# Chapter 15: Bremstrahlung, Method of Virtual Quanta, Radiative Beta Processes:

15.1 (14.65) "Energy radiated per unit solid angle per unit frequency"

$$\frac{d^2I}{d\omega d\Omega} = \frac{e^2}{4\pi^2 c}\left|\int_{-\infty}^{\infty} \frac{n\times[(n-\beta)\times\dot{\beta}]}{(1-n\cdot\beta)^2} e^{i\omega(t-n\cdot r(t)/c)} dt\right|^2$$

$$= \frac{e^2}{4\pi^2 c}\left|\int_{-\infty}^{\infty} \frac{n\times[(n-\beta)\times\dot{\beta}]}{(1-n\cdot\beta)^2} e^{i\omega t}\cdot e^{-ikr} dt\right|^2$$

$$= \frac{e^2}{4\pi^2 c}\left|\int_{-\infty}^{\infty} \frac{n\times[(n-\beta)\times\dot{\beta}]}{(1-n\cdot\beta)^2} e^{i\omega t}(1-ikr) dt\right|^2$$

Taylor Series: Exponential

$$e^x = 1 + x + \frac{x^2}{2!} +$$

(14.66) "Perfect Differential"

$$\frac{n\times[(n-\beta)\times\dot{\beta}]}{(1-n\cdot\beta)^2} = \frac{d}{dt}\left[\frac{n\times(n\times\beta)}{1-n\cdot\beta}\right]$$

$$\frac{d^2I}{d\omega d\Omega} = \frac{e^2}{4\pi^2 c}\left|\int_{-\infty}^{\infty} \frac{d}{dt}\frac{n\times[(n\times\beta)]}{1-n\cdot\beta} e^{i\omega t}(1-ikr) dt\right|^2$$

$$= \frac{e^2}{4\pi^2}\left|\int_{-\infty}^{\infty} \frac{d}{dt}\frac{\dot{\beta}_\perp - \dot{\beta}_\perp ikr}{(1-n\cdot\beta)} e^{i\omega t} dt\right|^2$$

(14.69) "Continuous distribution"

$$e\beta e^{-i(\omega/c)n\cdot r(t)} \Rightarrow \frac{1}{c}\int d^3x\, J(x,t)\, e^{-i(\omega/c)n\cdot r}$$

$$J = e\int dt\, e^{i\omega t}(\dot{\beta}_\perp - \dot{\beta}\, i k r)$$

b) (14.67) "Intensity distribution"

$$\frac{d^2 I}{d\omega\, d\Omega} = \frac{e^2\omega^2}{4\pi^2 c}\left|\int_{-\infty}^{\infty} n\times(n\times\beta)\, e^{i\omega(t - n\cdot r(t)/c)}\, dt\right|^2$$

(from problem) "Current Density"

$$J = e\int dt\, e^{i\omega t}\left(\dot{\beta}_\perp + \frac{d}{dt}(\beta_\parallel \dot{\beta}_\perp) - i k r \dot{\beta}_\perp + \cdots\right) e^{-r}$$

$$= \frac{1}{c}\int dt\, e^{i\omega t}\cdot q\left(\dot{\beta}_\perp + \frac{d}{dt}(\beta_\parallel \dot{\beta}_\perp) - i k r \beta_\perp + \cdots\right)\left(1 - r + r^2 - \frac{r^3}{6}\right)$$

$$= \frac{1}{c}\int dt\, e^{i\omega t}\left(-qr + q\frac{r^2\times\beta}{2} - \frac{1}{6c} q\frac{r^3}{2} + \cdots\right)$$

$$= \frac{1}{c}\int dt\, e^{i\omega t}\left(\frac{dP}{dt} + \frac{dm}{dt}\times n + \frac{1}{6c}\frac{dQ}{dt} + \cdots\right)$$

$\therefore$ when $q = e$, $p = q\cdot r$, $m = q\dfrac{r\times\beta}{2}$

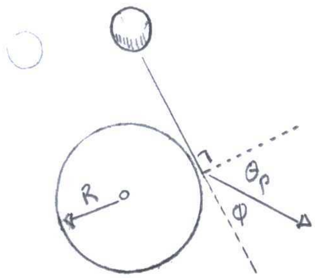
and $Q = q(3XX - r^2\delta_{ij})$

(14.69) "Continuous distribution"

$$e\dot{\beta}\, e^{-i(\omega/c)\cdot n\cdot r(t)} = \frac{1}{c}\int d^3x\, J(X,t)\, e^{-i(\omega/c)\, n\cdot r}$$

$$\frac{d^2 I}{d\omega\, d\Omega} = \frac{1}{4\pi c}\left|\int dt\, e^{i\omega t}\left(\frac{dp}{dt} + \frac{dm}{dt}\times n + \frac{1}{6c}\frac{dQ}{dt} + \cdots\right)\right.$$

15.2.




"nonrelativistic particle ... collides with a fixed smooth hard sphere ... for emission of photon per unit solid angle"

(      ) "Elastic differential/Cross section"

$$d\sigma_p = \frac{1}{4} R^2 d\Omega_p$$

(      ) "Velocity change"

$$\Delta\beta = 2\beta \sin\phi \left(\sin\phi\, \hat{e}_z + \cos\phi\, \hat{e}_x\right)$$

$$= 2\beta \cos\left(\frac{\theta_p}{2}\right)\left(\cos\left(\frac{\theta_p}{2}\right)\hat{e}_z + \sin\left(\frac{\theta_p}{2}\right)\hat{e}_x\right)$$

$$= \beta\left((\cos^2\theta_p + 1)\hat{e}_z + \sin\theta_p\, \hat{e}_x\right)$$

If $\phi = (\pi - \theta_p)|2$, then

$$\Delta\beta = \beta\left((\cos\theta_p + 1)\hat{e}_z + \sin\theta_p \cos\phi_p\, \hat{e}_x + \sin\theta_p \sin\phi_p\, \hat{e}_y\right)$$

(15.7) "radiated intensity"

$$\lim_{\omega\to 0} \frac{d^2 I_{rad}}{d\omega\, d\Omega} = \frac{z^2 e^2}{4\pi^2 c}\left|\epsilon^* \cdot \Delta\beta\right|^2$$

(15.4) "Soft photon energy per unit energy and per unit solid angle"

$$\frac{d^3\sigma}{d\Omega_p\, d(\hbar\omega)\, d\Omega_\gamma} = \left[\lim_{\hbar\omega\to 0} \frac{d^2 N}{d(\hbar\omega)\, d\Omega_\gamma}\right] \cdot \frac{d\sigma}{d\Omega_p}$$

(15.5) "invariant expression"

$$\frac{d^3 N}{(d^3k/k_0)} = \frac{c^2}{\hbar\omega} \frac{d^2 N}{d(\hbar\omega)\, d\Omega_\gamma} = \frac{c^2}{\hbar(\hbar\omega)} \cdot \frac{d^2 I}{d\omega\, d\Omega_\gamma}$$

$$\frac{d^2 I}{d\omega\, d\omega} = \frac{d^2\sigma}{d\Omega_p\, d(\hbar\omega)\, d\Omega_\gamma} \frac{1}{\hbar^2\omega} \cdot \frac{d\sigma_p}{d\Omega_p}$$

$$\frac{d^2\sigma}{d\Omega_p\, d(\hbar\omega)\, d\Omega_p} = \frac{z^2 e^3}{4\pi^2} \frac{1}{\hbar^2(\omega)}\left|\epsilon^* \cdot \Delta\beta\right|^2 \frac{d\sigma}{d\Omega_p}$$

If $Z=1$, then

$$\frac{d^2\sigma}{d\Omega_p d(\hbar\omega) d\Omega_p} = \frac{e^3}{4\pi^2} \cdot \frac{1}{\hbar^2(\omega)} \left| \epsilon^* \cdot \Delta\beta \right| \frac{d\sigma}{d\Omega_p}$$

... where $\vec{\epsilon}_1 = -\sin\theta\,\hat{e}_z + \cos\theta\,\hat{e}_x$ and $\vec{\epsilon}_2 = \hat{\epsilon}_y$

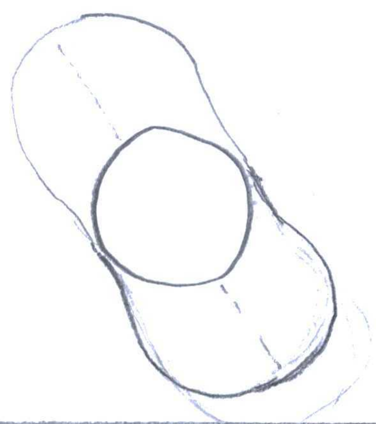
$$\frac{d^3\sigma}{d\Omega_p d(\hbar\omega) d\Omega_y} = \frac{e^2}{16\pi^2 c} \frac{\beta^2}{\hbar\omega} R^2 \Big[ \left(-\sin\theta(\cos\theta_p + 1) + \cos\theta \sin\theta_p \cos\phi_p\right)^2 + \sin^2\theta_p \sin^2\phi_p \Big]$$

$$\frac{d^2\sigma}{d(\hbar\omega) d\Omega_\gamma} = \int \frac{d^3\sigma}{d\Omega_p d(\hbar\omega) d\Omega_y} d\Omega_p$$

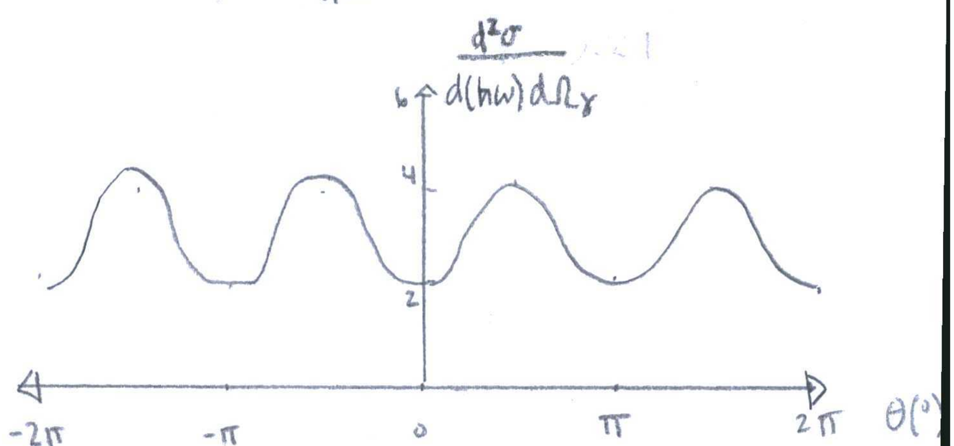
$$= \int_0^{2\pi} \int_0^{\pi} \frac{e^2}{16\pi^2 c} \cdot \frac{\beta^2}{\hbar\omega} R^2 \Big[ \left(-\sin\theta(\cos\theta_p + 1) + \cos\theta \sin\theta_p \cos\phi_p\right)^2 + \sin^2\theta_p \sin^2\phi_p \Big] d\theta_p d\phi_p$$

$$= \frac{R^2}{16\pi^2} \frac{e^2}{\hbar\omega} \frac{\beta^2}{\hbar\omega} \Big[ \sin^2\theta \left(\frac{4\pi}{3} + 4\pi\right) + \cos^2\theta \left(\frac{4\pi}{3} + \frac{4\pi}{3}\right) \Big]$$

$$\cong \frac{R^2}{12\pi} \frac{e^2}{\hbar c} \left(\frac{v}{c}\right)^2 \frac{1}{\hbar\omega} \left(2 + 3\sin^2\theta\right) \quad \ldots \text{if } \theta \ll 1$$



