Chapter 1: Introduction to Electrostatics:

(Gauss' Law of Electrostatics)

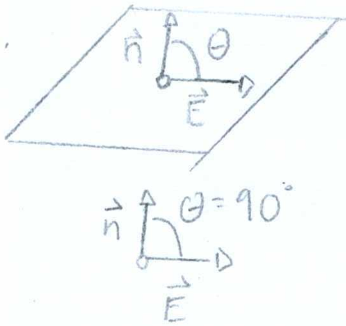
$\nabla \cdot \vec{E} = \rho / \varepsilon_0$ where $\vec{E}$ = Electric Field (V/m)

ρ = Charge Density (C/m^2)

ε_0 = Permittivity of Free Space (Farad/m)

1.1.

a.



$$\nabla \cdot E = \frac{1}{\varepsilon_0} \rho(x)$$

$$\int_V \nabla \cdot E \, d^3x = \frac{1}{\varepsilon_0} \int_V \rho(x) \, d^3x$$ Volume Integral

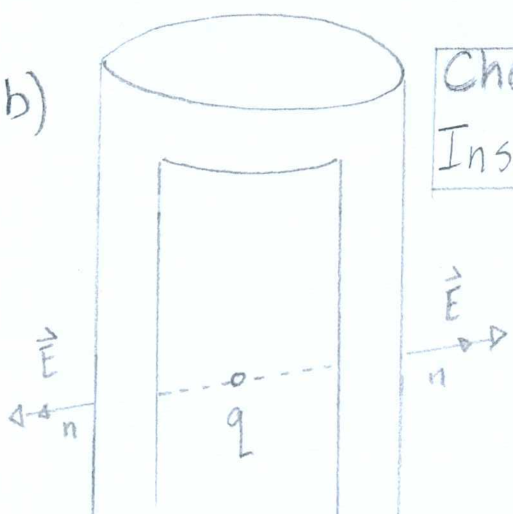
$$\int_V \nabla \cdot E \, d^3x = \frac{1}{\varepsilon_0} \oint_S E \cdot n \, d^2x$$ Area Integral

$$= \frac{1}{\varepsilon_0} \oint_S \rho(x) \, d^2x$$

$$= \frac{1}{4\pi\varepsilon_0} \oint_S \frac{q \cos\theta \, d^2r}{r^2}$$

$$= 0 \text{ when } \theta = 90°$$

b)

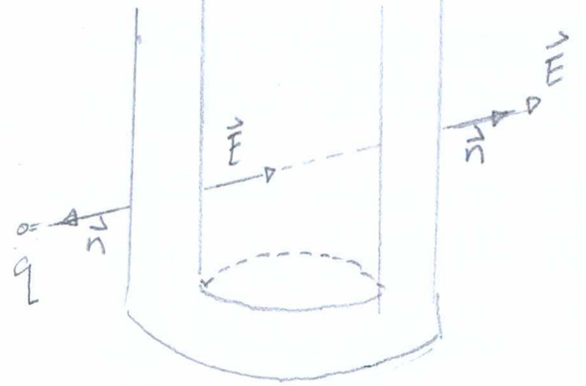


Charge Inside:

$$\oint_S E \cdot n \, d^2x = \frac{1}{\varepsilon_0} \oint_S \rho(x) \, d^2x$$

$$= \frac{1}{4\pi\varepsilon_0} \oint_S \frac{q \cos\theta \, d^2r}{r^2}$$

$$= \frac{1}{4\pi\varepsilon_0} \oint_S q \cos\theta \, d\Omega$$



$$= \frac{1}{4\pi\varepsilon_0}\left(\oint_S q\left[\cos(0^\circ)\right] + \oint_S q\left[\cos(0^\circ)\right]\right) d\Omega$$

$$= \frac{1}{2\pi\varepsilon_0}\oint_S q\, d\Omega$$

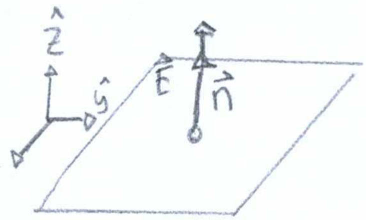
$$= \frac{q}{\varepsilon_0}$$ when cylinder circumference $= 2\pi$

$\boxed{\text{Charge Outside}}$: $$\oint_S E\cdot n\, d^2x = \frac{1}{\varepsilon_0}\oint_S \rho(x)\, d^2x$$

$$= \frac{1}{4\pi\varepsilon_0}\oint q\frac{\cos\theta\, d\vec{r}^2}{r^2}$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\oint q\cos(0^\circ)\, d\Omega + \oint q\cos(180^\circ)\, d\Omega\right]$$

$$= 0$$

c) 

$$\oint_S \nabla\cdot E\, d\vec{r}^3 = \frac{1}{\varepsilon_0}\oint_S \rho(x)\, d^3r$$

$$= \frac{1}{4\pi\varepsilon_0}\oint_S \frac{q_z\cos\theta}{r^3}\, d\vec{r}^3 \qquad \theta = 0^\circ$$

$$= \frac{1}{4\pi\varepsilon_0}\int_{-\infty}^{\infty}\frac{q\circ z}{(x^2+y^2+z^2)^{3/2}}\, dx\, dy$$

$$= \frac{q}{4\pi\varepsilon_0}\int_{-\infty}^{\infty}\frac{z}{(x^2+y^2+z^2)^{3/2}}\, dx\, dy$$

$$= \frac{2\pi}{4\pi\varepsilon_0}\, q$$

$$= \frac{q}{2\varepsilon_0}$$

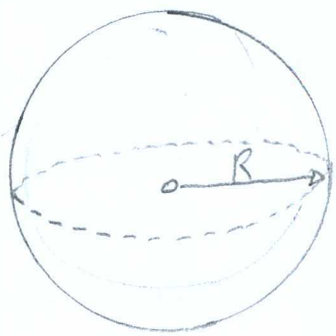
1.2. $D(\alpha, x, y, z) = (2\pi)^{-3/2} \circ \alpha^{-3} \circ \exp\left[-\frac{1}{2\alpha^2}(x^2+y^2+z^2)\right]$

$$= \frac{\exp\left[\frac{-x^2}{2\alpha^2}\right]}{(2\pi)^{1/2} \circ \alpha} \circ \frac{\exp\left[\frac{-y^2}{2\alpha^2}\right]}{(2\pi)^{1/2} \circ \alpha} \circ \frac{\exp\left[\frac{-z^2}{2\alpha^2}\right]}{(2\pi)^{1/2} \circ \alpha}$$

$$= \delta(u-u')\, U \circ \delta(v-v')\, V \circ \delta(w-w')\, W$$

$$\delta(x-x') = \delta(u-u')\, U \circ \delta(v-v')\, V \circ \delta(w-w')\, W$$

1.3. Three methods about Gauss' Law:



Conductor

Charge [Q]

Method #1:

A guess, charge (q) per surface area ($4\pi R^2$) is charge density ($\rho(x)$).

$$\rho(x) = \frac{q}{4\pi R^2}$$

Method #2:

Coefficient A

$$\oint \nabla \cdot E\, d^3x = \frac{1}{\varepsilon_0}\int \rho(x)\, d^2x$$

$$= \frac{A}{\varepsilon_0}\int \sigma(r)\, dr$$

$$= \frac{q}{\varepsilon_0}$$

$$q = A\int \sigma(r)\, dr$$

$$= A\int_0^{2\pi}\int_0^{\pi} R^2 \sin\theta\, d\theta\, d\phi$$

$$= A \circ 4\pi R^2$$

$$A = \frac{q}{4\pi R^2}$$

$$\rho(x) = \frac{q}{4\pi R^2}$$

Method #3:

"Schrödinger-esque"

$$\oint E\, ds = \frac{q}{\varepsilon_0}\int \rho(x)\, d^2x$$

$$E\oint ds = \frac{q}{\varepsilon_0}$$

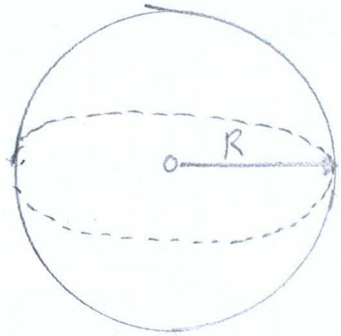
$$E = \frac{q}{\varepsilon_0 \oint ds}$$

$$= \frac{q}{\varepsilon_0 \int_0^{2\pi}\int_0^{\pi} R^2 \sin\theta\, d\theta\, d\phi}$$

$$= \frac{q}{\varepsilon_0\, 4\pi R^2}$$

$$\rho(x) = \frac{q}{4\pi R^2}$$

a)



Surface Conductor

Shape = Sphere
Dimension = Surface [2D]
Charge = q
Coordinates = Spherical

$$\oint E \cdot ds = \frac{1}{\varepsilon_0} \int \rho(x) d^3x$$

$$E \oint ds = \frac{q}{\varepsilon_0}$$

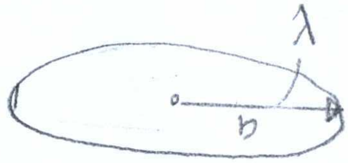
$$E = \frac{q}{\varepsilon_0 \int ds}$$

$$= \frac{q}{\varepsilon_0 \int_0^{2\pi} \int_0^{\pi} R^2 \sin\theta \, d\theta \, d\phi}$$

$$= \frac{q}{\varepsilon_0 4\pi R^2}$$

$$\rho(x) = \frac{q}{4\pi R^2}$$

b)



Cylindrical surface

Shape = Circle perimeter
Dimension = line [1D]
Charge = λ
Coordinates = cylindrical

$$\oint E \cdot dl = \frac{1}{\varepsilon_0} \int \rho(x) d x$$

$$E \oint dl = \frac{\lambda}{\varepsilon_0}$$

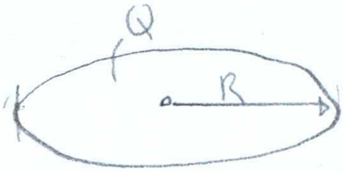
$$E = \frac{\lambda}{\varepsilon_0 \oint dl}$$

$$= \frac{\lambda}{\varepsilon_0 \int_0^{2\pi} \int_0^{b} d\rho \, d\phi}$$

$$= \frac{\lambda}{\varepsilon_0 \cdot 2\pi b}$$

$$\rho(x) = \frac{\lambda}{2\pi b}$$

c)



