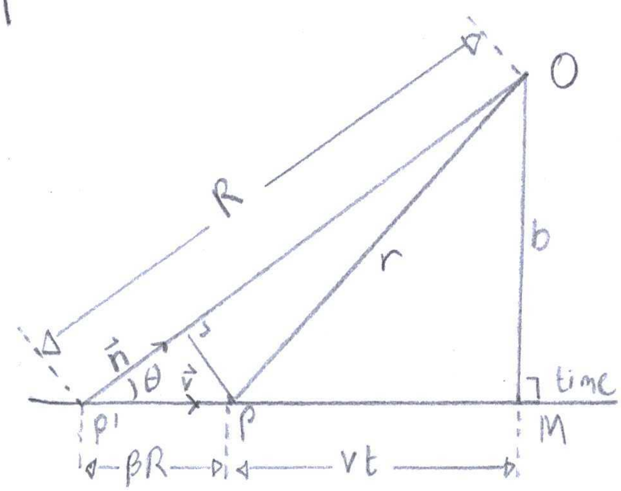
Chapter 14: Radiation by Moving Charges:

14.1



"Figure 14.2"

$\overline{P'Q} = \beta R\cos\theta$

$= \beta \cdot n R$

$\overline{OQ} = (1-\beta\cos\theta)R$

$= (1-\beta\cdot n)R$

$\overline{MO} = b$

$= R\sin\theta$

$\overline{OQ}^2 = \overline{OP}^2 - \overline{PQ}^2$

$= r^2 - \beta^2 R^2 \sin^2\theta$ ← It's the law, Pythagoreus' Law

$= b^2 + v^2t^2 - \beta^2 b^2$

$= \frac{1}{\gamma^2}(b^2 + \gamma^2 v^2 t^2)$

(11.152) "Transverse Component"

$$E = \frac{e\gamma b}{(b^2+\gamma^2 v^2 t^2)^{3/2}}$$

$$= e\left[\frac{b}{\gamma^2(1-\beta\cdot n)^3 R^3}\right]$$ ∴ when $\overline{OQ}^2$ is $(1-\beta\cdot n)R$

(14.14) "Velocity fields"

$$E(x,t) = e\left|\frac{n-\beta}{\gamma^2(1-\beta\cdot n)^3R^3}\right| + \frac{e}{c}\left[\frac{n\times[(n-\beta)\times\dot\beta]}{(1-\beta\cdot n)^3 R}\right]$$

14.2

a) "Taylor Expansion"

e

"Particle moving with nearly nonrelativistic motion"

$$X(t') = X + V(t'-t_0) + \frac{1}{2}a(t'-t_0)^2 + \cdots$$
$$\cong \frac{1}{2}a(t'-t_0)^2$$

$$\beta(t) = \frac{1}{c}a(t'-t_0) + \cdots$$

$$\dot{\beta}(t) = \frac{1}{c}a + \cdots$$

"retarded time"

$$t = t' + |X - r(t)|/c$$
$$= t' + X/c + \cdots$$

$$t' = t_0 - X/c$$

$$r = \frac{X^2}{2c^2}a \; ; \; \beta = -\frac{X}{c^2}a \; ; \; \dot{\beta} = \frac{1}{c}a$$

$$R = X - r$$
$$= X - \frac{X^2}{2c^2}a$$
$$= X\left(1 - \frac{X}{2c^2}\cdot a\right)$$

$$\hat{n} \equiv X - R$$
$$\equiv X - \frac{X}{2c^2}\left[a - \hat{x}(\hat{x}\cdot a)\right]$$

(14.14) "Velocity Fields"

$$E(X,t) = e\left[\frac{n-\beta}{\gamma^2(1-\beta\cdot n)^3 R^2}\right]_{ret} + \frac{e}{c}\left[\frac{n\times[(n-\beta)\times\dot{\beta}]}{(1-\beta\cdot n)^3 R}\right]_{ret}$$

(1.1a) "Faraday's Law"

$$\nabla \times E - \frac{1}{c}\frac{\partial B}{\partial t} = 0$$

$$\frac{\partial B}{\partial t} = -c\nabla \times E$$
$$= -\frac{e}{c}\frac{\hat{r} \times a}{r^2}$$
$$B = -\frac{e}{c}\frac{\hat{r} \times (a(t-t_0))}{r^2}$$
$$= -\frac{e}{c}\frac{\hat{r} \times v(t)}{r^2}$$
$$= \frac{e}{c}\frac{v(t) \times \hat{r}}{r^2}$$

(5.5) "Bio and Savart Law"

$$B = kq\frac{v \times x}{|x|^3}$$

$$= \frac{e}{c}\frac{v \times x}{x^{3/2}}$$

...(pg 176) $k = 1/c$ in empirical searches about the constant (k)

The problem derived a similar answer to the book.

<u>Curl of Magnetic field:</u>

$$\nabla \times E = \nabla \times \left(\frac{er}{r^3}\right) - \frac{e}{2c^2}\nabla \times \left(\frac{a}{r} + \frac{r(r \circ a)}{r^3}\right)$$
$$= -\frac{e}{2c^2}\left(\nabla\left(\frac{1}{r}\right) \times a + \frac{1}{r^2}\nabla(r \circ a) \times \hat{r}\right)$$
$$= -\frac{e}{2c^2x^2}\left(-\hat{r} \times a + a \times \hat{r}\right)$$
$$= \frac{e}{c^2x^2}\hat{x} \times a$$

$$= e\left[\frac{X - \frac{\hat{X}}{2c^2}[a - \hat{X}(\hat{X}\cdot a)] + \frac{X}{c^2}a}{X^2(1 - \frac{\hat{X}}{2c^2}\circ\hat{X}a)(1 + \frac{\hat{X}}{c^2}\hat{X}\circ a)}\right] + \frac{e}{c}\left[\frac{X\cdot(\hat{X}\cdot\frac{1}{c}a)}{\gamma}\right]$$

$$= \frac{e\hat{X}}{X^2} + \frac{e}{2e^2\gamma}[a - 3\hat{X}(\hat{X}\circ a)] - \frac{e}{c}\frac{\hat{X}\cdot(\hat{X}\cdot\frac{1}{c}a)}{\gamma}$$

$$= E_v + E_a$$

$$E_{TOT} = \frac{e\hat{X}}{X^2} - \frac{e}{2c^2\gamma}[a + \hat{X}(\hat{X}\circ a)]$$

"Static coulomb term $(1/X^2)$"

"Dynamic relativistic term"

b) (14.13) "Velocity field"

$$B = [n \times E]_{ret}$$

$$= \left(X - \frac{X}{2c^2}[a - \hat{X}(\hat{X}\circ a)]\right)\circ\left(\frac{e\hat{X}}{X^2} - \frac{e}{2c^2X}[a + \hat{X}(X\circ a)]\right)$$

$$= -\frac{e}{2c^2X}(a\circ X + X\circ a)$$

$$= 0$$

The particle has no magnetic induction.

c) Zero-divergence Electric field:

$$\nabla\circ\left(\frac{a + r(r\circ a)}{r}\right) = \nabla\circ\left(\frac{a}{r}\right) + \nabla\circ\left(\frac{r(r\circ a)}{r^3}\right)$$

$$= \nabla\left(\frac{1}{r}\right)\circ a + \nabla\left(\frac{1}{r^3}\right)(r(r\circ a))$$

$$+ \frac{1}{r^3}\nabla\circ(r(r\circ a))$$

$$= -\frac{1}{r^2}\hat{r}\circ a - \frac{3}{r^2}\hat{r}\circ a + \frac{4}{r^2}\hat{r}\circ a$$

$$+\frac{1}{(1-\beta\circ n)^3R^2}\Big[n\circ(n\circ\beta)^3-\beta^2(n\circ\beta)-\beta(n\circ\beta)^2+\beta^2\beta+$$

$$3n(n\circ\beta)^2-2\beta(n\circ\beta)-\beta n-3n(n\circ\beta)^3$$

$$+2\beta(n\circ\beta)^2+\beta^2 n(n\circ\beta)\Big]$$

$$=\frac{n\times(n-\beta)-\dot{\beta}}{c(1-\beta n)^3R}+\frac{1}{(1-\beta n)^3R^2}\Big[-2n(n\circ\beta)^3+\beta(n\circ\beta)^2+\beta^2\beta$$

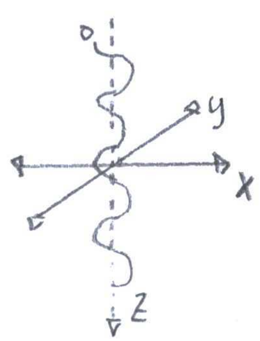
$$+3n(n\circ\beta)^2-2\beta(n\circ\beta)-\beta^2\circ n\Big]$$

$$E_{ret}=e\left[\frac{n}{R^2}\right]+e\left[\frac{R}{c}\right]+\frac{n(n\circ v)-v}{R}+e\,\frac{3n(n\circ\beta)-\beta}{(1-\beta\circ n)R^2}$$

$$=\frac{n-\beta}{\gamma^2(1-\beta\circ n)^2R^2}$$

$$E(x,t)=e\,E_{ret}+\frac{e}{c}\left[\frac{n\times(n-\beta)\times\dot{\beta}}{(1-\beta\circ n)^3R}\right]_{ret}$$

14.4



"particle on z-axis"

a) $z(t)=a\cos(\omega t)$

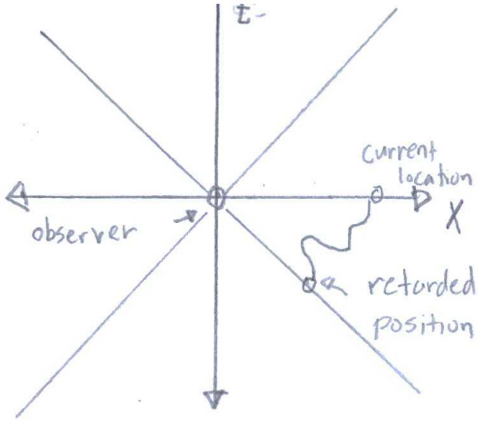
(14.21) "time-averaged Power radiated per solid angle"

$$\frac{dP(t)}{d\Omega}=\frac{e^2}{4\pi c^3}|\dot{v}|^2\sin^2\Theta$$

$$=\frac{e^2a^2\omega_0^4}{4\pi c^3}\cos^2(\omega_0 t)\sin^2\Theta$$

$$\left\langle\frac{dP(t)}{d\Omega}\right\rangle=\frac{e^2a^2\omega_0^4}{8\pi c^3}\sin^2\Theta \quad \ldots \text{ when } \langle\cos^2(\omega_0 t)\rangle=1/2$$

14.3



"retarded time"

(14.14) "Lienard-Wiechart"

$$E(x,t) = e\left[\frac{n-\beta}{\gamma^2(1-\beta\circ n)^3 R^2}\right]_{ret} + \frac{e}{c}\left[\frac{n\times[(n-\beta)\times\mathring{\beta}]}{(1-\beta\circ n)^3 R}\right]_{ret}$$

(14.13) "Lienard-Wiechart field"

$$B = [n\times E]_{ret}$$

$$\text{where } E_{ret} = e\left[\frac{n}{R^2}\right]_{ret} + e\left[\frac{R}{c}\right]_{ret} + \frac{d}{dt}\left[\frac{n}{R^2}\right]_{ret} + e\frac{1}{c^2}\frac{d^2}{d^2t}\left[n\right]_{ret}$$

From (14.3),

$$\frac{dt}{dt'} = 1-\beta\circ n$$

3rd term: $\frac{d}{dt}\left[\frac{n}{R^2}\right] = \frac{n(n\circ v)-v}{R}$

4th term: $\frac{1}{c^2}\frac{dt'}{dt}\frac{dn}{dt} = \frac{1}{c^2}\left[\frac{1}{1-\beta\circ n}\right]\left[\frac{d}{dt'}\frac{1}{1-\beta\circ n}\frac{d}{dt'}\hat{n}\right]$

$$= \frac{1}{c^2}\left[\frac{1}{1-\beta\circ n}\cdot(\mathring{\beta}\circ n + \beta\circ\mathring{n})\frac{d}{dt}n + \frac{1}{1-\beta\circ n}\frac{d^2}{d^2t}n\right]$$

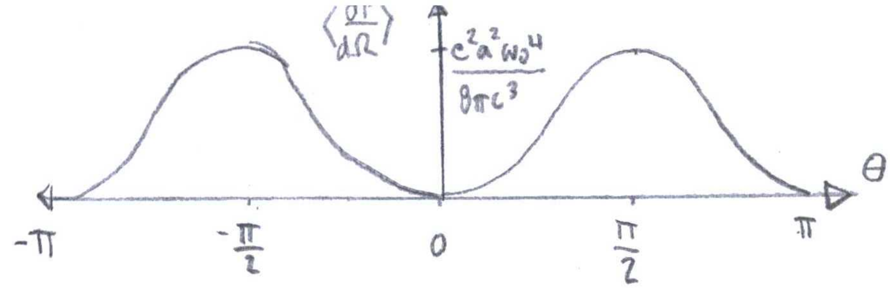
$$= \frac{1}{c^2}\left[\frac{1}{(1-\beta\circ n)^3}\left(n\circ\mathring{\beta} + \frac{(n(n\circ v)-v)}{R}\beta\right)\frac{n(n\circ v)-v}{R}\right.$$

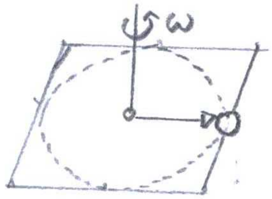
$$\left. + \frac{1}{(1-\beta\circ n)^2}\left[\frac{n\times(n\times v)}{R} + \frac{3n(n\circ v)^2 - 2v(n\circ v) - v^2 n}{R^2}\right]\right.$$

$$= \frac{1}{cR}\left[\frac{n\circ\mathring{\beta}}{(1-\beta\circ n)^3}\left[n(n\circ\beta)-\beta\right] + \frac{n\times(n\times\mathring{\beta})}{(1-\beta\circ n)^2}\right]$$

$$+ \frac{1}{R^2}\frac{[(n(n\circ\beta)-\beta)\beta][n(n\circ\beta)-\beta]}{(1-\beta\circ n)^3} + \frac{3n(n\circ\beta)^2 - 2\beta(n\circ\beta) - \beta^2 n}{(1-\beta\circ n)^2}$$

$$= \frac{1}{c(1-\beta\circ n)^3 R}\left[n\times(n\times\mathring{\beta}) - n\times(\beta\times\mathring{\beta})\right]$$





"particle in plane"

b. $x(t) = R\langle \cos(\omega_0 t), \sin(\omega_0 t)\rangle$

$$\mathring{v}(t) = -\omega_0^2 R \langle \cos(\omega_0 t), \sin(\omega_0 t)\rangle$$

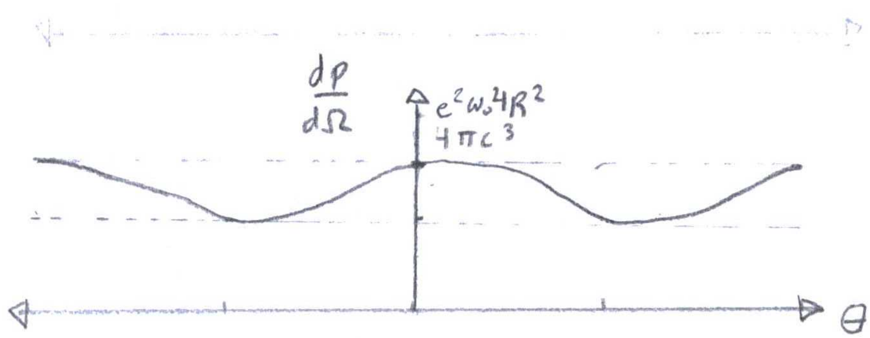
$$\frac{dP}{d\Omega} = \frac{e^2}{4\pi c^3} |\mathring{v}|^2 \sin^2\theta$$

$$= \frac{e^2 \omega_0^4 R^2}{4\pi c^3}\left(1 - \sin^2\theta \cdot \cos^2(\phi - \omega t)\right)$$

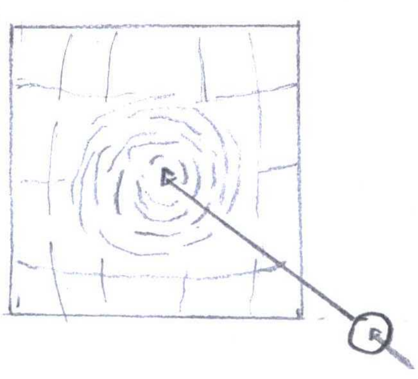
$$\left\langle \frac{dP}{d\Omega} \right\rangle = \frac{e^2 \omega_0^4 R^2}{4\pi c^3}\left(1 - \frac{1}{2}\sin^2\theta\right) \quad \ldots \text{when } \cos^2(\phi - \omega t) = 1/2$$

Total Power:

$$P = \int \frac{e^2 \omega_0^4 R^2}{4\pi c^3}\left(1 - \frac{1}{2}\sin^2\theta\right) d\Omega$$



14.5.