$\boxed{\text{Spectrum in polar coord.-dipole}}$




$\boxed{\text{Spectrum in Cartesion coord.}}$

## Total Bremsstrahlung cross section:

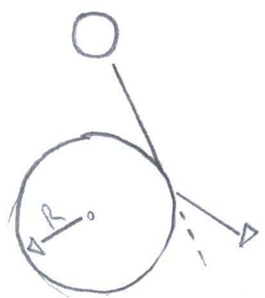
$$\frac{d\sigma}{d(\hbar\omega)} = \int \frac{d^2\sigma}{d(\hbar\omega)\,d\Omega_\gamma}\,d\Omega_\gamma$$

$$= \frac{R^2}{12\pi}\frac{e^2}{\hbar c}\frac{\beta^2}{\hbar\omega}\left(8\pi + 3\,\frac{8\pi}{3}\right)$$

$$= \frac{4R^2}{3}\frac{e^2}{\hbar c}\frac{\beta^2}{\hbar\omega}$$

Upper limit: $\omega R/c \ll 1 \;\therefore\; \beta \ll 1$

15.3



"elastic impact is negligable"

$$\beta_i = \beta\,\hat{\epsilon}_z$$

$$\beta_f = \beta(\sin\theta\cos\phi_p\,\hat{e}_x + \sin\theta_p\sin\phi_p\,\hat{e}_y + \cos\theta_p\,\hat{e}_z)$$

$$\hat{n} = \sin\theta\,\hat{e}_x + \cos\theta\,\hat{e}_z$$

$$\hat{\epsilon}_1 = \cos\theta\,\hat{\epsilon}_x - \sin\theta\,\hat{e}_z$$

$$\hat{\epsilon}_2 = \hat{\epsilon}_y$$

$$\beta\circ n = \beta(\sin\theta\sin\theta_p\cos\phi_p + \cos\theta\cos\theta_p)$$

$$\epsilon_1\circ\beta_1 = -\beta\sin\theta$$

$$\epsilon_1\cdot\beta_f = \beta(\cos\theta\sin\theta\cos\phi_p - \sin\theta\cos\theta_p)$$

$$\epsilon_2\circ\beta = 0$$

$$\epsilon_2\circ\beta_f = \beta\sin\theta_p\cdot\sin\phi_p$$

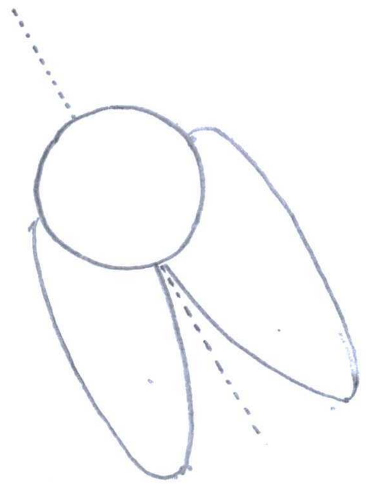
(15.2) "spectrum of radiation with polarization

$$\lim_{\omega\to 0}\frac{d^2I}{d\omega\,d\Omega} = \frac{z^2e^2}{4\pi^2c}\left|\epsilon^*\circ\left(\frac{\beta'}{1-n\circ\beta'} - \frac{\beta}{1-n\circ\beta}\right)\right|^2$$

$$= \frac{R^2}{4}\frac{e^2}{4\pi^2 c}\frac{\beta^2}{\hbar\omega}\left[\left(\frac{\cos\theta\sin\theta_p\cos\phi_p - \sin\theta\cos\theta_p}{1-\beta(\sin\theta\sin\theta_p\cos\phi_p+\cos\theta\cos\theta_p)} + \frac{\sin\theta}{1-\beta\sin\theta}\right)^2 + \frac{\sin^2\theta_p\sin^2\phi_p}{[1-\beta(\sin\theta\sin\theta_p\cos\phi_p+\cos\theta\cos\theta_p)]^2}\right]$$

"spectrum in polar coord - relativistic dipole"

$\boxed{\beta = 0.98}$

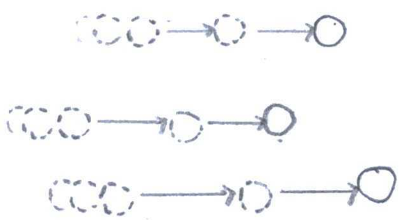


$$\frac{d^2\sigma}{d\Omega\, d(\hbar\omega)} = \int \frac{d^3\sigma}{d\Omega\, d(\hbar\omega)\, d\Omega}\, d\Omega$$

$$= \int_0^{2\pi}\int_0^{\pi} \frac{R^2}{4}\frac{e^2}{4\pi^2 c}\frac{\beta^2}{\hbar\omega}\left[\left(\frac{\cos\theta\sin\theta_p\cos\phi_p - \sin\theta\cos\theta_p}{1-\beta(\sin\theta\sin\theta_p\cos\phi_p+\cos\theta\cos\theta_p)} + \frac{\sin\theta}{1-\beta\sin\theta}\right)^2 + \frac{\sin^2\theta_p\sin^2\phi_p}{1-\beta(\sin\theta\sin\theta_p\cos\phi_p+\cos\theta\cos\theta_p)^2}\right] d\theta\, d\phi$$

$$= \frac{R^2}{4\pi}\frac{e^2}{\hbar c}\frac{\beta^2}{\hbar\omega}\left[\frac{\sin^2\theta}{(1-\beta\cos\theta)^2} + \frac{1}{\beta^3}\ln\left(\frac{1+\beta}{1-\beta}\right) - \frac{2}{\beta^2}\right]$$

15.4



"group of charged particles undergo... acceleration"

a) (14.14) "Energy at light speed - modified"

$$E_{rad} = \frac{1}{c}\sum_j e_j \left[\frac{n\times\{(n-\beta)\times\dot\beta\}}{(1-\beta\cdot n)^3 R}\right]_{ret}$$

$$\frac{d^2 I}{d\omega\, d\Omega} = \frac{1}{4\pi^2 c}\left|\epsilon^*\cdot\int_{-\infty}^{\infty}\sum_j e_j \frac{n\times[(n-\beta)\times\dot\beta]}{(1-\beta\cdot n)^2} e^{i\omega(t-n\cdot r(t))/c}\, dt\right|$$

(14.65) "Energy radiated per unit solid angle per unit frequency interval"

$$= \frac{1}{4\pi^2 c}\left|\epsilon^*\cdot\sum_j e_j\int_{-\infty}^{\infty}\frac{d}{dt}\left[\frac{n\times(n\times\beta)}{1-\beta\cdot n}\right] e^{i\omega(t-n\cdot r(t)/c)}\, dt\right|$$

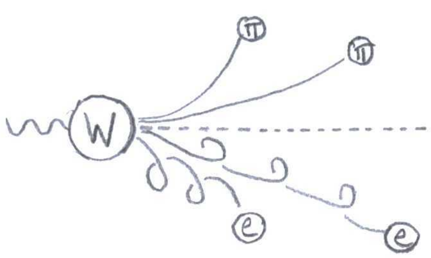
If $\omega t \ll 1$,

$$= \frac{1}{4\pi^2 c}\left|\epsilon^* \cdot \sum_j e_j\left(\frac{\beta_j'}{1-\beta_j'\cdot n} - \frac{\beta}{1-\beta\cdot n}\right)e^{i\omega(t-n\cdot r(t))/c}\right|^2$$

$$= \frac{1}{4\pi^2 c}\left|\epsilon^* \cdot E\right|^2$$

...where $E = \sum_j e_j\left(\frac{\beta_j'}{1-\beta_j'\cdot n} - \frac{\beta_j}{1-\beta\cdot n}\right)e^{i\omega(t-n\cdot r(t))/c}$

b) $$\frac{d^2 I}{d\omega d\Omega} = \frac{|\epsilon^*|^2}{4\pi^2 c}\left|\frac{\beta}{1-\beta\cdot n} - \frac{-\beta}{1+\beta\cdot n}\right|^2$$



"meson decays into pions and electrons"

Relationship: $\left(\frac{x}{1-ax} - \frac{-x}{1+ax}\right)^2 = \left|\frac{1}{1-(ax)^2}\right.$

$$= \frac{e^2}{\pi^2 c}\left|\epsilon^* \cdot \frac{\beta}{1-(n\cdot\beta)^2}\right|^2$$

$$= \frac{e^2}{\pi^2 c}\frac{\beta^2\sin^2\theta}{(1-\beta^2\cos\theta)^2}$$

$$\frac{dI}{d\omega} = \int \frac{d^2 I}{d\omega d\Omega} d\Omega$$

$$= \int_0^{2\pi}\int_0^{\pi} \frac{e^2}{\pi^2 c}\frac{\beta^2 \sin\theta}{(1-\beta^2\cos\theta)^2}\, d\theta\, d\phi$$

$$= \frac{2e^2}{\pi c}\int_{-1}^{1}\frac{\beta^2\sin^2\theta}{1-\beta^2\cos^2\theta}\, d\cos\theta$$

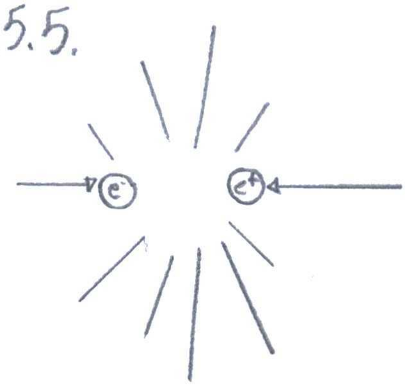
$$= \frac{e^2}{\pi c}\left[\frac{1+\beta^2}{\beta}\log\left(\frac{1+\beta}{1-\beta}\right) - 2\right]$$

Relativistic speeds: $\beta \sim 1$

$$\frac{dI}{d\omega} = \frac{e^2}{\pi c}\left(2\log(4\gamma^2) - 2\right)$$

$$= \frac{4e^2}{\pi c}\left(\log(2\gamma) - 1/2\right)$$

15.5.



