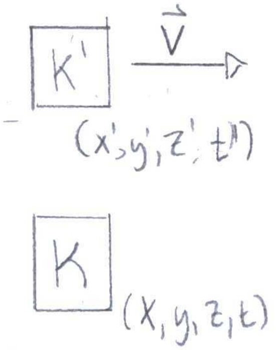
Chapter 11: Special Theory of Relativity:

11.1



"Two equivalent inertial frames K and K'.... K' moves with speed V... from K"

a) Isotropy of space-time: equivalent coordinates in every direction.

Homogenous space-time: equivalent coordinates at every point"

Ptolemy and Aristotle:

$x' = f_1 \cdot x + f_2 \cdot t$; $y' = y$; $z' = z$

$t' = g \cdot t + h \cdot x$

Galileo (1632): mechanisms are the same in all inertial frames, cite: "Dialogue concerning the Two Chief World System"

"Dialogo sopra i due massimi sistemi del mondo"

Isaac Newton: Philosophiae
(1687) Naturalis
(Principia Mathematica)

$x' = fx - vf_z t$; $y' = y$; $z' = z$

$t' = gt - vhx$

Einstein (1915) : General Relativity

$x' = f(v^2)x - vf(v^2)t$; $y' = y$; $z' = z$

$t' = g(v^2)t - vh(v^2)x$

Answer:

K' Frame $x' = f(v^2)x - vf(v^2)t$; $y' = y$; $z' = z$

K Frame $x = f(v^2)x' + vf(v^2)t$; $y = y'$; $z = z'$

K' Frame $t' = g(v^2)t - vh(v^2)x$

K Frame $t = g(v^2)t' + vh(v^2)x$

b) $x = fx' + vft$

$= f[fx - vft] + vf[gt - vhx]$

$= (f^2 - v^2 fh)x + (g - f)f \cdot v \cdot t$

$t = gt' + vhx$

$= g(gt - vhx) + vh(fx - vft)$

$= (g^2 - v^2 hf)t + (f - g)h \cdot v x$

A consistent solution requires similar results between both the lab frame (K) and center of mass frame (K').

If the speed limit (c) is $u' = u$, then

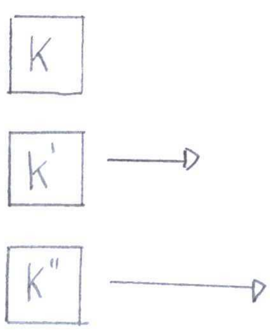
$$c = \frac{c+v}{1+vc\left(\frac{f-1/f}{fv^2}\right)}$$ rearranges to $$f = \pm\frac{c}{\sqrt{c^2-v^2}} = \pm\frac{1}{\sqrt{1-(v/c)^2}}$$

$$X' = \frac{1}{\sqrt{1-(v/c)^2}} \; ; \; t' = \frac{1}{\sqrt{1-(v/c)^2}} \cdot \left(t - \frac{vx}{c^2}\right)$$

$$X = \frac{1}{\sqrt{1-(v/c)^2}} \; ; \; t = \frac{1}{\sqrt{1-(v/c)^2}} \left(t' + \frac{vx}{c^2}\right)$$

...when $y' = y$; $y = y'$ and $z' = z$; $z = z'$

11.2



"three inertial frames.... movement relative"

Citation: "Lorentz Transformations in Steps"
Y. P. Terletskii, Paradoxes in theory of Relativity, pp 17-25 (1968)

$$x = f(v^2)(x' + v_1 t') = f(v^2)(x'' + v_3 t'')$$
$$y = y' = y''$$
$$z = z' = z''$$
$$t = f(v^2)\left(t' + v_1 x' \frac{f(v^2) - 1/f(v^2)}{f(v^2)\,v_1^2}\right) = f(v^2)\left(t'' + v_3 x'' \frac{f(v^2) - 1/f(v^2)}{f(v^2)\,v^2}\right)$$

$$x' = f(v^2)(x - v_1 t) = f(v^2)(x'' + v_2 t'')$$
$$y' = y = y''$$
$$z' = z = z''$$
$$t' = f(v^2)\left(t - v_1 x \frac{f(v^2) - 1/f(v^2)}{f(v^2)\,v_1^2}\right) = f(v^2)\left(t'' + v x'' \frac{f(v^2) - 1/f(v^2)}{f(v^2)\,v^2}\right)$$

$$x'' = f(v^2)(x' - v_2 t) = f(v^2)(x - v_3 t)$$
$$y'' = y = y'$$
$$z'' = z = z'$$
$$t'' = f(v^2)\left(t' - v_2 x' \frac{f(v^2) - 1/f(v^2)}{f(v^2)\,v_2^2}\right) = f(v^2)\left(t - v_3 x \frac{f(v^2) - 1/f(v^2)}{f(v^2)\,v^2}\right)$$

$1 = f^2 - v^2 \cdot f h$; $g - f = 1$

$h = \frac{f^2 - 1}{f v^2}$

So, $x' = f(v^2)(x - vt)$; $y' = y$; $z' = z$

$x = f(v^2)(x + vt)$; $y = y'$; $z = z$

$t' = f(v^2)(t - vx \circ \left(\frac{f(v^2) - 1/f}{f(v^2) \cdot v^2}\right) + t$

$t = f(v^2)(t' + vx'\left(\frac{f(v^2) - 1/f}{f(v^2) \cdot v^2}\right) + t$

c) Postulates of Relativity (pg 517-519)

1) Frame of reference are independent to translation

2) Speed of light is finite and independent of motion

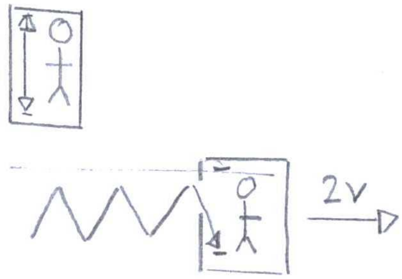
2') In every inertial frame, there is a finite universal speed (c) for physical entities.

$u = dx/dt$

$= \frac{f[dx' + v dt']}{f[dt' + v dx' \frac{f - 1/f}{f v^2}]}$

$= \frac{u' + v}{1 + v \cdot u \left(\frac{f - 1/f}{f v^2}\right)}$ ..if $f = 1/dt$

11.3.



"Two successive Lorentz transformations ... parallel velocity addition law"

Parallel Velocity Addition Formula

(Rapidity): "collinear velocities interpret as rapidity, the sum formula is simple addition" - Minkowski

"Es sei V ein Minkowski-Raum mit der"

"Ein Vektor $v \in V$ mit $\langle v, v \rangle < 0$ heißt zeitartig"

(11.16) "Lorentz transformation"

$$x_0' = \gamma (x_0 - \beta_1 x_1)$$

$$x_1' = \gamma (x_1 - \beta_1 x_0)$$

$$x_2' = x$$

$$x_3' = x_3$$

(11.17) "Symbols, boost, Lorentz shift"

$$\beta = v/c$$

$$\gamma = (1-\beta^2)^{-1/2}$$

Second frame:

$$x_0'' = \gamma_2 (x_0' - \beta_2 x_1')$$

$$x_1'' = \gamma_2 (x_1' - \beta_2 x_0')$$

... where $\gamma_2 = (1-\beta_2^2)^{-1/2}$; $\beta_2 = \frac{v}{c}$

Universal constant:

$$x = f(x' + vt')$$
$$= f(fx - vft) + v(gt - vhx)$$
$$= (f^2 - v^2 fh) + (g - f) f \cdot v \cdot t$$

$$f^2 - v^2 fh = 1 \quad \text{and} \quad 1 = g \cdot f$$

$$h = \frac{f^2 - 1}{fv^2}$$

$$u = \frac{dx}{dt}$$
$$= \frac{f[dx' + v dt']}{f[dt' + v dx' \frac{f - 1/f}{fv^2}]}$$

$$= \frac{u' + v}{1 + v \cdot u \left(\frac{f - 1/f}{fv^2}\right)} \qquad \therefore \text{ if } \quad u = c$$

$$c = \frac{u + v}{1 + v \cdot c \left(\frac{f - 1/f}{fv^2}\right)}$$

$$f = \pm \frac{c}{\sqrt{c^2 - v^2}}$$

$$= \frac{1}{\sqrt{1 - (v/c)^2}}$$

$$h = \frac{f^2 - 1}{fv^2}$$

$$= \frac{\left(\frac{1}{\sqrt{1 - (v/c)^2}}\right)^2 - 1}{\left(\frac{1}{\sqrt{1 - (v/c)^2}}\right) \cdot v^2}$$

$$= \frac{f}{c}$$

$$\frac{h(v^2)}{f(v^2)} = \frac{1}{c}$$

$$x_0'' = \gamma_2\left(\gamma_1(x_0-\beta_1 x_1) - \beta_2\gamma_1(x_1-\beta_1 x_0)\right)$$

$$x_1'' = \gamma_2\left(\gamma_1(x_1-\beta_1 x_0) - \beta_2\gamma_1(x_0-\beta_1 x_1)\right)$$

$$= \gamma_2\gamma_1\left((1+\beta_2\beta_1)x_0 - (\beta_1+\beta_2)x_1\right)$$

$$= \gamma_2\gamma_1\left((1+\beta_2\beta_1)x_1 - (\beta_1+\beta_2)x_0\right)$$

$$= \gamma(x_0 - \beta x_1)$$

$$= \gamma(x_1 - \beta x_0) \quad \therefore \quad \gamma = 1/\sqrt{1-v^2/c^2}$$

$$\beta = v/c$$

$$\boxed{\begin{array}{l}\underline{\text{Identities:}}\\ \gamma_2\gamma_1(1+\beta_2\beta_1) = \gamma \\ \gamma_2\gamma_1(\beta_1+\beta_2) = \beta\gamma\end{array}}$$

$$\gamma = \frac{1}{\sqrt{1-v^2/c^2}}$$

$$= \gamma_1\gamma_2(1+\beta_1\beta_2)$$

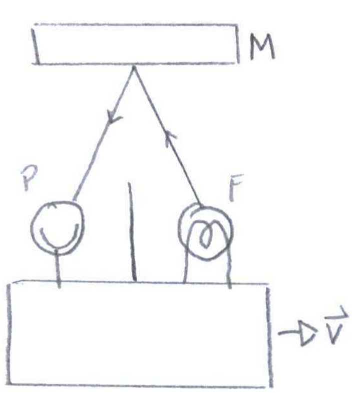
$$= \frac{1}{\sqrt{1-v_1^2/c^2}}\,\frac{1}{\sqrt{1-v_2^2/c^2}}\left(1+\frac{v_1v_2}{c^2}\right)$$

$$v = \sqrt{c^2 - \frac{(1-v_1^2/c^2)(1-v_2^2/c^2)c^2}{\left(1+\frac{v_1v_2}{c^2}\right)}}$$

$$= \sqrt{c^2 - \frac{c^2-v_1^2-v_2^2+v_1^2v_2^2/c^2}{1+v_1^2v_2^2/c^4+2v_1v_2/c^2}}$$

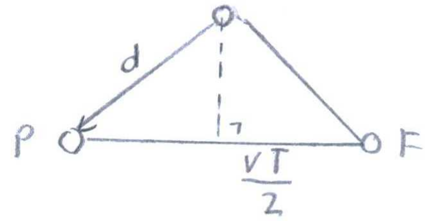
$$= \frac{v_1+v_2}{1+(v_1v_2/c^2)}$$

11.4



"clock, flash tube(F), photocell(P) shielded by a mirror M, a distance away"

a) $tick = 2d/c$



$(\text{Total Distance})^2 = d^2 + \left(\frac{vT}{2}\right)^2$

$D = \sqrt{d^2 + (vT/2)^2}$

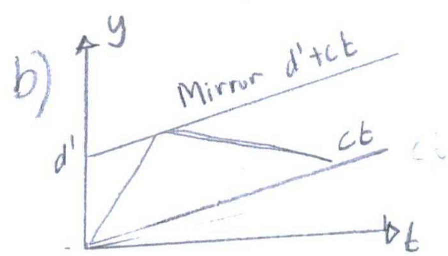
$= \sqrt{(2d)^2 + (vT)^2}$

$= cT$

If $T = \gamma T_0$ When $\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$; $T_0 = \frac{2d}{c}$

$T_0 = \frac{1}{\gamma}\sqrt{(2d)^2 + (vT)^2}$

b)



$y_r = d' + v t_r$

$= c t_r$

$= \frac{d'}{1 - v/c}$

$t' = \frac{d'/c}{1 - v/c}$

$y = y_r - c(t - t_r)$

$= 2y_r - ct$

$= \frac{2d'}{1 - v/c} - ct$

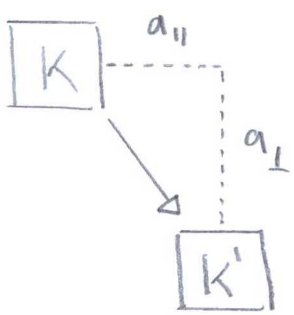
$vT = \frac{2d/c}{1 - v^2/c^2}$

$$= \gamma^2 2d'/c \quad \text{∘∘ when } d' = d/\gamma$$

$$= \gamma \frac{2d}{c}$$

$$= \gamma T_0$$

11.5.