Circular Disc

Shape = circle
Dimension = Area [2D]
Charge = Q
Coordinates = cylindrical

$$\oint E \cdot ds = \frac{1}{\varepsilon_0}\int \rho(x)\, d^2x$$

$$E \oint ds = \frac{Q}{\varepsilon_0}$$

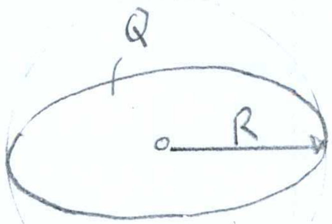
$$E = \frac{Q}{\varepsilon_0 \oint ds}$$

$$= \frac{Q}{\varepsilon_0 \int_0^{2\pi}\int_0^{R} \rho \, d\rho \, d\phi \, dz}$$

$$= \frac{Q}{\varepsilon_0 \pi R^2}$$

$$\rho(x) = \frac{Q}{\pi \cdot R^2}$$

d)



Circular Disc

Shape = circle
Dimension = Area [2D]
Charge = Q
Coordinates = spherical

$$\oint E \cdot ds = \frac{1}{\varepsilon_0}\int \rho(x)\, d^2x$$

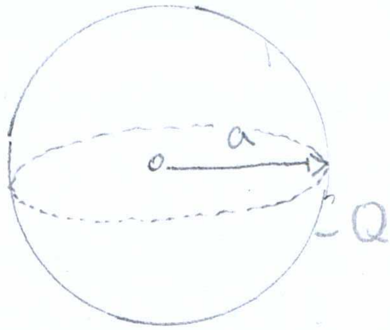
$$E \oint ds = \frac{Q}{\varepsilon_0}$$

$$E = \frac{Q}{\varepsilon_0 \oint ds}$$

$$= \frac{Q}{\varepsilon_0 \int_0^{2\pi}\int_0^{R} r \, dr \, d\theta}$$

$$= \frac{Q}{\varepsilon_0 \pi R^2}$$

1.4. Hollow Conductor



Shape = Sphere
Dimension = Area [2D]
Charge = Q

Inside: The charge inside a conductor had a resolution in Problem 1.1a by $\theta = 90°$ and $\vec{E} = 0$. How is a person to measure zero charge? The electric field inside a a conductor seems as a lemma.

Outside: $\oint E \cdot ds = \frac{1}{\varepsilon_0} \int \rho(r) d^2 r$

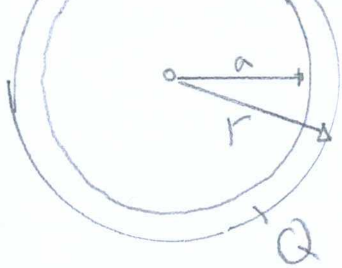
$$E \oint ds = \frac{q}{\varepsilon_0}$$

$$E = \frac{q}{\varepsilon_0 \oint ds}$$

$$= \frac{q}{\varepsilon_0 \int_0^{2\pi} \int_0^{\pi} a^2 \sin\theta \, d\theta \, d\phi}$$

$$= \frac{q}{\varepsilon_0 \cdot 4\pi a^2}$$

Uniform Charge Density in a sphere



$$\oint E \cdot d^3x = \frac{1}{\varepsilon_0} \int \rho(r) d^3 r$$

$$E \oint d^3x = \frac{q}{\varepsilon_0} \frac{\int_V \delta(r,\theta,\phi) \, d^3 r}{\int_{SA} \delta(r,\theta,\phi) \, d^2 r}$$

Shape = Sphere
Dimension = Volume [3D]
Charge = Q

$$E\int_0^{2\pi}\int_0^{\pi}\int_0^{a} r^2\sin\theta\, dr\, d\theta\, d\phi = \frac{q}{\varepsilon_0}\frac{\int_0^{2\pi}\int_0^{\pi}\int_0^{r} r^2\sin\theta\, dr\, d\theta\, d\phi}{\int_0^{2\pi}\int_0^{\pi} r^2\sin\theta\, d\theta\, d\phi}$$

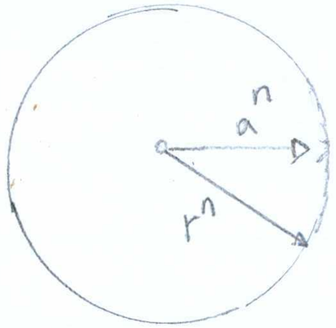
$$E\cdot\frac{4}{3}\pi a^3 = \frac{q}{\varepsilon_0}\frac{\frac{4}{3}\pi r^3}{4\pi r^2}$$

$$E = \frac{q\cdot r}{\varepsilon_0\cdot 4\pi a^3}$$

Symmetric charge density within the volume

$$\oint E\, dv = \frac{1}{\varepsilon_0}\int \rho(r)\, d^3r$$

$$E\cdot\oint dV = \frac{q}{\varepsilon_0}\frac{\int_V \delta(r,\theta,\phi)\, d^3r}{\int_{SA}\delta(r,\theta,\phi)\, d^2r}$$



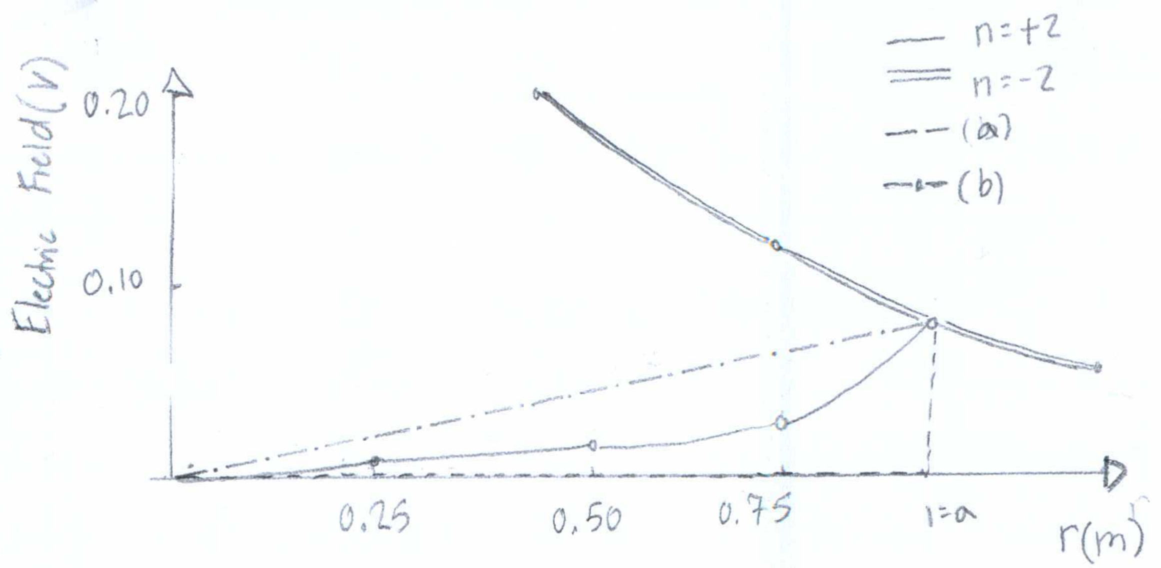
Shape = Sphere
Dimension = Volume [3D]
Charge = Q

$$E\int_0^{2\pi}\int_0^{\pi}\int_0^{a} r^n r^2\sin\theta\, dr\, d\theta\, d\phi = \frac{q}{\varepsilon_0}\frac{\int_0^{2\pi}\int_0^{\pi}\int_0^{r} r^n r^2\sin\theta\, dr\, d\theta\, d\phi}{\int_0^{2\pi}\int_0^{\pi} r^2\sin\theta\, d\theta\, d\phi}$$

$$E\cdot\frac{4}{3}\pi a^{n+3} = \frac{q}{\varepsilon_0}\frac{\frac{4}{3}\pi r^{n+3}}{4\pi r^2}$$

$$E = \frac{q}{\varepsilon_0\, 4\pi r^2}\left[\frac{r}{a}\right]^{n+3}$$

$$= \frac{q\cdot r^{n+1}}{\varepsilon_0\, 4\pi a^{n+3}}$$



(Poisson Equation)

$$\nabla \cdot E = \rho / \varepsilon_0 \longrightarrow \nabla \cdot (\nabla \cdot E) = -\rho / \varepsilon_0 \longrightarrow \nabla \phi^2 = -\rho / \varepsilon_0$$

"Gauss Law" "Poisson's Equation"

1.5. Time-averaged potential of a hydrogen atom:

$$\phi(r) = \phi - \frac{-q}{4\pi\varepsilon_0} \frac{e^{-\alpha r}}{r}\left(1 + \frac{\alpha r}{2}\right)$$

$$\rho(r) = -\varepsilon_0 \nabla^2 \phi(r)$$

$$= \frac{-\varepsilon_0}{r^2} \frac{d}{dr}\left(r^2 \frac{d\phi(x)}{dr}\right)$$

$$= \frac{-\varepsilon_0}{r^2} \frac{d}{dr}\left[\frac{1}{2} e^{-\alpha r}(-\alpha r(\alpha r + 2) - 2)\right]$$

$$= -\frac{q}{8\pi}\alpha^3 e^{-\alpha r}$$

$$\rho(r') = q\delta(r) + \rho(r)$$

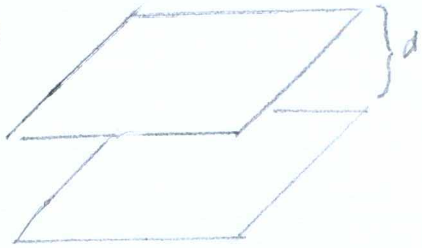
$$= q\delta(r) + q\frac{\alpha^3}{8\pi} e^{-\alpha r}$$

"Static" "Continuous"
"potential" "Kinetic"

Poisson's derivation has a second derivative of the electric field. Schrödinger's equation has a second derivative of total energy. One is static-and-continuous, the other, potential-and-kinetic.

