c^3} \tan^3 \frac{\theta}{2} \left[\frac{1}{6} (\pi - \theta) \left(1 + 3 \tan^2 \frac{\theta}{2} \right) - \tan \frac{\theta}{2} \right]$$

Only for $E \leq 13$

d) An attractive Coulomb potential is important to chemists and physicists. A common depiction is Lennard-Jones Potential. The term $(e^2/4\pi\epsilon_0 r)$ describes attraction between two charges, including O_2, H_2, N_2, F_2, Cl_2 and Br_2 .

14.7.

a) (14.22) "Larmor Power Formula"

$$P = \frac{2}{3} \frac{e^2}{c^3} |V|^2$$

(14.51) "General Power Formula"

$$\frac{dP}{d\Omega} = |A(t)|^2$$

(14.53) "Total energy radiated"

$$\frac{dW}{d\Omega} = \int_{-\infty}^{\infty} |A(t)|^2 dt$$

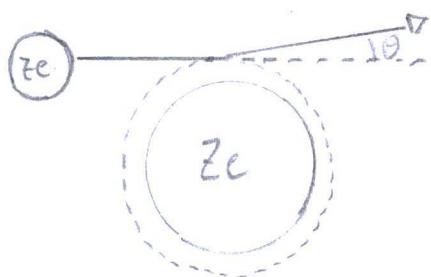
$$\Delta W = \int_{-\infty}^{\infty} P(t) dt$$

$$= 2 \int_{r_{min}}^{\infty} \frac{P}{dr/dt} dr$$

book calls this ΔW

If $E = K + V$

$$= \frac{1}{2} m \dot{r}^2 + V(r)$$



"A nonrelativistic
particle... incident...
impacts... particle
deflection... very small"

$$\frac{dr}{dt} = -\sqrt{\frac{2(E-V)}{m}}$$

$$\Delta W = \frac{2}{3} \frac{e^2}{c^3} \sqrt{\frac{m}{2}} \int_{r_{min}}^{r_0} \frac{Z^2 e^2}{r^2} \left| \frac{dr}{\sqrt{E-V}} \right|$$

$$= \frac{2}{3} \frac{e^2}{c^3} \sqrt{\frac{m}{2}} Z^2 e^2 \int_{r_{min}}^{r_0} \frac{1}{r^2} dr$$

$$= \frac{2}{3} \frac{e^2}{c^3} \sqrt{\frac{m Z^2 e^2}{2}} r_{min} \circ Z^2 e^2 \frac{1}{r_{min}} \quad \text{when } r = \frac{2 Z^2 e^2}{m v_0^2}$$

$$= \frac{4}{9} \frac{e^5 Z^3 E^3}{c^3 V_0} \frac{1}{r_{min}}$$

$$= \frac{4}{9} \frac{Z^3 e^5}{c^3 V_0} \frac{1}{b_{min}}$$

b) $r_c = 2 Z^2 e^2 / m v_0^2$

$$\Delta W = \frac{\pi Z^4 E^2 e^6}{3 m^2 c^3 V_0} \frac{1}{b^3}$$

$$= \frac{\pi Z^4 E^2 e^6}{3 m^2 c^3 V_0} \frac{m^3 V_0^5}{8 e^6 Z^3 E^3} = \frac{\pi Z m V_0^5}{24 c^3 Z}$$

(Problem 14.5) "Head-on collision"

$$\Delta W = \frac{8}{45} \frac{Z m V_0^5}{Z c^5}$$

$$\propto \text{constant} \cdot \frac{m v_0^5}{Z}$$

c) Radiation cross section:

$$\chi = \int \Delta W(b) \cdot 2\pi b db$$

$$= \frac{8 \pi^2 \cdot Z m V_0^5}{12 \cdot c^3 Z} \frac{1}{b_{min}}$$

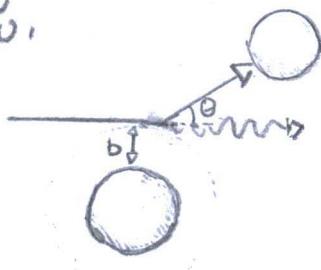
(15.30) "Bethe - Heitler formula"

$$\frac{dE_{\text{rad}}}{dx} = \frac{16}{3} N Z \left(\frac{Ze^2}{hc} \right) \frac{e^4 c^4}{mc^2} \int_0^1 \ln \left(\frac{1 + \sqrt{1-x}}{\sqrt{x}} \right) dx$$



Coefficient in cross section above.

14.0.



(12.1) "Equation of motion"

$$\begin{aligned}\frac{dP}{dt} &= e \left[E + \frac{u}{c} \times B \right] \\ &= \frac{d}{dt} (\gamma m v) \\ &= \frac{d}{dt} (\gamma m c \beta) \\ &= \gamma^3 m c \beta (\beta \cdot \dot{\beta}) + \gamma m c \ddot{\beta} \\ &= z e E\end{aligned}$$

$$\begin{aligned}\gamma m c (\gamma^3 \beta^2 + 1) (\beta \cdot \dot{\beta}) &= \gamma^3 m c \beta \cdot \ddot{\beta} \\ &= z e \beta \cdot E\end{aligned}$$

$$\beta \cdot \dot{\beta} = \frac{z e}{\gamma^3 m c} \beta \cdot E$$

$$\begin{aligned}\dot{\beta} &= (z e E - \gamma^2 m c \beta (\beta \cdot \dot{\beta})) / \gamma m c \\ &= \frac{z e}{\gamma m c} (E - \beta (\beta \cdot E))\end{aligned}$$

(14.26) "Lienard Result (1895)"

$$P = \frac{2}{3} \frac{e^2}{c} \gamma^2 [\dot{\beta}^2 - (\beta \times \dot{\beta})^2]$$

$$\dot{\beta}^2 = \frac{z^2 e^2}{\gamma^2 m^2 c^2} [|E|^2 - 2 |\beta \cdot E|^2 + \beta^2 |\beta \cdot E|^2]$$

$$\begin{aligned}|\beta \times \dot{\beta}|^2 &= \frac{z^2 c^2}{\gamma^2 m^2 c^3} |\beta \times E|^2 \\ &= \frac{z^2 e^2}{\gamma^2 m^2 c^2} [\beta^2 E^2 - |\beta \cdot E|^2]\end{aligned}$$

$$P(t) = \frac{2}{3} \frac{Z^4 e^4}{m^2 c^3} \gamma^4 \left[|E|^2 - 2|\beta \cdot E|^2 + \beta^2 |\beta \cdot E|^2 - \beta^2 |E|^2 + |\beta \cdot E|^2 \right]$$

$$= \frac{2}{3} \frac{Z^4 e^4}{m^2 c^3} \gamma^4 (1 - \beta) \left[|E|^2 - |\beta \cdot E|^2 \right]$$

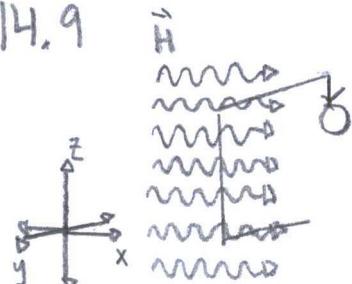
$$\text{If } |E| = \frac{Ze}{r^2 + b^2} \quad \text{and } |\beta \cdot E| = \beta E \cos \theta \\ = -\frac{ze}{r^2 + b^2} \frac{\beta r}{(r^2 + b^2)^{1/2}}$$

$$P(t) = \frac{2}{3} \frac{Z^4 Z^2 e^6}{m^2 c^3} \gamma^2 \frac{1}{(r^2 + b^2)^2} \left(1 - \frac{\beta^2 r^2}{r^2 + b^2} \right)$$

(14.51 & 14.53) "Total Power radiated"

$$\begin{aligned} \Delta W &= \int_{-\infty}^{\infty} P(t) dt \\ &= \int_{-\infty}^{\infty} \frac{2}{3} \frac{Z^4 Z^2 e^6}{m^2 c^3} \gamma^2 \frac{1}{(r^2 + b^2)^2} \left(1 - \frac{\beta^2 r^2}{r^2 + b^2} \right) dr \\ &= \frac{2}{3} \frac{Z^4 Z^2 e^6}{m^2 c^4 \beta} \gamma^2 \left[\frac{\pi}{2b^2} - \frac{\pi \beta^2}{8b^2} \right] \\ &= \frac{2}{3} \frac{Z^4 Z^2 e^6}{m^2 c^4 \beta} \gamma^2 \left[\frac{3}{8} + \frac{1 - \beta^2}{8} \right] \frac{1}{b^3} \\ &= \frac{-Z^4 Z^2 e^6 \pi}{m^2 c^4 \beta} \left[\frac{3}{8} + \frac{1}{8\gamma^2} \right] \frac{1}{b^3} \\ &= \frac{\pi Z^4 Z^2 e^6}{4m^2 c^4 \beta} \left[\gamma^2 + \frac{1}{3} \right] \frac{1}{b^3} \end{aligned}$$

14.9



"a particle moves in a plane perpendicular to a uniform, static magnetic field"

a) (14.26) "Lienard Result"

$$\begin{aligned} P &= \frac{2}{3} \frac{e^2}{c} \gamma^6 \left[\dot{\beta}^2 - (\beta \times \dot{\beta})^2 \right] \\ &= \frac{2}{3} \frac{e^2}{c} \gamma^6 \cdot \dot{\beta}^2 (1 - \beta^2) \\ &= \frac{2}{3} \frac{e^2}{c} \gamma^2 \cdot \dot{v}^2 \end{aligned}$$

(1.3) "Lorentz force"

$$F = q \left(E + \frac{v \times B}{c} \right)$$

$$\frac{dp}{dt} = \frac{qB}{c}$$

$$\dot{v} = \frac{qB}{\gamma m}$$

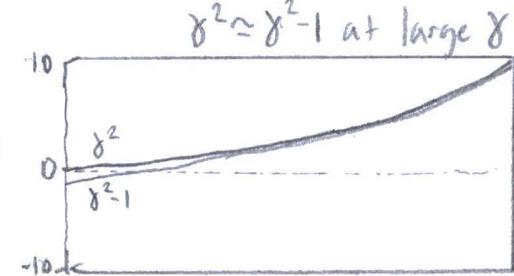
$$\begin{aligned} P &= \frac{2}{3} \frac{q^4}{c^3} \frac{\gamma^2}{m^2} B^2 \\ &= \frac{2q^4 B^2}{3m^2 c^3} (\gamma^2 - 1) \quad \text{... when } q = e \end{aligned}$$

b) $E_0 = \gamma_0 m c^2$

$$\frac{dE}{dt} = \frac{2q^4 B^2}{3m^2 c^2} dt$$

$$\int_{\gamma_0}^{\gamma} \frac{d\gamma}{\gamma^2 - 1} = - \frac{2q^4 B^2}{3m^2 c^2} \int_0^t dt$$

$$\int_{\gamma_0}^{\gamma} \frac{d\gamma}{\gamma^2} = \frac{1}{\gamma_0} - \frac{1}{\gamma}$$



$$= \frac{-2q^4 B^2}{3m^3 c^5} t$$

$$t = \frac{3m^3 c^5}{2q^4 B^2} \left(\frac{1}{\gamma} - \frac{1}{\gamma_0} \right)$$

c) Kinetic energy:

$$\dot{V} = \frac{qB^2}{8m}$$

$$T_0 = \frac{1}{2}mv^2$$

$$V = \sqrt{2T_0/m}$$

$$P = \frac{2}{3} \frac{q^2}{c^3} |V|^2$$

$$= \frac{4q^4 B^2 T_0}{3m^3 c^5}$$

$$T = T_0 - P \cdot t$$

$$= T_0 \left(1 - \frac{4q^4 B^2}{3m^3 c^5} t \right)$$

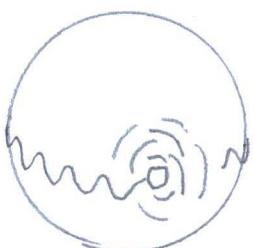
$$= T_0 \exp\left(-\frac{4q^4 B^2 t}{3m^3 c^5}\right)$$

Exponential Expansion:

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} +$$

- d) The particle radiates during large magnetic fields. At the equator, the acceleration is greatest in the function

$$P(t) = \frac{2e^2 \gamma^4}{c^3} \left| \frac{dv}{dt} \right|^2$$



"Particle trapped in
magnetic dipole field
spiraling back and
forth along a line, radiating"

