

Chapter 1: Introduction to Electrostatics

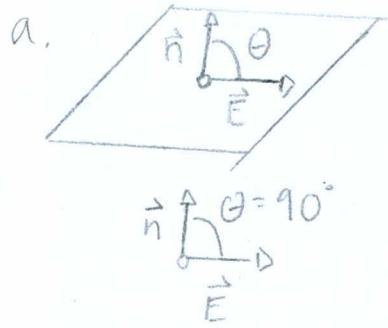
(Gauss' Law of Electrostatics)

$$\nabla \cdot \vec{E} = \rho / \epsilon_0 \quad \text{where} \quad \vec{E} = \text{Electric Field (V/m)}$$

ρ = Charge Density (C/m^3)

ϵ_0 = Permittivity of Free Space (Farad/m)

1.1.



$$\nabla \cdot \vec{E} = \frac{1}{\epsilon_0} \rho(x)$$

$$\int_V \nabla \cdot \vec{E} d^3x = \frac{1}{\epsilon_0} \int_V \rho(x) d^3x \quad \boxed{\text{Volume Integral}}$$

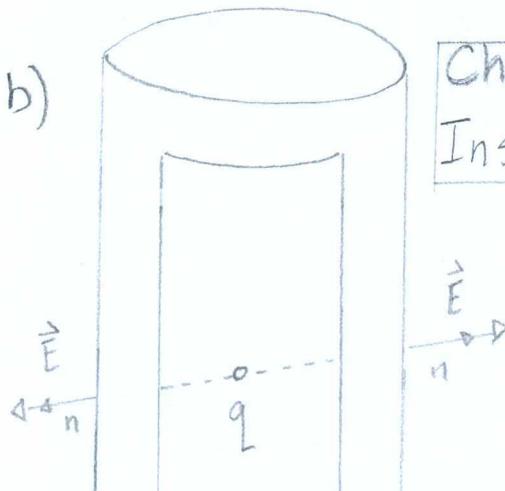
$$\int_V \nabla \cdot \vec{E} d^3x = \frac{1}{\epsilon_0} \int_S E \cdot \vec{n} d^2x \quad \boxed{\text{Area Integral}}$$

$$= \frac{1}{\epsilon_0} \int_S \rho(x) d^2x$$

$$= \frac{1}{4\pi\epsilon_0} \int_S q \cos\theta \frac{d^2r}{r^2}$$

$$= 0 \quad \text{when} \quad \theta = 90^\circ$$

b)

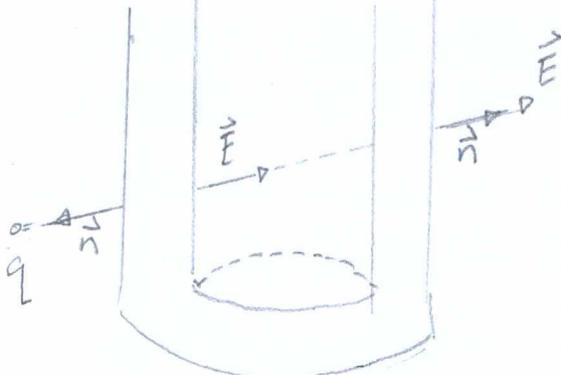


Charge Inside

$$\int_S E \cdot \vec{n} d^2x = \frac{1}{\epsilon_0} \int_S \rho(x) d^2x$$

$$= \frac{1}{4\pi\epsilon_0} \int_S q \cos\theta \frac{d^2r}{r^2}$$

$$= \frac{1}{4\pi\epsilon_0} \int_S q \cos\theta d\Omega$$



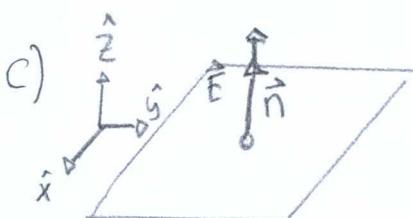
$$\begin{aligned}
 &= \frac{1}{4\pi\epsilon_0} \left(\int_S q [\cos(0^\circ)] + \int_S q [\cos(180^\circ)] \right) d\Omega \\
 &= \frac{1}{2\pi\epsilon_0} \int_S q d\Omega \\
 &= \frac{q}{\epsilon_0} \quad \text{when cylinder circumference} = 2\pi R
 \end{aligned}$$

Charge outside: $\int_S \vec{E} \cdot \vec{n} d^2x = \frac{1}{\epsilon_0} \int_S \rho(x) d^2x$

$$= \frac{1}{4\pi\epsilon_0} \int_S q \cos\theta \frac{dr^2}{r^2}$$

$$= \frac{1}{4\pi\epsilon_0} \left[\int_S q \cos(0^\circ) d\Omega + \int_S q \cos(180^\circ) d\Omega \right]$$

$$= 0$$



$$\int_S \nabla \cdot \vec{E} d^3r = \frac{1}{\epsilon_0} \int_S \rho(x) d^3r$$

$$= \frac{1}{4\pi\epsilon_0} \int_S \frac{q \cos\theta}{r^3} d^3r \quad \theta = 0^\circ$$

$$= \frac{1}{4\pi\epsilon_0} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{q z}{(x^2+y^2+z^2)^{3/2}} dx dy$$

$$= \frac{q}{4\pi\epsilon_0} \int_{-\infty}^{\infty} \frac{z}{(x^2+y^2+z^2)^{3/2}} dx dy$$

$$= \frac{2\pi}{4\pi\epsilon_0} q$$

$$= \frac{q}{2\epsilon_0}$$

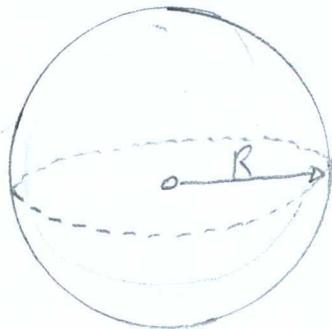
$$1.2. D(x, x_1, y, z) = (2\pi)^{-3/2} \cdot \alpha \cdot \exp\left[-\frac{1}{2\alpha^2}(x^2 + y^2 + z^2)\right]$$

$$= \frac{\exp\left[-\frac{x^2}{2\alpha^2}\right]}{(2\pi)^{1/2}} \cdot \frac{\exp\left[-\frac{y^2}{2\alpha^2}\right]}{(2\pi)^{1/2}} \cdot \frac{\exp\left[-\frac{z^2}{2\alpha^2}\right]}{(2\pi)^{1/2}}$$

$$= \delta(u-u') \cdot U \cdot \delta(v-v') \cdot V \cdot \delta(w-w') \cdot W$$

$$\delta(x-x') = \delta(u-u') U \cdot \delta(v-v') V \cdot \delta(w-w') W$$

1.3. Three methods about Gauss Law:



Conductor

charge [Q]

Method #1:

A guess, charge (q)

per surface area ($4\pi R^2$)

is charge density ($\rho(x)$). $A = \frac{q}{4\pi R^2}$

$$\rho(x) = \frac{q}{4\pi R^2}$$

$$\rho(x) = \frac{q}{4\pi R^2}$$

Method #2:

Coefficient A

$$\oint \nabla \cdot E d^3x = \frac{1}{\epsilon_0} \int \rho(x) d^3x$$

$$= \frac{A}{\epsilon_0} \int \delta(r) dr$$

$$= \frac{q}{\epsilon_0}$$

$$q = A \int \delta(r) dr$$

$$= A \int_0^{2\pi} \int_0^\pi R^2 \sin\theta d\theta d\phi$$

$$= A \cdot 4\pi R^2$$

Method #3:

"Schrödinger-esque"

$$\oint E ds = \frac{q}{\epsilon_0} \int \rho(x) d^3x$$

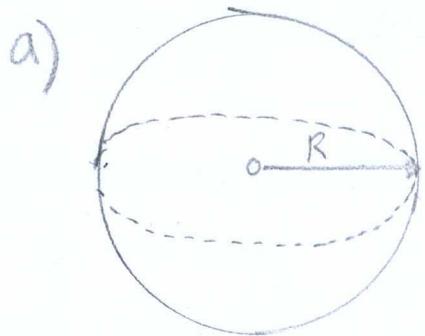
$$E \oint ds = \frac{q}{\epsilon_0}$$

$$E = \frac{q}{\epsilon_0 \oint ds}$$

$$= \frac{q}{\epsilon_0 \int_0^{2\pi} \int_0^\pi R^2 \sin\theta d\theta d\phi}$$

$$= \frac{q}{\epsilon_0 4\pi R^2}$$

$$\rho(x) = \frac{q}{4\pi R^2}$$



Surface Conductor

Shape = Sphere

Dimension = Surface [2D]

Charge = q

Coordinates = Spherical

$$\int \mathbf{E} \cdot d\mathbf{s} = \frac{1}{\epsilon_0} \int \rho(x) d^3x$$

$$E \int ds = \frac{q}{\epsilon_0}$$

$$E = \frac{q}{\epsilon_0 \int ds}$$

$$= \frac{q}{\epsilon_0 \int_0^{2\pi} \int_0^{\pi} R^2 \sin \theta d\theta d\phi}$$

$$= \frac{q}{\epsilon_0 4\pi R^2}$$

$$\rho(x) = \frac{q}{4\pi R^2}$$



Cylindrical Surface

Shape = Circle - perimeter

Dimension = Line [1D]