"Nonrelativistic particle ... head on collision with ... a force field"

a) (14.22) "Total Instantaneous Power radiated"

$$P\,dt = \frac{2}{3}\frac{z^2 e^2}{c^3}|\dot{v}|^2 dt$$

$$F = ma$$
$$= m\dot{v}$$
$$= -\nabla V$$
$$= -\frac{dV}{dr}\hat{e}_r$$

$$E = \frac{1}{2}mv^2 + V(r)$$
$$= V(r_{min})$$

$$v = \sqrt{\frac{2}{m}}\left(V(r_{min}) - V(r)\right)^{1/2}$$

$$P\,dt = \frac{2}{3}\frac{z^2 e^2}{m^2 c^3}\sqrt{\frac{m}{2}}\left|\frac{dV}{dr}\right|^2 \frac{dr}{\sqrt{V(r_{min}) - V(r)}}$$

(14.51 & 14.53) "Total energy radiated"-modified"

$$\Delta W = \ldots$$

$$= \frac{4}{3}\frac{z^2 e^2}{m^2 c^3}\sqrt{\frac{m}{2}}\int_{V_{min}}^{\infty}\left|\frac{dV}{dr}\right|^2 \frac{dr}{\sqrt{V(r_{min}) - V(r)}}$$

b) If $V(r) = zZe^2/r$

$$\Delta W = \frac{4}{3}\frac{z^2 e^2}{m^2 c^3}\sqrt{\frac{m}{2}}(zZe^2)^{3/2}\int_{r_{min}}^{\infty}\frac{1}{r^4}\frac{dr}{\sqrt{\frac{1}{r_{min}} - \frac{1}{r}}}$$

$$= \frac{4}{3}\frac{z^2e^2}{m^2c^3}\sqrt{\frac{m}{2}}(zZe^2)^{3/2} r_{min} \int_{r_{min}}^{\infty} r^{-7/2} \frac{dr}{\sqrt{\frac{1}{r_{min}} - \frac{1}{r}}}$$

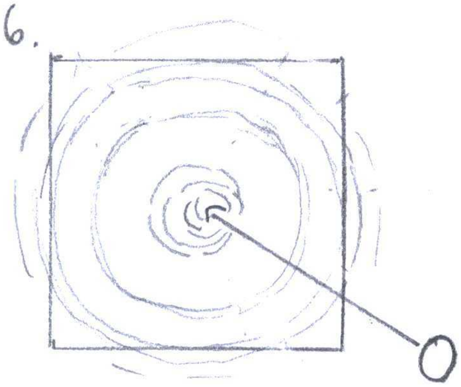
$$= \frac{4}{3}\frac{z^2e^2}{m^2c^3}\sqrt{\frac{m}{2}}(zZe^2)^{3/2} r_{min} \frac{16}{15 r_{min}^3}$$

$$\boxed{\text{Identity: } \int_a^{\infty} \frac{x^{-7/2}}{\sqrt{x-a}}dx = \frac{16}{15a^3}}$$

$$= \frac{64}{45}\sqrt{\frac{m}{2}}\frac{(zZe^2)^{3/2}}{m^2c^3}\left(\frac{2 \cdot zZe^2}{mv_0^2}\right)^{-5/2}$$

$$= \frac{8}{45}\frac{z m v_0^5}{Z c^3}$$

14.6.



"Nonrelativistic particle.. head on collision with ... a .. force field .. total energy radiated"

a) (Problem 14.6)

$$E = V - L^2/2mr^2$$

.. where $L = mbv_0$

$E = mv_0^2/2$

(Problem 14.5)

$$E = \frac{1}{2}m\dot{r}^2 + \frac{1}{2}mr^2\dot{\theta}^2 + V(r)$$

$$= \frac{1}{2}m\dot{r}^2 + \frac{L^2}{2mr^2} + V(r)$$

$$\dot{r} = \sqrt{\frac{2}{m}}\left[E - V(r) - \frac{L^2}{2mr^2}\right]^{1/2}$$

(14.51 & 14.53) "Total Power radiated - modified"

$$\Delta W = \frac{4ze^2}{3m^2c^3}\sqrt{\frac{m}{2}}\int_{r_{min}}^{\infty}\left|\frac{dV}{dr}\right|^2\left[E - V(r) - \frac{L^2}{2mr^2}\right]^{-1/2} dr$$

b) (Problem 14.6)

$$E = V - L^2/2mr^2$$

where $V = zZe^2/r$

$L = mbv_0$

$E = \frac{1}{2}mv_0^2$

$$\Delta W = \frac{4z^2e^2}{3m^2c^3}\sqrt{\frac{m}{2}}\int_{r_{min}}^{\infty}\left(\frac{dV}{dr}\right)^2\left(E - V(r) - \frac{L^2}{2mr^2}\right)^{-1/2} dr$$

$$= \frac{4z^4Z^2e^6}{3m^2c^3v_0}\int_{r_{min}}^{\infty} r^{-4}\left(1 - 2\frac{zZe^2}{mv_0^2 r} - \frac{b^2}{r^2}\right)^{-1/2} dr$$

$$= \frac{4zmv_0^5}{3Zc^3t^3}\int_{r_{min}}^{\infty}\left(\frac{b}{r}\right)^2\left(1 - 2\frac{b}{tr} - \frac{b^2}{r^2}\right)^{-1/2}\frac{b\,dr}{r^2} \quad \ldots \text{if } t = \frac{bmv_0^2}{zZe^2}$$

$$= \frac{4zmv_0^5}{3Zc^3t^3}\int_0^{x_{max}} \frac{x^2}{\sqrt{1 - 2(x/t) - x^2}}\,dx \quad \ldots \text{if } x = \frac{b}{r}$$

$$= \frac{4zmv_0^5}{3Zc^3t^3}\int_0^{-\frac{1}{t}+\sqrt{\frac{1}{t^2}+1}} \frac{x^2}{\sqrt{\left(x - \left(-\frac{1}{t} - \sqrt{\frac{1}{t^2}+1}\right)\right)\left(\left(-\frac{1}{t} + \sqrt{\frac{1}{t^2}+1}\right) - x\right)}}\,dx$$

$$= \frac{4zmv_0^5}{3Zc^3t^3}\left[-\frac{3}{2t} + \left(\frac{3}{t^2}+1\right)\tan^{-1}\left(-\frac{1}{t} + \sqrt{\frac{1}{t^2}+1}\right)\right]$$

$$= \frac{4zmv_0^5}{3Zc^3t^3}\left[-\frac{3}{2t} + \frac{1}{2}\left(\frac{3}{t^2}+1\right)\tan^{-1}t\right] \quad \ldots \text{if } t \gg 1$$

$$= \frac{2zmv_0^5}{Zc^3}\left(-\frac{1}{t^4} + \frac{1}{t^5}\left(1 + \frac{t^2}{3}\right)\tan^{-1}t\right)$$

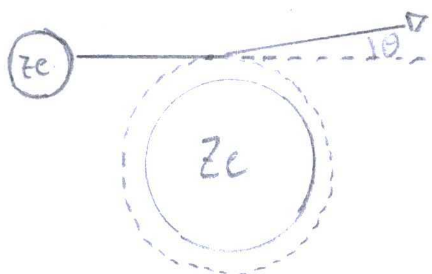
c) With $t = \cot\theta/2$; $\theta = 2\cdot\cot^{-1}(t)$

$$\cong \frac{2zmv_0^5}{Zc^3}\tan^3\frac{\theta}{2}\left[\frac{1}{6}(\pi - \theta)\left(1 + 3\tan^2\frac{\theta}{2}\right) - \tan\frac{\theta}{2}\right]$$

Only for $t \leq 1\mathring{o}$

d) An attractive Coulomb potential is important to chemists and physicists. A common depiction is Lennard-Jones Potential. The term $\left(e^2/4\pi\epsilon_0 r\right)$ describes attraction between two charges, including O_2, H_2, N_2, F_2, Cl_2 and Br_2.

14.7.



"A nonrelativistic particle... incident... impacts... particle deflection... very small"

a) (14.22) "Larmor Power Formula"

$$P = \frac{2}{3}\frac{e^2}{c^3}|\dot{v}|^2$$

(14.51) "General Power Formula"

$$\frac{dP}{d\Omega} = |A(t)|^2$$

(14.53) "Total energy radiated"

$$\frac{dW}{d\Omega} = \int_{-\infty}^{\infty} |A(t)|^2 dt$$

$$\Delta W = \int_{-\infty}^{\infty} P(t)\,dt$$

... book calls this ΔW

$$= 2\int_{r_{min}}^{\infty} \frac{P}{dr/dt}\,dr$$

If $E = K + V$

$$= \frac{1}{2}m\dot{r}^2 + V(r)$$

$$\frac{dr}{dt} = \sqrt{\frac{2(E-V)}{m}}$$

$$\Delta W = \frac{2}{3}\frac{e^2}{c^3}\sqrt{\frac{m}{2}}\int_{r_{min}}^{\infty}\left|\frac{zZe^2}{r^2}\right|\frac{dr}{\sqrt{E-V}}$$

$$= \frac{2}{3}\frac{e^2}{c^3}\sqrt{\frac{m}{2}}\, z^2Z^2e^2\int_{r_{min}}^{\infty}\frac{1}{r_4}\,dr$$

$$= \frac{2}{3}\frac{e^2}{c^3}\sqrt{\frac{m\, zZe^2}{2\, m}\cdot r_{min}}\cdot z^2Z^2e^2\,\frac{1}{r_{min}} \quad \therefore \text{when } r = \frac{ZzZe^2}{mv_0^2}$$

$$= \frac{4}{9}\frac{e^5z^3Z^3}{c^3\cdot V_0}\frac{1}{r_{min}}$$

$$= \frac{4}{9}\frac{z^3Z^3e^5}{c^3V_0}\frac{1}{b_{min}}$$

b) $r_c = 2zZe^2/mv_0^2$

$$\Delta W = \frac{\pi z^4 Z^2 e^6}{3m^2c^3V_0}\frac{1}{p^3}$$

$$= \frac{\pi z^4Z^2e^6}{3m^2c^3V_0}\frac{m^3V_0^5}{8e^6z^3Z^3} = \frac{\pi}{24}\frac{z\,m\,V_0^5}{c^3Z}$$

(Problem 14.5) "Head-on collision"

$$\Delta W = \frac{8}{45}\frac{zmv_0^5}{Zc^5}$$

$$\propto \text{constant}\cdot\frac{mv_0^5}{Z}$$

c) Radiation cross section:

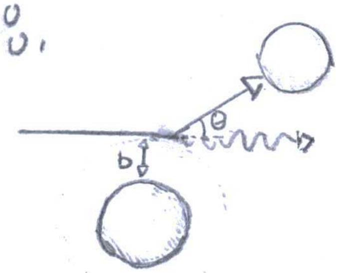
$$\chi = \int \Delta W(b)\cdot 2\pi b\, db$$

$$= \frac{E\pi^2\cdot z\,m\,V_0^5}{12\cdot c^3 Z}\frac{1}{b_{min}}$$

(15.30) "Bethe - Heitler formula"

$$\frac{dE_{rad}}{dx} = \frac{16}{3} N z \underbrace{\left(\frac{Ze^2}{\hbar c}\right) \frac{z^4 e^4}{mc^2}}_{\text{coefficient in cross section above.}} \int_0^1 \ln\left(\frac{1+\sqrt{1-x}}{\sqrt{x}}\right) dx$$

14.8.



"a swiftly moving particle... passes a fixed point charge... total energy radiated"

(12.1) "Equation of motion"

$$\frac{dP}{dt} = e\left[E + \frac{u}{c} \times B\right]$$

$$= \frac{d}{dt}(\gamma m v)$$

$$= \frac{d}{dt}(\gamma m c \beta)$$

$$= \gamma^3 m c \beta(\beta \cdot \dot{\beta}) + \gamma m c \dot{\beta}$$

$$= zeE$$

$$\gamma m c(\gamma^2 \beta^2 + 1)(\beta \cdot \dot{\beta}) = \gamma^3 m c \beta \cdot \dot{\beta}$$

$$= ze\, \beta \cdot E$$

$$\beta \cdot \dot{\beta} = \frac{ze}{\gamma^3 m c} \beta \cdot E$$

$$\dot{\beta} = (zeE - \gamma^2 m c \beta(\beta \cdot \dot{\beta})) / \gamma m c$$

$$= \frac{ze}{\gamma m c}(E - \beta(\beta \cdot E))$$

(14.26) "Lienard Result (1898)"

$$P = \frac{2}{3} \frac{e^2}{c} \gamma^2 \left[\dot{\beta}^2 - (\beta \times \dot{\beta})^2\right]$$

$$\dot{\beta}^2 = \frac{z^2 e^2}{\gamma^2 m^2 c^2}\left[|E|^2 - 2|\beta \cdot E|^2 + \beta^2 |\beta \cdot E|^2\right]$$

$$|\beta \times \dot{\beta}|^2 = \frac{z^2 e^2}{\gamma^2 m^2 c^3}|\beta \times E|^2$$

$$= \frac{z^2 e^2}{\gamma^2 m^2 c^2}\left[\beta^2 E^2 - |\beta \cdot E|^2\right]$$

$$P(t) = \frac{2}{3}\frac{z^4 e^4}{m^2 c^3}\gamma^4\left[|E|^2 - 2|\beta\cdot E|^2 + \beta^2|\beta\cdot E|^2 - \beta^2|E|^2 + |\beta\cdot E|^2\right]$$

$$= \frac{2}{3}\frac{z^4 e^4}{m^2 c^3}\gamma^4(1-\beta)\left[|E|^2 - |\beta\cdot E|^2\right]$$

If $|E| = \dfrac{Ze}{r^2+b^2}$ and $|\beta\cdot E| = \beta E\cos\theta$

$$= \frac{Ze}{r^2+b^2}\frac{\beta r}{(r^2+b^2)^{1/2}}$$

$$P(t) = \frac{2}{3}\frac{z^4 Z^2 e^6}{m^2 c^3}\gamma^2\frac{1}{(r^2+b^2)^2}\left(1 - \frac{\beta^2 r^2}{r^2+b^2}\right)$$

(14.51 & 14.53) "Total Power radiated"

$$\Delta W = \int_{-\infty}^{\infty} P(t)\,dt$$

$$= \int_{-\infty}^{\infty}\frac{2}{3}\frac{z^4 Z^2 e^6}{m^2 c^3}\gamma^2\frac{1}{(r^2+b^2)^2}\left(1 - \frac{\beta^2 r^2}{r^2+b^2}\right)dr$$