1.6

a)



Two, large, flat conducting sheets

Shape = Square
Dimension = Area [2D]
Charge = Q

$$\oint E \cdot ds = \frac{1}{\varepsilon_0} \int \rho(x) d^2x$$

$$E \oint ds = \frac{q}{\varepsilon_0}$$

$$E \int_0^a \int_0^a da = \frac{q}{\varepsilon_0}$$

$$E = \frac{q}{\varepsilon_0 a^2}$$

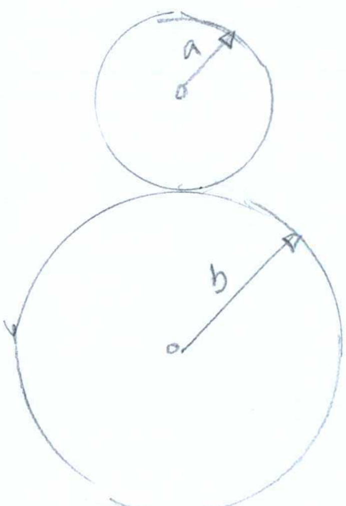
$$= \frac{V}{d}$$

$$C = \frac{q}{V}$$

$$= \frac{\varepsilon_0 a^2}{d}$$

$\boxed{\text{Capacitance } [C = q/V]}$

b)



Two concentric conducting spheres

Shape = Hollow sphere
Dimension = Surface Area [2D]
Charge = q

$$\oint E ds = \frac{1}{\varepsilon_0} \int \rho(x) d^2r$$

$$E \oint ds = \frac{q}{\varepsilon_0}$$

$$E \int_0^{2\pi} \int_0^{\pi} r^2 \sin\theta \, d\theta \, d\phi = \frac{q}{\varepsilon_0}$$

$$E = \frac{q}{\varepsilon_0 \cdot 4\pi r^2}$$

$$-\nabla \Phi = E$$

$$= \frac{q}{\varepsilon_0 \cdot 4\pi r^2}$$

$$\Phi = \int_a^b \frac{q}{\varepsilon_0 \cdot 4\pi r^2} dr$$

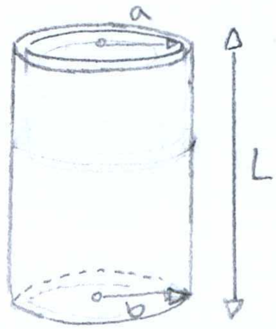
$$= \frac{q}{4\pi\varepsilon_0} \left(\frac{1}{b} - \frac{1}{a} \right)$$

$$= V$$

$$C = \frac{q}{V}$$

$$= \frac{4\pi \varepsilon_0 \cdot a \cdot b}{(b-a)}$$

c)



Two concentric Cylinders of length L

Shape = Cylinder
Dimension = Area [2D]
Charge = q

$$\oint E \cdot ds = \frac{1}{\varepsilon_0} \int \rho(x) d^2 r$$

$$E \oint ds = \frac{q}{\varepsilon_0}$$

$$E \int_0^{2\pi} \int_0^{r} \int_0^{L} dr\, dL\, dz = \frac{q}{\varepsilon_0}$$

$$E = \frac{q}{2\pi \cdot L \cdot r\, \varepsilon_0}$$

$$-\nabla \phi = E$$

$$= \frac{q}{2\pi L \cdot r\, \varepsilon_0}$$

$$\phi = -\int_a^b \frac{q}{2\pi \cdot L \cdot r \cdot \varepsilon_0} dr$$

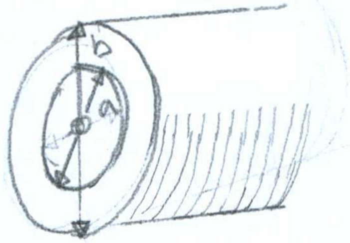
$$= \frac{q}{2\pi L \cdot \varepsilon_0} \ln\left(\frac{b}{a}\right)$$

$$= V$$

$$C = \frac{q}{V}$$

$$= \frac{2\pi L \varepsilon_0}{\ln(b/a)}$$

d)



$a = 1mm$

ε_0 = Permittivity of Free Space $= 8.85 \times 10^{-12}\ F/m$

$= 3 \times 10^{-11} F/m = \frac{C}{L}$

$= \frac{2\pi \varepsilon_0}{\ln\left(\frac{b}{a}\right)}$

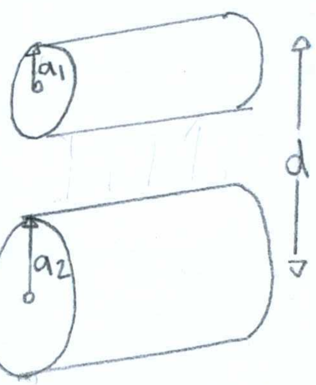
$2\pi \varepsilon_0 / C$

$b = a\, e^{\frac{2\pi(8.85 \times 10^{-12} F/m)}{(3 \times 10^{-12} F/m)}}$

$= 1mm \cdot e$

$\cong 1.x10^{8} mm$

1.7.



Two long, cylindrical Conductors

Shape = Cylinder
Dimension = Surface [2D]
Charge = q

$\oint E \cdot ds = \frac{1}{\varepsilon_0} \int \rho(r)\, d^2 r$

$E \cdot \oint ds = \frac{q}{\varepsilon_0}$

$E \int_0^{L} \int_0^{2\pi} \int_x^{x-d} d\rho\, d\theta\, dz = \frac{-q}{\varepsilon_0}$

$E = \frac{-q}{\varepsilon_0 \cdot 2\pi \cdot L} \cdot \left(\frac{1}{x} + \frac{1}{d-x}\right)$

$-\nabla \phi = E$

$= \frac{-q}{\varepsilon_0 2\pi L} \cdot \left(\frac{1}{x} + \frac{1}{d-x}\right)$

$\phi = -\int_{d-a_2}^{a_1} \frac{-q}{\varepsilon_0 2\pi L} \left(\frac{1}{x} + \frac{1}{d-x}\right) dx$

$= \frac{-q}{\varepsilon_0 \cdot 2\pi \cdot L} \ln\left(\frac{d-a_1}{a_2} \cdot \frac{d-a_2}{a_2}\right)$

$$\cong \frac{-q}{\varepsilon_0 \cdot 2\pi L} \ln\left(\frac{d^2}{a_1 \cdot a_2}\right)$$

$\boxed{\text{Geometric mean} \quad a = \sqrt{a_1 \cdot a_2}}$

$$\cong \frac{-q}{\varepsilon_0 \pi \cdot L} \ln\left(\frac{d}{a}\right)$$

$$C \cong \frac{-q}{V \cdot L}$$

$$\cong \pi \varepsilon_0 \ln\left(\frac{d}{a}\right)^{-1}$$

Diameter (d)	0.5cm	1.5cm	5.0cm
Width (a)	0.5mm	1.5mm	5.0mm

When $\frac{C}{L} = 1.2 \times 10^{-11}$ F/m.

1.8.

a)

Two large, flat Conducting sheets

Shape = Square
Dimension = Area [2D]
Charge = $-Q, +Q$

From 1.6a, $E = \frac{-q}{\varepsilon_0 \cdot a^2} = \frac{V}{d}$

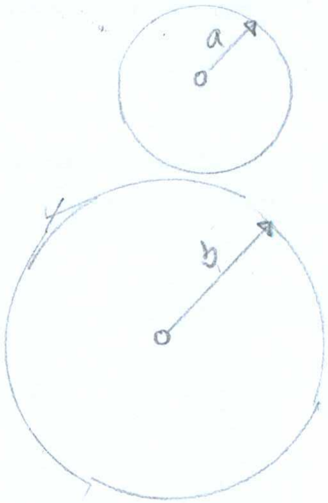
(Equation 1.62) $W = \frac{1}{2} QV$

$$= \frac{-Q \cdot Q}{2 \varepsilon_0 \cdot a^2}$$

— or —

$$= \frac{-V^2 a^2 \varepsilon_0}{2d}$$

The book describes the coefficient; ½, ¼, or ⅛; by half the amount, total potential or Dirac's symmetric derivation. No clue without measurement. Gauss' Law derives one-half the charges.



Two concentric conducting spheres

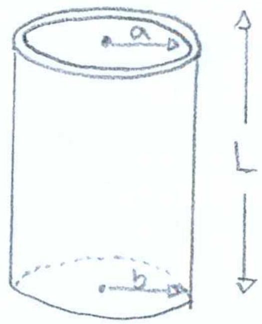
Shape = Hollow spheres
Dimension = Surface Area [2D]
Charge = $-Q$, $+Q$

From Problem 1.6b, $\phi = \dfrac{-q}{\varepsilon_0 \cdot 4\pi}\left(\dfrac{1}{a} - \dfrac{1}{b}\right) = V$

$$W = \frac{1}{2}\sum_{i=1}^{n} Q_i V_i$$

$$= -\frac{Q \cdot Q}{\varepsilon_0 8\pi}\left(\frac{1}{b} - \frac{1}{a}\right)$$

$$= \frac{V^2 \cdot 2\pi \varepsilon_0 \cdot a \cdot b}{a-b}$$



Two concentric cylinders of length L

Shape = cylinder
Dimension = Area [2D]
Charge = $-Q$, $+Q$

From Problem 1.6c, $\phi = \dfrac{-q}{2\pi L \cdot \varepsilon_0}\ln\left(\dfrac{b}{a}\right) = V$

$$W = \frac{1}{2}\sum_{i=1}^{n} Q_i V_i$$

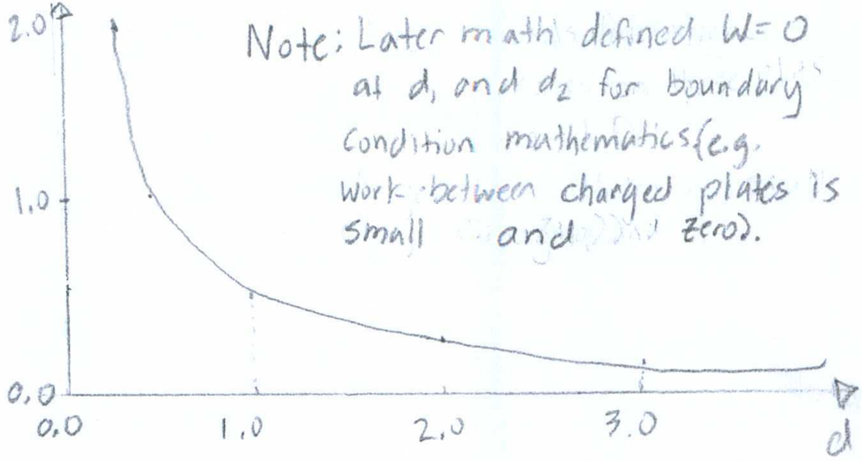
$$= \frac{Q \cdot Q}{4\pi L \varepsilon_0}\ln\left(\frac{a}{b}\right)$$

