

# Chapter 15: Bremsstrahlung, Method of Virtual Quanta, Radiative Beta Processes:

15.1

(14.65) "Energy radiated per unit solid angle per unit frequency"

$$\begin{aligned} \frac{d^2 I}{d\omega d\Omega} &= \frac{e^2}{4\pi^2 c} \left| \int_{-\infty}^{\infty} \frac{n x [(n - \beta) \times \dot{\beta}]}{(1 - n \cdot \beta)^2} e^{i\omega(t - n \cdot r(t)/c)} dt \right|^2 \\ &= \frac{e^2}{4\pi^2 c} \left| \int_{-\infty}^{\infty} \frac{n x [(n - \beta) \times \dot{\beta}]}{(1 - n \cdot \beta)^2} e^{i\omega t} e^{-ikr} dt \right|^2 \\ &= \frac{e^2}{4\pi^2 c} \left| \int_{-\infty}^{\infty} \frac{n x [(n - \beta) \times \dot{\beta}]}{(1 - n \cdot \beta)^2} e^{i\omega t} e^{i\omega(t + ikr)} dk \right|^2 \end{aligned}$$

(14.66) "Perfect Differential"

$$\frac{n x [(n - \beta) \times \dot{\beta}]}{(1 - n \cdot \beta)^2} = \frac{d}{dt} \left[ \frac{n x (n \times \beta)}{1 - n \cdot \beta} \right]$$

Taylor Series:  
Exponential

$$e^x = 1 + x + \frac{x^2}{2!} +$$

$$\begin{aligned} \frac{d^2 I}{d\omega d\Omega} &= \frac{e^2}{4\pi^2 c} \left| \int_{-\infty}^{\infty} \frac{d}{dt} \frac{n x [(n \times \beta)]}{1 - n \cdot \beta} e^{i\omega t} e^{i(k(t) + kr)} dt \right|^2 \\ &= \frac{e^2}{4\pi^2} \left| \int_{-\infty}^{\infty} \frac{d}{dt} \frac{\dot{\beta}_\perp - \dot{\beta}_\parallel kr}{(1 - n \cdot \beta)} e^{i\omega t} dt \right|^2 \end{aligned}$$

(14.69) "Continuous distribution"

$$e \beta e^{-i(\omega/c)n \cdot r(t)} \Rightarrow \frac{1}{c} \int d^3 X J(X, t) e^{-i(\omega/c)n \cdot r}$$

$$J = e \int dt e^{i\omega t} (\hat{p}_L - \hat{p}_R ikr)$$

b) (14.67) "Intensity distribution"

$$\frac{d^2 I}{d\omega d\Omega} = \frac{e^2 \omega^2}{4\pi^2 c} \left| \int_{-\infty}^{\infty} [n_x(n_x \beta)] e^{i\omega(t-n_x r(t)/c)} dt \right|^2$$

(From problem) "Current Density"

$$\begin{aligned} J &= e \int dt e^{i\omega t} \left( \hat{p}_L + \frac{d}{dt} (\hat{p}_R \hat{p}_L) - ikr \hat{p}_L + \dots \right) e^{-ir} \\ &= \frac{1}{c} \int dt e^{i\omega t} \cdot q \left( \hat{p}_L + \frac{d}{dt} (\hat{p}_R \hat{p}_L) - ikr \beta_L + \dots \right) (1 - r + r^2 - \frac{r^3}{6}) \\ &= \frac{1}{c} \int dt e^{i\omega t} \left( -qr + q \frac{r^2 \beta}{2} - \frac{1}{6c} q \frac{r^3}{2} + \dots \right) \\ &= \frac{1}{c} \int dt e^{i\omega t} \left( \frac{dp}{dt} + \frac{dm}{dt} x_n + \frac{1}{6c} \frac{dQ}{dt} + \dots \right) \end{aligned}$$

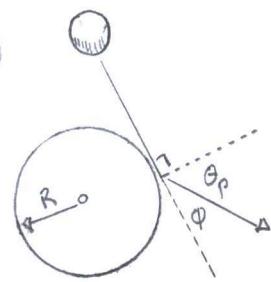
o, when  $q = e$ ,  $p = q \cdot r$ ,  $m = q \frac{rx\beta}{2}$   
and  $Q = q(3xx - r^2 \delta_{ij})$

(14.69) "Continuous distribution"

$$e^{\beta e^{-i(\omega/c)n_x r(t)}} = \frac{1}{c} \int d^3x J(x, t) e^{-i(\omega/c)n_x r}$$

$$\frac{d^2 I}{d\omega d\Omega} = \frac{1}{4\pi c} \left| \int dt e^{i\omega t} \left( \frac{dp}{dt} + \frac{dm}{dt} x_n + \frac{1}{6c} \frac{dQ}{dt} + \dots \right) \right|^2$$

15.2



"nonrelativistic particle  
... collides with a fixed  
smooth hard sphere ...  
for emission of photon  
per unit solid angle"

(15.2) "Elastic differential/cross section"

$$d\sigma_p = \frac{1}{4} R^2 d\Omega_p$$

(15.2) "Velocity change"

$$\Delta \beta = 2\beta \sin \phi (\sin \phi \hat{e}_z + \cos \phi \hat{e}_x)$$

$$= 2\beta \cos\left(\frac{\theta_p}{2}\right) \left( \cos\left(\frac{\theta_p}{2}\right) \hat{e}_z + \sin\left(\frac{\theta_p}{2}\right) \hat{e}_x \right)$$

$$= \beta ((\cos^2 \theta_p + 1) \hat{e}_z + \sin \theta_p \hat{e}_x)$$

If  $\phi = (\pi - \theta_p)/2$ , then

$$\Delta \beta = \beta ((\cos \theta_p + 1) \hat{e}_z + \sin \theta_p \cos \theta_p \hat{e}_x + \sin \theta_p \sin \theta_p \hat{e}_y)$$

(15.7) "radiated intensity"

$$\lim_{\omega \rightarrow 0} \frac{d^2 I_{\text{rad}}}{d\omega d\Omega} = \frac{e^2 c^2}{4\pi^2 c} |E^* \cdot \Delta \beta|^2$$

(15.4) "soft photon energy per unit energy  
and per unit solid angle"

$$\frac{d^3 \sigma}{d\Omega_p d(h\omega) d\Omega_\gamma} = \left[ \lim_{h\nu \rightarrow 0} \frac{d^2 N}{d(h\omega) d\Omega_\gamma} \right] \cdot \frac{d\sigma}{d\Omega_p}$$

(15.5) "invariant expression"

$$\frac{d^3 N}{(d^3 k/k_0)} = \frac{c^2}{h\nu} \frac{d^2 N}{d(h\omega) d\Omega_\gamma} = \frac{c^2}{h(h\nu)} \cdot \frac{d^2 I}{d\omega d\Omega_\gamma}$$

$$\frac{d^2 I}{d\omega d\nu} = \frac{d^2 \sigma}{d\Omega_p d(h\nu) d\Omega_\gamma} \frac{1}{h^2(\nu)} \cdot \frac{d\sigma_p}{d\Omega_p}$$

$$\frac{d^2 \sigma}{d\Omega_p d(h\nu) d\Omega_\gamma} = \frac{e^2 e^3}{4\pi^2} \frac{1}{h^2(\nu)} \left| E^* \cdot \Delta \beta \right|^2 \frac{d\sigma}{d\Omega_p}$$

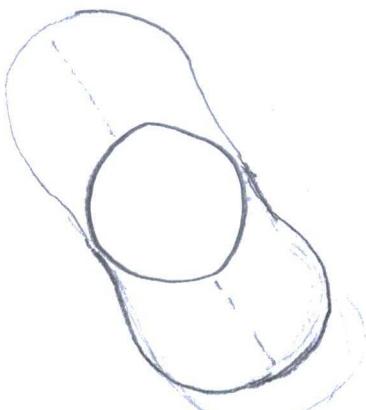
If  $Z=1$ , then

$$\frac{d^3\sigma}{d\Omega_p d(h\nu) d\Omega_y} = \frac{e^3}{4\pi^2} \cdot \frac{1}{h^2(\nu)} \left| E^* \cdot \Delta \beta \right| \frac{d\sigma}{d\Omega_p}$$

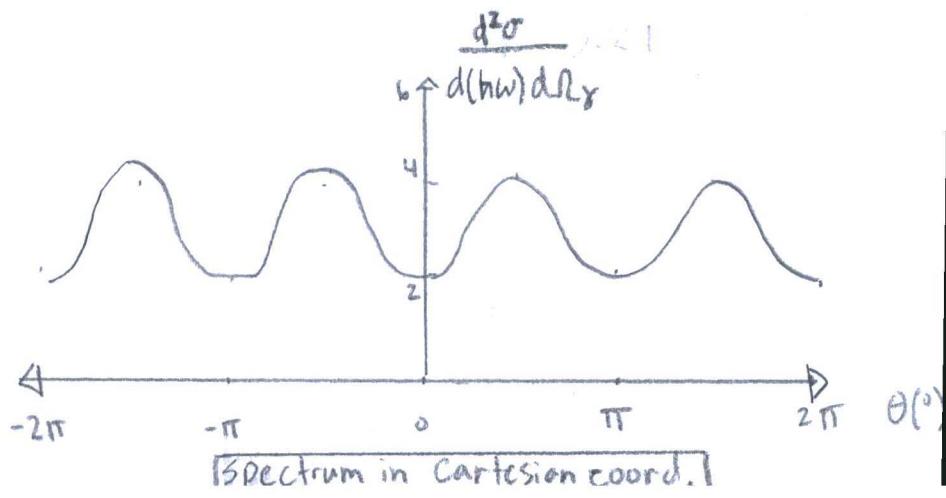
where  $\vec{E}_y = -\sin\theta \hat{e}_z + \cos\theta \hat{e}_x$  and  $\vec{E}_z = \hat{e}_y$

$$\begin{aligned} \frac{d^3\sigma}{d\Omega_p d(h\nu) d\Omega_y} &= \frac{e^2}{16\pi^2 c} \frac{\beta^2}{h\nu} R^2 [(-\sin\theta(\cos\theta_p + 1) \\ &\quad + \cos\theta \sin\theta_p \cos\phi_p)^2 \\ &\quad + \sin^2\theta_p \sin^2\phi_p] \end{aligned}$$

$$\begin{aligned} \frac{d^2\sigma}{d(h\nu) d\Omega_y} &= \int \frac{d^3\sigma}{d\Omega_p d(h\nu) d\Omega_y} d\Omega_p \\ &= \int_0^{2\pi} \int_0^\pi \frac{e^2}{16\pi^2 c} \frac{\beta^2}{h\nu} R^2 [(-\sin\theta(\cos\theta_p + 1) \\ &\quad + \cos\theta \sin\theta_p \cos\phi_p)^2 \\ &\quad + \sin^2\theta_p \sin^2\phi_p] d\theta_p d\phi_p \\ &= \frac{R^2}{16\pi^2 h\nu} \frac{e^2}{h\nu} \frac{\beta^2}{h\nu} \left[ \sin^2\theta \left( \frac{4\pi}{3} + 4\pi \right) \right. \\ &\quad \left. + \cos^2\theta \left( \frac{4\pi}{3} + \frac{4\pi}{3} \right) \right] \\ &\stackrel{\approx}{=} \frac{R^2}{12\pi} \frac{e^2}{h\nu} \left( \frac{V}{c} \right)^2 \frac{1}{h\nu} (2 + 3\sin^2\theta) \quad \dots \text{if } \theta \ll 1 \end{aligned}$$



Spectrum in polar coord.-dipole



Spectrum in Cartesian coord.