Charge = λ

Coordinates = cylindrical

$$\int \mathbf{E} \cdot dl = \frac{1}{\epsilon_0} \int \rho(x) d^3x$$

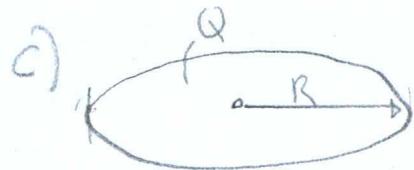
$$E \int dl = \frac{\lambda}{\epsilon_0}$$

$$E = \frac{\lambda}{\epsilon_0 \int dl}$$

$$= \frac{\lambda}{\epsilon_0 \int_0^{2\pi} \int_0^b dp d\phi}$$

$$= \frac{\lambda}{\epsilon_0 \cdot 2\pi b}$$

$$\rho(x) = \frac{\lambda}{2\pi b}$$



Circular Disc

Shape = circle
Dimension = Area [2D]
Charge = Q
Coordinates = cylindrical

$$\int E \cdot dS = \frac{1}{\epsilon_0} \int p(x) d^2x$$

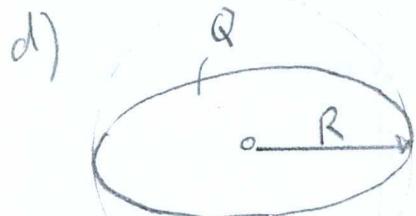
$$E \int dS = \frac{Q}{\epsilon_0}$$

$$E = \frac{Q}{\epsilon_0 \int dS}$$

$$= \frac{Q}{\epsilon_0 \int_0^{2\pi} \int_0^R p d\rho d\phi dz}$$

$$= \frac{Q}{\epsilon_0 \pi R^2}$$

$$p(x) = \frac{Q}{\pi R^2}$$



Circular Disc

Shape = circle
Dimension = Area [2D]
Charge = Q
Coordinates = spherical

$$\int E \cdot dS = \frac{1}{\epsilon_0} \int p(x) d^2x$$

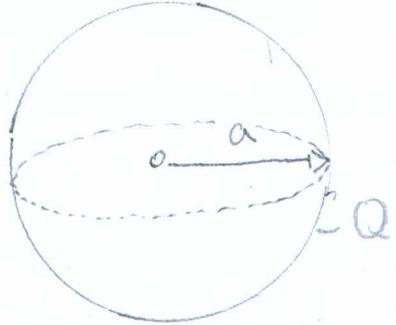
$$E \int dS = \frac{Q}{\epsilon_0}$$

$$E = \frac{Q}{\epsilon_0 \int dS}$$

$$= \frac{Q}{\epsilon_0 \int_0^{2\pi} \int_0^R r dr d\theta}$$

$$= \frac{Q}{\epsilon_0 \pi R^2}$$

1.4. Hollow Conductor



Shape = Sphere

Dimension = Area [2D]

Charge = Q

Inside: The charge inside a conductor had a resolution in Problem 1.1a by $\theta=90^\circ$ and $\vec{E}=0$. How is a person to measure zero charge? The electric field inside a conductor seems as a lemma.

$$\text{outside: } \oint \mathbf{E} \cdot d\mathbf{s} = \frac{1}{\epsilon_0} \int \rho(r) d^2r$$

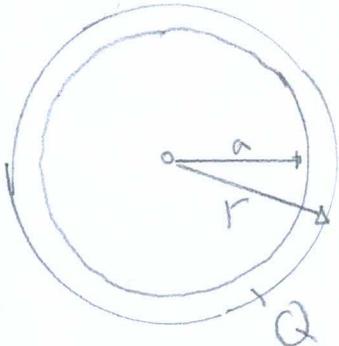
$$E \oint d\mathbf{s} = \frac{q}{\epsilon_0}$$

$$E = \frac{q}{\epsilon_0 \oint d\mathbf{s}}$$

$$= \frac{q}{\epsilon_0} \int_0^{2\pi} \int_0^{\pi} \alpha^2 \sin\theta d\theta d\phi$$

$$= -\frac{q}{\epsilon_0 \cdot 4\pi a^2}$$

Uniform Charge Density
in a sphere



$$\oint \mathbf{E} \cdot d^3x = \frac{1}{\epsilon_0} \int \rho(r) d^3x$$

$$E \oint d^3x = \frac{q}{\epsilon_0} \frac{\int_V \delta(r, \theta, \phi) d^3r}{\int_{SA} \delta(r, \theta, \phi) d^2r}$$

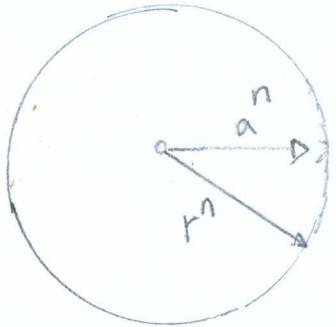
Shape = Sphere
 Dimension = Volume [3D]
 Charge = Q

$$E \iiint_{0..a} r^2 \sin\theta dr d\theta d\phi = \frac{q}{\epsilon_0} \frac{\int_{0..a} \int_{0..2\pi} r^2 \sin\theta dr d\theta d\phi}{\int_0^{2\pi} \int_0^\pi r^2 \sin\theta d\theta d\phi}$$

$$E \cdot \frac{4\pi a^3}{3} = \frac{q}{\epsilon_0} \frac{\frac{4}{3}\pi r^3}{4\pi r^2}$$

$$E = \frac{q \cdot r}{\epsilon_0 \cdot 4\pi a^3}$$

Symmetric charge density
within the volume



Shape = Sphere
 Dimension = Volume [3D]
 Charge = Q

$$\int EdV = \frac{1}{\epsilon_0} \int \rho(r) d^3r$$

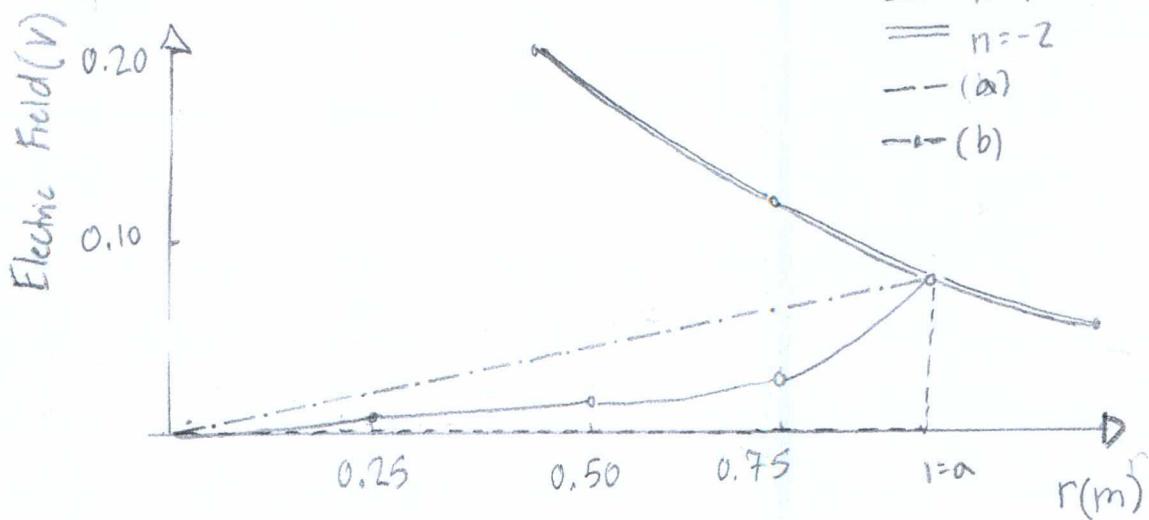
$$E \cdot \int dV = \frac{q}{\epsilon_0} \frac{\int_V \delta(r, \theta, \phi) d^3r}{\int_{SA} \delta(r, \theta, \phi) d^2r}$$

$$E \iiint_{0..a} r^n r^2 \sin\theta dr d\theta d\phi = \frac{q}{\epsilon_0} \frac{\int_0^{2\pi} \int_0^\pi r^n r^2 \sin\theta dr d\theta d\phi}{\int_0^{2\pi} \int_0^\pi r^2 \sin\theta d\theta d\phi}$$

$$E \cdot \frac{4}{3}\pi a^{n+3} = \frac{q}{\epsilon_0} \frac{\frac{4}{3}\pi r^{n+3}}{4\pi r^2}$$

$$E = \frac{q}{\epsilon_0 \cdot 4\pi r^2} \left[\frac{r}{a} \right]^{n+3}$$

$$= \frac{-q \cdot r^{n+1}}{\epsilon_0 \cdot 4\pi a^{n+3}}$$



(Poisson Equation)

$$\nabla \cdot E = \rho/\epsilon_0 \rightarrow \nabla \cdot (\nabla \cdot E) = -\rho/\epsilon_0 \rightarrow \nabla^2 \Phi = -\rho/\epsilon_0$$

"Poisson's Equation"

"Gauss Law"

1.5. Time-averaged potential of a hydrogen atom:

$$\phi(r) = \phi - \frac{q}{4\pi\epsilon_0} \frac{e^{-\alpha r}}{r} \left(1 + \frac{\alpha r}{2}\right)$$

$$\rho(r) = -\epsilon_0 \nabla^2 \phi(r)$$

$$= -\epsilon_0 \frac{d}{r^2 dr} \left(r^2 \frac{d\phi(r)}{dr} \right)$$

$$= -\frac{\epsilon_0}{r^2} \frac{d}{dr} \left[\frac{1}{2} e^{-\alpha r} (-\alpha r(\alpha r + 2) - 2) \right]$$

$$= -\frac{q\alpha^3}{8\pi} e^{-\alpha r}$$

$$\rho(r) = q\delta(r) + \rho(r)$$

$$= q\delta(r) + q\frac{\alpha^3}{8\pi} e^{-\alpha r}$$

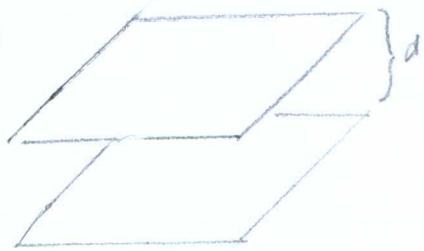
"Static"
"potential"

"Continuous"
"Kinetic"

Poisson's derivation has a second derivative of the electric field. Schrödinger's equation has a second derivative of total energy. One is static-and-continuous, the other, potential-and-kinetic.