14.10



"a particle of charge... moves at constant velocity ... but speed decreases"

a) (14.3a) "Power radiated per solid angle"

$$\frac{dP(t)}{d\Omega} = \frac{e^2 v^2}{4\pi c^3} \frac{\sin^2 \theta}{(1-\beta \cos \theta)^5}$$

$$\dot{\beta} = \frac{\beta}{\Delta t}$$

$$\beta(t) = \beta_0 (1 - t/\Delta t)$$

$$\frac{dE}{d\Omega} = \int_0^{\Delta t} \frac{dP}{d\Omega} dt$$

$$= \frac{e^2 \beta^2}{4\pi c (\Delta t)^2} \int_0^{\Delta t} \frac{\sin^2 \theta}{[1 - \beta \cos \theta + \frac{\beta \cos \theta}{\Delta t} t]^5} dt$$

$$= \frac{e^2 \beta^2}{4\pi c (\Delta t)^2 \beta \cos \theta} \left. \left(-\frac{1}{4} \left((1 - \beta \cos \theta + \frac{\beta \cos \theta}{\Delta t} t)^{-4} \right) \right) \right|_0^{\Delta t}$$

$$= \frac{e^2 \beta^2}{16\pi c \Delta t \beta \cos \theta} \frac{1}{(1 - \beta \cos \theta)^4} \left((1 - \beta \cos \theta)^{-4} - 1 \right) \sin^2 \theta$$

$$= \frac{e^2 \beta^2}{16\pi c \Delta t} \frac{(2 - \beta \cos \theta)[1 + (1 - \beta \cos \theta)^2]}{(1 - \beta \cos \theta)^4} \sin^2 \theta$$

$$b) \frac{dE}{d\Omega} = \frac{e^2 \beta^2}{16\pi c \Delta t} \frac{(2 - \beta \cos \theta)[1 + (1 - \beta \cos \theta)^2]}{(1 - \beta \cos \theta)^4} \sin^2 \theta$$

$$= \frac{e^2 \beta^2}{16\pi c \Delta t} \frac{[1 + \frac{1}{28}(1 + 8\theta^2)][1 + \frac{1}{48}(1 + 8\theta^2)^2]}{\frac{1}{16\theta^3} (1 + 8\theta^2)^4} \theta^2$$

When $\sin \theta \sim \theta$, $\cos \theta \sim 1 - \frac{\theta^2}{2}$

$$\beta = (1 - 1/8^2) \sim 1 - 1/2 \cdot 8^2$$

$$= \frac{e^2 \beta^2}{16\pi c \Delta t} \gamma^6 \frac{(\gamma \theta)^2}{(1+(\gamma \theta)^2)^4}$$

If $\xi = (\gamma \theta)^2$, then

$$\frac{dE}{d\xi} = \frac{d}{d\xi} \frac{e^2 \beta^2}{16\pi c \Delta t} \gamma^6 \frac{(\xi)^2}{(1+\xi^2)^4}$$

$$= \frac{e^2 \beta^2}{c \Delta t} \gamma^4 \frac{\xi}{(1+\xi^2)^4}$$

$$\langle \theta^2 \rangle = \frac{1}{\gamma^4} \langle \xi \rangle$$

$$= \frac{1}{\gamma^2} \frac{\int \xi \frac{dE}{d\xi} d\xi}{\int \frac{dE}{d\xi} d\xi}$$

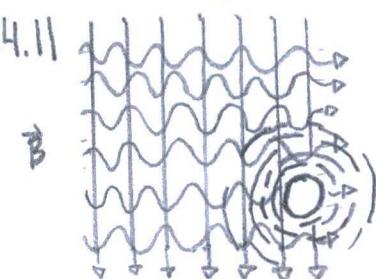
$$= \frac{1}{\gamma^2} \frac{\int \frac{\xi^2}{(1+\xi^2)^4} d\xi}{\int \frac{\xi}{(1+\xi^2)^4} d\xi}$$

$$= \frac{1}{\gamma^2} \frac{1/3}{1/6}$$

$$= \frac{2}{\gamma^2}$$

$$\langle \theta \rangle = \frac{\sqrt{2}}{\gamma}$$

\vec{E}



4.11 "a particle of charge in external electric and magnetic fields... instantaneous energy radiated"

a) (12.1) "Equation of motion"

$$\frac{dP}{dt} = e \left[\vec{E} + \frac{\vec{u}}{c} \times \vec{B} \right]$$

(12.2) "

$$\underline{dE} = eu \cdot E$$

(14.24) "Lorentz invariance generalization"

$$P = -\frac{2}{3} \frac{e^2}{m^2 c^3} \left(\frac{dp_H}{dT} \frac{dp^H}{dT} \right) \text{ where } dT = dt/\gamma$$

$$= -\frac{2}{3} \frac{Z^2 e^2}{m^4 c^3} \gamma^2 \cdot Z^2 \cdot e^2 \left(\left(\frac{V}{c} \cdot E \right)^2 - \left(E + \frac{V}{c} \cdot \beta \cdot B \right)^2 \right)$$

$$= \frac{2}{3} \frac{Z^4 e^4}{m^4 c^3} \gamma^2 \left[(E + \beta \cdot B) - (\beta \cdot E)^2 \right]$$

because $P^H = (E/c, p)$

b) Manifests:

$$P = \frac{2}{3} \frac{Z^4 e^4}{m^4 c^3} \gamma^2 \left[(E + \beta \cdot B) - (\beta \cdot E)^2 \right]$$

$$= \frac{2}{3} \frac{Z^4 r_0^2}{m^2 c} \left[(P^H \cdot E + p \cdot B) - (p \cdot E)^2 \right]$$

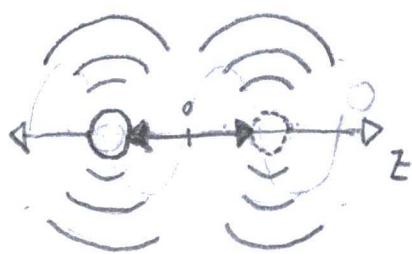
$$F^{xP} F_\beta = \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -B_z & B_y \\ E_y & B_z & 0 & -B_x \\ E_z & -B_y & B_x & 0 \end{pmatrix} \begin{pmatrix} P_0 \\ -P_x \\ -P_y \\ -P_z \end{pmatrix}$$

$$= \begin{pmatrix} P_0 E \\ P_0 E_x + P_x B_x \\ P_0 E_y + P_x B_y \\ P_0 E_z + P_x B_z \end{pmatrix}$$

$$g_{\mu\nu} F^{xP} P_\beta \cdot F^{\mu\nu} \cdot P_v = (P_0 E)^2 - (P^H \cdot E + p \cdot B)^2$$

$$P = \frac{2 Z^4 r_0^2}{3 m^2 c} F^{\mu\nu} P_\nu P^\lambda \cdot F_{\lambda\mu}$$

14.12.



"Charge moves in simple harmonic motion along z-axis"

Note:

Problem 14.4a had a similar function, $Z(t) = \cos(\omega t)$. Problem 14.12 is the correct traversal.

$$\begin{aligned}
 a) Z(t') &= a \cos(\omega_0 t') \hat{z} \\
 V(t') &= \dot{Z}(t') \\
 &= -a \omega_0 \sin(\omega_0 t') \hat{z} \\
 a(t') &= \ddot{Z}(t) \\
 &= -a \omega_0^2 \sin(\omega_0 t') \hat{z}
 \end{aligned}$$

(14.39) "Angular Distribution"

$$\begin{aligned}
 \frac{dP(t)}{d\Omega} &= \frac{e^2}{4\pi c} \frac{\sin^2 \theta}{(1 - \beta \cos \theta)^5} \\
 &= \frac{e^2}{4\pi c} \frac{|n \times n \times v|}{(1 - n \cdot v/c)^5} \\
 &= \frac{e^2}{4\pi c^3} \frac{(a \omega_0^2 \sin \theta \cos(\omega_0 t))}{(1 + \frac{a \omega_0}{c} \cos \theta \sin(\omega_0 t))^5} \\
 &= \frac{e^2 c \beta^4}{4\pi a^2} \frac{\sin^2 \theta \cos^2(\omega_0 t)}{(1 + \beta \cos \theta \sin(\omega_0 t))^5} \quad \text{when } v = a \omega_0 \\
 &\quad \beta = a \omega_0 / c
 \end{aligned}$$

b) (14.53) "Total energy radiated per unit solid angle"

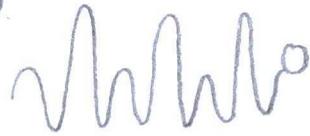
$$\frac{dW}{d\Omega} = \int_{-\infty}^{\infty} |A(t)|^2 dt$$

(14.51) "Power radiated per unit solid angle"

$$\frac{dP(t)}{d\Omega} = |A(t)|^2$$

$$\begin{aligned}
 \frac{dW}{d\Omega} &= \int_0^{2\pi \omega_0} \frac{dP(t)}{d\Omega} dt \\
 &= \frac{dP}{d\Omega} \cdot \frac{2\pi}{\omega_0}
 \end{aligned}$$

14.13



" motion of radiating
particle repeats
itself... continuous
frequency spectrum "

(14.53) "Total energy radiated per solid angle"

$$\frac{dW}{d\Omega} = \int_{-\infty}^{\infty} |A(t)|^2 dt$$

(14.54) "Fourier Transform modified"

$$A(t) = \sum_{m=-\infty}^{\infty} A_m e^{-im\omega_0 t} \quad \text{where } A_m = \frac{\omega_0}{2\pi} \int_0^{2\pi/\omega_0} A(t) e^{im\omega_0 t} dt$$

$$\begin{aligned} \frac{dW}{d\Omega} &= \int_{-\infty}^{\infty} \left| \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} A_m * A_n e^{i(m-n)\omega_0 t} \right|^2 dt \\ &= \frac{2\pi}{\omega_0} \sum_{m=-\infty}^{\infty} |A_m|^2 \end{aligned}$$

(14.55) "Power radiated per unit solid angle"

$$\left\langle \frac{dP}{d\Omega} \right\rangle = \frac{1}{T} \int_0^T \frac{dP(t)}{d\Omega} dt$$

$$= \frac{1}{T} \int_0^T |A(t)|^2 dt$$

$$= \sum_{m=-\infty}^{\infty} |A_m|^2$$

(14.52) "General Amplitude"

$$A(t) = \left(\frac{c}{4\pi} \right)^{1/2} [RF]_{ret}$$

(14.64) "Fourier Transform with trajectory
and distribution from origin"

$$A_m = \left(\frac{e^2}{4\pi c} \right)^{1/2} \frac{\omega_0}{2\pi} \int_0^{2\pi/\omega_0} \frac{d}{dt} \left[\frac{n \times (n \times p)}{1 - \beta \omega n} \right] e^{im\omega_0(t - n \cdot x(t)/c)} dt$$

$$\frac{dP}{d\Omega} = \frac{e^2 c \beta^4}{4 \pi a^2} \frac{\omega_0}{2\pi} \int_0^{2\pi/\omega_0} \frac{\sin^2 \theta \cos^2(\omega_0 t)}{(1 + \beta \cos \theta \sin(\omega_0 t))^5} dt'$$

$$t' = 2 \arctan(u)$$

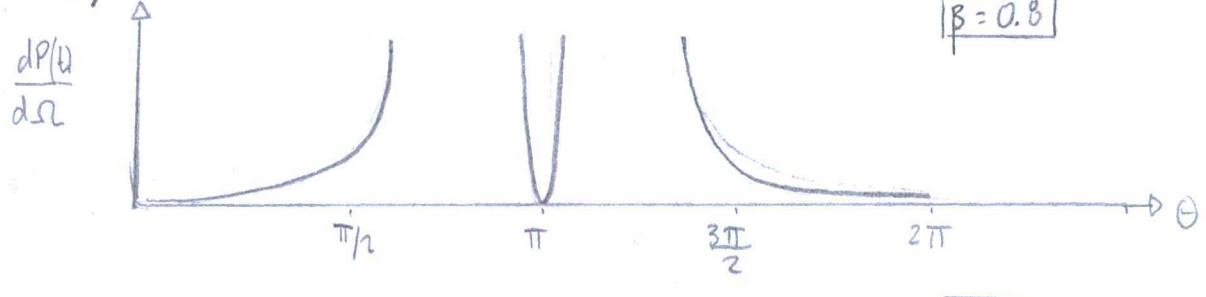
$$dt' = \frac{2}{1+u^2} du$$

$$= \frac{e^2 c \beta^4}{4 \pi a^2} \frac{\omega}{2\pi} \sin^2 \theta \int_{-\infty}^{\infty} \frac{\left(\frac{1-u^2}{1+u^2}\right)^2}{\left(1 + \beta \cos \theta \frac{2u}{1+u^2}\right)^5} \frac{2du}{1+u^2}$$

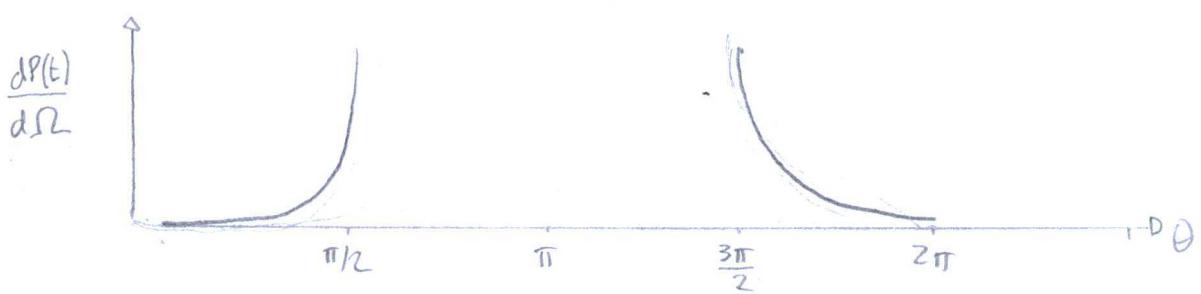
$$= \frac{e^2 c \beta^4}{4 \pi a^2} \frac{\omega}{2\pi} \sin^2 \theta \frac{\pi (4 + \beta^2 \cos^2 \theta)}{4 (1 - \beta^2 \cos^2 \theta)^{7/2}}$$

$$= \frac{e^2 c \beta^4}{32 \pi a^2} \frac{4 + \beta^2 \cos^2 \theta}{(1 - \beta^2 \cos^2 \theta)^{7/2}} \sin^2 \theta$$