Total Bremsstrahlung cross section:

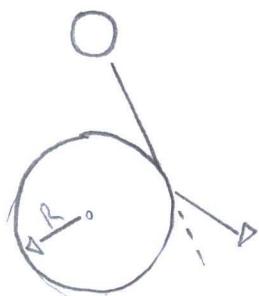
$$\frac{d\sigma}{d(\hbar\omega)} = \int \frac{d^2\sigma}{d(\hbar\omega)d\Omega_\gamma} d\Omega_\gamma$$

$$= \frac{R^2}{12\pi} \frac{e^2}{hc} \frac{\beta^2}{\hbar\omega} \left( 3\pi + 3 \frac{8\pi}{3} \right)$$

$$= \frac{4R^2}{3} \frac{e^2}{hc} \frac{\beta^2}{\hbar\omega}$$

Upper limit:  $\omega R/c \ll 1 \Rightarrow \beta \ll 1$

15.3



"elastic impact  
is negligible"

$$\beta_i = \beta \hat{E}_z$$

$$\beta_f = \beta (\sin \theta \cos \phi_p \hat{e}_x + \sin \theta_p \sin \phi_p \hat{e}_y + \cos \theta_p \hat{e}_z)$$

$$\hat{n} = \sin \theta \hat{e}_x + \cos \theta \hat{e}_z$$

$$\hat{E}_i = \cos \theta \hat{e}_x + \sin \theta \hat{e}_z$$

$$\hat{E}_i = \hat{e}_y$$

$$\beta \cdot n = \beta (\sin \theta \sin \theta_p \cos \phi_p + \cos \theta \cos \theta_p)$$

$$E_i \cdot \beta_i = -\beta \sin \theta$$

$$E_i \cdot \beta_f = \beta (\cos \theta \sin \theta \cos \phi_p + \sin \theta \cos \theta_p)$$

$$E_2 \cdot \beta = 0$$

$$E_2 \cdot \beta_p = \beta \sin \theta_p \cdot \sin \phi_p$$

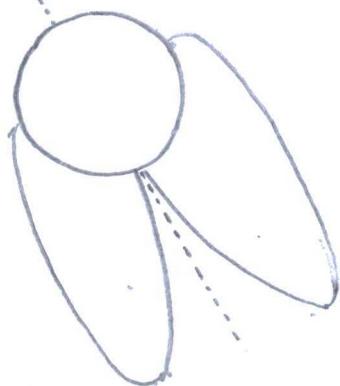
(15.2) "Spectrum of radiation with polarization

$$\lim_{\omega \rightarrow 0} \frac{d^2 I}{d\omega d\Omega} = \frac{z^2 e^2}{4\pi^2 c} \left| E^* \cdot \left( \frac{\beta'}{1 - n \cdot \beta'} - \frac{\beta}{1 - n \cdot \beta} \right) \right|^2$$

$$= \frac{R^2}{4} \frac{e^2}{4\pi^2 c} \frac{\beta^2}{\hbar\omega} \left[ \frac{\cos\theta \sin\theta_p \cos\phi_p - \sin\theta \cos\theta_p}{1 - \beta(\sin\theta \sin\theta_p \cos\phi_p + \cos\theta \cos\theta_p)} \right. \\ \left. + \frac{\sin\theta}{1 - \beta \sin\theta} \right]^2 + \frac{\sin^2\theta_p \sin^2\phi_p}{[1 - \beta(\sin\theta \sin\theta_p \cos\phi_p + \cos\theta \cos\theta_p)]^2}$$

"spectrum in polar  
coord.-relativistic dipole"

$\beta = 0.98$



$$\frac{d^2\sigma}{d\Omega d(\hbar\omega)} = \int \frac{d^3\sigma}{d\Omega d(\hbar\omega) d\Omega} d\Omega \\ = \int_0^{2\pi} \int_0^\pi \frac{R^2}{4} \frac{e^2}{4\pi^2 c} \frac{\beta^2}{\hbar\omega} \left[ \frac{\cos\theta \sin\theta_p \cos\phi_p - \sin\theta \cos\theta_p}{1 - \beta(\sin\theta \sin\theta_p \cos\phi_p + \cos\theta \cos\theta_p)} \right. \\ \left. + \frac{\sin\theta}{1 - \beta \sin\theta} \right]^2 + \frac{\sin^2\theta_p \sin^2\phi_p}{[1 - \beta(\sin\theta \sin\theta_p \cos\phi_p + \cos\theta \cos\theta_p)]^2} d\theta d\phi \\ = \frac{R^2}{4\pi} \frac{e^2}{\hbar c} \frac{\beta^2}{\hbar\omega} \left[ \frac{\sin^2\theta}{(1 - \beta \cos\theta)^2} + \frac{1}{\beta^3} \ln\left(\frac{1 + \beta}{1 - \beta}\right) - \frac{2}{\beta^2} \right]$$

15.4

$$\text{CCC} \rightarrow \bullet \rightarrow \bullet$$

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$$\text{CCC} \rightarrow \bullet \rightarrow \bullet$$

"group of charged  
particles undergo...  
acceleration"

a) (14.14) "Energy at light speed-modified"

$$E_{\text{rad}} = \frac{1}{c} \sum_j e_j \left[ \frac{n \times \{(n - \beta) \times \beta\}}{(1 - \beta \cdot n)^3 R} \right]_{\text{rect}}$$

$$\frac{d^2 I}{d\omega d\Omega} = \frac{1}{4\pi^2 c} \left| E^* \cdot \int_{-\infty}^{\infty} \sum_j e_j \frac{n \times \{(n - \beta) \times \beta\}}{(1 - \beta \cdot n)^2} e^{i\omega(t - n \cdot r(t))/c} dt \right|^2$$

(14.65) "Energy radiated per unit solid angle  
per unit frequency interval"

$$= \frac{1}{4\pi^2 c} \left| E^* \cdot \sum_j e_j \int_{-\infty}^{\infty} \frac{d}{dt} \left[ \frac{n \times (n \times \beta)}{1 - \beta \cdot n} \right] e^{i\omega(t - n \cdot r(t))/c} dt \right|^2$$

If  $\omega t \ll 1$ ,

$$= \frac{1}{4\pi^2 c} \left| \epsilon^* \cdot \sum_j e_j \left( \frac{\beta'_j}{1-\beta'_j \cdot n} - \frac{\beta_j}{1-\beta_j \cdot n} \right) e^{i\omega(t-n \cdot r(t))/c} \right|^2$$

$$= \frac{1}{4\pi^2 c} |\epsilon^* \cdot E|^2$$

$$\text{where } E = \sum_j e_j \left( \frac{\beta'_j}{1-\beta'_j \cdot n} - \frac{\beta_j}{1-\beta_j \cdot n} \right) e^{i\omega(t-n \cdot r(t))/c}$$

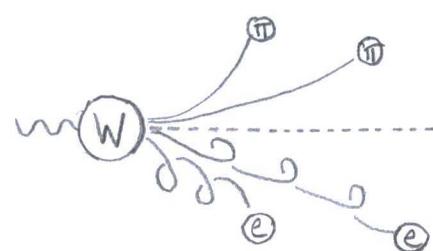
b)  $\frac{d^2 I}{dw d\Omega} = \frac{1 |\epsilon^*|^2}{4\pi^2 c} \left| \frac{\beta}{1-\beta \cdot n} - \frac{-\beta}{1+\beta \cdot n} \right|^2$

$$= \frac{e^2}{\pi^2 c} \left| \epsilon^* \cdot \frac{\beta}{1-(n \cdot \beta)^2} \right|^2$$

$$= \frac{e^2}{\pi^2 c} \frac{\beta^2 \sin^2 \theta}{(1-\beta^2 \cos \theta)^2}$$

Relationship:

$$\left( \frac{x}{1-\alpha x} - \frac{-x}{1+\alpha x} \right)^2 = \frac{1}{1-(\alpha x)^2}$$



"meson decays into pions and electrons"

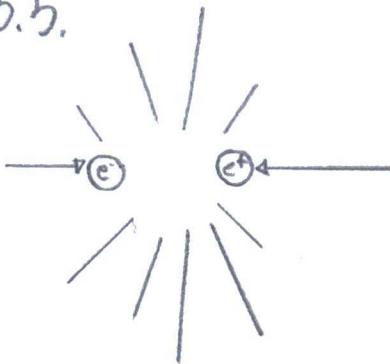
$$\begin{aligned} \frac{dI}{dw} &= \int \frac{d^2 I}{dw d\Omega} d\Omega \\ &= \int_0^{2\pi} \int_0^\pi \frac{e^2}{\pi^2 c} \frac{\beta^2 \sin \theta}{(1-\beta^2 \cos \theta)^2} d\theta d\phi \\ &= \frac{2e^2}{\pi c} \int_{-1}^1 \frac{\beta^2 \sin^2 \theta}{1-\beta^2 \cos^2 \theta} d\cos \theta \\ &= \frac{e^2}{\pi c} \left[ \frac{1+\beta^2}{\beta} \log \left( \frac{1+\beta}{1-\beta} \right) - 2 \right] \end{aligned}$$

Relativistic speeds:  $\beta \sim 1$

$$\frac{dI}{dw} = \frac{e^2}{\pi c} \left( 2 \log(4\gamma^2) - 2 \right)$$

$$= \frac{4e^2}{\pi c} \left( \log(2\gamma) - \frac{1}{2} \right)$$

15.5.