"emission of radiation by... annhilation of electrons and positrons"

(14.67) "Intensity distribution" - Electric"

$$\frac{d^2I}{d\omega d\Omega} = \frac{e^2\omega^2}{4\pi^2 c}\left|\int_{-\infty}^{\infty} n\times(n\times\beta)e^{i\omega(t-n\cdot r(t)/c)}dt\right|^2$$

(Problem 14.9a) "Intensity distribution - Magnetic"

$$\frac{d^2I}{d\omega d\Omega} = \frac{\omega^4}{4\pi^2 c^3}\left|\int_{-\infty}^{\infty} n\times\mu(t)e^{i\omega(t-n\cdot r(t)/c)}dt\right|^2$$

Total Intensity:

$$\frac{d^2I}{d\omega d\Omega} = \frac{d^2I_{elec}}{d\omega d\Omega} + \frac{d^2I_{mag}}{d\omega d\Omega}$$

$$= \frac{\omega^2}{4\pi^2 c^3}\left|\int_{-\infty}^{\infty}\left\{n\times(n\times\beta) + \frac{\omega}{c}n\times\mu(t)\right\}e^{i\omega(t-n\cdot r(t)/c)}dt\right|^2$$

$$= \frac{1}{\pi^2 c}\left|\frac{e^2\beta^2\sin\theta}{(1-\beta^2\cos^2\theta)^2} + \frac{\omega}{c}\frac{e^2\hbar^2/4m^2c^2}{(1-\beta^2\cos^2\theta)^2}\right|$$

$$= \frac{e^2}{\pi^2 c}\frac{\beta^2\sin^2\theta + \frac{\hbar^2\omega^2}{4m^2c^4}}{(1-\beta^2\cos^2\theta)^2}$$

(15.4 ) "Soft photon energy per unit energy and per unit solid angle"

$$\frac{d^3\sigma}{d\Omega_p\, d(\hbar\omega)\, d\Omega_\gamma} = \left[\lim_{\hbar\omega\to 0}\frac{d^2N}{d(\hbar\omega)\,d\Omega_\gamma}\right]\frac{d\sigma}{d\Omega_p}$$

(15.5) "invariant expression"

$$\frac{d^3N}{(d^3k/k_0)} = \frac{c^2}{\hbar\omega}\frac{d^2N}{d(\hbar\omega)d\Omega_\gamma} = \frac{c^2}{\hbar(\hbar\omega)}\frac{d^2I}{d\omega d\Omega_\gamma}$$

$$\cong \frac{4e^2}{\pi c}\left[\ln\left(\frac{M\omega}{m}\right) - \frac{1}{2}\right]$$

Total energy:

(15.68) "Total energy"

$$E_{rad} = \int_0^{\omega_{max}} \frac{dI}{d\omega}(\omega)\,d\omega = \frac{e^2}{\pi\hbar c}\left[\frac{1}{\beta}\ln\left(\frac{1+\beta}{1-\beta}\right) - 2\right]E$$

(15.69) "ratio of energy"

$$\frac{E_{rad}}{E} = \frac{2}{\pi}\frac{e^2}{\hbar c}\left[\ln\left(\frac{2E}{mc^2}\right) - 1\right]$$

$$E_{rad} = \frac{e^2}{\pi\hbar c}\left[\frac{1}{\beta}\ln\left(\frac{1+\beta}{1-\beta}\right) - 2\right]E$$

$$\cong \frac{4\alpha}{\pi}\left[\ln\left(\frac{M_\omega}{m}\right) - \frac{1}{2}\right]E$$

| Fine Structure Constant |
|---|
| $\alpha \cong e^2/\hbar c$ |
| (pg 710) |

$$\frac{E_{rad}}{E} = \frac{2}{\pi}\frac{e^2}{\hbar c}\left[\ln\left(\frac{2E}{mc^2}\right) - 1\right]$$

$$\cong \frac{2\alpha}{\pi}\left[\ln\left(\frac{2E}{mc^2}\right) - 1\right]$$

Mass Meson: 740 MeV/c

| particle | mass (MeV/c) | $E_{rad}(\alpha E)$ | $E_{rad}/E(\alpha)$ |
|---|---|---|---|
| $\pi^{\pm}$ | 139.570 | 1.561 | 1.561 |
| $e^{\pm}$ | 0.511 | 8.704 | 8.704 |

$$\frac{d^2\sigma}{d\omega d\Omega} = \frac{1}{\hbar^2\omega}\frac{d^2 I}{d\omega d\Omega} d\sigma$$

$$\simeq \frac{\alpha}{\pi^2}\frac{d\sigma}{\hbar\omega}\frac{\beta^2\sin^2\theta + \frac{\hbar^2\omega^2}{4m^2c^4}}{(1-\beta^2\cos^2\theta)^2}$$

| Fine-Structure constant |
| --- |
| $\alpha = e^2/\hbar c$ (pg 710) |

$$\frac{d\sigma}{d(\hbar\omega)} = \int \frac{d^2\sigma}{d\omega d\Omega} d\Omega$$

$$= \int_0^{2\pi}\int_0^{\pi} \frac{\alpha}{\pi^2}\frac{d\sigma}{\hbar\omega}\frac{\beta^2\sin^2\theta + \frac{\hbar^2\omega^2}{4m^2c^4}}{(1-\beta^2\cos^2\theta)^2} d\theta d\phi$$

**Integral Identities:**

$$\int_{-1}^{1} \frac{1-x^2}{(1-b^2x^2)^2} dx = \frac{1}{b^2}\left[\frac{1+b^2}{2b}\log\left(\frac{1+b}{1-b}\right) - 1\right]$$

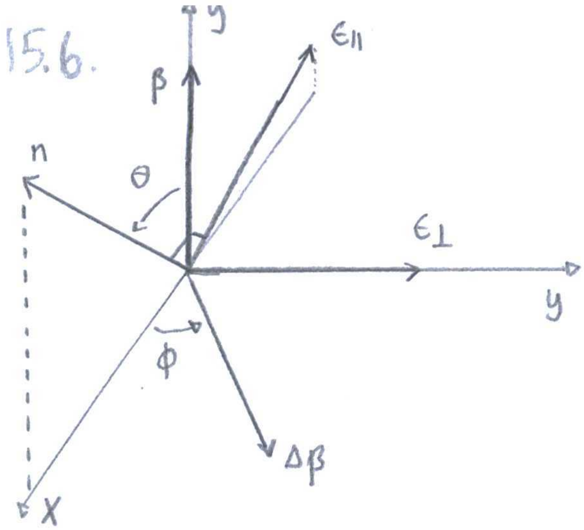
$$\int_{-1}^{1} \frac{dx}{(1-b^2x^2)^2} = \frac{1}{2b}\log\left(\frac{1+b}{1-b}\right) + \frac{1}{1-b^2}$$

$$\frac{d\sigma}{d(\hbar\omega)} = \frac{2\alpha}{\pi}\frac{d\sigma}{\hbar\omega}\left\{\frac{1+\beta^2}{2\beta}\log\left(\frac{1+\beta}{1-\beta}\right) - 1\right\}$$

$$+ \frac{\hbar^2\omega^2}{4\pi m^2c^4}\left[\frac{1}{2\beta}\log\left(\frac{1+\beta}{1-\beta}\right) + \frac{1}{1-\beta^2}\right]$$

The above equations are proportional to the quantum mechanical solution from the problem.

15.6.




"radiation emitted with small change in $\Delta\beta$"

(15.9) "Intensity distribution as velocity changes"

$$\lim_{\omega\to 0} \frac{d^2 I}{d\omega d\Omega} = \frac{z^2 e^2}{4\pi^2 c}\left| \epsilon^* \cdot \frac{\Delta\beta + n\times(\beta\times\Delta\beta)}{(1-n-\beta)^2}\right|^2$$

If $\vec{n} = \langle \sin\theta, 0, \cos\theta\rangle$

$\vec{\epsilon}_{\parallel} = \langle -\cos\theta, 0, \sin\theta\rangle$

$\vec{\epsilon}_{\perp} = \langle 0, 1, 0\rangle$

$\beta = \beta\cdot\hat{e}_z$

$\Delta\beta = |\Delta\beta|(\cos\theta\,\hat{e}_x + \sin\phi\,\hat{e}_y)$

$$n\times(\beta\times\Delta\beta) = \beta|\Delta\beta|(-\cos\theta\cos\phi\,\hat{e}_x - \cos\theta\sin\phi\,\hat{e}_y + \sin\theta\cos\phi\,\hat{e}_z)$$

$$\epsilon_{\parallel}\cdot[\Delta\beta + n\times(\beta\times\Delta\beta)] = |\Delta\beta|(\beta-\cos\theta)\cos\phi$$

$$\epsilon_{\perp}\cdot[\Delta\beta + n\times(\beta\times\Delta\beta)] = |\Delta\beta|(1-\beta\cos\theta)\sin\phi$$

"Dot product"

(15.10) "Intensity distribution with Polarization and angle"

$$\lim_{\omega\to 0} \frac{d^2 I}{d\omega d\Omega} = \frac{z^2 e^2}{8\pi^2 c}|\Delta\beta|^2 \frac{(\beta-\cos\theta)^2}{(1-\beta\cos\theta)^4}$$

$$\lim_{\omega\to 0} \frac{d^2 I}{d\omega d\Omega} = \frac{z^2 e^2}{8\pi^2 c}|\Delta\beta|^2 \frac{1}{(1-\beta\cos\theta)^2}$$

b) $\gamma \gg 1$, $\theta \ll 1$

(15.9) "Intensity distribution with polarization"

$$\lim_{\omega\to 0} \frac{d^2 I}{d\omega d\Omega} \simeq \frac{z^2 e^2}{4\pi^2 c} \left| \epsilon^* \cdot \left( \frac{\Delta\beta + n \times (\beta \times \Delta\beta)}{(1-n\cdot\beta)^2} \right) \right|^2$$

(15.10) "Intensity distribution with polarization and angle"

$$\lim_{\omega\to 0} \frac{d^2 I_{\parallel}}{d\omega d\Omega} \simeq \frac{z^2 e^2}{8\pi^2 c} |\Delta\beta| \frac{(\beta - \cos\theta)^2}{(1-\beta\cos\theta)^4}$$

$$\lim_{\omega\to 0} \frac{d^2 I_{\perp}}{d\omega d\Omega} \simeq \frac{z^2 e^2}{8\pi^2 c} |\Delta\beta| \frac{1}{(1-\beta\cos\theta)^2}$$

$$\lim_{\omega\to 0} \frac{d^2 I}{d\omega d\Omega} = \lim_{\omega\to 0} \left( \frac{d^2 I_{\parallel}}{d\omega d\Omega} + \frac{d^2 I_{\perp}}{d\omega d\Omega} \right)$$

$$= \frac{z^2 e^2 |\Delta\beta|^2}{8\pi^2 c} \left( \frac{(\beta - \cos\theta)^2}{(1-\beta\cos\theta)^4} + \frac{1}{(1-\beta\cos\theta)^2} \right)$$

$$= \frac{z^2 e^2 |\Delta\beta|^2}{8\pi^2 c} \left( \frac{\frac{1}{2\gamma}(\gamma^2\theta^2 - 1)\frac{1}{2}}{\left[\frac{1}{2\gamma}(1+\gamma^2\theta^2)^4\right.} + \frac{\frac{1}{2}}{\left(\frac{1}{2\gamma}(1+\gamma^2\theta^2)\right)^2} \right)$$

