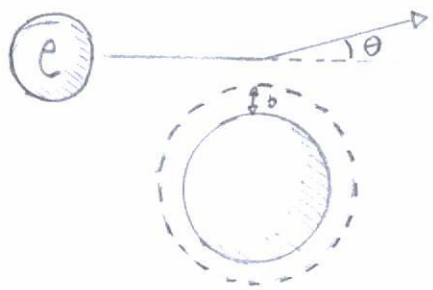
Chapter 13: Collision, Energy Loss, and Scattering of Charged Particles; Cherenkov and Transition Radiation:

13.1



"light particle (electron) .... Scattering through an angle"

Scattering Parameter:

$$b = \frac{ze^2}{pv} \cot \frac{\theta}{2}$$

Differential cross section:

$$\frac{d\sigma}{d\Omega} = \frac{b}{\sin\theta} \left| \frac{db}{d\theta} \right|$$

a) (Prob 11.26) "Invariant Momentum Transfer"

$$Q^2 = -(p-p')^2$$
$$= -(mc^2 - m'c^2)^2$$

Rest Frame:

$$Q^2 = -(mc)^2 + 2(mm'c^2) - (m'c)^2$$
$$= -2(pp' - m^2c^2)$$
$$= -2(\vec{E}\vec{E}'/c^2 - m^2c^2 - \vec{p}\vec{p}')$$

...when $p = \frac{\vec{E}}{c} + \vec{p}$

$p' = \frac{\vec{E}}{c} - \vec{p}'$

Center of Mass Frame:

$$Q^2 = 2(pp' - m^2c^2)$$

$$= 2\left(\frac{EE'}{c^2} - m^2c^2 - \vec{p}\vec{p}'\right)$$

$$= 2\left(\frac{\sqrt{m^2c^4+|p|^2c^2}\sqrt{m^2c^4+|p|^2c^2}}{c^2} - m^2c^2 - |p|^2\cos\theta\right)$$

$$= 2(m^2c^2 + |p|^2 - m^2c^2 - |p|^2\cos\theta)$$

$$= 2|p|^2(1-\cos\theta)$$

$$= 4|p|^2\sin^2\frac{\theta}{2}$$

Trigonometric Identities

$$\sin^2\theta = \frac{1-\cos 2\theta}{2}$$

$$1+\cot^2\theta = \operatorname{Cosec}^2\theta = \frac{1}{\sin^2\theta}$$

$$= 4|p|^2\frac{1}{1+\cot^2\left(\frac{\theta}{2}\right)}$$

$$= 4|p|^2\frac{1}{1+\frac{p^2v^2}{z^2e^4}b^2}$$

$$= \left(\frac{2ze}{v}\right)^2\frac{1}{\frac{z^2e^4}{p^2v^2}+b^2}$$

$$= \left(\frac{2ze}{v}\right)^2\frac{1}{b_{(min)}^2+b^2}$$

(pg 626) Impact Parameter

$$b_{(min)} = ze^2/pv$$

$$= 2M(E' - mc)$$

$$= 2MT$$

$$T = \frac{2z^2e^4}{mv^2}\frac{1}{b^2+b_{min}^2}$$

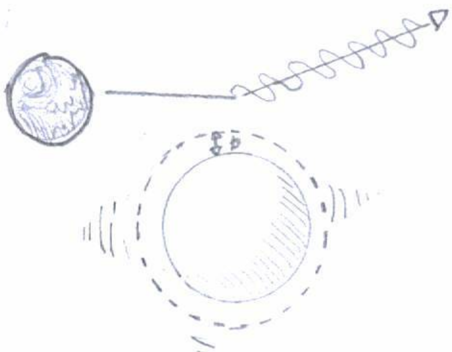
b) (11.152) "Transverse fields"

$$E_1 = E_1' = -\frac{-q\gamma v t}{(b^2+\gamma^2 v^2 t^2)^{3/2}}$$

$$E_2 = \gamma E_2' = \frac{\gamma q b}{(b^2+\gamma^2 v^2 t^2)^{3/2}}$$

$$B_3 = \gamma \beta E_2 = \beta E_2'$$

∴ when heavy particle ; $q = Ze$.



"Transverse impulse ∴ heavy particle ∴ passes large impact parameter"

$$E_\perp = E_2 - \gamma E_2'$$

$$= \frac{q\gamma b}{(b^2+\gamma^2 v^2 t^2)^{1/2}}$$

<u>Impulse</u>: $\Delta p = \int F\,dt$

$$= e\int E_\perp\,dt$$

$$= Ze^2\gamma b \int \frac{dt}{(b^2+\gamma^2 v^2 t^2)^{3/2}}$$

∴ when $q = Ze$

$t = (b/\gamma v)\tan\theta$

$$= Ze^2\gamma b \int \frac{dt}{(b^2+\gamma^2 v^2 (b/\gamma v)^2 \tan^2\theta)^{3/2}}$$

$$= Ze^2\gamma \int \frac{dt}{(1+\tan^2\theta)^{3/2}}$$

∴ by $\frac{dt}{d\theta} = (b/\gamma v)\sec^2\theta$

$$= \frac{Ze^2}{bv} \int \frac{\sec^2\theta \, d\theta}{(1+\tan^2\theta)^{3/2}}$$

$$= \frac{Ze^2}{bv} \int \frac{1}{\sqrt{1+\tan^2\theta}}$$

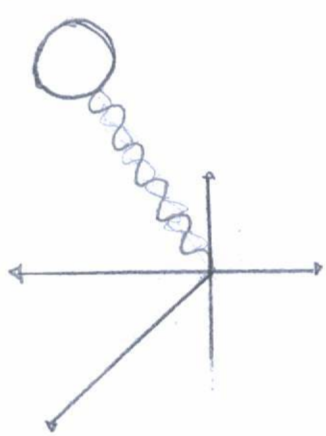
$$= 2Ze^2/bv$$

$$T = \frac{(\Delta p)^2}{2m}$$

$$= \frac{2z^2e^4}{mv^2}\frac{1}{b^2}$$

Parts a and b relate when b_{min} is zero. eg. mass is humangous in the denominator from a really heavy particle.

13.2.



"Time-varying electromagnetic field... bound to origin ... nonrelativistic response"

$$m\ddot{x} = -m\omega_0^2 x - m\Gamma\dddot{x} + eE(x,t) + \frac{e}{c}\dot{x} \times B(x,t)$$

x = position

$\dot{x}$ = velocity

$\ddot{x}$ = acceleration

$$\ddot{x} + m\Gamma\dddot{x} + \omega_0^2 x = \frac{e}{m}E(t) + \frac{e}{mc}\dot{x} \times B(x,t)$$

$$X(\omega) = \frac{e/m}{\omega_0^2 - \omega^2 - i\omega\Gamma} E(\omega)$$

(12.27) "Energy loss modified"

$$\Delta E = \int_{-\infty}^{\infty} F(t) \cdot \dot{x}(t)\, dt$$

$$= e \int_{-\infty}^{\infty} E(\omega)\, \dot{x}(\omega)\, d\omega$$

$$= \frac{e^2}{m} \int_{-\infty}^{\infty} \frac{-i\omega}{\omega_0^2 - \omega^2 - i\omega\Gamma} |E(\omega)|^2 d\omega$$

$$= \frac{e^2}{m} 2 \cdot \mathrm{Re} \int_0^{\infty} \frac{i\omega}{\omega_0^2 - \omega^2 - i\omega\Gamma} |E(\omega)|^2 d\omega$$

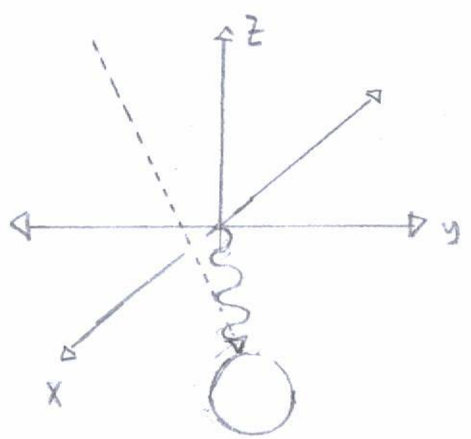
$$= \frac{2e^2}{m} \int_0^{\infty} \frac{\omega^2 \Gamma}{(\omega^2 - \omega_0^2) + \omega^2 \Gamma^2} |E(\omega)|^2 d\omega$$

$$\lim_{\Gamma \to 0} \Delta E = \lim_{\Gamma \to 0} \frac{2e^2}{m} \int_0^{\infty} \frac{\omega^2 \Gamma}{(\omega - \omega_0^2) + \omega^2 \Gamma^2} |E(\omega)|^2 d\omega$$

$$= \frac{2e^2}{m} \frac{\pi}{2} \left[\delta(\omega - \omega_0) + \delta(\omega + \omega_0) \right] |E(\omega)|^2$$

