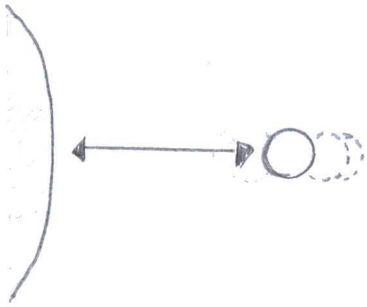
Chapter 16: Radiation Damping, Classical Models of charged particles:

16.1.



"particle of charge bound by linear isotropic force"

(16.13) "time-averaged energy"

$$\left\langle \frac{dE}{dt} \right\rangle = -\frac{\tau}{m}\left\langle \left(\frac{dV}{dr}\right)^2 \right\rangle$$

(16.16) "secular rate of change of angular momentum"

$$\left\langle \frac{dL}{dt} \right\rangle = -\frac{\tau}{m}\left\langle \frac{1}{r}\frac{dV}{dr} \right\rangle L$$

Energy of particles:

(16.3.5) "Mechanical energy of motion"

$$E_0 = m\omega_0^2 d^2$$

$$\left\langle \frac{dE}{dt} \right\rangle = -\frac{\tau}{m}\left\langle \left(\frac{dV}{dr}\right)^2 \right\rangle$$

$$= -\frac{\tau}{m}\left\langle \frac{d}{dr}\left(\frac{1}{2}E_0\right) \right\rangle^2$$

$$= -\frac{\tau}{m} \cdot m^2\omega_0^4 r^2$$ ... when $d=r$

$$= -\omega_0^2 \tau E_0$$

$$E(t) = E(0) \cdot e^{-\omega_0^2 \tau \cdot t}$$

$$= E(0) \cdot e^{-\Gamma t}$$ ... From problem $\Gamma = \omega_0^2 \tau$

Angular Momentum of Particle:

$$\left\langle \frac{dL}{dt} \right\rangle = -\frac{\tau}{m}\left\langle \frac{1}{r}\frac{dv}{dr} \right\rangle L$$

$$= -\frac{\tau}{m}\frac{1}{r} m w_0^2 r \cdot L$$

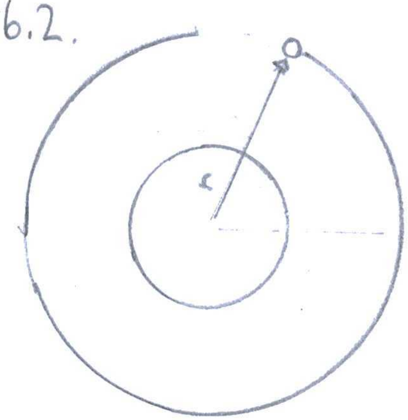
$$= -\tau \cdot w_0^2 \cdot L$$

$$L = L(0)\, e^{-w_0^2 \tau \cdot L}$$

$$= L(0)\, e^{-\Gamma \cdot t}$$

... when $\Gamma = w_0^2 \cdot \tau$

16.2.



"nonrelativistic electron ... bound in an attractive Coulomb potential .. moves in a circular orbit .. in absence of radiation"

Quote found?

> "Classical models addressed radiation, electrons, kinetic energy, damping, and characteristic times. Quantum electrodynamics ... successful in explaining... tiny radiative corrections ... in precision atomic experiments."

a) (16.13) "time-averaged energy"

$$\left\langle \frac{dE}{dt} \right\rangle = -\frac{\tau}{m}\left\langle \left(\frac{dV}{dr}\right)^2 \right\rangle$$

(16.16) "secular rate of change of angular momentum"

$$\left\langle \frac{dL}{dt} \right\rangle = -\frac{\tau}{m}\left\langle \frac{1}{r}\frac{dV}{dr} \right\rangle L$$

$$r^2 \cdot \frac{dr}{dt} = -\frac{\tau Z e^2}{m}$$

$$= -\frac{2 Z e^4}{3 m^2 c^3}$$

$$\int_0^t r^2\, dr = \int_0^{2t} -\frac{2 Z e^4}{3 m^2 c^3}\, dt$$

"Funny business"

$$r^3(t) = r^3(0) - \frac{4 Z e^4}{3 m^2 c^3} dt$$

$$= r^3(0) - 9 Z (c\tau)^3 \frac{1}{\tau}$$

b) $r = n^2 a_0 / Z$

$$\frac{dr}{dn} = \frac{2 n a_0}{Z}$$

$$\frac{dr}{dt} = \frac{2 n a_0}{Z} \frac{dn}{dt}$$

$$= -\frac{4 Z e^4}{3 m^2 r^2 c^3}$$

$$\frac{dn}{dt} = \frac{2 Z^2 e^4}{3 m^2 r^2 c^3} \frac{1}{n a_0}$$

$$= \frac{2 Z^4 e^4}{3 m^2 n^5 a_0^5 c^3}$$

$$= -\frac{2 Z^4 m e^{10}}{3 n^5 \hbar^6 c^3}$$

Bohr radius
$a_0 = \hbar^2 / m e^2$

Energy of particle:

$$\left\langle \frac{dE}{dt} \right\rangle = -\frac{\tau}{m}\left\langle\left(\frac{dV}{dr}\right)\right\rangle^2$$

$$= +\frac{\tau}{m}\left(\frac{Ze^2}{r^2}\right)^2$$

$$\boxed{\text{(from problem)} \quad V(r) = Z^2e^2/r}$$

$$= -\frac{Ze^2}{2r^2}\cdot\frac{dr}{dt}$$

$$\boxed{\text{(From problem)} \quad E(r) = -Ze^2/2r}$$

$$r^2\frac{dr}{dt} = -\frac{2Ze^2\tau}{m}$$

(16.3) "Characteristic time"