...where $\cos\theta = 1 - \theta^2/2$

$$\beta = \left(1 - \frac{1}{\gamma^2}\right)^{1/2}$$

$$\langle \cos^2\theta \rangle = \frac{1}{2\pi} \int_0^{2\pi} \cos^2\theta \, d\theta$$

$$= 1/2$$

$$\langle \sin^2\theta \rangle = \frac{1}{2\pi} \int_0^{2\pi} \sin^2\theta \, d\theta$$

$$= 1/2$$

$$= \frac{z^2 e^2 \gamma^2 |\Delta\beta|^2}{2\pi^2 c} \left( \frac{(\gamma^2\theta^2 - 1)^2}{(1+\gamma^2\theta^2)^4} + \frac{1}{(1+\gamma^2\theta^2)^2} \right)$$

(15.11) "Total intensity per unit frequency at arbitrary velocity"

$$\lim_{\omega \to 0} \frac{d^2 I}{d\omega\, d\Omega} = \frac{z^2 e^2 \gamma^4 |\Delta\beta|^2}{\pi^2 c} \frac{(1+\gamma^2\theta^4)}{(1+\gamma^2\theta^2)^4}$$

c) (pg 712) $P(\theta) \simeq 2\gamma^2\theta^2/(1+\gamma^4\theta^4)$

$P(0) = 0$

$P_{max}(\theta) = 1$

.. when $\theta^2\gamma^2 = 1$

d) $$\lim_{\omega \to 0} \frac{dI}{d\omega} = \int \lim_{\omega \to 0} \frac{d^2 I}{d\omega\, d\Omega} d\Omega$$

$$= 2\pi \int_0^{\pi} \lim_{\omega \to 0} \frac{d^2 I}{d\omega\, d\Omega} \sin\theta\, d\theta$$

$$\simeq \frac{2 z^2 e^2 \gamma^4}{\pi c} |\Delta\beta| \int_0^{\infty} \frac{1+\gamma^4\theta^4}{(1+\gamma^2\theta^2)^4} \theta\, d\theta$$

$$= \frac{2 z^2 e^2 \gamma^2}{\pi c} |\Delta\beta| \int_0^{\infty} \frac{1+\gamma^4\theta^4}{(1+\gamma^2\theta^2)^4} d(\gamma^2\theta^2)$$

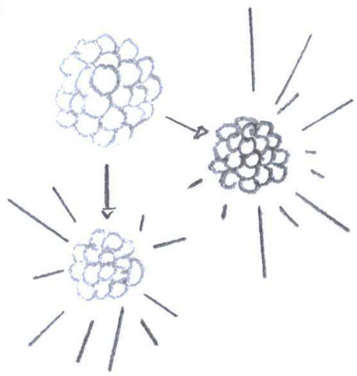
$$= \frac{2 z^2 e^2 \gamma^2}{\pi c} |\Delta\beta| \int_1^{\infty} \frac{1+(y-1)^2}{y^4} dy$$

... when $y = (1+\gamma^2\theta^2)$

$$= \frac{2z^2e^2\gamma^4|\Delta\beta|^2}{\pi c}\left(-\frac{2}{3y^3}+\frac{1}{y^3}-\frac{1}{y}\right)\Bigg|_1^\infty$$

$$= \frac{4z^2e^2\gamma^4|\Delta\beta|^2}{3\pi c}$$

15.7



"radiation emitted in nuclear fission"

$$E_{TOT} = \frac{Z\cdot e\circ\vec{\beta}}{1-\beta\cos\theta}$$

$$= \frac{Z_1\cdot e\circ\frac{A_2}{A_1+A_2}\beta}{1-\frac{A_2}{A_1+A_2}\beta\cos\theta} + \frac{Z_2 e\,\frac{A_1}{A_1+A_2}\cdot\beta}{1+\frac{A_1}{A_1+A_2}\beta\cos\theta}$$

$$= e\beta\left(\frac{Z_1A_2-Z_2A_1}{A_1+A_2}+\frac{Z_1A_2^2+Z_2A_1^2}{(A_1+A_2)^2}\beta\cos\theta+\cdots\right)$$

(Problem 15.4) "Intensity of radiation emitted with polarization"

$$\frac{d^2I}{d(\hbar\omega)d\Omega} = \frac{1}{4\pi^2c}|\epsilon^*\cdot E|^2$$

$$= \frac{e^2\beta^2\sin\theta}{4\pi^2\hbar c}\left|e\beta\left(\frac{Z_1A_2-Z_2A_1}{A_1+A_2}+\frac{Z_1A_2^2+Z_2A_1^2}{(A_1+A_2)^2}\beta\cos\theta\cdots\right)\right|^2$$

$$= \frac{\alpha^2\beta^2\sin^2\theta}{4\pi^2}|p+q\beta\cos\theta|^2$$

...where $\alpha = e^2/\hbar c$

$$p = \frac{Z_1A_2-Z_2A_1}{A_1+A_2}$$

$$q = \frac{Z_1A_2^2+Z_2A_1^2}{(A_1+A_2)^2}$$

$$\frac{dI}{d(\hbar\omega)} = \int \frac{\alpha\beta^2\sin^2\theta}{4\pi^2}|p+q\beta\cos\theta|^2 d\Omega$$

$$= \frac{\alpha\beta^2}{4\pi^2}\int_{-1}^{1}\int_{0}^{2\pi}\sin^2\theta\,(p^2+2pq\beta\cos\theta + q^2\beta^2\cos^2\theta)\sin\theta\, d\phi\, d\theta$$

$$= \frac{\alpha\beta^2}{2\pi}(2\pi)\int_{-1}^{1}\sin^2\theta\,(p^2+2pq\beta\cos\theta + q^2\beta^2\cos^2\theta)\, d\cos\theta$$

$$= \frac{\alpha\beta^2}{2\pi}\left(\frac{4}{3}p^2+\frac{4}{15}q^2\beta^2\right)$$

$$= \frac{2\alpha\beta^2}{3\pi}\left(p^2+\frac{\beta^2 p^2}{5}\right)$$

b) If $Z_1 = 36,\ A_1 = 95$ (krypton)

$Z_2 = 56,\ A_2 = 138$ (barium)

$E = 170$ MeV

$mc^2 = 931.5$ MeV

$$p = \frac{Z_1A_2 - Z_2A_1}{A_1+A_2} \qquad q = \frac{Z_1A_2^2+Z_2A_1^2}{(A_1+A_2)^2}$$

$$= -3.82 \qquad\qquad = 140.71$$

(from problem) "radiated energy"

$$\frac{dI}{d(\hbar\omega)} = \frac{2\alpha\beta^2}{3\pi}\left(p^2+\frac{\beta^2 q^2}{5}\right)$$

$$\frac{dI}{d(\hbar\omega)} = \int \frac{\alpha\beta^2\sin^2\theta}{4\pi^2}\left|p+q\beta\cos\theta\right|^2 d\Omega$$

$$= \frac{\alpha\beta^2}{4\pi^2}\int_{-1}^{1}\int_{0}^{2\pi}\sin^2\theta\,(p^2+2pq\beta\cos\theta + q^2\beta^2\cos^2\theta)\sin\theta\, d\phi\, d\theta$$

$$= \frac{\alpha\beta^2}{2\pi}(2\pi)\int_{-1}^{1}\sin^2\theta\,(p^2+2pq\beta\cos\theta + q^2\beta^2\cos^2\theta)\, d\cos\theta$$

$$= \frac{\alpha\beta^2}{2\pi}\left(\frac{4}{3}p^2+\frac{4}{15}q^2\beta^2\right)$$

$$= \frac{2\alpha\beta^2}{3\pi}\left(p^2+\frac{\beta^2 p^2}{5}\right)$$

b) If $Z_1 = 36,\ A_1 = 95$ (krypton)

$Z_2 = 56,\ A_2 = 138$ (barium)

$E = 170\ \text{MeV}$

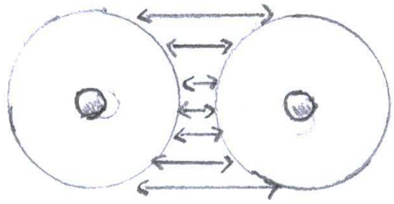
$mc^2 = 931.5\ \text{MeV}$

$$p = \frac{Z_1A_2 - Z_2A_1}{A_1+A_2} = -3.82 \qquad q = \frac{Z_1A_2^2 + Z_2A_1^2}{(A_1+A_2)^2} = 140.71$$

(from problem) "radiated energy"

$$\frac{dI}{d(\hbar\omega)} = \frac{2\alpha\beta^2}{3\pi}\left(p^2+\frac{\beta^2 q^2}{5}\right)$$

15.8.



"particles... interact by means of a short range interaction"

(15.4) "soft photon of energy per unit energy"

$$\frac{d^3\sigma}{d\Omega\, d(\hbar\omega)\, d\Omega_y} = \left[\lim_{\Delta\omega \to 0} \frac{d^2N}{d(\hbar\omega)\, d\Omega_y}\right] \frac{d\sigma}{d\Omega_p}$$

$$\frac{d^2\sigma}{d(\hbar\omega)\, d\Omega_y} = \left[\lim_{\Delta\omega \to 0} \frac{d^2N}{d(\hbar\omega)\, d\Omega_y}\right]$$

(15.5) "Lorentz invariant phase space"

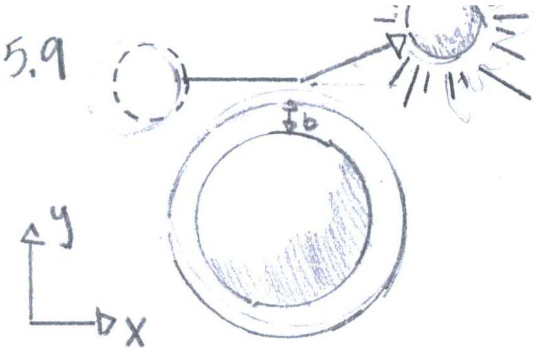
$$\frac{d^3N}{(d^3k/k_0)} = \frac{c^2}{\hbar\omega} \frac{d^2N}{d(\hbar\omega)\, d\Omega_y} = \frac{c^2}{\hbar(\hbar\omega)} \frac{d^2I}{d\omega\, d\Omega_y}$$

$$\frac{d^2\sigma}{d(\hbar\omega)\, d\Omega_y} = \left[\lim_{\Delta\omega \to 0} \frac{1}{\hbar^2\omega} \frac{d^2I}{d\omega\, d\Omega_y}\right]$$

$$\frac{d^2N}{d(\hbar\omega)\, d\Omega} = \frac{z^2e^2}{4\pi^2} \frac{1}{\hbar(\hbar\omega)} \left| \int \frac{d}{dt}\left[\frac{n\times(n\times\beta')}{1-n\cdot\beta}\right] e^{i\omega(t-n\cdot r(t)/2c} - \left[\frac{n\times(n\times\beta')}{1+n\cdot\beta}\right] e^{i\omega(t+n\cdot r(t)/2c}\, dt \right|^2$$

$$= \frac{z^2e^2}{4\pi^2} \frac{1}{\hbar(\hbar\omega)} \left| \int \frac{d}{dt}\left\{[n\times(n\times\beta')][1+n\cdot\beta]\, e^{i\omega t} \times\left(1 - \frac{i\omega}{2c} n\cdot r(t)\right) - [n\times(n\times\beta')][1-n\cdot\beta]\, e^{i\omega t} \times\left(1+\frac{i\omega}{2c} n\cdot r(t)\right) dt \right|^2$$