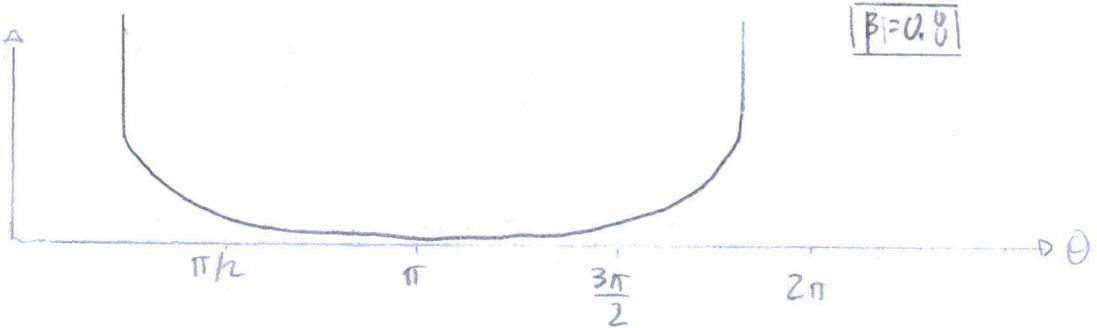
c)



Instantaneous

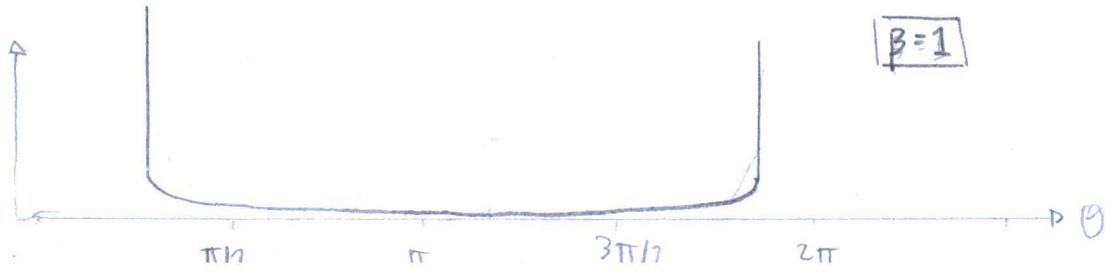


$\frac{dP}{d\Omega}$



Time averaged

$\frac{dP}{d\Omega}$



$$= - \left(\frac{e^2}{4\pi c} \right) \frac{im\omega_0^2}{2\pi} \int_0^{2\pi/\omega_0} [nx(nx\beta)] e^{im\omega_0(t-n\cdot x(t)/c)} dt$$

$$\left\langle \frac{dP(t)}{d\Omega} \right\rangle = \sum_{m=-\infty}^{\infty} \frac{e^2 m^2 \omega_0^4}{8\pi^3 c^3} \left| \int_0^{2\pi/\omega_0} (nx(nx\beta)) e^{im\omega_0(t-n\cdot x(t)/c)} dt \right|^2$$

$$= \sum_{m=-\infty}^{\infty} \frac{e^2 \omega_0^4 m^2}{(2\pi c)^3} \left| \int_0^{2\pi/\omega_0} (V(t) \times n) e^{im\omega_0(t-n\cdot x(t)/c)} dt \right|^2$$

$$= \sum_{m=-\infty}^{\infty} \frac{d\sigma}{d\Omega}$$

14.14. a) $X(t) = a \cos(\omega_0 t) \hat{z}$

$$V(t) = -a \omega_0 \sin(\omega_0 t) \hat{z}$$

(14.64) "Fourier Transform modified"

"charged particle radiating per unit solid angle in the m^{th} -harmonic"

$$A_m = \left(\frac{e^2}{4\pi c} \right)^{1/2} \frac{\omega_0}{2\pi} \int_0^{2\pi/\omega_0} \frac{d}{dt} \left[\frac{nx(nx\beta)}{1-\beta \cos\theta} \right] e^{im\omega_0(t-n\cdot x(t)/c)} dt$$

$$= \left(\frac{e^2}{4\pi c} \right)^{1/2} \frac{\omega_0}{2\pi} \int_0^{2\pi/\omega_0} V(t) \times \exp \left[i m \omega_0 (t - \frac{n \cdot x(t)}{c}) \right] dt$$

$$= \left(\frac{e^2}{4\pi c} \right)^{1/2} \frac{\omega_0}{2\pi} \int_0^{2\pi/\omega_0} \frac{\sin\theta \sin(\omega_0 t) \exp \left[i m \omega_0 (t - \frac{a \cos\theta \cos\omega t}{c}) \right]}{x \cdot a \cdot \omega_0} dt$$

Mathematical Identity:

$$\exp \left\{ -im \omega_0 \frac{a}{c} \cos\theta \cos(\omega_0 t) \right\} = \sum_{n=0}^{\infty} (-i)^n J_n(m \beta \cos\theta) e^{-in\omega_0 t}$$

.. when "a" units equal miles/hour
because $a/c = v/c = \beta$

$$= \left(\frac{e^2}{4\pi c} \right) \frac{\omega_0}{2\pi} \sin \theta \sum_{n=0}^{\infty} (-i)^n J_n(m\beta \cos \theta) \int_0^{2\pi/\omega_0} \sin(\omega t) e^{i(m-n)\omega t} dt$$

$$= \left(\frac{e^2}{4\pi c} \right) \frac{\omega_0}{2\pi} \sin \theta \sum_{n=0}^{\infty} (-i)^n J_n(m\beta \cos \theta) \frac{1}{2i} \frac{2\pi}{\omega_0} (J_{m+1,n} - J_{m-1,n})$$

$$= \frac{\pi}{\omega_0}$$

Recurrence Relation

$$= \left(\frac{e^2}{4\pi c} \right) \frac{\omega_0}{2\pi} \sin \theta \frac{\pi}{i \omega_0} \left[(-i)^{m+1} J_{m+1}(m\beta \cos \theta) - (-i)^{m-1} J_{m-1}(m\beta \cos \theta) \right]$$

$$= \left(\frac{e^2}{4\pi c} \right) \frac{\omega_0}{2\pi} \sin \theta - (-i)^m \frac{\pi}{\omega_0} \left[J_{m+1}(m\beta \cos \theta) + J_{m-1}(m\beta \cos \theta) \right]$$

Recurrence Relation

Note: Bessel's had more analysis in 1950-1960's.

Then the 2000's revitalized distributions,

interval values, and floating point arithmetic
with Bessel's.

$$= \left(\frac{e^2}{4\pi c} \right) \frac{\omega_0}{2\pi} \sin \theta - (-i)^m \frac{\pi}{\omega_0} \frac{2m}{m\beta \cos \theta} J_m(m\beta \cos \theta)$$

$$= \left(\frac{e^2}{4\pi c} \right) \frac{\omega_0}{2\pi} \sin \theta - (-i)^m \frac{2\pi}{\omega_0} \frac{J_m(m\beta \cos \theta)}{\beta \cos \theta}$$

(Problem 14.13) "Averaged power radiated per unit solid angle"

$$\frac{dP}{d\Omega} = \frac{e^2 \omega_0 4m^2}{(2\pi c)^3} \left| \int_0^{2\pi/\omega_0} r(t) \times n e^{im\omega_0(t - \frac{r \cdot x(t)}{c})} dt \right|^2$$

$$= \frac{e^2 \omega_0^4 m^2}{(2\pi c)^3} \frac{(2\pi)^2}{\omega_0^2} \underbrace{\frac{J_m(m\beta \cos\theta)^2}{\beta^2 \cos^2\theta}}_{\text{Components from 14.64, except coefficient.}} a^2 \omega_0^2 \sin^2\theta$$

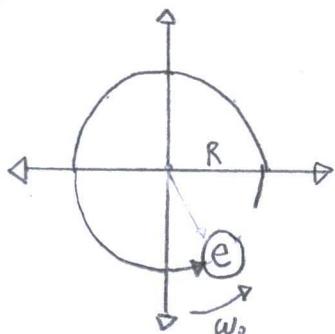
Components from 14.64, except coefficient.

$$= \frac{e^2 c \beta^2}{2\pi a^2} m^2 \tan^2\theta J_m(m\beta \cos\theta)^2$$

b) $v \ll c$, $J_1(x) \sim x^2/c$, $\frac{dP}{d\Omega} = \frac{e^2 c \beta^4}{8\pi a^2} \sin^2\theta$

$$\begin{aligned} P &= \int \frac{e^2 c \beta^4}{8\pi a^2} \sin^2\theta d\Omega \\ &= \frac{1}{3} \frac{e^2 c \beta^4}{a^2} \\ &= \frac{2}{3} \frac{e^2}{c^3} \omega_0^4 a^2 \end{aligned}$$

4.15 a) $\vec{n} = (\sin\theta, 0, \cos\theta)$



$$x(t) = R(\cos(\omega_0 t), \sin(\omega_0 t), 0)$$

$$v(t) = \omega_0 R(-\sin(\omega_0 t), \cos(\omega_0 t), 0)$$

$$n \cdot x(t) = R \sin\theta \cos(\omega_0 t)$$

$$v(t) \times n = \omega_0 R (\cos\theta \cos(\omega_0 t), \cos\theta \sin(\omega_0 t), -\sin\theta \cos(\omega_0 t))$$

"particle... moves
in a circular
path.. on x-y plane"

$$\frac{dP}{d\Omega} = \frac{e^2 \omega_0^4 m^2}{(2\pi c)^3} \left| \int_0^{2\pi/\omega_0} \begin{pmatrix} \cos\theta \cos(\omega_0 t) \\ \cos\theta \sin(\omega_0 t) \\ -\sin\theta \cos(\omega_0 t) \end{pmatrix} \omega_0 R \cdot e^{im\theta(t - \frac{R}{c} \sin\theta \cos(\omega_0 t))} dt \right|^2$$

Identity:

$$e^{-im\omega_0 t} \frac{R}{c} \sin\theta \cos(\omega_0 t) = \sum (-i)^k J_m(m) \frac{\omega_0 R}{c} \sin^{m+1}\theta \cos^{m-1}\theta e^{-ik\omega_0 t}$$

$$= \frac{e^2 \omega_0^4 m^2}{(2\pi c)^3} \left| \begin{pmatrix} \cos\theta & \\ \cos\theta & \\ -\sin\theta & \end{pmatrix} \int_0^{2\pi/\omega_0} \begin{pmatrix} \cos(\omega_0 t) & \\ \sin(\omega_0 t) & \\ \cos(\omega_0 t) & \end{pmatrix} e^{im\omega_0 t} \cdot \sum (-i)^k J_m(m) \frac{\omega_0 R}{c} \sin^{m+1}\theta \cos^{m-1}\theta \right. \\ \left. e^{-ik\omega_0 t} dt \right|^2$$

$$= \frac{e^2 \omega_0^4 m^2}{(2\pi c)^3} \left| \begin{pmatrix} \cos\theta & \\ \cos\theta & \\ -\sin\theta & \end{pmatrix} \int_0^{2\pi/\omega_0} \frac{1}{2} \begin{pmatrix} e^{i\omega_0 t} + e^{-i\omega_0 t} \\ e^{i\omega_0 t} - e^{-i\omega_0 t} \\ e^{i\omega_0 t} + e^{-i\omega_0 t} \end{pmatrix} e^{im\omega_0 t} \cdot \sum (-i)^k J_m(m) \frac{\omega_0 R}{c} \sin^{m+1}\theta \cos^{m-1}\theta \right. \\ \left. e^{-ik\omega_0 t} dt \right|^2$$

$$= \frac{e^2 \omega_0^4 m^2}{(2\pi c)^3} \left| \begin{pmatrix} \cos\theta & \\ \cos\theta & \\ -\sin\theta & \end{pmatrix} \int_0^{2\pi/\omega_0} \frac{1}{2} \begin{pmatrix} ((-i)^{m+1} \cdot J_{m+1}(m\beta \sin\theta) + (-i)^m \cdot J_{m-1}(m\beta \sin\theta)) \\ ((-i)^{m+1} \cdot J_{m+1}(m\beta \sin\theta) + (-i)^{m-1} \cdot J_{m-1}(m\beta \sin\theta)) \\ ((-i)^{m+1} \cdot J_{m+1}(m\beta \sin\theta) + (-i)^{m-1} \cdot J_{m-1}(m\beta \sin\theta)) \end{pmatrix} \right. \\ \left. \cdot \sum (-i)^k J_m(m) \frac{\omega_0 R}{c} \sin^{m+1}\theta \cos^{m-1}\theta \right. \\ \left. e^{-ik\omega_0 t} dt \right|^2$$