$$\tau \sim \frac{2}{3}\frac{e^2}{mc^3}$$

$$r^2\frac{dr}{dt} = -\frac{4Ze^4}{3m^2c^3}$$

$$\int_0^t r^2 dr = \int_0^t -\frac{4Ze^4}{3m^2c^3}dt$$

$$r(t)^3 - r(0)^3 = -\frac{4Ze^4}{m^2c^3}t$$

$$r^3(t) = r^3(0) - 9Z(c\tau)^3\frac{1}{\tau}$$

Angular Momentum:

$$\left\langle\frac{dL}{dt}\right\rangle = -\frac{\tau}{m}\left\langle\frac{1}{r}\frac{dV}{dr}\right\rangle L$$

$$\frac{d(r_0mv)}{dt} = -\frac{\tau}{m}\left\langle\frac{1}{r}\frac{Ze^2}{r^2}\right\rangle(r_0mv)$$

$$\boxed{\boxed{\text{Angular Momentum}} \quad L = r_0mv}$$

a) (16.13) "time-averaged energy"

$$\left\langle \frac{dE}{dt} \right\rangle = -\frac{\tau}{m}\left\langle \left(\frac{dV}{dr}\right)^2 \right\rangle$$

(16.16) "secular rate of change of angular momentum"

$$\left\langle \frac{dL}{dt} \right\rangle = -\frac{\tau}{m}\left\langle \frac{1}{r}\frac{dV}{dr} \right\rangle L$$

Energy of electron:

$$\frac{d\epsilon}{dt} = \left\langle -\frac{dE}{dt} \right\rangle = \frac{\tau}{m}\left\langle \left(\frac{dV}{dr}\right)^2 \right\rangle$$

$$= \frac{\tau}{m}\frac{1}{T}\int_0^T \left(\frac{dV}{dr}\right)^2 dt$$

$$T = \pi Z e^2 \sqrt{\frac{m}{2\epsilon^2}}$$

Cite: "Landau and Lifshitz

The Holy Series

"Mechanics"

Volume 1 of Course of Theoretical Physics

Also, Johannes Keplar, "Astronomia nova" (1609) Second Law of Planetary Motion.

(Landau 14.2) "Conservation of Angular Momentum"

$$M = mr^2\dot{\phi}$$

$$\int_0^T dt = \int_0^{2\pi} d\theta \, \frac{mr^2}{L} \quad \text{.. when } L = M$$

(Problem 14.21a).

$$\frac{1}{\tau} = \frac{2}{3}\frac{e^2}{\hbar c}\left(\frac{Ze^2}{\hbar c}\right)^4 \frac{mc^2}{\hbar}\frac{1}{n^5}$$ ...very similar.

c) $\underline{n=10 \rightarrow n=4:}$

$$-\frac{dn}{dt} = \frac{2}{3}\frac{Z^4 m e^{10}}{n^5 \hbar^6 c^3}$$

$$\int_{10}^{4} n^5 dn = \int_0^t \frac{2}{3}\frac{Z^4 m e^{10}}{\hbar^6 c^3} dt$$

$$t = -\frac{\hbar^6 c^3}{4 Z^4 m e^{10}}(4^6 - 10^6)$$

=

$\underline{n=10 \rightarrow n=1:}$

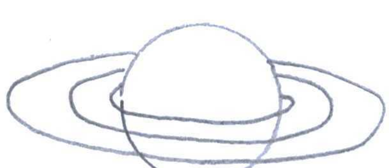
$$-\frac{dn}{dt} = \frac{2}{3}\frac{Z^4 m e^{10}}{n^5 \hbar^6 c^3}$$

$$\int_{10}^{1} n^5 dn = \int_0^t \frac{2}{3}\frac{Z^4 m e^{10}}{\hbar^6 c^3} dt$$

$$t = -\frac{\hbar^6 c^3}{4 Z^4 m e^{10}}(1^6 - 10^6)$$

=

16.3.



"electron moving in an attractive Coulomb field.. with binding energy and angular momentum ...has an elliptic orbit"

$$\frac{d\epsilon}{dt} = \frac{\tau}{m}\frac{1}{T}\int_0^{2\pi} \frac{mr^2}{L}\frac{Z^2e^4}{r^4}\,d\theta$$

(from Problem)
"Coulomb Potential"
$V = -Ze^2/r$

$$= \frac{2}{3}\frac{e^2}{m^2c^3}\sqrt{\frac{2\epsilon^2}{m}}\frac{1}{\pi Ze^2}\frac{1}{r^2}\int_0^{2\pi} d\theta$$

$$= \frac{2e^2}{3m^2c^3}\sqrt{\frac{2\epsilon^2}{m}}\frac{1}{\pi Ze^2}\frac{m}{L}Z^2e^4\frac{Z^2e^4m^2}{L^4}\left[2\pi + \pi - \pi\right]$$

$$= \frac{2^{3/2}}{3}\frac{Z^3e^8m^{1/2}}{c^3}\frac{\epsilon^{3/2}}{L^5}\left[3 - \left(\frac{2\epsilon L^2}{Z^2e^4m}\right)\right]$$

$$\boxed{\epsilon = \frac{Z^2e^4m}{2L^2}}$$
(from problem)