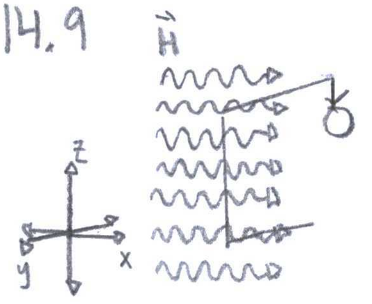
$$= \frac{2}{3}\frac{z^4 Z^2 e^6}{m^2 c^4 \beta}\gamma^2\left[\frac{\pi}{2b^2} - \frac{\pi\beta^2}{8b^2}\right]$$

$$= \frac{2}{3}\frac{z^4 Z^2 e^6}{m^2 c^4 \beta}\gamma^2\left[\frac{3}{8} + \frac{1-\beta^2}{8}\right]\frac{1}{b^3}$$

$$= \frac{z^4 Z^2 e^6 \pi}{m^2 c^4 \beta}\left[\frac{3}{8} + \frac{1}{8\gamma^2}\right]\frac{1}{b^3}$$

$$= \frac{\pi z^4 Z^2 e^6}{4 m^2 c^4 \beta}\left[\gamma^2 + \frac{1}{3}\right]\frac{1}{b^3}$$

14.9



"a particle moves in a plane perpendicular to a uniform, static magnetic field"

a) (14.26) "Lienard Result"

$$P = \frac{2}{3}\frac{e^2}{c}\gamma^6\left[\dot{\beta}^2 - (\beta \times \dot{\beta})^2\right]$$

$$= \frac{2}{3}\frac{e^2}{c}\gamma^6 \cdot \dot{\beta}^2(1-\beta)$$

$$= \frac{2}{3}\frac{e^2}{c}\gamma^4 |\dot{\beta}|^2$$

$$= \frac{2}{3}\frac{e^2}{c^3}\gamma^2 \cdot \dot{v}^2$$

(1.3) "Lorentz force"

$$F = q\left(E + \frac{v \times B}{c}\right)$$

$$\frac{dP}{dt} = \frac{vB}{c}$$

$$\dot{v} = \frac{q\beta B}{\gamma m}$$

$$P = \frac{2}{3}\frac{q^4}{c^3}\frac{\gamma^2}{m^2}\beta^2$$

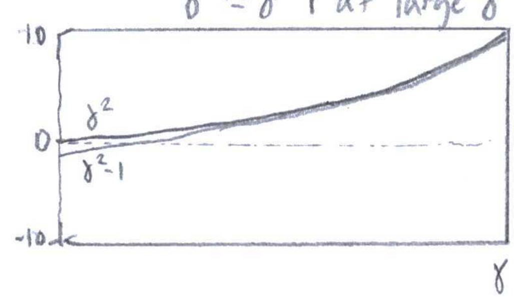
$$= \frac{2q^4 B^2}{3m^2c^3}(\gamma^2 - 1) \quad \ldots \text{ when } q = e$$

b) $E_0 = \gamma_0 mc^2$

$$\frac{dE}{\gamma^2 - 1} = -\frac{2q^4 B^2}{3m^2c^2}dt$$

$$\int_{\gamma_0}^{\gamma}\frac{d\gamma}{\gamma^2 - 1} = -\frac{2q^4 B^2}{3m^3c^5}\int_0^t dt$$

$\gamma^2 \simeq \gamma^2 - 1$ at large γ



$$\int_{\gamma_0}^{\gamma}\frac{d\gamma}{\gamma^2} = \frac{1}{\gamma_0} - \frac{1}{\gamma}$$

$$= \frac{-2q^4B^2}{3m^3c^5} t$$

$$t = \frac{3m^3c^5}{2q^4B^2}\left(\frac{1}{\gamma} - \frac{1}{\gamma_0}\right)$$

c) Kinetic energy:

$$\dot{v} = \frac{q\beta B}{\gamma m}$$

$$T_0 = \frac{1}{2}mv^2$$

$$v = \sqrt{2T_0/m}$$

$$P = \frac{2}{3}\frac{q^2}{c^3}|\dot{v}|^2$$

$$= \frac{4q^4B^2T_0}{3m^3c^5}$$

$$T = T_0 - P \cdot t$$

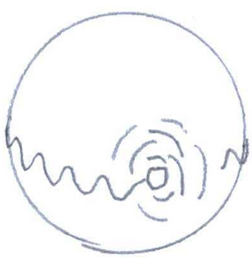
$$= T_0\left(1 - \frac{4q^4B^2}{3m^3c^5}t\right)$$

$$= T_0 \exp\left(\frac{-4q^4B^2t}{3m^3c^5}\right)$$

Exponential Expansion

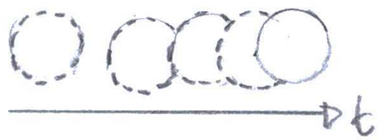
$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + $$

d) The particle radiates during large magnetic fields. At the equator, the acceleration is greatest in the function $P(t') = \frac{2e^2\gamma^4}{c^3}\left|\frac{dv}{dt}\right|^2$



"Particle .. trapped in .. magnetic dipole field .. spiraling back and fourth along a line .. radiating"

14.10



"a particle of charge...moves at constant velocity ...but speed decreases"

a) (14.39) "Power radiated per solid angle"

$$\frac{dP(t')}{d\Omega} = \frac{e^2\dot{v}^2}{4\pi c^3}\frac{\sin^2\theta}{(1-\beta_0\cos\theta)^5}$$

$$\dot{\beta} = \frac{\beta}{\Delta t}$$

$$\beta(t) = \beta_0(1 - t/\Delta t)$$

$$\frac{dE}{d\Omega} = \int_0^{\Delta t}\frac{dP}{d\Omega}dt$$

$$= \frac{e^2\beta^2}{4\pi c(\Delta t)^2}\int_0^{\Delta t}\frac{\sin^2\theta}{\left[1-\beta\cos\theta + \frac{\beta\cos\theta}{\Delta t}t\right]^5}dt$$

$$= \frac{e^2\beta^2}{4\pi c(\Delta t)^2}\frac{\Delta t}{\beta\cos\theta}\left(-\frac{1}{4}\left(1-\beta\cos\theta + \frac{\beta\cos\theta}{\Delta t}t\right)^{-4}\right)\Bigg|_0^{\Delta t}$$

$$= \frac{e^2\beta^2}{16\pi c\Delta t}\frac{1}{\beta\cos\theta}\left((1-\beta\cos\theta)^{-4}-1\right)\sin^2\theta$$

$$= \frac{e^2\beta^2}{16\pi c\Delta t}\frac{(2-\beta\cos\theta)\left[1+(1-\beta\cos\theta)^2\right]}{(1-\beta\cos\theta)^4}\sin^2\theta$$

b) $$\frac{dE}{d\Omega} = \frac{e^2\beta^2}{16\pi c\Delta t}\frac{(2-\beta\cos\theta)\left[1+(1-\beta\cos\theta)^2\right]\sin^2\theta}{(1-\beta\cos\theta)^4}$$

$$= \frac{e^2\beta^2}{16\pi c\Delta t}\frac{\left[1+\frac{1}{2\gamma^2}(1+\gamma^2\theta^2)\right]\left[1+\frac{1}{4\gamma^2}(1+\gamma^2\theta^2)^2\right]\theta^2}{\frac{1}{16\gamma^8}(1+\gamma^2\theta^2)^4}$$

...when $\sin\theta \sim \theta$, $\cos\theta \sim 1 - \frac{\theta^2}{2}$

$$\beta = (1-1/\gamma^2) \sim 1 - \tfrac{1}{2}\gamma^2$$

$$= \frac{e^2\beta^2}{16\pi c\Delta t}\gamma^6\frac{(\gamma\theta)^2}{(1+(\gamma\theta)^2)^4}$$

If $\xi=(\gamma\theta)^2$, then

$$\frac{dE}{d\xi} = \frac{d}{d\xi}\frac{e^2\beta^2}{\pi c\Delta t}\gamma^6\frac{(\xi)^2}{(1+\xi^2)^4}$$

$$= \frac{e^2\beta^2}{c\Delta t}\gamma^4\frac{\xi}{(1+\xi)^4}$$

$$\langle\theta^2\rangle = \frac{1}{\gamma^2}\langle\xi\rangle$$

$$= \frac{1}{\gamma^2}\frac{\int \xi\frac{dE}{d\xi}d\xi}{\int\frac{dE}{d\xi}d\xi}$$

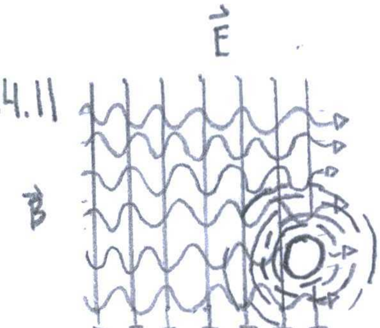
$$= \frac{1}{\gamma^2}\frac{\int\frac{\xi^2}{(1+\xi)^4}d\xi}{\int\frac{\xi}{(1+\xi)^4}d\xi}$$

$$= \frac{1}{\gamma^2}\frac{1/3}{1/6}$$

$$= \frac{2}{\gamma^2}$$

$$\langle\theta\rangle = \frac{\sqrt{2}}{\gamma}$$

4.11



"a particle of charge in external electric and magnetic fields... instantaneous energy radiated"

a) (12.1) "Equation of Motion"

$$\frac{dP}{dt} = e\left[E + \frac{u}{c}\times B\right]$$

(12.2) "

$$\frac{dE}{\ } = eu\cdot E$$

(14.24) "Lorentz invariance generalization"

$$P = -\frac{2}{3}\frac{e^2}{m^2c^3}\left(\frac{dp_\mu}{d\tau}\frac{dp^\mu}{d\tau}\right) \text{ where } d\tau = dt/\gamma$$

$$= -\frac{2}{3}\frac{Z^2e^2}{m^4c^3}\gamma^2\cdot Z^2\cdot e^2\left(\left(\frac{v}{c}\circ E\right)^2-\left(E+\frac{v}{c}\times B\right)^2\right)$$

$$= \frac{2}{3}\frac{Z^4e^4}{m^4\cdot c^3}\gamma^2\left[(E+\beta\times B)-(\beta\circ E)^2\right]$$

...because $p^\mu = (E/c, p)$

b) Manifests:

$$P = \frac{2}{3}\frac{Z^4e^4}{m^4c^3}\gamma^2\left[(E+\beta\times B)-(\beta\circ E)^2\right]$$

$$= \frac{2}{3}\frac{Z^4r_0^2}{m^2c}\left[(p^\mu\cdot E+p\times B)-(p\circ E)^2\right]$$

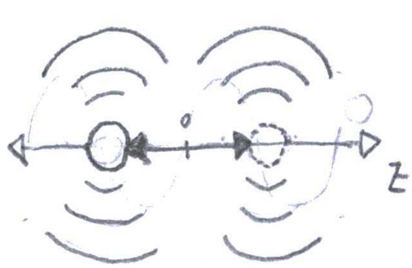
$$F^{\alpha\beta}F_\beta = \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -B_z & B_y \\ E_y & B_z & 0 & -B_x \\ E_z & -B_y & B_x & 0 \end{pmatrix}\begin{pmatrix} p_0 \\ -p_x \\ -p_y \\ -p_z \end{pmatrix}$$

$$= \begin{pmatrix} p_0 E \\ p_0E_x + P_xB_x \\ P_0E_y + P_xB_y \\ p_0E_z + P_xB_z \end{pmatrix}$$

$$g_{\alpha\mu}F^{\alpha\beta}p_\beta\cdot F^{\mu\nu}\cdot p_\nu = (p_0E)^2-(p^0E+p\times B)^2$$

$$P = \frac{2Z^4r_0^2}{3m^2c}F^{\mu\nu}\cdot P_\nu P^\lambda\cdot F_{\lambda\mu}$$

14.12.



"Charge moves in simple harmonic motion along z-axis"

Note:

Problem 14.4a had a similar function, $z(t)=a\cos(\omega t)$, Problem 14.12 is the correct traversal.

a) $Z(t') = a\cos(\omega_0 t')\,\hat{z}$

$$v(t') = \dot{z}(t')$$
$$= -a\omega_0 \sin(\omega_0 t')\,\hat{z}$$

$$a(t') = \ddot{z}(t)$$
$$= -a\omega_0^2 \sin(\omega_0 t')\,\hat{z}$$

(14.38) "Angular Distibution"

$$\frac{dP(t')}{d\Omega} = \frac{e^2}{4\pi c} \frac{\sin^2\theta}{(1-\beta\cos\theta)^5}$$

$$= \frac{e^2}{4\pi c} \frac{|n \times n \times \dot{v}|}{(1-n\cdot v/c)^5}$$

$$= \frac{e^2}{4\pi c^3} \frac{(a\omega_0^2 \sin\theta\cos(\omega_0 t)}{(1+\frac{a\omega_0}{c}\cos\theta\sin(\omega_0 t))^5}$$

$$= \frac{e^2 c\beta^4}{4\pi a^2} \frac{\sin^2\theta\cos^2(\omega_0 t')}{(1+\beta\cos\theta\sin(\omega_0 t'))^5}$$

$\therefore$ when $v = a\omega_0$

$\beta = a\omega_0/c$

b) (14.53) "Total energy radiated per unit solid angle"

$$\frac{dW}{d\Omega} = \int_{-\infty}^{\infty} |A(t)|^2 dt$$

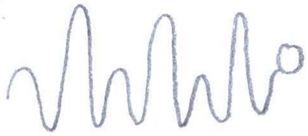
(14.51) "Power radiated per unit solid angle"

$$\frac{dP(t)}{d\Omega} = |A(t)|^2$$

$$\frac{dW}{d\Omega} = \int_0^{2\pi/\omega_0} \frac{dP(t)}{d\Omega} dt$$

$$= \frac{dP}{d\Omega} \cdot \frac{2\pi}{\omega_0}$$

14.13



"motion of radiating particle repeats itself... continuous frequency spectrum"

(14.53) "Total energy radiated per solid angle"

$$\frac{dW}{d\Omega} = \int_{-\infty}^{\infty} |A(t)|^2 dt$$

(14.54) "Fourier Transform modified"

$$A(t) = \sum_{m=-\infty}^{\infty} A_m e^{-im\omega_0 t} \quad \text{where } A_m = \frac{\omega_0}{2\pi}\int_0^{2\pi/\omega_0} A(t) e^{im\omega_0 t} dt$$

$$\frac{dW}{d\Omega} = \int_{-\infty}^{\infty} \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} A_m^* A_n e^{i(m-n)\omega_0 t} dt$$

$$= \frac{2\pi}{\omega_0}\sum_{m=-\infty}^{\infty} |A_m|^2$$

(14.51) "Power radiated per unit solid angle"

$$\left\langle \frac{dP}{d\Omega} \right\rangle = \frac{1}{T}\int_0^T \frac{dP(t)}{d\Omega} dt$$

$$= \frac{1}{T}\int_0^T |A(t)|^2$$

$$= \sum_{m=-\infty}^{\infty} |A_m|^2$$

(14.52) "General Amplitude"

$$A(t) = \left(\frac{c}{4\pi}\right)^{1/2} [RE]_{ret}$$

(14.64) "Fourier Transform with trajectory and distribution from origin"

$$A_m = \left(\frac{e^2}{4\pi c}\right)^{1/2} \frac{\omega_0}{2\pi} \int_0^{2\pi/\omega_0} \frac{d}{dt}\left[\frac{n \times (n \times \beta)}{1 - \beta \cdot n}\right] e^{im\omega_0 (t - n \cdot x(t)/c)} dt$$

$$\frac{dP}{d\Omega} = \frac{e^2 c\, \beta^4}{4\pi a^2} \frac{\omega_0}{2\pi} \int_0^{2\pi/\omega_0} \frac{\sin^2\theta \cos^2(\omega_0 t)}{(1+\beta\cos\theta\sin(\omega_0 t'))} dt'$$