"K' moves ∘∘∘ a velocity and acceleration"

(11.116) "Lorentz Transformation"

$$x_0' = \gamma(x_0 + \beta x_1) \; ; \; y' = y' \; ; \; z = z'$$

$$x_1' = \gamma(x_1 + \beta x_0) \; ; \; y' = y \; ; \; z' = z$$

<u>Frame K:</u>

$$u = c\frac{dx}{dx_0'} \; ; \; a = c\frac{du}{dx_0'}$$

<u>Frame K':</u>

$$u' = c\frac{dx_1}{dx_0'} \; ; \; a' = c\frac{du}{dx_0}$$

$$x_0' = \gamma(x_0 + \beta x_1(x_0))$$

$$\frac{dx_0'}{dx_0} = \gamma(1 + \beta u'x/c)$$

So, $$u_x = c\frac{dx_1}{dx_0'}$$

$$= c\frac{dx_0}{dx_0'}\frac{dx_1}{dx_0}$$

$$= \frac{c}{\gamma(1+\beta u_x'/c)}\frac{d}{dx_0}\gamma(x_1 + \beta x_0)$$

$$= \frac{ux' + c\beta}{1 + \beta ux'/c}$$

$$u_y = c\frac{dy}{dx_0'}$$

$$= c\frac{dx_0}{dx_0'}\frac{dy}{dx_0}$$

$$= \frac{c}{\gamma(1+\beta u_x'/c)} \cdot (u_y'/c)$$

$$= \frac{u_y'}{\gamma(1+\beta u_x/c)} \quad \ldots \beta u_x' = \beta \cdot u$$

$$u_{\parallel} = \frac{u_{\parallel}' + c\beta}{1 + \beta \cdot u'/c} \qquad u_{\perp} = \frac{u_{\perp}}{\gamma(1+\beta u'/c)}$$

$$a_x = c\frac{du_x}{dx_0'}$$

$$= \frac{c}{\gamma(1+\beta u_x/c)} \frac{d}{dx_0} \frac{u_x' + c/\beta}{1+\beta u_x'/c}$$

$$= \frac{c}{\gamma(1+\beta u_x/c)} \frac{(1+\beta u_x/c)(a_x/c) - (u_x + c\beta)(\beta u_x'/c^2)}{(1+\beta u_x/c)^2}$$

$$= \frac{(1-\beta^2)a_x'}{\gamma(1+\beta u_x/c)^3}$$

$$= \frac{a_x'}{\gamma^3(1+\beta u_x'/c)^3}$$

$$= \frac{a_{\parallel}}{\gamma^3(1+\beta u_x'/c)^3}$$

$$a_y = c\frac{du_y}{dx_0}$$

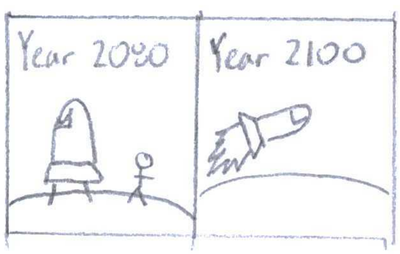
$$= \frac{c}{\gamma(1+\beta u_x'/c)} \frac{d}{dx_0} \frac{u_y}{\gamma(1+\beta u_x'/c)}$$

$$= \frac{c}{\gamma^2(1+\beta u_x'/c)} \frac{(1+\beta u_x'/c)(a_y'/c) - u_y'(\beta a_x'/c^2)}{(1+\beta u_x'/c)^2}$$

$$= \frac{a_y' + \beta(u_x' \cdot a_y' - u_y' a_x')/c}{\gamma^2(1+\beta u_x'/c)^3}$$

$$= \frac{a_\perp + a'(\beta u') - u(\beta \cdot a')/c}{\gamma^2(1+\beta \cdot u'/c)^3}$$

$$= \frac{a_\perp + \beta \times (a' \times u')/c}{\gamma^2(1+\beta \cdot u'/c)^3}$$

11.6. 

a) (11.33) "Velocities"

$$u = \frac{u'+v}{1+\frac{u'v}{c^2}}$$

$$a_{\parallel} = \frac{du}{dt}$$

$$= \frac{\left(1+\frac{u'v}{c^2}\right)\left(\frac{du'}{dt}+\frac{dv}{dt}\right) - (u'+v)\frac{1}{c^2}\left(\frac{du'}{dt}v + u'\frac{dv}{dt}\right)}{\left(1+\frac{u'v}{c^2}\right)^2}$$

$$= \frac{1-v^2/c^2}{\left(1+\frac{u'v}{c^2}\right)^2} \frac{du'}{dt'} \frac{dt'}{dt}$$

$$= \left(1-\frac{v^2}{c^2}\right)^{3/2} \frac{du}{dt'} \quad \ldots \text{when } u'=0 \text{ at } t=0$$

$$v = \frac{u'+v}{1+\frac{u'v}{c^2}}$$

$$a_{\parallel} = \frac{dv}{dt}$$

$$= \frac{d}{dt}\left[\frac{u'+v}{1+u'v/c^2}\right]$$

$$= (1 - v^2/c^2)\frac{dv'}{dt'}$$

$$\int \frac{dv}{(1-v^2/c^2)^{3/2}} = \int \frac{dv'}{dt'} dt$$

$$v = \frac{gt}{\sqrt{1+g^2t^2/c^2}}$$

$$t' = \int \sqrt{1-v^2/c^2}\, dt$$

$$= \int \sqrt{1 - \frac{g^2t^2}{c^2(1+g^2t^2/c^2)}}\, dt$$

$$= \int \left(1+\frac{g^2t^2}{c^2}\right)^{-1/2} dt$$

$$= \frac{c}{g}\operatorname{arcsinh}\left(\frac{gt'}{c}\right)$$

Integral Identity:

$$\int \frac{du}{\sqrt{1+u^2}} = \operatorname{arcsinh}(x) +$$

$$t = \frac{c}{g}\operatorname{arcsinh}\left(\frac{gt'}{c}\right)$$

If $g = 9.86 m/s^2$, $c = 3\times10^8 m/s$, $t' = 5$ years

$$t = \frac{c}{g}\operatorname{arcsinh}\left(\frac{gt'}{c}\right)$$

$= 86$ years

$t_{total} = 4 \cdot t$

$= 244$ years

Year on earth: 2444.

The "muggle" is 344 years older than the astronaut.

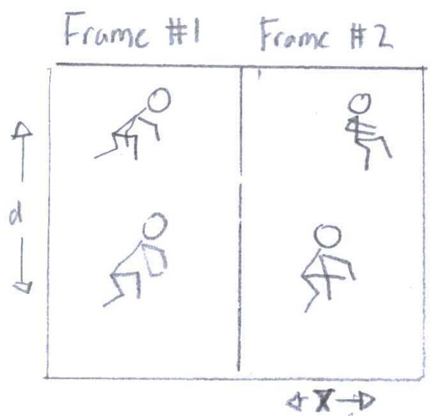
b) $$S = 2\int_0^T v \cdot dt$$

$$= \int \frac{gt}{\sqrt{1+g^2t^2/c^2}}$$

$$= \frac{c^2}{g}\sqrt{1+\frac{g^2t^2}{c^2}}\Bigg|_0^T$$

$$= 148 \text{ light years}$$

11.7.



"sprinters ... lined up ... starting positions slightly different times"

a) $$x' = \left(\sqrt{x_0^2+d^2}\right) - vT$$

$$0 \leq \left(\sqrt{x_0^2+d^2}\right) - vT$$

$$T \leq \frac{\sqrt{x_0^2+d^2}}{v}$$

"No difference between runners"

$$T \geq \frac{\sqrt{x_0^2+d^2}}{v}$$

"Time difference providing handicap"

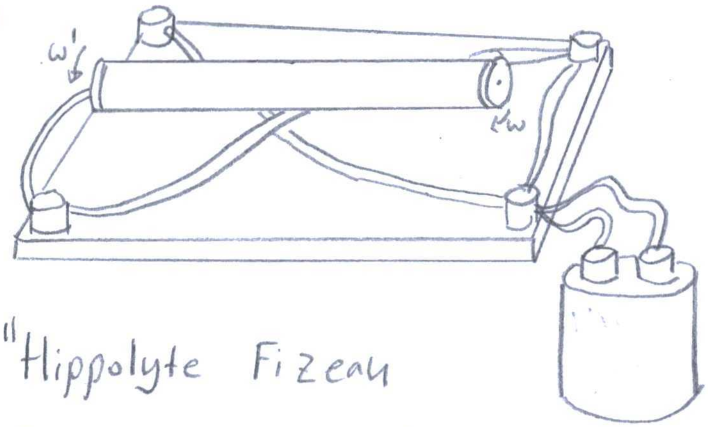
b) $$x' = \gamma(x - vt)$$

$$y' = y$$

$$z' = z$$

$$t' = \gamma\left(\frac{\sqrt{x_0^2+d^2}}{v} - \frac{vx}{c^2}\right)$$

11.8



"Hippolyte Fizeau experiment on velocity of propagation of light in liquid"

a) Velocity : $v_{\parallel} = \frac{dx}{dt}$

$= \frac{\omega}{k}$

$= \frac{c}{n}$

"Hippolyte" experiment, refraction about a rotating frame. He argued, "in a substance the frequency scales by a coefficient, index of refraction. Also, the scale by substance is constant"

$$V_{\parallel} = \frac{V + v_{\parallel}'}{1 + V \cdot v_p' / c^2}$$

$$= \frac{c}{n} \left\{ \frac{1 + \beta \cdot n}{1 + \beta / n} \right\}$$

$$= \frac{c}{n}$$

$$\cong \frac{c}{n(\omega)} + v \left\{ 1 - \frac{1}{n^2(\omega)} \right\}$$

Parameters: $(\omega' - \omega) = -k \cdot v$

$= -\frac{n}{c} \cdot \omega \cdot v$

$= -\beta \omega n$

$$\frac{1}{n(\omega')} = \frac{1}{n(\omega) + (\omega' - \omega) \frac{dn}{d\omega}}$$

$$\cong \frac{1}{n(\omega)} \frac{1 + \beta \omega \, dn/d\omega}{1 + \beta^2 \omega^2 n(\omega) \frac{d^2 n}{d\omega} - 2\beta(\omega) \frac{dn}{d\omega}}$$

$$X''^{\alpha} = (g^{\alpha\beta} + \epsilon'^{\alpha\beta}) X'_{\beta}$$

$$= (g^{\alpha\beta} + \epsilon'^{\alpha\beta}) g_{\beta\gamma} \cdot X'^{\gamma}$$ "Inverse Contraction"

$$= (g^{\alpha\beta} + \epsilon'^{\alpha\beta}) g_{\beta\gamma} (g^{\gamma\delta} + \epsilon^{\gamma\delta}) X_{\delta}$$

$$= (g^{\alpha\beta} + \epsilon'^{\alpha\beta}) g_{\beta\gamma} (g^{\gamma\delta} + \epsilon^{\gamma\delta}) g_{\delta\eta} \cdot X^{\eta}$$

$$= \delta^{\alpha}_{\eta} X^{\eta}$$

So, $\delta^{\alpha}_{\eta} = (g^{\alpha\beta} + \epsilon'^{\alpha\beta}) g_{\beta\gamma} (g^{\gamma\delta} + \epsilon^{\gamma\delta}) g_{\delta\eta}$

$$= (g^{\alpha\beta} \cdot g_{\beta\gamma} + \epsilon'^{\alpha\beta})(g^{\gamma\delta} + \epsilon^{\gamma\delta}) g_{\delta\eta}$$

$$= (g^{\alpha\beta} \cdot g_{\beta\gamma} \cdot g^{\gamma\delta} + g^{\alpha\beta} \cdot g_{\beta\gamma} \cdot \epsilon^{\gamma\delta} + \epsilon'^{\alpha\beta} g_{\beta\gamma} \cdot g^{\gamma\delta} + \epsilon'^{\alpha\beta} \cdot g_{\beta\gamma} \cdot \epsilon^{\gamma\delta}) g_{\delta\eta}$$

$$= (\delta^{\alpha}_{\gamma} (g^{\gamma\delta} + \epsilon^{\gamma\delta}) + \epsilon'^{\alpha\beta} \cdot g_{\beta\gamma} (g^{\gamma\delta} + \epsilon^{\gamma\delta})) g_{\gamma\eta}$$

$$= [g^{\alpha\delta} + \epsilon^{\alpha\delta} + \epsilon'^{\alpha\delta} + \epsilon'^{\alpha\beta} \cdot g_{\beta\gamma} \cdot \epsilon^{\gamma\delta}] g_{\delta\eta}$$

$$= [g^{\alpha\delta} + \epsilon^{\alpha\delta} + \epsilon'^{\alpha\beta}] g_{\delta\eta}$$

$$= \delta^{\alpha}_{\eta} + g_{\delta\eta} \underbrace{(\epsilon^{\alpha\delta} + \epsilon'^{\alpha\delta})}_{\epsilon^{\alpha\delta} = -\epsilon'^{\alpha\delta}}$$

b) Preservation:

$$X^{\alpha} X_{\alpha} = X'^{\alpha} \circ X'_{\alpha}$$

$$= (g^{\alpha\beta} + \epsilon^{\alpha\beta}) X_{\beta} \cdot X'_{\alpha}$$

$$= (g^{\alpha\beta} + \epsilon^{\alpha\beta}) X_{\beta} \cdot g_{\alpha\beta} \cdot X'^{\gamma}$$ "Contraction"

$$\cong \frac{1}{n(\omega)} + \beta\omega \, dn/d\omega$$

$$\frac{1}{n^2(\omega)} \cong \frac{1}{n^2(\omega') + 2(\omega'-\omega)\frac{dn}{d\omega}n^2(\omega)}$$

$$\cong \frac{1}{n^2(\omega)} \frac{1+2\beta\omega \, dn/d\omega}{1-4\beta\omega \, dn/d\omega + 4\beta^2\omega^2 dn^2/d\omega^2}$$

$$\cong \frac{1}{n^2(\omega)}\left\{1+2\beta\omega\frac{dn}{d\omega}\right\}$$

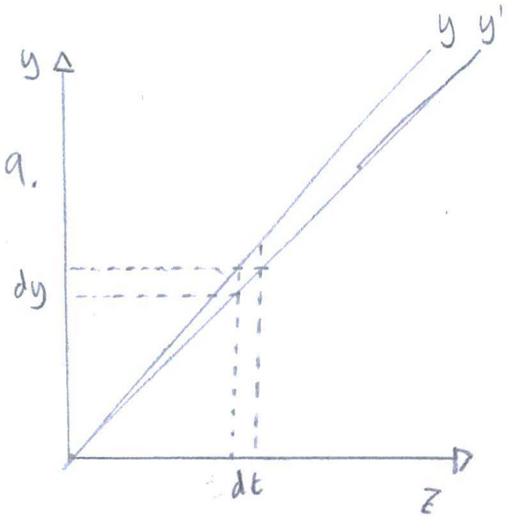
Final velocities:

$$v = \frac{c}{n(\omega)}\left\{1+\beta\frac{dn}{d\omega}\right\} \pm \left\{1 - \frac{1}{n^2(\omega)}\left(1+2\beta\omega\frac{dn}{d\omega}\right)\right\}$$

$$\cong \frac{c}{n(\omega)} \pm v\left\{1-\frac{1}{n(\omega)} + \frac{\omega}{n(\omega)}\frac{dn(\omega)}{d\omega}\right\}$$

b) Cite: W.M. Macek, J.R. Shneider, and R.M. Salamon J. Apple. Phys. 35, 2556(1964).

11.9.



"Infinitesmal Lorentz transformation"

$$\boxed{\begin{aligned} y &= ct \\ y' &= \gamma ct \end{aligned}}$$

a) (11.70) "Flat space-time"

$$g^{\alpha\beta} = g_{\alpha\beta}$$

(11.72) "Contraction"

$$x_\alpha = g_{\alpha\beta} \cdot x_\beta$$

(11.73) "Inverse Contraction"

$$x^\alpha = g^{\alpha\beta} \cdot x_\beta$$

(11.71) "Kronecker Delta"

$$g_{\alpha\gamma} \cdot g^{\gamma\beta} = \delta_\alpha{}^\beta$$

$$= (g^{\alpha\beta} + \epsilon^{\alpha\beta}) x_{\beta} \cdot g_{\alpha\gamma} (g^{\gamma\delta} + \epsilon^{\gamma\delta}) x_{\delta}$$

$$= (g^{\alpha\beta} + \epsilon^{\alpha\beta}) [g_{\alpha\gamma} (g^{\gamma\delta} + \epsilon^{\gamma\delta})] g_{\delta\eta} x_{\beta} x^{\eta}$$

$$= (g^{\alpha\beta} + \epsilon^{\alpha\beta}) (\delta_{\alpha}^{\delta} + g_{\alpha\gamma} \epsilon^{\gamma\delta}) g_{\delta\eta} \circ x_{\beta} \circ x^{\eta}$$

$$= (g^{\alpha\beta} + \epsilon^{\alpha\beta}) (g_{\alpha\eta} + g_{\alpha\gamma} \epsilon^{\gamma\delta} \cdot g_{\delta\eta}) x_{\beta} \cdot x^{\eta}$$

"Kronecker" Delta

$$= (\delta^{\beta}_{\eta} + \delta^{\beta}_{\gamma} \epsilon^{\gamma\delta} \cdot g_{\delta\eta} + \epsilon^{\alpha\beta} \cdot g_{\alpha\eta}) x_{\beta} \cdot x^{\eta}$$

"Inverse contraction"

$$= x_{\beta} x^{\beta} + (\epsilon^{\beta\delta} g_{\delta\eta} + \epsilon^{\alpha\beta} \cdot g_{\alpha\eta}) x_{\beta} \cdot x^{\eta}$$

$$= x_{\alpha} x^{\alpha} + (\epsilon^{\beta\alpha} + \epsilon^{\alpha\beta}) g_{\alpha\eta} \circ x_{\beta} \circ x^{\eta}$$

$$= x_{\alpha} x^{\alpha} + \underbrace{(\epsilon^{\beta\alpha} + \epsilon^{\alpha\beta})}_{-\epsilon^{\beta\alpha} = \epsilon^{\alpha\beta}} x_{\alpha} \cdot x_{\beta}$$

c) $x'^{\alpha} = (g^{\alpha\beta} + \epsilon^{\alpha\beta}) x_{\beta}$

$$= (g^{\alpha\beta} + \epsilon^{\alpha\beta}) g_{\beta\gamma} \cdot x^{\gamma}$$

$$= (\delta^{\alpha}_{\gamma} + \epsilon^{\alpha\beta} \cdot g_{\beta\gamma}) x^{\gamma}$$

(11.93) "Boost, Lorentz Transformation [3x3]

$$L = -\omega \cdot S - \xi \circ K$$

$$A = e^{-\omega \cdot S - \xi \cdot K}$$

(11.90) "General Lorentz"

$$L = \begin{pmatrix} 0 & L_{01} & L_{02} & L_{03} \\ L_{01} & 0 & L_{12} & L_{13} \\ L_{02} & -L_{21} & 0 & L_{23} \\ L_{03} & -L_{13} & -L_{23} & 0 \end{pmatrix}$$

$$= \begin{pmatrix} \cosh\xi & -\sinh\xi & 0 & 0 \\ -\sinh\xi & \cosh\xi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

where ξ = angle.