Angular Momentum of Particle:

$$\frac{dL}{dt} = -\frac{\tau}{m}\left\langle\frac{1}{r}\frac{dV}{dr}\right\rangle L$$

$$= -\frac{\tau}{m}\frac{1}{T}\int_0^{T}\frac{1}{r}\frac{dV}{dr}\cdot L\cdot dt$$

$$= -\frac{2e^2}{3m^2c^3}\sqrt{\frac{2\epsilon^3}{m}}\frac{1}{\pi Ze^2}\frac{m}{L}Ze^2\frac{L}{r}\cdot\int_0^{2\pi} d\theta$$

$$= -\frac{2e^2}{3m^2c^3}\sqrt{\frac{2\epsilon^3}{m}}\frac{1}{\pi Ze^2}mZe^2\frac{Ze^2m}{L}\cdot 2\pi$$

$$= -\frac{2^{5/2}}{3}\frac{Ze^4}{\sqrt{m}\,c^3}\frac{\epsilon^{3/2}}{L^2}$$

b) $$\frac{d\epsilon}{dL} = -\frac{1}{2}\frac{Z^2e^4m}{L^3}\left[3 - \left(\frac{2\epsilon L^2}{Z^2e^4m}\right)\right]$$

$$= -\frac{3}{2}\frac{Z^2e^4m}{L^3} + \frac{\epsilon}{L}$$

$$\frac{d}{dL}\left(\frac{\epsilon}{L}\right) = \frac{d}{dL}\left(\frac{d\epsilon}{dL} + \frac{3}{2}\frac{Z^2e^6m}{L^3}\right)$$

$$= -\frac{9}{2}\frac{Z^2e^4m}{L^4}$$

$$\int_{L_0}^{L} d\left(\frac{\epsilon}{L}\right) = \int_{L_0}^{L} -\frac{9}{2}\frac{Z^2e^4m}{L^4}\,dL$$

$$\frac{\epsilon(L)}{L} = \frac{\epsilon(L_0)}{L_0} = \frac{3}{2}\frac{Z^2e^4m}{L^3}\Bigg|_0^L$$

$$= \frac{3}{2}Z^2e^4m\left[\frac{1}{L^3} - \frac{1}{L_0^3}\right]$$

$$\epsilon(L) = \frac{3}{2}\frac{Z^2e^6m}{L^3}\left[1 - \left(\frac{L^3}{L_0^3}\right)\right] + \frac{\epsilon_0}{L_0}L$$

Note: Book shows 1/2 versus 3/2 - coefficient.

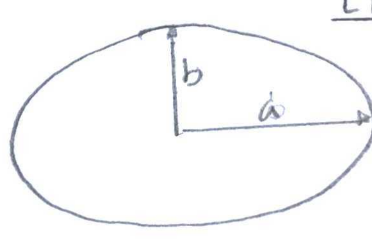
<u>Eccentricity of Ellipse:</u>

Cite: Wikipedia - eccentricity

$$\epsilon = \sqrt{1 - \frac{2EL^2}{m\alpha^2}}$$

$$\simeq \sqrt{1 - b^2/a^2}$$ ... where

<u>Ellipse</u>



a = semi-major axis
b = semi-minor axis

$$= \sqrt{1 - \frac{2L^2}{Z^2 e^4 m}\left[\frac{Z^2 e^4 m}{2L^2}\left[1 - \left(\frac{L}{L_0}\right)^3\right] - \frac{\epsilon_0}{L_0} L\right]}$$

$$= \sqrt{1 - \left[1 - \left(\frac{L}{L_0}\right)^3\right] - \frac{2L^2}{Z^2 e^4 m}\frac{\epsilon_0}{L_0} \cdot L}$$

$$= \sqrt{\left(\frac{L}{L_0}\right)^3 - \frac{2\epsilon_0 L_0^2}{Z^2 e^4 m}\left(\frac{L}{L_0}\right)^3}$$

$$= \left(\frac{L}{L_0}\right)^{3/2}\sqrt{1 - \frac{2\epsilon_0 L_0^2}{Z^2 e^6 m}}$$

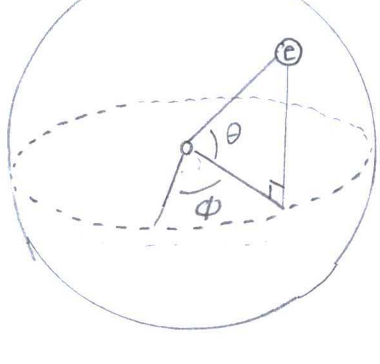
The eccentricity becomes circular when $\frac{2\epsilon_0 L_0^2}{Z^2 e^6 m}$.

c) Problem 16.2 demonstrates a circular orbit with $r = n^2 a_0 / Z = L^2/2m$.

$$L = \sqrt{\frac{2mn^2 a_0}{Z}}$$

$$\epsilon = \left(\frac{\sqrt{\frac{2mn^2 a_0}{Z}}}{L_0}\right)\sqrt{1 - \frac{2\epsilon_0 L^2}{Z^2 e^6 m}}$$

16.4.



"Classical model of an electron is a spherical shell of charge"

a) (16.30) "Effective mass"

$$M(\omega) = m_0 + \frac{2}{3c^3}\int d^3x \int d^3x' \rho(x) \frac{e^{i\omega R}}{R} \rho(x')$$

(9.98) "Greens Function"

$$\frac{e^{ik|x-x'|}}{4\pi|x-x'|} = ik \sum_{l=0}^{\infty} j_l(kr_<)\cdot h_l^{(1)}(kr_>) \sum_{m=-l}^{l} Y^*_{lm}(\theta',\phi') Y_{lm}(\theta,\phi)$$

$$M(\omega) = m_0 + \frac{2}{3c^2}\int d^3x \int d^3x' \cdot 4\pi i \frac{\omega}{c} \sum_{l=0} j_l(kr_>)\cdot h_l^{(1)}(kr_>)$$

$$\cdot \sum_{m=-l}^{l} Y^*_{lm}(\theta',\phi') Y_{lm}(\theta,\phi)$$

$$\cdot \rho(x)\cdot\rho(x')$$

If $\rho(x) = \frac{e}{4\pi a^2}\delta(r-a)$, then

$$= m_0 + \frac{2e^2}{3c^2} i \frac{\omega}{c} j_0\left(\frac{\omega a}{c}\right)\cdot h_l^{(1)}\left(\frac{\omega a}{c}\right)$$

$$= m_0 + \frac{2e^2}{3c^2} i \frac{\omega}{c} \frac{\sin(\omega a/c)}{\omega a/c} \frac{-i e^{i\omega a/c}}{\omega a/c}$$

$$= m_0 + \frac{2e^2}{3c^2} \frac{\sin(\omega a/c)\, e^{i\omega a/c}}{\omega a/c}$$

If $\xi = 2\omega a/c$, then

$$= m_0 + \frac{2e^2}{3ac^2} \frac{e^{i\xi}-1}{i\xi}$$

Spherical Bessel

$$j_0(x) = \frac{\sin(x)}{x}$$

Joke! You fit an image of Herman Hankel.