1.6

a)



Two, large, flat
conducting sheets

Shape = Square

Dimension = Area [2D]

Charge = Q

$$\int E \cdot ds = \frac{1}{\epsilon_0} \int \rho(x) d^2x$$

$$E \int ds = \frac{q}{\epsilon_0}$$

$$E \int_0^a \int_0^a da = \frac{q}{\epsilon_0}$$

$$E = \frac{q}{\epsilon_0 a^2}$$

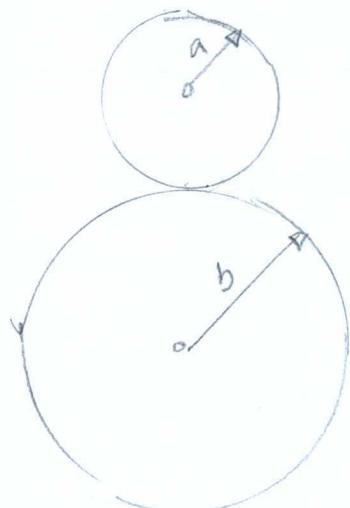
$$= \frac{V}{d}$$

$$C = \frac{q}{V}$$

$$= \frac{\epsilon_0 a^2}{d}$$

$$\boxed{\text{Capacitance } [C = q/V]}$$

b)



Two concentric
conducting spheres

Shape = Hollow sphere

Dimension = Surface
Area [2D]

Charge = q

$$\int E ds = \frac{1}{\epsilon_0} \int \rho(x) d^2r$$

$$E \int ds = \frac{q}{\epsilon_0}$$

$$E \int_0^{2\pi} \int_0^\pi r^2 \sin\theta d\theta d\phi = \frac{q}{\epsilon_0}$$

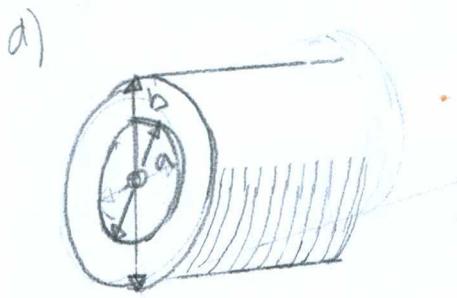
$$E = \frac{q}{\epsilon_0 \cdot 4\pi r^2}$$

$$-\nabla \phi = E \hat{r}$$

$$\phi = \frac{q}{\epsilon_0 \cdot 4\pi r^2}$$

$$\phi = \int_a^b \frac{q}{\epsilon_0 \cdot 4\pi r^2} dr$$

$$= \frac{q}{4\pi \epsilon_0} \left(\frac{1}{b} - \frac{1}{a} \right)$$



$$a = 1 \text{ mm}$$

ϵ_0 = Permittivity
of
Free Space
 $= 8.85 \times 10^{-12} \text{ F/m}$

$$C = 3 \times 10^{-11} \text{ F/m} = \frac{C}{L}$$

$$= \frac{2\pi\epsilon_0}{\ln(\frac{b}{a})}$$

$$2\pi\epsilon_0/C$$

$$b =$$

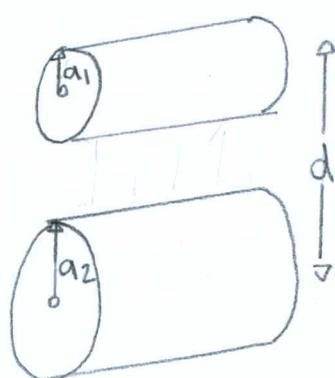
$$b = a e$$

$$\frac{2\pi(8.85 \times 10^{-12} \text{ F/m})}{(3 \times 10^{-12} \text{ F/m})}$$

$$= 1 \text{ mm} \circ e$$

$$\approx 1 \times 10^9 \text{ mm}$$

1.7.



Two long, cylindrical
conductors

Shape = Cylinders

Dimension = Surface [2D]

Charge = q

$$\int E \cdot dS = \frac{1}{\epsilon_0} \int p(r) d^2 r$$

$$E \cdot f ds = \frac{q}{\epsilon_0}$$

$$E \int_0^{L\pi} \int_0^{2\pi} \int_x^d dp d\theta dz = \frac{-q}{\epsilon_0}$$

$$E = \frac{-q}{\epsilon_0 \cdot 2\pi \cdot L} \left(\frac{1}{x} + \frac{1}{d-x} \right)$$

$$-\nabla \phi = E$$

$$= \frac{-q}{\epsilon_0 \cdot 2\pi \cdot L} \left(\frac{1}{x} + \frac{1}{d-x} \right)$$

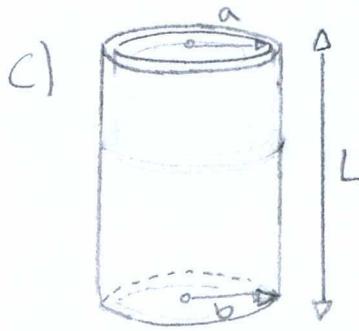
$$\phi = \int_{a-a_2}^{a_1+a_2} -\frac{q}{\epsilon_0 \cdot 2\pi \cdot L} \left(\frac{1}{x} + \frac{1}{d-x} \right) dx$$

$$= \frac{-q}{\epsilon_0 \cdot 2\pi \cdot L} \ln \left(\frac{d-a_1}{a_2} \cdot \frac{d-a_2}{a_1} \right)$$

$$= V$$

$$C = \frac{q}{V}$$

$$= \frac{4\pi\epsilon_0 \cdot a \cdot b}{(b-a)}$$



Two concentric
Cylinders of length L

Shape = cylinder

Dimension = Area [2D]

Charge = q

$$\int E \cdot ds = \frac{1}{\epsilon_0} \int \rho(x) d^2 r$$

$$E \int ds = \frac{q}{\epsilon_0 L}$$

$$E \int_0^{2\pi} \int_0^r \int_0^L dr dL dz = \frac{-q}{\epsilon_0}$$

$$E = -\frac{q}{2\pi \cdot L \cdot r \cdot \epsilon_0}$$

$$-\nabla \phi = E$$

$$= -\frac{q}{2\pi L \cdot r \cdot \epsilon_0}$$

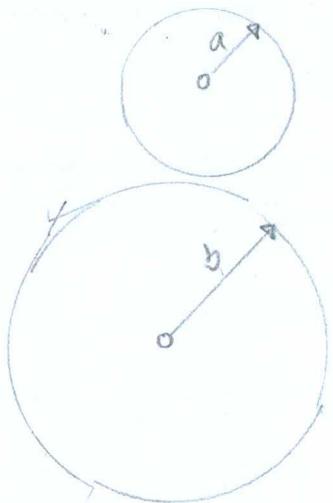
$$\phi = -\int_a^b \frac{-q}{2\pi \cdot L \cdot r \cdot \epsilon_0} dr$$

$$= -\frac{q}{2\pi L \cdot \epsilon_0} \ln\left(\frac{b}{a}\right)$$

$$= V$$

$$C = \frac{q}{V}$$

$$= \frac{2\pi L \cdot \epsilon_0}{\ln(b/a)}$$



Two concentric
conducting spheres

Shape = Hollow spheres

Dimension = Surface Area [2D]

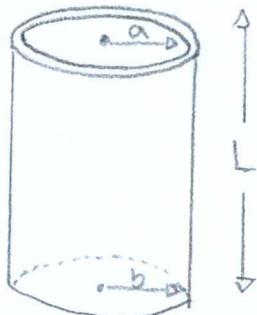
Charge = $-Q, +Q$

From Problem 1.6b, $\phi = -\frac{q}{\epsilon_0 \cdot 4\pi} \left(\frac{1}{a} - \frac{1}{b} \right)$

$$W = \frac{1}{2} \sum_{i=1}^n Q_i V_i = V \cdot d$$

$$= -\frac{Q \cdot Q}{\epsilon_0 \cdot 8\pi} \left(\frac{1}{b} - \frac{1}{a} \right)$$

$$= \frac{V^2 \cdot 2\pi \epsilon_0 \cdot a \cdot b}{a - b}$$



Two concentric
cylinders of length L

Shape = cylinder

Dimension = Area [2D]