(11.92) "Spatial Rotation"

$$S_1^2 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$S_2^2 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$S_3^2 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$K_1^2 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$K_2^2 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$K_3^2 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$L = A^{\alpha}{}_{\gamma} x^{\gamma}$$

$$= (\delta^{\alpha}_{\gamma} + L^{\alpha}_{\gamma}) x^{\gamma} \quad \ldots \leftarrow (11.93)$$

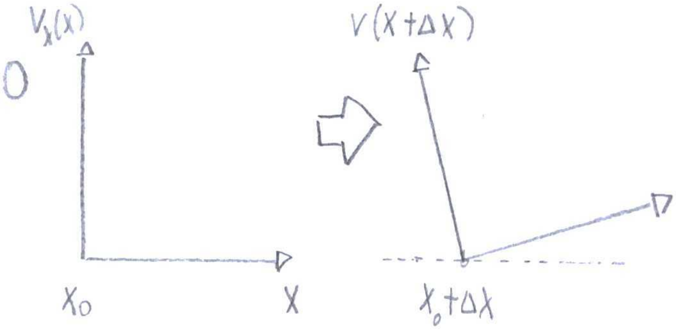
... when $L^{\alpha}{}_{\gamma} = \epsilon^{\alpha\beta} \cdot g_{\beta\gamma}$

$$L^{\alpha}{}_{\gamma} \cdot g^{\gamma\delta} = \epsilon^{\alpha\beta} \cdot g_{\beta\gamma} \cdot g^{\gamma\delta}$$

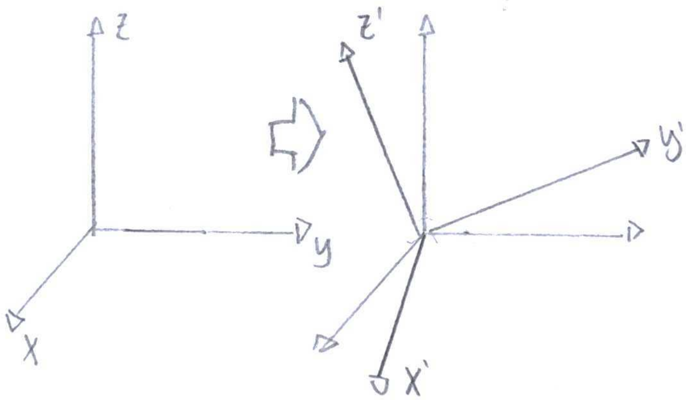
$$= \epsilon^{\alpha\beta} \cdot \delta_{\beta}{}^{\delta}$$

$$= \epsilon^{\alpha\delta}$$

11.10



"Lorentz Boost"



"Lorentz rotation"

a) $(\epsilon \circ S)^3 = -\epsilon \circ S$

$(\epsilon' \circ S)^3 = \epsilon' \circ K$

(11.91) "Six Fundamental Matrices"

... previous problem ....

$$\epsilon \circ S = \epsilon(S_1 + S_2 + S_3)$$

$$= \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -\epsilon_3 & \epsilon_2 \\ 0 & \epsilon_3 & 0 & -\epsilon_1 \\ 0 & -\epsilon_2 & \epsilon_1 & 0 \end{pmatrix}$$

$$(\epsilon \circ S)^2 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -\epsilon_2^2 - \epsilon_3^2 & \epsilon_1\epsilon_2 & \epsilon_1\epsilon_3 \\ 0 & \epsilon_1\epsilon_2 & -\epsilon_1^2 - \epsilon_3^2 & \epsilon_2\epsilon_3 \\ 0 & \epsilon_1\epsilon_3 & \epsilon_2\epsilon_3 & -\epsilon_1^2 - \epsilon_2^2 \end{pmatrix}$$

$$(\epsilon \circ S)^3 = (\epsilon \circ S)(\epsilon \circ S)^2$$

$$= -(\epsilon_1^2 + \epsilon_2^2 + \epsilon_3^2)\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -\epsilon_3 & \epsilon_2 \\ 0 & \epsilon_3 & 0 & -\epsilon_1 \\ 0 & -\epsilon_2 & \epsilon_1 & 0 \end{pmatrix}$$

$$= -\epsilon \circ S \;\ldots \text{when } \epsilon_1^2 + \epsilon_2^2 + \epsilon_3^2 = 1$$

$$\epsilon' \circ K = \epsilon \cdot (K_1 + K_2 + K_3)$$

$$= \epsilon \cdot \begin{pmatrix} 0 & \epsilon_1' & \epsilon_2' & \epsilon_3' \\ \epsilon_1' & 0 & 0 & 0 \\ \epsilon_2' & 0 & 0 & 0 \\ \epsilon_3' & 0 & 0 & 0 \end{pmatrix}$$

$$(\epsilon' \circ K)^2 = \begin{pmatrix} \epsilon_1^2 + \epsilon_2^2 + \epsilon_3^2 & 0 & 0 & 0 \\ 0 & \epsilon_1^2 & \epsilon_1\epsilon_2 & \epsilon_1\epsilon_3 \\ 0 & \epsilon_1\epsilon_2 & \epsilon_2^2 & \epsilon_2'\epsilon_3 \\ 0 & \epsilon_1\epsilon_3 & \epsilon_2'\epsilon_3 & \epsilon_3^2 \end{pmatrix}$$

$$(\epsilon' \circ K^3) = (\epsilon' \circ K)(\epsilon' \circ K)^2$$

$$= (\epsilon_1^2 + \epsilon_2^2 + \epsilon_3^2)\begin{pmatrix} 0 & \epsilon_1' & \epsilon_2' & \epsilon_3' \\ \epsilon_1' & 0 & 0 & 0 \\ \epsilon_2' & 0 & 0 & 0 \\ \epsilon_3' & 0 & 0 & 0 \end{pmatrix}$$

b) $$\exp(\xi \circ \beta \circ K) = \sum_{n=0}^{\infty} \frac{(-\xi)^n}{n!}(\beta \circ K)^n$$

$$= 1 - \beta \circ K \sum_{n=\text{odd}} \frac{\xi^n}{n!} + (\beta \circ K)^2 \sum_{n=\text{even}} \frac{\xi^n}{n!}$$

$$= 1 - \beta\circ K \sum_{n=0} \frac{\xi^{2n+1}}{(2n+1)!} + (\beta\circ K)^2 \sum_{n\geq 2} \frac{\xi^{2n}}{(2n)!}$$

$$= 1 - \beta\circ K \circ \sinh\xi + (\beta\circ K)^2(\cosh\xi - 1)$$

11.11

$$A_1 = e^{\lambda L} \qquad A_2 = e^{\lambda(L+\delta L)}$$

$$A(\lambda) = A_2 \circ A_1^{-1}$$
$$= e^{\lambda(L+\delta L)} \circ e^{-\lambda L}$$
$$= \sum_{n=0}^{\infty} \frac{\lambda}{n!} \Omega_n(L, \delta L) \quad \circ\circ\circ \text{where } \Omega_0 = 1$$

$$\Omega_1 = (L, \delta L) = \delta L$$
$$\Omega_n = (L, \delta L)$$
$$= [L, \Omega_{n-1}(L, \delta L)]$$

$$= \sum_{n=0}^{\infty} \frac{\lambda^n}{n!} A^{(n)}(0) \quad \circ\circ\circ \text{when } A^{(0)}(0) = \Omega_0$$

$$A^{(1)}(0) = \Omega_1(L, \delta L)$$
$$A^{(n)}(0) = \Omega_n(A, \beta)$$
$$= [L, \Omega_{n-1}(L, \delta L)]$$
$$= [L, A^{(n-1)}(0)]$$
$$= [L, \Omega_n(L, \delta L)$$
$$= \Omega_{n+1}(L, \delta L)$$

$$A(\lambda) = \frac{\lambda^0}{0!} A^{(0)}(0) + \frac{\lambda^1}{1!} A^{(1)}(0) + \frac{\lambda^2}{2!} A^{(2)}(0) + \frac{\lambda^3}{3!} A^{(3)}(0)$$

$$= 1 + \delta L + \frac{L + \delta L}{2!} + \frac{L + L + \delta L}{3!} \text{ tow}$$

11.12 Citation: "Shelupsky, David. Derivation of the Thomas Precession Formula' (1966)"

Thomas Precession: a relationship between relativity and spin

$L = \frac{\beta \circ K (\tanh^{-1}\beta)}{\beta}$ $L + \delta L = \frac{-(\beta + \delta\beta_1 + \delta\beta_2)(K \tanh^{-1}\beta)}{\beta'}$

...where $\beta' = \sqrt{(\beta + \delta\beta_1)^2 + (\delta\beta_\perp)^2}$

$\delta L = -\gamma^2 \delta\beta \circ K - \frac{\delta\beta_\perp \circ K \tanh^{-1}\beta}{\beta}$

(11.116) "Spin interaction Energy'

$U' = -\frac{ge}{2mc} s \circ B + \frac{g}{2m^2c^2}(s \circ L)\frac{1}{r}\frac{dv}{dr}$

..when-that-so-be-it:

$L = m(r \times v)$

$L + \delta L = \frac{\beta \circ k (\tanh^{-1}\beta)}{\beta} + \delta L$

$\delta L = \frac{\beta \circ k (\tanh^{-1}\beta)}{\beta} - \frac{(\beta + \delta\beta_{\parallel} + \delta\beta_\perp) \circ K (\tanh^{-1}\beta)}{\beta'}$

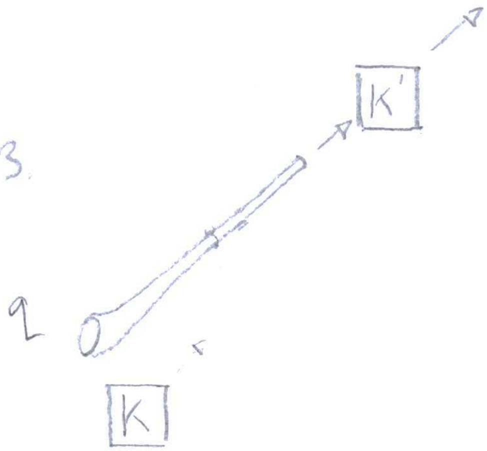
$= -\frac{\delta\beta_\perp \cdot k \tanh^{-1}(\beta)}{\beta'} - \frac{\delta\beta_\perp \cdot k (\tanh^{-1}\beta)}{\beta'}$

...when $\beta \simeq \beta' \gg \beta_{\parallel} \wedge \beta_\perp$

$$\cong I - (\gamma^2 \delta \beta_1) - \left(\frac{\tanh^{-1}\beta}{\beta}\right)^2 (\beta \times \delta \beta_\perp) \cdot S$$

$$\cong I - (\gamma^2 \delta \beta_1) \cdot K - \frac{\gamma^2}{\gamma+1} (\beta \times \delta \beta) \cdot S$$

11.13.



"an infinitely long straight wire... has charge... then moves parallel to rest frame"

a) (11.149) "Boosted Electric and Magnetic fields"

$$E = \gamma(E' - \beta \times B) - \frac{\gamma^2}{\gamma+1} \beta(\beta \cdot E)$$

$$B = \gamma(B' + \beta \times E) - \frac{\gamma^2}{\gamma+1} \beta(\beta \cdot B)$$

$$E = \gamma E' = \frac{2\gamma q_0}{\rho'} \hat{\rho}$$

$$B = \gamma \beta \times E = \frac{2v\gamma q_0}{c\rho'} \hat{\phi}$$

"No contraction in transverse direction"

$$E = \frac{2\gamma q_0}{\rho} \hat{\rho} \qquad \ldots \text{when } \rho' = \rho$$

$$B = \frac{2v\gamma q_0}{c\rho} \hat{\phi}$$

"charge density"

$$\rho = \frac{q_0 \delta(\rho)}{2\pi\rho'}$$

(11.128) "Current densities"

$$J'^\mu = \left(c q_0 \frac{\delta(\rho')}{2\pi\rho'}, 0\right)$$

b) $C_1 = [L, \delta L]$

$= L \circ \delta L$

$= \delta L \circ L$

$$= \frac{\beta \circ K(\tanh^{-1}\beta)}{\beta}\left[-\gamma^2 \delta\beta_\| K - \frac{\delta\beta_\perp \cdot K(\tanh^{-1}(\beta))}{\beta}\right]$$

$$= \beta \circ \frac{\beta K \tanh^{-1}(\beta)}{\beta}$$

$$= \left(\frac{\tanh^{-1}\beta}{\beta}\right)^2 (\beta \times \delta\beta_\perp) \circ S$$

$C_2 = [L_1, C_1]$

$= L \circ C_1 - C_1 \circ L$

$= (\tanh^{-1}\beta)^2 \delta L$

$C_3 = [L, C_2]$

$= L \circ C_2 - C_2 - L$

$= (\tanh^{-1}\beta)^2 C_1$

$C_4 = [L_1, C_3]$

$= (\tanh^{-1}\beta)^4 \delta L_\perp \quad \ldots$ when $\delta L_\perp = \delta L$

c) $A_T = A_2 A_1$

$$= I + \delta L + \frac{1}{2!}[L, \delta L] + \frac{1}{3!}[L,[L, \delta L]] + \ldots$$

$$= I - \gamma^2 \delta\beta_\| K - \frac{\delta\beta_\perp \cdot K \tanh^{-1}\beta}{\beta}$$

$$- \frac{1}{2!}\left(-\left(\frac{\tanh^{-1}\beta}{\beta}\right)^2\right)(\beta + \delta\beta_\perp) \circ S$$

Charge Density boosted;

$$\rho = \gamma q_0 \frac{\delta(\rho)}{2\pi\rho}$$

Current Density boosted;

$$J^{\mu} = (\gamma J, \beta\gamma J'^{0})$$

$$\doteq \left(c\gamma q_0 \frac{\delta(\rho)}{2\pi\rho}, v\gamma q_0 \frac{\delta(\rho)}{2\pi\rho}\right)$$

$$J = v\gamma q_0 \frac{\delta(\rho)}{2\pi\rho}$$

c) If $I = v\gamma q_0$,

(1.1b) "Gauss' Law"

$$\nabla \cdot E = \rho/\epsilon_0$$

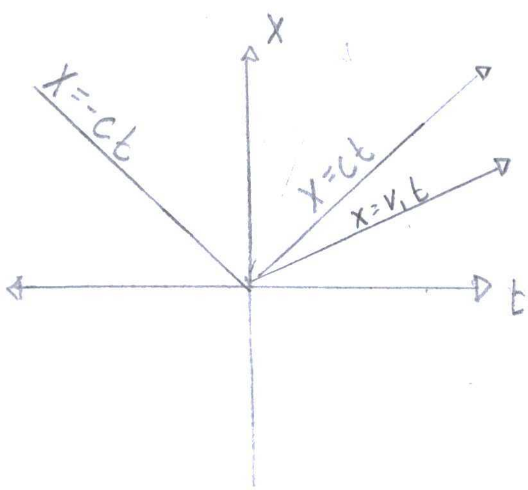
$$= \frac{2\gamma q_0}{\rho}\hat{\rho}$$

(1.1b) "Ampere's Law"

$$\nabla \times B = \mu_0 J + \frac{1}{c^2}\frac{\partial E}{\partial t}$$

$$B = \frac{2v\gamma q_0}{c\rho}\hat{\phi}$$

11.14



"Lorentz scalars" in the Hermann Minkowski spacetime.