(Problem 16.4) "Effective mass"

$$\boxed{h_0(x) = j_0(x) + i n_0(x) = -i\frac{e^{ix}}{x}}$$

$$m = m_0 + 2e^2/3ac^2$$

$$M(\omega) = m + \frac{2e^2}{3ac^2}\left(\frac{e^{i\xi}-1}{i\xi} - 1\right)$$

$$= m + \frac{2e^2}{3ac^2}\left(\frac{e^{i\xi}-1-i\xi}{i\xi}\right)$$

b)

$$M(\omega) = m + \frac{2e^2}{3ac^2}\left(\frac{1+i\xi-\xi^2/2-1-i\xi}{i\xi}\right)$$

$$\boxed{\text{Taylor Series @ } a=0 \quad f(x) = \sum_{n!}^{\infty} \frac{1}{n!}\frac{d^n f(x)}{dx^n}(x-a)^n}$$

$$= m + i\frac{2e^2}{3c^2}\cdot\omega$$

$$= m(1+i\omega\tau) \qquad \text{...when } \tau = 2e^2/3mc^2$$

At zero effective mass the characteristic time becomes complex.

c) Two simultaneous equations:

$$\omega M(\omega) = \omega\left(m + \frac{2}{3}\frac{e^2}{ac^2}\left(\frac{e^{i\xi}-1-i\xi}{i\xi}\right)\right)$$

$$= 0$$

① $i\xi m + \dfrac{2e^2}{3ac^2}\left(e^{i\xi}-1-i\xi\right) = 0$

If $x = \mathrm{Re}(\xi)$ and $y = \mathrm{Im}(\xi)$ when $\xi = x+iy$, then

$$\boxed{\text{Complex numbers} \quad \xi = x+iy}$$

② $i(x+iy) + \dfrac{c\tau}{a}\left(e^{i(x+iy)}+1-i(x+iy)\right) = 0$

③ $iX - y + \frac{c\tau}{a}\left(e^{-y}\cos(X) + ie^{-y}\sin(X) - 1 - iX + y\right) = 0$

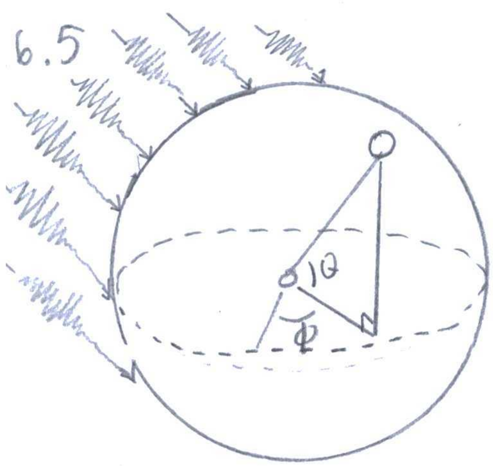
④ $e^{-y}\cos(X) - 1 + y(1 - a/c\tau) = 0$

$e^{-y}\sin(X) - X(1 - a/c\tau) = 0$

○○○.○○ but only when

$0 < y < -\log \frac{X}{\sin(X)}(1 - a/c\tau)$

6.5



"Particle at rest in a spatially uniform but time varying electric field."

a) (16.29) "Force Equation"

$$-i\omega M(\omega)v(\omega) = F_{ext}(\omega)$$

$$v(\omega) = \frac{i F_{ext}(\omega)}{\omega M(\omega)}$$

$$F(\omega) = \int_{-\infty}^{\infty} eE(t)e^{-i\omega t}\,dt$$

$$= e\int_{-\infty}^{\infty} E_0\,\Theta(t)\,e^{-i\omega t}\,dt$$

$$= eE_0\int_{0}^{\infty} e^{-i\omega t}\,dt$$

$$= \frac{ieE_0}{\omega}$$

(16.28.5) "Fourier Transform of Velocity"

$$v(t) = \frac{1}{\sqrt{2\pi}}\int v(\omega)e^{-i\omega t}\,d\omega$$

$$= \frac{eE_0}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{e^{-i\omega t}}{\omega^2 M(\omega)} d\omega$$

$$= -\frac{eE_0 a}{\sqrt{\pi}\, c} \int d\xi \, \frac{e^{-i\xi T}}{\xi M(\omega)}$$

...book has a π versus $\sqrt{\pi}$ coefficient, from the Fourier Transform personal error, or mistake by author.

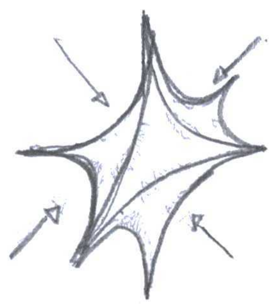
b) For $t \le 0$, then $\xi \ge 0$

and

$$\lim_{\xi \to 0} V(t) = \lim_{\xi \to 0} -\frac{eE_0 a}{\sqrt{\pi} c} \int d\xi \frac{e^{-i\xi T}}{\xi M(\omega)}$$

$$= 0$$

16.6



"particle of bare mass... and charge has a change in density"

a) $\rho(x) = e\, e^{-r/a} / 4\pi a^2 r$

$$f(k) = \frac{1}{e} \int f(x) e^{-ikx} d^3x$$

$$= \frac{1}{4\pi a^2} \int \frac{e^{-r/a}}{r} e^{-ikx} d^3x$$

$$= \frac{1}{a} \frac{1}{(1 + a^2 k^2)}$$

Laplace Transform Formula (Exponential Pulse)

$$X(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

$$= \frac{2\alpha}{\alpha^2 + \omega^2}$$

b) (16.33) "Physical mass"

$$m = m_0 + \frac{e^2}{3\pi c^2} \int d^3k \, \frac{|f(k)|^2}{k^2}$$

$$= m_0 + \frac{e^2}{3\pi^2 c} \int_0^{\infty} \frac{1}{a^2} \frac{dk}{[1 + k^2 a^2]^2}$$

$$= m_0 + \frac{e^2}{3\pi c a^2} \frac{\pi}{4a}$$

Necessary identities to integral:

$u = \arctan(ak)$

$\tan^2 u + 1 = \sec^2 u$

$$= m_0 + \frac{mc\tau}{2a} \quad \ldots \text{when} \quad \tau = \frac{2e^2c^2}{3a^2m}$$

Note: Chapter 16 applied classical theories to electrodynamics with characteristic times, force, mass, and damping.

c) (16.34) "Effective Mass"

$$M(\omega) = m + \frac{e^2\omega^2}{3\pi^2 c} \int d^3k \frac{|f(k)|^2}{k^2[k^2-(\omega/c)^2]}$$

$$= m + \frac{4e\omega^2}{3\pi c^2} \int_{-\infty}^{\infty} \frac{dk}{[1+k^2a^2]^2[k^2(\omega/c)^2]}$$

$$= m$$

6.7

Classical Theory of Radiating Electrons

- Fields associated to the electron
- Application to equation of motion
- Motion of an electron disturbed by a pulse.

Cite: Paul Adrien Maurice Dirac "Classical Theory of radiating electrons" The Royal Society (1938)

$$\frac{dp_\mu}{d\tau} = F_\mu^{ext} + F_\mu^{rad} \quad \ldots \text{with} \quad p_\mu = \text{4-momentum}$$

τ = Proper time

(16.8) "Radiation reaction force"

F_μ = covariant force

$$F_{rad} = \frac{2}{3}\frac{e^2}{c^3}\ddot{v}$$

$$= m\tau\ddot{v}$$

$$= \tau \frac{d^2 p}{d^2 t}$$

$$F_\mu^{rad} = \tau\left(\frac{d^2 p}{d^2 t} + A_\mu\right)$$

$$= \tau\left(\frac{d^2 p}{d^2 t} + cP_\mu\right)$$

$$F_\mu^{rad} p^\mu = p^\mu\left(\frac{d^2 p}{d^2 t} + cP_\mu\right)$$

$$= p^\mu \frac{d^2 p}{d^2 t} + cp^2$$

$$= 0$$

$\circ\circ\circ$ Satisfies $F_\mu p^\mu = 0$ from problem

$$C = -\frac{1}{p^2} p^\mu \frac{d^2 p}{d^2 \tau}$$

$$= \frac{1}{m^2c^2} \frac{dp_\mu}{d\tau} \frac{dp^\mu}{d\tau}$$

$$F_\mu^{rad} = \frac{2e^2}{3mc^3}\left[\frac{d^2 p_\mu}{d\tau^2} + \frac{p_\mu}{m^2c^2}\left(\frac{dp_\mu}{d\tau}\frac{dp^\mu}{d\tau}\right)\right]$$

Note: Dirac connected fields through momentum.

16.8.

$$\frac{dP'}{d\tau} = F_{ext}(t) + \frac{2e^2}{3mc^3}\left[\frac{d^2 p}{d\tau^2} + \frac{p'}{m^2c^2}\left(\frac{dp}{d\tau}\right)^2\right]$$

$$= F_{ext}(t) + \frac{2e^2}{3mc^3}\left[\frac{d^2 p}{d\tau^2} + \frac{p'}{m^2c^2}\left[\left(\frac{dp}{d\tau}\right)^2 - \left(\frac{dp'}{d\tau}\right)^2\right]\right]$$

$$= F_{ext}(t) + \frac{2e^2}{3mc^3}\left[\frac{d^2p}{d\tau^2} + \frac{p'}{m^2c^2}\left[\left(\frac{d}{d\tau}\sqrt{(p')^2+m^2c^2}\right)^2 - \left(\frac{dp'}{d\tau}\right)\right]\right]$$

$$= F_{ext}(\tau) + \frac{2e^2}{3mc^3}\left[\frac{d^2p}{d\tau^2} + \frac{p'}{m^2c^2}\left[\frac{(p')^2}{(p')^2+m^2c^2}\left(\frac{dp'}{d\tau}\right)^2 - \left(\frac{dp'}{d\tau}\right)^2\right]\right]$$

$$= F_{ext}(\tau) + \frac{2e^2}{3mc^3}\left[\frac{d^2p}{d\tau^2} + \frac{p'}{m^2c^2}\left[-\frac{m^2c^2}{(p')^2+m^2c^2}\left(\frac{dp'}{d\tau}\right)^2\right]\right]$$

$$F_{ext} = \dot{p} - \frac{2e^2}{3mc^2}\left[\ddot{p} - \frac{p\dot{p}^2}{p^2+m^2c^2}\right]$$

If $f(\tau) = dp/d\tau$, then $F_{ext}(\tau) = \text{constant} \cdot f(\tau)$

$$= \gamma f(\tau)$$

$$= \sqrt{1+p^2/m^2c^2}\, f(\tau)$$

$$\dot{p} - \frac{2e^2}{3mc^2}\left[\ddot{p} - \frac{p\dot{p}^2}{p^2+m^2c^2}\right] = \sqrt{1+p^2/m^2c^2} \cdot f(\tau)$$

b) If $p = mc\sinh y\, \dot{y}$

$$\dot{p} = mc\cosh y\, \dot{y}$$

$$\ddot{p} = mc\cosh y\, \ddot{y} + mc\sinh y\, \dot{y}$$

$$\dot{p} - \frac{2e^2}{3mc^2}\left[\dot{p} - \frac{p\dot{p}^2}{p^2+m^2c^2}\right] \doteq 0$$

$$\dot{y}\cosh y - \frac{2e^2}{3mc^3}\left[\ddot{y}\, mc\cosh y + \dot{y}\, mc\sinh y - \frac{mc\sinh y}{m^2c^2\cos^2(y)}\dot{y}\, m^2c^2\cosh^2(y)\right]$$