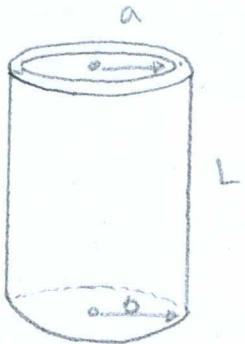
Charge = $-Q, +Q$

From Problem 1.6c, $\phi = -\frac{q}{2\pi L \cdot \epsilon_0} \ln \left(\frac{b}{a} \right) = V$

$$W = \frac{1}{2} \sum_{i=1}^n Q_i V_i$$

$$= \frac{Q \cdot Q}{4\pi L \epsilon_0} \ln \left(\frac{a}{b} \right)$$

$$= \frac{V^2 \pi \epsilon_0 L}{\ln(a/b)}$$



Parallel Cylinder Capacitor

Shape = cylinder

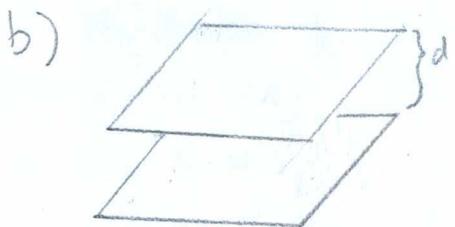
Dimension = Area [2D]

Charge : Fixed

Force = charge \cdot Electric Field Strength

$$\text{From Problem 1.6c, } E = \frac{q}{2\pi L \cdot r \cdot \epsilon_0} = \frac{V}{d}$$

$$F = q \cdot E \\ = -q \frac{V^2}{2\pi L \cdot r \cdot \epsilon_0}$$



Parallel Plate Conductor

Shape = square

Dimension = Area [2D]

Charge : Fixed

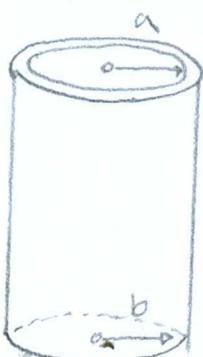
Force = charge \cdot Potential per distance

$$\text{From Problem 1.6a, } E = \frac{q}{\epsilon_0 \cdot a^2} = \frac{V}{d}$$

$$F_{\text{for}} = q \cdot \left(\frac{\phi}{L} \right) \\ = \frac{\epsilon_0 a^2 V^2}{d^2}$$

Force = charge \times Potential per distance

$$\text{From Problem 1.6c, } E = \frac{q}{\epsilon_0 \cdot \pi \cdot L} \ln\left(\frac{d}{a}\right) = \frac{V}{d}$$



Parallel Cylinder Capacitor

Shape = cylinder

Dimension = Area [2D]

Charge : Fixed

$$F_{\text{for}} = q \left(\frac{\phi}{L} \right) \\ = \frac{\epsilon_0 \pi L \cdot V^2}{\ln(d/a) \cdot d}$$

$$\approx -\frac{q}{\epsilon_0 \cdot 2\pi L} \ln\left(\frac{d^2}{a_1 \cdot a_2}\right)$$

$$\approx -\frac{q}{\epsilon_0 \cdot \pi \cdot L} \ln\left(\frac{d}{a}\right)$$

$$C = \frac{q}{V \cdot L}$$

$$\approx \pi \epsilon_0 \ln\left(\frac{d}{a}\right)^{-1}$$

Diameter (d)	0.5 cm	1.5 cm	5.0 cm
Width (a)	0.5 mm	1.5 mm	5.0 mm

When $\frac{C}{L} = 1.2 \times 10^{-11} \text{ F/m}$,

1.0,

a)



$$\text{From } 1.6a, E = \frac{q}{\epsilon_0 \cdot a^2} = \frac{V}{d}$$

Two large, flat
conducting sheets

$$(Equation 1.62) W = \frac{1}{2} Q \cdot V$$

$$= -\frac{Q \cdot Q \cdot d}{2 \epsilon_0 \cdot a^2}$$

- or -

$$= -\frac{V^2 a^2 \epsilon_0}{2 d}$$

Shape = Square
Dimension = Area [2D]
Charge = $-Q, +Q$

The book describes the coefficient is $\frac{1}{2}, \frac{1}{4}, \text{ or } \frac{1}{3}$ by half the amount, total potential or Dirac's symmetric derivation. No clue without measurement. Gauss' Law derives one-half the charges.

$$\frac{1}{E} \frac{\partial E}{\partial n \Delta d} = \frac{-(R_1 + R_2)}{(R_1 + n \Delta d)(R_2 + n \Delta d)}$$

$$\lim_{\Delta d \rightarrow 0} \frac{1}{E} \circ \frac{\partial E}{\partial n \Delta d} = \lim_{\Delta d \rightarrow 0} \frac{-(R_1 + R_2)}{(R_1 + n \Delta d)(R_2 + n \Delta d)}$$

$$= -\frac{1}{R_1} - \frac{1}{R_2}$$

(Green's Theorem)

$$\int_V (\phi \nabla^2 \phi - 4 \nabla^2 \phi) d^3x = \int_S [\phi \frac{\partial^2 \phi}{\partial n^2} - 4 \frac{\partial \phi}{\partial n}] da$$

"Work / Potential difference in a volume" = "Work / potential difference in surface area"

$$1.12. \quad \int_V (\phi \nabla^2 \phi - 4 \nabla^2 \phi) d^3x = \int_{SA} [\phi \frac{\partial^2 \phi}{\partial n^2} - 4 \frac{\partial \phi}{\partial n}] da$$

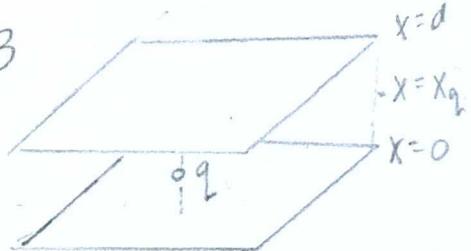
$$\int_V (\phi \frac{f}{\epsilon_0} - 4 \frac{f}{\epsilon_0}) d^3x = \int_{SA} [\phi \frac{\sigma}{\epsilon_0} - 4 \frac{\sigma}{\epsilon_0}] da$$

$$\int_V \phi \frac{f}{\epsilon_0} dv - \int_{SA} \phi \frac{\sigma}{\epsilon_0} da = \int_V 4 \frac{f}{\epsilon_0} dv - \int_{SA} 4 \frac{\sigma}{\epsilon_0} da$$

$$\int_V \rho \phi dv + \int_{SA} \sigma \phi da = \int_V 4 \rho dv + \int_{SA} 4 \sigma da$$

Note: Surface integrals are independent of parameters; signs change.

1.13 (Green's Theorem)



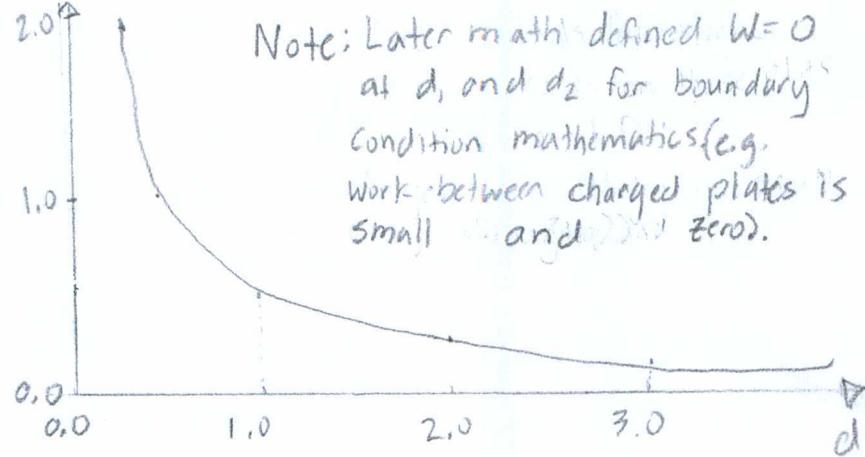
$$\int_V (\phi \nabla^2 \phi - 4 \nabla^2 \phi) dv = \int_S (\phi \frac{\partial^2 \phi}{\partial n^2} - 4 \frac{\partial \phi}{\partial n}) ds$$

Two infinitely grounded Parallel Plates

b) Conducting Sheets

Conducting sheet

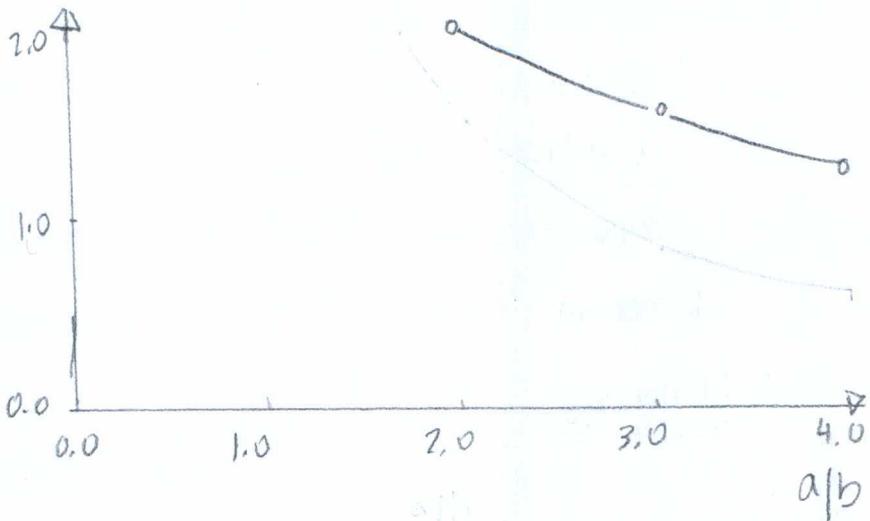
$$|W| = \left| \frac{V^2 a^2 \epsilon_0}{2d} \right|$$



Note: Later in math it's defined $W=0$ at d_1 and d_2 for boundary condition mathematics (e.g. work between charged plates is small and zero).