(11.137) "Field-strength tensor"

$$F_{\alpha\beta} = \begin{pmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & -B_z & B_y \\ -E_y & B_z & 0 & -B_x \\ -E_z & -B_y & B_x & 0 \end{pmatrix}$$

$$F_{\alpha\beta} = \begin{pmatrix} 0 & B_x & B_y & B_z \\ -B_x & 0 & E_z & -E_y \\ -B_y & -E_z & 0 & E_x \\ -B_z & E_z & -E_x & 0 \end{pmatrix}$$

(11.137) "Index raised field-strength tensor"

$$F^{\alpha\beta} = \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -B_z & B_y \\ E_y & B_z & 0 & -B_x \\ E_z & -B_y & B_x & 0 \end{pmatrix}$$

$$F_{\alpha\beta} = \begin{pmatrix} 0 & -B_x & -B_y & -B_z \\ B_x & 0 & E_z & -E_y \\ B_y & -E_z & 0 & E_x \\ B_z & E_y & -E_x & 0 \end{pmatrix}$$

$$F^{\alpha\beta} \cdot F_{\alpha\beta} = -F^{\alpha\beta} \cdot F_{\alpha\beta}$$

$$= -2(E^2 - B^2)$$

$$= -4 E \cdot B$$ "unsure about invariants"

b)

$$E' = \gamma(E + \beta \times B) - \frac{\gamma^2}{\gamma+1}\beta(\beta \circ E)$$

$$B' = \gamma(B - \beta \times E) - \frac{\gamma^2}{\gamma+1}\beta(\beta \circ B)$$

If $E \neq 0$ and $B = 0$, then,

$$E' = \gamma E - \frac{\gamma^2}{\gamma+1}\beta(\beta \circ E)$$

$$B' = -\gamma \beta \times E$$

When $E' = 0$, then,

$$E = \frac{\gamma}{\gamma+1}\beta(\beta \circ E)$$

$$= \frac{\gamma}{\gamma+1}\frac{v^2}{c^2}E$$

$$v = c$$

The magnetic field becomes the electric field at light speed, generally, not possible by ordinary objects.

If $E' = 0$, no electric field in moving frame:

$$E' = \gamma(E + \beta \times B) - \frac{\gamma^2}{\gamma+1}\beta(\beta \circ E)$$

$$= 0$$

$$\beta \times B = \frac{\gamma}{\gamma+1}\beta(\beta \circ E)$$

c)

$$G^{\alpha\beta} = \begin{bmatrix} 0 & -D_x & -D_y & -D_z \\ D_x & 0 & -H_z & H_y \\ D_y & H_z & 0 & -H_x \\ D_z & -H_y & H_x & 0 \end{bmatrix}$$

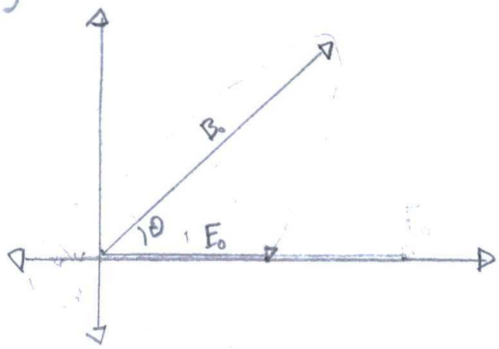
$$G^{\alpha\beta}G_{\alpha\beta} = \pm 2(|H|^2 - |D|^2)$$

$$= -4 \cdot DH$$

$$F^{\alpha\beta} \cdot G_{\alpha\beta} = \pm(B \cdot H - ED)$$

$$= \pm(D \cdot D + EH)$$

11.15



"Static, uniform, electric field, ... static, uniform, magnetic field ... angle theta"

$B_0 = 2E_0$

$E_1' = \gamma(E_0 - \beta B_2)$ $\qquad B_1 = \gamma(B_1 + \gamma E_2)$

$E_2' = \gamma(E_2 - \beta B_1)$ $\qquad B_2 = \gamma(B_2 + \gamma E_1)$

$= \gamma\beta B_1$

$E_3' = E_3 = 0$

If $E \times B = 0$, then $E_1' B_2' = E_2 B_1$,

$(E_0 - \beta B_2)(B_2 - \beta E_0) = \beta B_1$

$2\beta^2 \sin\theta - 5\beta + 2\sin\theta = 0$

$\beta^2 - \frac{5}{2\sin\theta}\beta + 1 = 0$

$\beta^2 - \frac{5}{2\sin\theta}\beta + \frac{25}{16\sin^2\theta} = \frac{25}{16\sin^2\theta} - 1$

$\left(\beta - \frac{5}{4\sin\theta}\right)^2 = \frac{25}{16\sin^2\theta} - 1$

$\beta = \frac{5}{4\sin\theta} \pm \sqrt{\frac{25}{16\sin^2\theta} - \frac{16\sin^2\theta}{16\sin^2\theta}}$

$= \frac{5}{4\sin\theta} \pm \sqrt{\frac{(5-4\sin\theta)(5+4\sin\theta)}{16\sin^2\theta}}$

$= \frac{5 \pm \sqrt{25 - 16\sin^2\theta}}{4\sin\theta}$

If θ approaches zero,

$\lim_{\theta\to 0} E_x' = \lim_{\theta\to 0}\left(E_0 - \beta B_2\right)$

$= \lim_{\theta\to 0}\left(E_0 - \frac{5 \pm \sqrt{25 - 16\sin^2\theta}}{4\sin\theta} \cdot B_2\right)$

$= E_0\left(1 - \frac{18}{25}\theta^2\right)$

$\lim_{\theta\to 0} B_x' = \lim_{\theta\to 0}\left(B_2 - \beta E_0\right)$

$= 2(1-21/50\cdot\theta^2)E_0$

$E_y \approx \frac{4}{3}\theta E_0$ $\qquad$ $B_y' = \frac{8}{3}\theta E_0$

$B'-E^2 = 3E_0^2$ $\qquad$ $B'\cdot E' = 2E_0\cos\theta$

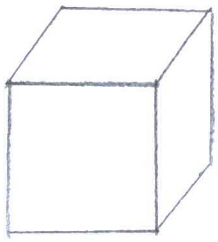
If $\theta \to \pi/2$, then $\sin\theta \to 1$

$E_x = 0$ $\qquad$ $B_x = 0$

$E_y = 0$ $\qquad$ $B_y = \sqrt{3}E_0$

$B^2 - E^2 = 3E_0^2$ $\qquad$ $B_0 \cdot E = 0$

11.16



"Conducting medium at rest"

a) Convection current and conduction currents described in medium, but not for heat, rather electrical current.

Rest frame: $J' = \sigma E$

(11.36) "4-velocity of 11.36"

$U'^\mu = (c, 0)$

$F^{\mu\nu} \cdot U_\nu' = (0, cE')$

$= (0, \sigma E)$

$= \frac{\sigma}{c} F'^{\mu\nu} \cdot U_\nu'$

$(0, J) = \frac{\sigma}{c} F'^{\mu\nu} \cdot U_\nu'$ ∴ if $J'^\mu \cdot U_\nu' = c^2\rho$

$J^\mu - \frac{1}{c^2}(J^\nu \cdot U_\nu)U^\mu = \frac{\sigma}{c} F^{\mu\nu} \cdot U_\nu$

$$J^\mu = \underbrace{\frac{\sigma}{c} F^{\mu\nu} \cdot U_\nu}_{\text{"conduction current"}} + \underbrace{\frac{1}{c^2}(J^\nu \cdot U_\nu)U^\mu}_{\text{"convection current"}}$$

b) $v = c\beta$

Lab Frame: $U^\mu = (\gamma c, \gamma v)$; $J^\mu = (\rho c, J)$

$$J^\mu - \frac{1}{c^2}(J^\nu U_\nu)U^\mu = \frac{\sigma}{c} F^{\mu\nu} \cdot U_\nu$$

$$\frac{1}{c^2} J^\nu \cdot U_\nu = -\frac{\sigma}{c} F^{\mu\nu} \cdot U_\nu + J^\mu$$

$$= J^\mu U_\mu - \frac{\sigma}{c} F^{\mu\nu} \cdot U_\nu \cdot U_\mu$$

$$= \gamma \rho - \frac{\gamma}{c^2} v \cdot J$$

$$= \gamma\left(\rho - \frac{1}{c^2} v \cdot J\right)$$

Time-component Equation:

$$\frac{\gamma}{c^2} J^\nu \cdot U_\nu = \gamma^2\left(\rho - \frac{1}{c^2} v \cdot J\right)$$

Space component Equation:

$$\gamma \sigma E \cdot v = c^2 \gamma^2 \rho - v J \gamma^2$$

$$J - \gamma^2 \rho v + \frac{\gamma^2}{c^2} v (v \cdot J) = \sigma \gamma \left(E + \frac{1}{c} v \times B\right)$$

$$v \cdot J = \frac{\sigma \gamma v \cdot E + v^2 \gamma^2 \rho}{\gamma^2}$$

$$J = \gamma^2 \rho v + \frac{\gamma^2}{c^2} v \left(\frac{\sigma \gamma v \cdot E + v^2 \gamma^2 \rho}{\gamma^2}\right)$$

$$+ \sigma \gamma \left(E + \frac{v \times B}{c}\right)$$

$$= \sigma \gamma \left[E + \frac{v}{\beta} \times B - \frac{v}{\beta}\left(\frac{v}{\beta} \cdot E\right)\right] + \frac{\gamma^2 \rho v^2}{c^2} + \gamma^2 \rho v$$

$$= \sigma \gamma \left[E + \beta \times B - \beta(\beta \cdot E)\right] + \rho v$$

c) If $\rho' = 0$, then $J^\nu U_\nu = 0$

and $J^\mu = \frac{\sigma}{c} F^{\mu\nu} U_\nu$

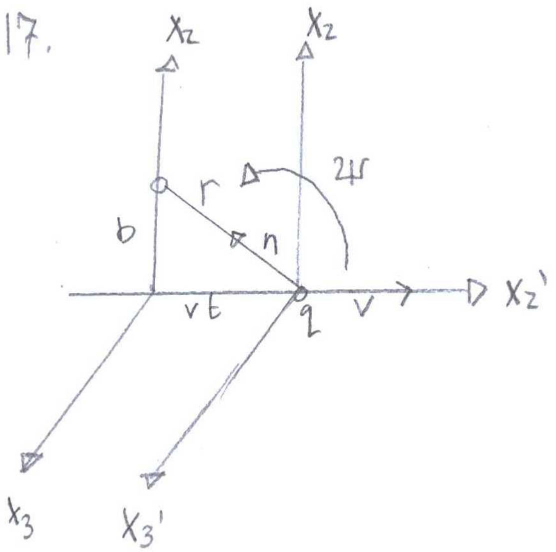
$$\mathbf{J} = \sigma\gamma\left[\mathbf{E} + \beta \times \mathbf{B} - \beta(\beta\cdot\mathbf{E})\right] + \rho v$$

$$= \sigma\gamma\left[\mathbf{E} + \beta\times\mathbf{B}\right] - \gamma\beta(\beta\cdot\mathbf{E}) + \rho v$$

$$\rho' = 0$$

$$\rho = \frac{\sigma\gamma}{c}(\beta\cdot\mathbf{E})$$

11.17.



"Figure 11.8"

a) $X_\rho^\circ = (ct, b)$

$X_q^\alpha = (ct, vt)$

$b\cdot v = 0$

(pg 563) "Electromagnetic field tensor"

$F^{\alpha\beta} = k(X^\alpha U^\beta - X^\beta U^\alpha)$

Rest frame:

$U^\mu = (c, 0, 0, 0)$

$X^\mu = (ct, x, y, z)$

$E^i = F^{0i} c$

$F^{0i} = k(X^0 U^i - X^i U^0)$

$= k(ct \cdot 0 - x^i c)$

$= -kc x^i$

$E^i = -kc^2 x^i$

Coulombs Law: $E^i = \frac{q}{4\pi\epsilon_0 r^3} x^i$

$$= \frac{q}{4\pi\epsilon_0 r^3} X_i$$

$$k = \frac{-q}{4\pi\epsilon_0 R^3}$$

$$r^2 \sqrt{-Y^\mu Y_\mu} \qquad R^2 = (Y^\mu U_\mu)^2 + c^2 Y^\mu Y_\mu$$

$$F^{\alpha\beta} = \frac{q}{c} \frac{(Y^\alpha U^\beta - Y^\beta \cdot U^\alpha)}{(Y_\alpha \cdot Y^\alpha)}$$

c) $X_e^\alpha = (ct, b)$

$$X_q^\alpha = [ct ; R^i ; \beta(ct \cdot R)]$$

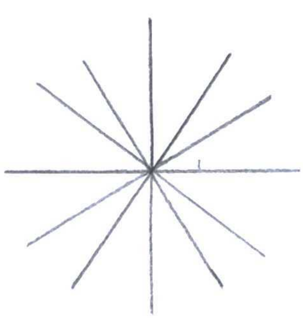
$$Z^\alpha = [R, b - \beta(ct \cdot R)]$$

$$r^2 = \sqrt{U_\alpha U^\alpha} \; ; \; R^2 = (U_\alpha Z^\alpha)^2 + c\, U_\alpha U^\alpha$$

$$F^{\alpha\beta} = \frac{q}{c} \frac{(U_\alpha Z^\alpha - Z^\alpha U^\alpha)}{(U_\alpha Z^\alpha)}$$

11.18

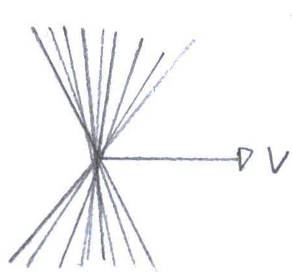
(11.152) "transformed fields"



$$E_1 = E_1' = -\frac{qvt}{(b^2 + \gamma^2 v^2 t^2)^{3/2}}$$

$$E_2 = \gamma E_2' = \frac{\gamma q b}{(b^2 + \gamma^2 v^2 t^2)^{3/2}}$$

$$B_3 = \gamma \beta E_2' = \beta E_2$$



"Figure 11.9"