$= \cosh y \, F(\tau)$

$$m\left(\mathring{y} - \frac{2e^2}{3mc^3}\ddot{\mathring{y}}\right) = F(\tau) \qquad \ldots \text{when } p_0 = p(\tau=0)$$

in turn $\sinh y = 0$

(16.9) "Abraham - Lorentz Equation of motion"

$$m(\mathring{V} - \tau \ddot{V}) = F_{ext}$$

16.9 a) $$F_\mu^{rad} = \frac{2e^2}{3mc^3}\left[\frac{d^2 p}{d^2\tau} + \frac{p_\mu}{m^2c^2}\left(\frac{dp_\mu}{d\tau}\frac{dp^\mu}{d\tau}\right)\right]$$

$$= \frac{2e^2}{3mc^3}\left[g_{\mu\nu} - \frac{p_\mu p_\nu}{m^2c^2}\right]\frac{d^2p^\nu}{d\tau^2}$$

b) When $d^2p^\nu/d\tau^2 = g^{\nu\lambda} dF_\lambda^{ext}/d\tau$

$$F_\mu^{rad} = \tau\left[\frac{d^2p^\mu}{d\tau^2} + \frac{p_\mu}{m^2c^2}\frac{dp_\nu}{d\tau}\frac{dp^\nu}{d\tau}\right] \qquad \ldots \tau = 2e^2/3mc^2$$

$$= \tau\left[\frac{dF_\mu^{ext}}{d\tau} + \frac{p_\mu}{m^2c^2}F_\lambda^{ext}\frac{dp}{d\tau}\right]$$

If $F^{ext} = \gamma F$,

$$\frac{dF}{d\tau} = \gamma\frac{d}{dt}(\gamma F)$$

$$= \gamma^2\frac{dF}{d\tau} + \gamma F\frac{d\gamma}{dt}$$

$$= \gamma^2\frac{dF}{dt} + \gamma^6 F(\beta \cdot \mathring{\beta}) \qquad \ldots \text{where } \mathring{\beta} = \frac{1}{c}\frac{dv}{dt}$$

(14.12) "Field - Strength Tensor"

$$\frac{dv^{\alpha}}{d\tau} = \left[c\gamma^4 \beta \circ \mathring{\beta},\ c\gamma^2 \beta + c\gamma^4 \beta(\beta \circ \mathring{\beta}) \right]$$

$$F_v^{ext} = m\left(F_{\circ}^{ext} \circ p_j^{\circ} - F_v^{ext} \circ p^{3}\right)$$

$$= m\left[c\gamma^4 F^{ext} \beta \circ \mathring{\beta},\ \left[-c\gamma^2 \beta^{\circ} - c\gamma^4 \beta \cdot (\beta \circ \dot{\beta}) \right] F^{ext} \right]$$

$$= -mc\gamma^2 F^{ext} \circ \beta$$

$$= -mc\gamma^3 F \circ \beta$$

$$F^{rad} = \tau \left[\gamma^2 \frac{dF}{dt} + \gamma^4 F(\beta \circ \dot{\beta}) - \frac{\gamma m v}{m^2 c^2} mc\gamma^3 F \circ \beta \right]$$

$$= \tau \left[\gamma^2 \frac{dF}{dt} + \frac{\gamma^4}{c^2} F\left(v \circ \frac{dv}{dt}\right) - \frac{\gamma^4}{c^2} v\left(F \circ \frac{dv}{dt}\right) \right]$$

$$= \tau \left[\gamma^2 \frac{dF}{dt} - \frac{\gamma^4}{c^2} \frac{dv}{dt} \gamma (v \times F) \right]$$

Cite: G. W. Ford, R. F. O'Connell "Relativistic Form of Radiation Reaction" Cedex, France (1993)

(Problem 16.7)

$$\frac{dp_{\mu}}{d\tau} = F_{\mu}^{ext} + F_{\mu}^{rad}$$

$$= \gamma F_T\, \tau \left[\gamma^2 \frac{dF}{dt} - \frac{\gamma^4}{c^2} \frac{dv}{dt} \times (v \times F) \right]$$

16.10

(16.9) "Abraham Lorentz Equation of Motion"

$$m(\dot{v} - \tau \ddot{v}) = F_{ext}$$

a) $m\tau(\dot{v}/\tau - \ddot{v}) = F$

$$\ddot{v} - \dot{v}/\tau = -\frac{1}{m\tau} F$$

$$(\ddot{v} - \dot{v}/\tau) e^{-t/\tau} = -\frac{1}{m\tau} F e^{-t/\tau}$$ Exponential

$$\frac{d}{dt}\left(e^{-t/\tau} \cdot \dot{v}\right) = -\frac{1}{m\tau} F e^{-t/\tau}$$ Derivative Factored

$$e^{-t/\tau} \cdot \dot{v} = -\frac{1}{m\tau} \int_t^\infty F(u) e^{-u/\tau} du$$ u-substitution

$$m\dot{v}(t) = -\frac{1}{\tau} \int_t^\infty F(u) e^{(t-u)/\tau} du$$ Inverse exponential

$$= \int_0^\infty e^{-s} F(t + \tau s) ds$$ .. when $u = t + \tau s$

b) $m\dot{v}(t) = \sum_{n=0}^{\infty} \tau^n \cdot \frac{d^n F(t)}{dt^n}$

$$= F(t) + \tau \frac{dF(\tau)}{dt}$$

Gamma Function

$$\Gamma(n) = \int_0^\infty s^{n-1} \cdot e^{-s} ds$$

(16.10) "radiation reaction"