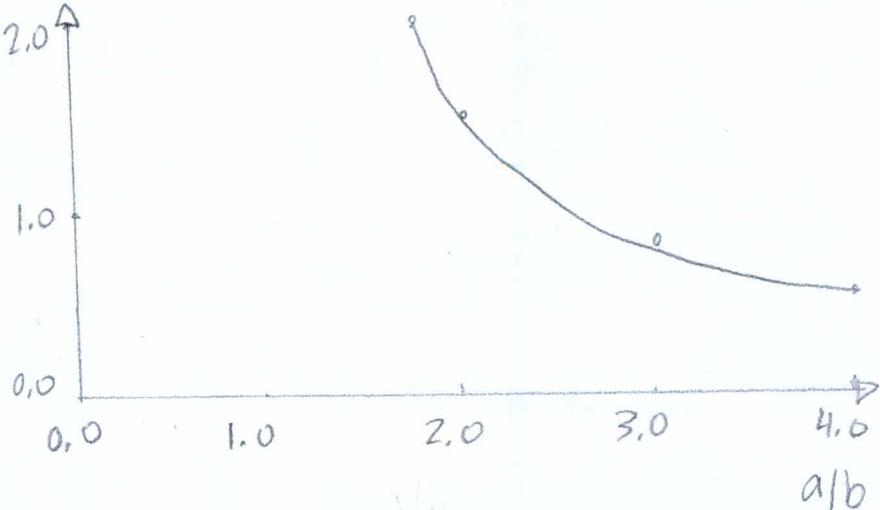
Concentric Spheres

$$|W| = \left| \frac{V^2 2\pi \epsilon_0 a \cdot b}{\ln(a/b)} \right|$$



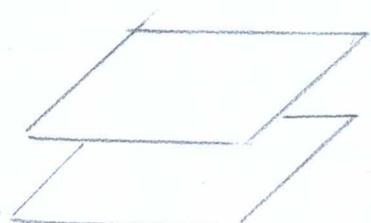
Concentric cylinders

$$|W| = \left| \frac{V^2 \pi \epsilon_0 \cdot L}{\ln(a/b)} \right|$$



1.9

a)



$$(Equation 1.17b) F = qE$$

$$\text{From Problem 1.6a, } E = \frac{q}{\epsilon_0 a^2} = \frac{V}{d}$$

Parallel plate
Capacitor

Shape = Square

Dimension = Area [2D]

Charge = Fixed

$$F = q \cdot E_{\text{tot}}$$

$$= q \cdot \frac{q}{\epsilon_0 a^2}$$

Ambiguous Terminology:

E represents both

1) Electric Field [V/m]

2) Electric Field Strength [V/m]

$$b) \quad \phi(x) = \langle \phi \rangle_s + \frac{1}{4\pi\epsilon_0} \int_V \rho(x') G_n(x, x') d^3x + \frac{1}{4\pi} \oint_S \frac{\partial \phi}{\partial n} G_n da$$

$$\phi(x) - \langle \phi \rangle_s = \frac{1}{4\pi} \oint_S \frac{\partial \phi}{\partial n} G_n da \text{ "Not symmetric."}$$

$$= \frac{1}{4\pi} \oint_S \frac{\partial \phi}{\partial n} G(x, y) da - \frac{1}{4\pi} \oint_S \frac{\partial \phi}{\partial n} G(y, x) da \text{ "symmetric"}$$

$$c) \quad \phi(x) - \langle \phi \rangle_s = \frac{1}{4\pi\epsilon_0} \int_V \rho(x') G_n d^3x + \frac{1}{4\pi} \oint_S (G_n \frac{\partial \phi}{\partial n}) da,$$

$$- \frac{1}{4\pi\epsilon_0} \int_V \rho(x') F(x) d^3x + \frac{1}{4\pi} \oint_S F(x) \frac{\partial \phi}{\partial n} da$$

$$= \phi(x) + \frac{1}{4\pi} F(x) \left[\frac{1}{\epsilon_0} \int_V \rho(x) d^3x + \oint_S \frac{\partial \phi}{\partial n} da \right]$$

$$= \phi(x) + \frac{1}{4\pi} F(x) \left[\oint_S E \cdot n da + \oint_S \left(\frac{\partial \phi}{\partial n} \right) da \right]$$

$$= \phi(x) + \frac{1}{4\pi} F(x) \left[- \oint_S \left(\frac{\partial \phi}{\partial n} \right) da + \oint_S \left(\frac{\partial \phi}{\partial n} \right) da \right]$$

$$\phi(x) = \langle \phi \rangle_s$$

$$1.15. \quad U = \sum_{n=1}^N \left\{ \frac{1}{2} \int \rho \phi dV_{int}^{(n)} + \frac{1}{2} \int \sigma \phi dS^{(n)} + \lambda \left(Q^{(n)} - \int \rho dV_{int}^{(n)} - \int \sigma dS^{(n)} \right) \right\}$$

$$\frac{\partial U}{\partial r} = \sum_{n=1}^N \left\{ \frac{1}{2} \int \rho' (\phi - \lambda^{(n)} + 4\pi \alpha^{(n)}) + \phi' (\rho + \nabla^2 \alpha^{(n)}) dV_{int}^{(n)} \right.$$

$$\left. + \frac{1}{2} \int \sigma' (\phi - \lambda^{(n)} + 4\pi \beta^{(n)}) + \phi' (\sigma - \hat{n} \cdot (\nabla \beta^{(n)} - \nabla^- \beta^{(n)})) dS^{(n)} \right)$$

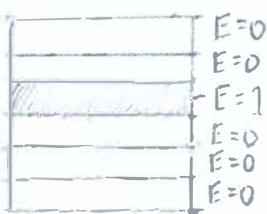
$$= 0$$

$\phi = \lambda^{(n)} - 4\pi \alpha^{(n)}$ Where α -term: Internal charge Density
 β -term: Surface charge Interaction
 $\nabla^2 \phi = -4\pi \nabla^2 \alpha$ Also, Lagrange multipliers of unknown value

$$\begin{aligned}
 &= \frac{Q_1}{4\pi\epsilon_0} \left(\frac{1}{r_1^2} - \frac{1}{r_7^2} \right) + \frac{Q_2}{4\pi\epsilon_0} \left(\frac{1}{r_2^2} - \frac{1}{r_9^2} \right) \\
 &+ \frac{Q_3}{4\pi\epsilon_0} \left(\frac{1}{r_3^2} - \frac{1}{r_{10}^2} \right) + \frac{(Q_1 - Q_2)}{4\pi\epsilon_0 r_5^2} + \frac{(Q_1 - Q_3)}{4\pi\epsilon_0 r_6^2} \\
 &+ \frac{(Q_2 - Q_3)}{4\pi\epsilon_0 r_8^2}
 \end{aligned}$$

The electrostatic potential for one insulator in the presence of other conductors is lower by the distance to the conductor.

1.17.



Several Separate Conducting Surfaces

Shape = Layers

Dimension = Surface

Area [2D]

Charge = ??

$$a) \text{ (Equation 1.54)} \quad W = \frac{\epsilon_0}{2} \int |E|^2 d^3x$$

$$(Equation 1.62) \quad W = \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n C_{ir} V_i V_j$$

$$W = \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n C_{ir} V_i V_j = \frac{\epsilon_0}{2} \int |E|^2 d^3x$$

$$C \leq \epsilon_0 \int |E|^2 d^3x$$

$$\leq \epsilon_0 \int |\nabla \phi|^2 d^3x$$

$$b) W = \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n C_{ir} V_i V_j = \frac{\epsilon_0}{2} \int |E|^2 d^3x$$

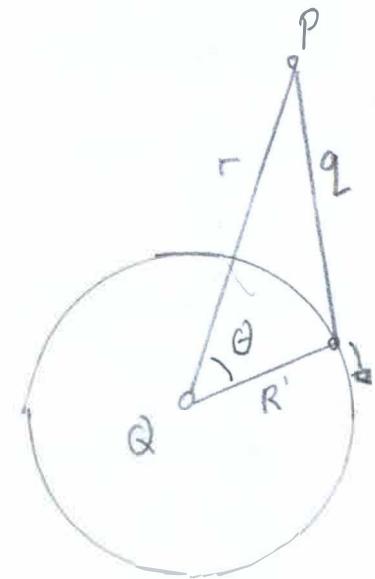
$$C[4r] = \frac{\epsilon_0 \int |E|^2 d^3x}{V^2}$$

$$\leq \epsilon_0 \int |\nabla \phi|^2 d^3x$$

$$1.10 \quad \bar{\Phi} = \frac{1}{4\pi R^2} \int_{SA} \bar{\Phi} dS = \frac{1}{4\pi R^2} \int_0^{2\pi} \int_0^\pi \bar{\Phi} r^2 \sin\theta d\theta d\phi$$

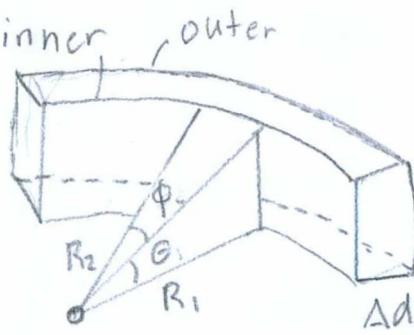
$$= \bar{\Phi}_{\text{central}}$$

Note: Other averages derive a 2D model about a circle's charge from the law of cosines as a representation for a sphere's charge. A similar 3D derivation needs a 3D law of cosines.