$$= \frac{\pi e^2}{m} |E(\omega)|^2$$

13.3



"Time-varying electromagnetic fields ... bounded ... and charge passing the origin"

$$= \frac{e}{m} E(t) \quad \ldots \text{ when } \quad \dot{x} \times B(t) = 0$$

$$x(\omega) = \frac{e/m}{\omega_0^2 - \omega^2 - i\omega\Gamma} E(\omega)$$

$$\Delta E = \int_{-\infty}^{\infty} F(t) \cdot \dot{x}(t)\, dt$$

$$= e \int_{-\infty}^{\infty} E^*(\omega) \cdot \dot{x}(\omega)\, d\omega$$

$$= \frac{e^2}{m} 2 \cdot \mathrm{Re} \int_0^{\infty} \frac{-i\omega}{\omega_0^2 - \omega^2 - i\omega\Gamma} |E(\omega)|^2 d\omega$$

$$= \frac{2e^2}{m} \int_0^{\infty} \frac{\omega^2 \Gamma}{(\omega^2 - \omega_0^2)^2 + \omega^2 \Gamma^2} |E(\omega)|^2 d\omega$$

$$\lim_{\Gamma \to 0} \Delta E = \lim_{\Gamma \to 0} \frac{2e^2}{m} \int_0^{\infty} \frac{\omega^2 \Gamma}{(\omega^2 - \omega_0^2)^2 + \omega^2 \Gamma^2} |E(\omega)|^2 d\omega$$

$$= \frac{2e^2}{m} \frac{\pi}{2} \int_0^{\infty} \left[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)\right] |E(\omega)|^2 d\omega$$

$$= \frac{\pi e^2}{m} |E(\omega_0)|^2$$

(11.152) "Transverse fields"

$$E_1 = E_1' = -\frac{q\gamma vt}{(b^2+\gamma^2 v^2 t^2)^{3/2}}$$

"Modified Bessels"

$$K_0(x) = \int_0^\infty \frac{\cos(xt)}{(t^2+1)^{1/2}}\,dt$$

$$K_1(x) = \int_0^\infty \frac{t\sin(xt)}{(t^2+1)^{1/2}}\,dt$$

Thus,

$$K_0 = \frac{1}{2}\int_{-\infty}^{\infty} \frac{e^{ixt}}{(t^2+1)^{1/2}}\,dt$$

$$K_1 = \frac{1}{2i}\int_{-\infty}^{\infty} \frac{te^{ixt}}{(t^2+1)^{1/2}}\,dt$$

Huzzah! Exponential Identities

$$\boxed{\begin{aligned} \cos x &= \frac{e^{ix}+e^{-ix}}{2} \\ \sin x &= \frac{e^{ix}-e^{-ix}}{2} \end{aligned}}$$

$$E_\perp(\omega) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} \frac{ze\gamma b}{(b^2+\gamma^2 v^2 t^2)^{3/2}}\, e^{i\omega t}\,dt$$

$$= \frac{ze\gamma}{b^2\sqrt{2\pi}}\int_{-\infty}^{\infty} \frac{e^{i\omega t}}{(1+(\gamma vt/b)^2)^{3/2}}\,dt$$

$$= \frac{ze}{bv\sqrt{2\pi}}\int_{-\infty}^{\infty} \frac{e^{i\xi t}}{(1+t')^{3/2}}\,dt' \qquad \therefore\ t' = \gamma vt/b, \quad \xi = \omega b/\gamma v$$

$$= \frac{ze}{bv}\sqrt{\frac{2}{\pi}}\,\xi K_1(\xi)$$

$$E_{\parallel}(\omega) = -\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{Ze\gamma vt}{(b^2+\gamma^2 v^2 t^2)^{3/2}} e^{i\omega t} dt$$

$$= -\frac{Ze\gamma v}{b\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{t e^{i\omega t}}{(1+(\gamma v t/b)^2)^{3/2}} dt$$

$$= -\frac{Ze}{\gamma b v \sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{t' e^{i\xi t'}}{(1+t'^2)^{3/2}} dt$$

$$= -i \frac{Ze}{\gamma b v} \sqrt{\frac{2}{\pi}}\, \xi K_0(\xi)$$

b) (Problem 13.2)

$$\Delta E = \frac{\pi e^2}{m} |E(\omega_0)|^2$$

$$= \frac{\pi e^2}{m}\left[E_{\parallel}^2 + E_{\perp}^2\right]$$

$$= \frac{2Z^2 e^4}{mv^2} \frac{\xi_0^2 [K_1(\xi_0)^2 + \gamma^2 K_0^2(\xi)]}{b^2}$$

If $b \ll b_{max}$, $K_0(\xi) = -\ln\left(\frac{\xi e^{\gamma}}{2}\right) + \ldots$

$$K_1(\xi) = \frac{1}{\xi} + \ldots$$

$$\Delta E \approx \frac{2Z^2 e^4}{mv^2} \frac{1 + (\gamma^{-1} \xi \ln(\xi e^{\gamma}/2))^2}{b^2}$$

$$\lim_{\xi \to 0} \Delta E = \lim_{\xi \to 0} \frac{2Z^2 e^4}{mv^2} \frac{1 + (\gamma^{-1} \xi \ln(\xi e^{\gamma}/2))^2}{b^2}$$

$$= \frac{2Z^2 e^4}{mv^2} \frac{1}{b^2}$$

If $b \gg b_{max}$, $K_0(\xi) = \sqrt{\frac{\pi}{2\xi}}\, e^{-\xi}$

$$K_1(\xi) = \sqrt{\frac{\pi}{2\xi}}\, e^{-\xi}$$

$$\Delta E \approx \frac{\pi z^2 e^4}{m v^2} \frac{(1+\gamma^{-2}) e^{-b/b_{max}}}{b/b_{max}}$$

13.4

"Kinetic energy losses"

(13.14) "Bethe Formula
Total Energy Loss per unit length"

$$\frac{dE}{dX} = 4\pi N Z \frac{z^2 e^4}{mc^2\beta^2}\left[\ln B_q - \beta^2\right]$$

(13.15) where $B_q = \frac{2\gamma^2\beta^2 mc^2}{\hbar \langle \omega \rangle}$

N = Avogadro's number

Z = Atomic #

z = charge

m = mass

Air: $\rho_{Ar} = 1.2 \times 10^{-3}\ g/cm^3$

$Z_{Air} \approx Z_{N_2} \approx 7$

$m_{Air} \approx m_{N_2} \approx 14\ g/mol$

$\hbar \langle \omega \rangle = 12 \cdot Z\ eV$

$= 84\ eV$

10 MeV: $v = 1.13 \times 10^7$ m/s ; $\frac{dE}{dx} = -0.0014$ MeV/cm

100 MeV: $v = 3.58 \times 10^7$ m/s ; $\frac{dE}{dx} = -0.0142$ MeV/cm

1000 MeV: $v = 1.13 \times 10^8$ m/s ; $\frac{dE}{dx} = -0.1419$ MeV/cm

Aluminum: $\rho = 2.70 g/cm^3$

$Z_{Al} = 13$

$m_{Al} = 26.98 g/mol$

$\hbar\langle\omega\rangle = 12 \cdot 13 eV$

$= 156 eV$

10 MeV: $v = 8.46 \times 10^6$ m/s ; $\frac{dE}{dx} = -0.0008$ MeV/cm

100 MeV: $v = 2.67 \times 10^7$ m/s ; $\frac{dE}{dx} = -0.0079$ MeV/cm

1000 MeV: $v = 8.46 \times 10^7$ m/s ; $\frac{dE}{dx} = -0.0795$ MeV/cm

Copper: $\rho = 8.96 g/cm^3$

$Z_{Cu} = 29$

$m_{Cu} = 63.55 g/mol$

$\hbar\langle\omega\rangle = 12 \cdot 29 eV$

$= 348 eV$

10 MeV; $v = 5.5 \times 10^6$ m/s ; $\frac{dE}{dx} = -0.0003$ MeV/cm

100 MeV: $v = 1.74 \times 10^7$ m/s ; $\frac{dE}{dx} = -0.0034$ MeV/cm

1000 MeV: $v = 5.51 \times 10^7$ m/s ; $\frac{dE}{dx} = -0.0337$ MeV/cm

Lead: $\rho \simeq 11.34 g/cm^3$

$Z = 82$

$m_{Pb} = 207.2 g/mol$

$\hbar\langle\omega\rangle = 12 \cdot 82 eV$

$= 984 eV$

10 MeV: $V = 3.05 \times 10^6 m/s$; $\frac{dE}{dx} = -0.0001$ MeV/cm

100 MeV: $V = 9.65 \times 10^6 m/s$; $\frac{dE}{dx} = -0.0010$ MeV/cm

1000 MeV: $V = 3.05 \times 10^7 m/s$; $\frac{dE}{dx} = -0.0103$ MeV/cm

Note: A table is a better representation, but without. Light elements lost most energy per centimeter. Heavy atoms kept momentum and energy per centimeter.