$$m\dot{v} = F_{ext} + \tau \frac{dF_{ext}}{dt}$$

$$= F_{ext} + \tau \left[\frac{dF_{ext}}{dt} + (v \cdot \nabla) F_{ext}\right]$$

c) $F(t) = F_0 \Theta(t)$

$\underline{t<0}$, $F(t+\tau s) = F_0$

$$m\dot{v}(t) = F_0 \int_0^\infty e^{-s}\, ds$$

$$= F_0$$

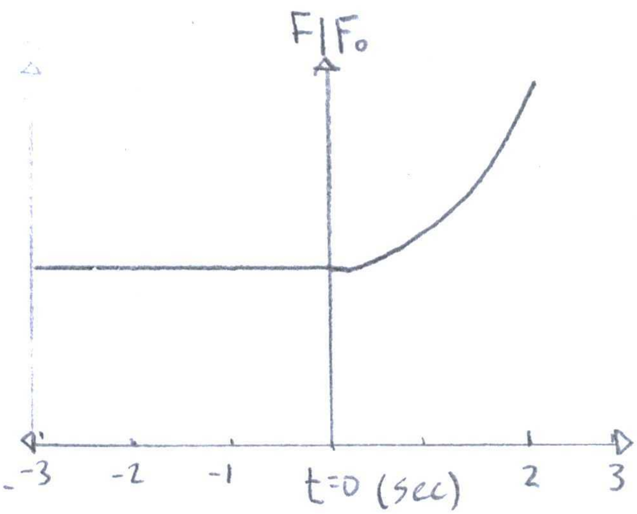
$$\dot{v}(t) = F_0/m$$

$\underline{t>0}$:

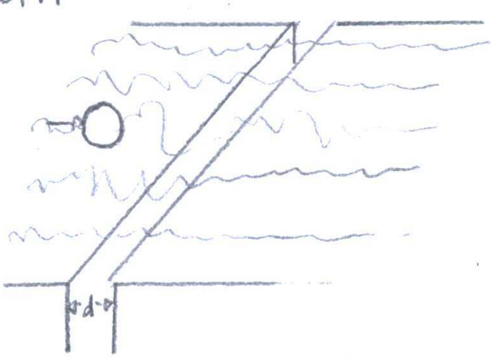
$$m\dot{v}(t) = F_0 \int_{-t/\tau}^{\infty} e^{-s}\, ds$$

$$= F_0 e^{t/\tau}$$

$$\dot{v}(t) = \frac{F_0 e^{t/\tau}}{m}$$



16.11



"Nonrelativistic particle ... accelerated in one-dimensional motion across a gap... by constant electric field"

Initial Velocity: V_0

Uniform acceleration: $T = (-V_0/\alpha) + \sqrt{(v^2/\alpha^2) + (2d/}$

Final Velocity: $V_1 = \sqrt{v^2 - 2\alpha d}$

a) $m\dot{v}(t) = \int_0^\infty F(t+\tau s) e^{-s}\, ds$

$\underline{t \le 0}$: $v(t) = V_0 + \int_{-\infty}^{0} \dot{v}(u)\, du$

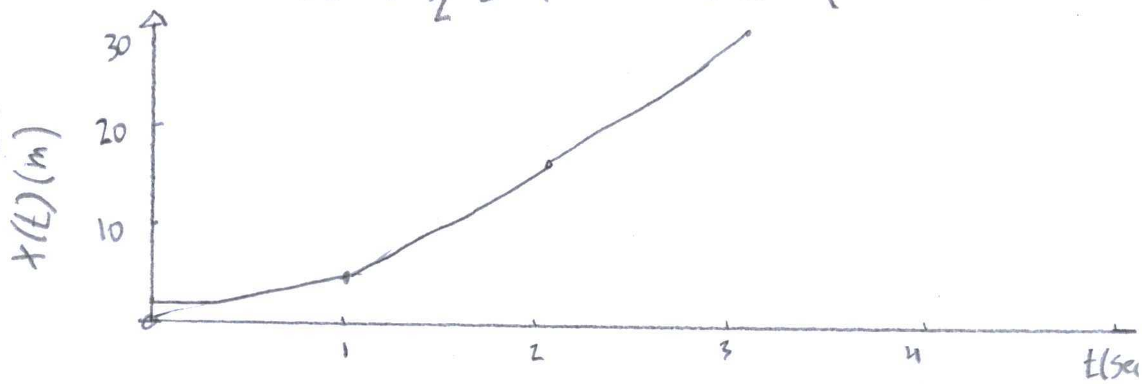
$$= V_0 + \alpha\tau(1 - e^{-t/\tau})$$

$\underline{t>0}$: $v(t) = V_0 + \int_0^t \dot{v}(u)\, du$ $\quad \dot{v}(t) = \alpha e^{t/\tau}(1 - e^{-T/\tau})$

$$= V_0 + \alpha \cdot \tau(1 - e^{-t/\tau}) + d\tau + d\tau(e^{-T/\tau} - e^{-(T-t)/\tau}$$

$$x(t) = \int_0^b v(u)\,du$$

$$= V_0 t + \frac{\alpha}{2} t^2 + \alpha \tau t + \alpha \tau^2 (e^{-T/\tau} - e^{-(T-t)/\tau})$$



b) $x(t) = V_0 t + \frac{\alpha}{2} t^2 + \alpha \tau t + \alpha \tau^2 (e^{-T/\tau} - e^{-(T-t)/\tau})$

$$\simeq V_0 T' + \frac{\alpha}{2} T'^2 + \alpha \tau T'$$

$$\simeq \frac{\alpha}{2} T'^2 + (V_0 + \alpha \tau) T'$$

If gap distance $= d$, then

$$d \simeq \frac{\alpha}{2} T'^2 + (V_0 + \alpha \tau) T'$$

$$0 \simeq \frac{\alpha}{2} T'^2 + (V_0 + \alpha \tau) T' - d$$

$$T' = \sqrt{\left(\frac{V_0}{\alpha} + \tau\right)^2 + \frac{2d}{\alpha}} - \left(\frac{V_0}{d} + \tau\right)$$