"emission of radiation  
by... annihilation of  
electrons and positrons"

$$(14.67) \text{ "Intensity distribution - Electric"} \\ \frac{d^2 I}{d\omega d\Omega} = \frac{e^2 \omega^2}{4\pi^2 c} \left| \int_{-\infty}^{\infty} n_x (n_x \beta) e^{i\omega(t-n \cdot r(t)/c)} dt \right|^2$$

(Problem 14.9a) "Intensity distribution - Magnetic"

$$\frac{d^2 I}{d\omega d\Omega} = \frac{\omega^4}{4\pi^2 c^3} \left| \int_{-\infty}^{\infty} n_x \mu(t) e^{i\omega(t-n \cdot r(t)/c)} dt \right|^2$$

Total Intensity:

$$\frac{d^2 I}{d\omega d\Omega} = \frac{d^2 I_{elec}}{d\omega d\Omega} + \frac{d^2 I_{mag}}{d\omega d\Omega}$$

$$= \frac{\omega^2}{4\pi^2 c^3} \left| \int_{-\infty}^{\infty} \left\{ n_x (n_x \beta) + \frac{\omega}{c} n_x \mu(t) \right\} e^{i\omega(t-n \cdot r(t)/c)} dt \right|^2$$

$$= \frac{1}{\pi^2 c} \left| \frac{e^2 \beta^2 \sin \theta}{(1 - \beta^2 \cos^2 \theta)^2} + \frac{\omega}{c} \frac{e^2 h^2 / 4m^2 c^2}{(1 - \beta^2 \cos^2 \theta)^2} \right|^2$$

$$= \frac{e^2}{\pi^2 c} \frac{\beta^2 \sin^2 \theta + \frac{h^2 \omega^2}{4m^2 c^4}}{(1 - \beta^2 \cos^2 \theta)^2}$$

(15.4) "Soft photon energy per unit energy and per unit solid angle"

$$\frac{d^3 \sigma}{d\Omega_p d(h\omega) d\Omega_\gamma} = \left[ \lim_{h\omega \rightarrow 0} \frac{d^2 N}{d(h\omega) d\Omega_\gamma} \right] \frac{d\sigma}{d\Omega_p}$$

(15.5) "invariant expression"

$$\frac{d^3 N}{(d^3 k/k_0) d(h\omega) d\Omega_\gamma} = \frac{c^2}{h\omega} \frac{d^2 N}{d(h\omega) d\Omega_\gamma} = \frac{c^2}{h(h\omega)} \frac{d^2 I}{d\omega d\Omega_\gamma}$$

$$\cong \frac{4e^2}{\pi c} \left[ \ln \left( \frac{M_w}{m} \right) - \frac{1}{2} \right]$$

Total energy:

(15.68) "Total energy"

$$E_{\text{rad}} = \int_0^{W_{\text{max}}} \frac{dI}{dW}(w) dw = \frac{e^2}{\pi \hbar c} \left[ \frac{1}{\beta} \ln \left( \frac{1+\beta}{1-\beta} \right) - 2 \right] E$$

(15.69) "ratio of energy"

$$\frac{E_{\text{rad}}}{E} = \frac{2}{\pi} \frac{e^2}{\hbar c} \left[ \ln \left( \frac{2E}{mc^2} \right) - 1 \right]$$

$$\frac{E_{\text{rad}}}{E} = \frac{e^2}{\pi \hbar c} \left[ \frac{1}{\beta} \ln \left( \frac{1+\beta}{1-\beta} \right) - 2 \right] E$$

$$\frac{E_{\text{rad}}}{E} \cong \frac{4\alpha}{\pi} \left[ \ln \left( \frac{M_w}{m} \right) - \frac{1}{2} \right] E$$

Fine Structure
Constants
$\alpha \cong e^2/\hbar c$
(pg 710)

$$\frac{E_{\text{rad}}}{E} = \frac{2}{\pi} \frac{e^2}{\hbar c} \left[ \ln \left( \frac{2E}{mc^2} \right) - 1 \right]$$

$$\cong \frac{2\alpha}{\pi} \left[ \ln \left( \frac{2E}{mc^2} \right) - 1 \right]$$

Mass Meson: 740 MeV/c

particle	mass (MeV/c)	$E_{\text{rad}}/\langle E \rangle$	$E_{\text{rad}}/E(\alpha)$
$\pi^\pm$	139.570	1.561	1.561
$e^\pm$	0.511	8.704	8.704

$$\frac{d^2\sigma}{d\omega d\Omega} = \frac{1}{\hbar^2 w} \frac{d^2 I}{d\omega d\Omega} d\sigma$$

$$\simeq \frac{\alpha}{\pi^2} \frac{d\sigma}{\hbar w} \frac{\beta^2 \sin^2 \theta + \frac{\hbar^2 w^2}{4m^2 c^4}}{(1 - \beta^2 \cos^2 \theta)^2}$$

Fine-Structure  
Constant

$$\alpha = e^2 / \hbar c$$

(pg 710)

$$\begin{aligned} \frac{d\sigma}{d(\hbar w)} &= \int \frac{d^2\sigma}{d\omega d\Omega} d\Omega \\ &= \int_0^{2\pi} \int_0^\pi \frac{\alpha}{\pi^2} \frac{d\sigma}{\hbar w} \frac{\beta^2 \sin^2 \theta + \frac{\hbar^2 w^2}{4m^2 c^4}}{(1 - \beta^2 \cos^2 \theta)^2} d\theta d\phi \end{aligned}$$

### Integral Identities:

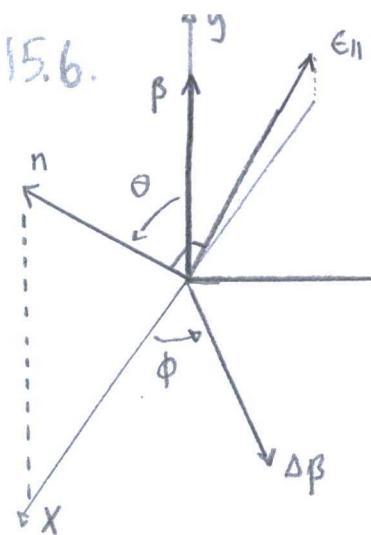
$$\int_{-1}^1 \frac{1-x^2}{(1-b^2x^2)^2} dx = \frac{1}{b^2} \left[ \frac{1+b^2}{2b} \log\left(\frac{1+b}{1-b}\right) - 1 \right]$$

$$\int_{-1}^1 \frac{dx}{(1-b^2x^2)^2} = \frac{1}{2b} \log\left(\frac{1+b}{1-b}\right) + \frac{1}{1-b^2}$$

$$\frac{d\sigma}{d(\hbar w)} = \frac{2\alpha}{\pi} \frac{d\sigma}{\hbar w} \left\{ \frac{1+\beta^2}{2\beta} \log\left(\frac{1+\beta}{1-\beta}\right) - 1 \right\}$$

$$+ \frac{\hbar^2 w^2}{4\pi m^2 c^4} \left[ \frac{1}{2\beta} \log\left(\frac{1+\beta}{1-\beta}\right) + \frac{1}{1-\beta^2} \right]$$

The above equations are proportional to the quantum mechanical solution from the problem.



"radiation emitted  
with small change  
in  $\Delta\beta$ "

(15.9) "Intensity distribution as  
velocity changes"

$$\lim_{\omega \rightarrow 0} \frac{d^2 I}{d\omega d\Omega} = \frac{Z^2 e^2}{4\pi^2 c} \left| \vec{E}_{||}^* \cdot \frac{\Delta\beta + n \times (\beta \times \Delta\beta)}{(1 - n \cdot \beta)^2} \right|^2$$

$$\text{If } \vec{n} = \langle \sin\theta, 0, \cos\theta \rangle$$

$$\vec{E}_{||} = \langle -\cos\theta, 0, \sin\theta \rangle$$

$$\vec{E}_{\perp} = \langle 0, 1, 0 \rangle$$

$$\beta = \beta \cdot \hat{e}_z$$

$$\Delta\beta = |\Delta\beta| (\cos\theta \hat{e}_x + \sin\phi \hat{e}_y)$$

$$\begin{aligned} n \times (\beta \times \Delta\beta) &= \beta |\Delta\beta| (-\cos\theta \cos\phi \hat{e}_x \\ &\quad - \cos\theta \sin\phi \hat{e}_y \\ &\quad + \sin\theta \cos\phi \hat{e}_z) \end{aligned}$$

$$\vec{E}_{||} \cdot [\Delta\beta + n \times (\beta \times \Delta\beta)] = |\Delta\beta| (\beta - \cos\theta) \cos\phi$$

$$\vec{E}_{\perp} \cdot [\Delta\beta + n \times (\beta \times \Delta\beta)] = |\Delta\beta| (1 - \beta \cos\theta) \sin\phi$$

"Dot product"

(15.10) "Intensity distribution with  
Polarization and angle"

$$\lim_{\omega \rightarrow 0} \frac{d^2 I}{d\omega d\Omega} = \frac{Z^2 e^2}{8\pi^2 c} |\Delta\beta|^2 \frac{(\beta - \cos\theta)^2}{(1 - \beta \cos\theta)^4}$$

$$\lim_{\omega \rightarrow 0} \frac{d^2 I}{d\omega d\Omega} = \frac{Z^2 e^2}{8\pi^2 c} |\Delta\beta|^2 \frac{1}{(1 - \beta \cos\theta)^2}$$

b)  $\gamma \gg 1$ ,  $\theta \ll 1$

(15.9) "Intensity distribution with polarization"

$$\lim_{\omega \rightarrow 0} \frac{d^2 I}{d\omega d\Omega} \simeq \frac{z^2 e^2}{4\pi^2 c} \left| \epsilon^* \cdot \left( \frac{\Delta \beta + n \times (\beta \times \Delta \beta)}{(1 - n \cdot \beta)^2} \right) \right|^2$$

(15.10) "Intensity distribution with polarization and angle"

$$\lim_{\omega \rightarrow 0} \frac{d^2 I_{||}}{d\omega d\Omega} \simeq \frac{z^2 e^2}{8\pi^2 c} |\Delta \beta| \frac{(\beta - \cos \theta)^2}{(1 - \beta \cos \theta)^4}$$

$$\lim_{\omega \rightarrow 0} \frac{d^2 I_{\perp}}{d\omega d\Omega} \simeq \frac{z^2 e^2}{8\pi^2 c} |\Delta \beta| \frac{1}{(1 - \beta \cos \theta)^2}$$

$$\lim_{\omega \rightarrow 0} \frac{d^2 I}{d\omega d\Omega} = \lim_{\omega \rightarrow 0} \left( \frac{d^2 I_{||}}{d\omega d\Omega} + \frac{d^2 I_{\perp}}{d\omega d\Omega} \right)$$

$$= \frac{z^2 e^2 |\Delta \beta|^2}{8\pi^2 c} \left( \frac{(\beta - \cos \theta)^2}{(1 - \beta \cos \theta)^4} + \frac{1}{(1 - \beta \cos \theta)^2} \right)$$

$$= \frac{z^2 e^2 |\Delta \beta|^2}{8\pi^2 c} \left( \frac{\frac{1}{28} (8\theta^2 - 1) \frac{1}{2}}{\left[ \frac{1}{28} (1 + 8\theta^2)^4 \right]} + \frac{\frac{1}{2}}{\left( \frac{1}{28} (1 + 8\theta^2) \right)^2} \right)$$

where  $\cos \theta = 1 - \theta^2/2$

$$\beta = \left( 1 - \frac{1}{\theta^2} \right)^{1/2}$$

$$\langle \cos^2 \theta \rangle = \frac{1}{2\pi} \int_0^{2\pi} \cos^2 \theta d\theta$$

$$= 1/2$$

$$\langle \sin^2 \theta \rangle = \frac{1}{2\pi} \int_0^{2\pi} \sin^2 \theta d\theta$$

$$= 1/2$$

$$= \frac{Z^2 e^2 \gamma^2 |\Delta\beta|^2}{2\pi^2 c} \left( \frac{(\gamma^2 \theta^2 - 1)^2}{(1 + \gamma^2 \theta^2)^4} + \frac{1}{(1 + \gamma^2 \theta^2)^2} \right)$$