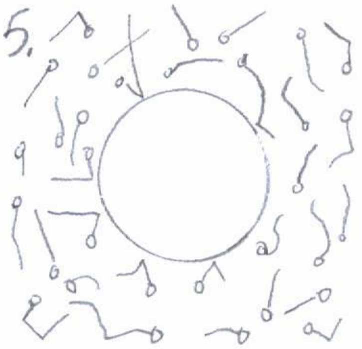
b)

Substance	10 MeV	100 MeV	1000 MeV
Air	-1.7×10^{-6} MeV·cm²/g	-1.7×10^{-5} MeV cm²/g	-1.7×10^{-4} MeV cm²/g
Aluminum	-2.14×10^{-3} MeV cm²/g	-2.13×10^{-2} MeV·cm²/g	-2.1×10^{-1} MeV·cm²/g
Copper	-3.0×10^{-3} MeV cm²/g	-3×10^{-2} MeV cm²/g	-3×10^{-1} MeV cm²/g
Lead	-1.1×10^{-3} MeV cm²/g	-1.1×10^{-2} MeV cm²/g	-1.1×10^{-1} MeV cm²/g

A factor of two $\longrightarrow$

because 10 MeV, 100 MeV, 1000 MeV scale.

13.5.



"energy loss by Close collisions of a fast... heavy particle... through electronic plasma"

a) $V(r) = Ze^2 \exp(-k_D \cdot r)/r$

k_D = Debye screening parameter

(Table 6.1) "Kinetic energy"

$$\Delta E = \frac{p^2}{2m}$$

$$= \int_{-\infty}^{\infty} \frac{F_\perp dt}{2m}$$

$$F = \nabla E$$

$$= Ze^2 e^{-k_D \cdot r} \frac{(1 + k_D \cdot r)}{r^2}$$

$$F_\perp = F \sin\theta$$

$$= F_r \frac{b}{r}$$

$$= \frac{Ze^2 b\, e^{-k_D r}(1 + k_D r)}{r^3}$$ ...where $\sqrt{b^2 + \gamma^2} = \sqrt{b^2 + v^2 t^2}$

$$\Delta p_\perp = Zeb \int_{-\infty}^{\infty} \frac{e^{-k_D \cdot r}(1 + k_D \cdot r)}{r^3} dt$$

$$= \frac{2Ze^2 b}{v} \int_b^{\infty} \frac{e^{-k_D r}(1 + k_D r)}{r^2 \cdot \sqrt{r^2 - b^2}} dr$$

$$= \frac{2Ze^2}{vb} \int_0^{\infty} \frac{e^{-\xi \cosh t}(1 + \xi \cosh t)}{\cosh^2 t} dt$$ .. $\xi = k_D b$

$$f(\xi) = \int_0^{\infty} \frac{2Ze^2\, e^{-\xi \cosh t}(1 + \xi \cosh t)}{vb \cdot \cosh^2 t} dt$$

$$f''(\xi) = -\xi \int_0^\infty e^{-\xi \cosh t}\, dt$$

$$= -\xi K_0(\xi)$$

$$= (\xi K_1(\xi))'$$

So, $f(\xi) = \xi K_1(\xi)$

$$\Delta p_\perp = \frac{2ze^2}{vb} \xi K_1(\xi)$$

$$\Delta E = \frac{\Delta p^2}{2m}$$

$$= \frac{2(ze^2)}{mv^2 b^2} \xi^2 K_1^2(\xi)$$

$$= \frac{2(ze^2)}{mv^2} k_D^2 K_1^2(k_D b)$$

b) (13.35) "Energy loss per unit distance ... with impact parameter"

$$\frac{dE}{dx} = 2\pi N \int_{b_{min}}^{\infty} \Delta E\, b\, db$$

$$= \frac{4\pi N (ze^2)^2}{mv^2} \int_{\xi_{min}}^{0} \xi K_1^2(\xi)\, d\xi$$

$$= \frac{4\pi N (ze^2)^2}{mv^2} \left[\frac{\xi^2}{2} (K_1^2(\xi) - K_0(\xi) K_2(\xi)) \right]_0^\infty$$

$$= \frac{(ze)^2}{v^2} \omega_p^2 \xi_{min}^2 \frac{(K_0(\xi_{min}) K_2(\xi_{min}) - K_1^2(\xi_{min}))}{2}$$

... where $\omega_p^2 = 4\pi N e^2/m$

If $\xi < b/b_{max}$,

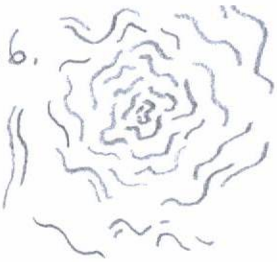
$$K_0(\xi) = \log\frac{2}{\xi} - \gamma, \quad K_1(\xi) \approx 1/\xi, \quad K_2(\xi) \approx 2/\xi^2$$

$$\frac{dE}{dx} = \frac{4\pi N (ze^2)^2}{mv^2}\left[\log\frac{2}{\xi_{min}} - \gamma - \frac{1}{2}\right]$$

$$= \frac{4\pi N (ze^2)^2}{mv^2} \log \frac{2e^{-\gamma - 1/2}}{\xi_{min}}$$

$$= \frac{4\pi N (ze^2)^2}{mv^2} \log \frac{1}{1.468\, \xi_{min}}$$

$$= \frac{(ze)^2}{v^2} \omega_p^2 \log \frac{1}{1.468 \cdot k_D \cdot b_{min}}$$

13.6. 

"Energy loss in a plasma from distant collisions"

(7.59) "Dielectric constant"

$$\frac{\epsilon(\omega)}{\epsilon_0} = 1 - \frac{\omega_p^2}{\omega^2}$$

where $\omega_p^2 = \dfrac{NZe^2}{\epsilon_0 m}$

(13.36) "Energy loss per unit distance"

$$\left(\frac{dE}{dx}\right) = \frac{2}{\pi}\frac{(Ze)^2}{v^2} Re \int_0^\infty i\omega \lambda^* a\, K_1(\lambda^* a) K_0(\lambda a)\left(\frac{1}{\epsilon(\omega)} - \beta^2\right) d\omega$$

where (13.30) wavelength

$$\lambda^2 = \frac{\omega^2}{v^2} - \frac{\omega^2}{c^2}\epsilon(\omega)$$

$$= \frac{\omega^2}{v^2}\left[1 - \beta^2 \epsilon(\omega)\right]$$

(7.59) "Dielectric with dampening"

$$\epsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2 + i\omega\Gamma}$$

a) $\boxed{\text{Citation: Wolfram Research, Inc.}}$ $a K_1(a)\cdot K_0(a) = \log_e\left(\frac{1.123}{a}\right)$

$$\frac{dE}{dx} = \frac{2}{\pi}\frac{Z^2 e^2}{v^2} Re \int_0^\infty \left(\frac{i\omega}{\epsilon(\omega)}\right)\log_e\left(\frac{1.123}{\lambda a}\right) d\omega$$

$$= \frac{2}{\pi}\frac{Z^2 e^2}{v^2} Re \int_0^\infty \left(\frac{i\omega}{\epsilon(\omega)}\right)\log_e\left(\frac{1.123 \cdot v}{\omega \cdot a\left[1-\beta^2\epsilon(\omega)\right]}\right) \quad \ldots \text{when } \frac{k}{\omega} = \frac{n}{c}$$

$$= \frac{2 Z^2 e^2}{\pi \cdot v^2} Re \int_0^\infty \left(\frac{i\omega}{\epsilon(\omega)}\right)\log_e\left(\frac{1.123 \cdot v\, k_0}{\omega}\right) \quad \ldots \text{if } k_0 = \frac{1}{a\left[1-\beta^2\epsilon(\omega)\right]}$$

b) If $\Gamma \ll \omega_p$, $\dfrac{1}{\epsilon(\omega)} = \dfrac{\omega^2 + i\omega\Gamma}{\omega^2 + \omega_p^2 + i\omega\Gamma}$

$$\lim_{\tau \to 0} \frac{1}{\epsilon(\omega)} = \frac{\omega^2/\omega_p^2}{\omega^2/\omega_p^2 - 1}$$

$$\frac{dE}{dx} = \frac{2z^2e^2}{\pi v^2}\int_0^\infty \omega \pi \frac{\omega^2}{\omega_p^2}\left(\frac{\omega^2}{\omega_p^2} - 1\right) \cdot \ln\left(\frac{1.123}{\lambda a}\right) d\omega$$

$$= \frac{2z^2e^2}{v^2}\int_0^\infty \omega \frac{\omega^2}{\omega_p^2}\frac{\omega_p^2}{2\omega} \delta(\omega^2 - \omega_p^2) \cdot \ln\left(\frac{1.123}{\lambda a}\right) d\omega$$

$$= \frac{z^2e^2}{v^2}\omega_p^2 \ln\left(\frac{1.123}{\lambda a}\right)$$

(Problem 13.5) "Energy per distance from collision"

$$\left(\frac{dE}{dx}\right) = \frac{(ze)^2}{v^2}\omega_p^2 \ln\left(\frac{1}{1.47 \cdot k_0 \cdot b_{min}}\right)$$