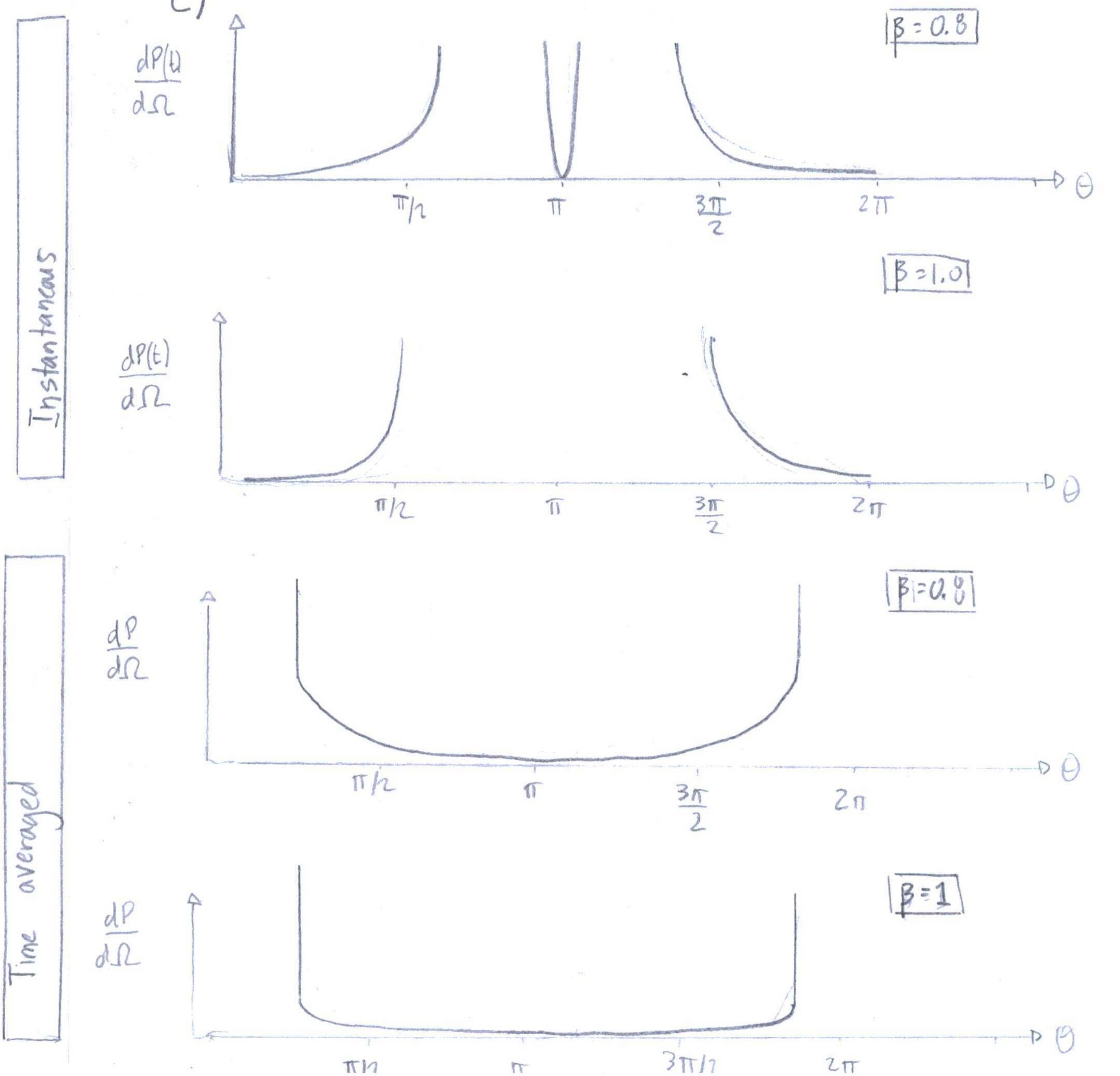
$$t' = 2\arctan(u)$$
$$dt' = \frac{2}{1+u^2}du$$

$$= \frac{e^2 c\, \beta^4}{4\pi a^2} \frac{\omega}{2\pi} \sin^2\theta \int_{-\infty}^{\infty} \frac{\left(\frac{1-u^2}{1+u^2}\right)^2}{\left(1+\beta\cos\theta \frac{2u}{1+u^2}\right)^5} \frac{2du}{1+u^2}$$

$$= \frac{e^2 c\, \beta^4}{4\pi \cdot a^2} \frac{\omega}{2\pi} \sin^2\theta \, \frac{\pi(4+\beta^2\cos^2\theta)}{4(1-\beta^2\cos^2\theta)^{7/2}}$$

$$= \frac{e^2 c\, \beta^4}{32\pi a^2} \frac{4+\beta^2\cos^2\theta}{(1-\beta^2\cos^2\theta)^{7/2}} \sin^2\theta$$

c)



$$= -\left(\frac{e^2}{4\pi c}\right)\frac{im\omega_0^2}{2\pi}\int_0^{2\pi/\omega_0}\left[n\times(n\times\beta)\right]e^{im\omega_0(t-n\cdot x(t)/c)}dt$$

$$\left\langle\frac{dP(t)}{d\Omega}\right\rangle = \sum_{-\infty}^{\infty}\frac{e^2m^2\omega_0^4}{8\pi^3c^3}\left|\int_0^{2\pi/\omega_0}(n\times(n\times v))\,e^{im\omega_0(t-n\cdot x(t)/c)}dt\right|^2$$

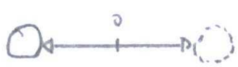
$$= \sum_{m=-\infty}^{\infty}\frac{e^2\omega_0^4m^2}{(2\pi c)^3}\left|\int_0^{2\pi/\omega_0}(v(t)\times n)\,e^{im\omega_0(t-n\cdot x(t)/c)}dt\right|^2$$

$$= \sum_{m=-\infty}^{\infty}\frac{d\sigma}{d\Omega}$$

14.14.

a) $x(t) = a\cos(\omega_0 t)\,\hat{z}$

$v(t) = -a\omega_0\sin(\omega_0 t)\,\hat{z}$



"charged particle radiating per unit solid angle in the m^{th}-harmonic"

(14.64) "Fourier Transform modified"

$$A_m = \left(\frac{e^2}{4\pi c}\right)^{1/2}\frac{\omega_0}{2\pi}\int_0^{2\pi/\omega_0}\frac{d}{dt}\left[\frac{n\times(n\times\beta)}{1-\beta\cdot n}\right]e^{im\omega_0(t-n\cdot x(t)/c)}dt$$

$$= \left(\frac{e^2}{4\pi c}\right)^{1/2}\frac{\omega_0}{2\pi}\int_0^{2\pi/\omega_0}v(t)\times e\exp^{im\omega_0(t-n\cdot x(t)/c)}dt$$

$$= \left(\frac{e^2}{4\pi c}\right)^{1/2}\frac{\omega_0}{2\pi}\int_0^{2\pi/\omega_0}\frac{\sin\theta\sin(\omega_0 t)\exp^{im\omega_0(t-\frac{a}{c}\cos\theta\cos\omega t)}}{x\cdot a\cdot\omega_0}dt$$

Mathematical Identity:

$$\exp\left\{-im\omega_0\frac{a}{c}\cos\theta\cos(\omega_0 t)\right\} = \sum_{-\infty}^{\infty}(-i)^nJ_n(m\beta\cos\theta)\,e^{-in\omega_0 t}$$

..when "a" units equal miles/hour

because $a/c = v/c = \beta$

$$=\left(\frac{e^2}{4\pi c}\right)\frac{\omega_0}{2\pi}\sin\theta \cdot \sum_{-\infty}^{\infty}(-i)^n J_n(m\beta\cos\theta)\int_0^{2\pi/\omega_0}\sin(\omega t)e^{i(m-n)\omega_0 t}\,dt$$

$$=\left(\frac{e^2}{4\pi c}\right)\frac{\omega_0}{2\pi}\sin\theta \sum_{-\infty}^{\infty}(-i)^n J_n(m\beta\cos\theta)\frac{1}{2i}\frac{2\pi}{\omega_0}\underbrace{\left(J^{(\cdot)}_{m+1,n}-J^{(\cdot)}_{m-1,n}\right)}_{\text{Recurrence Relation}}$$

$$=\left(\frac{e^2}{4\pi c}\right)\frac{\omega_0}{2\pi}\sin\theta\,\frac{\pi}{i\omega_0}\left[(-i)^{m+1}J_{m+1}(m\beta\cos\theta)-(-i)^{m-1}J_{m-1}(m\beta\cos\theta)\right]$$

$$=\left(\frac{e^2}{4\pi c}\right)\frac{\omega_0}{2\pi}\sin\theta\,-(-i)^m\frac{\pi}{\omega_0}\underbrace{\left[J_{m+1}(m\beta\cos\theta)+J_{m+1}(m\beta\cos\theta)\right]}_{\text{Recurrence Relation}}$$

Note: Bessel's had more analysis in 1950-1960's. Then the 2000's revitalized distributions, pilevalues, and floating point arithmetic with Bessel's.

$$=\left(\frac{e^2}{4\pi c}\right)\frac{\omega_0}{2\pi}\sin\theta\,-(-i)^m\frac{\pi}{\omega_0}\frac{2m}{m\beta\cos\theta}J_m(m\beta\cos\theta)$$

$$=\left(\frac{e^2}{4\pi c}\right)\frac{\omega_0}{2\pi}\sin\theta\,-(-i)^m\frac{2\pi}{\omega_0}\frac{J_m(m\beta\cos\theta)}{\beta\cos\theta}$$

(Problem 14.13) "Averaged power radiated per unit solid angle"

$$\frac{dP}{d\Omega}=\frac{e^2\omega_0^4m^2}{(2\pi c)^3}\left|\int_0^{2\pi/\omega_0}v(t)\times n\,e^{im\omega_0\left(t-\frac{n\cdot x(t)}{c}\right)}dt\right|^2$$

$$= \frac{e^2 \omega_0^4 m^2}{(2\pi c)^3} \underbrace{\frac{(2\pi)^2}{\omega_0^2} \frac{J_m(m\beta\cos\theta)^2}{\beta^2\cos^2\theta} a^2 \omega_0^2 \sin^2\theta}$$

Components from 14.64, except coefficient.

$$= \frac{e^2 c \beta^2}{2\pi a^2} m^2 \tan^2\theta \, J_m(m\beta\cos\theta)^2$$

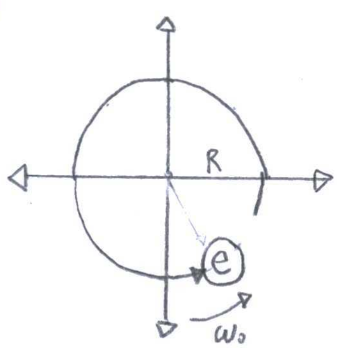
b) $v \ll c$, $J_1(x) \sim x^2/c$, $\frac{dP}{d\Omega} = \frac{e^2 c \beta^4}{8\pi a^2} \sin^2\theta$

$$P = \int \frac{e^2 c \beta^4}{8\pi a^2} \sin^2\theta \, d\Omega$$

$$= \frac{1}{3} \frac{e^2 c \beta^4}{a^2}$$

$$= \frac{2}{3} \frac{e^2}{c^3} \omega_0^4 a^2$$

4.15



"particle... moves in a circular path... on x-y plane"

a) $\vec{n} = (\sin\theta, 0, \cos\theta)$

$X(t) = R(\cos(\omega_0 t), \sin(\omega_0 t), 0)$

$v(t) = \omega_0 R(-\sin(\omega_0 t), \cos(\omega_0 t), 0)$

$n \cdot X(t) = R \sin\theta \cos(\omega_0 t)$

$v(t) \times n = \omega_0 R(\cos\theta\cos(\omega_0 t), \cos\theta\sin(\omega_0 t), -\sin\theta\cos(\omega_0 t))$

$$\frac{dP}{d\Omega} = \frac{e^2 \omega_0^4 m^2}{(2\pi c)^3} \left| \int_0^{2\pi/\omega_0} \begin{pmatrix} \cos\theta\cos(\omega_0 t) \\ \cos\theta\sin(\omega_0 t) \\ -\sin\theta\cos(\omega_0 t) \end{pmatrix} \omega_0 R \cdot e^{im\omega_0\left(t - \frac{R}{c}\sin\theta\cos(\omega_0 t)\right)} dt \right|^2$$

$$\boxed{\begin{array}{l}\underline{\text{Identity:}}\\ e^{-im\omega_0 \frac{R}{c}\sin\theta\cos(\omega_0 t)} = \sum (-i)^k J_m\left(m\frac{\omega_0 R}{c}\sin\theta\right) \\ \qquad\qquad\qquad\qquad \times e^{-ik\omega_0 t}\end{array}}$$

$$= \frac{e^2\omega_0^4 m^2}{(2\pi c)^3}\left|\begin{pmatrix}\cos\theta\\ \cos\theta\\ -\sin\theta\end{pmatrix}\int_0^{2\pi/\omega_0}\begin{pmatrix}\cos(\omega_0 t)\\ \sin(\omega_0 t)\\ \cos(\omega_0 t)\end{pmatrix} e^{im\omega_0 t}\cdot\sum(-i)^k J_m\left(m\frac{\omega_0 R}{c}\sin\theta\right) \times e^{-ik\omega_0 t}\,dt\right|^2$$

$$= \frac{e^2\omega_0^4 m^2}{(2\pi c)^3}\left|\begin{pmatrix}\cos\theta\\ \cos\theta\\ -\sin\theta\end{pmatrix}\int_0^{2\pi/\omega_0}\begin{pmatrix}\frac{1}{2}\left(e^{i\omega_0 t}+e^{-i\omega_0 t}\right)\\ \frac{1}{2i}\left(e^{i\omega_0 t}-e^{-i\omega_0 t}\right)\\ \frac{1}{2}\left(e^{i\omega_0 t}+e^{-i\omega_0 t}\right)\end{pmatrix} e^{im\omega_0 t}\circ\sum(-i)^k J_m\left(m\frac{\omega_0 R}{c}\sin\theta\right) \times e^{-ik\omega_0 t}\cdot dt\right|^2$$

$$= \frac{e^2\omega_0^4 m^2}{(2\pi c)^3}\left|\begin{pmatrix}\cos\theta\\ \cos\theta\\ -\sin\theta\end{pmatrix}\int_0^{2\pi/\omega_0}\begin{pmatrix}\frac{1}{2}\left((-i)^{m+1}\cdot J_{m+1}(m\beta\sin\theta)+(-i)^{m-1}\cdot J_{m-1}(m\beta\sin\theta)\right)\\ \frac{1}{2i}\left((-i)^{m+1} J_{m+1}(m\beta\sin\theta)+(-i)^{m-1} J_{m-1}(m\beta\sin\theta)\right)\\ \frac{1}{2}\left((-i)^{m+1} J_{m+1}(m\beta\sin\theta)+(-i)^{m-1} J_{m-1}(m\beta\sin\theta)\right)\end{pmatrix}\circ\sum(-i)^k J_m\left(m\frac{\omega_0 R}{c}\sin\theta\right) \times e^{-ik\omega_0 t}\,dt\right|^2$$

$$= \frac{e^2\omega_0^4 m^2}{(2\pi c)^3}\left|\begin{pmatrix}\cos\theta\\ \cos\theta\\ -\sin\theta\end{pmatrix}\frac{2\pi}{\omega_0}\int_0^{2\pi/\omega_0}\begin{pmatrix}\frac{1}{2}(-i)^{m+1}\, 2\left.\frac{dJ_m(x)}{dx}\right|_{x=m\beta\sin\theta}\\ \frac{1}{2i}(-i)^{m+2}\,\frac{2m}{m\beta\sin\theta} J_m(m\beta\sin\theta)\\ \frac{1}{2}(-i)^{m+1}\, 2\left.\frac{dJ_m(x)}{dx}\right|_{x=m\beta\sin\theta}\end{pmatrix}dt\right|^2$$

$$= \frac{e^2\omega_0^4 R^2}{2\pi c^3} m^2\left|\begin{pmatrix}\cos\theta\, dJ_m/dx|_{x=m\beta\sin\theta}\\ (-i)\cos\theta\, J_m(m\beta\sin\theta)/\beta\sin\theta\\ -\sin\theta\, dJ_m/dx|_{x=m\beta\sin\theta}\end{pmatrix}\right|^2$$

$$= \frac{e^2 \omega_0^4 R^2}{2\pi c^3} m^2 \left\{ \left[\frac{dJ_m(m\beta \sin\theta)}{d(m\beta\sin\theta)} \right]^2 + \frac{\cot^2\theta}{\beta^2} J_m^2(m\beta\sin\theta) \right\}$$

b) $J_1(z) \sim z/2$, $dJ_1(z)/dz = 1/2$

$$\frac{dP}{d\Omega} = \frac{dP_1}{d\Omega} = \frac{e^2\omega_0^4 R^2}{2\pi c^3}\left(\frac{1}{4} + \frac{1}{4}\cos^2\theta\right)$$

$$= \frac{e^2\omega_0^4 R^2}{4\pi c^3}\left(1 - \frac{1}{2}\sin^2\theta\right)$$

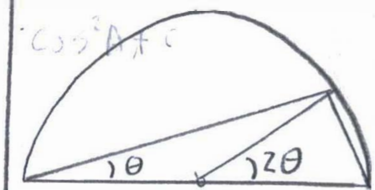
$$= \frac{2}{3}\frac{e^2}{c^3}\omega_0^4 \bar{a}^2 \quad \text{(Problem 14.14b)}$$

When $\bar{a}^2 = \frac{3R^2}{8\pi}\left(1 - \frac{1}{2}\sin^2\theta\right)$

<u>Double-Angle Identity:</u>

$\cos(2A) = 2\cos^2 A - 1$

$\Rightarrow \frac{1}{2} + \frac{1}{2}\cos(2A) = \cos^2 A$



c) <u>Citation</u>: Watson, G. N. "Theory of Bessel Functions," 2nd edition, Cambridge University Press. Cambridge (1952)., pg 79, 249

14.16.