Citation: R. Jackiw, D. Kabut
M. Ortiz, Phys. letters. B
277, 143. (1992)

"Star Trek warp drive"

$$= -kc^2 x^i$$

$$r = \sqrt{-X^\alpha X_\alpha} \; ; \; r^2 = -X^\alpha X_\alpha$$

General Frame:

$$R^2 = (X^\mu \cdot U_\mu)^2 + c^2 X^\mu X_\mu$$

$$F^{\alpha\beta} = \frac{-q}{4\pi\epsilon_0 c^2 R^3}(X^\alpha U^\beta - X^\beta \cdot U^\alpha)$$

$$= \frac{-q}{4\pi\epsilon_0 c^2 R^3} \frac{X^\alpha U^\beta - X^\beta \cdot U^\alpha}{\left[\frac{1}{c^2}(U_\alpha X^\alpha)^2 - X_\alpha X^\alpha\right]^{3/2}}$$

$$\simeq \frac{q}{c} \frac{X^\alpha U^\beta - X^\beta \cdot U^\alpha}{\left[\frac{1}{c^2}(U_\alpha X^\alpha)^2 - X_\alpha X^\alpha\right]^{3/2}}$$

(11.152) "System transformed"

$$E_1 = E_1' = -\frac{q\gamma vt}{(b^2+\gamma^2 v^2 t^2)^{3/2}}$$

$$E_2 = \gamma E_2' = \frac{\gamma q b}{(b^2+\gamma^2 v^2 t^2)^{3/2}}$$

$$B_3 = \gamma\beta E_2'$$
$$= \beta E_2$$

b) $X_P'^\alpha = (ct', b \cdot vt)$

$$X_q'^\alpha = (ct, 0)$$

If $Y'^\alpha = X_P'^\alpha - X_q'^\alpha$, then

$$F^{\mu\nu} = k(X^\mu \cdot U^\nu - X^\nu U^\mu)$$

$$E^i = -kc^2 \cdot x^i$$

c) (11.135) "Potentials"

$$E = -\frac{1}{c}\frac{\partial A_\gamma}{\partial t} - \frac{\partial \phi}{\partial \gamma} \qquad B_x = \frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}$$

$$A^0 = A^z$$
$$= -2q\delta(ct-z)\ln(\lambda r_\perp) \qquad A_\perp = 0$$
$$= 0$$

$$A_\perp = -2q\Theta(ct-z)\nabla_\perp \ln(\lambda r_\perp)$$

$$E = -\frac{1}{c}\frac{\partial A_\gamma}{\partial t} - \frac{\partial \phi}{\partial \gamma}$$
$$= 2q\nabla[\delta(z-ct)\ln(\lambda r_\perp)] + \frac{2q}{c} z \frac{\partial}{\partial t}[\delta(z-ct)\ln(\lambda r_\perp)]$$
$$- 2q(z-ct)\nabla_\perp \ln(\lambda r_\perp) + 2qz\delta(z-ct)\ln(\lambda r_\perp)$$
$$- 2qz'\delta(z-ct)\ln(\lambda r_\perp)$$
$$= 2q\delta(z-ct)\, r_\perp / r_\perp^2$$

$$B = \nabla \times A$$
$$= -2q\nabla \times [\hat{z}\delta(z-ct)\ln(r_\perp)]$$
$$= -2q\frac{r_\perp}{r_\perp^2} \times \hat{z}\delta(z-ct)$$
$$= 2q\frac{v \times r_\perp}{r_\perp^2}\delta(z-ct)$$

$$A_\perp = -2q\Theta(ct-z)\nabla_\perp \ln(\lambda r_\perp)$$
$$= -2q\Theta(ct-z)\, r_\perp / r_\perp^2$$

$$E = -\frac{1}{c}\frac{\partial A}{\partial t}$$
$$= 2q\delta(ct-z)\frac{r_\perp}{r_\perp^2}$$

$$B = \nabla \times A \; \cdots$$

$$= 2q\,\delta(ct-z)\,\hat{z}\times r_\perp / r_\perp^2$$

$$= -2q\,\theta(ct-z)\,\nabla\times(r_\perp/r_\perp^2)$$

$$= 2q\,\frac{v\times r}{r_\perp^2}\,\delta(ct-z)$$

<u>Potential Difference:</u>

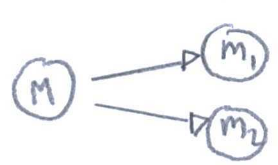
$$A^\mu_{(1)} - A^\mu_{(2)} = \partial^\mu X$$

$$= 2q\left(-\delta(ct-z)\ln(\lambda r_\perp),\ \theta(ct-z)\nabla_\perp \ln(r_\perp\right.$$
$$-\delta(ct-z)\ln(\lambda r_\perp)$$

$$= 2q\left(-\frac{\partial}{\partial ct}\theta(ct-z)\ln(\lambda r_\perp),\right.$$
$$\theta(ct-z)\nabla_\perp \ln(\lambda r_\perp)$$
$$\left.+\hat{z}\,\frac{\partial}{\partial z}\theta(ct-z)\ln(\lambda r_\perp)\right]$$

$$= 2q\left(\frac{\partial}{\partial x_0}, -\nabla\right)\theta(ct-z)\ln(\lambda r_\perp)$$

11.19



"a particle... decays into two particles"

a) $E_1^2 = m_1^2 + m_2^2$

$$E_2^2 = (E_{TOT} - E_1)^2$$
$$= E_{TOT}^2 - 2E_{TOT}\cdot E_1 + E_1^2$$
$$= m_2^2 + p_2^2$$

$$E_1^2 = (E_{TOT} - E_2)^2$$
$$= E_{TOT}^2 - 2E_{TOT}E_2 + E_2^2$$
$$= E_{TOT}^2 - 2E_{TOT}E_2 + m_2 + p_2^2$$
$$= m_1^2 + p_1^2$$

If $m_1 = m_2$, then $p_1 = p_2$

$$= 2q \frac{r_\perp}{r_\perp^2} \delta(z-ct)$$

$$B = \beta \times E$$

$$= 2q \frac{v \times r_\perp}{r_\perp^2} \delta(z-ct)$$

b) (11.141) "4-current"

$$\partial_\alpha F^{\alpha\beta} = \frac{4\pi}{c} J^\beta$$

$$\frac{4\pi}{c} J^0 = \nabla \cdot E$$

$$= 2q \nabla_\perp \cdot \left(\frac{r_\perp}{r_\perp^2} \right) \delta(z-ct)$$

$$\nabla_\perp \cdot \left(\frac{r_\perp}{r_\perp^2} \right) = 2\pi \delta^{(2)}(r_\perp)$$

$$\frac{4\pi}{c} J = -\frac{1}{c} \frac{\partial E}{\partial t} + \nabla \times B$$

$$= 2q \frac{r_\perp}{r_\perp^2} \delta'(z-ct) + 2q \nabla \times \left[z \times \left(\frac{r_\perp}{r_\perp^2} \right) \delta(z-ct) \right]$$

$$= 2q \frac{r_\perp}{r_\perp^2} \delta'(z-ct) + 2q \hat{z} \times \left[z \times \left(\frac{r_\perp}{r_\perp^2} \right) \right] \delta(z-ct)$$

$$+ 2q \left[z \nabla \cdot \left(\frac{r_\perp}{r_\perp^2} \right) - (z \cdot \nabla) \left(\frac{r_\perp}{r_\perp^2} \right) \right] \delta(z-ct)$$

$$= 4\pi q z \delta^{(2)}(r_\perp) \delta(z-ct)$$

$$J = cqv\delta^{(2)}(r_\perp) \cdot \delta(z-ct)$$

$$J^\mu = cq(1, v) \delta^{(2)}(r_\perp) \delta(z-ct)$$

$$= qcv^\alpha \delta^{(2)}(r_\perp) \delta(ct-z)$$

$\circ\circ\circ$ when $v^\alpha = (1, v)$

a) Observation Point: (x, y, z)

Transverse Vector: $r_\perp$

Rest Frame: $E = \frac{qr}{r^3}$ $\quad B = 0$

Lab Frame: $E = \gamma E - \frac{\gamma^2}{\gamma+1} \beta - \beta \cdot E$

Transformation: $x' = x$

$y' = y$

$z' = \gamma(z - vt)$

$r = x + y + z$

$= r_\perp + \gamma(z - vt)\, z$

So, $E' = q \frac{(r_\perp + \gamma(z-vt) z)}{(r_\perp^2 + \gamma^2 (z-vt)^2))^{3/2}}$

$$\lim_{\gamma \to \infty} \gamma E' = \lim_{\gamma \to \infty} \left[\gamma q \frac{(r_\perp + \gamma(z-vt) z)}{(r_\perp^2 + \gamma^2 (z-vt)^2))^{3/2}} \right]$$

$$= \frac{1}{r_\perp} \cdot 2\delta(z - vt)$$

$$E' = \frac{2q(r_\perp + (z-vt)\hat{z})}{r_\perp^2} \delta(z - vt)$$

$$= \frac{2q r_\perp}{r_\perp^2} \delta(z - vt)$$

$$\lim_{v \to c} E' = \lim_{v \to c} \left[q \frac{(r_\perp + \gamma(z-vt) z)}{(r_\perp^2 + \gamma^2 (z-vt)^2)^{3/2}} \right]$$

$$E_2 = \frac{E_{TOT}^2 + m_2^2 - m_1^2}{2E_{TOT}} =$$

$$E_1 = \frac{E_{TOT}^2 + m_1^2 + m_2^2}{2E_{TOT}}$$

Note: Book prefers M_{TOT} rather than E_{TOT}. The math is similar.

b) $$\Delta M\left(1 - \frac{m_i}{M} - \frac{\Delta M}{2M}\right) = (M - m_1 - m_2)\left(1 - \frac{m_i}{M} - \frac{M - m_1 - m_2}{2M}\right)$$

$$= (M - m_1 - m_2)\left(\frac{2M - 2m_i - M - m_1 - m_2}{2M}\right)$$

$$= \frac{(M - m_1 - m_2)(M - m_i + m_j)}{2M} \quad i \neq j = 1,2$$

$$= \frac{(M^2 + m_i^2 - m_j^2) - 2Mm_i}{2M}$$

$$= E_i - m_i$$

$$= \text{Kinetic Energy}$$

$$\Delta T = \Delta M\left(1 - \frac{m_i}{M} - \frac{\Delta M}{2M}\right)$$

c) Pi meson [$\pi^{+/-}$] ($M = 139.6$ MeV)

Mu meson [$\mu^{+/-}$] ($M = 105.7$ MeV)

Reaction: $\pi \to \mu + \nu$

$$E_1 = \frac{M^2 + m_1^2 - m_2^2}{2M} \qquad E_\mu = \frac{(139.6)^2 + (105.7)^2 - 0^2}{2(139.6)} \text{ MeV}$$

$$= 109.8 \text{ MeV}$$

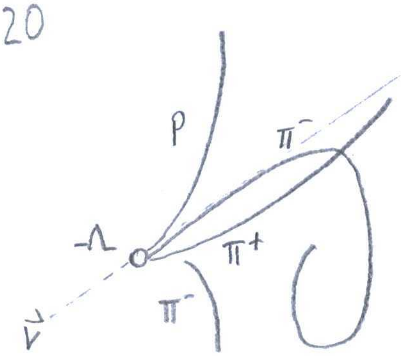
$$T_1 = T_\mu = E_\mu - m_\mu$$
$$= (109.8 - 105.7)\,\text{MeV}$$
$$= 4.1\,\text{MeV}$$

$$T_2 = T_\pi = E_\pi - E_\mu + m_2$$
$$= (139.6) - (109.8) - 0$$
$$= 29.8\,\text{MeV}$$

Notes about relativistic energy:

- General calculations consider number of particles in collision
- Common calculations involve both the rest frame (lab frame) and center of mass frame.
- Two regular energies are threshold and maximum.
- Lastly, how elastic is the collision?

11.20



"Lambda particle.... decays in a cloud chamber"

$$\Lambda\,(M = 1115\,\text{MeV}) \xrightarrow{\tau = 2.9\times10^{10}\,\text{s}} n\,(m = 139\,\text{MeV}) + \pi\,(m_3 = 140\,\text{MeV})$$

a) $P=(E,p)$; $m_1=m_2$

$P=p_1+p_2$

$P^2=(p_1+p_2)^2$

$=p_1^2+2p_1p_2+p_2^2$

$=M^2$

$M^2=m_1^2+m_2^2+2p_1\cdot p_2$

$=m_1^2+m_2^2+2(E_1E_2-P_1P_2\cos\theta)$

$=m_1^2+m_2^2+2E_1E_2-2P_1P_2\cos\theta$

b) Λ(10 GeV)

Distance:

$\gamma=E/m_\Lambda\cdot c^2$

$=\dfrac{10\,GeV}{1.1157\,GeV}$

$\cong 8.96$

$=\sqrt{1-(\frac{v}{c})^2}$

$V=c\sqrt{1-\dfrac{1}{(8.96)^2}}$

$\cong 0.9938c$

$\gamma v\tau=0.9938c\cdot 2.9\times10^{-10}\,s$

$=0.081\,m$

Angle:

$\tan\dfrac{\theta}{2}=\dfrac{P_{trans}}{P_{long}}$

$=\dfrac{m\cdot u}{\gamma m\cdot u}$

$=\dfrac{1}{\gamma}$

$$\Theta_{max} = 2\arctan\left(\frac{1}{\gamma}\right)$$

$$= 2\arctan\left(\frac{1}{0.56}\right)$$

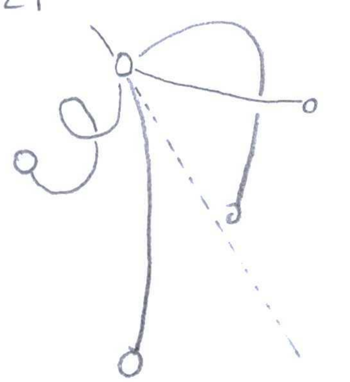
$$= 0.22\,\text{rad}$$

$$= 12.6^\circ$$

Note: Particle decay models changed during generations, noticeable by meson, positronium, and composite particles.