15.9




"A particle.. deflected in a screened Coulomb field... and consequently emits radiation"

a) $V(r) = \dfrac{Zze^2 e^{-\alpha r}}{r}$

$$\nabla V(r) = Zze^2\left(-\alpha - \frac{1}{r}\right)\frac{e^{-\alpha r}}{r}$$

$$F = m\ddot{x}(t)$$

$$= Zze^2\left(\alpha + \frac{1}{r}\right)\frac{e^{-\alpha r}}{r}\frac{x}{\sqrt{x^2+y^2}}$$

$$F = m\ddot{y}(t)$$

$$= Zze^2\left(\alpha + \frac{1}{r}\right)\frac{e^{-\alpha r}}{r}\frac{y}{\sqrt{x^2+y^2}}$$

...when $y = b$

$$\ddot{x}(\omega) = \int_{-\infty}^{\infty} \ddot{x}(t)\, e^{i\omega t}\, dt$$

$$= \frac{Zze^2}{m}\int_{-\infty}^{\infty}\left(\alpha + \frac{1}{r}\right)\frac{e^{-\alpha r}}{r}\frac{x}{\sqrt{x^2+y^2}}\, e^{i\omega t}\, dt$$

$$= \frac{\alpha Zze^2}{m}\int_{-\infty}^{\infty}\left(\frac{1}{x^2+y^2} + \frac{1}{\alpha(x^2+y^2)^{3/2}}\right)e^{-\alpha(x^2+y^2)} \cdot x \cdot e^{i\omega t}\, dt$$

$$= \frac{\alpha Zze^2}{m\cdot v}\int_{-\infty}^{\infty}\left(\frac{1}{x^2+y^2} + \frac{1}{\alpha(x^2+y^2)^{3/2}}\right)e^{-\alpha(x^2+y^2)} \circ$$

$$= \frac{Z^2e^2}{4\pi^2}\frac{1}{\hbar(\hbar\omega)}\left|\int \frac{d}{dt}\left\{[n\times(n\times\beta)'][1+n\cdot\beta]e^{i\omega t}\right.\right.$$

$$\times\left(1-\frac{i\omega}{2c}n\cdot r(t)\right)-[n\times(n\times\beta)][1-n\cdot\beta]e^{i\omega t}$$

$$\left.\left.\times\left(1+\frac{i\omega}{2c}n\cdot r(t)\right)dt\right\}\right|^2$$

$$= \frac{Z^2e^2}{\pi}\frac{1}{\hbar(\hbar\omega)}\left|\int \frac{d}{dt}\left\{[n\times(n\times\beta')][n\cdot\beta']-[n\times(n\times\beta')]\right.\right.$$

$$\left.\left.\frac{i\omega}{2c}n\cdot r(t)\right\}e^{i\omega t}dt\right|^2$$

(15.4) "Soft photon energy per unit energy interval and per unit solid angle"

$$\frac{d^3\sigma}{d\Omega_p d(\hbar\omega)d\Omega} = \frac{Z^2e^2}{\pi^2 c}\frac{1}{\hbar(\hbar\omega)}\left|\int \frac{d}{dt}\left\{[n\times(n\times\beta')][n\cdot\beta']\right.\right.$$

$$\left.\left.-[n\times(n\times\beta')]\cdot\frac{i\omega t}{2c}n\cdot r(t)\right\}e^{i\omega t}\right|\frac{d\sigma}{d\Omega_p}$$

$$= \left(\frac{q^2}{\hbar c}\right)\frac{\beta^2\beta^2}{4\pi}\frac{1}{\hbar\omega}\left|\left\{\beta(n\times n)-\frac{i\omega}{2c}\beta(n\times n)n\cdot r\right\}e^{i\omega t}\right|$$

$$= \left(\frac{q^2}{\hbar c}\right)\frac{\beta^2\beta^2}{4\pi}\frac{1}{\hbar\omega}\left|\beta^2\left(\frac{2}{3}+3\sin^2\theta\cos^2\theta\right)\right.$$

$$\left.+\left(\frac{\omega R}{c}\right)^2\left(1-\frac{3}{8}\sin^4\theta\right)\right|$$

... I checked. Pure neglect.

$$\frac{d\sigma}{d(\hbar\omega)} = \int_0^{2\pi}\int_0^{\pi}\left(\frac{q^2}{\hbar c}\right)\frac{\beta^2 R^2}{48\pi}\frac{1}{\hbar\omega}\left[\beta^2\left(\frac{2}{5}+3\sin^2\theta\cos^2\theta\right)\right.$$

$$\left.+\left(\frac{\omega R}{c}\right)^2\left(1-\frac{3}{8}\sin^4\theta\right)\right]\sin\theta$$

$$d\theta\, d\phi$$

$$= \left(\frac{q^2}{\hbar c}\right)\frac{\beta^2 R^2}{15}\frac{1}{\hbar\omega}\left[\beta^2+\left(\frac{\omega R}{c}\right)^2\right]$$

$(\cos(wt) + i\sin(wt))\,dt$

**Cite:** I. S. Gradshteyn, I. M. Ryzhik

"Table of Integrals, series, and products"

-Trigonometric functions and exponentials-

(3.914.5)

$$\int_0^\infty \left(\frac{1}{\beta(\gamma^2+x^2)^{3/2}} + \frac{1}{\gamma^2+x^2}\right) e^{-\beta\sqrt{\gamma^2+x^2}} \cdot \cos(bx)\,dx$$

$$= \frac{1}{\beta\cdot\gamma}\sqrt{\beta^2+b^2}\cdot K_1\left(\gamma\sqrt{\beta^2+b^2}\right)$$

(3.914.5)

$$\int_0^\infty \left(\frac{1}{\beta(\gamma^2+x^2)^{3/2}} + \frac{1}{\gamma^2+x^2}\right) e^{-\beta\sqrt{\gamma^2+x^2}} \cdot x\cdot\sin(bx)\,dx$$

$$= \frac{b}{\beta} K_0\left(\gamma\sqrt{\beta^2+b^2}\right)$$

$$\ddot{x}(w) = \frac{2iw\,Zze^2}{mv^2} K_0\left(b\sqrt{\alpha^2+\frac{w^2}{v^2}}\right)$$

$$\ddot{y}(w) = \frac{\alpha b Zze^2}{mv}\int_{-\infty}^{\infty}\left(\frac{1}{x^2+b^2} + \frac{1}{\alpha(x^2+b^2)^{3/2}}\right)\cdot e^{-\alpha\sqrt{x^2+b^2}} \cdot\cos(wt)\,dx$$

$$= \frac{2\sqrt{\alpha^2+\omega^2/v^2}}{mv} Zze^2 K_1\left(b\cdot\sqrt{\alpha^2+\omega^2/v^2}\right)$$

$$\frac{dI}{d\omega}(\omega,b) = \frac{2z^2e^2}{3\pi c^3}\left(\ddot{r}(\omega)^2\right)$$

$$= \frac{2z^2e^2}{3\pi c^3}\left(\ddot{x}(\omega)^2 + \ddot{y}(\omega)^2\right)$$

$$= \frac{8}{3\pi^3}\left(\frac{Zz^2e^3}{mv}\right)\left[\frac{\omega^2}{v^2}K_0^2\left(b\sqrt{\alpha^2+\omega^2/v^2}\right) + \left(\alpha^2+\frac{\omega^2}{v^2}\right)K_1^2\left(b\sqrt{\alpha^2+\omega^2/v^2}\right)\right]$$

If $\omega \gg v/b$, then

$$= \frac{8}{3\pi^2c^3}\left(\frac{Zz^2e^3}{mv}\right)\alpha^2\cdot K_1(\alpha b)$$

$$= \frac{8}{3\pi}\frac{Z^2e^2}{c}\left(\frac{z^2e^2}{mc^2}\right)\left(\frac{c}{v}\right)^2\alpha^2 K_1^2(\alpha b)$$

b) (15.20) "Differential Radiation cross section"

$$\frac{d^2\chi}{d\omega\, dQ} = \frac{dI(\omega,Q)}{d\omega}\cdot\frac{d\sigma}{dQ}(Q)$$

where $dI(\omega,Q)/d\omega$ = energy radiated per unit frequency interval in a collision with momentum transfer $Q$.

(15.2?) "radiation cross section integrated over momentum transfer"

$$\frac{d\chi}{d\omega} = \frac{16}{3}\frac{Z^2e^2}{c}\left(\frac{z^2e^2}{mc^2}\right)^2\frac{1}{\beta^2}\int^{Q_{max}}\frac{dQ}{Q}$$

$$= \int \frac{dI}{dw}(w,b)\, d\sigma$$

$$= \int_{b_{min}}^{b_{max}} \int_0^{2\pi} \frac{dI}{dw}(w,b)\, b\, d\phi\, db$$

$$= 2\pi \int_{b_{min}}^{b_{max}} \frac{dI}{dw}(w,b)\, b\, db$$

$$= 2\pi \int_{b_{min}}^{b_{max}} \frac{8}{3\pi} \frac{z^2e^2}{c}\left(\frac{z^2e^2}{mc^2}\right)\left(\frac{c}{v}\right)^2 \alpha^2 K_1^2(\alpha b) \cdot b\, db$$

$$= \frac{16}{3} \frac{z^2e^2}{c}\left(\frac{z^2e^2}{mc^2}\right)\left(\frac{c}{v}\right)^2 \int_{b_{min}}^{b_{max}} \alpha^2 K_1^2(\alpha b) \cdot b \cdot db$$

$$= \frac{16}{3} \frac{z^2e^2}{c}\left(\frac{z^2e^2}{mc^2}\right)\left(\frac{c}{v}\right)^2 \int_{b_{min}}^{b_{max}} \alpha^2 \cdot K_1^2(x) \cdot x\, dx$$

$$= \frac{16}{3} \frac{z^2e^2}{c}\left(\frac{z^2e^2}{mc^2}\right)\left(\frac{c}{v}\right)^2 \left\{ \frac{x^2}{2}\left[K_0^2(x) - K_1^2(x) + \frac{2 \cdot K_0(x) K_1(x)}{x}\right]_{x_1}^{x_2} \right\}$$

where $x_1 = \alpha b_{min}$

$x_2 = \alpha b_{max}$

c) If $b_{min} = \hbar/mv$, $b_{max} = v/w$, $\alpha' = 1.4 a_0 Z^{1/3}$

If $x_2 \ll 1$, $\alpha b_{max} \ll 1$

$$\frac{dX}{dw} = -\frac{16}{3} \frac{z^2e^2}{c}\left(\frac{z^2e^2}{mc^2}\right)\left(\frac{c}{v}\right)^2 \frac{(1.4 a_0 Z^{1/3} \hbar)^2}{2 m^2 v^2}\Big[K_0^2(1.4 a_0 Z^{1/3} \hbar/mv)$$

$$- K_1^2(1.4 a_0 Z^{1/3} \hbar/mv) + \frac{2 K_0(1.4 a_0 Z^{1/3} \hbar/mv) K_1(1.4 a_0 Z^{1/3} \hbar/mv)}{1.4 a_0 Z^{1/3} \hbar/mv}$$