(14.79) "Intensity Distibution"

$$\frac{d^2 I}{d\omega d\Omega} = \frac{e^2}{3\pi^2 c}\left(\frac{\omega \rho}{c}\right)^2\left(\frac{1}{\gamma^2}+\theta^2\right)^2 \cdot \left[K_{2/3}^2(\xi) + \frac{\theta^2}{(1/\gamma^2)+\theta^2} K_{1/3}^2(\xi)\right]$$

(pg 678) "Approximation"

$$\omega \sim (c/\rho)$$

$$\frac{d^2 I}{d\omega d\Omega} = \frac{e^2}{3\pi^2 c}\frac{1}{\gamma^4}\left(1+\gamma^2\theta^2\right)^2 \cdot \left[K_{2/3}^2(\xi) + \frac{\theta^2}{\frac{1}{\gamma^2}(1+\gamma^2\theta^2)} K_{1/3}^2(\xi)\right]$$

(14.76) "Parameter"

$$\xi = \frac{\omega \rho}{3c}\left(\frac{1}{\gamma^2}+\theta^2\right)^{1/2}$$

$$= \frac{\omega\rho}{3c}\frac{1}{\gamma}\left(1+\gamma^2\theta^2\right)^{1/2}$$

$$= \frac{2\sqrt{2}}{3m}\frac{(p\cdot k)^{3/2}}{(|d^2(p\cdot k)/d\tau^2|)^{1/2}}$$ ...from (Problem 14.16)

$$\frac{(1+\gamma^2\theta^2)}{\gamma^2} = \frac{8c^2}{\omega^2\rho^2}\frac{(p\cdot k)^3}{(|d^2(p\cdot k)/d\tau^2|)}$$

$$= 8\frac{(p\cdot k)^3}{(|d^2(p\cdot k)/d\tau^2|)}$$

$$\frac{(1+\gamma^2\theta^2)^2}{\gamma^4} = 64\frac{(p\cdot k)^6}{(|d^2(p\cdot k)/d\tau^2|)^2}$$

$$\frac{d^2 I}{d\omega d\Omega} = \frac{e^2}{3\pi^2 c} 64 \cdot \frac{(p\cdot k)^6}{(|d^2(p\cdot k)/d\tau^2|)^2}\left[K_{2/3}^2(\xi) + \frac{\theta^2}{\left(\frac{8(p\cdot k)^3}{(|d^2(p\cdot k)/d\tau^2|)}\right)} \cdot K_{1/3}^2(\xi)\right]$$

(14.85) "Angle"

$$\theta_c \simeq \left(\frac{3c}{\omega\rho}\right)^{1/3} = \frac{1}{\gamma}\left(\frac{2\omega_c}{\omega}\right)^{1/3}$$

$$\simeq \left(\frac{3c}{\omega\rho}\right)^{1/2}$$ ...@ critical angle 1/e (pg 680)

$$\theta^2 = \left(\frac{3c}{\omega\rho}\right)$$

$$\frac{d^2I}{d\omega\, d\Omega} = \frac{4e^2}{3\pi^2 c} \cdot \left(\frac{3}{4}\right)(p\circ k)^2 \left[\frac{4}{3}\frac{(p\circ k)^4}{(|d^2(p\circ k)/d\tau^2|)^2} K_{2/3}^2(\xi) + \frac{(p\circ k)}{2\cdot(|d^2(p\circ k)/d\tau^2|)^2} K_{1/3}^2(\xi)\right]$$

(pg 683) "Number of photons... obtained by $\hbar\omega$"

$$\hbar\omega \frac{d^2N}{d\omega\, d\Omega} = \frac{4e^2}{3\pi^2 c} \cdot \left(\frac{3}{4}\right)(p\circ k)^2 \left[\frac{4}{3}\frac{(p\circ k)^4}{(|d^2(p\circ k)/d\tau^2|)^2} K_{2/3}^2(\xi) + \frac{(p\circ k)}{2\cdot(|d^2(p\circ k)/d\tau^2|)^2} K_{1/3}^2(\xi)\right]$$

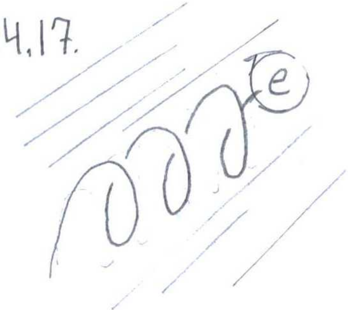
$$\hbar\omega \frac{d^2N}{dk\, d\Omega} = \frac{4e^2}{3\pi^2 c} \left(\frac{3}{4}\right) k\cdot c\,(p\circ k)^2 \left[\frac{4}{3}\frac{(p\circ k)^4}{(|d^2(p\circ k)/d\tau^2|)^2} K_{2/3}^2(\xi) + \frac{(p\circ k)}{2\cdot(|d^2(p\circ k)/d\tau^2|)^2} K_{1/3}^2(\xi)\right]$$

$$\hbar\omega \frac{d^3N}{d^3k} = \frac{4e^2}{3\pi^2 c} \left(\frac{3}{4}\right) k^2 (p\circ k)^2 \left[\frac{4}{3}\frac{(p\circ k)^4}{(|d^2(p\circ k)/d\tau^2|)^2} K_{2/3}^2(\xi) + \frac{(p\circ k)}{2(|d^2(p\circ k)/d\tau^2|)^2} K_{1/3}^2(\xi)\right]$$

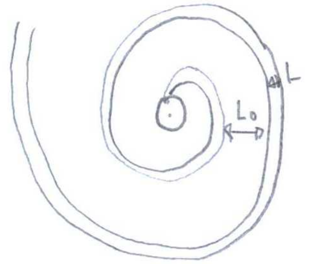
$$= \frac{4e^2}{3\pi^2 c} \left(\frac{3}{4}\right) \left[\frac{4}{3}\frac{(p\circ k)^4}{(|d^2(p\circ k)/d\tau^2|)^2} \cdot (\epsilon\circ p)^2 K_{2/3}^2(\xi) + \frac{(p\circ k)(\epsilon\circ p)^2}{2(|d^2(p\circ k)/d\tau^2|)^2} K_{1/3}^2(\xi)\right]$$

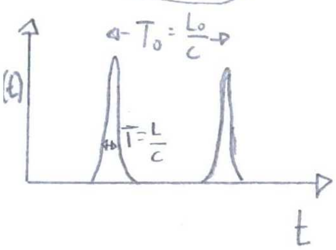
...a couple proportions were incorrect. Critical pieces in the problem include ξ-parameter relationship, θ-angle, and third-order derivative"

14.17.



"Particle... moves relativistically in a helical path in a uniform magnetic field"





a) (12.41.5) "Transverse Momentum"

$$cp_\perp = eBr$$

$$c\gamma m v\cos\alpha = eBr$$

$$r = \frac{\gamma mc}{eB} v\cos\alpha$$

$$= \frac{v\cos\alpha}{\omega_B} \quad \text{when } \omega_B = \frac{eB}{\gamma mc}$$

(14.4a) "Pulse Length"

$$L = D - d = \left(\frac{1}{\beta} - 1\right)\frac{\rho}{\gamma}$$

$$\cong \frac{\rho}{2\gamma^3}$$

$$L\cos\alpha = \frac{v}{2\gamma^3} \circ \frac{\cos^2\alpha}{\omega_B}$$

"Fundamental Frequency: $\frac{\text{Helicity}}{\text{rotation angle}}$"

$$x(t) = r\cos(\omega_B t)$$

$$y(t) = r\sin(\omega_B t)$$

$$z(t) = v\sin\alpha\, t$$

$$= \frac{v\sin\alpha}{\omega_B}$$

$$= c$$

$$\rho = \frac{z^2 + c^2}{\overset{\circ}{z}} = \frac{v^2/\omega_B^2}{v\cos\alpha/\omega_B} = \frac{v}{\omega_B\cos\alpha}$$

(14.48) "Curvature"

$$\rho = \frac{v^2}{\dot{v}_\perp} = \frac{\frac{v^2\cos^2\alpha}{\omega_B^2} + \frac{v^2\sin^2\alpha}{\omega_B^2}}{v\cos\alpha/\omega_B}$$

$$= \frac{v^2/\omega_B^2}{v\cos\alpha/\omega_B}$$

$$= \frac{v}{\omega_B\cos\alpha}$$

(14.81) "critical frequency - helix"

$$\omega_c = \frac{3}{2}\gamma^3\left(\frac{c}{\rho}\right)$$

$$= \frac{3}{2}\left(\frac{E}{mc^2}\right)\frac{c}{\rho}$$

$$= \frac{3}{2}\gamma^3\left(\frac{c\,\omega_B\cos\alpha}{v}\right)$$

$$= \frac{3}{2}\gamma^3\omega_B\cos\alpha$$ ... when $v \approx c$ at light speeds.

b) (14.79) "Energy radiated per unit frequency interval per unit solid angle"

$$\frac{d^2I}{d\omega\, d\Omega} = \frac{e^2}{3\pi^2 c}\left(\frac{\omega\rho}{c}\right)^2\left(\frac{1}{\gamma^2} + \theta^2\right)\left[K_{2/3}^2(\xi) + \frac{\theta^2}{(1/\gamma^2)+\theta^2}K_{1/3}^2(\xi)\right]$$

$$\frac{d^2P}{d\omega\, d\Omega} = \frac{1}{T}\frac{d^2I}{d\omega\, d\Omega}$$

$$= \frac{\omega_0}{2\pi}\frac{d^2I}{d\omega\, d\Omega}$$

$$= \frac{1}{2\pi}\frac{\omega_B}{\cos^2\alpha}\frac{e^2}{3\pi^2 c}\left(\frac{\omega}{\omega_c}\frac{3}{2}\gamma^3\right)^2\left(\frac{1}{\gamma^2} + \psi^2\right)^2\left[K_{2/3}^2(\xi) + \frac{\gamma^2\psi^2}{1+\gamma^2\psi^2}K_{1/3}^2(\xi)\right]$$

$$= \frac{3e^2\gamma^2}{8\pi^3 c}\left(\frac{\omega}{\omega_c}\right)^2 \frac{\omega_B}{\cos\alpha}(1+\gamma^2\psi^2)\left[K_{2/3}^2(\xi) + \frac{\gamma^2\psi^2}{1+\gamma^2\psi^2}K_{1/3}^2(\xi)\right]$$

...where $\xi = \frac{\omega}{2\omega_c}(1+\gamma^2\psi^2)^{3/2}$

14.18

a) (14.91) "Frequency distribution of total power"

$$\frac{dI}{d\omega} = \sqrt{3}\frac{e^2}{c}\gamma\frac{\omega}{\omega_c}\int_{\omega/\omega_c}^{\infty} K_{5/3}(x)\,dx$$

(14.79) "Energy radiated per unit frequency per unit solid angle"

$$\frac{d^2I}{d\omega d\Omega} = \frac{e^2}{3\pi^2 c}\left(\frac{\omega\rho}{c}\right)^2\left(\frac{1}{\gamma^2}+\theta^2\right)^2\left[K_{2/3}^2(\xi) + \frac{\theta^2}{(1/\gamma^2)+\theta^2}K_{1/3}^2(\xi)\right]$$

$$\frac{dI}{d\Omega} = \frac{e^2}{3\pi^2}\frac{9\gamma^6}{4}\left(\frac{\omega}{\omega_c}\right)^2\frac{1}{\gamma^4}(1+\gamma^2\theta^2)\left[K_{2/3}^2(\xi) + \frac{\gamma^2\theta^2}{(1+\gamma^2\theta^2)}K_{1/3}^2(\xi)\right]$$

where $\xi = \frac{\omega\rho}{3c}\left(\frac{1}{\gamma^2}+\theta^2\right)^{3/2}$

$= \frac{\omega}{2\omega_c}(1+\gamma^2\theta^2)^{3/2}$

$$= \frac{\sqrt{3}e^2\gamma}{c}\frac{\sqrt{3}\gamma}{4\pi^2}\left(\frac{\omega}{\omega_c}\right)^2(1+\gamma^2\theta^2)^2\left[K_{2/3}^2(\xi) + \frac{\gamma^2\theta^2}{(1+\gamma^2\theta^2)}K_{1/3}^2(\xi)\right]$$

(Problem 14.17b)

$$\frac{d^2P}{d\omega d\Omega} = \frac{3e^2\gamma^2}{8\pi^3 c}\frac{\omega_B}{\cos^2\alpha}\left(\frac{\omega}{\omega_c}\right)^3(1+\gamma^2\psi^2)^2\left[K_{2/3}^2(\xi) + \frac{\gamma^2\theta^2}{(1+\gamma^2\theta^2)}K_{1/3}^2(\xi)\right]$$

If $\theta = \pi/2$ and $d\Omega = \sin\theta\, d\theta\, d\phi$

$= \cos\alpha\, d\theta\, d\phi$ where $\alpha = \theta + \pi/2$

$$\frac{dP}{d\omega} = \frac{\sqrt{3}e^2\gamma^2}{2\pi c}\frac{\omega_B}{\cos^2\alpha}\cos\alpha\left(\frac{\omega}{\omega_c}\right)\int_{\omega/\omega_c}^{\infty}K_{5/3}(y)\,dy = \left(\frac{\sqrt{3}e^2\gamma\omega_B}{2\pi\cos\alpha}\right)G\left(\frac{\omega}{\omega_c}\right)$$

from $G(x) = x\int_0^\infty K_{5/3}(t)\,dt$

Certainly two uncommon integrals were 'fractional variables of integration' and 'Dirac integrals' in school.

b) $$P = \frac{\sqrt{3}e^2\gamma\omega_B}{2\pi\cos\alpha}\cdot\omega_c\int_0^\infty G\left(\frac{\omega}{\omega_c}\right)d\left(\frac{\omega}{\omega_c}\right)$$

$$= \frac{\sqrt{3}e^2\gamma\omega_B}{2\pi\cos\alpha}\cdot\omega_c\int_0^\infty dx\cdot x\cdot\int_x^\infty dy\cdot K_{5/3}(y)$$

$$= \frac{\sqrt{3}e^2\gamma\omega_B}{2\pi\cos\alpha}\,\omega_c\cdot\frac{1}{2}\int_0^\infty y^2 K_{5/3}(y)\,dy$$

$$= \frac{\sqrt{3}e^2\gamma\omega_B}{2\pi\cos\alpha}\,\frac{3}{2}\gamma^3\omega_B\cos\alpha\cdot\frac{8\pi}{9\sqrt{3}}$$

$$= \frac{2e^2\omega_B^2\gamma^4}{3c}$$

(14.31) "Power radiated"

$$P = \frac{2}{3}\frac{e^2}{m^2c^3}\gamma^2\omega^2|p|^2$$

$$= \frac{2}{3}\frac{e^2c}{\rho^2}\beta^4\gamma^4$$

Total power in linear relativistic particles is proportional to helical paths. A relativistic helical particle emits total power independent~~th~~ pitch and angle.