2D-Example With q's average length around the circle, as average surface charge.

1.11.



$$\int_{\text{Outer}} E \cdot dS = \int_{\text{Inner}} E \cdot dS$$

$$E \int_{\text{Outer}} dS = E \int_{\text{Inner}} dS$$

$$E(R_1 + n\Delta d)(R_2 + n\Delta d) \int_0^{2\pi} \int_0^\pi d\theta d\phi = E \cdot R_1 R_2 \int_0^{2\pi} \int_0^\pi d\theta d\phi$$

$$E(R_1 + n\Delta d)(R_2 + n\Delta d) = E \cdot R_1 R_2$$

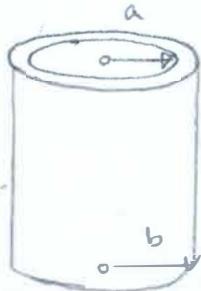
$$E = \frac{ER_1 R_2}{(R_1 + n\Delta d)(R_2 + n\Delta d)}$$

$$\frac{\partial E}{\partial n\Delta d} = \frac{-R_1 - R_2 - n\Delta d}{(R_1 + n\Delta d)(R_2 + n\Delta d)} E$$

$$= \frac{\int da \int da' \sigma(x) G(x, x') \sigma(x')}{4\pi\epsilon_0 [\int \sigma(x) da]^2}$$

The variational principle above is a lower bound because the equality - "?" Jackson delineates many work derivations by $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}$, and $\frac{1}{16}$. Reason's seemed arbitrary, such as half the distance, total potential or Dirac's symmetry. Other possible reasons include Green's theorem's without symmetry.

1.19.



Two concentric cylinders of Length L

Shape = cylinder

Dimensions = Area [2D]

Charge: Fixed

$$\psi_i(p) = \frac{(b-p)}{(b-a)}$$

$$C[\psi] = \epsilon_0 \int_a^b |\nabla \psi|^2 d^3x$$

$$= \epsilon_0 \int_a^b \frac{1}{(b-a)^2} d^3x$$

$$= \frac{2\pi L \epsilon_0}{(b-a)^2} \int_a^b p dp$$

$$= \frac{\pi L \epsilon_0 (b^2 - a^2)}{(b-a)^2}$$

$$= \frac{\pi L \epsilon_0 (b+a)}{(b-a)}$$

$$= \frac{\pi L \epsilon_0 (b/a + 1)}{(b/a - 1)}$$

$$0 + \frac{4}{\epsilon_0} q = \frac{1}{\epsilon_0} \sigma_1 - \frac{1}{\epsilon_0} \sigma_3$$

$$[x_0 + (1-x_0)] \frac{q}{\epsilon_0} = 0$$

$$(1-x_0) q = -x_0 q$$

$$q = \frac{-x_0}{1-x_0} q$$

"From Green's reciprocity theorem, the charge never changes between two plates."

(.14.

a) For a Dirichlet Boundary condition,

$$\int_V (\phi \nabla^2 \phi - 4 \nabla^2 \phi) d^3x = \int_S [\phi \frac{\partial \phi}{\partial n} - 4 \frac{\partial \phi}{\partial n}] da$$

$$\int_V [\phi (-4\pi \delta(x-x')) + \frac{1}{\epsilon_0 R} p(x')] d^3x = \int_S [\phi(\frac{1}{R}) + (\frac{1}{R}) \frac{\partial \phi}{\partial n}] da$$

$$\phi(x) = \frac{1}{4\pi\epsilon_0} \int_V p(x) G(x, x') d^3x + \frac{1}{4\pi} \underbrace{\int_S [G(x, x') \frac{\partial \phi}{\partial n} - \phi(x) \frac{\partial G(x, x')}{\partial n}]}_{=0 \text{ because Dirichlet's Boundary.}} da$$

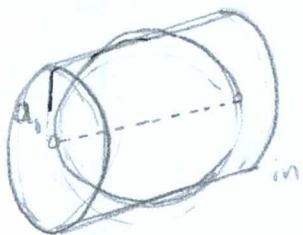
$$= \frac{1}{4\pi\epsilon_0} \int_V p(x) G(x, x') d^3x - \frac{1}{4\pi} \int_S \phi(x) \frac{\partial G(x, x')}{\partial n} da$$

$$\phi(x) - \bar{\phi}(x') = \frac{1}{4\pi\epsilon_0} \int_V p(x) G(x, x') d^3x - \frac{1}{4\pi} \int_S \phi(x) \frac{\partial G(x, x')}{\partial n} da$$

$$- \frac{1}{4\pi\epsilon_0} \int_V p(x) G(x, x') d^3x + \frac{1}{4\pi} \int_S \phi(x) \frac{\partial G(x, x')}{\partial n} da$$

$$= 0$$

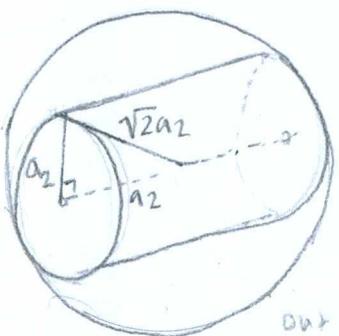
$\phi(x) = \phi(x')$ "Symmetric"



d

c) Lower bound

$$\pi \epsilon_0 C_{in}^{-1} \leq C_{Avg} \leq C_{out}$$



Two long cylindrical
conductors

Shape = cylinder

Dimension = Surface [2D]

Charge = q

Average Upper Bound

$$\pi \epsilon_0 C_{in}^{-1} \leq C_{Avg} \leq C_{out}$$

$$\pi \epsilon_0 \ln\left(\frac{d}{\sqrt{a_1 a_2}}\right)^{-1} \leq C_{Avg} \leq \pi \epsilon_0 \ln\left(\frac{d}{\sqrt[4]{a_1 a_2}}\right)^{-1}$$

$$C_{Avg} = \frac{\pi \epsilon_0}{2} \left[\ln\left(\frac{d}{\sqrt{a_1 a_2}}\right)^{-1} + \ln\left(\frac{d}{\sqrt[4]{a_1 a_2}}\right)^{-1} \right]$$

$$= \frac{\pi \epsilon_0}{2} \left[\ln\left(\frac{d}{a}\right)^{-1} + \ln\left(\frac{d}{\sqrt{a}}\right)^{-1} \right]$$

$$= 1.05 \cdot C_{in}$$

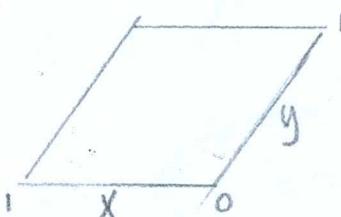
≈ 5% change to average

$$C_{Avg} \approx 0.95 \cdot C_{out}$$

≈ 5% change to upper bound

A 10% whopping change.

1.21.



Unit square Area

a) (Equation 1.63)

$$I[\psi] = \frac{1}{2} \int \nabla \psi \cdot \nabla \psi d^3x - \int g \psi d^3x$$

$$\psi(x,y) = A \cdot x(1-x)y(1-y)$$

$$\nabla \psi(x,y) = A [(1-2x)y(1-y) + x(1-x)(1-2y)]$$

$$I[\psi] = \frac{1}{2} \int_0^1 \int_0^1 \left[A^2 \left[(-2x)^2 y^2 (1-y)^2 + x^2 (1-x)^2 (1-2y)^2 \right] \right] dx dy$$

$$- \int_0^1 \int_0^1 g A x(1-x)y(1-y) dx dy$$

Shape: Square

Dimension: Area

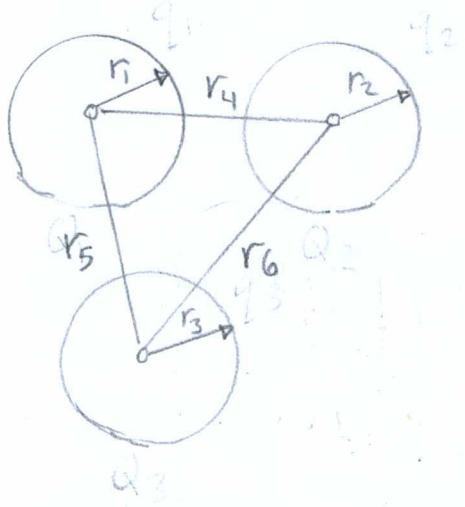
charge: q

$\rho = -\nabla^2 \Phi^{(n)}$ "Each electrostatic energy is exactly equipotential."