If $X_2 \gg 1$, then

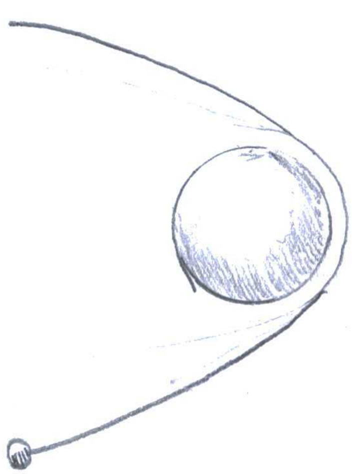
$$\frac{dX}{d\omega} = \frac{16}{3}\frac{Z^2e^2}{c}\left(\frac{Z^2e^2}{mc^2}\right)\left(\frac{c}{v}\right)\left[\frac{(1.4a_0 Z^{1/3} v/\omega)}{2}\right]^2 \times \Bigg\{$$

$$\Bigg[K_0^2(1.4a_0 Z^{1/3} v/\omega) - K_1^2(1.4a_0 Z^{1/3} v/\omega) + \frac{2K_0(1.4a_0 Z^{1/3} v/\omega)K_1(1.4a_0 Z^{1/3} v/\omega)}{1.4a_0 Z^{1/3} v/\omega}\Bigg]$$

$$-\Bigg[K_0^2(1.4a_0 Z^{1/3} \hbar/mv) - K_1^2(1.4a_0 Z^{1/3} \hbar/mv) + \frac{2K_0(1.4a_0 Z^{1/3} \hbar/mv)K_1(1.4a_0 Z^{1/3} \hbar/mv)}{1.4a_0 Z^{1/3} \hbar/mv}\Bigg]\Bigg\}$$

Section 15.3 - Screening effects; Relativistic Radiative Energy Loss has a proportional equation, specifically (15.47).

15.10



"Particle ... deflected in a hyperbolic path .. by repulsive Coulomb potential"

a) (14.60) "Frequency distribution"

$$\frac{d^2I}{d\omega\, d\Omega} = 2\left|A(\omega)\right|^2$$

(14.51) "Power radiated per solid angle"

$$\frac{dP(t)}{d\Omega} = \left|A(t)\right|^2$$

$$\frac{dI}{d\omega} = 2 \cdot P(t)$$

If $x=(\epsilon+\cosh\xi)$ ; $y=b\sinh\xi$ ; $\omega_0 t=\xi+\epsilon\sinh\xi$

where $a=Zze^2/mv^2$ ; $\epsilon=\sqrt{1+(b/a)^2}$ ; $\omega_0=v/a$

$$x=\frac{i}{\omega_0 n\tau}\int_0^{2\pi} e^{i\omega_0 n t}\cdot\dot{x}\,dt$$

$$=\frac{ia}{2\pi n}\int_0^{2\pi} e^{in(\xi+\epsilon\sinh\xi)}\cdot\sinh(\xi)\,d\xi$$

$$y=\frac{i}{\omega_0 n\tau}\int_0^{2\pi} e^{i\omega_0 n t}\cdot\dot{y}\,dt$$

$$=\frac{-ib}{2\pi n}\int_0^{2\pi} e^{in(\xi+\epsilon\sinh\xi)}\cdot\cosh(\xi)\,d\xi$$

Cite: Landau, Lifshitz

"The Classical Theory of Fields"

Pg 199,7 (1971)

Identity #1:

$$\int_{-\infty}^{\infty} e^{p\xi-ix\sinh\xi}\,d\xi=i\pi H_p^{(1)}(ix)$$

$$x=\frac{\pi a}{\omega}H^{(1)}_{i\omega/\omega_0}(i\omega\epsilon/\omega_0)$$

$$=+\frac{2ai}{\omega}e^{\frac{\omega}{\omega_0}\frac{i}{2}}K_{i\omega/\omega_0}\left(\frac{\omega\epsilon}{\omega_0}\right)$$

Identity #2:

$$K_\nu(z)=-\frac{1}{2}\pi i e^{-\nu\pi i/2}\cdot H_\nu^{(2)}(ze^{-\pi i/2})$$

$$y=-\pi a\frac{\sqrt{\epsilon^2-1}}{\omega\epsilon}H_{i\omega/\omega_0}(i\omega\epsilon/\omega_0)$$

$$=-\frac{2ia}{\omega}e^{\frac{\omega}{\omega_0}\frac{i}{2}}\frac{\sqrt{\epsilon^2-1}}{\epsilon}K_{i\omega/\omega_0}\left(\frac{\omega\epsilon}{\omega_0}\right)$$

$$P_n = \frac{4e^2}{3c^3}(n\omega_0)^4 \left| x^2 + y^2 \right|$$

$$= \frac{4e^2}{3c^3}(n\omega_0)^4 \left[ \left| \frac{2i}{\omega} a e^{i\pi\epsilon\omega/\omega_0} K'_{i\omega/\omega_0}\left(\frac{\omega}{\omega_0}\epsilon\right) \right|^2 \right.$$

$$\left. + \left| \frac{2i}{\omega} a e^{i\pi\epsilon\omega/\omega_0} \frac{\sqrt{\epsilon^2-1}}{\epsilon} K_{i\omega/\omega_0}\left(\frac{\omega}{\omega_0}\epsilon\right) \right|^2 \right]$$

$$= \frac{8e^2}{3c^3} \frac{(n\omega_0)^4 e^{i\pi\epsilon\omega/\omega_0} a^2}{\omega_0^2} \left[ K'_{i\omega/\omega_0}\left(\frac{\omega\epsilon}{\omega_0}\right)^2 + \frac{\epsilon^2-1}{\epsilon^2} \left[ K_{i\omega/\omega_0}\left(\frac{\omega\epsilon}{\omega_0}\right) \right]^2 \right]$$

$$= \frac{8}{3\pi} \frac{(zea\omega)^2}{c^3} e^{-i\pi\epsilon\omega/\omega_0} \left[ K'_{i\omega/\omega_0}\left(\frac{\omega\epsilon}{\omega_0}\right)^2 + \frac{\epsilon^2-1}{\epsilon^2} K_{i\omega/\omega_0}\left(\frac{\omega\epsilon}{\omega_0}\right)^2 \right]$$

$$= \mathrm{Re}\left[ \frac{8}{3\pi} \frac{(zea\omega)^2}{c^3} e^{-\pi\epsilon\omega/\omega_0} \left[ K'_{i\omega/\omega_0}\left(\frac{\omega\epsilon}{\omega_0}\right) + \frac{\epsilon^2-1}{\epsilon^2} K_{i\omega/\omega_0}\left(\frac{\omega\epsilon}{\omega_0}\right) \right]^2 \right]$$

b) (15.20) "Differential Cross Section"

$$\frac{d^2\chi}{d\omega\, dQ} = \frac{dI(\omega, Q)}{d\omega} \frac{d\sigma}{dQ}(Q)$$

$$\frac{d\chi}{d\omega} = \int \frac{dI}{d\omega}(\omega, b)\, d\sigma$$

$$= \int_{b_{min}}^{b_{max}} \int_0^{2\pi} \frac{dI(\omega, b)}{d\omega} b\, d\phi\, db$$

$$= 2\pi \int_{b_{min}}^{b_{max}} \frac{dI(\omega, b)}{d\omega} b\, db$$

$$= \frac{16}{3} \frac{(zea\omega)^2}{c^3} e^{-\pi\omega/\omega_0} \int_0^\infty \left[ K'_{i\omega/\omega_0}\left(\frac{\omega\epsilon}{\omega_0}\right) + \frac{\epsilon^2-1}{\epsilon^2} K_{i\omega/\omega_0}\left(\frac{\omega\epsilon}{\omega_0}\right) \right]^2 \cdot b\, db$$

$$= \frac{16}{3} \frac{(Zea\omega)^2}{c^3} e^{-\pi\omega/\omega_0} \cdot a^2 \int_1^\infty \left[ K_{i\omega/\omega_0}\left(\frac{\omega\epsilon}{\omega_0}\right)^2 + \frac{\epsilon^2-1}{\epsilon} K_{i\omega/\omega_0}\left(\frac{\omega\epsilon}{\omega_0}\right)^2 \right] \cdot \epsilon \, d\epsilon$$

...when $\epsilon = \sqrt{1+(b/a)^2}$

If $k = \frac{\omega}{\omega_0}\epsilon$, then

$$= \frac{16}{3} \frac{(Zea\omega)^2}{c^3} e^{-\pi\omega/\omega_0} \cdot a^2 \frac{\omega^2}{\omega_0^2} \int_{\omega/\omega_0}^\infty \left[ K'_{i\omega/\omega_0}(k)^2 + \left(1 - \frac{\omega_0^2}{k^2\omega^2}\right) K_{i\omega/\omega_0}(k)^2 \right] \cdot k \, dk$$

$$= \frac{16}{3} \frac{(Zea\omega)^2}{c^3} e^{-\pi\omega/\omega_0} \cdot a^2 \frac{\omega^2}{\omega_0^2} \left[ \int_{\omega/\omega_0}^\infty K'_{i\omega/\omega_0}(k) \, k \cdot dk' + \left(1 - \frac{\omega_0^2}{k^2\omega^2}\right) \int_{\omega/\omega_0}^\infty K_{i\omega/\omega_0}(k) \cdot k \cdot dk \right]$$

$$= \frac{16}{3} \frac{(Zea\omega)^2}{c^3} e^{-\pi\omega/\omega_0} \cdot a^2 \frac{\omega^2}{\omega_0^2} \left\{ \Big[ k \cdot K_{i\omega/\omega_0}(k) \cdot K_{i\omega/\omega_0}(k) \Big]_{\omega/\omega_0}^\infty - \int_{\omega/\omega_0}^\infty K_{i\omega/\omega_0}(k) \cdot k \, dk \right\}$$

$$\boxed{\begin{array}{l} \underline{\text{Bessel Identity:}} \\ \frac{d}{dx}\left[k \cdot K_v'(k)\right] = \left(1 + \frac{v^2}{x^2}\right) K_v(x) \end{array}}$$

$$= \frac{16}{3} \frac{(Zea\omega)^2}{c^3} e^{\pi\omega/\omega_0} \frac{\omega}{\omega_0} K_{i\omega/\omega_0}\left(\frac{\omega}{\omega_0}\right) \left[ -K'_{i\omega/\omega_0}\left(\frac{\omega}{\omega_0}\right) \right]$$

$$= \frac{2}{\pi}\frac{e^2}{\hbar c}\int_{\omega}^{E/\hbar} \frac{A}{2\pi}\frac{c^2}{Mc}\frac{\omega_0}{\omega_0^2+2\omega_0^2}\ln\left(\frac{k\gamma^2 mc}{\hbar\omega}\right)\frac{d\omega}{\omega}$$

$$= \frac{2}{\pi}\frac{e^2}{\hbar c}\int_{\omega}^{E/k} \frac{A}{2\pi}\frac{e^2}{Mc}\frac{1}{2\omega_0}\ln\left(\frac{E^2}{\hbar\omega_0}\right)$$

$$= \frac{2}{\pi}\left(\frac{e^2}{\hbar c}\right)\frac{Ac^2}{Mc}\frac{1}{4\pi\omega_0}\ln\left(\frac{E^2}{\hbar\omega}\right)$$

c) (from problem) "Photonuclear cross section"

$$\sigma_{photo}(\omega) = (Ac^2/Mc)\,\delta(\omega-\omega_0)$$

(15.47) "Radiation cross section"