14.19. a) (14.70) "Intensity distribution"

$$\frac{d^2I}{d\omega\,d\Omega} = \frac{\omega^2}{4\pi^2c}\left|\int dt\int d^3x\cdot n\times[n\times J(x,t)]\,e^{i\omega[t-(n\cdot x)/c]}\right|^2$$

$J_{eff} = c\nabla \times M$; $M = \mu(t) \cdot \delta|x - r(t)|$

$$= \frac{\omega_0^2}{4\pi^2 c}\left[\oint dt \int d^3x \cdot n \times \left[n \times (\nabla \times (\mu(t)\delta(x - r(t)))\right] e^{i\omega(t - n\cdot x/c)}\right]^2$$

$$= \frac{\omega_0^4}{4\pi^2 c^3}\left[\oint dt \cdot n \times \left[n \times (n \times \mu(t))\right] e^{i\omega(t - n\cdot r(t)/c)}\right]^2$$

$$= \frac{\omega_0^4}{4\pi^2 c^3}\left[\oint dt \cdot n \times \mu(t) \cdot e^{i\omega(t - n\cdot r(t)/c)}\right]^2$$

b) $\mu_x = \mu_0 \sin\omega_0 t$ $\quad \mu_z = \mu\cos\omega_0 t$ $\quad \mu_y = 0$

$$= \frac{\omega_0^4}{4\pi^2 c^3}\left[\int_{-T/2}^{T/2} \mu_0 \begin{pmatrix} \cos(\omega_0 t)\sin\theta\sin\phi \\ \sin(\omega_0 t)\cos\theta - \cos(\omega_0 t)\sin\theta\cos\phi \\ -\sin(\omega_0 t)\cdot\sin\theta\sin\phi \end{pmatrix} e^{i\omega(t - n\cdot r(t)/c)}\right]^2$$

$$= \frac{\omega_0^4}{4\pi^2 c^3}\mu_0^2 \frac{T^2}{4}\begin{pmatrix} \sin\theta\sin\phi \\ -i\cos\theta - \sin\theta\cos\phi \\ i\sin\theta\sin\phi \end{pmatrix}$$

Intensity distribution:

$$\frac{dI}{d\Omega} = \frac{\omega_0^4}{4\pi^2 c^3}\mu_0^2 \frac{T}{2}\int \begin{pmatrix} \sin\theta\sin\phi \\ -i\cos\theta - \sin\theta\cos\phi \\ i\sin\theta\sin\phi \end{pmatrix} d\omega$$

$$= \frac{\omega_0^4}{4\pi^2 c^3}\mu_0^2 \frac{T^2}{4}\left(2\sin^2\theta\sin^2\phi + \cos^2\theta + \sin^2\theta\cos^2\phi\right)$$

$$= \frac{\omega_0^4}{4\pi^2 c^3}\mu_0^2 \frac{T^2}{4}\left(1 + \sin^2\theta\sin^2\phi\right)$$

$\boxed{\text{Pythagorean Identity: } \sin\theta + \cos\theta = 1}$

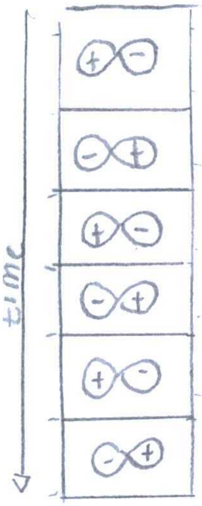
Total Power Radiated:

$$I = \int \frac{\omega_0^4}{4\pi^2 c^3}\mu_0^2 \frac{T^2}{4}\left(1 + \sin^2\theta\sin^2\phi\right) d\Omega$$

$$= \frac{\omega_0^4}{3\pi c^3}\mu_0 T^2$$

$$= \frac{1}{2\pi}\langle P\rangle T^2 \rightarrow \langle P\rangle = \frac{2}{3}\frac{\omega_0^4}{3\pi c^3}\mu_0 \quad \text{because } P = I\frac{2\pi}{T}$$

14.20



"radiation emitted by a magnetic moment at the origin flipping"

a) $\mu_z = \mu_0 \tanh(\nu t)$ $\quad \mu_x = \mu_0 \,\mathrm{sech}(\nu t)$ $\quad \mu_y = 0$

where ν^{-1} is the characteristic flip.

(Problem 14.19 a)

$$\frac{d^2 I}{d\omega\, d\Omega} = \frac{\omega^4}{4\pi^2 c}\left| \int dt \cdot n \times \mu(t)\, e^{i\omega(t - n\cdot r(t)/c)} \right|^2$$

$$= \frac{\omega^4}{4\pi^2 c}\mu_0^2 \left| \int \begin{pmatrix} \tanh(\nu t)\sin\theta\sin\phi \\ \mathrm{sech}(\nu t)\cos\theta - \tanh(\nu t)\sin\theta\cos\phi \\ -\mathrm{sech}(\nu t)\sin\theta\sin\phi \end{pmatrix} \cdot e^{i\omega t}\cdot dt \right|^2$$

$$= \frac{\omega^4}{4\pi^2 c}\mu_0^2 \left| \begin{pmatrix} i\pi/\nu \cdot \mathrm{cosech}\left(\frac{\omega\pi}{2\nu}\right)\sin\theta\sin\phi \\ \frac{\pi}{\nu}\,\mathrm{sech}\left(\frac{\omega\pi}{2\nu}\right)\cos\theta - \frac{\pi}{\nu}\,\mathrm{cosech}\left(\frac{\omega\pi}{2\nu}\right)\sin\theta\cos\phi \\ -\frac{\pi}{\nu}\,\mathrm{sech}\left(\frac{\omega\pi}{2\nu}\right)\sin\theta\sin\phi \end{pmatrix} \right|$$

Citation: Spiegal, Murray.
"Mathematical Handbook of Formulas and Tables"

Identities: $\displaystyle\int_0^\infty \frac{\cos ax}{\cosh \beta x}\,dx = \int_0^\infty \frac{\text{exponential}}{\text{exponential}}\,dx$

$$= \int_0^\infty \frac{\text{exponential}}{\text{Complex Series}}\,dx$$

$$= \int^\infty \frac{1}{\text{Complex Series}}\,dx$$

Modulus of a limit

$$= \sum \frac{1}{\text{complex series}}$$

$$= \frac{\pi}{2\beta} \operatorname{sech}\left(\frac{a\pi}{2\beta}\right)$$

$$\int_0^\infty \sin(ax) \frac{\sinh(\beta x)}{\cosh(\gamma x)} dx = \frac{\pi}{\gamma} \frac{\sin\left(\frac{\beta\pi}{2\gamma}\right)\sinh\left(\frac{a\pi}{2\gamma}\right)}{\cosh\left(\frac{a\pi}{\gamma}\right) + \cos\left(\frac{\beta\pi}{\gamma}\right)}$$

$$\frac{dI}{d\omega} = \frac{\omega^4}{4\pi^2 c^3} \mu_0^2 \left(\frac{\pi}{v}\right)^2 \left[\operatorname{cosech}^2\left(\frac{\omega\pi}{2v}\right)\frac{8\pi}{3} + \operatorname{sech}^2\left(\frac{\omega\pi}{2v}\right)\left(\frac{4\pi}{3} + \frac{4\pi}{3}\right)\right]$$

$$= \frac{2\pi\omega^4}{3v^2c^3} \mu_0^2 \left(\operatorname{cosech}^2\left(\frac{\omega\pi}{2v}\right) + \operatorname{sech}^2\left(\frac{\omega\pi}{2v}\right)\right)$$

If $x = \pi\omega/2v$, then

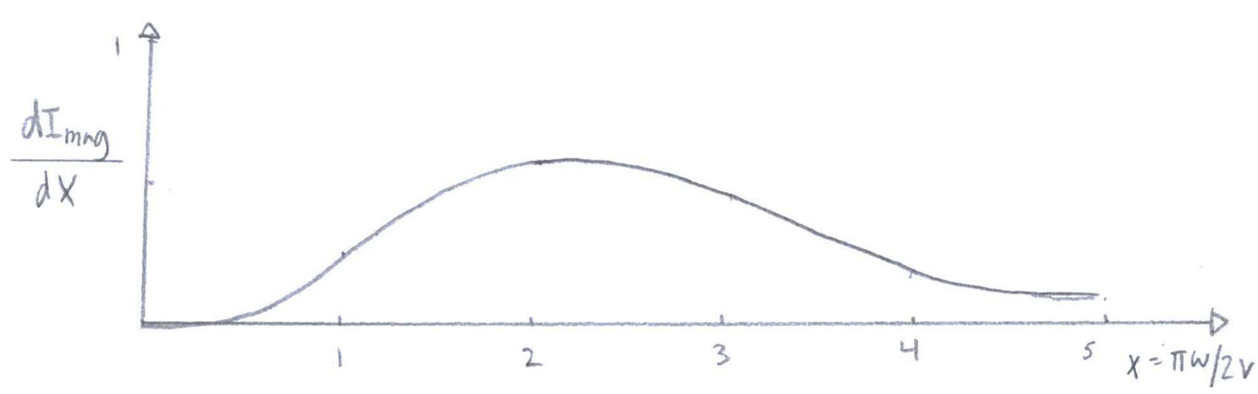
$$\frac{dI_{mag}}{dx} = \frac{dI}{d\omega}\frac{d\omega}{dx}$$

$$= \frac{2\pi^2\omega^4}{3v^2c^3} \mu_0^2 \left(\operatorname{cosech}^2(x) + \operatorname{sech}^2(x)\right) \cdot \frac{2v}{\pi}$$

$$= \frac{4\omega^4}{3v^2c^3} \mu_0^2 \left(\operatorname{cosech}^2(x) + \operatorname{sech}^2(x)\right)$$

$$\frac{dI_{mag}}{dx} = \frac{4}{3} \frac{\mu_0^2}{v \cdot c^3} \left(\frac{2v}{\pi}x\right)^4 \left(\operatorname{cosech}^2(x) + \operatorname{sech}^2(x)\right)$$

$$= \frac{4}{3}\left(\frac{v}{c}\right)^3 \mu_0^2 \left[16\left(\frac{x}{\pi}\right)^4 \left(\operatorname{cosech}^2(x) + \operatorname{sech}^2(x)\right)\right]$$



$$\langle x \rangle = \frac{\int_0^\infty x \frac{dI}{dx}\, dx}{\int_0^\infty \frac{dI}{dx}\, dx}$$

$$= \frac{465}{32} \Big/ \frac{\pi^4}{16}$$

$$= 2.387$$

b) Gauss units: $Z_0 = 4\pi/c$

$$\frac{dP(t)}{d\Omega} = \frac{Z_0}{16\pi^2 c^2} \left| \left[n \times \frac{d^2 p}{d^2 t}(t') \right] \times n \right|^2$$

$$= \frac{1}{4\pi c^3} \Big| \mu_0 v^2 \langle \operatorname{sech}(vt') \circ \tanh^2(vt') - \operatorname{sech}^2(vt),\ 0,$$
$$-2\tanh(vt')\operatorname{sech}^2(vt') \rangle \times \cdots$$
$$\langle \sin\theta \sin\phi,\ \cos\theta - \sin\theta\cos\phi,\ \sin\theta\sin\phi \rangle$$

$$= \frac{1}{4\pi c^3} \Big| \mu_0 v^2 (-2\tanh(vt')\operatorname{sech}(vt') \sin\theta\sin\phi)\,\hat{i}$$
$$+ (\operatorname{sech}(vt')(\tanh^2(vt')\operatorname{sech}^2(vt'))\cos\theta$$
$$+ 2\tanh(vt')\operatorname{sech}^2(vt)\sin\theta\cos\phi)\,\hat{j}$$
$$+ (\operatorname{sech}(vt')(\tanh^2(vt) - \operatorname{sech}^2(vt')\sin\theta\sin\phi)\,\hat{k} \Big|$$

$$= \frac{1}{4\pi c^3} \Big(\mu_0^2 v^4 \big(4\tanh^2(vt')\operatorname{sech}^4(vt')\sin^2\theta$$
$$+ \operatorname{sech}^2(vt')(\tanh^2(vt') - \operatorname{sech}^2(vt'))^2$$
$$\times (\cos^2\theta + \sin^2\theta\sin^2\phi) \big) \Big)$$

$$P(t) = \int_0^{''}\!\!\int_0^{''} \frac{1}{4\pi c^3}\Big(\mu_0^2 v^4 \big(4\tanh^2(vt')\,\mathrm{sech}^4(vt')\sin^2\theta$$
$$+ \mathrm{sech}^2(vt')\left(\tanh^2(vt') - \mathrm{sech}^2(vt')\right)^2$$
$$\times \left(\cos^2\theta + \sin^2\theta\sin^2\phi\right)\sin\theta\, d\theta\, d\phi$$

$$= \frac{2v^4}{3c^3}\mu_0^2\left(4\tanh^2(vt')\,\mathrm{sech}^4(vt') + \mathrm{sech}^2(vt')\right)$$
$$\times\left(\tanh^2(vt') + \mathrm{sech}^4(vt')\right)^2\Big)$$

Total Power radiated:

"Intensity."

$$I = \int_{-\infty}^{\infty} P(t)\,dt$$

$$= \frac{2v^4}{3c^3}\mu_0^2 \int_{-\infty}^{\infty}\left(4\tanh^2(vt')\,\mathrm{sech}^4(vt') + \mathrm{sech}^2(vt')\right)$$
$$\times\left(\tanh^2(vt') + \mathrm{sech}^4(vt')\right)^2 dt'$$

$$= \frac{4}{3}\left(\frac{v}{c}\right)^3 \mu_0^2$$ ⊲– coefficient to part a.