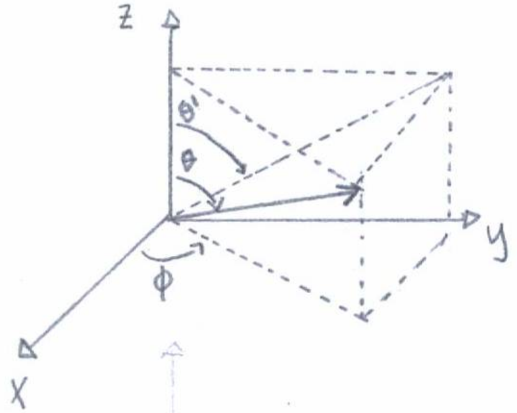
$$\left(\frac{dE}{dx}\right)_{Total} = \left(\frac{dE}{dx}\right)_{plasma} + \left(\frac{dE}{dx}\right)_{collision}$$

$$= \frac{z^2e^2}{v^2}\omega_p^2 \ln\left(\frac{1.123}{\lambda a}\right) + \frac{z^2e^2}{v^2}\omega_p^2 \ln\left(\frac{1}{1.47 \cdot k_0 \cdot b_{min}}\right)$$

$$= \frac{z^2e^2}{v^2}\omega_p^2 \ln\left(\frac{1.123 \cdot v}{\omega_p b_{min}}\right)$$

where $\boxed{\omega_p = 2\pi\nu \quad \lambda = \frac{2\pi}{k}}$

13.7.



"Projected transverse displacement.. of an incident particle"

$$P(y)\,dy = A\exp\left[-y^2/2\langle y^2\rangle\right]dy$$

...when $\langle y^2\rangle = \langle x^2/6\rangle\langle\theta\rangle$

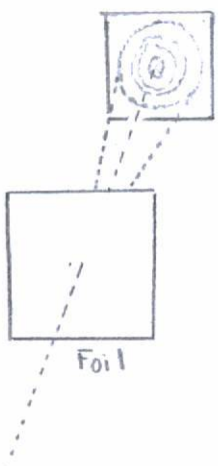
(13.61) "Approximations"

$$\langle\theta^2\rangle = 2\theta_{min}^2 \ln\left(\frac{\theta_{max}}{\theta_{min}}\right)$$

$$= 2\cdot\langle y^2\rangle \ln\left(\frac{1}{P(y)}\right)$$

$$P(y) = A\exp\left[\frac{-y^2}{2\langle y^2\rangle}\right]$$

13.8.



"finite sized nucleus... single scattering"

a) (13.66) "Mean square angle"

$$\langle\theta^2\rangle \cong 4\pi N\left(\frac{2zZe^2}{pv}\right)^2 \ln(204\cdot Z^{-1/3})\cdot t$$

(13.70) "Relativistic projected angle"

$$\alpha = \theta^\circ / \langle\theta^2\rangle^{1/2}$$

(pg 645) "...seemingly statistical cutoff [z-table]"

$\alpha \approx 2.5$

<u>Thickness:</u>

If $t = x$, Lead: $$X = \left(\frac{\theta^\circ}{2.5}\right)^{2}\frac{1}{4\pi N\left(\frac{2zZe^2}{pv}\right)^2\ln(204Z^{-1/3})}$$

$$= 33\text{cm}\cdot\theta^2$$

Aluminum: $x = \left(\frac{\theta^{\circ}}{2.5}\right)^2 \frac{1}{4\pi N \left(\frac{2zZe^2}{pv}\right)^2 \ln\left(204 \cdot Z^{-1/3}\right)}$

$= 97\,cm^2 \cdot \theta^2$

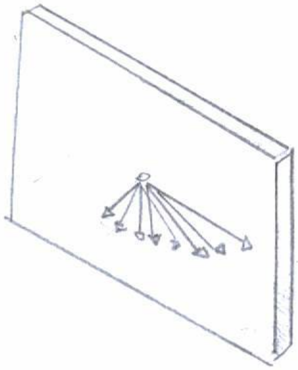
b) Unsure, (13.13) "Bethe Equation"

$$B_q = \frac{2\gamma^2\beta^2 mc^2}{\hbar\langle\omega\rangle}$$

but without a $\hbar\langle\omega\rangle$ for

collision number.

13.9



"Plexiglass... angle of emission.. for electrons and protons"

(13.50) "the angle of emission of Cherenkov radiation - far field"

$$\cos\theta_c = \frac{1}{\beta\sqrt{\epsilon(\omega)}}$$

(13.4) "Kinetic energy max"

$$T = E - mc^2 = (\gamma - 1)mc^2$$
$$= \left(\frac{1}{\sqrt{1-\beta^2}} - 1\right)mc^2$$

$$\beta = \frac{\sqrt{(T/mc^2)(2+T/mc^2)}}{1+T/mc^2}$$

$$\cos\theta_c = \frac{1+T/mc^2}{\sqrt{(T/mc^2)(2+T/mc^2)}\sqrt{\epsilon(\omega)}}$$
$$= \frac{1+T/mc^2}{\sqrt{(T/mc^2)(2+T/mc^2)}\cdot n}$$

If $\theta_c = 0°$, then

$$\frac{T_{min}}{mc^2} = \frac{n}{\sqrt{n^2-1}} - 1 \qquad \text{... when } n > 1$$

If $n = 1.5$, $T_{min}/mc^2 = 0.341$

$$\theta_{max} = \cos^{-1}\left(\frac{1}{n}\right)$$
$$= 48°$$

$m_e = 0.511$ MeV

$m_p = 938$ MeV

Quanta per wavelengths (4000 - 6000Å) per cm at 1MeV:

(13.48) "Cherenkov radiation per unit distance"

$$\left(\frac{dE}{dx}\right)_{rad} = \frac{(ze)^2}{c^2}\int_{\epsilon(\omega)>1/\beta^2} \omega\left(1-\frac{1}{\beta^2\epsilon(\omega)}\right)d\omega$$

$$= \int \frac{(ze)^2}{\hbar c^2}\left(1-\frac{1}{\beta^2\epsilon(\omega)}\right)d\omega$$

$$= \frac{z^2e^2}{\hbar c^2}\left(1-\frac{1}{\beta^2\epsilon(\omega)}\right)\left[\omega_{max}-\omega_{min}\right]$$

$$= \frac{z^2e^2}{\hbar c^2}\left(1-\frac{1}{\beta^2\epsilon(\omega)}\right)\left[\frac{2\pi c}{n\lambda_{min}}-\frac{2\pi c}{n\lambda_{max}}\right]$$

$$= \frac{2\pi z^2e^2}{n\hbar c}\left(1-\frac{1}{\beta^2\epsilon(\omega)}\right)\left[\frac{1}{\lambda_{min}}-\frac{1}{\lambda_{max}}\right]$$

If $\lambda_{min} = 4000$Å and $\lambda_{max} = 6000$Å, then

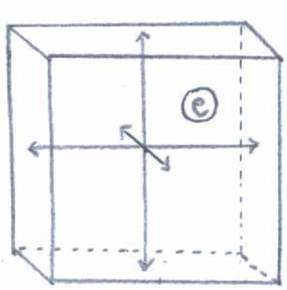
$$\left(\frac{dE}{dx}\right) = 283\left(1-\frac{1}{1.5^2\beta^2}\right)cm^{-1}$$

@1MeV, $\beta = 0.941$ $dE/dx = 149$ photons per cm

@500MeV, $\beta = 0.758$ $dE/dx = 64$ photons per cm

@5000MeV, $\beta = 0.937$ $dE/dx = 154$ photons per cm

13.10



"Particle of charge moves along... in medium"

a) (13.25) "Fourier transforms of the potentials"

$$\Phi(k,\omega) = \frac{2ze}{\epsilon(\omega)} \cdot \frac{\delta(\omega - k\cdot v)}{k^2 - \frac{\omega^2}{c^2}\epsilon(\omega)}$$

$$A(k,\omega) = \epsilon(\omega)\frac{v}{c}\Phi(k,\omega)$$

(13.28) "Fourier Transform"

$$E(\omega) = \frac{1}{(2\pi)^{3/2}}\int d^3k\, E(k,\omega) e^{ikx}$$

$$\Phi(x,\omega) = \frac{1}{(2\pi)^{3/2}}\int d^3k\, e^{ikx}\,\Phi(k,\omega)$$

$$= \frac{1}{(2\pi)^{3/2}}\frac{2ze}{\epsilon(\omega)}\iiint \frac{e^{i(k_x x + k_y y + k_z z)} \cdot \delta(\omega - k_z v)}{k^2 - \frac{\omega^2}{c^2}\epsilon(\omega)}\, dk_x\, dk_y\, dk_z$$

$$= \frac{ze}{v\epsilon(\omega)}\frac{1}{\sqrt{2\pi^3}}\iint \frac{e^{i\omega z/v} \cdot e^{i(k_x x + k_y y)}}{k_x^2 + k_y^2 - \frac{\omega^2}{c^2}\epsilon(\omega)}\, dk_x\, dk_y$$