$$= \frac{V^2 \pi \varepsilon_0 L}{\ln(a/b)}$$

b $\boxed{\text{Conducting Sheets}}$

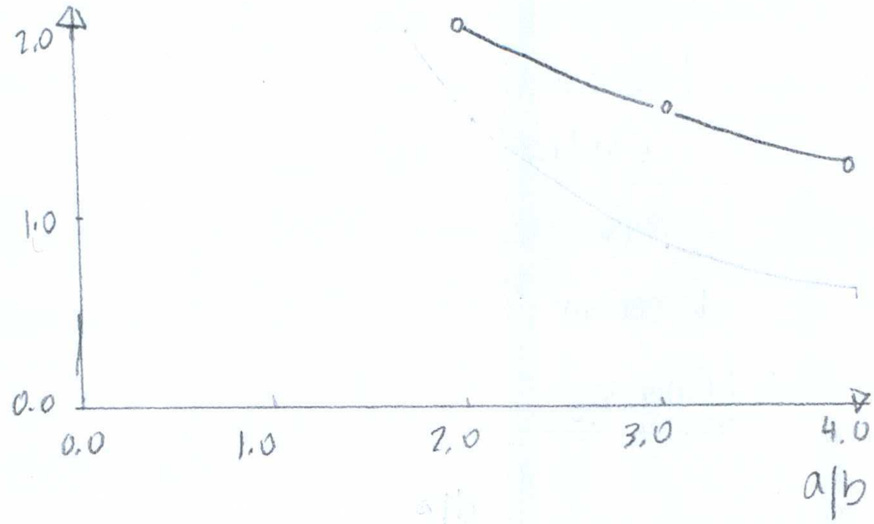
Note: Later math defined $W=0$ at d_1 and d_2 for boundary condition mathematics (e.g. work between charged plates is small and zero).

$$|W| = \left| \frac{V^2 a^2 \varepsilon_0}{2d} \right|$$



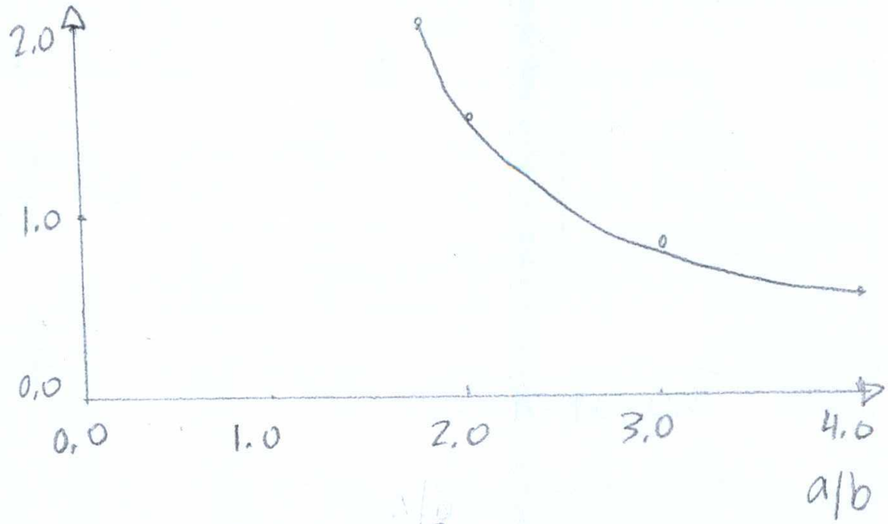
$\boxed{\text{Concentric spheres}}$

$$|W| = \left| \frac{V^2 2\pi \varepsilon_0 \, a \cdot b}{a - b} \right|$$



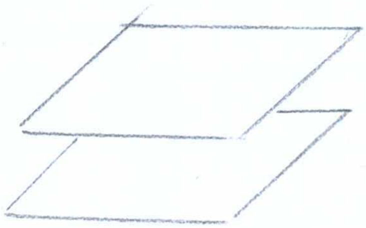
$\boxed{\text{Concentric cylinders}}$

$$|W| = \left| \frac{V^2 \pi \varepsilon_0 \cdot L}{\ln(a/b)} \right|$$



1.9

a)



Parallel plate Capacitor

Shape = Square
Dimension = Area [2D]
Charge = Fixed

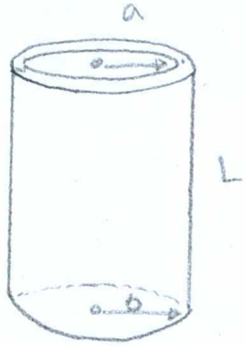
(Equation 1.17b) $F = qE$

From Problem 1.6a, $E = \frac{q}{\varepsilon_0 a^2} = \frac{V}{d}$

$$F_{TOT} = q \cdot E_{TOT}$$

$$= q \cdot \frac{q}{\varepsilon_0 a^2}$$

Ambiguous Terminology:
E represents both
1) Electric Field [V]
2) Electric Field Strength [V/m]



Parallel Cylinder Capacitor

Shape = cylinder
Dimension = Area [2D]
Charge: Fixed

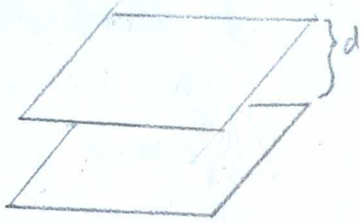
Force = charge · Electric Field Strength

From Problem 1.6c, $E = \frac{q}{2\pi L \cdot r \cdot \varepsilon_0} = \frac{V}{d}$

$$F = q \cdot E$$

$$= \frac{q^2}{2\pi L \cdot r \cdot \varepsilon_0}$$

b)



Parallel Plate Conductor

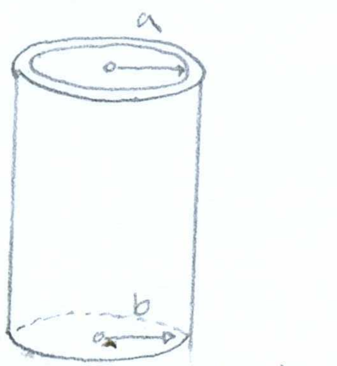
Shape = square
Dimension = Area [2D]
Charge: Fixed

Force = charge · Potential per distance

From Problem 1.6a, $E = \frac{q}{\varepsilon_0 \cdot a^2} = \frac{V}{d}$

$$F_{TOT} = q \cdot \left(\frac{\Phi}{L}\right)$$

$$= \frac{\varepsilon_0 a^2 V^2}{d^2}$$



Parallel Cylinder Capacitor

Shape = Cylinder
Dimension = Area [2D]
Charge = Fixed

Force = charge × Potential per distance

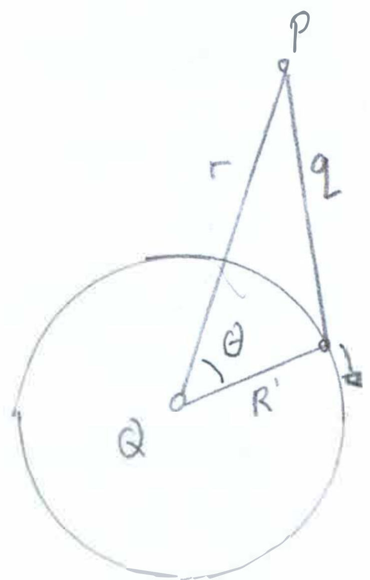
From Problem 1.6c, $E = \frac{q}{\varepsilon_0 \cdot \pi \cdot L} \ln\left(\frac{d}{a}\right) = \frac{V}{d}$

$$F_{TOT} = q\left(\frac{\Phi}{L}\right)$$

$$= \frac{\varepsilon_0 \pi L \cdot V^2}{\ln(d/a) \cdot d}$$

1.10 $\bar{\Phi} = \frac{1}{4\pi R^2}\int_{SA} \Phi \, ds = \frac{1}{4\pi R^2}\int_0^{2\pi}\int_0^{\pi} \Phi \, r^2 \sin\theta \, d\theta \, d\phi$

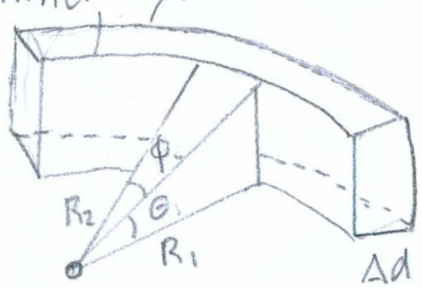
$= \Phi_{central}$



Note: Other averages derive a 2D model about a circle's charge from the law of cosines as a representation for a sphere's charge. A similar 3D-derivation needs a 3D law of cosines.

2D-Example with q's average length around the circle, as average surface charge.