$$\cdot \sum (-i)^k J_m(m) \frac{\omega_0 R}{c} \sin^{m+1}\theta \cos^{m-1}\theta \\ \left. \cdot e^{-ik\omega_0 t} dt \right|^2$$

$$= \frac{e^2 \omega_0^4 m^2}{(2\pi c)^3} \left| \begin{pmatrix} \cos\theta & \\ \cos\theta & \\ -\sin\theta & \end{pmatrix} \frac{2\pi}{\omega_0} \int_0^{2\pi/\omega_0} \frac{1}{2} \begin{pmatrix} (-i)^{m+1} 2 \frac{dJ_m(x)}{dx} \Big|_{x=m\beta \sin\theta} \\ (-i)^{m+2} \frac{2m}{m\beta \sin\theta} J_m(m\beta \sin\theta) \\ (-i)^{m+1} 2 \frac{dJ_m(x)}{dx} \Big|_{x=m\beta \sin\theta} \end{pmatrix} dt \right|^2$$

$$= \frac{e^2 \omega_0^4 R^2}{2\pi c^3} m^2 \left| \begin{pmatrix} \cos\theta dJ_m/dx \Big|_{x=m\beta \sin\theta} \\ (-i)\cos\theta J_m(m\beta \sin\theta) / \beta \sin\theta \\ -\sin\theta dJ_m/dx \Big|_{x=m\beta \sin\theta} \end{pmatrix} \right|^2$$

$$= \frac{e^2 w_0^4 R^2}{2\pi c^3} m^2 \left\{ \left[\frac{d J_m(m \beta \sin \theta)}{d(m \beta \sin \theta)} \right]^2 + \frac{\cot^2 \theta}{\beta^2} J_m^2(m \beta \sin \theta) \right\}$$

b) $J_1(z) \sim z/2$, $d J_1(z)/dz = 1/2$

$$\frac{dP}{d\Omega} = \frac{dP_1}{d\Omega} = \frac{e^2 w_0^4 R^2}{2\pi c^3} \left(\frac{1}{4} + \frac{1}{4} \cos^2 \theta \right)$$

$$= \frac{e^2 w_0^4 R^2}{4\pi c^3} \left(1 - \frac{1}{2} \sin^2 \theta \right)$$

$$= \frac{2}{3} \frac{e^2}{c^3} w_0^4 \bar{a}^2 \quad (\text{Problem 14.11b})$$

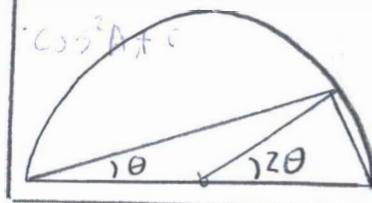
$$\text{When } \bar{a}^2 = \frac{3R^2}{8\pi} \left(1 - \frac{1}{2} \sin^2 \theta \right)$$

Double-Angle

Identity:

$$\cos(2A) = 2\cos^2 A - 1$$

$$\Rightarrow \frac{1}{2} + \frac{1}{2} \cos(2A) = \cos^2 A$$



c) Citation: Watson, G.N. "Theory of Bessel Functions"

2nd edition, Cambridge University Press.

Cambridge (1952), pg 79, 249.

14.16.

(14.79) "Intensity Distribution"

$$\frac{d^2 I}{d\omega d\Omega} = \frac{e^2}{3\pi^2 c} \left(\frac{\omega p}{c}\right)^2 \left(\frac{1}{\gamma^2} + \Theta^2\right)^2 \cdot \left[K_{2/3}^2(\xi) + \frac{\Theta^2}{(1/\gamma^2) + \Theta^2} K_{1/3}^2(\xi) \right]$$

(pg 678) "Approximation"

$$\omega \sim (c/p)$$

$$\frac{d^2 I}{d\omega d\Omega} = \frac{e^2}{3\pi^2 c} \frac{1}{\gamma^4} \left(1 + \gamma^2 \Theta^2\right)^2 \left[K_{2/3}^2(\xi) + \frac{\Theta^2}{\gamma^2 (1 + \gamma^2 \Theta^2)} K_{1/3}^2(\xi) \right]$$

(14.76) "Parameter"

$$\xi = \frac{\omega p}{3c} \left(\frac{1}{\gamma^2} + \Theta^2\right)^{1/2}$$

$$= \frac{\omega p}{3c} \frac{1}{\gamma} \left(1 + \gamma^2 \Theta^2\right)^{1/2}$$

$$= \frac{2\sqrt{2}}{3m} \frac{(p_0 k)^{3/2}}{(|d^2(p_0 k)/dT^2|)^{1/2}} \quad \text{... from (Problem 14.16)}$$

$$\frac{(1 + \gamma^2 \Theta^2)}{\gamma^2} = \frac{8 c^{2/3} (p_0 k)^3}{\omega^2 p^2 (|d^2(p_0 k)/dT^2|)^3}$$

$$= \xi^2 \frac{(p_0 k)^3}{(|d^2(p_0 k)/dT^2|)}$$

$$\frac{(1 + \gamma^2 \Theta^2)^2}{\gamma^4} = 64 \frac{(p_0 k)^6}{(|d^2(p_0 k)/dT^2|)^2}$$

$$\frac{d^2 I}{d\omega d\Omega} = \frac{e^2}{3\pi^2 c} 64 \frac{(p_0 k)^6}{(|d^2(p_0 k)/dT^2|)^2} \left[K_{2/3}^2(\xi) + \frac{\Theta^2}{\left(\frac{8(p_0 k)^3}{(|d^2(p_0 k)/dT^2|)}\right)} K_{1/3}^2(\xi) \right]$$

(14.95) "Angle"

$$\Theta_c \approx \left(\frac{3c}{\omega p}\right)^{1/3} = \frac{1}{\gamma} \left(\frac{2\omega_c}{\omega}\right)^{1/3}$$

$$\approx \left(\frac{3c}{\omega p}\right)^{1/2}$$

... @ critical angle 1/e (pg 680)

$$\Theta^2 = \left(\frac{3c}{\omega p}\right)$$

$$\frac{d^2I}{d\omega d\Omega} = \frac{4e^2}{3\pi^2 c} \left(\frac{3}{4} \right) (p_0 k)^2 \left[\frac{4}{3} \frac{(p_0 k)^4}{(d^2(p_0 k)/d\zeta^2))^2} K_{2/3}^2(\xi) + \frac{(p_0 k)}{2 \cdot (d^2(p_0 k)/d\zeta^2))^2} K_{1/3}^2(\xi) \right]$$

(pg 683) "Number of photons... obtained by $\hbar\omega$ "

$$\hbar\omega \frac{d^2N}{d\omega d\Omega} = \frac{4e^2}{3\pi^2 c} \left(\frac{3}{4} \right) (p_0 k)^2 \left[\frac{4}{3} \frac{(p_0 k)^4}{(d^2(p_0 k)/d\zeta^2))^2} K_{2/3}^2(\xi) + \frac{(p_0 k)}{2 \cdot (d^2(p_0 k)/d\zeta^2))^2} K_{1/3}^2(\xi) \right]$$

$$\hbar\omega \frac{d^2N}{dk d\Omega} = \frac{4e^2}{3\pi^2 c} \left(\frac{3}{4} \right) k \cdot c (p_0 k)^2 \left[\frac{4}{3} \frac{(p_0 k)^4}{(d^2(p_0 k)/d\zeta^2))^2} K_{2/3}^2(\xi) + \frac{(p_0 k)}{2 \cdot (d^2(p_0 k)/d\zeta^2))^2} K_{1/3}^2(\xi) \right]$$

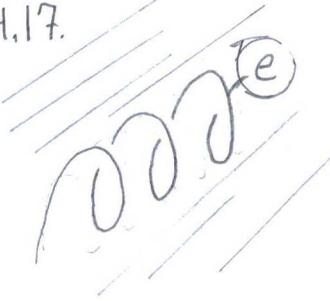
$$\hbar\omega \frac{d^3N}{d^3k} = \frac{4e^2}{3\pi^2 c} \left(\frac{3}{4} \right) k^2 (p_0 k)^2 \left[\frac{4}{3} \frac{(p_0 k)^4}{(d^2(p_0 k)/d\zeta^2))^2} K_{2/3}^2(\xi) + \frac{(p_0 k)}{2 \cdot (d^2(p_0 k)/d\zeta^2))^2} K_{1/3}^2(\xi) \right]$$

$$= \frac{4e^2}{3\pi^2 c} \left(\frac{3}{4} \right) \left[\frac{4}{3} \frac{(p_0 k)^4}{(d^2(p_0 k)/d\zeta^2))^2} \cdot (E_0 p)^2 K_{2/3}^2(\xi) + \frac{(p_0 k)(E_0 p)^2}{2 \cdot (d^2(p_0 k)/d\zeta^2))^2} K_{1/3}^2(\xi) \right]$$

... a couple proportions were incorrect.

Critical pieces in the problem include ξ -parameter relationship, Θ -angle, and third-order derivative"

14.17.



a) (12.41.5) "Transverse Momentum"

$$c p_{\perp} = e Br$$

()

$$c \gamma m v \cos \alpha = e Br$$

"Particle moves relativistically in a helical path in a uniform magnetic field"

$$r = \frac{\gamma mc}{eB} v \cos \alpha$$

$$= \frac{v \cos \alpha}{\omega_B} \quad \text{when } \omega_B = \frac{eB}{\gamma mc}$$

(14.4a) "Pulse Length"

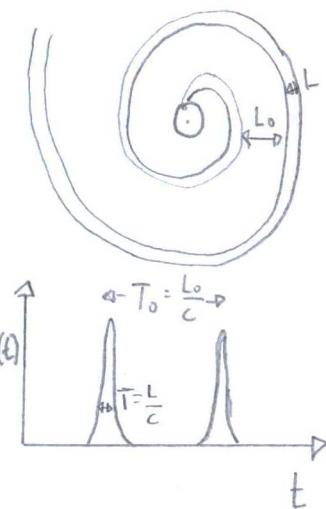
$$L = D - d = \left(\frac{1}{\beta} - 1 \right) \frac{p}{\gamma}$$

$$\approx \frac{p}{2\gamma^3}$$

$$L \cos \alpha = \frac{v}{2\gamma^3} \cdot \frac{\cos^2 \alpha}{\omega_B}$$

in

"Fundamental Frequency": Helicity
rotation angle



$$X(t) = r \cos(\omega_B t)$$

$$y(t) = r \sin(\omega_B t)$$

$$z(t) = v \sin \alpha t$$

$$= \frac{v \sin \alpha}{\omega_B}$$

$$= c$$

$$p = \frac{z^2 + c^2}{\frac{1}{2}} = \frac{v^2 / \omega_B^2}{v \cos \alpha / \omega_B} = \frac{v}{\omega_B \cos \alpha}$$

(14.48) "curvature"

$$\rho = \frac{V^2}{\dot{V}_1} = \frac{\frac{V^2 \cos^2 \alpha}{WB^2} + \frac{V^2 \sin^2 \alpha}{WB^2}}{V \cos \alpha / WB}$$

$$= \frac{V^2 / WB^2}{V \cos \alpha / WB}$$

$$= \frac{V}{WB \cos \alpha}$$

(14.81) "critical frequency - helix"

$$\omega_c = \frac{3}{2} \gamma^3 \left(\frac{c}{\rho} \right)$$

$$= \frac{3}{2} \left(\frac{F}{mc^2} \right) \frac{c}{\rho}$$

$$= \frac{3}{2} \gamma^3 \left(\frac{c WB \cos \alpha}{V} \right)$$

$$\Gamma = \frac{3}{2} \gamma^3 WB \cos \alpha \quad \text{... when } V \approx c \text{ at light speeds.}$$

b) (14.79) "Energy radiated per unit frequency interval per unit solid angle"

$$\frac{d^2 I}{d\omega d\Omega} = \frac{e^2}{3\pi^2 c} \left(\frac{W_p}{c} \right)^2 \left(\frac{1}{\gamma^2} + \theta^2 \right) \left[K_{2/3}^2(\xi) + \frac{\theta^2}{(1/\gamma^2) + \theta^2} K_{1/3}^2(\xi) \right]$$

$$\frac{d^2 P}{d\omega d\Omega} = \frac{1}{B} \frac{d^2 I}{d\omega d\Omega} \left(\frac{\omega}{\omega_0} \right)^2 \frac{B}{\omega_0}$$

$$= \frac{\omega_0}{2\pi} \frac{d^2 I}{d\omega d\Omega}$$

$$= \frac{1}{2\pi} \frac{WB}{\cos \alpha} \frac{e^2}{3\pi^2 c} \left(\frac{\omega}{\omega_c} \frac{3}{2} \gamma^3 \right)^2 \left(\frac{1}{\gamma^2} + 4\mu^2 \right)^2 \left[K_{2/3}^2(\xi) + \frac{\theta^2 4\mu^2}{1 + 8\mu^2} K_{1/3}^2(\xi) \right]$$

$$= \frac{3e^2 \gamma^2}{8\pi^3 c} \left(\frac{\omega}{\omega_c} \right) \frac{w_B}{\cos \alpha} (1 + \gamma^2 4r^2) \left[K_{2/3}^2(\xi) + \frac{\gamma^2 \theta^2}{1 + \gamma^2 \theta^2} K_{1/3}^2(\xi) \right]$$

where $\xi = \frac{\omega}{2\omega_c} (1 + \gamma^2 \theta^2)^{3/2}$

a) (14.91) "Frequency distribution of total power"

$$\frac{dI}{d\omega} = \sqrt{3} \frac{e^2}{c} \gamma \frac{\omega}{\omega_c} \int_{\omega/\omega_c}^{\infty} K_{5/3}(x) dx$$