1.16. (Equation 1.16) $E = -\nabla \Phi$

(Equation 1.43)

$$\Phi(x_i) = \frac{1}{4\pi\epsilon_0} \sum_{j=1}^{n-1} \frac{-q_j}{|x_i - x_j|}$$



Shape: A number of conducting surfaces

Dimension: Surface Area [2D]

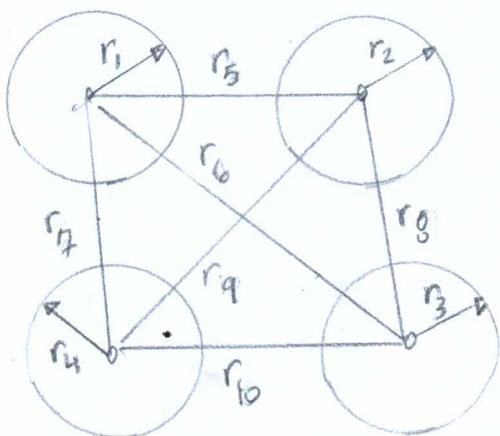
Charge: Q_1, Q_2, Q_3

$$E = -\nabla \Phi$$

$$= \frac{1}{4\pi\epsilon_0} \sum_{j=1}^{n-1} \frac{-q_j}{|x_i - x_j|^2}$$

$$= \frac{Q_1}{4\pi\epsilon_0 r_1^2} + \frac{Q_2}{4\pi\epsilon_0 r_2^2} + \frac{Q_3}{4\pi\epsilon_0 r_3^2}$$

$$+ \frac{(Q_1 - Q_2)}{4\pi\epsilon_0 r_4^2} - \frac{(Q_1 - Q_3)}{4\pi\epsilon_0 r_5^2} - \frac{(Q_2 - Q_3)}{4\pi\epsilon_0 r_6^2}$$



Shape: A number of conducting spheres with an insulator

Dimension: Surface Area [2D]

Charge: $Q_1, Q_2, Q_3, Q_4 = 0$

$$E = -\nabla \Phi$$

$$= \frac{1}{4\pi\epsilon_0} \sum_{j=1}^{n-1} \frac{-q_j}{|x_i - x_j|^2}$$

$$= \frac{Q_1}{4\pi\epsilon_0 r_1^2} + \frac{Q_2}{4\pi\epsilon_0 r_2^2} + \frac{Q_3}{4\pi\epsilon_0 r_3^2} + \frac{Q_4}{4\pi\epsilon_0 r_4^2}$$

$$+ \frac{(Q_1 - Q_2)}{4\pi\epsilon_0 r_5^2} - \frac{(Q_1 - Q_3)}{4\pi\epsilon_0 r_6^2} - \frac{(Q_1 - Q_4)}{4\pi\epsilon_0 r_7^2} + \frac{(Q_2 - Q_3)}{4\pi\epsilon_0 r_8^2}$$

$$+ \frac{(Q_2 - Q_4)}{4\pi\epsilon_0 r_9^2} - \frac{(Q_3 - Q_4)}{4\pi\epsilon_0 r_{10}^2}$$

1.22.

$$\text{a) } F(h, 0) = F(0, 0) + h \cdot \nabla F_x + \frac{h^2 \nabla^2 F_x}{2!} - \frac{h^3 \nabla^3 F_x}{3!} + \frac{h^4 \nabla^4 F_x}{4!} + \dots$$

$$F(-h, 0) = F(0, 0) - h \cdot \nabla F_x + \frac{h^2 \nabla^2 F_x}{2!} - \frac{h^3 \nabla^3 F_x}{3!} + \frac{h^4 \nabla^4 F_x}{4!} + \dots$$

$$F(0, h) = F(0, 0) + h \cdot \nabla F_y + \frac{h^2 \nabla^2 F_y}{2!} - \frac{h^3 \nabla^3 F_y}{3!} + \frac{h^4 \nabla^4 F_y}{4!} + \dots$$

$$F(0, -h) = F(0, 0) - h \cdot \nabla F_y + \frac{h^2 \nabla^2 F_y}{2!} - \frac{h^3 \nabla^3 F_y}{3!} + \frac{h^4 \nabla^4 F_y}{4!} + \dots$$

$$S_C = F(h, 0) + F(0, h) + F(-h, 0) + F(0, -h)$$

$$= 4F(0, 0) + h^2 \nabla^2 F + \frac{h^4}{12} (F_{xxxx} + F_{yyyy}) + O(h^6)$$

Why? Maclaurin Series : $F(x) = \sum_{i=1}^n \frac{d^i F(0)}{d^i x} \frac{x^i}{i!}$

$$\text{b) } F(h, h) = F(0, 0) + h(\nabla_x F + \nabla_y F) + \frac{h^2 (\nabla_x F + \nabla_y F)^2}{2!} + \frac{h^3 (\nabla_x F + \nabla_y F)^3}{3!} + \dots$$

$$+ \frac{h^4 (\nabla_x F + \nabla_y F)^4}{4!} + \dots$$

$$F(-h, h) = F(0, 0) + h(-\nabla_x F + \nabla_y F) + h^2 (-\nabla_x F + \nabla_y F)^2 + h^3 (-\nabla_x F + \nabla_y F)^3 + \dots$$

$$+ \frac{h^4 (-\nabla_x F + \nabla_y F)^4}{4!} + \dots$$

$$F(h, -h) = F(0, 0) + h(\nabla_x F - \nabla_y F) + \frac{h^2 (\nabla_x F - \nabla_y F)^2}{2!} + \frac{h^3 (\nabla_x F - \nabla_y F)^3}{3!} + \dots$$

$$+ \frac{h^4 (\nabla_x F - \nabla_y F)^4}{4!} + \dots$$

$$F(-h, -h) = F(0, 0) - h(\nabla_x F + \nabla_y F) + \frac{h^2 (\nabla_x F + \nabla_y F)^2}{2!} - \frac{h^3 (\nabla_x F + \nabla_y F)^3}{3!} + \dots$$

1.18

	$F=0$
	$E=0$
	$E=0$
	$E=0$
	$F=0$
	$E=0$

a) (Equation 1.49) $\phi(x_i) = \frac{1}{4\pi\epsilon_0} \sum_{j=1}^{n-1} \frac{-q}{|x_i - x_j|}$

$$= \frac{1}{4\pi\epsilon_0} \int_{S_1} p(x) G_D(x, x') d^3x$$

$$= \frac{1}{4\pi\epsilon_0} \int_{S_1} \sigma_i(x) G(x, x') d^3x$$

Several Separate
Conducting Surfaces

Shape = Layers

Dimension = Surface Area

Charge = q

(Equation 1.52) $W = \frac{1}{8\pi\epsilon_0} \iint \frac{p(x)p(x')}{|x - x'|} d^3x d^3x'$

$$= \frac{1}{8\pi\epsilon_0} \iint \frac{\sigma_i(x)\sigma_i(x')}{|x - x'|} da da$$

$$= \frac{1}{8\pi\epsilon_0} \int_{S_1} da \int_{S_1} da' \sigma_i(x) G(x, x') \sigma_i(x')$$

b) (Equation 1.47)

$$W = q_i \phi(x) \stackrel{?}{=} \frac{1}{2} q \phi(x)$$

$$= \frac{1}{2} q \cdot V$$

$$= \frac{1}{2} q \left(\frac{-q}{C} \right)$$

$$\frac{1}{C} = \frac{2W}{q^2}$$

$$\frac{1}{C} = \frac{2}{4\pi\epsilon_0} \int_{S_1} da \int_{S_1} da' \sigma_i(x) G(x, x') \sigma_i(x')$$

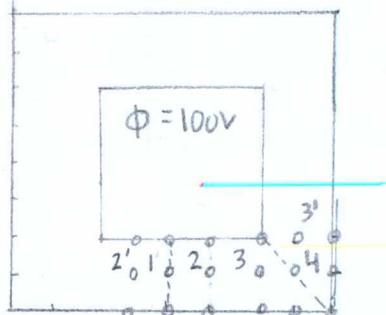
$$\left[\int_{S_1} \sigma(x) da \right]^2$$

$$+ \frac{h^4 (\nabla_x F + \nabla_y F)^4}{4!} + \dots$$

$$S_s = F(h, h) + F(-h, h) + F(h, -h) + F(-h, -h)$$

$$= 4F(0,0) + 2h^2 \nabla^2 F - \frac{h^4}{3} (\nabla_x^4 F + \nabla_y^4 F) + \frac{h^4}{2} \nabla^4 F$$