	Fermions			Bosons	
Quarks	2.16 MeV/c² 2/3 1/2 u up	1.273 GeV/c² 2/3 1/2 c charm	172.57 GeV/c² 2/3 1/2 t top	0 0 1 g gluon	125.26 GeV/c² 0 0 H Higgs
Quarks	4.7 MeV/c² -1/3 1/2 d down	93.511 MeV/c² -1/3 1/2 s strange	4.163 GeV/c² -1/3 1/2 b bottom	0 0 1 γ photon	
Leptons	0.511 MeV/c² -1 1/2 e electron	105.6 MeV/c² -1 1/2 μ muon	1.77 GeV -1 1/2 τ tau	91.1 GeV 0 1 Z Z boson	
Leptons	0.8 eV/c² 0 1/2 ν_e electron neutrino	<0.17 MeV/c² 0 1/2 ν_μ muon neutrino	18.2 MeV/c² 0 1/2 ν_τ tau neutrino	80.36 GeV/c² 0 ±1 W W boson	

11.21



"a system... decays or transforms... into a number of particles"

a) $P = (E, p)$; $m_1 = m_2$

$$P = P_1 + P_2$$

$$P^2 = (P_1 + P_2)^2$$

$$= P_1^2 + 2P_1P_2 + P_2^2$$

$$= M^2$$

$$M^2 = m_1^2 + m_2^2 + 2P_1 \cdot P_2$$

$$= m_1^2 + m_2^2 + 2(E_1E_2 - p_1p_2\cos\theta)$$

$$= m_1^2 + m_2^2 + 2E_1E_2 - 2p_1p_2\cos\theta$$

b) $\mu(M = 105.6\,MeV/c^2) \longrightarrow e(0.511\,MeV/c^2) + \bar{\nu}(0.8\,eV/c^2) + \nu(0.17\,MeV/c^2)$

Note: $\bar{\nu}$ is ν_e in modern models.
ν is ν_μ by recent physics.

$$\Delta M = (m_e + m_\nu + m_{\bar{\nu}} - m_\mu)$$
$$= (0.511\,\text{MeV} + 0.17\,\text{MeV} + 0.8\,\text{eV} - 105.6\,\text{MeV})/c^2$$
$$= -105\,\text{MeV}/c^2$$

$$M = (0.511\,\text{MeV} + 0.17\,\text{MeV} + 0.8\,\text{eV})/c^2$$
$$= 0.681\,\text{MeV}$$

$$T_{max,e} = \Delta M\left(1 - \frac{m_e}{M} - \frac{\Delta M}{2M}\right)$$
$$= -105\,\frac{\text{MeV}}{c^2}\left(1 - \frac{0.511\,\text{MeV}}{0.681\,\text{MeV}} - \frac{-105\,\text{MeV}}{2\cdot 0.681\,\text{MeV}}\right)$$
$$= -8{,}137\,\text{MeV}/c^2$$

$$T_{max,\nu} = \Delta M\left(1 - \frac{m_\nu}{M} - \frac{\Delta M}{2M}\right)$$
$$= -105\,\frac{\text{MeV}}{c^2}\left(1 - \frac{0.17\,\text{MeV}}{0.681\,\text{MeV}} - \frac{-105\,\text{MeV}}{2\cdot 0.681\,\text{MeV}}\right)$$
$$= -8{,}178\,\text{MeV}/c^2$$

$$T_{max,\bar{\nu}} = \Delta M\left(1 - \frac{m_{\bar{\nu}}}{M} - \frac{\Delta M}{2M}\right)$$
$$= -105\,\frac{\text{MeV}}{c^2}\left(1 - \frac{0.8\,\text{eV}}{0.681\,\text{MeV}} - \frac{-105\,\text{MeV}}{2\cdot 0.681\,\text{MeV}}\right)$$
$$= -8{,}419\,\text{MeV}/c^2$$

$$\frac{T_{max,e}}{\Delta M} = 78$$

$$\frac{T_{max,\nu}}{\Delta M} = 78$$

$$\frac{T_{max,\bar{\nu}}}{\Delta M} = 78$$

"The collision lost mass energy into kinetic energy by 78:1."

Note: Composite Particles: Kaon(K) = $\begin{cases} K^{\pm}: 493\ MeV/c^2 \\ K^{\circ}: 497\ MeV/c^2 \end{cases}$

Pion(π) = $\begin{cases} \pi^{\pm}: 139\ MeV/c^2 \\ \pi^{\circ}: 135\ MeV/c^2 \end{cases}$

$$\boxed{K^-(M=493\,MeV) \longrightarrow \pi^-(139\,MeV) + \pi(135\,MeV) + \pi^+(139\,MeV)}$$

$$\Delta M = (m_{\pi^-} + m_{\pi} + m_{\pi^+} - m_{K^-})/c^2$$
$$= (139\,MeV + 135\,MeV + 139\,MeV - 493\,MeV)/c^2$$
$$= -80\ MeV/c^2$$

$$M = (m_{\pi^-} + m_{\pi} + m_{\pi^+})$$
$$= (139\,MeV - 135\,MeV - 139\,MeV)/c^2$$
$$= 413\ MeV/c^2$$

$$T_{max,\pi^-} = \Delta M \left(1 - \frac{m_{\pi^-}}{M} - \frac{\Delta M}{2M}\right)$$
$$= -80\ \frac{MeV}{c^2}\left(1 - \frac{139\,MeV}{413\ MeV} - \frac{-80\ MeV}{2 \cdot 413\ MeV}\right)$$
$$= -60\ \frac{MeV}{c^2}$$

$$T_{max,\pi} = \Delta M \left(1 - \frac{m_{\pi}}{M} - \frac{\Delta M}{2 \cdot M}\right)$$
$$= -80\ \frac{MeV}{c^2}\left(1 - \frac{135\,MeV}{413\ MeV} - \frac{-80\ MeV}{2 \cdot 413\ MeV}\right)$$
$$= -61\ \frac{MeV}{c^2}$$

$$T_{max,\pi^+} = \Delta M\left(1 - \frac{m_{\pi^+}}{M} - \frac{\Delta M}{2 \cdot M}\right)$$

$$= -80\ \frac{MeV}{c^2}\left(1 - \frac{139 MeV}{413 MeV} - \frac{-80\ MeV}{2 \cdot 413\ MeV}\right)$$

$$= -61\ \frac{MeV}{c^2}$$

$$\frac{T_{max,\pi^-}}{\Delta M} = 0.76$$

"Mostly mass change in the collision"

$$\frac{T_{max,\pi}}{\Delta M} = 0.76$$

$$\frac{T_{max,\pi^+}}{\Delta M} = 0.76$$

$$\boxed{K^+(M = 497 MeV/c^2) \longrightarrow e^-(0.511 MeV/c^2) + \pi^0(135 MeV) + \nu(0.17 MeV/c^2)}$$

$$\Delta M = (m_{e^-} + m_{\pi^0} + m_\nu - m_{K^+})$$

$$= (0.511 MeV + 0.135 MeV + 0.17 MeV - 497 MeV)/c^2$$

$$= -496\ MeV$$

$$M = (0.511 MeV + 0.135 MeV + 0.17 MeV)$$

$$= 0.816\ MeV$$

$$T_{max,e^-} = \Delta M\left(1 - \frac{m_{e^-}}{\Delta M} - \frac{\Delta M}{2M}\right)$$

$$= -496\ \frac{MeV}{c^2}\left(1 - \frac{0.511 MeV}{0.816 MeV} - \frac{-496\ MeV}{2 \cdot 0.816\ MeV}\right)$$

$$= -150{,}240\ \frac{MeV}{c^2}$$

$$T_{max,\pi^0} = \Delta M\left(1 - \frac{m_{\pi^0}}{\Delta M} - \frac{\Delta M}{2 \cdot M}\right)$$

$$= -496\ \frac{MeV}{c^2}\left(1 - \frac{0.135\,MeV}{0.816\ MeV} - \frac{-496\ MeV}{2 \cdot 0.816\ MeV}\right)$$

$$= -151{,}159\ MeV/c^2$$

$$T_{max,v} = \Delta M\left(1 - \frac{m_v}{\Delta M} - \frac{\Delta M}{2 \cdot M}\right)$$

$$= -496\ \frac{MeV}{c^2}\left(1 - \frac{0.135\,MeV}{0.816\ MeV} - \frac{-496\ MeV}{2 \cdot 0.816\ MeV}\right)$$

$$= -151{,}\ \ MeV/c^2$$

$$\frac{T_{max,e^-}}{\Delta M} = 304$$

"Very little mass changes and mostly Kinetic energy transfer"

$$\frac{T_{max,\pi^0}}{\Delta M} = 304$$

$$\frac{T_{max,v}}{\Delta M} = 304$$

$$\boxed{K^{\pm}(M = 497\,MeV/c^2) \longrightarrow \mu^+(105.6\,MeV) + \pi^0(135\,MeV) + v(0.17\,MeV/c^2)}$$

$$\Delta M = (m_{\mu^+} + m_{\pi^0} + m_v - m_{K^\pm})$$

$$= (105.6\,MeV + 135\,MeV + 0.17\,MeV - 497\,MeV)/c^2$$

$$= -256.23\,MeV/c^2$$

$$M = (m_{\mu^+} + m_{\pi^0} + m_v)$$

$$= (105.6\,MeV + 135\,MeV + 0.17\,MeV)$$

$$= 240\ MeV/c^2$$

$$T_{max,\mu^+} = \Delta M\left(1 - \frac{m_{\mu^+}}{\Delta M} - \frac{\Delta M}{2 \cdot M}\right)$$

$$= -256\ \frac{MeV}{c^2}\left(1 - \frac{105.6\ MeV}{240\ MeV} - \frac{-256\ MeV}{2 \cdot 240\ MeV}\right)$$

$$= -297\ MeV/c^2$$

$$T_{max,\pi^+} = \Delta M\left(1 - \frac{m_{\pi^+}}{\Delta M} - \frac{\Delta M}{2 \cdot M}\right)$$

$$= -256\ \frac{MeV}{c^2}\left(1 - \frac{135\ MeV}{240\ MeV} - \frac{-256\ MeV}{2 \cdot 240\ MeV}\right)$$

$$= -249\ MeV/c^2$$

$$T_{max,\nu} = \Delta M\left(1 - \frac{m_\nu}{\Delta M} - \frac{\Delta M}{2 \cdot M}\right)$$

$$= -256\ \frac{MeV}{c^2}\left(1 - \frac{0.17\ MeV}{240\ MeV} - \frac{-256\ MeV}{2 \cdot 240\ MeV}\right)$$

$$= -392\ MeV/c^2$$

$$\frac{T_{max,\mu^+}}{\Delta M} = 1.1$$

$$\frac{T_{max,\pi^0}}{\Delta M} = 1.0$$

$$\frac{T_{max,\nu}}{\Delta M} = 1.5$$

$$\boxed{p(M=938\,MeV)+p^-(938\,MeV)\longrightarrow 2\pi^+(139\,MeV)+2\pi^-(139\,MeV)+\pi^\circ(135\,MeV)}$$

$$\Delta M=(2\cdot m_{\pi^+}+2\cdot m_{\pi^-}+m_{\pi^0}-m_p-m_{p^-})$$

$$=(2\cdot 139\,MeV+2\cdot 139\,MeV+135\,MeV-938\,MeV-938\,MeV)$$

$$=-1185\,MeV$$

$$M=(2\cdot m_{\pi^+}+2m_{\pi^-}+m_{\pi^0})$$

$$=(2\cdot 139\,MeV+2\cdot 139\,MeV+135\,MeV)$$

$$=691\,MeV$$

$$T_{max,\pi^+}=\Delta M\left(1-\frac{m_{\pi^+}}{\Delta M}-\frac{\Delta M}{2\cdot M}\right)$$

$$=-1{,}185\,\frac{MeV}{c^2}\left(1-\frac{139\,MeV}{691\,MeV}-\frac{-1{,}185\,MeV}{2\cdot 691\,MeV}\right)$$

$$=-1{,}193\,MeV/c^2$$

$$T_{max,\pi^-}=\Delta M\left(1-\frac{m_{\pi^-}}{\Delta M}-\frac{\Delta M}{2\cdot M}\right)$$

$$=-1{,}185\,\frac{MeV}{c^2}\left(1-\frac{139\,MeV}{691\,MeV}-\frac{-1{,}185\,MeV}{2\cdot 691\,MeV}\right)$$

$$=-1{,}193\,MeV/c^2$$

$$T_{max,\pi^0}=\Delta M\left(1-\frac{m_{\pi^0}}{\Delta M}-\frac{\Delta M}{2\cdot M}\right)$$

$$=-1{,}185\,\frac{MeV}{c^2}\left(1-\frac{139\,MeV}{691\,MeV}-\frac{-1{,}185\,MeV}{2\cdot 691\,MeV}\right)$$

$$=-1{,}193\,MeV/c^2$$

$$\frac{T_{max,\pi^+}}{\Delta M} = 1.7$$

$$\frac{T_{max,\pi^-}}{\Delta M} = 1.7$$

$$\frac{T_{max,\pi^0}}{\Delta M} = 1.7$$

$$\boxed{p(938 MeV) + \bar{p}(938 MeV) \rightarrow K^+(493 MeV) + K(497 MeV) + 3 \cdot \pi^0(135 MeV}$$

$$\Delta M = (m_{K^+} + m_K + 3m_{\pi^0} - m_p - m_{\bar{p}}) -$$

$$= (493 MeV + 497 MeV + 3 \cdot 135 MeV - 938 MeV - 938 MeV$$

$$= -481 MeV/c^2$$

$$M = (m_{K^+} + m_K + 3 \cdot m_{\pi^0})$$

$$= (493 MeV + 497 MeV + 3 \cdot 135 MeV)/c^2$$

$$= 1,395 MeV/c^2$$

$$T_{max,K^+} = \Delta M \left(1 - \frac{m_{K^+}}{\Delta M} - \frac{\Delta M}{2 \cdot M}\right)$$

$$= -48 \frac{MeV}{c^2}\left(1 - \frac{493 MeV}{1395 MeV} - \frac{-48 MeV}{2 \cdot 1395 MeV}\right)$$

$$= -32 MeV/c^2$$

$$T_{max,K} = \Delta M \left(1 - \frac{m_K}{\Delta M} - \frac{\Delta M}{2 \cdot M}\right)$$

$$= -48\ \frac{\text{MeV}}{c^2}\left(1 - \frac{497\,\text{MeV}}{1395\,\text{MeV}} - \frac{-48\,\text{MeV}}{2 \cdot 1395\,\text{MeV}}\right.$$

$$= -32\,\text{MeV}/c^2$$

$$T_{max,\pi^0} = \Delta M\left(1 - \frac{m_{\pi^0}}{\Delta M} - \frac{\Delta M}{2 \cdot M}\right)$$

$$= -48\ \frac{\text{MeV}}{c^2}\left(1 - \frac{135\,\text{MeV}}{1395\,\text{MeV}} - \frac{+48\,\text{MeV}}{2 \cdot 1395\,\text{MeV}}\right)$$

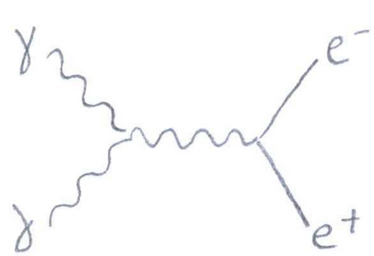
$$= -44\,\text{MeV}/c^2$$

$$\frac{T_{max\,K^+}}{\Delta M} = 0.66$$

$$\frac{T_{max,K}}{\Delta M} = 0.66$$

$$\frac{T_{max,\pi}}{\Delta M} = 0.92$$

11.22



"Photon-photon collision"

a) $$E_{thresh} = \frac{\sum m_i^2 - m_a^2 - m_b^2}{2m_b} \cdot c^2$$

$$= \frac{m_{e^-}^2 + m_{e^+}^2 - m_\gamma^2 - m_\gamma^2}{2 \cdot m_\gamma} \cdot c^2$$

$$= \frac{(0.511\,\text{MeV})^2 + (0.511\,\text{MeV})^2 - (8.3\times10^{-9}\,\text{eV})^2 - (8.3\times10^{-9}\,\text{eV})^2}{2 \cdot (8.3\times10^{-9}\,\text{eV})}$$

$$= 0.47 eV/c^2$$

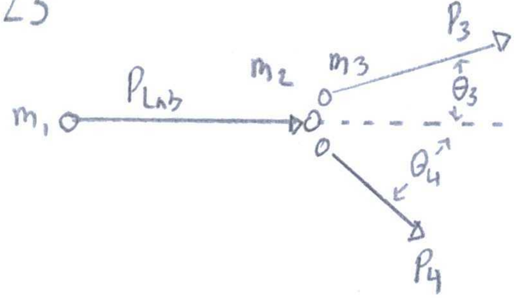
b) $$E_{threshold} = \frac{\sum_i m_i^2 - m_a^2 - m_b^2}{2 \cdot m_b} c^2$$

$$= \frac{m_{e^-}^2 + m_{e^+}^2 - m_\gamma^2 - m_\gamma^2}{2 \cdot m_\gamma} c^2$$