(14.79) "Energy radiated per unit frequency per unit solid angle"

$$\frac{d^2 I}{d\omega d\Omega} = \frac{e^2}{3\pi^2 c} \left(\frac{w_B}{c} \right) \left(\frac{1}{\gamma^2} + \theta^2 \right)^2 \left[K_{2/3}^2(\xi) + \frac{\theta^2}{(1/\gamma^2) + \theta^2} K_{1/3}^2(\xi) \right]$$

$$\frac{dI}{d\Omega} = \frac{e^2}{3\pi^2 c} \frac{q\gamma^6}{4} \left(\frac{\omega}{\omega_c} \right)^2 \frac{1}{\gamma^4} (1 + \gamma^2 \theta^2) \left[K_{2/3}^2(\xi) + \frac{\gamma^2 \theta^2}{(1 + \gamma^2 \theta^2)} K_{1/3}^2(\xi) \right]$$

where $\xi = \frac{\omega^2}{3c} \left(\frac{1}{\gamma^2} + \theta^2 \right)^{3/2}$

$$= \frac{\omega}{2\omega_c} (1 + \gamma^2 \theta^2)^{3/2}$$

$$= \frac{\sqrt{3} e^2 \gamma}{c} \frac{\sqrt{3} \gamma}{4\pi^2} \left(\frac{\omega}{\omega_c} \right)^2 (1 + \gamma^2 \theta^2)^2 \left[K_{2/3}^2(\xi) + \frac{\gamma^2 \theta^2}{(1 + \gamma^2 \theta^2)} K_{1/3}^2(\xi) \right]$$

(Problem 14.17b)

$$\frac{d^2 P}{d\omega d\Omega} = \frac{3c^2 \gamma^2}{8\pi^3 c} \frac{w_B}{\cos^2 \alpha} \left(\frac{\omega}{\omega_c} \right)^3 (1 + \gamma^2 4r^2)^2 \left[K_{2/3}^2(\xi) + \frac{\gamma^2 \theta^2}{(1 + \gamma^2 \theta^2)} K_{1/3}^2(\xi) \right]$$

IF $\theta = \pi/2$ and $d\Omega = \sin \theta d\theta d\phi$
 $= \cos \alpha d\theta d\psi$ where $\alpha = \theta + \pi/2$

$$\frac{dP}{d\omega} = \frac{\sqrt{3} e^2 \gamma^2}{2\pi c} \frac{w_B}{\cos^2 \alpha} \cos \alpha \left(\frac{\omega}{\omega_c} \right) \int_{\omega/\omega_c}^{\infty} K_{5/3}(y) dy = \left(\frac{\sqrt{3} e^2 \gamma w_B}{2\pi \cos \alpha} \right) G \left(\frac{\omega}{\omega_c} \right)$$

$$\text{from } G(x) = x \int_0^\infty K_{5/3}(t) dt$$

$$\begin{aligned}
 b) P &= \frac{\sqrt{3} e^2 \gamma w_B}{2\pi \cos \alpha} \cdot w_c \int_0^\infty G\left(\frac{w}{w_c}\right) d\left(\frac{w}{w_c}\right) \\
 &= \frac{\sqrt{3} e^2 \gamma w_B}{2\pi \cos \alpha} \cdot w_c \int_0^\infty dx \cdot x \cdot \int_x^\infty dy \cdot K_{5/3}(y) \\
 &= \frac{\sqrt{3} e^2 \gamma w_B}{2\pi \cos \alpha} w_c \cdot \left\{ \frac{1}{2} \int_0^\infty y^2 K_{5/3}(y) dy \right\} \\
 &= \frac{\sqrt{3} e^2 \gamma w_B}{2\pi \cos \alpha} \frac{3}{2} \gamma^3 w_B \cos \alpha \cdot \frac{8\pi}{9\sqrt{3}} \\
 &= \frac{2 e^2 w_B^2 \gamma^4}{3c}
 \end{aligned}$$

(14.31) "Power radiated"

$$\begin{aligned}
 P &= \frac{2}{3} \frac{e^2}{m^2 c^3} \gamma^2 w^2 |P|^2 \\
 &= \frac{2}{3} \frac{e^2 c}{\rho^2} \beta^4 \gamma^4
 \end{aligned}$$

Total power in linear relativistic particles is proportional to helical paths. A relativistic helical particle emits total power independently of pitch and angle.

a) (14.70) "Intensity distribution"

$$\frac{d^2 I}{d\omega d\Omega} = \frac{\omega^2}{4\pi^2 c} \left| \int dt \int d^3 x \cdot n \times [n \times J(x, t)] e^{i\omega[t - (n \cdot x)/c]} \right|^2$$

Certainly two uncommon integrals were 'fractional variables of integration' and 'Dirac integrals' in school.

$$\begin{aligned}
 J_{\text{eff}} &= c \nabla \times M ; \quad M = \mu(t) \cdot \delta(x - r(t)) \\
 &= \frac{\omega_0^2}{4\pi^2 c} \left[\int dt \int d^3x \cdot n \chi \left[n \chi (\nabla \times \mu(t)) \delta(x - r(t)) \right] e^{i\omega(t - n \cdot r(t)/c)} \right]^2 \\
 &= \frac{\omega_0^4}{4\pi^2 c^3} \left[\int dt \cdot n \chi \left[n \chi (n \chi \mu(t)) \right] e^{i\omega(t - n \cdot r(t)/c)} \right]^2 \\
 &= \frac{\omega_0^4}{4\pi^2 c^3} \left[\int dt \cdot n \chi \mu(t) \cdot e^{i\omega(t - n \cdot r(t)/c)} \right]^2
 \end{aligned}$$

b) $\mu_x = \mu_0 \sin \omega_0 t \quad \mu_z = \mu_0 \cos \omega_0 t \quad \mu_y = 0$

$$\begin{aligned}
 &= \frac{\omega_0^4}{4\pi^2 c^3} \left[\int_{-\pi/2}^{\pi/2} \mu_0 \begin{pmatrix} \cos(\omega_0 t) \sin \theta \sin \phi \\ \sin(\omega_0 t) \cos \theta - \cos(\omega_0 t) \sin \theta \cos \phi \\ -\sin(\omega_0 t) \cdot \sin \theta \sin \phi \end{pmatrix} e^{i\omega(t - n \cdot r(t)/c)} \right]^2 \\
 &= \frac{\omega_0^4}{4\pi^2 c^3} \mu_0^2 \frac{T^2}{4} \begin{pmatrix} \sin \theta \sin \phi \\ -i \cos \theta - \sin \theta \cos \phi \\ i \sin \theta \sin \phi \end{pmatrix}
 \end{aligned}$$

Intensity distribution:

$$\begin{aligned}
 \frac{dI}{d\Omega} &= \frac{\omega_0^4}{4\pi^2 c^3} \mu_0^2 \frac{T^2}{4} \begin{pmatrix} \sin \theta \sin \phi \\ -i \cos \theta - \sin \theta \cos \phi \\ i \sin \theta \sin \phi \end{pmatrix} d\omega \\
 &= \frac{\omega_0^4}{4\pi^2 c^3} \mu_0^2 \frac{T^2}{4} (2 \sin^2 \theta \sin^2 \phi + \cos^2 \theta + \sin^2 \theta \cos^2 \phi) \\
 &= \frac{\omega_0^4}{4\pi^2 c^3} \mu_0^2 \frac{T^2}{4} (1 + \sin^2 \theta \sin^2 \phi)
 \end{aligned}$$

Pythagorean Identity:
 $\sin^2 \theta + \cos^2 \theta = 1$

Total Power Radiated:

$$\begin{aligned}
 I &= \int \frac{\omega_0^4}{4\pi^2 c^3} \mu_0^2 \frac{T^2}{4} (1 + \sin^2 \theta \sin^2 \phi) d\Omega \\
 &= \frac{\omega_0^4}{3\pi c^3} \mu_0 T^2 \\
 &= \frac{1}{2\pi} \langle P \rangle T^2 \rightarrow \langle P \rangle = \frac{2}{3} \frac{\omega_0^4}{3\pi c^3} \mu_0 \quad \text{because } P = I \frac{2\pi}{T}
 \end{aligned}$$

14.20

time	↑
0	∞
1	∞+
2	+
3	-
4	+
5	-
6	+
7	-
8	+
9	-
10	∞

"radiation emitted by a magnetic moment at the origin flipping"

$$a) \mu_z = \mu_0 \tanh(\nu t) \quad \mu_x = \mu_0 \operatorname{sech}(\nu t) \quad \mu_y = 0$$

where ν^{-1} is the characteristic flip.

(Problem 14.19 a)

$$\frac{d^2 I}{d\omega d\Omega} = \frac{\omega^4}{4\pi^2 c} \left| \int dt \cdot n \times \mu(t) e^{i\omega(t - n \cdot r(t))/c} \right|^2$$

$$= \frac{\omega^4}{4\pi^2 c} \mu_0^2 \left| \begin{array}{l} \tanh(\nu t) \sin \theta \sin \phi \\ \operatorname{Sech}(\nu t) \cos \theta - \tanh(\nu t) \sin \theta \cos \phi \\ - \operatorname{Sech}(\nu t) \sin \theta \sin \phi \end{array} \right|^2$$

$$x e^{i\omega t} \cdot dt$$

$$= \frac{\omega^4}{4\pi^2 c} \mu_0^2 \left| \begin{array}{l} i\pi/\nu \operatorname{cosech}\left(\frac{w\pi}{8\nu}\right) \sin \theta \sin \phi \\ \frac{\pi}{\nu} \operatorname{sech}\left(\frac{w\pi}{8\nu}\right) \cos \theta - \frac{\pi}{\nu} \operatorname{cosech}\left(\frac{w\pi}{8\nu}\right) \sin \theta \cos \phi \\ - \frac{\pi}{\nu} \operatorname{sech}\left(\frac{w\pi}{8\nu}\right) \sin \theta \sin \phi \end{array} \right|^2$$

Citation: Spiegel, Murray.

"Mathematical Handbook of Formulas and Tables"

Identities: $\int_0^\infty \frac{\cos ax}{\cosh \beta x} dx = \int_0^\infty \frac{\text{exponential}}{\text{exponential}} dx$

$$= \int_0^\infty \frac{\text{exponential}}{\text{Complex Series}} dx$$

$$= \int_0^\infty \frac{1}{\text{Complex Series}} dx$$

Modulus
of
a
limit

$$= \sum_{\text{complex series}} \frac{1}{}$$

$$= \frac{\pi}{2\beta} \operatorname{Sech}\left(\frac{\alpha\pi}{2\beta}\right)$$

$$\int_0^{\infty} \sin(\alpha x) \frac{\sinh(\beta x)}{\cosh(\gamma x)} dx = \frac{\pi}{\gamma} \frac{\sin\left(\frac{\beta\pi}{2\gamma}\right) \sinh\left(\frac{\alpha\pi}{2\gamma}\right)}{\cosh\left(\frac{\alpha\pi}{\gamma}\right) + \cos\left(\frac{\beta\pi}{\gamma}\right)}$$

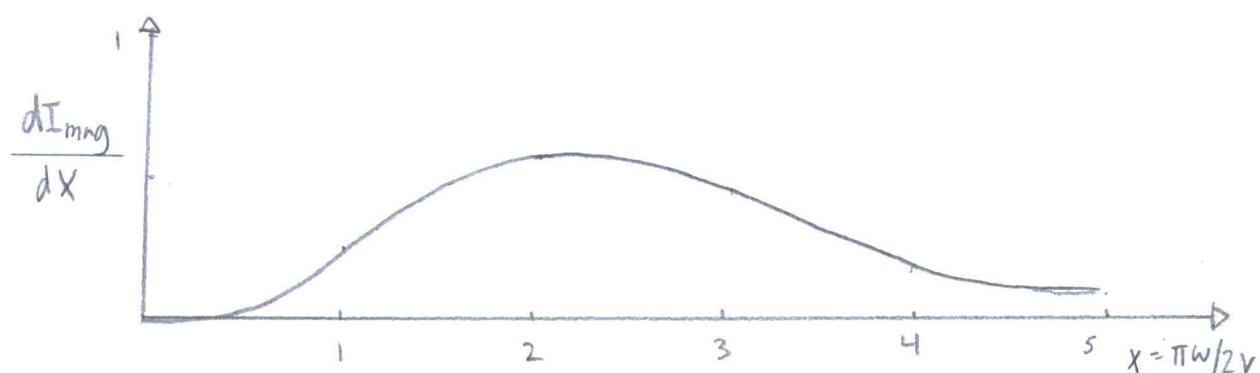
$$\frac{dI}{dw} = \frac{\omega^4}{4\pi^2 c^3} H_0^2 \left(\frac{\pi}{V} \right)^2 \left[\coth^2\left(\frac{\omega\pi}{2V}\right) \frac{8\pi}{3} + \operatorname{sech}^2\left(\frac{\omega\pi}{2V}\right) \left(\frac{4\pi}{3} + \frac{4\pi}{3} \right) \right]$$

$$= \frac{2\pi\omega^4}{3V^2 c^3} H_0^2 \left(\coth^2\left(\frac{\omega\pi}{2V}\right) + \operatorname{sech}^2\left(\frac{\omega\pi}{2V}\right) \right)$$