(15.11) "Total intensity per unit frequency  
at arbitrary velocity"

$$\lim_{w \rightarrow 0} \frac{d^2 I}{dw d\Omega} = \frac{Z^2 e^2 \gamma^4 |\Delta\beta|^2}{\pi^2 c} \frac{(1 + \gamma^2 \theta^4)}{(1 + \gamma^2 \theta^2)^4}$$

c) (pg 712)  $P(\theta) \approx 2\gamma^2 \theta^2 / (1 + \gamma^4 \theta^4)$

$$P(0) = 0$$

$$P_{\max}(\theta) = 1$$

.. when  $\theta^2 \gamma^2 = 1$

$$\begin{aligned} d) \lim_{w \rightarrow 0} \frac{dI}{dw} &= \int \lim_{w \rightarrow 0} \frac{d^2 I}{dw d\Omega} d\Omega \\ &= 2\pi \int_0^\pi \lim_{w \rightarrow 0} \frac{d^2 I}{dw d\Omega} \sin\theta d\theta \\ &= \frac{2Z^2 e^2 \gamma^4}{\pi c} |\Delta\beta| \int_0^\infty \frac{1 + \gamma^4 \theta^4}{(1 + \gamma^2 \theta^2)^4} \theta d\theta \\ &= \frac{2Z^2 e^2 \gamma^2}{\pi c} |\Delta\beta| \int_0^\infty \frac{1 + \gamma^4 \theta^4}{(1 + \gamma^2 \theta^2)^4} d(\gamma^2 \theta^2) \end{aligned}$$

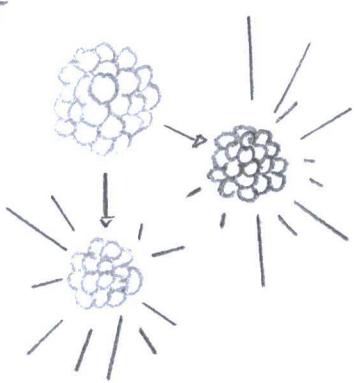
$$= \frac{2Z^2 e^2 \gamma^2}{\pi c} |\Delta\beta| \int_1^\infty \frac{1 + (y-1)^2}{y^4} dy$$

.. when  $y = (1 + \gamma^2 \theta^2)$

$$= \frac{2z^2 e^2 \gamma^4 |\Delta\beta|^2}{\pi c} \left( -\frac{z}{3y^3} + \frac{1}{y^3} - \frac{1}{y} \right) \Big|_1^\infty$$

$$= \frac{4z^2 e^2 \gamma^4 |\Delta\beta|^2}{3\pi c}$$

15.7



"radiation emitted  
in nuclear fission"

$$\begin{aligned} E_{TOT} &= \frac{Z_1 e \cdot \vec{B}}{1 - \beta \cos\theta} \\ &= \frac{Z_1 e \cdot \frac{A_2}{A_1 + A_2} \beta}{1 - \frac{A_2}{A_1 + A_2} \beta \cos\theta} + \frac{Z_2 e \cdot \frac{A_1}{A_1 + A_2} \beta}{1 + \frac{A_1}{A_1 + A_2} \beta \cos\theta} \end{aligned}$$

$$= e\beta \left( \frac{Z_1 A_2 - Z_2 A_1}{A_1 + A_2} + \frac{Z_1 A_2^2 + Z_2 A_1^2}{(A_1 + A_2)^2} \beta \cos\theta + \dots \right)$$

(Problem 15.4) "Intensity of radiation emitted with polarization"

$$\frac{d^2 I}{d(\hbar\omega) d\Omega} = \frac{1}{4\pi^2 c} |E^* \cdot E|^2$$

$$= \frac{e^2 \beta^2 \sin\theta}{4\pi^2 \hbar c} \left| e\beta \left( \frac{Z_1 A_2 - Z_2 A_1}{A_1 + A_2} + \frac{Z_1 A_2^2 + Z_2 A_1^2}{(A_1 + A_2)^2} \beta \cos\theta + \dots \right) \right|^2$$

$$= \frac{\alpha^2 \beta^2 \sin^2\theta}{4\pi^2} |P + q\beta \cos\theta|^2$$

where  $\alpha = e^2/\hbar c$

$$P = \frac{Z_1 A_2 - Z_2 A_1}{A_1 + A_2}$$

$$q = \frac{Z_1 A_2^2 + Z_2 A_1^2}{(A_1 + A_2)^2}$$

$$\begin{aligned}
 \frac{dI}{d(\hbar\omega)} &= \int \frac{\alpha \beta^2 \sin^2 \theta}{4\pi^2} |p + q\beta \cos \theta|^2 d\Omega \\
 &= \frac{\alpha \beta^2}{4\pi^2} \int_{-1}^1 \int_0^{2\pi} \sin^2 \theta (p^2 + 2pq\beta \cos \theta \\
 &\quad + q^2 \beta^2 \cos^2 \theta) \sin \theta d\phi d\theta \\
 &= \frac{\alpha \beta^2}{2\pi} (2\pi) \int_{-1}^1 \sin^2 \theta (p^2 + 2pq\beta \cos \theta \\
 &\quad + q^2 \beta^2 \cos^2 \theta) d\cos \theta \\
 &= \frac{\alpha \beta^2}{2\pi} \left( \frac{4}{3} p^2 + \frac{4}{15} q^2 \beta^2 \right) \\
 &= \frac{2\alpha \beta^2}{3\pi} \left( p^2 + \frac{\beta^2 p^2}{5} \right)
 \end{aligned}$$

b) If  $Z_1 = 36, A_1 = 95$  (Krypton)

$Z_2 = 56, A_2 = 130$  (barium)

$$E = 170 \text{ MeV}$$

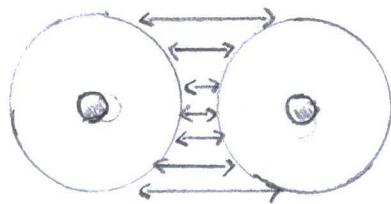
$$mc^2 = 931.5 \text{ MeV}$$

$$\begin{aligned}
 p &= \frac{Z_1 A_2 - Z_2 A_1}{A_1 + A_2} & q &= \frac{Z_1 A_2^2 + Z_2 A_1^2}{(A_1 + A_2)^2} \\
 &= -3.82 & &= 140.71
 \end{aligned}$$

(from problem) "radiated energy"

$$\frac{dI}{d(\hbar\omega)} = \frac{2\alpha \beta^2}{3\pi} \left( p^2 + \frac{\beta^2 q^2}{5} \right)$$

15.8.



"particles... interact by means of a short range interaction"

(15.4) "soft photon of energy per unit energy"

$$\frac{d^3\sigma}{d\Omega d(\hbar\omega) d\Omega_y} = \left[ \lim_{\Delta\omega \rightarrow 0} \frac{d^2N}{d(\hbar\omega) d\Omega_y} \right] \frac{d\sigma}{d\Omega_p}$$

$$\frac{d^2\sigma}{d(\hbar\omega) d\Omega_y} = \left[ \lim_{\Delta\omega \rightarrow 0} \frac{d^2N}{d(\hbar\omega) d\Omega_y} \right]$$

(15.5) "Lorentz invariant phase space"

$$\frac{d^3N}{(d^3k/k_0)} = \frac{c^2}{\hbar\omega} \frac{d^2N}{d(\hbar\omega) d\Omega_y} = \frac{c^2}{\hbar(\hbar\omega)} \frac{d^2I}{d\omega d\Omega_y}$$

$$\frac{d^2\sigma}{d(\hbar\omega) d\Omega_y} = \left[ \lim_{\Delta\omega \rightarrow 0} \frac{1}{\hbar^2\omega} \frac{d^2I}{d\omega d\Omega_y} \right]$$

$$\frac{d^2N}{d(\hbar\omega) d\Omega} = \frac{Z^2 c^2}{4\pi^2 \hbar(\hbar\omega)} \left| \int dt \left[ \frac{nx(nx\beta')}{1-n\cdot\beta} \right] e^{i\omega(t-n\cdot r(t))/2c} \right|^2$$

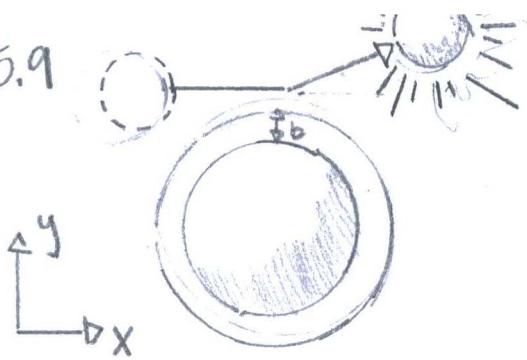
$$- \left[ \frac{nx(nx\beta')}{1+n\cdot\beta} \right] e^{i\omega(t+n\cdot r(t))/2c} \int dt \left| \frac{d}{dt} \left[ \frac{nx(nx\beta')}{1+n\cdot\beta} \right] \right|^2$$

$$= \frac{Z^2 e^2}{4\pi^2} \frac{1}{\hbar(\hbar\omega)} \left| \int dt \left\{ [nx(nx\beta')] [1+n\cdot\beta] e^{i\omega t} \right. \right.$$

$$\left. \left. x \left( 1 - \frac{i\omega}{2c} n \cdot r(t) \right) - [nx(nx\beta')] [1-n\cdot\beta] \right\} e^{i\omega t} \right|^2$$

$$x \left( 1 + \frac{i\omega}{2c} n \cdot r(t) \right) dt \left| \frac{d}{dt} \left[ \frac{nx(nx\beta')}{1+n\cdot\beta} \right] \right|^2$$