$\therefore$ if $n=1$, $k_z = \frac{\omega}{v^2}$

$$= \frac{ze}{v\epsilon(\omega)}\frac{1}{\sqrt{2\pi^3}}\int_0^{2\pi}\int_0^{\infty}\frac{k_\perp e^{ik_\perp \rho\cos\theta}}{k_\perp^2 + \frac{\omega^2}{v^2}(1-\beta^2\epsilon(\omega))}\, e^{i\omega z/v}\, d\theta\, dk_\perp$$

$\therefore$ $k_\perp \rho\cos\theta = k_x x + k_y y$

$$= \frac{ze}{v\epsilon(\omega)}\sqrt{\frac{2}{\pi}}\int_0^{\infty}\frac{k_\perp J_0(k_\perp \rho)}{k_\perp^2 + \frac{\omega^2}{v^2}(1-\beta^2\epsilon(\omega))}\, e^{i\omega z/v}\, dk_\perp$$

$$= \frac{ze}{v\epsilon(\omega)}\sqrt{\frac{2}{\pi}}\, K_0\left(\frac{|\omega|\rho}{v}\sqrt{1-\beta^2\epsilon(\omega)}\right) e^{i\omega z/v}$$

Identity:

$$\int_0^{\infty}\frac{x J_0(ax)}{x^2+b^2}\, dx = K_0(ab)$$

$$e^{i\rho\cos\theta} = \sum_{-\infty}^{\infty} i^n J_n(\rho\theta) e^{in\theta}$$

b) $\phi(x,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} d\omega\, \phi(x,\omega)\, e^{-i\omega t}$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} d\omega\, \frac{ze}{v\epsilon(\omega)} \sqrt{\frac{2}{\pi}}\, K_0\!\left(\frac{|\omega|\rho}{v}\sqrt{1-\beta^2\epsilon(\omega)}\right) e^{i\omega(z/v - t)}$$

$$= \frac{2ze}{v\epsilon(r)\pi} \int_0^{\infty} d\omega\, K_0\!\left(\frac{\omega\rho}{v}\sqrt{1-\beta\epsilon(\omega)}\right) \cos\!\left(\omega\left(\frac{z}{v}-t\right)\right)$$

$$= \frac{2ze}{v\epsilon(r)\pi} \frac{\pi}{2} \left[\frac{\rho^2}{v^2}\left(1-\beta^2\epsilon(\omega)\right) + \left(\frac{z}{v}-t\right)^2\right]^{-1/2}$$

$$= \frac{ze}{v\epsilon(r)} \frac{1}{\sqrt{\frac{\rho^2}{v^2}(1-\beta^2\epsilon(\omega)) + \left(\frac{z}{v}-t\right)^2}}$$

Identity:

$$\int_0^{\infty} dx\, K_0(ax)\cos(\beta x) = \frac{\pi}{2\sqrt{a^2+b^2}}$$

If $\Gamma = (1-\beta^2\epsilon(\omega))^{-1/2}$, then

$$\phi(x,t) = \frac{ze\Gamma}{\epsilon\left[\rho^2 + \Gamma^2(z-vt)^2\right]^{1/2}}$$

∴ which is (11.152) when $E = -\nabla\phi(x,t)$.

c) If $\beta^2\epsilon > 1$, $\phi(x,\omega) = \frac{ze}{v\epsilon(\omega)}\sqrt{\frac{2}{\pi}}\, e^{i\omega z/v} \cdot K_0\!\left(\pm \frac{i|\omega|\rho}{v}\sqrt{\rho^2\epsilon - 1}\right)$

$$= \frac{ze}{v\epsilon(\omega)}\left[-N_0\!\left(\frac{|\omega|\rho}{v}\sqrt{\rho^2\epsilon-1}\right) \mp i J_0\!\left(\frac{i|\omega|\rho}{v}\sqrt{\rho^2\epsilon-1}\right.\right.$$

$$\phi(x,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \phi(x,\omega)\, e^{-i\omega t}\, d\omega$$

$$= \frac{ze}{v\cdot\epsilon(\omega)} \int_{-0}^{\infty} \left[-N_0\!\left(\frac{|\omega|\rho}{v}\sqrt{\rho^2\epsilon-1}\right) \mp i J_0\!\left(\frac{i|\omega|\rho}{v}\sqrt{\rho^2\epsilon-1}\right)\right] \cdot e^{i\omega(z/v-t)}\, d\omega$$

$$= \frac{Ze}{v\epsilon(\omega)} \int_0^\infty \Big[-N_0\left(\frac{|\omega|\rho}{v}\sqrt{\beta^2\epsilon - 1}\right) \cos\left(\omega\left(\frac{z}{v} - t\right)\right)$$

$$+ J_0\left(\frac{|\omega|\rho}{v}\sqrt{\beta^2\epsilon-1}\right) \sin\left(\omega\left(\frac{z}{v}-t\right)\right)\Big] d\omega$$

Identities:

$$\int_0^\infty J_0(ax)\sin(bx)\,dx = \begin{cases} 0 & 0<b<a \\ \frac{1}{\sqrt{b^2-a^2}} & 0<a<b \end{cases}$$

$$\int_0^\infty N_0(ax)\cos(bx)\,dx = \begin{cases} 0 & 0<b<a \\ \frac{1}{\sqrt{b^2-a^2}} & 0<a<1 \end{cases}$$

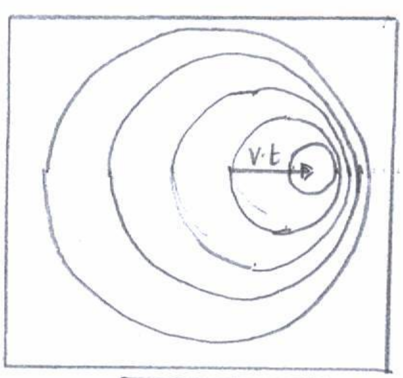
$$\phi(x,t) = \frac{2Ze}{v\epsilon(\omega)} \left[\left(\frac{z}{v} - t\right)^2 - \frac{\rho^2}{v^2}\left(\beta^2\epsilon - 1\right)\right]^{-1/2}$$

$$= \frac{2ze/\epsilon}{\sqrt{(z-vt)^2 - (\beta^2\epsilon-1)\rho^2}}$$

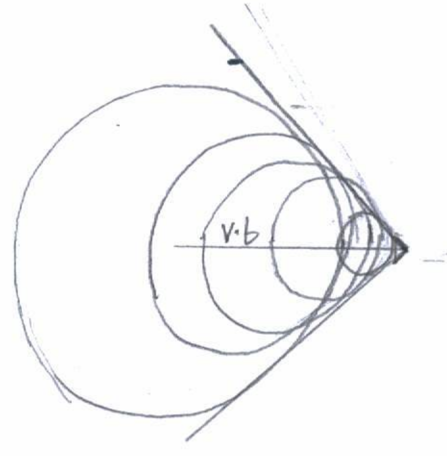
(13.51) "Potential ahead of particle"

$$A(x,t) = \beta \frac{2ze}{\sqrt{(x-vt)^2 - (\beta^2\epsilon-1)^2\rho^2}}$$

$$= \beta\phi(x,t)$$

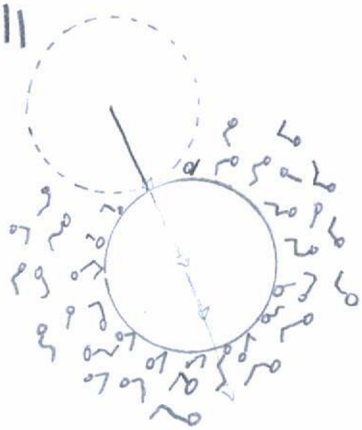


$\boxed{\beta^2\epsilon<1}$ "Low velocity"



$\boxed{\beta^2\epsilon>1}$ "High velocity"