If $x = \pi w / 2V$, then

$$\begin{aligned} \frac{dI_{\max}}{dx} &= \frac{dI}{dw} \frac{dw}{dx} \\ &= \frac{2\pi^2 \omega^4}{3V^2 c^3} H_0^2 \left(\coth^2(x) + \operatorname{sech}^2(x) \right) \cdot \frac{2V}{\pi} \\ &= \frac{4\omega^4}{3V^2 c^3} H_0^2 \left(\coth^2(x) + \operatorname{sech}^2(x) \right) \end{aligned}$$

$$\begin{aligned} \frac{dI_{\max}}{dx} &= \frac{4}{3} \frac{H_0^2}{V \cdot c^3} \left(\frac{2V}{\pi} x \right)^4 \left(\coth^2(x) + \operatorname{sech}^2(x) \right) \\ &= \frac{4}{3} \left(\frac{V}{c} \right)^3 H_0^2 \left[16 \left(\frac{x}{\pi} \right)^4 \left(\coth^2(x) + \operatorname{sech}^2(x) \right) \right] \end{aligned}$$



$$\langle x \rangle = \frac{\int_0^{\infty} x \frac{dI}{dx} dx}{\int_0^{\infty} \frac{dI}{dx} dx}$$

$$= \frac{465}{32} / \frac{\pi^4}{16}$$

$$= 2.337$$

b) Gauss units: $Z_0 = 4\pi/c$

$$\frac{dP(t)}{d\Omega} = \frac{Z_0}{16\pi^2 c^2} \left| \left[n \times \frac{d^2 P}{dt^2}(t') \right] \times n \right|^2$$

$$= \frac{1}{4\pi c^3} \left| \mu_0 v^2 \left\langle \operatorname{sech}(vt') \cdot \tanh^2(vt') - \operatorname{sech}^2(vt'), 0, -2\tanh(vt') \operatorname{sech}^2(vt') \right\rangle \times \right.$$

$$\left. \langle \sin\theta \sin\phi, \cos\theta - \sin\theta \cos\phi, \sin\theta \sin\phi \rangle \right.$$

$$= \frac{1}{4\pi c^3} \left| \mu_0 v^2 (-2\tanh(vt') \operatorname{sech}(vt') \sin\theta \sin\phi) \hat{i} \right.$$

$$+ (\operatorname{sech}(vt') (\tanh^2(vt') \operatorname{sech}^2(vt'))) \cos\theta$$

$$+ 2\tanh(vt') \operatorname{sech}^2(vt') \sin\theta \cos\phi \hat{j} \right|$$

$$+ (\operatorname{sech}(vt') (\tanh^2(vt) - \operatorname{sech}^2(vt) \sin\theta \sin\phi)) \hat{k} \right|$$

$$= \frac{1}{4\pi c^3} \left(\mu_0^2 v^4 (4\tanh^2(vt') \operatorname{sech}^4(vt') \sin^2\theta \right.$$

$$+ \operatorname{sech}^2(vt') (\tanh^2(vt') - \operatorname{sech}^2(vt'))^2 \right)^2$$

$$\times (\cos^2\theta + \sin^2\theta \sin^2\phi))$$

$$\begin{aligned}
 P(E) &= \int_0^{\infty} \frac{1}{4\pi c^3} \left(\mu_0^2 v^4 (4 \tanh^2(vE') \operatorname{sech}^4(vE') \sin^2\theta \right. \\
 &\quad \left. + \operatorname{sech}^2(vE') (\tanh^2(vE) - \operatorname{sech}^2(vE'))^2 \right. \\
 &\quad \left. \times (\cos^2\theta + \sin^2\theta \sin^2\phi) \sin\theta d\theta d\phi \right) \\
 &= \frac{2v^4}{3c^3} \mu_0^2 (4 \tanh^2(vE') \operatorname{sech}^4(vE') + \operatorname{sech}^2(vE')) \\
 &\quad \times (\tanh^2(vE) + \operatorname{sech}^4(vE'))^2
 \end{aligned}$$

Total Power radiated:

"Intensity."

$$\begin{aligned}
 I &= \int_{-\infty}^{\infty} P(E) dE \\
 &= \frac{2v^4}{3c^3} \mu_0^2 \int_{-\infty}^{\infty} (4 \tanh^2(vE') \operatorname{sech}^4(vE') + \operatorname{sech}^2(vE')) \\
 &\quad \times (\tanh^2(vE) + \operatorname{sech}^4(vE'))^2 dE' \\
 &= \frac{4}{3} \left(\frac{v}{c} \right)^3 \mu_0^2 \text{ coefficient to part a.}
 \end{aligned}$$

4.21

a) Hydrogen-like Transition Probability:



"Bohr Correspondence principle"

(Problem 14.22b)

$$P_n = \frac{4e^2}{3c^3} (n\omega_0)^4 a^2 \left\{ \frac{1}{n^2} \left[J_K^2(n\epsilon) + \left(\frac{1-\epsilon}{\epsilon} \right) J_K^2(n\epsilon) \right] \right\}$$

If $E \geq 0$, then circular orbit by electron.

$$\lim_{\epsilon \rightarrow 0} J_n(n\epsilon) = \left(\frac{n\epsilon}{2}\right)^n$$

$$\lim_{\epsilon \rightarrow 0} J_n'(n\epsilon) = \frac{n}{2} \left(\frac{n\epsilon}{2}\right)^{n-1}$$

$$\lim_{\epsilon \rightarrow 0} P_n = \lim_{\epsilon \rightarrow 0} \frac{4e^2}{3c^3} (n\omega_0)^4 a^2 \left\{ \frac{1}{n^2} \left[J_n'(n\epsilon)^2 + \left(\frac{1-\epsilon^2}{\epsilon^2} \right) J_n(n\epsilon)^2 \right] \right\}$$

$$= \frac{4e^2}{3c^3} \omega_0^4 a^2 \left(\frac{1}{4} + \frac{1}{4} \right)$$

$$= \frac{2e^2}{3c^3} \omega_0^4 a^2$$

$$\omega_0 = \frac{Ze^2}{m a^2}$$

$$= \frac{Ze^2}{m} \left(\frac{Zme^2}{n^2 h^2} \right)^3$$

$$= \frac{Z^4 m^2 e^8}{n^6 h^6}$$

$$= \frac{Z^2 m e^4}{n^3 h^3}$$

"Mean lifetime"

$$\frac{1}{T} = P_i / \hbar \omega_0$$

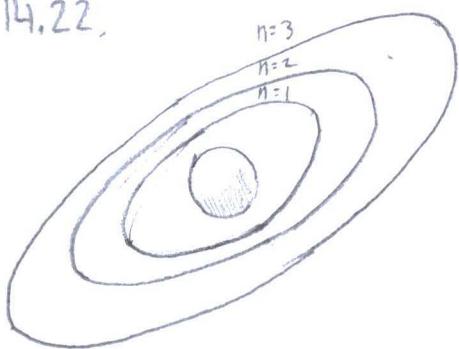
$$= \frac{2e^2}{3c^3} \frac{1}{\hbar} \omega_0^2 a^2$$

$$= \frac{Ze^2}{3c^3} \frac{1}{\hbar} \left(\frac{Z^2 m e^4}{m^3 h^3} \right) \left(\frac{n^2 h^2}{Z m c^2} \right)^2$$

$$= \frac{2}{3} \frac{e^2}{\hbar c} \left(\frac{Z e^2}{\hbar c} \right)^4 \frac{mc^2}{\hbar} \frac{1}{n^5}$$

b)	Transition:	Quantum Mechanical Values:	Classical Values:
	$2p \rightarrow 1s$	$1.6 \times 10^{-9} s$	$3.2 \times 10^{-9} s$
	$4f \rightarrow 3d$	$7.3 \times 10^{-9} s$	$1.2 \times 10^{-7} s$
	$6h \rightarrow 5g$	$6.1 \times 10^{-7} s$	$7.8 \times 10^{-7} s$

14.22.



"Elliptic Motion
in hydrogen atom"

$$x = a(\cos u - E)$$

$$y = a\sqrt{1-E^2} \sin u$$

$$\omega_0 t = u - E \sin u$$

where a = semi-major axis

$$= e^2 / 2B$$

E = eccentricity

$$= \sqrt{1 - \frac{2BL^2}{me^4}}$$

ω_0 = orbital frequency

$$= \frac{8B^3}{me^4}$$

u = angle in polar

$$= \tan(u/2)$$

$$= \sqrt{(1-E)(1+E)} \tan(\theta/2)$$

a) Citation: Landau, Lifshitz

"The Classical Theory of
Fields"

Academy of Sciences, U.S.S.R.

Pg 195, "Radiation in the case of
Coulomb interaction"

$$P_n = \frac{4e^2}{3c^3} (n\omega_0)^4 |r_i|^2$$

$$= \frac{4e^2}{3c^3} (n\omega_0)^4 [|x|^2 + |y|^2]$$

$$x = \int_0^{2\pi/\omega_0} \dot{x}(t) dt$$

$$= \frac{\omega_0}{2\pi} \int_0^{2\pi/\omega_0} \ddot{x}(t) dt$$

$$= \frac{\omega_0}{2\pi} \int_0^{2\pi/\omega_0} e^{in\omega_0 t} \dot{x}(t) dt$$

$$= -\frac{i}{2\pi n} \int_0^{2\pi/\omega_0} e^{in(u-E\sin u)} (-a\sin u) du$$

$$y = \int_0^t \dot{y}(t) dt$$

$$= \frac{\omega_0}{2\pi} \int_0^{2\pi/\omega_0} \dot{y}(t) dt$$

$$= \frac{\omega_0}{2\pi} \int_0^{2\pi/\omega_0} e^{in\omega_0 t} \dot{y}(t) dt$$

$$= -\frac{i}{2\pi n} \int_0^{2\pi/\omega_0} e^{in(u-E\sin u)} (a\sqrt{1-r^2}\cos u) du$$

Note: Second pass at
the problem. Chapter
16 had hyperbolic
motion by an
electron, rather
than elliptical.

The first three
lines build from:

$$\textcircled{1} \quad x = \int \frac{dx}{dt} dt$$

$$\textcircled{2} \quad = \frac{1}{t} \int_0^t \frac{dx}{dt} dt$$

$$\textcircled{3} \quad = \frac{1}{t} \int_0^t e^{izut} \frac{dx}{dt} dt$$

Identity:

$$\begin{aligned} \sin \alpha &= i \cos(\alpha + \pi/2) \\ e^{-i\phi} &= e^{i\phi} \\ &= \sum_{m=-\infty}^{\infty} i^m J_m(x) e^{im\phi} \end{aligned}$$

Alternative Identity
"Hansen-Bessel Formula"

$$r = |x|^2 + |y|^2$$

$$= + \frac{1}{(2\pi n)^2} \left(\int_0^{2\pi/\omega_0} e^{in(u-E\sin u)} (-a\sin u) du \right)^2 + \left[\int_0^{2\pi/\omega_0} e^{in(u-E\sin u)} (a\sqrt{1-r^2}\cos u) du \right]^2$$

$$= + \frac{a^2}{(2\pi n)^2} \left(J_{n+1}(nE)(-i) e^{(n+1-i)(n+1)\pi/2} - J_{n+1}(nE)(-i) e^{(n+1-i)(n+1)\pi/2} \right)^2 + \left(\frac{ia\sqrt{1-r^2}}{E} (-i) J_{n+1}(nE) \right)^2$$

$$= + \frac{a^2}{K^3} \left[J_K(kE)^2 + \left(\frac{1-E^2}{E^2} \right) J_K(KE)^2 \right]$$

.. when $k=n$

$$P_n = \frac{4e^2}{3c^3} (R\omega_0)^4 a^2 \left\{ \frac{1}{k^3} \left[(J_K(n\epsilon))^2 + \left(\frac{1-\epsilon^2}{\epsilon^2} \right) J_K(n\epsilon)^2 \right] \right\}$$

b) Eccentricity:

Circle	$E=0$
Ellipse	$E = \frac{\sqrt{a^2-b^2}}{a}, -\frac{\sqrt{b^2-a^2}}{b}$
Parabola	$E=1$
Hyperbola	$E = \frac{\sqrt{a^2+b^2}}{a}$

$$\lim_{E \rightarrow 0} J_K(n\epsilon) = \left(\frac{n\epsilon}{2} \right)^n \quad \lim_{E \rightarrow 0} J_K'(n\epsilon) = \frac{n}{2} \left(\frac{n\epsilon}{2} \right)^{n-1}$$

$$\lim_{E \rightarrow 0} P_n = \lim_{E \rightarrow 0} \frac{4e^2}{3c^3} (R\omega_0)^4 a^2 \left\{ \frac{1}{k^3} \left[J_K(n\epsilon)^2 + \left(\frac{1-\epsilon^2}{\epsilon^2} \right) J_K(n\epsilon)^2 \right] \right\}$$

$$= \frac{2e^2}{3c^2} \omega_0^4 a^2 \quad \text{when } a = \frac{n^2 a_0}{Z}$$

$$= \frac{n^2 h^2}{Z m c^2}$$

$$\omega_0 = \frac{Z^2 m e^4}{m a^3}$$

$$= \frac{2e^2}{m} \left(\frac{Z m e^2}{n^2 h^2} \right)^3$$

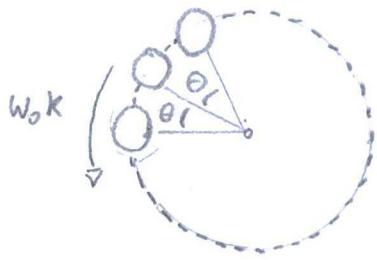
$$= \frac{Z^4 m^2 e^9}{n^6 h^6}$$

$$\frac{1}{C} = P_1 / \hbar \omega_0$$

$$= \frac{2e^2}{3c^3} \frac{1}{\hbar} \omega_0^2 a^2$$

$$= \frac{2}{3} \frac{e^2}{\hbar c} \left(\frac{Z e^3}{\hbar c} \right)^4 \frac{m c^2}{\hbar} \frac{1}{n^5}$$

14.23.



"N charges... move with fixed relative positions... around the same circle"

a) (Problem 14.13a) "Power radiated per unit angle"

$$\frac{dP_m}{d\Omega} = \frac{e^2 \omega_0^4 m^2}{(2\pi c)^3} \left| \int_0^{2\pi/\omega_0} r(t) \times n \exp^{im\omega_0(t - \frac{n\omega_0 t}{c})} dt \right|^2$$

(Problem 14.15a) "Angular distribution of polar radiation into mth-multipole"

$$\frac{dP_m}{d\Omega} = \frac{e^2 \omega_0^4 R^2 m^2}{2\pi c^3} \left\{ \left[\frac{dJ_m(m\beta \sin \theta)}{d(m\beta \sin \theta)} \right]^2 + \frac{\cot^2 \theta}{\beta^2} J_m^2(m\beta \sin \theta) \right\}$$

$$\text{where } \beta = \omega_0 R / c$$

$$\text{If } X(t) = R(\cos(\omega_0 t + \phi), \sin(\omega_0 t + \phi), 0)$$

$$V(t) = \omega_0 R(-\sin(\omega_0 t + \phi), \cos(\omega_0 t + \phi), 0)$$

$$n = (\cos \theta, \cos \theta, -\sin \theta)$$

$$\frac{dP_m}{d\Omega} = \frac{\omega_0^4 m^2}{(2\pi c)^3} \left| \int_0^{2\pi/\omega_0} \begin{pmatrix} \cos \theta \cos(\omega_0 t + \phi) \\ \cos \theta \sin(\omega_0 t + \phi) \\ -\sin \theta \cos(\omega_0 t + \phi) \end{pmatrix} \omega_0 R e^{im\omega_0 t} \right. \\ \left. - ik(\omega_0 t + \phi) dt \right|^2$$

$$= \frac{\omega_0^4 m^2}{(2\pi c)^3} \left| e^{-im\phi_0} \int_0^{2\pi/\omega_0} \begin{pmatrix} \cos \theta \cos(\omega_0 t + \phi) \\ \cos \theta \sin(\omega_0 t + \phi) \\ -\sin \theta \cos(\omega_0 t + \phi) \end{pmatrix} \omega_0 R \right. \\ \left. dt \right|^2$$

$$\times e^{im\omega_0 t} \sum_{k=-\infty}^{\infty} (-i)^k J_k(m\beta \sin \theta) e^{-ik\omega_0 t} dt \Big|^2$$

$$\text{If } F_m(N) = \left| \sum_{j=1}^N q_j e^{im\theta} \right|^2, \text{ then}$$

$$\frac{dP_m(N)}{d\Omega} = \frac{\omega_0^4 m^2}{(2\pi c)^3} \left| \int_0^{2\pi/\omega_0} \begin{pmatrix} \cos \theta \cos(\omega_0 t + \phi) \\ \cos \theta \sin(\omega_0 t + \phi) \\ -\sin \theta \cos(\omega_0 t + \phi) \end{pmatrix} \omega_0 R \right. \\ \left. dt \right|^2$$

d) If $N \gg \delta^3$

$$\frac{dP_m(N)}{d\Omega} = \frac{\omega_0^4 m^2}{(2\pi c)^3} \left| \sum_{j=1}^N q_j e^{-im\phi} \int_0^{2\pi/\omega_0} \begin{pmatrix} \cos\theta \cos(\omega_0 t) \\ \cos\theta \sin(\omega_0 t) \\ \sin\theta \cos(\omega_0 t) \end{pmatrix} w_0 k e^{-ik_0 \omega t} \right|^2 \times \sum_{k=-\infty}^{\infty} (-i)^k J_k(m \beta \sin\theta) e^{-ik_0 \omega t} dt \right|^2$$

Structure Factor:

$$F_m(N) = \left| \sum_{j=1}^N q_j e^{-im\phi} \right|^2 = \left| \sum_{j=1}^N q_j e^{-im2\pi j/N} \right|^2 = \frac{-iZ \cdot N / 3\delta^3}{}$$

\propto constant $\times e$

when $m = \frac{1}{3}, j = N$

$$\lim_{N \rightarrow \infty} \frac{dP_m(N)}{d\Omega} = 0$$

e) Above, the radiation pattern describes a steady state current in a circular path. A common place with charges in a circular path is the cyclotron.

14.24.



"N identical charges... move with constant speed"

"Steady-state current flowing in a circuit"

$$x e^{im\omega t} \sum_{k=-\infty}^{\infty} (-i)^k J_k(m\beta \sin \theta) e^{-iR\omega t} dt \Big|^2$$

$$= \frac{dP_m}{d\Omega} \cdot F_m(N)$$

b) If the coefficient $F_m(N)$ describes the m^{th} multipole, Two charges evaluates to

$$F_m(N) = q^2 \left| \sum_{j=1}^2 e^{-im\theta_j} \right|^2$$

Proof by induction:

$$\textcircled{1} \text{ Base case: } F_m(1) = q \cdot e^{-im2\pi m 2\theta}$$

$$\textcircled{2} \text{ Inductive hypothesis: } F_m(N) = q \cdot N \cdot e^{z_0 z_1 2\pi m 2\theta}$$

$$\textcircled{3} \text{ Inductive step: } F_m(N+1) = q \cdot (N+1) \cdot e^{z_0 z_1 2\pi (m+1) 2\theta} \\ = q^2 N^2 e^{z_0 z_1 2\pi m 2\theta} + \dots$$

c) When $v \ll c$ and $\beta \ll 1$,

$$\lim_{\beta \rightarrow 0} \frac{dP(l)}{d\Omega} = \lim_{\beta \rightarrow 0} \frac{dP_m}{d\Omega} F_m(N)$$

$$= \beta^{2k} \frac{\omega_0^4 m^2}{(2\pi c)^3} \left(\sum_{j=1}^2 q e^{-im\theta_j} \int_0^{2\pi/\omega_0} \begin{pmatrix} \cos \theta \cos(\omega_0 t + \phi) \\ \cos \theta \sin(\omega_0 t + \phi) \\ \sin \theta \cos(\omega_0 t + \phi) \end{pmatrix} \cdot \omega_0 R dt \right)$$

$$e^{im\omega t} \sum_{k=-\infty}^{\infty} (-i)^k (m\beta \sin \theta)^k e^{-iR\omega t} dt \Big|^2$$

As N approaches infinity, k approaches infinity, and $\frac{dP(l)}{d\Omega} = 0$.

(14.8) "Lienard Wiechart Potential"

$$\Phi(x_1, t) = \begin{bmatrix} e \\ (1-\beta \cdot n)R \end{bmatrix} \quad A(x_1, t) = \begin{bmatrix} e\beta \\ (1-\beta \cdot n)R \end{bmatrix}$$

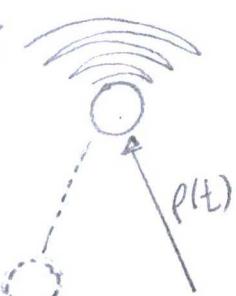
$$\Phi_N(x_1, t) = \sum_{n=1}^N q^n \begin{bmatrix} e \\ (1-\beta \cdot n)R \end{bmatrix} \quad A(x_1, t) = \sum_{n=1}^N q^n \begin{bmatrix} e\beta \\ (1-\beta \cdot n)R \end{bmatrix}$$

$$\lim_{q \rightarrow 0} \Phi_N(x_1, t) = \lim_{q \rightarrow 0} \sum_{n=1}^N q^n \begin{bmatrix} e \\ (1-\beta \cdot n)R \end{bmatrix}$$

$$= 0$$

$$\lim_{q \rightarrow 0} A(x_1, t) = \lim_{q \rightarrow 0} \sum_{n=1}^N q^n \begin{bmatrix} e\beta \\ (1-\beta \cdot n)R \end{bmatrix}$$

$$= 0$$

14.25  a) (14.79) "Energy radiated per unit frequency per unit solid angle"

$$\frac{d^2 I}{d\omega d\Omega} = \frac{e^2 (\omega c)^2}{3\pi^2 c} \left(\frac{1}{8c} + \theta^2 \right)^2 \left[K_{2/3}^2(\xi) + \frac{\theta^2}{(1/8c + \theta^2)} K_{1/3}^2(\xi) \right]$$

"relativistic particle
moving in a path
with radiation with
positive and negative
helicity"

(14.67) "Intensity distribution"

$$\frac{d^2 I}{d\omega d\Omega} = \frac{e^2 \omega^2}{4\pi^2 c} \left| \int_{-\infty}^{\infty} n \times (n \lambda \beta) e^{i\omega(t - n \cdot r(t)/c)} dt \right|^2$$

(14.71) "Vector"

$$n \times (n \times \beta) = \beta \left[-E_{||} \sin\left(\frac{vt}{\rho}\right) + E_{\perp} \cos\left(\frac{vt}{\rho}\right) \sin\theta \right]$$

(14.72) "Argument in exponential"

$$\omega\left(t - \frac{\eta \cdot n(t)}{c}\right) = \omega\left[t - \frac{e}{c} \sin\left(\frac{vt}{e}\right) \cos\theta\right]$$

Taylor Expansion:

$$\sin\left(\frac{vt}{e}\right) = \frac{vt}{e} - \left(\frac{vt}{e}\right)^3 \frac{1}{3!} + \dots$$

$$\cos\theta = 1 + \frac{\theta^2}{2!} - \dots$$

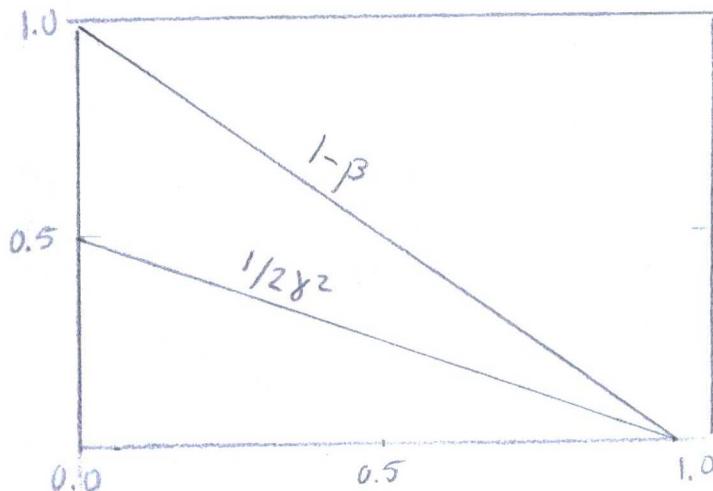
$$\approx \omega\left[t - \beta t + \beta \frac{t^2 \theta^2}{2} + \frac{e}{c} \left(\frac{vt}{e}\right)^3 \frac{1}{3!} + \dots\right]$$

when $\beta = \frac{v}{c}$

$$\approx \omega\left[t(1-\beta) + \beta \frac{t^2 \theta^2}{2} + \frac{\beta^3 t^3 c^2}{p^2 e^3} + \dots\right]$$

if $\beta \approx 1$ (relativistic speeds)

then $\frac{1}{2\beta^2} \approx 1-\beta$



$$\beta = \frac{v}{c}$$

$$\simeq \frac{\omega}{2} \left[\left(\frac{1}{\gamma^2} + \theta^2 \right) t + \frac{c^2}{\beta^3 \rho^2} t^3 \right]$$

so as if $\theta \ll 1$ "put to unity wherever possible" (pg 677)

and $\beta \sim 1$

(14.74) "Energy distribution approximation"

$$\frac{d^2 I}{d\omega d\Omega} = \frac{e^2 \omega^2}{4\pi^2 c} \left| -E_{\parallel} A_{\parallel}(w) + E_{\perp} A_{\perp}(w) \right|^2$$