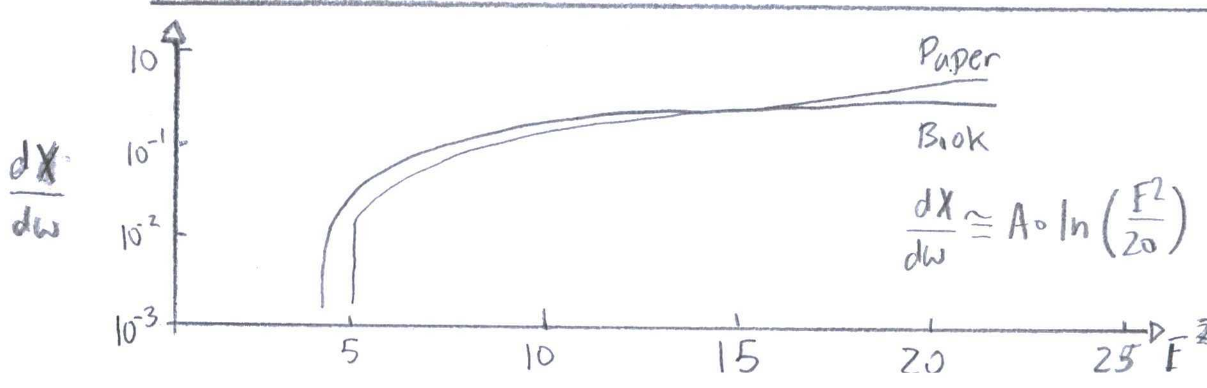
$$\frac{d\chi}{d\omega} = \frac{16}{3}\frac{Z^2e^2}{c^2}\left(\frac{Z^2e^2}{Mc^2}\right)^2\ln\left(\frac{233M}{mZ^{1/3}}\right)$$

$$\frac{d\chi}{d\omega}\frac{Ae^2}{Mc\hbar\omega_0} = \frac{16}{3}\frac{Z^2e^2}{c}\frac{Ae^2}{Mc\hbar\omega_0}\left(\frac{Z^2e^2}{Mc^2}\right)\ln\left(\frac{233M}{mZ^{1/3}}\right)$$

Cite: E. Wolynec

"Alpha decay of the giant quadropole Resonance" in $^{233}U$"

Universidade de S. Paulo (1975)



c) $\omega \ll \omega_0$, If $K_\nu(b) = -\log\left(\frac{x}{2}\right)$

and $K_\nu'(x) = -1/x$

$$\frac{dX(\omega)}{d\omega} = \frac{16}{3}\frac{(zea\omega)^2}{c^3}\frac{\omega_0}{\omega}\left[-\log(\omega/2\omega_0)\right]\left(\frac{\omega}{\omega_0}\right)$$

$$= \frac{16}{3}\frac{(zea\omega)^2}{c^3}\log(2\omega_0/\omega_0)$$

$$= \frac{16}{3}\frac{z^2e^2}{c^3}\left(\frac{z^2e^2}{m}\right)\frac{1}{(\beta c)^2}\log\left(\frac{2mv^3}{Zze^2\omega}\right)$$

...when $\omega_0 = v/a$

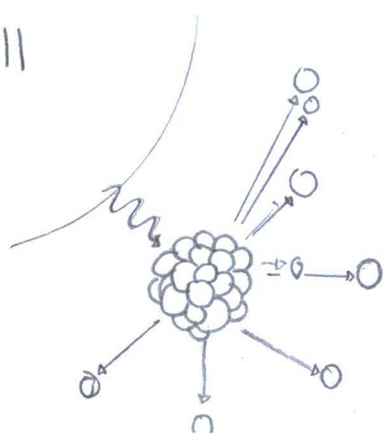
-close- $$= \frac{16}{3}\frac{z^2e^2}{c}\left(\frac{Z^2e^2}{mc^2}\right)\frac{1}{\beta^2}\log\left(\frac{2mv^3}{Zze^2\omega}\right)$$

(15.25) "Classical radiation cross section"

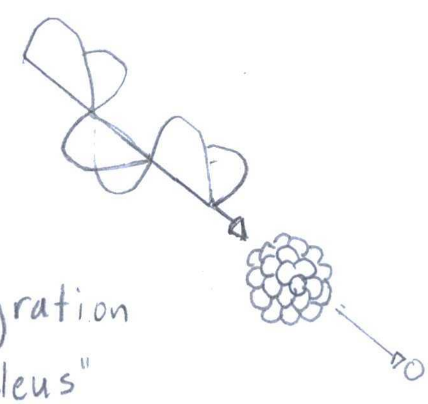
$$\frac{dX_c}{d\omega} \simeq \frac{16}{3}\frac{z^2e^2}{c}\left(\frac{z^2e^2}{Mc^2}\right)\frac{1}{\beta^2}\cdot\ln\left(\frac{\lambda Mv^3}{zZe^2\omega}\right)$$

d) An attractive Coulomb Potential has a circular orbit, rather than hyperbolic. In this case, x and y change to cos(x) and sin(x).

15.11



"Photodisintegration of a nucleus"



"electrodisintegration of a nucleus"

a) If $E = \gamma mc^2 \gg mc^2$, then ways to measuring:

<u>Cite #1</u>: Gallagher, Brion, Samson, Langhoff
"Absolute cross section for Molecular Photoabsorption, Photo-ionization, and Ionic Photofragmentation Process"
University of Colorado (1987)

<u>Intensity</u>:

① Beer-Lambert Law: $I = I_0 e^{-\sigma n l}$

② Cross-section: ; $\sigma = \frac{1}{nl} \ln(I/I_0)$

③ Cross-section per angle:

$$\frac{d\sigma}{d\Omega} = (2\pi e^2/hc) \cdot h\nu \underbrace{|\langle \phi_j(k) | \epsilon \cdot \mu | \phi \rangle|^2}$$

Actually, logarithm of intensity as photo-transition in sample.

<u>Cite #2</u>: Von H. Bethe
"Zur Theoric des Durchgangs Schnella Korpuskularstrablem durch Materie"
Munich, Germany (1930)

Wavenumber:

① De Broglie Equation: $\lambda = \frac{h}{m_0 v} = \frac{2\pi}{k}$

Wavenumber

② Atomic series:

| Z | 1 | 2 | 7 | 8 | 10 |
|---|---|---|---|---|---|
| | H | He | N | O | Ne |

③ Velocity change: $v = \sqrt{2E/m}$

and $\lambda = \frac{h}{\sqrt{+2mE_1}} = \frac{2\pi}{k}$

④ Bethe reviewed past work in "Quantum Mechanics of one- and two-electron atoms" (1957) with Salpester:

Cross-section (69.2): $\sigma = \frac{2\pi e^2 \hbar^2}{m^2 c v} |D_{wb}|^2$

...relates to photoelectric effect, cross-section and photodissociation in a transition.

Cite #3: Skopik, Murphy, Shiu "Total cross section results for deuterium electrodisintegration"

University of Saskatchewan (1970)

Intensity, but with coefficients:

(2) Cross section per angle:

$$\left.\frac{d\sigma}{d\Omega}\right|_{\gamma} = a_\gamma + b_\gamma \sin^2\theta + c_\gamma \sin^2\theta\cos\theta + d_\gamma \sin^2\theta\cos^2\theta$$

(1) Total cross section from intensity:

$$\sigma = \int \frac{2}{\pi}\frac{e^2}{hc} E_0 \ln(I/I_0)\, d\Omega$$

$$= \frac{2}{\pi}\frac{e^2}{\hbar c}\int E_0 \ln\left(\frac{\gamma^2 mc}{\hbar\omega}\right) d\Omega$$

$$= \frac{2}{\pi}\frac{e^2}{\hbar c}\int_{\omega}^{E/k} \sigma_{photo}' \ln\left(\frac{k\gamma^2 mc}{\hbar\omega}\right)\frac{d\omega}{\omega}$$ ... k from problem in unity.

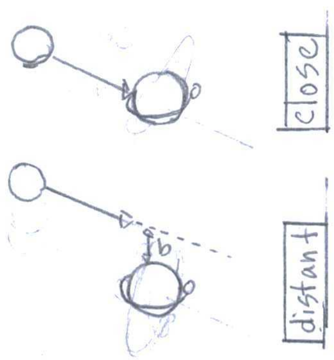
b) (from book) "cross section of resonance"

$$\sigma_{photo}(\omega) = \frac{A}{2\pi}\frac{e^2}{Mc}\frac{\Gamma}{[(\omega-\omega_0)^2 + (\Gamma/2)^2]}$$

$$\sigma_{el}(E) = \frac{2}{\pi}\frac{e^2}{\hbar c}\int_{\omega}^{E/k} \sigma_{photo} \ln\left(\frac{k\gamma^2 mc^2}{\hbar\omega}\right)\frac{d\omega}{\omega}$$

$$= \frac{2}{\pi}\frac{e^2}{\hbar c}\int_{\omega}^{E/k} \frac{Ac^2\ \Gamma}{2\pi Mc\,[(\omega-\omega_0)^2 + (\Gamma/2)^2]} \ln\left(\frac{k\gamma^2 mc^2}{\hbar\omega}\right)\frac{d\omega}{\omega}$$

15.12.