15.9



"A particle...  
deflected in a  
screened Coulomb  
field... and  
consequently emits  
radiation"

$$a) V(r) = \frac{Zze^2 e^{-\alpha r}}{r}$$

$$\nabla V(r) = Zze^2 \left( -\alpha - \frac{1}{r} \right) \frac{e^{-\alpha r}}{r}$$

$$F = m \ddot{x}(t)$$

$$= Zze^2 \left( \alpha + \frac{1}{r} \right) \frac{e^{-\alpha r}}{r} \frac{x}{\sqrt{x^2 + y^2}}$$

$$F = m \ddot{y}(t)$$

$$= Zze^2 \left( \alpha + \frac{1}{r} \right) \frac{e^{-\alpha r}}{r} \frac{y}{\sqrt{x^2 + y^2}}$$

when  $y = b$

$$( )$$

X

$$\ddot{x}(w) = \int_{-\infty}^{\infty} \ddot{x}(t) e^{iwt} dt$$

$$= \frac{Zze^2}{m} \int_{-\infty}^{\infty} \left( \alpha + \frac{1}{r} \right) \frac{e^{-\alpha r}}{r} \frac{x}{\sqrt{x^2 + y^2}} e^{iwt} dt$$

$$= \frac{\alpha Zze^2}{m} \int_{-\infty}^{\infty} \left( \frac{1}{x^2 + y^2} + \frac{1}{\alpha(x^2 + y^2)^{3/2}} \right) e^{-\alpha(x^2 + y^2)} x e^{iwt} dt$$

$$= \frac{\alpha Zze^2}{m \cdot V} \int_{-\infty}^{\infty} \left( \frac{1}{x^2 + y^2} + \frac{1}{\alpha(x^2 + y^2)^{3/2}} \right) e^{-\alpha(x^2 + y^2)} dt$$

$$= \frac{Z^2 e^2}{4\pi^2} \frac{1}{\hbar(\hbar\omega)} \left| \int \frac{d}{dt} \left\{ [nx(nx\beta)][1+n\cdot\beta] e^{i\omega t} \right. \right.$$

$$\left. \times \left( 1 - \frac{i\omega}{2c} n \cdot r(t) \right) - [nx(nx\beta)][1-n\cdot\beta] e^{i\omega t} \right. \\ \left. \times \left( 1 + \frac{i\omega}{2c} n \cdot r(t) \right) dt \right\}^2$$

$$= \frac{Z^2 e^2}{\pi} \frac{1}{\hbar(\hbar\omega)} \left| \int \frac{d}{dt} \left\{ [nx(nx\beta)][n\cdot\beta] - [nx(nx\beta')] \right. \right. \\ \left. \left. \frac{i\omega}{2c} \{n \cdot r(t)\} e^{i\omega t} dt \right\}^2 \right|$$

(15.4) "Soft photon energy per unit energy interval and per unit solid angle"

$$\frac{d^3\sigma}{d\Omega_p d(\hbar\omega) d\Omega} = \frac{Z^2 e^2}{\pi^2 c} \frac{1}{\hbar(\hbar\omega)} \left| \int \frac{d}{dt} \left\{ [nx(nx\beta)][n\cdot\beta] \right. \right. \\ \left. \left. - [nx(nx\beta')] \cdot \frac{i\omega t}{2c} n \cdot r(t) \right\} e^{i\omega t} \right| \frac{d\sigma}{d\Omega_p}$$

$$= \left( \frac{q^2}{hc} \right) \frac{\beta^2 R^2}{4\pi} \frac{1}{\hbar\omega} \left| \left\{ \beta(n \times n) - \frac{i\omega}{2c} \beta(n \times n) n \cdot r \right\} e^{i\omega t} \right|$$

$$= \left( \frac{q^2}{hc} \right) \frac{\beta^2 R^2}{4\pi} \frac{1}{\hbar\omega} \left| \beta^2 \left( \frac{2}{3} + 3 \sin^2 \theta \cos^2 \theta \right) \right. \\ \left. + \left( \frac{\omega R}{c} \right)^2 \left( 1 - \frac{3}{8} \sin^4 \theta \right) \right|$$

oo. I checked & pure neglect.

$$\frac{d\sigma}{d(\hbar\omega)} = \int_0^{2\pi} \int_0^{\pi} \left( \frac{q^2}{\hbar c} \right) \frac{\beta^2 R^2}{48\pi} \frac{1}{\hbar\omega} \left[ \beta^2 \left( \frac{2}{5} + 3 \sin^2\theta \cos^2\theta \right) \right. \\ \left. + \left( \frac{\omega R}{c} \right)^2 \left( 1 - \frac{3}{8} \sin^4\theta \right) \right] \sin\theta d\theta dd$$

$$= \left( \frac{q^2}{\hbar c} \right) \frac{\beta^2 R^2}{15} \frac{1}{\hbar\omega} \left[ \beta^2 + \left( \frac{\omega R}{c} \right)^2 \right]$$

$$(\cos(\omega t) + i \sin(\omega t)) dt$$

Cite: I.S. Gradshteyn, I.M. Ryzhik

"Table of Integrals, series,  
and products"

- Trigonometric Functions  
and exponentials -

(3.914.5)

$$\int_0^\infty \left( \frac{1}{\beta(\gamma^2+x^2)^{3/2}} + \frac{1}{\gamma^2+x^2} \right) e^{-\beta\sqrt{\gamma^2+x^2}} \cdot \cos(bx) dx \\ = \frac{1}{\beta \cdot \gamma} \sqrt{\beta^2 + b^2} \cdot K_1\left(\gamma \sqrt{\beta^2 + b^2}\right)$$

(3.914.5)

$$\int_0^\infty \left( \frac{1}{\beta(\gamma^2+x^2)^{3/2}} + \frac{1}{\gamma^2+x^2} \right) e^{-\beta\sqrt{\gamma^2+x^2}} \cdot x \cdot \sin(bx) dx \\ = \frac{b}{\beta} K_0\left(\gamma \sqrt{\beta^2 + b^2}\right)$$

$$\ddot{x}(w) = \frac{2i w^2 z e^2}{m v^2} K_0\left(b \sqrt{x^2 + \frac{w^2}{v^2}}\right)$$

$$\ddot{y}(w) = \frac{x b z e^2}{m v} \int_{-\infty}^{\infty} \left( \frac{1}{x^2 + b^2} + \frac{1}{\alpha(x^2 + b^2)^{3/2}} \right) \cdot e^{-\alpha\sqrt{x^2+b^2}} \cdot \cos(wt) dx$$

$$= \frac{2\sqrt{\alpha^2 + \omega^2/v^2}}{mv} ZZe^2 K_1(b \cdot \sqrt{\alpha^2 + \omega^2/v^2})$$

$$\begin{aligned}\frac{dI}{dw}(w, b) &= \frac{2Z^2e^2}{3\pi c^3} (\overset{\circ}{r}(w))^2 \\ &= \frac{2Z^2e^2}{3\pi c^3} (\overset{\circ}{x}(w)^2 + \overset{\circ}{y}(w)^2)\end{aligned}$$

$$\begin{aligned}&= \frac{8}{3\pi^3} \left( \frac{ZZ^2e^3}{mv} \right) \left[ \frac{\omega^2}{v^2} K_0^2(b\sqrt{\alpha^2 + \omega^2/v^2}) \right. \\ &\quad \left. + \left(\alpha^2 + \frac{\omega^2}{v^2}\right) K_1^2(b\sqrt{\alpha^2 + \omega^2/v^2}) \right]\end{aligned}$$

If  $\omega \gg v/b$ , then

$$\begin{aligned}&= \frac{8}{3\pi^3 c^3} \left( \frac{ZZ^2e^3}{mv} \right) \alpha^2 \cdot K_1(\alpha b) \\ &= \frac{8}{3\pi} \frac{Z^2 e^2}{c} \left( \frac{Z^2 e^2}{mc^2} \right) \left( \frac{c}{v} \right)^2 \alpha^2 K_1^2(\alpha b)\end{aligned}$$

b) (15.20) "Differential Radiation cross section"

$$\frac{d^2\chi}{dwdQ} = \frac{dI(w, Q)}{dw} \cdot \frac{d\sigma}{dQ}(Q)$$

where  $dI(w, Q)/dw$  = energy radiated per unit frequency interval in a collision with momentum transfer  $Q$ .

(15.22) "radiation cross section"

"integrated over momentum transfer"

$$\frac{dX}{dw} = \frac{16}{3} \frac{Z^2 e^2}{c} \left( \frac{Z^2 e^2}{mc^2} \right)^{2/3} \frac{1}{\beta} \int_{-\infty}^{Q_{max}} \frac{d\chi}{dx}$$

$$\begin{aligned}
&= \int \frac{dI}{dw} (w, b) dw \\
&= \int_{b_{\min}}^{b_{\max}} \int_0^{2\pi} \frac{dI}{dw} (w, b) b d\phi db \\
&= 2\pi \int_{b_{\min}}^{b_{\max}} \frac{dI}{dw} (w, b) b db \\
&= 2\pi \int_{b_{\min}}^{b_{\max}} \frac{8}{3\pi} \frac{Z^2 e^2}{c} \left( \frac{Z^2 e^2}{mc^2} \right) \left( \frac{c}{v} \right)^2 \alpha^2 K_1^2(\alpha b) \circ b db \\
&= \frac{16}{3} \frac{Z^2 e^2}{c} \left( \frac{Z^2 e^2}{mc^2} \right) \left( \frac{c}{v} \right)^2 \int_{b_{\min}}^{b_{\max}} \alpha^2 K_1^2(\alpha b) \circ b db \\
&= \frac{16}{3} \frac{Z^2 e^2}{c} \left( \frac{Z^2 e^2}{mc^2} \right) \left( \frac{c}{v} \right)^2 \int_{b_{\min}}^{b_{\max}} \alpha^2 \cdot K_1^2(x) \circ x dx \\
&= \frac{16}{3} \frac{Z^2 e^2}{c} \left( \frac{Z^2 e^2}{mc^2} \right) \left( \frac{c}{v} \right)^2 \left\{ \frac{x^2}{2} \left[ K_0^2(x) - K_1^2(x) + \frac{2 \cdot K_0(x) K_1(x)}{x} \right] \Big|_{x_1}^{x_2} \right\}
\end{aligned}$$

where  $x_1 = \alpha b_{\min}$

$x_2 = \alpha b_{\max}$

c) If  $b_{\min} = \hbar/mv$ ,  $b_{\max} = v/w$ ,  $\alpha' = 1.4 a_0 Z^{1/3}$

If  $x_2 \ll 1$ ,  $\alpha b_{\max} \ll 1$

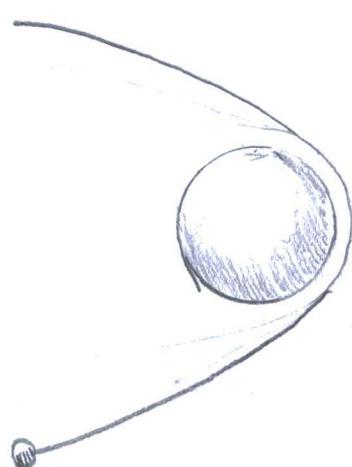
$$\begin{aligned}
\frac{dx}{dw} &= \frac{16}{3} \frac{Z^2 e^2}{c} \left( \frac{Z^2 e^2}{mc^2} \right) \left( \frac{c}{v} \right)^2 \frac{(1.4 a_0 Z^{1/3} \hbar)^2}{2 m^2 v^2} \left[ K_0^2(1.4 a_0 Z^{1/3} \hbar/mv) \right. \\
&\quad \left. - K_1^2(1.4 a_0 Z^{1/3} \hbar/mv) + \frac{2 K_0(1.4 a_0 Z^{1/3} \hbar/mv) K_1(1.4 a_0 Z^{1/3} \hbar/mv)}{1.4 a_0 Z^{1/3} \hbar/mv} \right]
\end{aligned}$$