1.11.



$$\int_{outer} E \cdot ds = \int_{Inner} E \cdot ds$$

$$E\int_{outer} ds = E\int_{Inner} ds$$

$$E(R_1 + n\Delta d)(R_2 + n\Delta d)\int_0^{2\pi}\int_0^{\pi} d\theta \, d\phi \overset{?}{=} E \cdot R_1 R_2 \int_0^{2\pi}\int_0^{\pi} d\theta \, d\phi$$

$$E(R_1 + n\Delta d)(R_2 + n\Delta d) = E \cdot R_1 \cdot R_2$$

$$E = \frac{E R_1 R_2}{(R_1 + n\Delta d)(R_2 + n\Delta d)}$$

$$\frac{\partial E}{\partial n\Delta d} = \frac{-R_1 - R_2 - n\Delta d}{(R_1 + n\Delta d)(R_2 + n\Delta d)} E$$

$$\frac{1}{E}\frac{\partial E}{\partial n\Delta d} = \frac{-(R_1+R_2)}{(R_1+n\Delta d)(R_2+n\Delta d)}$$

$$\frac{1}{E}\cdot\frac{\partial E}{\partial n} = \lim_{\Delta d\to 0}\frac{-(R_1+R_2)}{(R_1+n\Delta d)(R_2+n\Delta d)}$$

$$= -\frac{1}{R_1} - \frac{1}{R_2}$$

(Green's Theorem)

$$\int_V (\phi\nabla^2\psi - \psi\nabla^2\phi)\,d^3x = \oint_S \left[\phi\frac{\partial\psi}{\partial n} - \psi\frac{\partial\phi}{\partial n}\right] da$$

"Work / Potential difference in a volume" = "Work / Potential difference in surface area"

1.12.
$$\int_V (\phi\nabla^2\psi - \psi\nabla^2\phi)\,d^3x = \oint_{SA}\left[\phi\frac{\partial\psi}{\partial n} - \psi\frac{\partial\phi}{\partial n}\right] da$$

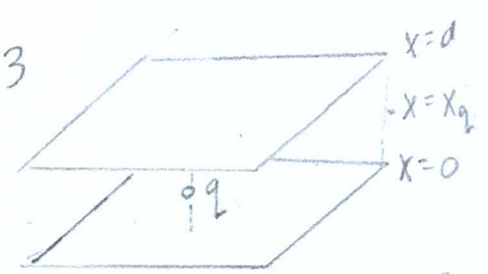
$$\int_V \left(\phi\frac{\rho}{\varepsilon_0} - \psi\frac{\rho}{\varepsilon_0}\right) d^3x = \oint_{SA}\left[\phi\frac{\sigma}{\varepsilon_0} - \psi\frac{\sigma}{\varepsilon_0}\right] da$$

$$\int_V \phi\frac{\rho}{\varepsilon_0}\,dv - \int_{SA}\phi\frac{\sigma}{\varepsilon_0}\,da = \int_V \psi\frac{\rho}{\varepsilon_0}\,dv - \oint_{SA}\psi\frac{\sigma}{\varepsilon_0}\,da$$

$$\int_V \rho\phi\,dv + \oint_{SA}\sigma\phi\,da = \int_V \rho\psi\,dv + \oint_{SA}\sigma\psi\,da$$

Note: Surface integrals are independent of parameters; signs change.

1.13



Two infinitely grounded Parallel Plates

(Greens Theorem)

$$\int_V (\phi\nabla^2\psi - \psi\nabla^2\phi)\,dv = \oint_S \left(\phi\frac{\partial\psi}{\partial n} - \psi\frac{\partial\phi}{\partial n}\right) ds$$

$$0 + \frac{\Psi q}{\varepsilon_0} = \frac{1}{\varepsilon_0}\sigma_1 - \frac{1}{\varepsilon_0}\sigma_1$$

$$[X_0 + (1-X_0)]\frac{q}{\varepsilon_0} = 0$$

$$(1-X_0)\, q = -X_0 q$$

$$q = -\frac{X_0}{1-X_0} q$$

"From Green's reciprocation theorem, the charge never changes between two plates."

1.14.

a) For a Dirichlet Boundary condition,

$$\int_V (\phi\nabla^2\Psi - \Psi\nabla^2\phi)\, d^3x = \oint_S \left[\phi\frac{\partial\Psi}{\partial n} - \Psi\frac{\partial\phi}{\partial n}\right] da$$

$$\int_V \left[\phi(-4\pi\delta(x-x')) + \frac{1}{\varepsilon_0 R}\rho(x')\right] d^3x = \oint_S \left[\phi\left(\frac{1}{R}\right) + \left(\frac{1}{R}\right)\frac{\partial\phi}{\partial n}\right] da$$

$$\phi(x) = \frac{1}{4\pi\varepsilon_0}\int_V \rho(x)G(x,x')d^3x + \frac{1}{4\pi}\oint_S \left[\underbrace{G(x,x')\frac{\partial\phi}{\partial n}}_{=0 \text{ because Dirichlet's Boundary.}} - \phi(x)\frac{\partial G(x,x')}{\partial n}\right] da$$

$$= \frac{1}{4\pi\varepsilon_0}\int_V \rho(x)G(x,x')d^3x - \frac{1}{4\pi}\oint_S \phi(x)\frac{\partial G(x,x')}{\partial n}\, da$$

$$\phi(x) - \phi(x') = \frac{1}{4\pi\varepsilon_0}\int_V \rho(x)G(x,x')d^3x - \frac{1}{4\pi}\oint_S \phi(x)\frac{\partial G(x,x')}{\partial n}\, da$$

$$- \frac{1}{4\pi\varepsilon_0}\int_V \rho(x)G(x,x')d^3x + \frac{1}{4\pi}\oint_S \phi(x)\frac{\partial G(x,x')}{\partial n}\, da$$

$$= 0$$

$\phi(x) = \phi(x')$ "Symmetric"

b) $$\phi(x) = \langle\phi\rangle_s + \frac{1}{4\pi\varepsilon_0}\int_V \rho(x')G_n(x,x')d^3x + \frac{1}{4\pi}\oint_S \frac{\partial\Phi}{\partial n'}G_n da$$

$$\phi(x)-\phi(x') = \frac{1}{4\pi}\oint \frac{d\Phi}{dn}G_n da$$ "Not symmetric."

$$= \frac{1}{4\pi}\oint \frac{d\phi}{dn}G(x,y)da - \frac{1}{4\pi}\oint \frac{d\Phi}{dn}G(x',y)da$$ "symmetric"

c) $$\Phi(x)-\phi(x') = \frac{1}{4\pi\varepsilon_0}\int \rho(x')G_n d^3x + \frac{1}{4\pi}\oint_S \left(G_n \frac{d\phi}{dn}\right)da'$$

$$-\frac{1}{4\pi\varepsilon_0}\int \rho(x')F(x)d^3x + \frac{1}{4\pi}\oint_S F(x)\frac{d\Phi}{dn}da'$$

$$= \phi(x) + \frac{1}{4\pi}F(x)\left[\frac{1}{\varepsilon_0}\int \rho(x)d^3x + \oint_S \frac{d\Phi}{dn}da\right]$$

$$= \Phi(x) + \frac{1}{4\pi}F(x)\left[\oint E\cdot n\, da + \oint_S \left(\frac{d\phi}{dn}\right)da'\right]$$

$$= \phi(X) + \frac{1}{4\pi}F(X)\left[-\oint_S \left(\frac{d\Phi}{dn}\right)da + \oint_S \left(\frac{d\phi}{dn}\right)da\right]$$

$$\Phi(x') = \phi(x)$$

1.15. $$U = \sum_{1}^{n}\left\{\frac{1}{2}\int \rho\phi\, dV_{int}^{(n)} + \frac{1}{2}\int \sigma\Phi\, ds^{(n)} + \lambda^{(n)}\left(Q^{(n)} - \int \rho dV_{int}^{(n)} - \int \sigma ds^{(n)}\right)\right\}$$

$$\frac{\partial U}{\partial r} = \sum_{1}^{n}\left\{\frac{1}{2}\int \rho'(\phi - \lambda^{(n)} + \underline{4\pi\alpha^{(n)}}) + \phi'(\rho + \underline{\nabla^2\alpha^{(n)}})dV_{int}^{(n)}\right.$$

$$\left. + \frac{1}{2}\int \sigma'(\Phi - \lambda^{(n)} + \underline{4\pi\beta^{(n)}}) + \Phi'(\sigma - \hat{n}\cdot(\nabla \underline{\overset{+}{\beta}^{(n)}} - \nabla \underline{\overset{-}{\beta}^{(n)}}))dS\right\}$$

$$= 0$$

$$\Phi = \lambda^{(n)} - 4\pi\alpha^{(n)}$$

$$\nabla^2\Phi = -4\pi\nabla^2\alpha$$

Where α-term: Internal charge Density

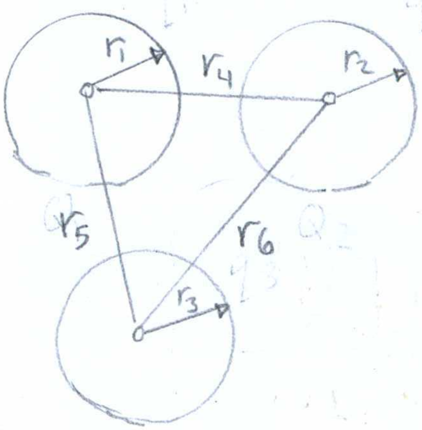
β-term: Surface charge Interaction

Also, Lagrange multipliers of unknown value

$\rho = -\nabla^2 \alpha^{(n)}$ "Each electrostatic energy is exactly equipotential."

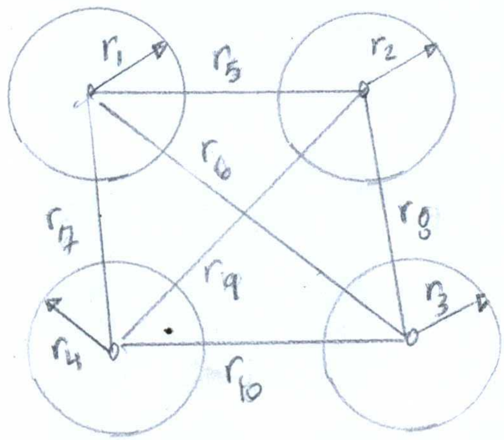
1.16. (Equation 1.16) $E = -\nabla \Phi$

(Equation 1.48) $\phi(x_i) = \frac{1}{4\pi\varepsilon_0} \sum_{j=1}^{n-1} \frac{q_j}{|x_i - x_j|}$



$$E = -\nabla \Phi$$
$$= \frac{1}{4\pi\varepsilon_0} \sum_{j=1}^{n-1} \frac{q_i}{|x_i - x_j|^2}$$
$$= \frac{Q_1}{4\pi\varepsilon_0 r_1^2} + \frac{Q_2}{4\pi\varepsilon_0 r_2^2} + \frac{Q_3}{4\pi\varepsilon_0 r_3^2}$$
$$- \frac{(Q_1 - Q_2)}{4\pi\varepsilon_0 r_4^2} - \frac{(Q_1 - Q_3)}{4\pi\varepsilon_0 r_5^2} - \frac{(Q_2 - Q_3)}{4\pi\varepsilon_0 r_6^2}$$