"A fast particle... collides with a hydrogen-like atom with one electron"

a) "Rutherford Cross Section" (d>b) - close"

(15.17) "Rutherford cross section"

$$\frac{d\sigma_s}{d\Omega_s} = \left(\frac{2zZe^2}{pv}\right)^2 \cdot \frac{1}{(2\sin\theta'/2)^4}$$

(15.18) "Momentum transfer"

$$Q^2 = 4p^2\sin^2(\theta'/2)$$

$$\frac{d\sigma}{dQ} = \frac{d\sigma_s}{d\Omega}\frac{d\Omega}{dQ} = \left(\frac{2zZe^2}{pv}\right)^2 \frac{1}{(2\sin\theta'/2)^4} \cdot \frac{2\pi\sin\theta}{dQ/d\Omega}$$

$$= \left(\frac{2zZe^2}{pv}\right)^2 \frac{1}{(2\sin\theta'/4)^4} \frac{2\pi\sin\theta}{8p^2 \cdot 2\sin(\theta'/2)\cdot\cos(\theta'/2)\cdot 2}$$

$$= \left(\frac{2zZe^2}{pv}\right)^2 \cdot \frac{1}{(2\sin\theta'/4)^4} \frac{2\pi\sin\theta}{8p^2\sin(\theta')}$$

$$= \frac{1}{2^2}\left(\frac{zZe^2}{pv}\right)^2 \frac{1}{Q^2} \frac{\pi}{(\sin\theta'/2)^2}$$

Trigonometric Identity:

$\sin(\theta)\cos(\theta) = \sin(2\theta)$

$$= \frac{1}{2^2}\left(\frac{zZe^2}{pv}\right)^2 \frac{1}{Q^2} \frac{\pi}{(\sin\theta'/2)^2}$$

$$= \frac{1}{2^6}\left(\frac{zZe^2}{mv}\right)^2 \frac{1}{Q^4} \frac{\pi}{10^3 I^2}$$

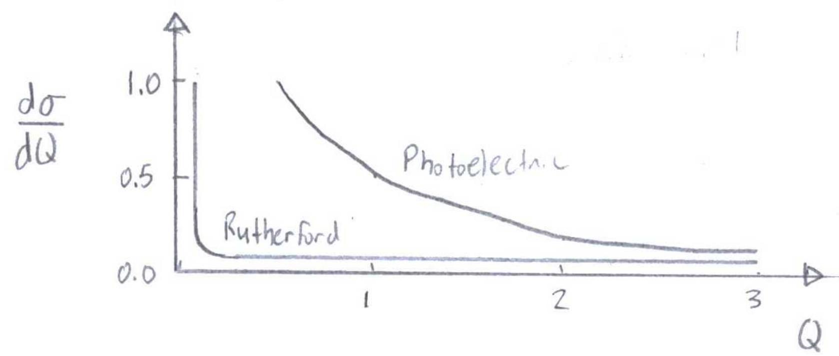
Photoelectric Effect (d<b) - distant"

$$\sigma_T(Q) = \frac{8\pi^2}{137}\left(\frac{a_0}{Z}\right)^2\left(\frac{I}{Q}\right)^3$$

$$\frac{d\sigma}{dQ} = \frac{24\pi^2}{137}\left(\frac{a_0}{Z}\right)^2\left(\frac{I}{Q}\right)^3\frac{1}{Q}$$

$$\frac{d\sigma}{dQ}\left(\text{in units} = \frac{2\pi Z^2 e^4}{mv^2 I}\right) = \frac{12\pi}{137}\,\frac{mv^2}{Z^4 e^4}\left(\frac{I}{Q}\right)^4$$

$$= \frac{12\pi}{137}\,m\left(\frac{v}{Z^2e^2}\right)^2\left(\frac{I}{Q}\right)^4$$




b) <u>Number of Collisions by integration:</u>

close - $\sigma = \int \frac{12\pi}{137}\,m\left(\frac{v}{Z^2e^2}\right)\left(\frac{I}{Q}\right)^4 dQ$

$\propto 1/Q^3$

Distant - $\sigma = \int \frac{11}{26}\left(\frac{1}{mI}\right)\frac{1}{Q^4}\,dQ$

$\propto 1/Q^3$

where $Q_s = 4p^2\sin^2(\theta/2) > Q_{max} = 2p$ ... (15.23)

$$\frac{Q_s}{Q_{max}} = 4p\sin(\theta/2)$$

(13.14) "Bethe formula - Total Energy Loss per unit length"

$$\frac{dE}{dx} = 4\pi N z \frac{z^2 e^4}{mc^2\beta^2}\left[\ln(\beta) - \beta^2\right]$$

(15.29.5) "Total energy loss"

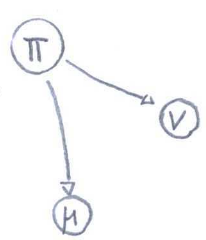
$$\frac{dE}{dx} = N \int_0^{\omega_{max}} \frac{d\chi(\omega)}{d\omega} d\omega$$

(15.29)

$$\frac{d\chi_{nm}}{d\omega} = \frac{16}{3} \frac{z^2 e^2}{c} \left(\frac{z^2 e^2}{Mc^2}\right) \frac{1}{\beta^2} \ln\left[\frac{\chi' Q_{max}}{Q_{mn}}\right]$$

$$\simeq \ln\left[\lambda' 4 p \sin\theta/2\right]$$

15.13.




"In decay of a pi meson at rest a mu meson and neutrino are created"

Cite: Primakoff Ho

"Anomalous π-μ Decay"

Washington University, St. Louis, MO.

$$E_\mu = E_\mu^{(0)}(1 - \epsilon/\epsilon_{max})$$

$$P(\epsilon)d\epsilon = A(\epsilon/\epsilon_{max})^A d\epsilon/\epsilon \qquad \int_0^\infty P(t)d\epsilon = 1$$

$$A = (2/3\pi)\cdot(1/137)\cdot 2E_\mu^{(0)}/m_\mu$$

301 decays!

Number of quanta emitted per unit energy:

(15.67) "Typical Bremsstralung Spectrum"

$$N(\hbar w) = \frac{e^2}{\pi \hbar c} \frac{1}{\hbar w} \left[ \frac{1}{\beta} \ln\left( \frac{1+\beta}{1-\beta} \right) + 2 \right]$$

Maximum photon E... when $V = 3/10 C$ and $\hbar w = 34$ MeV

Maximum photon Energy:

$$\epsilon_{max} = \frac{1}{2}(m_\pi - m_\mu)$$

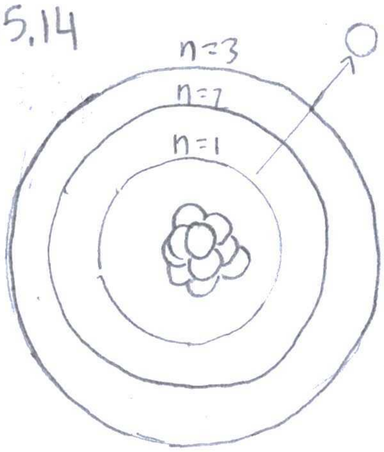
$$= 16.95 \text{ MeV}/c^2$$

Quantum emitted greater than 1/10 maximum:

$$17 \text{ MeV} \xrightarrow[\left(\frac{x-\bar{x}}{\sigma}\right) = 0.1]{} 18.7 \text{ MeV}$$ ... which is (15.67) with $\hbar w = 18.7$ MeV

$$T = \hbar w - \phi$$

15.14




"A nucleus makes a transition ... and an orbital electron is ejected."

(15.67) Typical Bremmstralung spectrum"

$$N(\hbar w) = \frac{e^2}{\pi \hbar c} \left( \frac{1}{\hbar w} \right) \left[ \frac{1}{\beta} \ln\left( \frac{1+\beta}{1-\beta} \right) - 2 \right]$$

$$N_{99\%}(\hbar w = 0.99 \cdot 1 \text{MeV}) = \phi / \hbar w$$

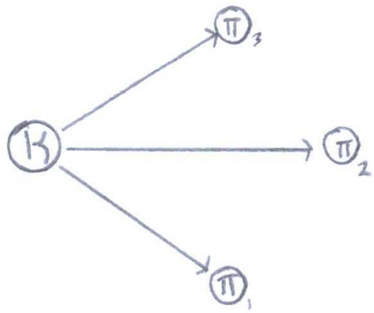
$$= \frac{1}{100} \cdot 10^6$$

$$= 10,000$$

The threshold is visible at a low intensity. Normally distributed at 1% has 5/1000 original intensity.

15.15.

a) $K^- \longrightarrow \pi^- + \pi^+ + \pi^+$ $\qquad \Delta E = 75\ \text{MeV}$




"one of the decay modes of the $K^-$ meson is the three point decay"

(15.63) "Intensity distribution of radiation"

$$\frac{dI}{d\omega\, d\Omega} = \frac{e^2}{4\pi^2 c}\left|\frac{\epsilon^* \cdot \vec{\beta}}{1 - n \cdot \vec{\beta}}\right|^2$$

$$\frac{dI_{TOT}}{d\omega\, d\Omega} = \frac{e^2}{4\pi^2 c}\sum_{i=1}^{3}\left|\frac{\epsilon_i^* \cdot \vec{\beta}_i}{1 - n \cdot \vec{\beta}_i}\right|^2$$

$$= \frac{e^2}{4\pi^2 c}\left(\left|\frac{\epsilon_1 \vec{\beta}_1}{1 - n\vec{\beta}_1}\right|^2 + \left|\frac{\epsilon_2 \vec{\beta}_2}{1 - n\vec{\beta}_2}\right|^2 + \left|\frac{\epsilon_3 \vec{\beta}_3}{1 - n\vec{\beta}_3}\right|^2\right)$$

$$\simeq \frac{e^2}{4\pi^2 c}\left(\left|\epsilon_1 \cdot \vec{\beta}_1\right|^2 + \left|\epsilon_2 \cdot \vec{\beta}_2\right|^2 + \left|\epsilon_3 \cdot \vec{\beta}_3\right|^2\right)$$

... nonrelativistic decay, $\beta \ll 1$

$$\cong \frac{e^2}{4\pi^2 c}\left|+2 \cdot \beta \sin\theta\right|^2$$

... when $\epsilon_1 = \epsilon_2 = -\epsilon_3$

$\beta_1 + \beta_2 = -\beta_3$

$$\cong \frac{e^2}{\pi^2 c}\beta^2 \sin^2\theta$$

$$\cong \frac{2e^2}{\pi^2 c}\frac{T}{mc^2}\sin^2\theta$$

Kinetic Energy

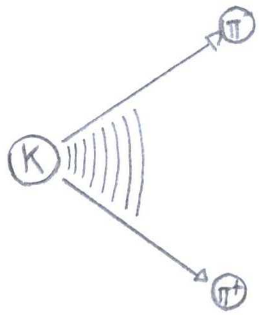
$T = \frac{1}{2} m v^2$

$= \frac{1}{2} m c^2 \cdot \beta^2$

b) $\Delta = 1\ \text{MeV}$ $\quad \frac{dI}{d\omega\, d\Omega} = 28 \times 10^3\ C/(m/s)$

$\Delta = 10\ \text{MeV}$ $\quad \frac{dI}{d\omega\, d\Omega} = 28 \times 10^4\ C/(m/s)$

15.16.




"one of the decay modes of the charged meson... inner Bremmstrahlung is emitted."

a) (Chapter 11) "Maximum Kinetic energy"

$$KE = \frac{M^2 - m^2}{2M} \quad \text{in units MeV/c}$$

$$= \frac{(493.7\,\text{MeV/c})^2 - (139.6\,\text{MeV/c})^2}{2 \cdot 493.7\,\text{MeV/c}}$$

$$= 227\,\text{MeV/c}$$

∴ then, I checked units in the book

(15.82) "Angular frequency spectrum"

$$\frac{d^2I}{d\omega d\Omega} = \frac{e^2}{8\pi^2 c}\left(\frac{\hbar\omega}{mc^2}\right)^2$$

∴ the units are not watts/area.