13.11



"magnetic monopole with magnetic charge ...loses energy by collisions with electrons"

a) (11.152) "Transformed fields"

$$E_1 = E_1' = -\frac{q\gamma v t}{(b^2+\gamma^2 v^2 t^2)^{3/2}}$$

$$E_2 = \gamma E_2' = \frac{\gamma q b}{(b^2+\gamma^2 v^2 t^2)^{3/2}}$$

$$B_3 = \gamma \beta E_2 = \beta E_2$$

If monopole, $E = B$:

$$B_1 = E_1 = \frac{q\gamma v t}{(b^2+\gamma^2 v^2 t^2)^{3/2}}$$

$$B_2 = \frac{\gamma q b}{(b^2+\gamma^2 v^2 t^2)^{3/2}}$$

$$E_3 = \beta B_2 = \frac{\gamma(\beta g) b}{(b^2+\gamma^2 v^2 t^2)^{3/2}}$$

(pg 626) "Energy Transfer"

$$T(b) = \frac{2z^2e^4}{mv^2}\frac{1}{b^2+b_{min}^2}$$

$$= \frac{2z^2e^4}{mc^2}\cdot\frac{1}{b^2} \quad \text{... when } b_{min}=0$$

(13.7) "Bethe Energy, Energy loss from movement"

$$\frac{dE}{dx}(T>\epsilon) = 2\pi N z \int_0^{b_{max}} T(b)\, b\, db$$

$$= 2\pi N z \int_0^{b_{max}} \frac{2ze^4}{mc^2}\frac{1}{b}\, db$$

$$= \frac{4\pi N z z^2 e^4}{mc^2} \ln\left\{ \frac{2\gamma^2\beta^2 mc^2}{\hbar\langle\omega\rangle} \right\}$$

because $b_{max} = \sqrt{T_{max}}$

and $T_{max} = E - mc^2$ ... (13.4)

$= \gamma(E' + \beta c p')$ ... (pg 625)

$= 2\gamma^2\beta^2 mc^2$

$$T_{max,\epsilon} = \frac{2\gamma^2\beta^2 mc^2}{\hbar\langle\omega\rangle}$$

units of $\hbar\langle\omega\rangle$

$$\frac{dE}{dX} = 4\pi N z \frac{z^2 e^4}{mc^2}\left\{ 2\ln(\gamma\beta) + \ln\left(\frac{2mc^2}{\hbar\langle\omega\rangle}\right)\right\}$$

$$= 4\pi N z \frac{z^2 e^4}{mc^2\beta^2} \ln\left\{\frac{2\gamma^2\beta mc}{\hbar\langle\omega\rangle}\right\}$$

$$= 4\pi N z \frac{g^2 e^2}{mc^2} \ln\left\{\frac{2\gamma^2\beta mc}{\hbar\langle\omega\rangle}\right\}$$

(Section 13.1) "Atoms never bind to electrons"

$$ze = \beta g$$

b) (6.153) "Magnetic Monopole quantization"

$$\frac{eg}{4\pi\hbar} = \frac{\alpha g}{z_0 e} = \frac{n}{2} \quad (n = 0, \pm 1, \pm 2 \ldots)$$

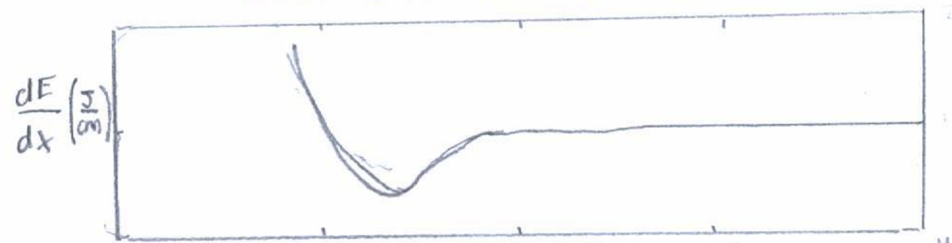
Where, so, that $\alpha = \frac{e^2}{4\pi\epsilon_0 \hbar c}$

≈ 1.137

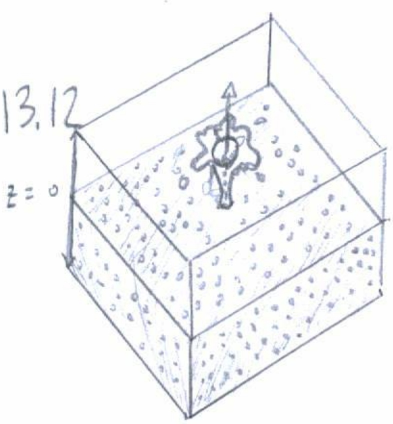
and $z = 137 \frac{n}{z}$

$$\left(\frac{dE}{dx}\right) = \frac{4\pi N z}{mc^2}\left\{\frac{137}{z}n\right\}^2 \ln\left\{\frac{2\gamma^2\beta^2 mc}{\hbar \langle w \rangle}\right\}$$

∴ Atomic number necessary for energy loss and ionization.



13.12



"relativistic particle moves... with constant speed... half space.. uniform isotropic medium [plasma]..... with similar medium"

a) (13.78) "Energy radiated"

$$E_{rad} = \frac{e^{ikr}}{r}\left(-\frac{\omega_p^2}{4\pi c^2}\right)\int_{z>0}^{\infty}(\hat{k} \times E) \times \hat{k}\, e^{-ikx}\, d^3x$$

(13.83) "Output to Fourier Integral"

$$E = \epsilon_0 4\sqrt{2\pi}\, \frac{ze}{c}\left(\frac{c}{\omega_p}\right)^2 \frac{\gamma}{\nu^2} \frac{\sqrt{\eta}}{(1+\frac{1}{\nu^2}+\eta)(1+\eta)}$$

...where $\nu = \frac{\omega}{\gamma\omega_p}$, $\eta = (\gamma\theta)^2$

$$= \epsilon_0 4\sqrt{2\pi}\, \frac{ze}{c}\left(\frac{c}{\omega_p}\right)^2 \gamma\sqrt{\eta}\left[\frac{1}{(1+\frac{1}{\nu^2}+\eta)} - \frac{1}{(1+\eta)}\right.$$

If $z>0$,

$$E = \epsilon_0 \left(-\sqrt{\frac{2}{\pi}}\right) \frac{ze\theta}{c} \left(\frac{1}{\frac{1}{\gamma^2} + \theta^2} - \frac{1}{\frac{1}{\gamma^2} + \frac{\omega_{p2}^2}{\omega} + \theta^2} \right) \frac{e^{ikr}}{r}$$

If $z<0$,

$$E = \epsilon_0 \sqrt{\frac{2}{\pi}} \frac{ze\theta}{c} \left(\frac{1}{\frac{1}{\gamma^2} + \theta^2} - \frac{1}{\frac{1}{\gamma^2} + \frac{\omega_{p1}^2}{\omega} + \theta^2} \right) \frac{e^{ikr}}{r}$$

Note: ω_{p1} and ω_{p2} describe plasma frequencys in two mediums.

$$E_{rad} = E_{z>0} + E_{z<0}$$

$$= \epsilon_0 \sqrt{\frac{2}{\pi}} \frac{ze\theta}{c} \left(\frac{1}{\frac{1}{\gamma^2} + \frac{\omega_{p1}^2}{\omega} + \theta^2} - \frac{1}{\frac{1}{\gamma^2} + \frac{\omega_{p2}^2}{\omega} + \theta^2} \right)$$

(13.79) "Differential spectrum"

$$\frac{d^2 I}{d\omega d\Omega} = \frac{c}{32\pi^2} \left(\frac{\omega_p}{c}\right)^4 \left| \int_{z>0} [\hat{k} \times E(x,\omega)] \times \hat{k} e^{-ikx} \cdot d^3x \right|^2$$

$$= \frac{z^2 e^2 \theta^2}{\pi^2 \cdot c} \left| \frac{1}{\frac{1}{\gamma^2} + \frac{\omega_{p1}^2}{\omega} + \theta^2} - \frac{1}{\frac{1}{\gamma^2} + \frac{\omega_p^2}{\omega} + \theta^2} \right|^2$$

b) $$I = \int_0^\infty d\omega \int d\Omega \frac{d^2 I}{d\omega d\Omega}$$

$$= \frac{2z^2e^2}{\pi c} \int_0^\infty \theta^3 \cdot \left| \frac{1}{\frac{1}{\gamma^2} + \frac{\omega_{p1}^2}{\omega} + \theta^2} - \frac{1}{\frac{1}{\gamma^2} + \frac{\omega_{p2}^2}{\omega} + \theta^2} \right|^2 d\theta$$

$$= \frac{Z^2 e^2}{\pi c} \int_0^\infty d\omega \left[\frac{\omega_1^2 + \omega_p^2 + 2\omega^2/\gamma^2}{\omega_{p_1}^2 - \omega_{p_2}^2} \right] \log\left(\frac{\omega_{p_1}^2 + \omega^2/\gamma^2}{\omega_{p_2}^2 + \omega^2/\gamma^2} \right) - 2$$

$$= \frac{Z^2 e^2}{\pi c} \frac{\gamma}{3(\omega_{p_1}^2 - \omega_{p_2}^2)} \Bigg[2\left(\omega_{p_1}^3 + 3\omega_{p_1}\omega_{p_2}^2\right) \arctan\left(\frac{y}{\omega_{p_1}}\right)$$

$$- 2\left(3\omega_{p_1}^2 \omega_{p_2} + \omega_{p_2}^3\right) \arctan\left(\frac{y}{\omega_{p_2}}\right)$$

$$+ y\Bigg(\left(3\omega_{p_1}^2 + 3\omega_{p_2}^2 + 2y^2\right) \log\left(\frac{\omega_{p_1}^2 + y^2}{\omega_{p_2}^2 + y^2}\right)$$

$$- 2\left(\omega_{p_1}^2 - \omega_{p_2}^2\right)\Bigg)\Bigg] \Bigg|_0^\infty$$

$$= \frac{Z^2 e^2}{\pi c} \frac{\gamma}{3(\omega_{p_1}^2 - \omega_{p_2}^2)} \left[\pi\left(\omega_{p_1}^3 - 3\omega_{p_1}^2 \omega_{p_2} + 3\omega_{p_1}\omega_{p_2}^2 - \omega_{p_2}^3\right) \right]$$