If  $X_2 \gg 1$ , then

$$\frac{dX}{dw} = \frac{16}{3} \frac{Z^2 e^2}{c} \left( \frac{Z^2 e^2}{mc^2} \right) \left( \frac{C}{V} \right) \left\{ \begin{aligned} & \left[ K_0^2 (1.4a_0 Z^{1/3} v/w) - K_1^2 (1.4a_0 Z^{1/3} v/w) \right. \\ & \left. + \frac{2K_0 (1.4a_0 Z^{1/3} v/w) K_1 (1.4a_0 Z^{1/3} v/w)}{1.4a_0 Z^{1/3} v/w} \right] \\ & - \left[ K_0^2 (1.4a_0 Z^{1/3} h/mv) - K_1^2 (1.4a_0 Z^{1/3} h/mv) \right. \\ & \left. + \frac{2K_0 (1.4a_0 Z^{1/3} h/mv) K_1 (1.4a_0 Z^{1/3} h/mv)}{1.4a_0 Z^{1/3} h/mv} \right] \end{aligned} \right\}$$

Section 15.3 - Screening effects: Relativistic Radiative Energy Loss has an proportional equation, specifically (15.47).

15.10.



"Particle ... deflected in a hyperbolic path by repulsive Coulomb potential"

a) (14.60) "Frequency distribution"

$$\frac{d^2 I}{dw d\Omega} = 2 |A(w)|^2$$

(14.51) "Power radiated per solid angle"

$$\frac{dP(t)}{d\Omega} = |A(t)|^2$$

$$\frac{dI}{dw} = 2 \cdot P(t)$$

IF  $X = (E + \cosh \xi)$ ;  $y = b \sinh \xi$ ;  $\omega_0 t = \xi + E \sinh \xi$

where  $a = Ze^2/mv^2$ ;  $E = \sqrt{1 + (b/a)^2}$ ;  $\omega_0 = V/a$

$$X = \frac{i}{\omega_0 n \pi} \int_0^{2\pi} e^{i\omega_0 n t} \cdot \dot{X} dt$$

$$= \frac{ia}{2\pi n} \int_0^{2\pi} e^{in(\xi + E \sinh \xi)} \cdot \sinh(\xi) d\xi$$

$$y = \frac{i}{\omega_0 n \pi} \int_0^{2\pi} e^{i\omega_0 n t} \cdot \dot{y} dt$$

$$= \frac{-ib}{2\pi n} \int_0^{2\pi} e^{in(\xi + E \sinh \xi)} \cdot \cosh(\xi) d\xi$$

Cite: Landau, Lifshitz

"The Classical Theory  
of Fields"

pg 199, 7 (1971)

Identity #1:

$$\int_{-\infty}^{\infty} e^{ps - ix \sinh \xi} d\xi = i\pi H_p^{(1)}(ix)$$

$$X = \frac{\pi a}{\omega} H_{i\omega/\omega_0}^{(1)}(i\omega E/\omega_0)$$

$$= + \frac{2ai}{\omega} e^{\frac{\omega i}{\omega_0^2}} K_{i\omega/\omega_0} \left( \frac{\omega E}{\omega_0} \right)$$

Identity #2:

$$K_V(z) = -\frac{1}{2} \pi i e^{-\frac{V\pi i}{2}} H_V^{(2)}(ze^{-\frac{\pi i}{2}})$$

$$y = -\pi a \frac{\sqrt{\epsilon^2 - 1}}{\omega E} H_{i\omega/\omega_0}^{(1)}(i\omega E/\omega_0)$$

$$= -\frac{2ia}{\omega} e^{\frac{\omega i}{\omega_0^2}} \frac{\sqrt{\epsilon^2 - 1}}{\epsilon} K_{i\omega/\omega_0} \left( \frac{\omega E}{\omega_0} \right)$$

$$\begin{aligned}
 P_n &= \frac{4e^2}{3c^3} (n\omega_0)^4 \left| X^2 + Y^2 \right| \\
 &= \frac{4e^2}{3c^3} (n\omega_0)^4 \left[ \left| \frac{2i}{\omega} a e^{i\pi\epsilon w/\omega_0} K_{i\omega/\omega_0}^{-1} \left( \frac{\omega}{\omega_0} \epsilon \right) \right|^2 \right. \\
 &\quad \left. + \left| \frac{2i}{\omega} a e^{-i\pi\epsilon w/\omega_0} \frac{\sqrt{\epsilon^2 - 1}}{\epsilon} K_{i\omega/\omega_0} \left( \frac{\omega}{\omega_0} \epsilon \right) \right|^2 \right] \\
 &= \frac{8e^2}{3c^3} \frac{(n\omega_0)^4}{\omega_0^2} e^{i\pi\epsilon w/\omega_0} \left[ K_{i\omega/\omega_0}^{-1} \left( \frac{\omega}{\omega_0} \right)^2 + \frac{\epsilon^2 - 1}{\epsilon^2} \left[ K_{i\omega/\omega_0} \left( \frac{\omega}{\omega_0} \right) \right]^2 \right] \\
 &= \frac{8}{3\pi} \frac{(zeaw)^2}{c^3} e^{-i\pi\epsilon w/\omega_0} \left[ K_{i\omega/\omega_0}^{-1} \left( \frac{\omega}{\omega_0} \right)^2 + \frac{\epsilon^2 - 1}{\epsilon^2} K_{i\omega/\omega_0} \left( \frac{\omega}{\omega_0} \right)^2 \right] \\
 &= \text{Re} \left[ \frac{8}{3\pi} \frac{(zeaw)^2}{c^3} e^{-i\pi\epsilon w/\omega_0} \left[ K_{i\omega/\omega_0}^{-1} \left( \frac{\omega}{\omega_0} \right) + \frac{\epsilon^2 - 1}{\epsilon^2} K_{i\omega/\omega_0} \left( \frac{\omega}{\omega_0} \right)^2 \right] \right]
 \end{aligned}$$

b) (15.20) "Differential Cross Section"

$$\begin{aligned}
 \frac{d^2\chi}{d\omega dQ} &= \frac{dI(\omega, Q)}{d\omega} \frac{d\sigma}{dQ}(Q) \\
 \frac{d\chi}{d\omega} &= \int \frac{dI}{d\omega}(w, b) d\sigma \\
 &= \int_{b_{\min}}^{b_{\max}} \int_0^{2\pi} \frac{dI(w, b)}{dw} b d\phi db \\
 &= 2\pi \int_{b_{\min}}^{b_{\max}} \frac{dI(w, b)}{dw} b db \\
 &= \frac{16}{3} \frac{(zeaw)^2}{c^3} e^{-i\pi\omega/\omega_0} \int_0^{\omega_0} \left[ K_{i\omega/\omega_0}^{-1} \left( \frac{\omega}{\omega_0} \right) + \frac{\epsilon^2 - 1}{\epsilon^2} K_{i\omega/\omega_0} \left( \frac{\omega}{\omega_0} \right)^2 \right] b db
 \end{aligned}$$

$$= \frac{16}{3} \frac{(zeaw)^2}{c^3} e^{-\alpha^2 \int_{w/w_0}^{\infty} [K_{iw/w_0}(\frac{w}{w_0})^2 + \frac{\epsilon^2 - 1}{\epsilon} K_{iw/w_0}(\frac{w}{w_0})^2]} \cdot E dE$$

when  $\epsilon = \sqrt{1 + (b/a)^2}$

If  $k = \frac{w}{w_0} \epsilon$ , then

$$= \frac{16}{3} \frac{(zeaw)^2}{c^3} e^{-\alpha^2 \int_{w/w_0}^{\infty} [K'_{iw/w_0}(k)^2 + (1 - \frac{w_0^2}{k^2 w^2}) K_{iw/w_0}(k)^2]} \cdot k dk$$

$$= \frac{16}{3} \frac{(zeaw)^2}{c^3} e^{-\alpha^2 \int_{w/w_0}^{\infty} [K'_{iw/w_0}(k) k \cdot dk' + (1 - \frac{w_0^2}{k^2 w^2}) K_{iw/w_0}(k) \circ k \cdot dk']}$$

$$+ (1 - \frac{w_0^2}{k^2 w^2}) \int_{w/w_0}^{\infty} K_{iw/w_0}(k) \circ k \cdot dk$$

$$= \frac{16}{3} \frac{(zeaw)^2}{c^3} e^{-\alpha^2 \int_{w/w_0}^{\infty} [k \circ K_{iw/w_0}(k) \circ K_{iw/w_0}(k)]} \int_{w/w_0}^{\infty}$$

$$- \int_{w/w_0}^{\infty} K_{iw/w_0}(k) \circ k \cdot dk$$

Bessel Identity:

$$\frac{d}{dx} \left[ k \circ K_v(k) \right] = \left( 1 + \frac{v^2}{x^2} \right) K_v(x)$$

$$= \frac{16}{3} \frac{(zeaw)^2}{c^3} e^{\pi w/w_0} \frac{w}{w_0} K_{iw/w_0} \left( \frac{w}{w_0} \right) \left[ -K'_{iw/w_0} \left( \frac{w}{w_0} \right) \right]$$

$$\begin{aligned}
 &= \frac{2}{\pi \hbar c} \int_{w_0}^{E/F} \frac{Ae^2}{2\pi Mc} \frac{c^2}{\omega_0^2 + 2\omega_0^2} \ln \left( \frac{k\gamma^2 mc}{\hbar\omega} \right) \frac{dw}{\omega} \\
 &= \frac{2}{\pi \hbar c} \int_{w_0}^{E/F} \frac{A}{2\pi Mc} \frac{e^2}{2\omega_0} \frac{1}{\omega} \ln \left( \frac{E^2}{\hbar\omega_0} \right) \\
 &= \frac{2}{\pi} \left( \frac{e^2}{\hbar c} \right) \frac{Ac^2}{Mc} \frac{1}{4\pi\omega_0} \ln \left( \frac{E^2}{\hbar\omega} \right)
 \end{aligned}$$

c) (from problem) "Photonuclear cross section"

$$\sigma_{\text{photo}}(\omega) = (Ac^2/Mc) \delta(\omega - \omega_0)$$

(15.47) "Radiation cross section"