Shape: A number of conducting surfaces
Dimension: Surface Area [2D]
Charge: Q_1 Q_2 Q_3



$$E = -\nabla \Phi$$
$$= \frac{1}{4\pi\varepsilon_0} \sum_{j=1}^{n-1} \frac{q_i}{|x_i - x_j|^2}$$
$$= \frac{Q_1}{4\pi\varepsilon_0 r_1^2} + \frac{Q_2}{4\pi\varepsilon_0 r_2^2} + \frac{Q_3}{4\pi\varepsilon_0 r_3^2} + \frac{Q_4}{4\pi\varepsilon_0 r_4^2}$$
$$- \frac{(Q_1 - Q_2)}{4\pi\varepsilon_0 r_5^2} - \frac{(Q_1 - Q_3)}{4\pi\varepsilon_0 r_6^2} - \frac{(Q_1 - Q_4)}{4\pi\varepsilon_0 r_7^2} - \frac{(Q_2 - Q_3)}{4\pi\varepsilon_0 r_8^2}$$
$$- \frac{(Q_2 - Q_4)}{4\pi\varepsilon_0 r_9^2} - \frac{(Q_3 - Q_4)}{4\pi\varepsilon_0 r_{10}^2}$$

Shape: A number of conducting spheres with an insulator
Dimension: Surface Area [2D]
Charge: Q_1, Q_2, Q_3, $Q_4 = 0$

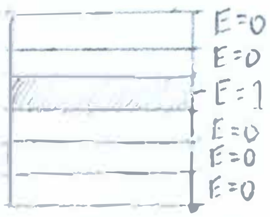
$$= \frac{Q_1}{4\pi\varepsilon_0}\left(\frac{1}{r_1^2}-\frac{1}{r_7^2}\right)+\frac{Q_2}{4\pi\varepsilon_0}\left(\frac{1}{r_2^2}-\frac{1}{r_9^2}\right)$$

$$+\frac{Q_3}{4\pi\varepsilon_0}\left(\frac{1}{r_3^2}-\frac{1}{r_{10}^2}\right)+\frac{(Q_1-Q_2)}{4\pi\varepsilon_0 r_5^2}+\frac{(Q_1-Q_3)}{4\pi\varepsilon_0 r_6^2}$$

$$+\frac{(Q_2-Q_3)}{4\pi\varepsilon_0 r_8^2}$$

The electrostatic potential for one insulator in the presence of other conductors is lower by the distance to the conductor.

1.17.



Several Separate Conducting Surfaces

Shape = Layers
Dimension = Surface Area [2D]
Charge = ??

a) (Equation 1.54) $W = \frac{\varepsilon_0}{2}\int |E|^2 d^3x$

(Equation 1.62) $W = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} C_{iv}\cdot V_i V_j$

$$W = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} C_{iv} V_i V_j = \frac{\varepsilon_0}{2}\int |E|^2 d^3x$$

$$C \leq \varepsilon_0 \int |E|^2 d^3x$$

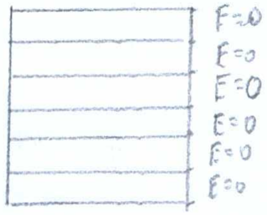
$$\leq \varepsilon_0 \int |\nabla\Phi|^2 d^3x$$

b) $$W = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} C_{iv} V_i V_j = \frac{\varepsilon_0}{2}\int |E|^2 d^3x$$

$$C[\Psi] = \varepsilon_0 \int \frac{|E|^2 d^3x}{V^2}$$

$$\leq \varepsilon_0 \int |\nabla\Psi|^2 d^3x$$

1.18



Several Separate Conducting Surfaces

Shape = Layers
Dimension = Surface Area
Charge = q

a) (Equation 1.49) $\Phi(x_i) = \frac{1}{4\pi\varepsilon_0}\sum_{j=1}^{n-1} \frac{q}{|x_i - x_j|}$

$$= \frac{1}{4\pi\varepsilon_0}\oint_{S_1} \rho(x)\, G_D(x,x')\, d^3x$$

$$= \frac{1}{4\pi\varepsilon_0}\oint \sigma_1(x)\, G(x,x')\, d^3x$$

(Equation 1.52) $W = \frac{1}{8\pi\varepsilon_0}\iint \frac{\rho(x)\rho(x')}{|x-x'|} d^3x\, d^3x$

$$= \frac{1}{8\pi\varepsilon_0}\oint\oint \frac{\sigma_1(x)\sigma_1(x)}{|x-x'|}\, da\, da$$

$$= \frac{1}{8\pi\varepsilon_0}\oint_{S_1} da \oint_{S_1} da'\, \sigma_1(x)\, G(x,x')\, \sigma_1(x')$$

b) (Equation 1.47)

$$W = q_i\,\Phi(x) \overset{?}{=} \frac{1}{2} q\,\Phi(x)$$

$$= \frac{1}{2} q \cdot V$$

$$= \frac{1}{2} q \left(\frac{q}{C}\right)$$

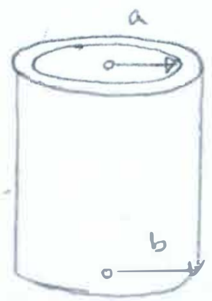
$$\frac{1}{C} = \frac{2W}{q^2}$$

$$= \frac{2\,\frac{1}{4\pi\varepsilon_0}\oint_{S_1} da \oint_{S_1} da\, \sigma_1(x)\, G(x,x')\, \sigma_1(x')}{\left[\oint_{S_1} \sigma(x)\, da\right]^2}$$

$$= \frac{\oint da \oint da\, \sigma(x)\, G(x,x')\, \sigma(x)}{4\pi\varepsilon_0 \left[\oint \sigma(x)\, da\right]^2}$$

The variational principle above is a lower bound because the equality -"?" Jackson delineates many work derivations by 1, 1/2, 1/4, and 1/8. Reason's seemed arbitrary, such as half the distance, total potential or Dirac's symmetry. Other possible reasons include Green's theorem's without symmetry.

1.19.



Two concentric cylinders of Length L

Shape = Cylinder
Dimension = Area [2D]
Charge: Fixed

$$\Psi_1(\rho) = \frac{(b-\rho)}{(b-a)}$$

$$C[\Psi] = \varepsilon_0 \int_a^b |\nabla \Psi|^2 d^3x$$

$$= \varepsilon_0 \int_a^b \frac{1}{(b-a)^2}\, d^3x$$

$$= \frac{2\pi L \varepsilon_0}{(b-a)^2} \int_a^b \rho\, d\rho$$

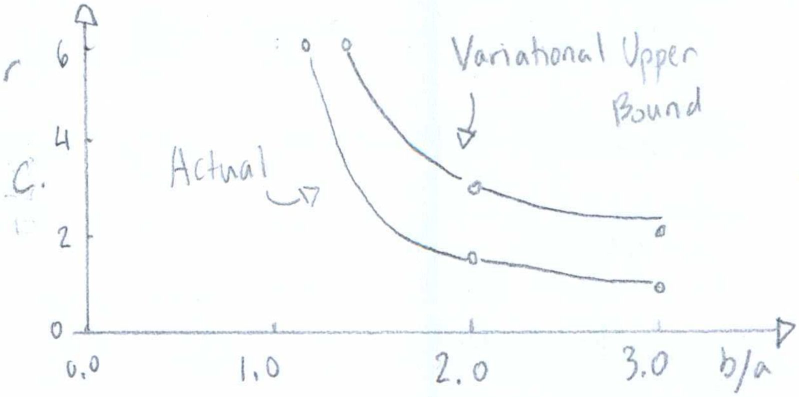
$$= \frac{\pi L \varepsilon_0 (b^2 - a^2)}{(b-a)^2}$$

$$= \frac{\pi L \varepsilon_0 (b+a)}{(b-a)}$$

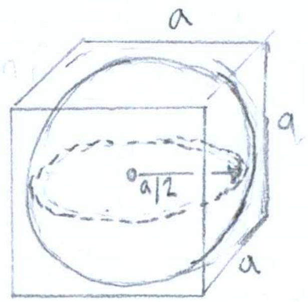
$$= \frac{\pi L \varepsilon_0 (b/a + 1)}{(b/a - 1)}$$

Type	Equation	$\frac{b}{a}=1.5$	$\frac{b}{a}=2.0$	$\frac{b}{a}=3.0$
Variational	$C=\frac{\pi\cdot L\cdot\varepsilon_0(b/a+1)}{(b/a-1)}$	$5\pi L\varepsilon_0$	$3\pi L\varepsilon_0$	$2\pi L\varepsilon_0$
Actual	$C=\frac{\pi\cdot L\cdot\varepsilon_0}{\ln(b/a)}$	$\frac{5\pi L\varepsilon_0}{2}$	$\frac{29\pi L\varepsilon_0}{20}$	$\frac{9\pi L\varepsilon_0}{10}$

A graph provided better due diligence about variational upper bound.



1.20



Configuration of Capacitors with closed surface

a) $C' \le C$

$$\varepsilon_0\int_V |\nabla\psi'|^2 d^3x \le \varepsilon_0\int_{V'} |\nabla\psi'|^2 d^3x$$

b) Lower bound Average Upper bound.

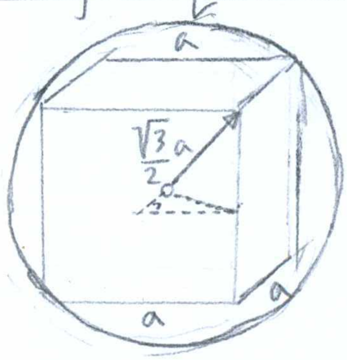
$$\frac{\varepsilon_0}{2}\int_V |\nabla\psi'|^2 d^3x \le C_{Avg} \le \frac{\sqrt{3}\,\varepsilon_0}{2}\int_V |\nabla\psi'|^2 d^3x$$

$$\frac{4\pi\varepsilon_0 a}{2} \le C_{Avg} \le \frac{\sqrt{3}\,4\pi\varepsilon_0 a}{2}$$

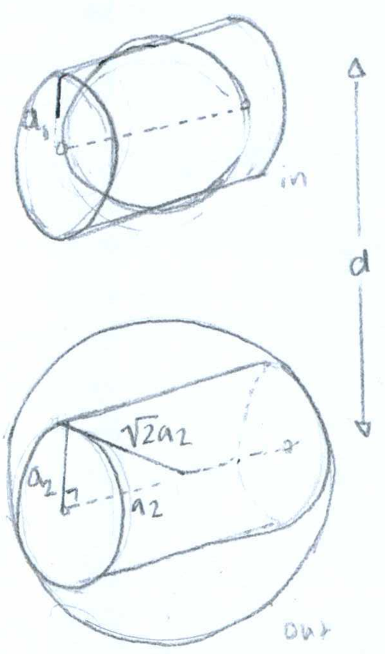
Shape = Sphere
Dimension = Surface[2D]
Charge = q

$$C_{Avg} = \frac{\left[\frac{1}{2}+\frac{\sqrt{3}}{2}\right]}{2}4\pi\varepsilon_0 a$$

$$= 0.683 \times (4\pi\varepsilon_0 a)$$



Configuration of Capacitors within a shell.