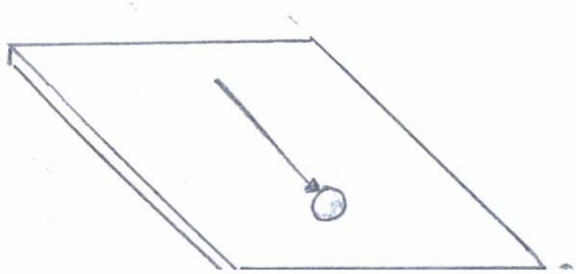
$$+ \lim_{y \to \infty} y \Bigg[\left(3\omega_{p_1}^2 + 3\omega_{p_2}^2 + 2y^2\right) \log\left(\frac{\omega_{p_1}^2 + y^2}{\omega_{p_2}^2 + y^2} \right)$$

$$- 2\left(\omega_{p_1}^2 - \omega_{p_2}^2\right) \Bigg]$$

$$= \frac{Z^2 e^2}{\pi c} \frac{\gamma \pi (\omega_{p_1} - \omega_{p_2})^3}{3(\omega_{p_1}^2 - \omega_{p_2}^2)}$$

$$= \frac{Z^2 e^2}{3c} \frac{(\omega_{p_1} - \omega_{p_2})^2}{\omega_{p_1} + \omega_{p_2}} \cdot \gamma$$

13.13.



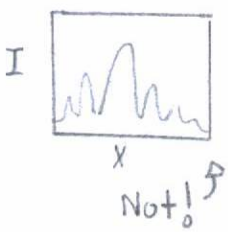
"relativistic particle traverses ... electric foil"

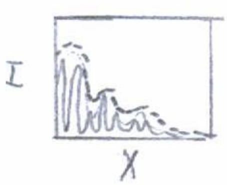
$$= \frac{v}{2\theta}\left(1 + \frac{1}{v^2} + \eta\right)$$

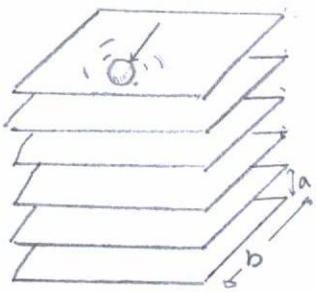
$$F = \sin^2)\left[\frac{v}{2\theta}\left(1 + \frac{1}{v^2} + \eta\right)\frac{a}{4D}\right]$$

$$= \sin^2\left[\frac{a}{2}\left(\frac{\omega}{v} - k\cos\theta\right)\right]$$

13.14







"Transition radiation ... emitted by a relativistic particle... array .... dielectric foil"

a) "Multipole reflections"

$$\left|\frac{n(\omega) - 1}{n(\omega) + 1}\right| \cong \frac{\omega_p^2}{4\omega^2} << 1$$

$$\epsilon(\omega, z) = 1 - (\omega_p^2/\omega^2)\,\rho(x)$$

(13.84) "Energy distribution"

$$\frac{d^2I}{d\nu\, d\eta} = \frac{\pi}{\gamma^2}\gamma\omega_p \frac{d^2I}{d\omega\, d\Omega}$$

$$\cong \frac{z^2e^2\gamma\omega_p}{\pi c}\left[\frac{\eta}{\nu^4\left(1 + \frac{1}{\nu^2} + \eta\right)(1+\eta)^2}\right]$$

... where $\eta = (\gamma\theta)^2$

Citation: G.M. Gariban, Z.h. EKsp Teor "Contributions to the theory of formation of transition X-radiation in a stack of plates" Fig 60.39 (1970)

$$(13.84)\quad \frac{d^2I}{d\nu\,d\eta} = \frac{\pi}{\gamma^2}\gamma\omega_p\frac{d^2I}{d\omega\,d\Omega}$$

$$= \frac{z^2e^2\gamma\omega_p}{\pi c}\left[\frac{\eta}{\nu^4\left(1+\frac{1}{\nu^2}+\eta\right)^2(1+\eta)^2}\right]$$

Book calls the function angular distribution or power spectrum. Preference resides with "current per angle scaled by velocity"

$F = 4\sin^2\theta$ with $\theta = \nu\left(1+\frac{1}{\nu^2}+\eta\right)\frac{a}{4D}$

... where $D = \gamma c/\omega_p$, $\nu = \omega/\gamma\omega_p$, $\eta = (\gamma\theta)^2$

If $a \gg D$, $\gamma \gg 1$, $\theta \ll 1$, $\omega \gg \omega_p$

and $\frac{\omega}{v} - \cos\theta = k\left(\frac{\omega}{kv} - 1 + \frac{\theta^2}{2}\right) = k\left(\frac{1}{\beta\sqrt{\epsilon(\omega)}} - 1 + \frac{\theta^2}{2}\right)$

$$= k\left(\left[(1-1/\gamma^2)\left(1-\frac{\omega_p^2}{\omega^2}\right)\right]^{1/2} - 1 + \theta^2/2\right)$$

$$= k\left(1 + \frac{1}{2\theta^2} + \frac{\omega_p^2}{2\omega^2} - 1 + \frac{\theta^2}{2}\right)$$

$$= \frac{k}{2\gamma^2}\left(1 + \frac{\gamma^2\omega_p^2}{\omega^2} + \gamma^2\theta^2\right)$$

$$= \frac{\omega}{2\gamma^2 c}\left(1 + \frac{1}{\nu^2} + \eta\right)$$

$$= \frac{\omega_p}{2\gamma c}\frac{\omega}{2\omega_p}\left(1 + \frac{1}{\nu^2} + \eta\right)$$

Important considerations:

1) Frequency is far from plasma frequency

2) No backward emissions

3) No reflection between layers

4) Fast and Furious velocity.

In equations, $\omega \gg \omega_p$, $\gamma \gg 1$

13.15.

a) (13.85) "Energy spectrums"

$$\frac{dI}{d\nu} = \frac{ze^2 \gamma \omega_p}{\pi c}\left[(1+2\nu^2)\ln\left(1+\frac{1}{\nu^2}\right) - 2\right]$$

If $\gamma \gg 1$, $\frac{dN}{d\nu} = \frac{1}{\hbar\omega}\frac{dI}{d\nu}$

$$N = \int_{1/\gamma}^{\infty} \frac{ze^2}{\hbar \pi c}\frac{1}{\nu}\left[(1+2\nu^2)\ln\left(1+\frac{1}{\nu^2}\right)-2\right]d\nu$$

$$\int_a^{\infty}\left[(1+2x^2)\log\left(1+\frac{1}{x^2}\right)-2\right]\frac{dx}{x} = 1 - \frac{1}{2}\mathrm{Li}_2\left(-\frac{1}{a^2}\right) - (1+a^2)\log\left(1+\frac{1}{a^2}\right)$$

where $\mathrm{Li}_2 = -\frac{1}{2}\log^2(-z) - \frac{\pi^2}{6} - \sum_{k=1}^{\infty}\frac{1}{k^2 x^k}$

"Dilogarithmic Function"

$$N_\gamma = \frac{z^2e^2}{\hbar\pi c}\left[1 - \frac{1}{8}\mathrm{Li}_2(-\gamma^2) - \left(1+\frac{1}{\gamma^2}\right)\log(1+\gamma^2)\right]$$

$$= \frac{z^2e^2}{\hbar\pi c}\left[1 + \frac{1}{4}\log^2(\gamma^2) + \frac{\pi^2}{12} + \frac{1}{2}\sum_{k=1}^{\infty}\frac{(-1)^k}{k^2 x^k} - \left(1+\frac{1}{\gamma^2}\right)\log(1+\gamma^2)\right]$$

Citation: X. Artu, G. B. Yodh, and G. Mennessi on "Practiced theory of the multiplied transition relativistic detector." Phys. Rev D. 12 1289 (1975)

If (13.84) indicates a single foil diffraction,

$$\left(\frac{d^2I}{d\nu d\eta}\right)_{multiple} = \left(\frac{d^2I}{d\nu d\eta}\right)_{single} \cdot F$$

interface factor

$$F = \left| \mu \int dz\, \rho(z) \cdot e^{i w z v} \cdot e^{-i\cos\theta \int_0^z R(z) \cdot dz} \right|^2$$

b) (Problem 13.13) "Frequency spectrum"

$$F = 4\sin^2\theta \quad \text{where } \theta = \nu\left(1+\frac{1}{\nu^2}+\eta\right)\frac{a}{4D}$$