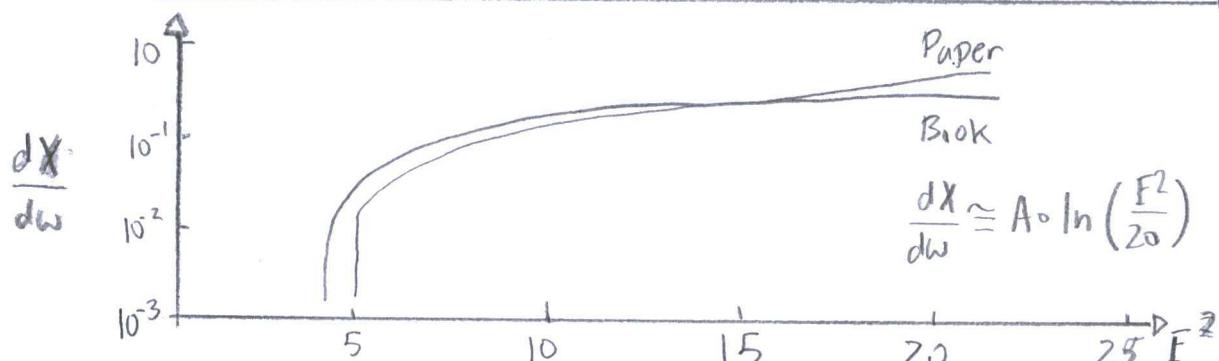
$$\frac{dX}{dw} = \frac{16}{3} \frac{Z^2 e^2}{c^2} \left( \frac{Z^2 e^2}{Mc^2} \right)^2 \ln \left( \frac{233M}{m Z^{1/3}} \right)$$

$$\frac{dX}{dw} \frac{Ac^2}{Mc\hbar\omega} = \frac{16}{3} \frac{Z^2 e^2}{c} \frac{Ac^2}{Mc\hbar\omega} \left( \frac{Z^2 e^2}{Mc^2} \right) \ln \left( \frac{233M}{m Z^{1/3}} \right)$$

Cite: E. Wolyniec

"Alpha decay of the giant quadrupole Resonance" in  $^{233}\text{U}$

Universidade de S. Paulo (1975)



c)  $\omega \ll \omega_0$ , If  $K_V(b) = -\log\left(\frac{x}{z}\right)$

and  $K_V'(x) = -1/x$

$$\frac{dX(w)}{dw} = \frac{16}{3} \frac{(zeaw)^2}{c^3} \frac{\omega_0}{w} \left[ -\log(w/2\omega_0) \right] \left( \frac{w}{\omega_0} \right)$$

$$= \frac{16}{3} \frac{(zeaw)^2}{c^3} \log(2\omega_0/w_0)$$

$$= \frac{16}{3} \frac{Z^2 e^2}{c^3} \left( \frac{Z^2 e^2}{m} \right) \frac{1}{(\beta c)^2} \log\left( \frac{2mv^3}{Zze^2 w} \right)$$

when  $w_0 = v/a$

$$\xrightarrow{-\text{close-}} = \frac{16}{3} \frac{Z^2 e^2}{c} \left( \frac{Z^2 e^2}{mc^2} \right) \frac{1}{\beta^2} \log\left( \frac{Zmv^3}{Zze^2 w} \right)$$

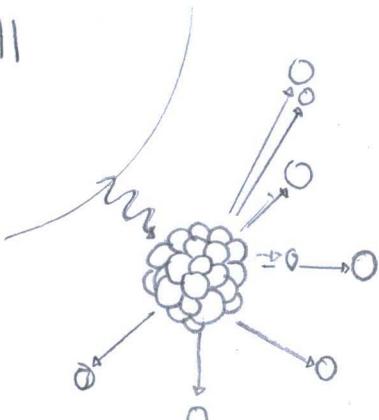
(15.25) "Classical radiation cross section"

$$\frac{dX_c}{dw} \sim \frac{16}{3} \frac{Z^2 e^2}{c} \left( \frac{Z^2 e^2}{mc^2} \right) \frac{1}{\beta^2} \cdot \ln\left( \frac{Zmv^3}{Zze^2 w} \right)$$

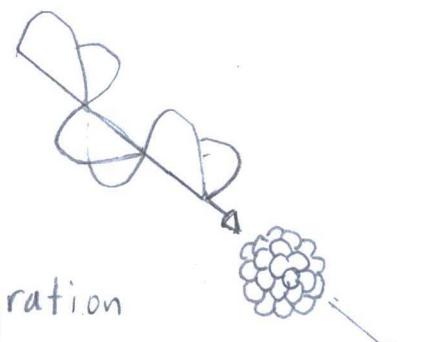
d) An attractive Coulomb Potential has a circular orbit, rather than hyperbolic.

In this case,  $x$  and  $y$  change to  $\cos(x)$  and  $\sin(x)$ .

15.11



"Photodisintegration  
of a nucleus"



"electrodisintegration  
of a nucleus"

a) If  $E = \gamma mc^2 \gg mc^2$ , then ways to measuring:

Cite #1: Gallagher, Brion, Samson, Langhoff

"Absolute cross section for  
Molecular Photoabsorption, Photo-  
ionization, and Ioniz Photofragment-  
ation Process"

University of Colorado (1987)

Intensity:

① Beer-Lambert Law :  $I = I_0 e^{-\sigma n L}$

② Cross-section :  $\sigma = \frac{1}{nL} \ln(I/I_0)$

③ Cross-section per angle:

$$\frac{d\sigma}{d\Omega} = (2\pi c^2 / hc) \cdot h\nu \underbrace{[K\phi_j(k) |e_0 \mu| \phi]_j^2}_{}$$

Actually, logarithm of  
intensity as photo-  
transition in sample.

Cite #2: Von H. Bethe

"Zur Theorie des Durchgangs  
Schnella Korpuskularstrahlen  
durch Materie"

Munich, Germany (1930)

## Wavenumber equation:

$$\textcircled{1} \text{ De Broglie Equation: } \lambda = \frac{h}{m v} = \frac{2\pi}{K}$$

Wavenumber

\textcircled{2} Atomic Series:

Z	1	2	7	9	10
	H	He	N	O	Ne

\textcircled{3} Velocity change:  $v = \sqrt{Z E / m}$

$$\text{and } \lambda = \frac{h}{\sqrt{2mE}} = \frac{2\pi}{K}$$

\textcircled{4} Bethe reviewed past work in

"Quantum Mechanics of one- and two-electron atoms" (1957)

with Salpeter:

$$\text{Cross-section (69.2): } \sigma = \frac{2\pi e^2 h}{m^2 c v} |D_{Nb}|^2$$

...relates to photoelectric effect,

cross-section and photodissociation in a transition.

Cite #3: Skopik, Murphy, Shiu

"Total cross section results for deuterium electrodisintegration"

Intensity, but with coefficients:

② Cross Section per angle:

$$\left. \frac{d\sigma}{d\Omega} \right|_{\gamma} = a_{\gamma} + b_{\gamma} \sin^2 \theta + c_{\gamma} \sin^2 \theta \cos 2\theta + d_{\gamma} \sin^2 \theta \cos^2 \theta$$

① Total cross section from intensity:

$$\sigma = \int \frac{2}{\pi} \frac{e^2}{hc} E \cdot \ln \left( \frac{I}{I_0} \right) d\Omega$$

$$= \frac{2}{\pi} \frac{e^2}{hc} \int E \cdot \ln \left( \frac{\delta^2 mc}{\hbar w} \right) d\Omega$$

$$= \frac{2}{\pi} \frac{e^2}{hc} \int_w^{E/k} \sigma_{\text{photo}} \cdot \ln \left( \frac{k \delta^2 mc}{\hbar w} \right) \frac{dw}{w}$$

... k from problem in unity.

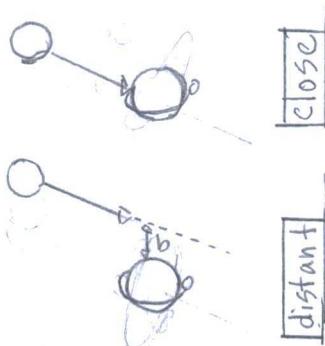
b) (from book) "Cross Section of resonance"

$$\sigma_{\text{photo}}(\omega) = \frac{A}{2\pi} \frac{e^2}{mc} \frac{T}{[(\omega - \omega_0)^2 + (T/2)^2]}$$

$$\sigma_{\text{el}}(E) = \frac{2}{\pi} \frac{e^2}{hc} \int_w^{E/k} \sigma_{\text{photo}} \ln \left( \frac{k \delta^2 mc^2}{\hbar w} \right) \frac{dw}{w}$$

$$= \frac{2}{\pi} \frac{e^2}{hc} \int_w^{E/k} \frac{Ac^2}{2\pi mc} \frac{T}{[(\omega - \omega_0)^2 + (T/2)^2]} \ln \left( \frac{k \delta^2 mc^2}{\hbar w} \right) \frac{dw}{w}$$

15.12.

a) "Rutherford Cross Section" ( $d > b$ ) - close"

$$d\sigma = 137(Z)^2 \cdot (Q^2)^2$$

(15.17) "Rutherford cross section"

$$\frac{d\sigma}{d\Omega_s} = \left( \frac{Z^2 e^2}{p v} \right)^2 \cdot \frac{1}{(2 \sin \theta'/2)^4}$$

(15.18) "Momentum transfer"

"A fast particle... collides with a hydrogen-like atom with one electron"

$$Q^2 = 4 p^2 \sin^2(\theta'/2)$$

$$\frac{d\sigma}{dQ} = \frac{d\sigma}{d\Omega} \frac{d\Omega}{dQ} = \left( \frac{Z^2 e^2}{p v} \right)^2 \frac{1}{(2 \sin \theta'/2)^4} \cdot \frac{2\pi \sin \theta}{dQ/d\Omega}$$

$$= \left( \frac{Z^2 e^2}{p v} \right)^2 \frac{1}{(2 \sin \theta'/4)^4} \frac{2\pi \sin \theta}{8 p^2 \cdot 2 \sin(\theta'/2) \cdot \cos(\theta'/2) \cdot 2}$$

$$= \left( \frac{Z^2 e^2}{p v} \right)^2 \frac{1}{(2 \sin \theta'/4)^4} \frac{2\pi \sin \theta}{8 p^2 \sin(\theta')}$$

$$= \frac{1}{2^2} \left( \frac{Z^2 e^2}{p v} \right)^2 \frac{1}{Q^2} \frac{\pi}{(\sin \theta'/2)^2}$$