Two long cylindrical Conductors

Shape = cylinder
Dimension = surface [2D]
Charge = q

c) Lower bound | Average | Upper Bound

$$C_{in} \leq C_{Avg} \leq C_{out}$$

$$\pi \varepsilon_0 \ln\left(\frac{d}{\sqrt{a_1 a_2}}\right)^{-1} \leq C_{Avg} \leq \pi \varepsilon_0 \ln\left(\frac{d}{\sqrt{\sqrt{2} a_1 a_2}}\right)^{-1}$$

$$C_{Avg} = \frac{\pi \varepsilon_0}{2}\left[\ln\left(\frac{d}{\sqrt{a_1 a_2}}\right)^{-1} + \ln\left(\frac{d}{\sqrt{\sqrt{2} a_1 a_2}}\right)^{-1}\right]$$

$$= \frac{\pi \varepsilon_0}{2}\left[\ln\left(\frac{d}{a}\right)^{-1} + \ln\left(\frac{d}{a}\right)^{-1}\right]$$

$$= 1.05 \cdot C_{in}$$

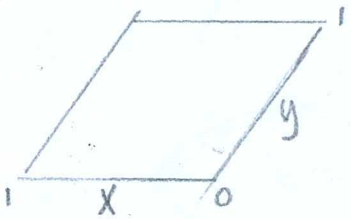
$\approx 5\%$ change to on average

$C_{Avg} = 0.95 \cdot C_{out}$

$\approx 5\%$ change to upper bound

$\boxed{\text{A 10\% whopping change.}}$

1.21.



Unit square Area

Shape: Square
Dimension: Area
charge: q

a) (Equation 1.63)

$$I[\psi] = \frac{1}{2}\int_V \nabla\psi \cdot \nabla\psi \, d^3x - \int_V g\,\psi \, d^3x$$

$$\psi(x,y) = A \cdot x(1-x)\,y(1-y)$$

$$\nabla\psi(x,y) = A\left[(1-2x)\,y(1-y) + x(1-x)(1-2y)\right]$$

$$I[\psi] = \frac{1}{2}\int_0^1\int_0^1 A^2\left[(1-2x)^2 y^2(1-y)^2 + x^2(1-x)^2(1-2y)^2\right] dx\,dy$$

$$- \int_0^1\int_0^1 g\,A\,x(1-x)\,y(1-y)\,dx\,dy$$

$$= 0.0111A^2 - g\frac{A}{36}$$

$$\underset{A}{\operatorname{argmin}}\{I[\psi]\} = \underset{A}{\operatorname{argmin}}\left\{0.0111A^2 - g\frac{A}{36}\right\}$$

$$A^* = 1.25g$$

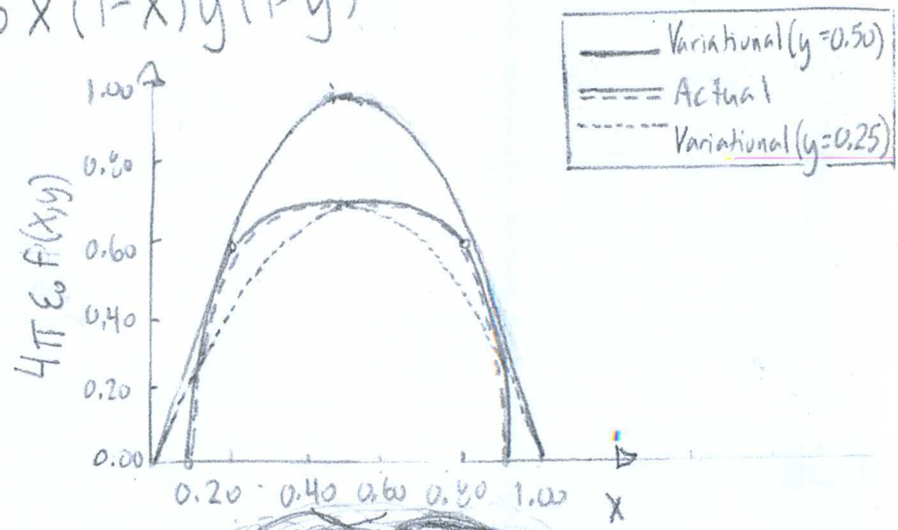
$$\psi(x,y) = 1.25g\,x(1-x)\,y(1-y)$$

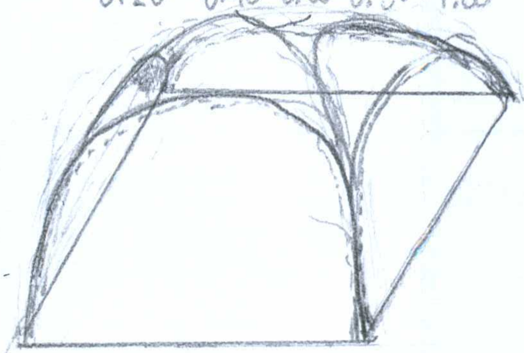
b) <u>Actual Equation:</u>

$$4\pi\varepsilon_0\,\Phi(x,y) = \frac{16}{\pi^2}\sum_{m=0}^{n}\frac{\sin[(2m+1)\pi x]}{(2m+1)^3}\left\{1 - \frac{\cosh[(2m+1)\pi(y-\frac{1}{2})]}{\cosh[(2m+1)\pi/2]}\right\}$$

<u>Variational Equation:</u>

$$4\pi\varepsilon_0\,\psi(x,y) = 5x(1-x)\,y(1-y)$$





"Graph represents potential gradient above a square"

1.22.

a) $F(h,0) = F(0,0) + h\cdot\nabla F_x + \frac{h^2\nabla^2 F_x}{2!} - \frac{h^3\nabla^3 F_x}{3!} + \frac{h^4\nabla^4 F_x}{4!} - \cdots$

$F(-h,0) = F(0,0) - h\cdot\nabla F_x + \frac{h^2\nabla^2 F_x}{2!} - \frac{h^3\nabla^3 F_x}{3!} + \frac{h^4\nabla^4 F_x}{4!} - \cdots$

$F(0,h) = F(0,0) + h\nabla F_y + \frac{h^2\nabla^2 F_y}{2!} - \frac{h^3\nabla^3 F_y}{3!} + \frac{h^4\nabla^4 F_y}{4!} - \cdots$

$F(0,-h) = F(0,0) - h\nabla F_y + \frac{h^2\nabla^2 F_y}{2!} - \frac{h^3\nabla^3 F_y}{3!} + \frac{h^4\nabla^4 F_y}{4!} - \cdots$

$S_c = F(h,0) + F(0,h) + F(-h,0) + F(0,-h)$

$= 4F(0,0) + h^2\nabla^2 F + \frac{h^4}{12}(F_{xxxx} + F_{yyyy}) + O(h^6)$

Why? Maclaurin Series : $F(x) = \sum_{i=1}^{n} \frac{d^i F(0)}{d^i x}\frac{x^i}{i!}$

b) $F(h,h) = F(0,0) + h(\nabla_x F + \nabla_y F) + \frac{h^2(\nabla_x F + \nabla_y F)^2}{2!} + \frac{h^3(\nabla_x F + \nabla F_y)^3}{3!}$

$+ \frac{h^4(\nabla_x F + \nabla_y F)^4}{4!} + \cdots$

$F(-h,h) = F(0,0) + h(-\nabla_x F + \nabla_y F) + h^2(-\nabla_x F + \nabla_y F)^2 + h^3(-\nabla_x F + \nabla_y F)^3$

$+ \frac{h^4(-\nabla_x F + \nabla_y F)^4}{4!} + \cdots$

$F(h,-h) = F(0,0) + h(\nabla_x F - \nabla_y F) + \frac{h^2(\nabla_x F - \nabla_y F)^2}{2!} + \frac{h^3(\nabla_x F - \nabla_y F)^3}{3!}$

$+ \frac{h^4(\nabla_x F - \nabla_y F)^2}{4!} + \cdots$

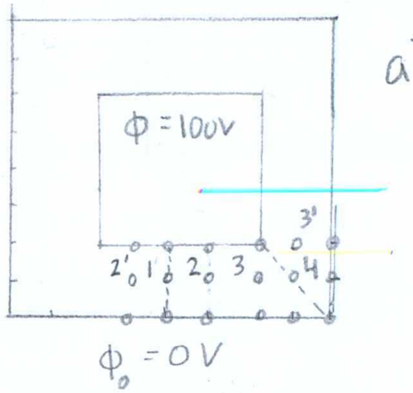
$F(-h,-h) = F(0,0) - h(\nabla_x F + \nabla_y F) + \frac{h^2(\nabla_x F + \nabla_y F)^2}{2!} - \frac{h^3(\nabla_x F + \nabla_y F)^3}{3!}$

$$+\frac{h^4(\nabla_x F+\nabla_y F)^4}{4!}+\cdots$$

$$S_s = F(h,h)+F(-h,h)+F(h,-h)+F(-h,-h)$$

$$=4F(0,0)+2h^2\nabla^2 F-\frac{h^4}{3}(\nabla_x^4 F+\nabla_y^4 F)+\frac{h^4}{2}\nabla^4 F$$