4.21

a) Hydrogen-like Transition Probability:

"Bohr correspondence principle"

$$a = \frac{n^2 \cdot a_0}{Z} = \frac{n^2\hbar^2}{Zme^2}$$

(Problem 14.22b)

$$P_n = \frac{4e^2}{3c^3}(n\,\omega_0)^4 a^2\left\{\frac{1}{n^2}\left[J_n'^2(n\epsilon) + \left(\frac{1-\epsilon}{\epsilon}\right) J_n^2(n\epsilon)\right]\right\}$$

If $\epsilon \to 0$, then circular orbit by electron.

$$\lim_{\epsilon\to 0} J_n(n\epsilon) = \left(\frac{n\epsilon}{2}\right)^n$$

$$\lim_{\epsilon\to 0} J_n'(n\epsilon) = \frac{n}{2}\left(\frac{n\epsilon}{2}\right)^{n-1}$$

$$\lim_{\epsilon\to 0} P_n = \lim_{\epsilon\to 0} \frac{4e^2}{3c^3}(n\omega_0)^4 a^2 \left\{\frac{1}{n^2}\left[J_n'(n\epsilon)^2 + \left(\frac{1-\epsilon^2}{\epsilon^2}\right)J_n(n\epsilon)^2\right]\right\}$$

$$= \frac{4e^2}{3c^3}\omega_0^4 a^2\left(\frac{1}{4}+\frac{1}{4}\right)$$

$$= \frac{2e^2}{3c^3}\omega_0^4 a^2$$

$$\omega_0 = \frac{Ze^2}{ma^2}$$

$$= \frac{Ze^2}{m}\left(\frac{Zme^2}{n^2\hbar^2}\right)^3$$

$$= \frac{Z^4m^2e^8}{n^6\hbar^6}$$

$$= \frac{Z^2me^4}{n^3\hbar^3}$$

"Mean lifetime"

$$\frac{1}{\tau} = P_1/\hbar\omega_0$$

$$= \frac{2e^2}{3c^3}\frac{1}{\hbar}\omega_0^2a^2$$

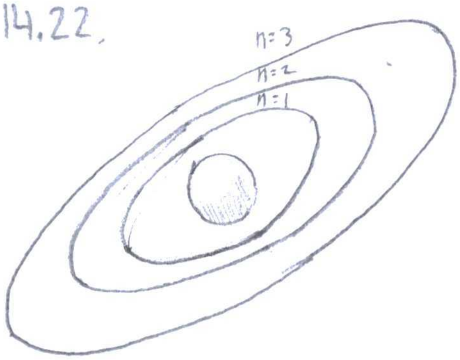
$$= \frac{Ze^2}{3c^3}\frac{1}{\hbar}\left(\frac{Z^2me^4}{m^3\hbar^3}\right)^3\left(\frac{n^2\hbar^2}{Zmc^2}\right)^2$$

$$= \frac{2}{3} \frac{e^2}{\hbar c} \left(\frac{Ze^2}{\hbar c} \right)^4 \frac{mc^2}{\hbar} \frac{1}{n^5}$$

b)

Transition:	Quantum Mechanical Values:	Classical Values:
$2p \longrightarrow 1s$	1.6×10^{-9} s	3.2×10^{-9} s
$4f \longrightarrow 3d$	7.3×10^{-8} s	1.2×10^{-7} s
$6h \longrightarrow 5g$	6.1×10^{-7} s	7.8×10^{-7} s

14.22.



"Elliptic motion in hydrogen atom"

$$x = a(\cos u - \epsilon)$$

$$y = a\sqrt{1-\epsilon^2}\, \sin u$$

$$\omega_0 t = u - \epsilon \sin u$$

where a = semi-major axis

$$= e^2/2B$$

ϵ = eccentricity

$$= \sqrt{1 - \frac{2BL^2}{me^4}}$$

ω_0 = orbital frequency

$$= \frac{8B^3}{me^4}$$

u = angle in polar

$$= \tan(u/2)$$

$$= \sqrt{(1-\epsilon)(1+\epsilon)}\ \tan(\theta/2)$$

a) Citation: Landau, Lifshitz

"The Classical Theory of Fields"

Academy of Sciences, U.S.S.R.

pg 195, "Radiation in the case of Coulomb interaction"

$$P_n = \frac{4e^2}{3c^3}(n\omega_0)^4 |r_n|^2 \quad \text{where} \quad r(t) = re^{-in\omega_0 t}$$

$$\dot{r}(t) = -in\omega_0 r e^{-in\omega_0 t}$$

$$r_n = \frac{\omega_0}{2\pi}\int_0^{2\pi/\omega_0} r(t) e^{in\omega_0 t} dt$$

$$= \frac{\omega_0}{2\pi}\int_0^{2\pi/\omega_0} e^{in\omega_0 t} \cdot \dot{r}(t) dt$$

$$= \frac{\omega_0}{2\pi}\int_0^{2\pi} e^{in(u-\epsilon\sin u)} dr$$

$$= \frac{\omega_0}{2\pi}\int_0^{2\pi} e^{in(u-\epsilon\sin u)} \begin{Bmatrix} -a\sin u \\ a\sqrt{1-\epsilon^2}\cos u \end{Bmatrix} du$$

where $r = a(\cos u - \epsilon) + a\sqrt{1-\epsilon^2}\sin u$

$$dr = (-a\sin u + a\sqrt{1-\epsilon^2}\cos u) du$$

$$= \frac{\dot{i}}{2\pi n}\int_0^{2\pi} e^{in(u-\epsilon\sin u)} \begin{Bmatrix} -a\sin u \\ a\sqrt{1-\epsilon^2}\cos u \end{Bmatrix} du$$

Identity:
$$e^{i\pi\cos\phi} = \sum_{m=-\infty}^{\infty} i^m J_n(x) e^{im\phi}$$

$$= \frac{-\dot{i}}{2\pi n}\int_0^{2\pi} e^{inu} \cdot \sum_{m=-\infty}^{\infty} J_m(n\epsilon)(-1)^m \exp\{-im(u+\pi/2)\} \begin{Bmatrix} -a\sin u \\ a\sqrt{1-\epsilon^2}\cos u \end{Bmatrix} du$$

$$= -\frac{a}{2n}\left\{J_{n+1}(n\epsilon)(-\dot{i})^{n+1} e^{-i(n+1)\pi/2} - J_{n+1}(n\epsilon)(-\dot{i})^{n-1} e^{-i(n-1)\pi/2}\right.$$

$$\left. + \frac{\dot{i}a\sqrt{1-\epsilon^2}}{2n}(-1)^{n+1} \cdot \{J_n(n\epsilon)\}\right\}$$

$$P_n = \frac{4e^2}{3c^3}(k\omega_0)^4 a^2 \left\{ \frac{1}{k^3}\left[\left(J_k'(k\epsilon)\right)^2 + \left(\frac{1-\epsilon^2}{\epsilon^2}\right) J_k(k\epsilon)^2 \right] \right\}$$

b) Eccentricity :

Circle	$\epsilon = 0$
Ellipse	$\epsilon = \frac{\sqrt{a^2-b^2}}{a}$, $\frac{\sqrt{b^2-a^2}}{b}$
Parabola	$\epsilon = 1$
Hyperbola	$\epsilon = \frac{\sqrt{a^2+b^2}}{a}$

$$\lim_{\epsilon \to 0} J_k(n\epsilon) = \left(\frac{n\epsilon}{2}\right)^n \qquad \lim_{\epsilon \to 0} J_k'(n\epsilon) = \frac{n}{2}\left(\frac{n\epsilon}{2}\right)^{n-1}$$

$$\lim_{\epsilon \to 0} P_n = \lim_{\epsilon \to 0} \frac{4e^2}{3c^3}(k\omega_0)^4 a^2 \left\{ \frac{1}{k^3}\left[J_k(k\epsilon)^2 + \left(\frac{1-\epsilon^2}{\epsilon^2}\right) J_k^2(k\epsilon) \right] \right.$$

$$= \frac{2e^2}{3c^2}\omega_0^4 a^2 \quad \ldots \text{when} \quad a = \frac{n^2 a_0}{Z}$$

$$= \frac{n^2 \hbar^2}{Z m c^2}$$

$$\omega_0 = \frac{Z^2 m e^4}{m a^3}$$

$$= \frac{Ze^2}{m}\left(\frac{Zme^2}{n^2\hbar^2}\right)^3$$

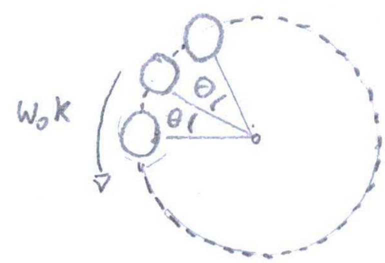
$$= \frac{Z^4 m^2 e^8}{n^6 \hbar^6}$$

$$\frac{1}{\tau} = P_1 / \hbar\omega_0$$

$$= \frac{2e^2}{3c^3}\frac{1}{\hbar}\omega_0^2 a^2$$

$$= \frac{2}{3}\frac{e^2}{\hbar c}\left(\frac{Ze^3}{\hbar c}\right)^4 \frac{mc^2}{\hbar}\frac{1}{n^5}$$

14.23.



"N charges... move with fixed relative positions... around the same circle"

a) (Problem 14.13a) "Power radiated per unit angle"

$$\frac{dP_m}{d\Omega} = \frac{e^2\omega_0^4 m^2}{(2\pi c)^3}\left|\int_0^{2\pi/\omega_0} v(t)\times n \exp^{im\omega_0\left(t-\frac{n\cdot x(t)}{c}\right)} dt\right|^2$$

(Problem 14.15a) "Angular distribution of polar radiation into mth-multipole"

$$\frac{dP_m}{d\Omega} = \frac{e^2\omega_0^4 R^2 m^2}{2\pi c^3}\left\{\left[\frac{dJ_m(m\beta\sin\theta)}{d(m\beta\sin\theta)}\right]^2 + \frac{\cot^2\theta}{\beta^2} J_m^2(m\beta\sin\theta)\right\}$$

where $\beta = \omega_0 R/c$

If $x(t) = R(\cos(\omega_0 t+\phi), \sin(\omega t+\phi), 0)$

$v(t) = \omega_0 R(-\sin(\omega_0 t+\phi), \cos(\omega_0 t+\phi), 0)$

$n = (\cos\theta, \cos\theta, -\sin\theta)$

$$\frac{dP_m}{d\Omega} = \frac{\omega_0^4 m^2}{(2\pi c)^3}\left|\int_0^{2\pi/\omega_0}\begin{pmatrix}\cos\theta\cos(\omega_0 t+\phi)\\ \cos\theta\sin(\omega_0 t+\phi)\\ -\sin\theta\cos(\omega_0 t+\phi)\end{pmatrix}\omega_0 R e^{im\omega_0 t} \circ e^{-ik(\omega_0 t+\phi)} dt\right|^2$$

$$= \frac{\omega_0^4 m^2}{(2\pi c)^3}\left| e^{-im\phi_0}\int_0^{2\pi/\omega_0}\begin{pmatrix}\cos\theta\cos(\omega_0 t+\phi)\\ \cos\theta\sin(\omega_0 t+\phi)\\ -\sin\theta\cos(\omega_0 t+\phi)\end{pmatrix}\omega_0 R \times e^{im\omega_0 t}\sum_{k=-\infty}^{\infty}(-i)^k J_k(m\beta\sin\theta) e^{-ik\omega_0 t} dt\right|^2$$

If $F_m(N) = \left|\sum_{j=1}^{N} q_j e^{im\theta}\right|^2$, then

$$\frac{dP_{m(N)}}{d\Omega} = \frac{\omega_0^4 m^2}{(2\pi c)^3}\left|\int_0^{2\pi/\omega_0}\begin{pmatrix}\cos\theta\cos(\omega_0 t+\phi)\\ \cos\theta\sin(\omega_0 t+\phi)\\ -\sin\theta\cos(\omega_0 t+\phi)\end{pmatrix}\omega_0 R\right.$$

d) If $N >> \gamma^3$

$$\frac{dP_m(N)}{d\Omega} = \frac{\omega_0^4 m^2}{(2\pi c)^3}\left|\sum_{j=1}^{N} q_j e^{-im\phi}\int_0^{2\pi/\omega_0}\begin{pmatrix}\cos\theta\cos(\omega_0 t)\\ \cos\theta\sin(\omega_0 t)\\ -\sin\theta\cos(\omega_0 t)\end{pmatrix}\omega_0 k\, e^{im\omega_0 t}\right.$$

$$\left. \times \sum_k (-i)^k J_k(m\beta\sin\theta)\, e^{-ik_0\omega t}\, dt\right|^2$$

Structure Factor:

$$F_m(N) = \left|\sum_{j=1}^{\gamma^3} q_j e^{-im\phi}\right|^2$$

$$= \left|\sum_{j=1}^{\gamma^3} q_j e^{-im2\pi j/N}\right|^2$$

$$\propto \text{constant} \times e^{-i2\cdot N/3\gamma^3} \qquad \text{when } m = \tfrac{1}{3},\ j = N$$

$$\lim_{N\to\infty} \frac{dP_m(N)}{d\Omega} = 0$$

e) Above, the radiation pattern describes a steady state current in a circular path. A common place with charges in a circular path is the cyclotron.

14.24.

"N identical charges.. move with constant speed"

"Steady-state current flowing in a circuit"

$$\times e^{im\omega_0 t} \sum_{k=-\infty}^{\infty} (-i)^k J_k(m\beta \sin\theta) e^{-ik\omega_0 t} \, dt \Bigg|^2$$

$$= \frac{dP_m}{d\Omega} \cdot F_m(N)$$

b) The coefficient $F_m(N)$ describes the mth multipole. Two charges evaluates to

$$F_m(N) = q^2 \left| \sum_{j=1}^{2} e^{-im\theta} \right|^2$$

<u>Proof by induction:</u>

① Base case: $F_m(1) = q \cdot e^{-im2\theta}$

② Inductive hypothesis: $F_m(N) = q^2 \cdot N e^{-im2\theta}$

③ Inductive step: $F_m(N+1) = q^2 \cdot (N+1)^2 e^{-im2\theta}$

$$= q^2 N^2 e^{-im2\theta} + \ldots$$

c) When $v \ll c$ and $\beta \ll 1$,

$$\lim_{\beta \to 0} \frac{dP(\ell)}{d\Omega} = \lim_{\beta \to 0} \frac{dP_m}{d\Omega} F_m(N)$$

$$= \beta^{2k} \frac{\omega_0^4 m^2}{(2\pi c)^3} \left(\sum q e^{-im\phi} \int_0^{2\pi/\omega_0} \begin{pmatrix} \cos\theta \cos(\omega_0 t + \phi) \\ \cos\theta \sin(\omega_0 t + \phi) \\ -\sin\theta \cos(\omega_0 t + \phi) \end{pmatrix} \cdot \omega_0 R \right.$$

$$\left. e^{im\omega_0 t} \sum_{k=-\infty}^{\infty} (-i)^k (m\beta \sin\theta)^k e^{-ik\omega_0 t} \cdot dt \right|^2$$

As N approaches infinity, k approaches infinity, and $\frac{dP(\ell)}{d\Omega} = 0$.