Papers extend F as :

$$F = \left| \frac{1-e^{i(\theta+\psi)N}}{1-e^{i(\theta+\psi)}} \right|^2 = 4\sin^2\theta \frac{\sin^2(N(\theta+\psi))}{\sin^2(\theta+\psi)}$$

when $\psi = \nu(1+\eta)(b/4D)$

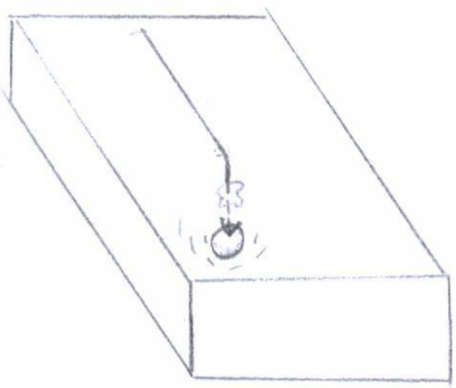
$$= \frac{z^2 e^2}{\hbar \pi c}\left[1 + \log^2\gamma + \frac{\pi^2}{12} - 2\log\gamma + O\left(\frac{1}{\gamma^2}\right)\right] - \frac{z^2 e^2}{\hbar \pi c}\left[\log(\nu-1)^2 + \frac{\pi^2}{12}\right]$$

b) If $\hbar\omega_p = 20\text{eV}$, $\gamma = 10^3$, $N=$

$\gamma = 10^4$, $N=$

$\gamma = 10^5$, $N=$

13.16.



"A highly relativistic neutral particle... emits radiation as it crosses at right angles a plane interface... into dielectric medium"

a) (13.79) "Energy radiated"

$$\frac{d^2 I}{d\omega\, d\Omega} = \frac{c}{32\pi^3}\left(\frac{\omega_p}{c}\right)^4 \left| \int_{z>0} \left[k \times E(x,\omega)\right] \times \hat{k}\, e^{-ik\cdot x}\, d^3x \right|^2$$

(13.80) "Incident fields"

$$E_\rho(x,\omega) = \sqrt{\frac{2}{\pi}}\frac{ze\omega}{\gamma v^2} e^{i\omega z/v} \cdot K_1\left(\frac{\omega\rho}{\gamma v}\right)$$

$$E_z(x,\omega) = -i\sqrt{\frac{2}{\pi}}\frac{ze\omega}{\gamma^2 v^2} e^{i\omega z/v} \cdot K_0\left(\frac{\omega\rho}{\gamma v}\right)$$

( )

$$B = \mu \frac{3n(n\cdot z) - z}{r'^3}$$

where $n' = \frac{1}{r'}(\rho', 0, z')$, $r' = \sqrt{\rho'^2 + z'^2}$

$$= \frac{\mu}{r'^3}\left[\frac{3}{r'}(\rho', 0, z') \cdot \frac{z}{r} - (0,0,1)\right]$$

$$= \frac{\mu}{r'^3}\left[3\rho' z', 0, 3z'^2 - r'^2\right]$$

$$B_\rho' = \mu \frac{3\rho' z'}{[\rho'^3 + z^2]^{3/2}} \qquad B_{z'} = \frac{\mu 3z'^2 + \rho'^2}{[\rho'^2 + z'^2]^{3/2}}$$

Rest Frame:

$$B_\rho' = \mu \frac{3\rho(\gamma)(z v t)}{[\rho^2 + \gamma^2[z - vt]^2]^{5/2}} \qquad B_\rho = \gamma B_\rho'$$

$$E_\phi = -\beta B_\rho$$

$$= -\beta B_\rho$$

$$= -\beta \mu \frac{3\rho \gamma'(z - vt)}{[\rho^2 + \gamma^2(z - vt)^2]^{3/2}}$$

$$E_\phi(\kappa, \omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} E_\phi(x, t)\, \rho^{i\omega t}\, dt$$

$$= \beta\mu\left(\frac{\partial}{\partial z}\right)\left\{\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{\rho e^{i\omega t}}{\rho^2 + \gamma^2(z - vt)^{3/2}}\, dt\right\}$$

$$= \beta\mu\left(\frac{\partial}{\partial z}\right)\left\{\sqrt{\frac{2}{\pi}}\, e^{i\omega z/v} K_1\left(\frac{\omega\rho}{\gamma v}\right)\right\}$$

$$= \frac{\beta\mu}{\gamma z e}\left(\frac{\partial}{\partial z}\right)\left\{\sqrt{\frac{2}{\pi}}\, e^{i\omega t/v} \frac{z e \omega}{\gamma v^2} K_{11}\left(\frac{\omega\rho}{\gamma v}\right)\right\}$$

Note: Proportional derivation with B-field in (13.80).

Identity:

$$\int_0^\infty \frac{\cos(dt)}{(b^2 + t^2)^{3/2}}\, dt = \frac{d}{b} K_1(ab)$$

b) Bohr Magneton: $\mu_B = e\hbar / 2m_e c$

Citation: Bohr, N. (1913)
"On the conservation of atoms and molecules
London, Edinburgh, and Dublin. 26 (151) 1-25

$\hat{\epsilon}_a = \cos\theta\, e_x - \sin\theta\, e_z \qquad e_b = \hat{e}_y$

$\vec{E}_\phi = E_\phi \hat{\Phi}$

$= -E_\phi \sin\phi\, e_x + E_\phi \cos\phi\, e_y$

$= -E_\phi \cos\theta \sin\phi\, \hat{e}_a + E_\phi \cos\phi\, \hat{\epsilon}_b$

(13.90.5) "Field"

$$F = \int_{z>w} [\vec{k} \times E(x,\omega)] \times \hat{k}\, e^{-ikx} d^3x$$

where $[\vec{k} \times E(x,\omega)] \times \hat{k} = -E_\phi \cos\theta \sin\phi\, \epsilon_a + E_\phi \cos\phi\, \epsilon_b$

$$E_\phi = -\frac{\beta\mu}{\gamma ze} \frac{i\omega}{v} \sqrt{\frac{2}{\pi}} \frac{ze\omega}{\gamma v^2} K_1\left(\frac{\omega p}{\gamma v}\right)$$

$$F = \frac{i}{\frac{\omega}{v} - k\cos\theta} \iint dx\,dy\, [\vec{k} \times E] \times \hat{k}\, e^{-ikx\sin\theta}$$

$$= \frac{-\mu\omega/\gamma zec}{\frac{\omega}{v} - k\cos\theta} \iint dx\,dy\, e^{-ikx\sin\theta} \left[-\cos\theta\sin\phi\,\epsilon_a + \cos\phi\,\epsilon_b\right]$$

$$\times \sqrt{\frac{2}{\pi}} \frac{z\omega}{\gamma v^2} K_1\left(\frac{\omega}{\gamma v}\right)$$

$$= \epsilon_b \left(\frac{-\mu\omega}{\gamma zec}\right) \frac{2\sqrt{2\pi}\, ze \sin\theta k}{v\left(\frac{\omega}{v} - k\sin\theta\right)\left(\frac{\omega^2}{\gamma^2 v^2} + k^2\sin^2\theta\right)}$$

(13.93) "Field"

$$F = \epsilon_a 4\sqrt{2\pi} \frac{ze}{c} \left(\frac{c}{\omega_p}\right)^2 \frac{\gamma}{v^2} \frac{\sqrt{\eta}}{\left(1 + \frac{1}{v^2} + \eta\right)(1+\eta)}$$

where $\eta = (\gamma\theta)^2$

The derivation above and (13.92) match when $k \approx k\cos\theta - \omega/v\gamma^2$ and with a multiple from the problem $(\mu\omega/ze\gamma c)^2$

c) If $z=1$,

$$\hbar\omega_0 = e^2/a_0 = 27.2\,eV$$

$$v = \omega/\gamma\omega_p$$

$$\frac{dI_H}{dI_v} = \left(\frac{\mu\omega}{e\gamma c}\right)^2$$

$$= \left(\frac{\mu}{\mu_B}\right)^2 v^2 \left(\frac{\mu_B\,\omega_p}{ec}\right)^2$$

$$= \frac{\alpha^4}{4}\left(\frac{\mu}{\mu_B}\right)^2\left(\frac{\hbar\omega_p}{\hbar\omega_0}\right)^2 v^2$$

d) $$\frac{I_H}{I_v} = \frac{\alpha^4}{4}\left(\frac{\mu}{\mu_B}\right)^2\left(\frac{\hbar\omega_p}{\hbar\omega_0}\right)^2 v^2$$

If $G = v^2/5$,

$$\frac{I_H}{I_v} = \frac{\alpha^4}{20}\left(\frac{\mu}{\mu_B}\right)^2\left(\frac{\hbar\omega_p}{\hbar\omega_0}\right)^2 G(v_{max})$$