1.23



Hollow Transmission Line with a square conductor

Shape: Square
Dimension: Lattice
Charge: Q

a) $$\psi_1=\frac{1}{4}[\psi_{2'}+\Phi_0+\psi_2+\Phi]$$

$$=\frac{1}{4}[2\psi_2+\Phi]$$

$$\psi_2=\frac{1}{4}[\psi_1+\psi_0+\psi_3+\Phi]$$

$$=\frac{1}{4}[\psi_1+\psi_3+\Phi]$$

$$\psi_3=\frac{1}{4}[\psi_1+\Phi_0+\psi_4+\Phi]$$

$$=\frac{1}{4}[\psi_1+\psi_4+\Phi]$$

$$\psi_4=\frac{1}{4}[\psi_3+\Phi_0+\Phi_0+\psi_{3'}]$$

$$=\frac{1}{4}[2\psi_3]$$

$$\begin{bmatrix}0&2&0&0&1\\1&0&1&0&1\\1&0&0&1&1\\0&0&2&0&0\\0&0&0&0&1\end{bmatrix}\begin{bmatrix}\psi_1\\\psi_2\\\psi_3\\\psi_4\\\Phi\end{bmatrix}=\begin{bmatrix}4\psi_1\\4\psi_2\\4\psi_3\\4\psi_4\\100\end{bmatrix}$$

$\psi_1=48.93V$, $\psi_2=47.87V$, $\psi_3=42.55V$, $\psi_4=21.28V$

b) $\frac{q}{\varepsilon_0} = \nabla\phi\, dl$

$$q = \varepsilon_0 \nabla \phi\, dl$$

$$= \varepsilon_0 \frac{\Delta\phi}{\Delta y} \Delta x$$

$$= \varepsilon_0 \Delta\phi$$

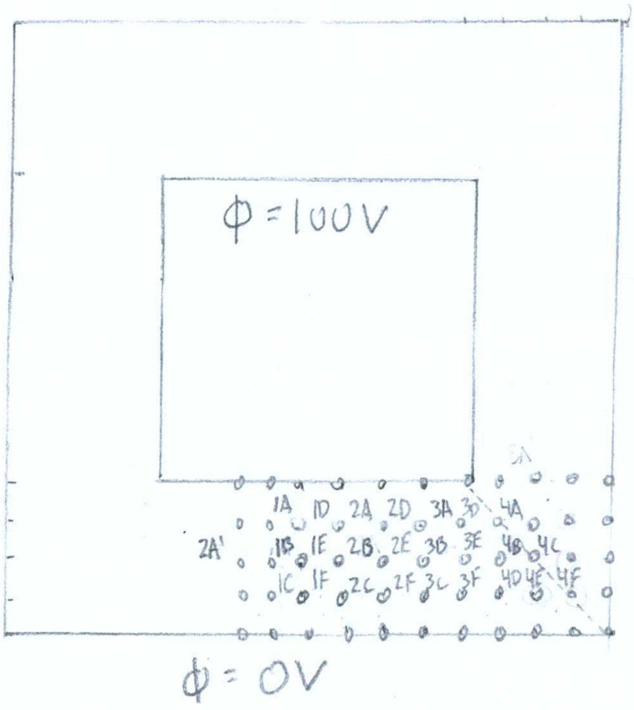
$$= \varepsilon_0 [-4_2 - 4_1 - 4_2 - 4_3 - 4_4 + \phi + \phi + \phi + \phi + \phi]$$

$$= \varepsilon_0 (270\ C/m)$$

$$C = \frac{4Q}{\phi} = \frac{\varepsilon_0 (1081.9\ C/m)}{100V}$$

$$= 10.81\ F/m$$

c)



Book suggests 21 interior points. For the relaxation equation. This model has 24 interior points with no solution, although a personal record in matrix size.

Greater Resolution Hollow Transmission Line with a Square conductor at center

$$
\begin{bmatrix}
0&1&0&2&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1\\
1&0&1&0&2&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&1&0&0&0&2&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
1&0&0&0&1&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1\\
0&1&0&1&0&1&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&1&0&1&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&1&0&0&0&1&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1\\
0&0&0&0&1&0&1&0&1&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&1&0&1&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&1&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&1\\
0&0&0&0&0&0&0&1&0&1&0&1&0&1&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&1&0&1&0&0&0&1&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&1&0&0&0&1&0&1&0&0&0&0&0&0&0&0&1\\
0&0&0&0&0&0&0&0&0&0&1&0&1&0&1&0&1&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&1&0&1&0&0&0&1&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&1&0&1&0&0&0&0&0&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&1&0&1&0&1&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&1&0&0&0&0&1&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&2&0&0&0&2&0&0&0&0&0\\
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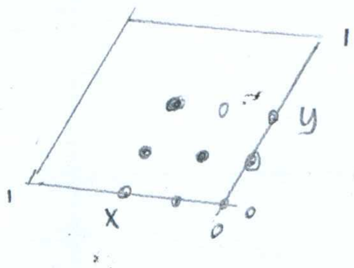
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&2&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1
\end{bmatrix}
\times
\begin{bmatrix}
4_{1A}\\ 4_{1B}\\ 4_{1C}\\ 4_{1D}\\ 4_{1E}\\ 4_{1F}\\ 4_{2A}\\ 4_{2B}\\ 4_{2C}\\ 4_{2D}\\ 4_{2E}\\ 4_{2F}\\ 4_{3A}\\ 4_{3B}\\ 4_{3C}\\ 4_{3D}\\ 4_{3E}\\ 4_{3F}\\ 4_{4A}\\ 4_{4B}\\ 4_{4C}\\ 4_{4D}\\ 4_{4E}\\ 4_{4F}\\ \Phi
\end{bmatrix}
=
\begin{bmatrix}
44_{1A}\\ 44_{1B}\\ 44_{1C}\\ 44_{1D}\\ 44_{1E}\\ 44_{1F}\\ 44_{2A}\\ 44_{2B}\\ 44_{2C}\\ 44_{2D}\\ 44_{2E}\\ 44_{2F}\\ 44_{3A}\\ 44_{3B}\\ 44_{3C}\\ 44_{3D}\\ 44_{3E}\\ 44_{3F}\\ 44_{4A}\\ 44_{4B}\\ 44_{4C}\\ 44_{4D}\\ 44_{4E}\\ 44_{4F}\\ 100
\end{bmatrix}
$$

1.24



Unit Square

Shape = Square
Dimension = Area [2D]
Charge = q

a) From Problem 1.21(b),

$$4\pi\varepsilon_0 \phi(x,y) = \frac{16}{\pi^2}\sum_{m=0}^{n} \frac{\sin[(2m+1)\pi x]}{(2m+1)^3}\left\{1 - \frac{\cosh((2m+1)\pi(y-\frac{1}{2}))}{\cosh((2m+1)\pi/2)}\right\}$$

Actual Function		Iteration (#)					
x	y	0	1	2	3	4	5
0.25	0.25	1.00	0.73	0.72	0.72	0.72	0.72
0.25	0.50	1.00	0.73	0.72	0.72	0.72	0.72
0.50	0.50	1.00	0.92	0.93	0.92	0.92	0.93

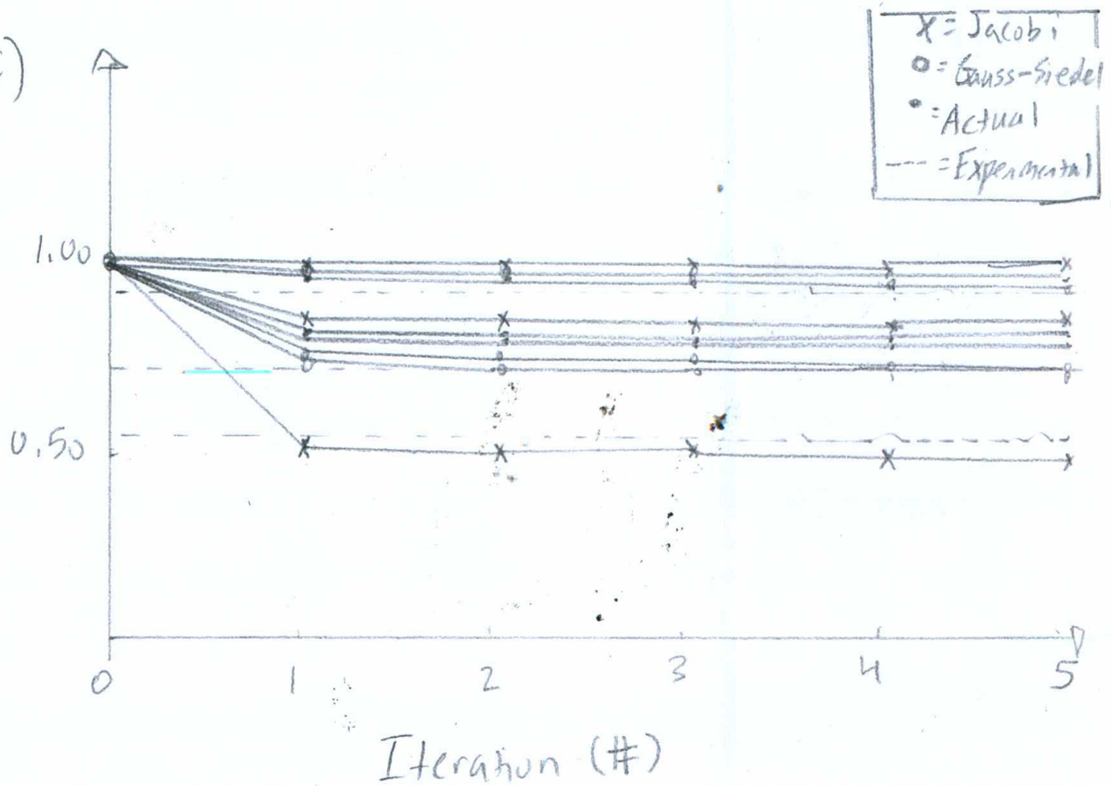
b)

Jacobi Method		Iteration (#)					
x	y	0	1	2	3	4	5
0.25	0.25	1.00	0.48	0.47	0.48	0.47	0.47
0.25	0.50	1.00	0.83	0.83	0.83	0.83	0.83
0.50	0.50	1.00	0.92	0.94	0.93	0.94	0.93

b)

Gauss-Siedel		Iteration (#)					
x	y	0	1	2	3	4	5
0.25	0.25	1.00	0.68	0.72	0.71	0.72	0.72
0.25	0.50	1.00	0.68	0.73	0.72	0.72	0.72
0.50	0.50	1.00	0.97	0.91	0.92	0.92	0.93

c)


X = Jacobi
O = Gauss-Siedel
• = Actual
--- = Experimental
1.00
0.50
0
1
2
3
4
5
Iteration (#)