(14.8) "Lienard Wiechart Potential"

$$\Phi(X,t) = \left[\frac{e}{(1-\beta\cdot n)R}\right] \qquad A(X,t) = \left[\frac{e\beta}{(1-\beta\cdot n)R}\right]$$

$$\Phi_N(X,t) = \sum_{n=1}^{N} q^n \left[\frac{e}{(1-\beta\cdot n)R}\right] \qquad A(X,t) = \sum_{n=1}^{N} q^n \left[\frac{e\beta}{(1-\beta\cdot n)R}\right]$$

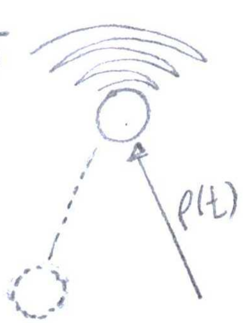
$$\lim_{q\to 0} \Phi_N(X,t) = \lim_{q\to 0} \sum_{n=1}^{N} q^n \left[\frac{e}{(1-\beta\cdot n)R}\right]$$

$$= 0$$

$$\lim_{q\to 0} A(X,t) = \lim_{q\to 0} \sum_{n=1}^{N} q^n \left[\frac{e\beta}{(1-\beta\cdot n)R}\right]$$

$$= 0$$

14.25



"relativistic particle moving in a path ... radiation with positive and negative helicity"

a) (14.79) "Energy radiated per unit frequency per unit solid angle"

$$\frac{d^2 I}{d\omega\, d\Omega} = \frac{e^2}{3\pi^2 c}\left(\frac{\omega_e}{c}\right)^2\left(\frac{1}{\gamma^2}+\theta^2\right)^2\left[K_{2/3}^2(\xi) + \frac{\theta^2}{(1/\gamma^2)+\theta^2}K_{1/3}^2(\xi)\right]$$

(14.67) "Intensity distribution"

$$\frac{d^2 I}{d\omega\, d\Omega} = \frac{e^2\omega^2}{4\pi^2 c}\left|\int_{-\infty}^{\infty} n\times(n\times\beta)\, e^{i\omega(t - n\cdot r(t)/c)} \, dt\right|^2$$

(14.71) "Vector"

$$n\times(n\times\beta) = \beta\left[-\epsilon_{\parallel}\sin\left(\frac{vt}{\rho}\right) + \epsilon_{\perp}\cos\left(\frac{vt}{\rho}\right)\sin\theta\right]$$

(14.72) "Argument in exponential"

$$\omega\left(t - \frac{n \cdot r(t)}{c}\right) = \omega\left[t - \frac{\rho}{c}\sin\left(\frac{vt}{\rho}\right)\cos\theta\right]$$

$$\boxed{\begin{array}{l}\underline{\text{Taylor Expansion:}} \\ \sin\left(\frac{vt}{\rho}\right) = \frac{vt}{\rho} - \left(\frac{vt}{\rho}\right)^3 \frac{1}{3!} + \dots \\ \cos\theta = 1 + \frac{\theta^2}{2!} - \dots\end{array}}$$

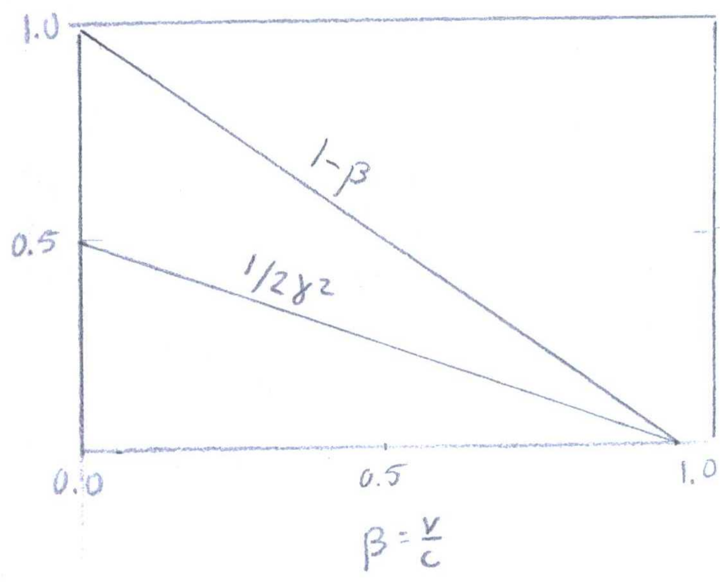
$$\cong \omega\left[t - \beta t + \beta \frac{t^2\theta^2}{2} + \frac{\rho}{c}\left(\frac{vt}{\rho}\right)^3 \frac{1}{3!} + \dots\right]$$

... when $\beta = \frac{v}{c}$

$$\cong \omega\left[t(1-\beta) + \beta \frac{t\theta^2}{2} + \beta \frac{^3 t^3 c^2}{\rho^2 c^3} + \dots\right]$$

... if $\beta \sim 1$ (relativistic speeds)

then $\frac{1}{2\gamma^2} \cong 1-\beta$



$$\simeq \frac{\omega}{2}\left[\left(\frac{1}{\gamma^2}+\theta^2\right)t + \frac{c^2}{\beta^3\rho^2}t^3\right]$$

$\therefore$ as if $\theta \ll 1$ "put to unity wherever possible" (pg 677)

and $\beta \sim 1$

(14.74) "Energy distribution approximation"

$$\frac{d^2I}{d\omega d\Omega} = \frac{e^2\omega^2}{4\pi^2 c}\left|-\epsilon_\parallel A_\parallel(\omega) + \epsilon_\perp A_\perp(\omega)\right|^2$$

(14.75) "Amplitudes"

$$A_\parallel(\omega) \simeq \frac{c}{\rho}\int_{-\infty}^{\infty} t \exp^{i\frac{\omega}{2}\left[\left(\frac{1}{\gamma^2}+\theta^2\right)t + \frac{c^2 t^3}{3\rho^2}\right]} dt$$

$$A_\perp(\omega) \simeq \theta \int_{-\infty}^{\infty} \exp^{i\frac{\omega}{2}\left[\left(\frac{1}{\gamma^2}+\theta^2\right)t + \frac{c^2 t^3}{3\rho^2}\right]} dt$$

(14.76) "Transform Integrals"

$$A_\parallel(\omega) = \frac{\rho}{c}\left(\frac{1}{\gamma^2}+\theta^2\right)\int_{-\infty}^{\infty} x \exp^{i\frac{3}{2}\xi\left(x+\frac{1}{3}x^3\right)} dx$$

$$A_\perp(\omega) = \frac{\rho\theta}{c}\left(\frac{1}{\gamma^2}+\theta^2\right)^{1/2}\int_{-\infty}^{\infty} \exp^{i\frac{3}{2}\xi\left(x+\frac{1}{3}x^3\right)} dx$$

$\therefore$ from $x = \dfrac{ct}{\rho\left(\frac{1}{\gamma^2}+\theta^2\right)^{1/2}}$

and $\xi = \dfrac{\omega\rho}{3c}\left(\frac{1}{\gamma^2}+\theta\right)^{1/2}$

Euler-Identity:

$e^{ix} = \cos(x) + i\sin(x)$

Airy integral as Modified Bessels:

$$Ai(x) = \frac{1}{\pi}\int_0^\infty \cos\left(xt + \frac{1}{3}t^3\right)dt$$

$$= \frac{1}{\pi}\sqrt{\frac{x}{3}}\, K_{\pm 1/3}(\xi)$$

$$Ai'(x) = \frac{-1}{\pi}\int_0^\infty x\sin\left(xt + \frac{1}{3}t^3\right)dt$$

$$= \frac{-1}{\pi}\left(\frac{x}{\sqrt{3}}\right) K_{\pm 2/3}(\xi)$$

when $\xi = \frac{2}{3} z^{3/2}$

N.I.S.T. - Digital Library of Mathematical Functions

(14.79) "Energy radiated per unit frequency interval per unit solid angle"

$$\frac{d^2 I}{d\omega\, d\Omega} = \frac{e^2}{3\pi^2 c}\left(\frac{\omega\rho}{c}\right)^2\left(\frac{1}{\gamma^2} + \theta^2\right)^2\left[K_{2/3}^2(\xi) + \frac{\theta^2}{(1/\gamma^2) + \theta^2} K_{1/3}^2(\xi)\right]$$

b) (14.81) "Critical frequency"

$$\omega_c = \frac{3}{2}\gamma^3\left(\frac{c}{\rho}\right) = \frac{3}{2}\left(\frac{E}{mc^2}\right)^3 \frac{c}{\rho}$$

1) High frequency: $\omega > \omega_c$:

$$\frac{d^2 I}{d\omega d\Omega} = \frac{3}{4\pi}\frac{e^2}{c}\gamma^2 \frac{\omega}{\omega_c} e^{-\omega/\omega_c}$$

2) Intermediate and Low Frequency ($\omega < \omega_c$):

$$\frac{d^2 I}{d\omega d\Omega} = \frac{e^2}{3\pi^2 c}\left(\frac{\omega \rho}{c}\right)^2 \left(\frac{2\theta^2}{\gamma^2} + \theta^4\right)\left[K_{2/3}^2(\xi) + \frac{\theta^2}{(1/\gamma^2)+\theta^2} K_{1/3}^2(\xi)\right]$$

3) Intermediate and Low Frequency at very small angles:

$$\left.\frac{d^2 I}{d\omega d\Omega}\right|_{\theta=0} = \frac{e^2}{c}\left[\frac{\Gamma\left(\frac{2}{3}\right)}{\pi}\right]\left(\frac{3}{4}\right)^{1/3}\left(\frac{\omega \rho}{c}\right)^{2/3}$$

c) Citation: P. Joos Phys Rev. Letters 4, 558, (1960)
"Measurement of the Polarization of Synchrotron Radiation"
Ithaca, New York. Cornell

The guy plotted parallel and perpendicular polarization by a synchrotron against theoretical values.

Peter modelled Intensity vs. angle in a 700MeV and RCA-6342 photomultiplier. He liked Bessels as the approximation:

$$J_{\parallel} = -\frac{ec}{\pi\sqrt{3}} i x^2 K_{2/3}\left(\frac{\omega}{3\omega_0}\gamma^3\right) \quad \text{(parallel)}$$

$$J_{\perp} = -\frac{ec}{\pi\sqrt{3}} x^{2\sqrt{}} K_{1/3}\left(\frac{\omega}{3\omega_0}\cdot x^3\right) \quad \text{(perpendicular}$$

The function is proportional to (14.79).

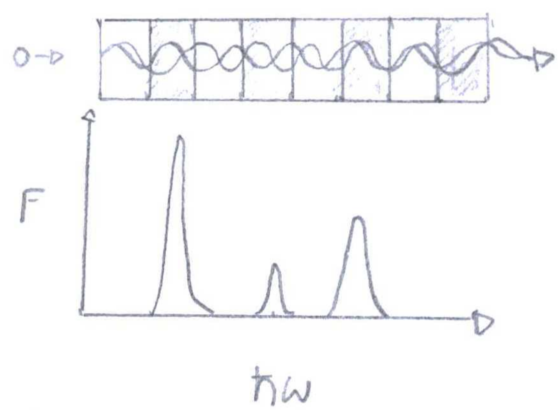
$$\frac{d^2 I}{d\omega d\Omega} = \frac{e^2}{3\pi^2 c}\left(\frac{\omega\rho}{c}\right)\left(\frac{1}{\gamma^2}+\theta^2\right)^2\left[K_{2/3}^2(\xi) + \frac{\theta^2}{(1/\gamma^2)+\theta^2} K_{1/3}^2(\xi\right.$$

Errors arose in the experiment such as electron frequency, bandwidth, and slit-width.

14.26

"Synchrotron radiation from the crab nebula"

14.27



"radiation emitted at twice the fundamental frequency ... in a sinusoidal undulator"

a) (pg 688)

$$x'(t') = \frac{K}{\gamma k_0} \sin(\theta(t')) = a \sin \theta(t')$$

$$z'(t) = \frac{\gamma K^2}{8\gamma^2 k_0} \sin(2\theta(t')) = \frac{Ka}{8\sqrt{1+K^2/2}} \sin(2\theta(t'))$$

If $k' = 2\gamma k_0$,

(9.4) "Magnetic field"

$$H = \frac{1}{\mu_0} \nabla \times A$$

(9.16) "Vector Potential"

$$A(x) = -\frac{i\mu_0 \omega}{4\pi} p \frac{e^{ikr}}{r}$$

(9.17) "Electric Dipole Moment"

$$p = \int x' \rho(x') d^3x'$$

(1.10) "Magnetic Induction - only one term"

$$H = \mu_0 B$$

$$= \nabla \times A$$

$$= \nabla \times \left(-\frac{i\omega}{4\pi} p \frac{e^{ikr}}{r} \right)$$

$$= \nabla \times \left(\frac{-ik'}{4\pi c} p \frac{e^{ik'(r'-ct')}}{r'} \right)$$

$$= \frac{\mu_0 k'^2}{4\pi c}(n \times p)\frac{e^{ik'(r-ct)}}{r'}\left(1-\frac{1}{ik'(r)}\right)$$

$$= \frac{k'^2}{4\pi c}\frac{e^{ik'(r-ct)}}{r'}\left(1-\frac{1}{k'r'}\right)(n \times p)$$

$$= \underbrace{\frac{k'^2}{4\pi c}\frac{e^{ik'(r-ct)}}{r'}}_{\text{"Far (radiation)" Zone}}\left(1-\underbrace{\frac{1}{k'r'}}_{\text{"near (radiation)" Zone}}\right) n \times \int x \rho(x)\, d^3x$$

(pg 408) "Far (radiation)" Zone "near (radiation)" Zone

<u>In the near zone, (pg 411):</u>

$$= \frac{k'^2}{4\pi c}\frac{e^{ik'(r-ct)}}{r'}\, n \times \int x \rho(x)\, d^3x$$

$$= \frac{k'^2}{4\pi c}\frac{e^{ik'(r-ct)}}{r'}\, n \times \int [\hat{z} - 4\hat{x}(n\cdot\hat{x})]\, d^3x$$

(9.42)

$$= \frac{k'^2}{4\pi c}\frac{\cos(k'(r-ct)) + i\sin(k'(r-ct))}{r'}\, n \times \int [\hat{z} - 4\hat{x}(n\cdot\hat{x})]\, d^3x$$

$$\cong \frac{ik'^2}{8} a \frac{k}{\sqrt{1+k^2/2}}\, n \times \int [\hat{z} - 4\hat{x}(n\cdot\hat{x})]\cdot d^3x$$

: when only sin term considered as $z'(t')$

b) $$\frac{dP'}{d\Omega} = \frac{e^2 c}{8\pi}\frac{k^2}{(1+k^2/2)}\frac{a^2}{64} S'$$

(pg 689) "Power distribution of an undulator"

$$\frac{dP'}{d\Omega} = \frac{e^2 c}{8\pi} k'^4 a^2 \sin^2\theta$$

$$= \frac{e^2 c}{8\pi} \frac{k'^2}{(1+K^2/2)} \frac{a^2}{64} S'$$

c) $$d^3P' = \left[\frac{e^2 c^2}{8\pi} \frac{k''}{\sqrt{1+K^2/2}} \frac{a^2}{64} S' \frac{\delta(k'-\bar{\gamma}k_0)}{\bar{\gamma}k_0}\right] \frac{d^3k}{\omega'}$$

$$\frac{d^3P}{dk\,d\Omega} = \frac{e^2 c}{8\pi\gamma^3} \frac{k}{\sqrt{1+K^2/2}} \frac{a^2}{64} S' \delta(k'-\gamma k_0)$$

where $k_y = k\sin\theta\sin\phi$

$k_y' = k\sin\theta\sin\phi$

$k_z' = \bar{\gamma}k(\cos\theta - \bar{\beta})$

$k' = \bar{\gamma}k(1-\bar{\beta}\cos\theta)$

If $\eta = (\bar{\gamma}\theta^2)$, $\gamma \gg 1$

$$\frac{d^2P}{d\eta\,dk\,d\phi} = \frac{e^2 c\,\bar{\gamma}^2}{2\pi} \frac{k}{\sqrt{1+K^2/2}} \frac{a^2}{64} S' \left[\frac{(1-\eta)^2 + 4\eta\sin^2\phi}{(1+\eta)^4}\right] \delta(k(1+\eta) - 2\bar{\gamma}^2 k_0)$$

$$\frac{d^2P}{d\eta\,d\phi} = \frac{e^2 c\gamma^2}{2\pi} \frac{k}{\sqrt{1+K^2/2}} \frac{a^2}{64} S' \left[\frac{(1-\eta)^2 + 4\eta\sin^2\phi}{(1+\eta)^5}\right]$$

$$\frac{dP}{d\phi} = \int \frac{e^2 c\gamma^2}{2\pi} \frac{k}{\sqrt{1+K^2/2}} \frac{a^2}{64} S' \left[\frac{(1-\eta)^2 + 4\eta\sin^2\phi}{(1+\eta)^5}\right] d\eta$$

$$= \frac{3P}{16}\left[\frac{K^2}{(1+K^2/2)^2}\right] \cdot \nu^2 (10 - 21\nu + 20\nu^2 - 6\nu^2)$$

where $\nu = \frac{k}{2\bar{\gamma}^2 k_0}$

and $P = \frac{e^2 c \bar{\gamma}^2 K^2 k_0^2}{3}$

Total Power radiated:

$\nu_{min} = 2/(4\eta_2)$ $\qquad \nu_{max} = 2/(1+\eta_1)$

Notes: Chapter 14 describes functions in high-speed objects, such as a boat or car. At lightspeed, the car's headlights narrow into a point, change to pastel color, and spiral into a helix at specific angles.

During a turn, the headlights fan outward horizontally and wobble vertically. Technically, at lightspeed, everything goes dark when on top the hood and peering back into the lights.