(14.75) "Amplitudes"

$$A_{\parallel}(w) \simeq \frac{c}{\rho} \int_{-\infty}^{\infty} t \exp \left[i \frac{\omega}{2} \left(\frac{1}{\gamma^2} + \theta^2 \right) t + \frac{c^2 t^3}{3 \rho^2} \right] dt$$

$$A_{\perp}(w) \simeq \theta \int_{-\infty}^{\infty} \exp \left[i \frac{\omega}{2} \left(\frac{1}{\gamma^2} + \theta^2 \right) t + \frac{c^2 t^3}{3 \rho^2} \right] dt$$

(14.76) "Transform Integrals"

$$A_{\parallel}(w) = \frac{\ell}{c} \left(\frac{1}{\gamma^2} + \theta^2 \right) \int_{-\infty}^{\infty} x \exp \left[i \frac{3}{2} \xi \left(x + \frac{1}{3} x^3 \right) \right] dx$$

$$A_{\perp}(w) = \frac{\rho \theta}{c} \left(\frac{1}{\gamma^2} + \theta^2 \right)^{1/2} \int_{-\infty}^{\infty} \exp \left[i \frac{3}{2} \xi \left(x + \frac{1}{3} x^3 \right) \right] dx$$

so from $x = \frac{ct}{\rho \left(\frac{1}{\gamma^2} + \theta^2 \right)^{1/2}}$

and $\xi = \frac{\omega \rho}{3c} \left(\frac{1}{\gamma^2} + \theta^2 \right)^{1/2}$

Euler-Identity:

$$e^{ix} = \cos(x) + i\sin(x)$$

Airy integrals as

Modified Bessel's:

$$Ai(x) = \frac{1}{\pi} \int_0^\infty \cos\left(xt + \frac{1}{3}t^3\right) dt$$

$$= \frac{1}{\pi} \sqrt{\frac{x}{3}} K_{\pm 1/3}(\xi)$$

$$Ai'(x) = \frac{-1}{\pi} \int_0^\infty x \sin\left(xt + \frac{1}{3}t^3\right) dt$$

$$= \frac{-1}{\pi} \left(\frac{x}{\sqrt{3}}\right) K_{\pm 2/3}(\xi)$$

$$\text{when } \xi = \frac{2}{3} E^{3/2}$$

N. I. S. T. - Digital
Library of Mathematical
Functions

(14.79) "Energy radiated
per unit frequency interval per unit
solid angle"

$$\frac{d^2I}{d\omega d\Omega} = \frac{e^2}{3\pi^2 c} \left(\frac{W\rho}{c} \right) \left(\frac{1}{\gamma^2} + \Theta^2 \right)^2 \left[K_{2/3}^2(\xi) + \frac{\Theta^2}{(1/\gamma^2) + \Theta^2} K_{1/3}^2(\xi) \right]$$

b) (14.81) "critical frequency"

$$\omega_c = \frac{3}{2} \gamma^3 \left(\frac{c}{\rho} \right) = \frac{3}{2} \left(\frac{E}{mc^2} \right)^3 \frac{c}{\rho}$$

1) High frequency: $\omega > \omega_c$:

$$\frac{d^2 I}{d\omega d\Omega} = \frac{3}{4\pi} \frac{e^2}{c} \gamma^2 \frac{\omega}{\omega_c} e^{-\omega/\omega_c}$$

2) Intermediate and Low Frequency ($\omega < \omega_c$):

$$\frac{d^2 I}{d\omega d\Omega} = \frac{e^2}{3\pi c} \left(\frac{\omega_p}{c} \right)^2 \left(\frac{2\theta^2}{\gamma^2} + \theta^4 \right) \left[K_{2/3}^2(\xi) + \frac{\theta^2}{(1/\gamma^2) + \theta^2} K_{1/3}^2(\xi) \right]$$

3) Intermediate and Low Frequency at very small angles:

$$\left. \frac{d^2 I}{d\omega d\Omega} \right|_{\theta=0} = \frac{e^2}{c} \left[\frac{\Gamma(\frac{2}{3})}{\pi} \right] \left(\frac{3}{4} \right)^{1/3} \left(\frac{\omega_p}{c} \right)^{2/3}$$

c) Citation: P. Joos Phys Rev. Letters
4, 55, (1960)

"Measurement of the Polarization
of Synchrotron Radiation"

Ithaca, New York, Cornell

The guy plotted parallel and
perpendicular polarization by a
synchrotron against theoretical values.

Peter modelled Intensity vs. angle
in a 700 MeV and RCA-6342
photomultiplier. He liked Bessel's
as the approximation:

$$J_{11} = -\frac{ec}{\pi\sqrt{3}} ix^2 K_{2/3} \left(\frac{\omega}{3w_0} \gamma^3 \right) \quad (\text{parallel})$$

$$J_1 = -\frac{ec}{\pi\sqrt{3}} x^2 K_{1/3} \left(\frac{\omega}{3w_0} \cdot x^3 \right) \quad (\text{perpendicular})$$

The function is proportional to
(14.79).

$$\frac{d^2 I}{d\omega d\Omega} = \frac{e^2}{3\pi^2 c} \left(\frac{w\ell}{c} \right) \left(\frac{1}{\gamma^2} + \theta^2 \right)^2 \left[K_{2/3}^2(\xi) + \frac{\theta^2}{(1/\gamma^2) + \theta^2} K_{1/3}^2(\xi) \right]$$

Errors arose in the experiment such
as electron frequency, bandwidth,
and slit-width.



"Synchrotron
radiation
from the
Crab nebula"

14.27

a) (pg 699)

$$x'(t') = \frac{k}{8R_0} \sin(\theta(t')) = \alpha \sin \theta(t')$$

$$z'(t) = \frac{8k^2}{8\gamma^2 R_0} \sin(2\theta(t)) = \frac{ka}{g\sqrt{1+k^2}/2} \sin(2\theta(t))$$

If $k' = 28R_0$,

(9.4) "Magnetic Field"

$$H = \frac{1}{\mu_0} \nabla \times A$$

(9.16) "Vector Potential"

$$A(x) = -\frac{i\mu_0 \omega}{4\pi} p \frac{e^{ikr}}{r}$$

(9.17) "Electric Dipole Moment"

$$P = \int x' p(x') d^3 x'$$

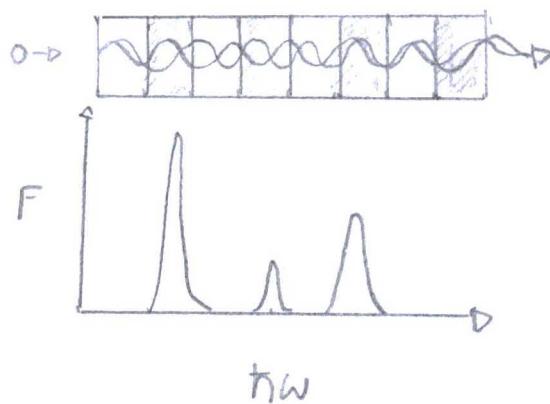
(1.10) "Magnetic Induction - only one term"

$$H = \mu_0 B$$

$$= \nabla \times A$$

$$= \nabla \times \left(-\frac{i\omega}{4\pi} p \frac{e^{ikr}}{r} \right)$$

$$= \nabla \times \left(-\frac{ik'}{4\pi c} p \frac{e^{ik'(r'-ct')}}{r'} \right)$$



"Radiation emitted
at twice the
fundamental frequency
in a sinusoidal
undulator"

$$\begin{aligned}
 &= \frac{16\pi k^2}{4\pi c} (n \times p) \frac{e^{ik'(r-ct)}}{r^1} \left(1 - \frac{1}{ik'(r)} \right) \\
 &= \frac{k'^2}{4\pi c} \frac{e^{ik'(r-ct)}}{r^1} \left(1 - \frac{1}{k'r^1} \right) (n \times p) \\
 &= \frac{k'^2}{4\pi c} \frac{e^{ik'(r-ct)}}{r^1} \left(1 - \frac{1}{k'r^1} \right) n \times \underbrace{\int x \rho(x) d^3x}_{\text{"near radiation zone"}^1} \\
 &\quad \text{(pg 408)} \quad \text{"Far(radiation)"^2} \quad \text{"near (radiation)"^3} \\
 &\quad \text{Zone}
 \end{aligned}$$

In the near zone, (pg 411)

$$\begin{aligned}
 &= \frac{k'^2}{4\pi c} \frac{e^{ik'(r-ct)}}{r^1} n \times \int x \rho(x) d^3x \\
 &= \frac{k'^2}{4\pi c} \frac{e^{ik'(r-ct)}}{r^1} n \times \int [\hat{z} - 4\hat{x}(n \cdot \hat{x})] d^3x
 \end{aligned} \tag{9.42}$$

$$= \frac{k'^2}{4\pi c} \frac{\cos(k'(r-ct)) + i \sin(k'(r-ct))}{r^1} n \times \int [\hat{z} - 4\hat{x}(n \cdot \hat{x})] d^3x$$

$$\approx \frac{i k'^2}{8} \alpha \frac{k}{\sqrt{1+k'^2/2}} n \times \int [\hat{z} - 4\hat{x}(n \cdot \hat{x})] d^3x$$

When only sin term
considered as $Z'(t')$

$$b) \frac{dP'}{d\omega} = \frac{e^2 c}{8\pi} \frac{K^2}{(1+K^2/2)} \frac{a^2}{64} s^1$$

(pg 689) "Power distribution of an undulator"

$$\frac{dP}{d\Omega} = \frac{e^2 c}{8\pi} k^{14} a^2 \sin^2 \theta$$

$$= \frac{e^2 c}{8\pi} \frac{k^{12}}{(1+k^2/2)} \frac{a^2}{64} s^1$$

C) $d^3 P = \left[\frac{e^2 c^2}{8\pi} \frac{k}{\sqrt{1+k^2/2}} \frac{a^2}{64} s^1 \frac{\delta(k^1 - \bar{K}k_0)}{\bar{K}k_0} \right] \frac{d^3 k}{\omega}$

$$\frac{d^3 p}{dk dk_0} = \frac{e^2 c}{8\pi \bar{K}^3} \frac{k}{\sqrt{1+k^2/2}} \frac{a^2}{64} s^1 \delta(k^1 - \bar{K}k_0)$$

where $k_y^1 = k \sin \theta \sin \phi$

$$k_{y'}^1 = k \sin \theta \sin \phi$$

$$k_z^1 = \bar{K} k (\cos \theta - \bar{\beta})$$

$$k^1 = \bar{K} k (1 - \bar{\beta} \cos \theta)$$

If $\eta = (\bar{\gamma} \theta^2)$, $\bar{\gamma} > 1$

$$\frac{d^2 P}{d\eta dk d\phi} = \frac{e^2 c \bar{\gamma}^2}{2\pi} \frac{k}{\sqrt{1+k^2/2}} \frac{a^2}{64} s^1 \left[\frac{(1-\eta)^2 + 4\eta \sin^2 \phi}{(1+\eta)^4} \right] \delta(k(1-\eta))$$

$$- 2\bar{\gamma}^2 k$$

$$\frac{d^2 P}{d\eta d\phi} = \frac{e^2 c \bar{\gamma}^2}{2\pi} \frac{k}{\sqrt{1+k^2/2}} \frac{a^2}{64} s^1 \left[\frac{(1-\eta)^2 + 4\eta \sin^2 \phi}{(1+\eta)^5} \right]$$

$$\frac{dP}{d\phi} = \int \frac{e^2 c \bar{\gamma}^2}{2\pi} \frac{k}{\sqrt{1+k^2/2}} \frac{a^2}{64} s^1 \left[\frac{(1-\eta)^2 + 4\eta \sin^2 \phi}{(1+\eta)^5} \right] d\eta$$

$$= \frac{3P}{16} \left[\frac{\kappa^2}{(1+\kappa^2/2)^2} \right] \cdot v^2 (10 - 21v + 20v^2 - 6v^4)$$

where $v = \frac{\kappa}{2\delta^2} R_0$

and $P = \frac{e^2 c \delta^2 \kappa^2 R_0^2}{3}$

Total Power radiated:

$$V_{min} = 2 / (4n_2) \quad V_{max} = 2 / (1+n_1)$$

Notes: Chapter 14 describes functions in high-speed objects, such as a boat or car. At lightspeed, the car's headlights narrow into a point, change to pastel color, and spiral into a helix at specific angles. During a turn, the headlights fan outward horizontally and wobble vertically. Technically, at lightspeed, everything goes dark when on top the hood and peering back into the lights.