Trigonometric Identity:

$$\sin(\theta) \cos(\theta) = \sin(2\theta)$$

$$= \frac{1}{2^2} \left( \frac{Z^2 e^2}{p v} \right)^2 \frac{1}{Q^2} \frac{\pi}{(\sin \theta'/2)^2}$$

$$= \frac{1}{2^6} \left( \frac{Z^2 e^2}{p v} \right)^2 \frac{1}{Q^4} \frac{\pi}{10^3 I^2}$$

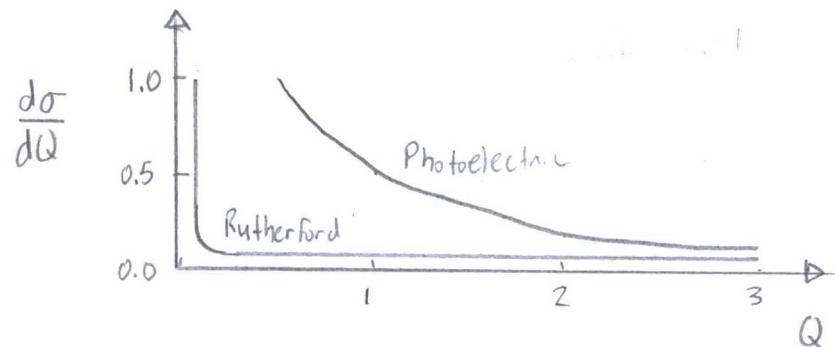
Photoelectric Effect ( $d < b$ ) - distant

$$\sigma_T(Q) = \frac{8\pi^2}{137} \left( \frac{a_0}{Z} \right)^2 \left( \frac{I}{Q} \right)^3$$

$$\frac{d\sigma}{dQ} = \frac{24\pi^2}{137} \left(\frac{a_0}{Z}\right)^2 \left(\frac{I}{Q}\right)^3 \frac{1}{Q}$$

$$\frac{d\sigma}{dQ} \left( \text{in units } = \frac{2\pi Z^2 e^4}{mv^2 I} \right) = \frac{12\pi}{137} \frac{m v^2}{Z^4 e^4} \left(\frac{I}{Q}\right)^4$$

$$= \frac{12\pi}{137} m \left(\frac{v}{Z^2 e^2}\right)^2 \left(\frac{I}{Q}\right)^4$$



b) Number of Collisions by integration:

$$\text{close - } \sigma = \int \frac{12\pi}{137} m \left(\frac{v}{Z^2 e^2}\right) \left(\frac{I}{Q}\right)^4 dQ$$

$$\propto \frac{1}{Q^3}$$

$$\text{Distant - } \sigma = \int \frac{12\pi}{137} \left(\frac{e E_{kin}}{m I}\right) \frac{1}{Q^4} dQ$$

$$\propto \frac{1}{Q^3}$$

$$\text{where } Q_s = 4p^2 \sin^2(\theta/2) > Q_{\max} = 2p. \quad (15.23)$$

$$\frac{Q_s}{Q_{\max}} = 4p \sin(\theta/2)$$

(13.14) "Bethe formula - Total Energy

Loss per unit length"

$$\frac{dE}{dx} = 4\pi N z \frac{z^2 e^4}{mc^2 \beta^2} \left[ \ln(\beta) - \beta^2 \right]$$

(15.29.5) "Total energy loss"

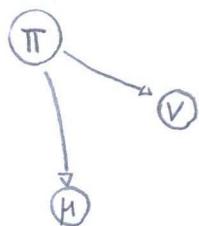
$$\frac{dE}{dx} = N \int_0^{W_{\max}} \frac{dX(w)}{dw} dw$$

(15.29)

$$\frac{dX_{nm}}{dw} = \frac{16}{3} \frac{z^2 e^2 (Z^2 e^2)}{c} \frac{1}{\beta^2} \ln \left[ \frac{x' Q_{\max}}{Q_{mn}} \right]$$

$$\simeq \ln \left[ \lambda'^4 p \sin \theta / 2 \right]$$

5.13.



"In decay of a pi meson at rest a mu meson and neutrino are created"

Cite: Primakoff Ho

"Anomalous  $\pi$ - $\mu$  Decay"

Washington University, St. Louis, MO.

$$E_\mu = E_\mu^{(0)} (1 - \epsilon / \epsilon_{\max})$$

$$P(\epsilon) d\epsilon = A(\epsilon / \epsilon_{\max})^A d\epsilon / \epsilon \int_0^\infty P(t) dt = 1$$

$$A = (2/3\pi)^0 (1/137) \cdot 2E_\mu^{(0)} / m\mu$$

301 decays!

Number of quanta emitted per unit energy:

(15.67) "Typical Bremsstrahlung Spectrum"

$$N(\hbar\omega) = \frac{e^2}{\pi\hbar c} \frac{1}{\hbar\omega} \left[ \frac{1}{\beta} \ln\left(\frac{1+\beta}{1-\beta}\right) + 2 \right]$$

1 Maximum photon Energy when  $V=3/10C$  and  $\hbar\omega=34\text{MeV}$

Maximum photon Energy:

$$E_{\max} = \frac{1}{2}(m_{\pi} - m_{\mu})$$

$$= 16.95 \text{ MeV}/c^2$$

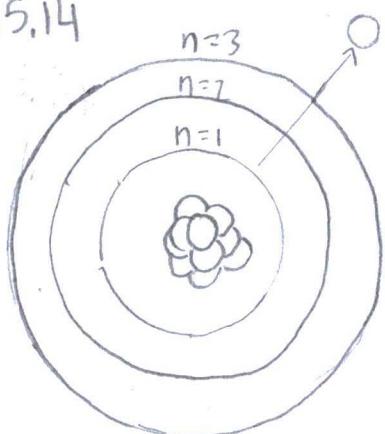
Quantum emitted greater than 1/10 maximum

$$17 \text{ MeV} \xrightarrow{\left(\frac{x-\bar{x}}{\sigma}\right) = 0.1} 18.7 \text{ MeV} \quad \text{which is } (15.67)$$

with  $\hbar\omega = 13.7 \text{ MeV}$

$$T = \hbar\omega - \phi$$

15.14



"A nucleus makes a transition... and an orbital electron is ejected."

$$N(\hbar\omega) = \frac{e^2}{\pi\hbar c} \left( \frac{1}{\hbar\omega} \right) \left[ \frac{1}{\beta} \ln\left(\frac{1+\beta}{1-\beta}\right) - 2 \right]$$

$$N_{99\%} (\hbar\omega = 0.99 \cdot 1 \text{ MeV}) = \phi / \hbar\omega$$

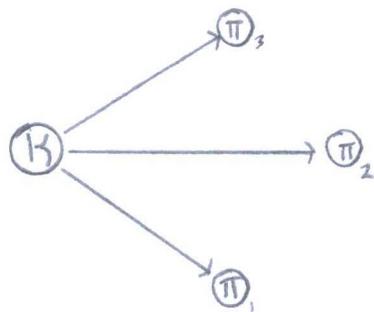
$$= \frac{1}{100} \cdot 10^6 \text{ s}^{-1}$$

$$= 10,000$$

The threshold is visible at a low intensity. Normally distributed at 1% has 5/1000 original intensity.

15.15.

$$a) K^- \rightarrow \pi^+ \pi^+ \pi^- \quad \Delta E = 75 \text{ MeV}$$



"One of the decay modes of the  $K^-$  meson is the three point decay"

(15.63) "Intensity distribution of radiation"

$$\frac{dI}{d\omega d\Omega} = \frac{e^2}{4\pi^2 c} \left| \frac{\epsilon^* \cdot \vec{\beta}}{1 - n \cdot \vec{\beta}} \right|^2$$

$$\frac{dI_{\text{tot}}}{d\omega d\Omega} = \frac{e^2}{4\pi^2 c} \sum_{i=1}^3 \left| \frac{\epsilon_i^* \cdot \vec{\beta}_i}{1 - n \cdot \vec{\beta}_i} \right|^2$$

$$= \frac{e^2}{4\pi^2 c} \left( \left| \frac{\epsilon_1 \cdot \vec{\beta}_1}{1 - n \cdot \vec{\beta}_1} \right|^2 + \left| \frac{\epsilon_2 \cdot \vec{\beta}_2}{1 - n \cdot \vec{\beta}_2} \right|^2 + \left| \frac{\epsilon_3 \cdot \vec{\beta}_3}{1 - n \cdot \vec{\beta}_3} \right|^2 \right)$$

$$\approx \frac{e^2}{4\pi^2 c} \left( \left| \epsilon_1 \cdot \vec{\beta}_1 \right|^2 + \left| \epsilon_2 \cdot \vec{\beta}_2 \right|^2 + \left| \epsilon_3 \cdot \vec{\beta}_3 \right|^2 \right)$$

... Nonrelativistic decay,  $\beta \ll 1$

$$\approx \frac{e^2}{4\pi^2 c} \cdot \left| 1 + 2 \cdot \beta \sin \theta \right|^2$$

... when  $\epsilon_1 = \epsilon_2 = -\epsilon_3$   
 $\beta_1 + \beta_2 = -\beta_3$

$$\approx \frac{e^2}{\pi^2 c} \beta^2 \sin^2 \theta$$

$$\approx \frac{2e^2}{\pi^2 c} \frac{T}{mc^2} \sin^2 \theta$$

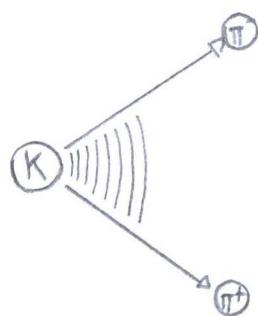
$$b) \Delta = 10 \text{ MeV} \quad \frac{dI}{d\omega d\Omega} = 20 \times 10^3 \text{ C/(m/s)}$$

Kinetic Energy	
$T$	$= \frac{1}{2} m v^2$
$m$	$= \frac{1}{2} m c^2 \cdot \beta^2$

$$\Delta = 10 \text{ MeV} \quad \frac{dI}{d\omega d\Omega} = 20 \times 10^4 \text{ C/(m/s)}$$

15.16.

a) (Chapter 11) "Maximum kinetic energy"



$$KE = \frac{M^2 - m^2}{2M} \text{ in units } MeV/c$$

$$= \frac{(493.7 \text{ MeV}/c)^2 - (139.6 \text{ MeV}/c)^2}{2}$$

$$\approx 493.7 \text{ MeV}/c$$

$$= 227 \text{ MeV}/c$$

"one of the decay  
modes of the  
charged meson...  
inner Bremsstrahlung  
is emitted."

oo then, I checked units in the book

(15.32) "Angular frequency spectrum"

$$\frac{d^2I}{d\omega d\Omega} = \frac{e^2}{9\pi^2 c} \left( \frac{\hbar\omega}{mc^2} \right)^2$$

oo the units are not watts/area.