$$\doteq T' - \frac{\tau T}{\left[\left(\frac{V_0}{2}\right)^2 + 2\left(\frac{d}{\alpha}\right)\right]^{1/2}}$$

$$v_1 = \sqrt{v_0^2 + 2\alpha d} \quad \text{and} \quad v_1 - v_0 = \alpha T$$

$$T' = T - \frac{\alpha \tau T}{v_1}$$

$$= T - \tau\left(1 - \frac{v_0}{v_1}\right)$$

$$v_1' = v_1 - \frac{\alpha^2 \tau}{v_1} T$$

c) $$\Delta T = \frac{1}{2} m (v_1'^2 - v_2^2)$$

$$= \frac{1}{2}(v_1^2 - v_0^2 - 2\alpha^2 \tau T)$$

$$= m\alpha d - m\alpha^2 \tau T$$

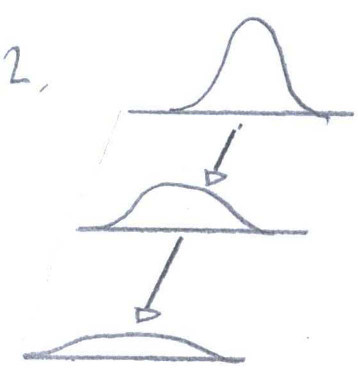
$$\Delta E = \tau m \int_0^T v(t)^2 dt$$

$$= \tau m \alpha^2 T$$

$$\Delta W = \Delta T + \Delta E$$

$$= m\alpha d$$

16.12.



"collision broadening of spectral lines"

a) (16.69) "Charged particle oscillator"

$$m\ddot{x} = -m\omega_0^2 x - m\omega_0^2 \tau \dot{x}$$

(From problem) "Shift"

$$v e^{-v T} = e^{-v(T+d)}$$

(16.71.5) "Fourier Transform"

$$E(\omega) \propto \int_0^{\infty} e^{-\alpha t} e^{i\omega t} dt$$

$$= \frac{1}{\alpha - i\omega}$$

$$I(\omega) = I_0 |E(\omega)|^2$$

$$= I_0 \left| \frac{1}{\sqrt{2\pi}} \int_0^{\infty} e^{-v(\Gamma + c)} \cdot e^{i\omega t} dt \right|^2$$

$$= \frac{I_0}{2\pi} \frac{\Gamma + 2v}{(\omega - \omega_0)^2 + \left(\frac{\Gamma}{2} + v\right)^2}$$

b) Na-Doublet (5893Å), f = "Oscillator Strength"

= 0.975

$\Delta\lambda$ = "Line width"

$= 1.2 \times 10^{-4}$ Å

$T = 500\,K$

(16.72.5) "Line Width"

$$\Delta\lambda = 2\pi \frac{c}{\omega_0^2} \Gamma$$

$$\Gamma = \frac{1.2 \times 10^{-4} A}{2\pi} \frac{\omega_0^2}{(2.998 \times 10^8 m/s)} \frac{1m}{10^{10} Å}$$

$= 1.59\ \mu m$

$\Gamma_a = f_{ij} \Gamma$

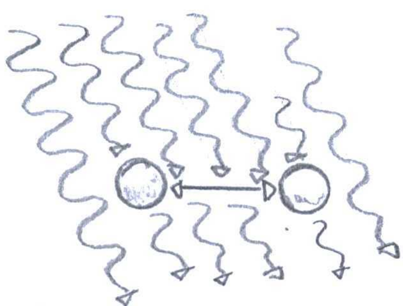
$= 1.55 \mu m$

$$\sigma_{scat} = 6\pi \lambda_o^2 (\Gamma/\Gamma_o)^2$$

$$\Gamma_t = \sqrt{\frac{6\pi (c/\omega_o)^2 \Gamma^2}{\sigma}}$$

$= 34784 \mu m$

16.13.



"oscillator under the action of an applied electric field"

Displacement: $p = \alpha(\omega) E_o e^{-i\omega t}$

a) Total Dipole Cross Section:

(10.146.1) "Displacement in action"

$$p = \frac{e^2}{m} \sum_j f_j (\omega_j^2 - \omega^2 - i\omega\gamma)^{-1} E_o \epsilon_o$$

(10.146.3) "Forward scattering Amplitude"

$$\epsilon_o^* \cdot f(k = k_o) = \frac{e^2 k^2}{4\pi\epsilon_o m} \sum_j f_j (\omega_j^2 - \omega^2 - i\omega\gamma)^{-1}$$

(10.139) "Total cross Section"

$$\sigma_t = \frac{4\pi}{k} \left| \epsilon_o^* \cdot f(k = k_o) \right|$$

$$\epsilon_o^* \cdot f(k = k_o) = \frac{k}{4\pi\epsilon_o} \frac{p}{E_o \epsilon_o}$$

$$\sigma_t = \frac{4\pi}{k} \left| \frac{k}{4\pi\epsilon_o} \frac{p}{E_o \epsilon_o} \right|$$

$$= \epsilon_0 \left| \alpha(\omega) e^{-i\omega t} \right|$$

$$= \epsilon_0 \left| \alpha(\omega)\left(1 - i\omega t + \frac{(i\omega t)^2}{2!} - \dots\right) \right|$$

$$= \epsilon_0 \left| -i\omega \alpha(\omega) t + c.c \right|$$

b) $$\int_0^\infty \sigma_t(\omega) d\omega = \int_0^\infty \frac{2\pi}{c}\left[-i\omega \alpha(\omega) t + c.c\right] d\omega$$

$$= \frac{2\pi}{c} \int_0^\infty \left[i\omega \frac{e^2}{m\omega^2} \right] d\omega$$

$$= \infty - \frac{2\pi e^2}{mc}$$

"not exact in the upper integral"