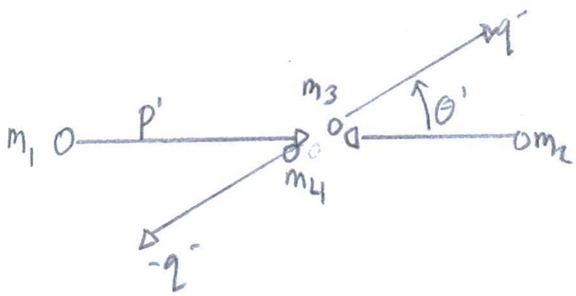
$$= \frac{(0.511 MeV)^2 + (0.511 MeV)^2 - (0.16 MeV)^2 - (0.16 MeV)^2}{2 \cdot (0.16 MeV)}$$

$$= 1.47 MeV/c^2$$

11.23



"Laboratory Frame"



"Center of mass Frame"

a) $$W = (p_1 + p_2)^2$$

$$= p_1^2 + p_2^2 + 2 p_1 \cdot p_2$$

equals zero

$$= p_1^2 + p_2^2 + 2(E_1 E_2 - \vec{p}_1 \cdot \vec{p}_2)$$

$$= m_1^2 + m_2^2 + 2 \cdot E_1 m_2 \quad \text{.. by } p_1^2 = m_1^2$$

$$p_2^2 = m_2^2$$

$$p' = \sqrt{E_2^2 - m_2^2}$$

$$= \frac{\sqrt{(W^2 - (m_1+m_2)^2)(W^2 - (m_1 - m_2)^2)}}{2W}$$

$$= \frac{m_2 \cdot \sqrt{E_{lab}^2 - m_1^2}}{W}$$

$$= \frac{m_2 \cdot p_{lab}}{W}$$

b) (11.17) "Lorentz Factor"

$$\gamma_{cm} = \frac{1}{\sqrt{1 - \beta^2}}$$

$$\beta_{cm} = \frac{p'}{m_2}$$

$$= \frac{p_{lab}}{m_2 + E_{lab}}$$

$$\gamma_{cm} = \frac{m_2 + E_{lab}}{W}$$

c) Non relativistic limit ; Before $\leftrightarrow$ After

$$\frac{m_1 v_1^2}{2} + \frac{m_2 v_2^2}{2} = \frac{m_1 v_1^2}{2} + \frac{m_2 v_2^2}{2}$$

$$E_{lab} \simeq m_1 + \frac{p_1^2}{2m_1},$$

$$W^2 \cong m_1^2 + m_2^2 + 2m_2\left(m_1 + \frac{p_1^2}{2m_1^2}\right)$$

$$= (m_1+m_2)^2\left[1 + \frac{m_2}{(m_1+m_2)^2} \cdot \frac{p_1^2}{m_1}\right]$$

$$W \cong (m_1+m_2)\sqrt{1 + \frac{m_2}{(m_1+m_2)^2}\,\frac{p_1^2}{m_1}}$$

$$\cong (m_1+m_2)\left[1 + \frac{m_2}{(m_1+m_2)^2} \cdot \frac{p_1^2}{2m}\right]$$

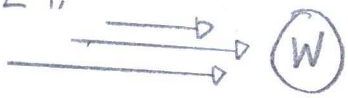
$$\cong m_1 + m_2 + \frac{m}{m_1+m_2} \cdot \frac{p_1^2}{2 \cdot m_1}$$

$$p' = \frac{m_2 p_{lab}}{W}$$

$$\cong \frac{m_2}{m_1+m_2}\, p_1$$

$$\beta_{cm} = \frac{p'}{m_2 + E_{lab}} \cong \frac{p_1}{m_1+m_2}$$

11.24.



"threshold
Kinetic energy
... center of mass"

a) Pi-meson Photoproduction:

$$\gamma p \longrightarrow \pi^0 p$$

$m_p = 938.5\,MeV$

$m_{\pi^0} = 135.0\,MeV$

is equivalent to

$$\gamma + p(938.5\,MeV) \longrightarrow \pi^0(135.0\,MeV) + n(939\,MeV)$$

Derivation threshold Kinetic Energy:

$$a + b \longrightarrow d + e + f + \ldots$$

Lab frame:

$P_{Lab} = P_a + P_b$ where $P_a = (p_a, E_a/c)$

$P_b = (0,0,0,m_b c)$

Center of mass frame:

$$P_{cm} = (0,0,0,E_{cm}/c)$$

$$P_{Lab}^2 = (P_a + P_b)^2$$

$$= P_a^2 + P_b^2 + 2 \cdot P_a \cdot P_b$$

$$= m_a^2 \cdot c^2 + m_b^2 \cdot c^2 + 2 \cdot E_a \cdot m_b$$

$$= P_{cm}^2$$

$$\frac{E_{cm}^2}{c^2} = m_a^2 c^2 + m_b^2 c^2 + 2 \cdot E_a \cdot m_b$$

$$E_a = \frac{E_{cm}^2 - m_a^2 \cdot c^2 - m_b^2 \cdot c^2}{2 \cdot m_b\, c^2}$$

$$= \frac{\sum m_i^2 - m_a^2 - m_b^2}{2 \cdot m_b}$$ "Two particle collision"

$$KE_{\text{threshold, pi-meson}} = \frac{(m_{\pi^-} + m_p)^2 - m_\gamma^2 - m_p^2}{2 \cdot m_p}$$

$$= \frac{(135\text{MeV} + (939\text{MeV})^2 - 0 - (939\text{MeV})^2}{2 \cdot 938\text{MeV}}$$

$$=$$

b) $pp \longrightarrow ppp\bar{p}$

is equivalent to

$p(935\text{MeV}) + p(935\text{MeV}) \longrightarrow p(935\text{MeV}) + p(935\text{MeV}) + p(935\text{MeV}) + \bar{p}(935\text{MeV})$

$$KE_{\text{threshold proton proton}} = \frac{(m_p + m_p)^2 - m_p^2 - m_p^2 - m_p^2 - m_{\bar{p}}^2}{2 \cdot m_p}$$

$$= \frac{(935\text{MeV} + 935\text{MeV})^2 - (935\text{MeV})^2 - (935\text{MeV})^2 - (935\text{MeV})^2 - (935\text{MeV}}{2 \cdot 935\text{MeV}}$$

$$=$$

Note: Antiproton calculated, but not visible in collision, emulsion, or gas until the tevatron (1955)

(1955... 1981... 1993, '94, '95, '97... 2002, 2004, 2006)

"tevatron"... "CERN"

The anti-proton accumulates in an electric field, sensed by inductors, but requires no interaction with chemical matter.

c) Nucleon-Antinucleon pair

$$e^{+}e^{-} \longrightarrow p\bar{p}$$

is equivalent to

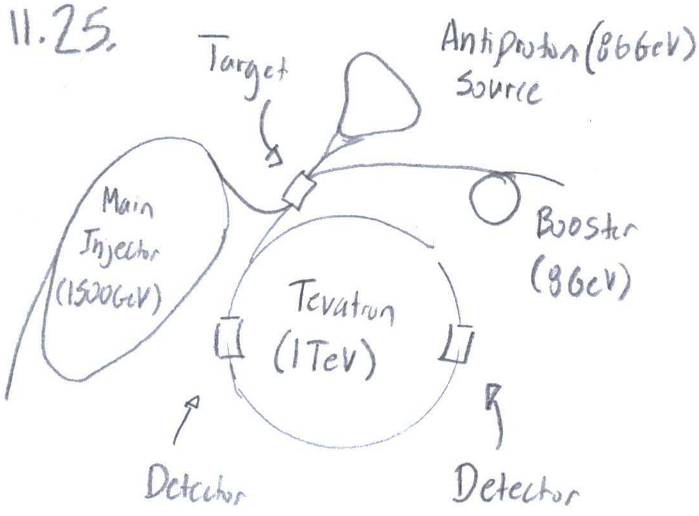
$$e^{+}(0.511\,MeV) + e^{-}(0.511\,MeV) \longrightarrow p(939\,MeV) + \bar{p}(939\,MeV)$$

$$KE_{\text{threshold, nucleon antinucleon}} = \frac{(m_p + m_p)^2 - m_{e^-}^2 - m_{e^+}^2}{2m_{e^-}}$$

$$= \frac{(939\,MeV + 939\,MeV)^2 - (0.511\,MeV)^2 - (0.511\,MeV)^2}{2 \cdot 935\,MeV} \frac{1}{c^2}$$

$$= 1{,}886\,\frac{MeV}{c^2}$$

11.25.



"Tevatron at Fermilab"

a) $W_{TOT}^2 = (p_1+p_2)^2$

$= m_1^2 + m_2^2 + 2p_1\cdot p_2$

$p_1\cdot p_2 = E_1\cdot E_2 + p_1\cdot p_2\cos\theta$

$= \left(\sqrt{p_1^2+m_1^2}\right)\left(\sqrt{p_2^2+m_2^2}\right) - p_1 p_2\cos\theta$

$= \left(p_1\sqrt{1+\frac{m_1^2}{p_1^2}}\right)\left(p_2\sqrt{1+\frac{m_2^2}{p_2^2}}\right) - p_1 p_2\cos\theta$

$\cong \left(p_1\left(1+\frac{m_1^2}{2p_1^2}\right)\right)\left(p_2\left(1+\frac{m_2^2}{2p_2^2}\right)\right) - p_1 p_2\cos\theta$

$W^2 = m_1^2 + m_2^2 + 2p_1\cdot p_2$

$\cong m_1^2 + m_2^2 + 2\left(p_1 p_2\left(\left(1+\frac{m_1^2}{2p_1^2}+\frac{m_2^2}{2p_2^2}+\frac{m_1^2 m_2^2}{4p_1^2 p_2^2}\right)\right) - \cos\theta\right)$

$\cong 2p_1 p_2(1-\cos\theta) + m_1^2 + m_2^2 + p_2 m_1^2 + p_1^2 m$

$\cong 4p_1 p_2\sin^2\frac{\theta}{2} + (p_1+p_2)\left(m_1^2(p_1^2+1) + m_2^2(p_2^2+1)\right)$

Book states $\cos^2\frac{\theta}{2}$

Also, the Book says, "$\left(\frac{m_1^2}{p_1}+\frac{m_2^2}{p_2}\right)$"

b) (From 11.23b)

$\beta_{cm} = \frac{p_{lab}}{m_2 + E_{lab}}$

$\cong \frac{p_{lab}}{E_{lab}}$

$$\cong \cancel{\frac{(p_1+p_2)\sin\frac{\theta}{2}}{(E_1+E_2)\sin\alpha}}$$

c) (From part 11.23b)

$$\beta_{cm} = \frac{p_{lab}}{m_2 + E_{lab}}$$

$$\cong \frac{(p_1+p_2)\sin\frac{\theta}{2}}{(E_1+E_2)\sin\alpha}$$

d) If $\theta^\circ = 20^\circ$ and $p_1 = p_2 = 100\,\text{GeV}/c$

$$W^2 = 4\left(100\frac{\text{GeV}}{c}\right)^2 \cos^2\frac{20^\circ}{2} + (100+100)\frac{\text{GeV}}{c}\left(\frac{10\text{GeV}^2}{100\text{GeV}} + \frac{10\text{GeV}^2}{100\text{GeV}}\right)$$

$$= 40.4\ \frac{\text{MeV}}{c^2}$$

11.26

a) (Problem 11.23b) Equation #1:

$$\beta_{cm} = \frac{P_{lab}}{m_2 + E_{lab}}$$

$$\gamma_{cm} = \frac{m_2 + E_{lab}}{W}$$

$$\gamma_{cm} \cdot \beta_{cm} = \frac{P_{lab}}{W}$$

$$p' = -m\,\gamma_{cm} \cdot \beta_{cm}$$

$$= -m \frac{P_{lab}}{W}$$

$$E_{cm,2} = \gamma_{cm}(m_2 - \beta_{cm} \cdot v)$$

$$= \gamma_{cm} \cdot m_2 \quad \ldots \text{ when } \quad v = 0$$

$$E_{lab,4} = \gamma[E_2^* + \beta P]$$

$$= \gamma[\gamma_{cm} \cdot m_2 + \beta_{cm}(m_2\,\gamma_{cm} \cdot \beta_{cm})]$$

$$= m_2\,\gamma_{cm}^2 - m_2\,\beta_{cm}^2\,\gamma_{cm}^2 \cdot \cos\theta$$

$$\Delta E = E_4 - m_2$$

$$= m_2\,\gamma_{cm}^2 - m_2\,\beta_{cm}^2 \cdot \gamma_{cm}^2 \cdot \cos\theta - m_2$$

$$= m_2(\gamma_{cm}^2 - 1) - m_2\,\beta_{cm}^2 \cdot \gamma_{cm}^2 \cdot \cos\theta$$

$$= m_2\,\gamma_{cm}^2\,\beta_{cm}^2\,(1 - \cos\theta)$$

$$= \frac{m_2}{W} P_{lab}^2 (1 - \cos\theta)$$

Equation #2:

$$T = \Delta E$$

$$= \frac{p^2}{2m} = \frac{Q^2}{2m_2}$$

Equation #3

$$(m_1 v_1' \sin\theta_3)^2 = (m_2 v_2' \sin\theta_4)^2$$

$$(m_1 v_1')^2 (\cos^2\theta + \sin^2\theta) = (m_1 v_1)^2 - 2m_1 v_1 m_2 v_2' \cos\theta_4 + (m_2 v_2')^2(\cos^2\theta_4 + \sin^2\theta_4)$$

... when $m_1 v_1^2 = m_1 v_1'^2 + m_2 v_2'^2$

$m_1 v_1'^2 = m_1 v_1^2 - m_2 v_2'^2$

$$v_2' = \frac{2m_1 v_1 \cos\theta_4}{m_1 + m_2}$$

$$\Delta E = \frac{1}{2} m v_2'^2 = \frac{1}{2} m_2 \left(\frac{2m_1 v_1 \cos\theta_4}{m_1 + m_2}\right)^2 = \frac{2m_2 p_{lab}^2 \cos^2\theta_4}{W^2 + p_{lab}^2 \sin^2\theta_4}$$

.. if $\tan\theta_2 = \dfrac{\sin 2\theta_4}{\left(\frac{m_1}{m_2} - \cos\theta_4\right)}$

$$\frac{m_1}{m_2} = \frac{\sin(2\theta_4 + \theta)}{\sin\theta_2}$$

b) (From part a)

$$\Delta E = \frac{m_2}{W^2} p_{lab}^2 (1 - \cos\theta)$$

$$= \frac{m_2}{m_1^2 + m_2^2 + 2m_2 E_{lab}} p_{lab}^2 (1 - \cos\theta)$$

$$= \frac{m_2}{m_1} \gamma^2 \beta^2 m_1 \cdot (2) \qquad @\ \theta = \pi \text{ rads}$$

$$\cong 2 m_2 \gamma^2 \beta^2$$

c) (From part a)

$$\Delta E = m_2 \cdot \gamma_{cm}^2 \cdot \beta_{cm}^2 (1 - \cos\theta)$$

$$= m_2 (\gamma^2 - 1)(1 - \cos\theta)$$

$$= m_2 (\gamma^2 - 1) \quad \text{... at} \quad \theta = \pi/2.$$

11.27

(1.10-1.11) "charge"

$$q' = \int \rho' d^3x$$

(9.17) "electric dipole moment"

$$p = \int x' \rho' d^3x$$

(11.139) "Current 4-vector"

$$J^{\mu} = (\gamma c \rho', \gamma \beta c \rho')$$

If $x^0 = \gamma(x'^0 + \beta \cdot x')$

$$\vec{x} = \vec{x}' + \frac{\gamma - 1}{\beta^2}\beta(\beta \cdot x) + \gamma \beta x^0$$

$$\rho(x^0, x) = \gamma \rho'(x')$$

$$J(x^0, x) = \gamma v \rho'(x)$$

<u>Lab Frame:</u>

$$x^0 = 0$$

$$\vec{x} = \vec{x}' + \frac{\gamma - 1}{\beta^2}\beta(\beta \cdot x)$$

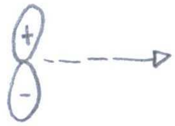
<u>Center of Mass Frame:</u>

$$\vec{x} = \vec{x}' - \frac{\gamma}{\gamma + 1}\beta(\beta \cdot x)$$

$$m = \frac{1}{2c}\int x \times J d^3x$$

$$= \frac{1}{2c}\int \Big(x - \underbrace{\frac{\gamma}{\gamma+1}\beta(\beta \cdot x)}_{<<1}\Big) \times (\gamma \beta c \rho') \frac{d^3x}{\gamma}$$

11.28



"Dipole initially at rest... has potential ... with uniform velocity"

a) $\phi = \gamma(\phi' + \beta \cdot A')$

$= \gamma\left(\frac{p \cdot r}{r'^3} + \beta \cdot 0\right)$

$= \gamma \frac{p \cdot r}{r'^3}$

$\cong \frac{p \cdot R}{R^3}$ ... because $\gamma = 1$ at rest

$A(x) = \beta\phi' + A'(x)$

$= \beta\phi'$

$= \beta \cdot \frac{p \cdot R}{R'^3}$

b) $\phi(x,t) = \frac{p \cdot R(x,t)}{R(x,t)^3}$

$= \frac{p\sqrt{(x-vt)^2 + y^2 + t^2}}{\sqrt{(x-vt)^2 + y^2 + t^2}^{\,3}}$

$= \frac{p\,(x-vt)}{|x-vt|^3}$

$A(x,t) = \beta\,\phi(x,t)$

$= \frac{\beta \cdot p(x-vt)}{|x-vt|^3}$

(11.78, 11.131) "Lorentz condition"

$\frac{1}{c}\frac{\partial \phi}{\partial t} + \nabla \cdot A = \gamma \frac{\partial}{\partial t}\left(\frac{1}{c} \cdot \frac{\partial \phi}{\partial r}\frac{\partial r}{\partial t} + \nabla\left(\beta \frac{\beta \cdot R}{R^3}\right)\right)$

$\frac{\partial r}{dt} = -\frac{v(x-vt)}{r'}$

$= -\frac{v\,\beta \cdot r_t}{r'}$

$$= \frac{1}{2}\int x' \times \beta \rho' d^3x$$

$$= \frac{1}{2}\int x\rho' d^3x \times \beta$$

$$= \frac{1}{2} p \times \beta$$

$$p = \int x\rho d^3x$$

$$= \int \left(x - \frac{\gamma}{\gamma+1}\beta(\beta\cdot x)\right)\rho' d^3x$$

$$= p' - \frac{\gamma}{\gamma+1}\beta(\beta\cdot p')$$

b) $J'^{\mu} = (0, J)$

$$J^{\mu} = \left(\gamma\beta\cdot J\cdot J + \frac{\gamma-1}{\beta^2}\beta(J\cdot J)\right)$$

$$p = \int x\rho d^3x$$

$$= \int \left(x - \frac{\gamma}{\gamma+1}\beta(\beta\cdot x)\right)\frac{\gamma}{c}\beta\cdot J\frac{d^3x}{\gamma}$$

$$= \frac{1}{c}\int \left(x - \frac{\gamma}{\gamma+1}\right)\beta(\beta\cdot x)\Big)\beta\cdot J' d^3x$$

$$m = \frac{1}{2c}\int x \times J d^3x$$

$$= \frac{1}{2c}\int \left(x - \frac{\gamma}{\gamma+1}\beta(\beta\cdot x)\right)\times\left(J + \frac{\gamma-1}{\beta^2}\beta(\beta\cdot J)\right)\frac{d^3x'}{\gamma}$$

$$= \frac{1}{2\gamma}m' + \frac{1}{2\gamma}\left(\frac{\gamma-1}{\beta^2} - \frac{\gamma}{\gamma+1}\right)(\beta^2 m - \beta(\beta\cdot m))$$

$$= \left(1 - \frac{1}{2}\beta^2\right)m' - \frac{\gamma-1}{2(\gamma+1)}\beta(\beta\cdot m)$$

$$\cong m' + \cdots$$

$$\frac{d\phi}{dt} = \frac{\gamma p \frac{\partial r}{\partial t} r'^3 - r'3r'^2 \frac{\partial r}{\partial t}}{r'^6}$$

$$= \gamma p\left(\frac{-\beta r'^3 + 3r(\beta \circ r')}{r'^6}\right)$$

$$\nabla \circ A = \gamma \circ \beta \cdot \nabla\left(\frac{p \circ r}{r'^3}\right)$$

$$= \gamma\beta\left(\frac{\beta \cdot p r'^3 - 3(\beta \cdot r)(p \cdot r')}{r'^6}\right)$$

$$\frac{1}{c}\frac{\partial \phi}{\partial t} + \nabla A = \gamma\left(\frac{\cancel{-p \cdot \beta r'^3 + 3(p \circ r)(\beta \circ r') + \beta p r'^3 - 3(\beta \cdot r)(p}}{r'^6}\right)$$

$$= 0$$

c) $E = -\nabla\phi$

$$= -\frac{\partial \phi}{\partial r}$$

$$= -\frac{\partial}{\partial r}\left[\frac{p \circ r}{r^3}\right]$$

$$= \frac{p \circ r'^3 + 3r'^3}{r^6}$$

$$= p\left(\frac{1+3}{r^3}\right)$$

$$= \frac{4p}{r^3}$$

$$B = \beta \times E$$

$$= \beta \times \frac{4p}{r^3}$$

Where is the effective dipole moment of problem 6.21 or 11.27a?

(Problem 6.21) "Magnetic dipole"

$\vec{m} = \frac{1}{2} p \times v$ -or- $m = (p \times \beta)/2$ from 11.27a

$$B = \beta \times \frac{4p}{r^3}$$

$$= \frac{8}{cr^3} \circ \vec{m} \quad \text{-or-} \quad = \frac{8}{r^3} \circ \vec{m}$$

Problems 6.21 and 11.27a dipoles differ by a 1/c factor.

11.29.

"Magnetic Moment"

a) $\phi' = 0$; $A' = \frac{m \times r'}{r'^3}$

$$\phi = \gamma(\phi' + \beta A')$$

$$= \gamma\beta \frac{m \times r}{r'^3}$$

$$\cong \beta \frac{m \times r}{r'^3}$$

$$A = \beta\phi' + A'(x)$$

$$= \frac{m \times r}{r'^3}$$

b) (11.131) "Lorentz condition"

$$\frac{1}{c}\frac{\partial \Phi}{\partial t} + \nabla A = \gamma\left(\frac{-r'^3 + 3\cdot(m \times r)\, r'}{r'^6}\right) + \gamma\left(\frac{r'^3 - 3\cdot m(m \times r)\, r'}{r'^6}\right)$$

$$= 0$$

$$E = -\nabla \Phi$$

$$= -\nabla \beta \frac{m \times r}{r'^3}$$

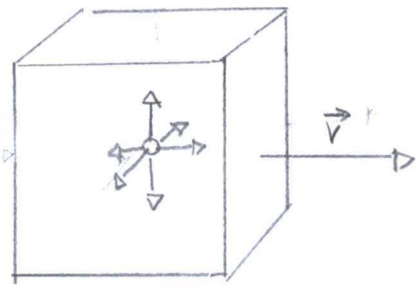
$$= -\frac{\beta \times r}{r'^3} - \frac{\beta \times m}{r'^3} - m \times \left(\frac{3\cdot r(r' \circ \beta) - \beta R^2}{R^6}\right)$$

$$= E_{dipole}(p_{eff} = m) - m \times \left(\frac{3 \circ n(n' \circ \beta) - \beta}{R^4} \right)$$

$$B = \frac{E}{\beta}$$

$$= E_{dipole}\left(p_{eff} = \frac{\beta}{2} m\right) + \frac{3}{2} n \left[\frac{m(n \cdot p) + \beta(n \circ m)}{R^3} \right]$$

11.30



"isotropic linear medium... translation"

a) ( )

$$E' = \gamma(E + \beta \times B) - \frac{\gamma^2}{\gamma+1} \beta(\beta \cdot E)$$

$$B' = \gamma(B - \beta \times E) + \frac{\gamma^2}{\gamma+1} \beta(\beta \circ E)$$

$$E' = \gamma(E_\perp + \beta \times B) + \beta(\beta \cdot E)$$

$$B' = \gamma(B_\perp - \beta \times E) + \beta(\beta \circ B)$$

...when $E_\perp = E - \beta(\beta \cdot E)$

$= -\beta \times (\beta \times E)$

$$D' = \gamma(D_\perp + \beta \times H) + \beta(\beta \circ D)$$

$$H' = \gamma(H_\perp - \beta \times D) + \beta(\beta \circ H)$$

If $\beta \to -\beta$, then

$$D' = \gamma(D'_\perp - \beta \times H') + \beta(\beta \cdot D')$$

$$H' = \gamma(H'_\perp + \beta \times D') + \beta(\beta \circ H)$$

$$= 0$$

$$B_z = \mu\epsilon \frac{\left(\frac{v}{c}E - \frac{v^2}{c^2}B - \frac{v}{c\mu\epsilon}E + \frac{v^2}{c^2}B\right)}{1-\omega^2\rho^2/c^2}$$

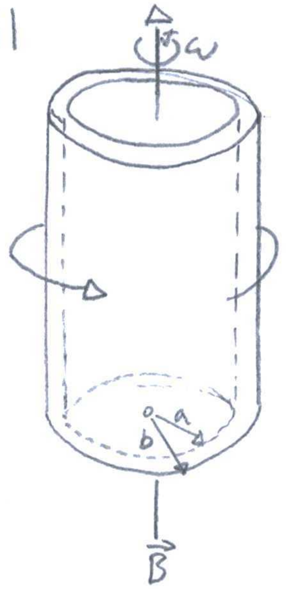
$$= \frac{\mu B_0\left(B - \omega^2\rho^2/c^2\mu\epsilon\right)}{1-\omega^2\rho^2/c^2}$$

$$V = -E \cdot d$$

$$= \frac{\mu\omega \cdot \rho B_0}{c(1-\omega^2\rho^2/c^2)}\left(1-\frac{1}{\mu\epsilon}\right) \cdot \frac{(b^2-a^2)}{2}$$

$$= \frac{\mu\omega\rho}{2c} B_0 (b^2-a^2)\left(1-\frac{1}{\mu\epsilon}\right) \quad \ldots \text{when } \frac{\omega b}{c} \ll 1$$

11.31



"hollow right-cylinder ... at angular speed (ω) in .. magnetic field"

(Problem 11.30)

$$D = \epsilon E + \gamma^2\left(\epsilon - \frac{1}{\mu}\right)\left[\beta^2 E_\perp + \beta \times \beta\right]$$

$$H = \frac{1}{\mu} B + \gamma^2\left(\epsilon - \frac{1}{\mu}\right)\left[-\beta^2 B_\perp + \beta \times E\right]$$

.. where $v = \omega\rho\hat{\phi}$

$E_\phi = E_z = B_\rho = B_\phi = 0$

$$\nabla \cdot D = \nabla\left[\epsilon E + \gamma^2\left(\epsilon - \frac{1}{\mu}\right)\left[\beta^2 E_\perp + \beta \times \beta\right]\right]$$

$$= \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho E_\rho) + \frac{1}{\rho}\frac{\partial}{\partial\phi}E_\phi + \frac{\partial}{\partial z}E_z$$

(Appendix: Cylindrical)

$$\nabla \times H = \left(\frac{1}{\rho}\frac{\partial E_\rho}{\partial\phi} - \frac{\partial E_\phi}{\partial z}\right) + \left(\frac{\partial E_\rho}{\partial z} - \frac{\partial E_z}{\partial\rho}\right) + \frac{1}{\rho}\left(\frac{\partial}{\partial\rho}(\rho E_\phi) - \frac{\partial E_\rho}{\partial\phi}\right)$$

$\nabla \cdot D = 0$, no free charge

$\nabla \times H = 0$, stationary magnetic field

$$D = \epsilon E + \gamma^2\left(\epsilon - \frac{1}{\mu}\right)\left[\beta^2 E_\perp + \beta \times \beta\right]$$

$$= \epsilon E_\rho + \gamma^2\left(\epsilon - \frac{1}{\mu}\right)\left[\beta \times B\right] \text{ ... when}$$

$$E_\rho = \frac{-\gamma^2(\epsilon + 1/\mu)\,\beta \times B}{\epsilon}$$

$$= -\frac{\mu\omega\rho B_0}{c(1 - \omega^2\rho^2/c^2)}\left(1 - \frac{1}{\mu\epsilon}\right)$$

$$H = \frac{1}{\mu} B + \gamma^2\left(\epsilon - \frac{1}{\mu}\right)\left[-\beta^2 B_\perp + \beta \times E\right]$$

$$D = \gamma\left(\epsilon E_\perp - \frac{1}{\mu}\beta \times B'\right) + \epsilon\beta(\beta\cdot E)$$

$$H = \gamma\left(\frac{1}{\mu}B_\perp + \epsilon\beta \times E'\right) + \frac{1}{\mu}\beta(\beta\cdot B')$$

$$\therefore D' = \epsilon E' \text{ and } \mu H' = B'$$

$$D = \gamma^2\left(\epsilon(E_\perp + \beta\times B) - \frac{1}{\mu}\left(\beta\times B + \beta^2 E_\perp\right)\right) + \epsilon\beta(\beta\cdot E)$$

$$H = \gamma^2\left(\frac{1}{\mu}(B_\perp - \beta\times E) + \epsilon\left(\beta\times E - \beta^2 B_\perp\right)\right) + \frac{1}{\mu}\beta(\beta\cdot E)$$

$$D = \epsilon E + \gamma^2\left(\epsilon - \frac{1}{\mu}\right)\left(\beta^2 E_\perp + \beta\times B\right)$$

$$H = \frac{1}{\mu}B + \gamma^2\left(\epsilon - \frac{1}{\mu}\right)\left(-\beta^2 B_\perp + \beta\times E\right)$$