1.23



$$\Phi_0 = 0V$$

Hollow Transmission
Line with a square
conductor

$$a) 4_1 = \frac{1}{4} [4_{21} + \Phi_0 + 4_2 + \Phi]$$

$$= \frac{1}{4} [24_2 + \Phi]$$

$$4_2 = \frac{1}{4} [4_1 + 4_0 + 4_3 + \Phi]$$

$$= \frac{1}{4} [4_1 + 4_3 + \Phi]$$

$$4_3 = \frac{1}{4} [4_1 + \Phi_0 + 4_4 + \Phi]$$

$$= \frac{1}{4} [4_1 + 4_4 + \Phi]$$

$$4_4 = \frac{1}{4} [4_3 + \Phi_0 + \Phi_0 + 4_{31}]$$

$$= \frac{1}{4} [24_3]$$

$$\begin{bmatrix} 0 & 2 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 4_1 \\ 4_2 \\ 4_3 \\ 4_4 \\ \Phi \end{bmatrix} = \begin{bmatrix} 44_1 \\ 44_2 \\ 44_3 \\ 44_4 \\ 100 \end{bmatrix}$$

$$4_1 = 49.93V, 4_2 = 47.97V, 4_3 = 42.55V, 4_4 = 21.29V$$

$$b) \frac{q}{\epsilon_0} = \nabla \phi \cdot d\ell$$

$$q = \epsilon_0 \nabla \phi \cdot d\ell$$

$$= \epsilon_0 \frac{\Delta \phi}{\Delta y} \Delta x$$

$$= \epsilon_0 \Delta \phi$$

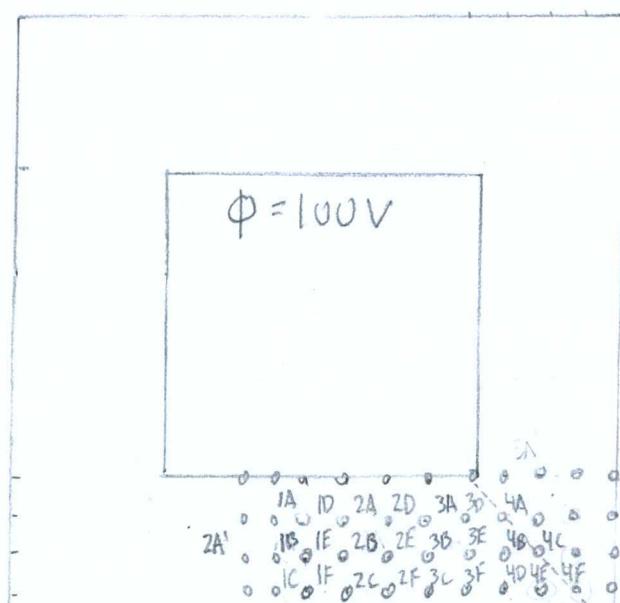
$$= \epsilon_0 [-4_2 - 4_1 - 4_2 - 4_3 - 4_4 + \phi + \phi + \phi + \phi + \phi]$$

$$= \epsilon_0 (270 \text{ C/m})$$

$$C = \frac{4Q}{\phi} = \frac{\epsilon_0 (1081.9 \text{ C/m})}{100V}$$

$$= 10.81 \text{ F/m}$$

c)



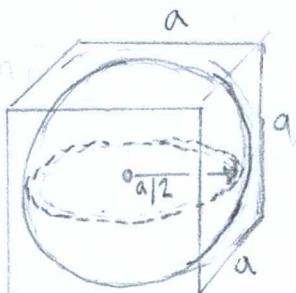
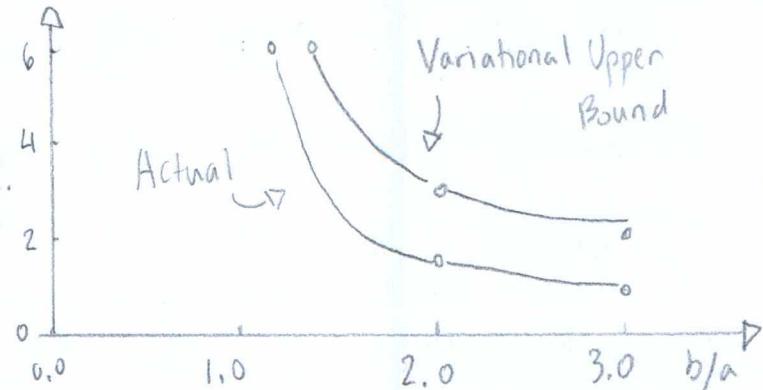
$$\phi = 0V$$

Book suggests 21 interior points.
For the relaxation equation,
This model has 24 interior
points with no solution; although
a personal record in matrix
size.

Greater Resolution Hollow
Transmission Line with a
Square conductor at center

Type	Equation	$\frac{b}{a} = 1.5$	$\frac{b}{a} = 2.0$	$\frac{b}{a} = 3.0$
Variational	$C = \frac{\pi \cdot L \cdot \epsilon_0 (b/a + 1)}{(b/a - 1)}$	$5\pi L \epsilon_0$	$3\pi L \epsilon_0$	$2\pi L \epsilon_0$
Actual	$C = \frac{\pi \cdot L \cdot \epsilon_0}{lh(b/a)}$	$\frac{5\pi L \epsilon_0}{2}$	$\frac{29\pi L \epsilon_0}{20}$	$\frac{9\pi L \epsilon_0}{10}$

A graph provided better
due diligence about C.
Variational upper bound.

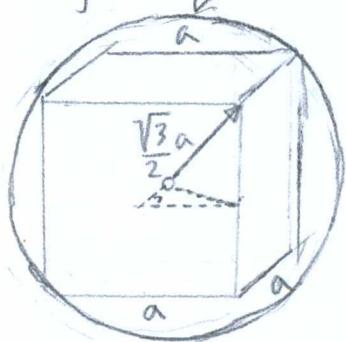


Configuration of
Capacitors Within
closed Surface

Shape = Sphere

Dimension = Surface[2D]

Charge = q



Configuration of
Capacitors Within
a shell.

a) $C' \leq C$

$$\epsilon_0 \int_V |\nabla V|^2 d^3x \leq \epsilon_0 \int_V |\nabla V'|^2 d^3x$$

b) Lower bound

Average Upper bound.

$$\frac{\epsilon_0}{2} \int_V |\nabla V|^2 d^3x \leq C_{Avg} \leq \frac{\sqrt{3}\epsilon_0}{2} \int_V |\nabla V'|^2 d^3x$$

$$\frac{4\pi \epsilon_0 a}{2} \leq C_{Avg} \leq \frac{\sqrt{3} 4\pi \epsilon_0 a}{2}$$

$$C_{Avg} = \frac{\left[\frac{1}{2} + \frac{\sqrt{3}}{2}\right]}{2} 4\pi \epsilon_0 a$$

$$= 0.683 \times (4\pi \epsilon_0 a)$$

$$= 0.0111 A^2 \times -\frac{gA}{36}$$

$$\underset{A}{\operatorname{argmin}} \{ I[4f] \} = \underset{A}{\operatorname{argmin}} \left\{ 0.0111 A^2 \cdot -\frac{gA}{36} \right\}$$

$$A^* = 1.25g$$

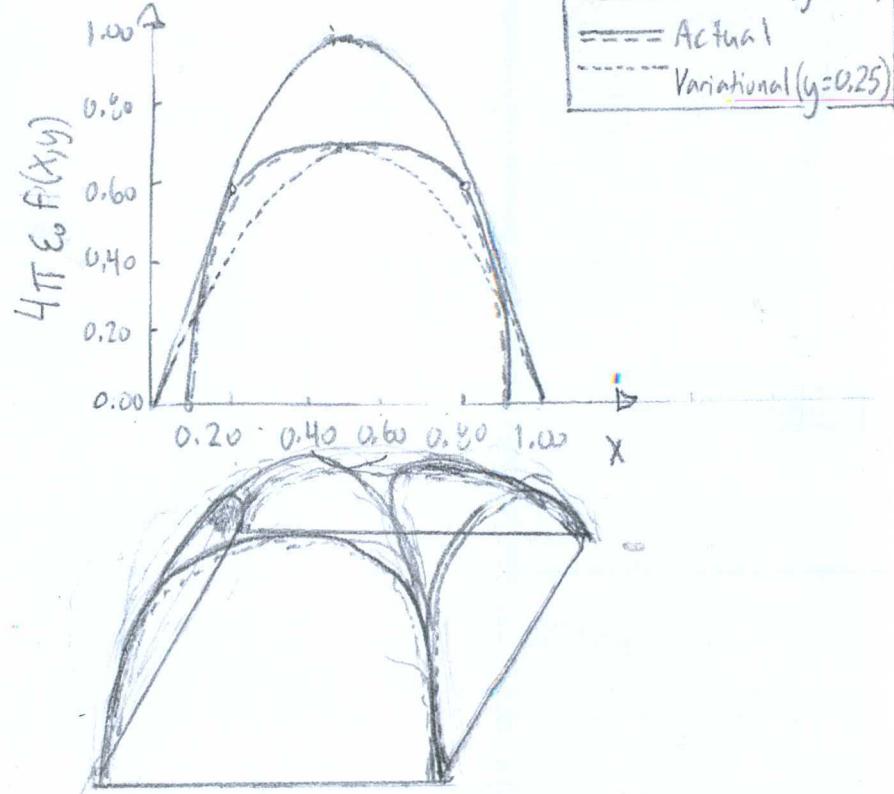
$$4f(x,y) = 1.25g \times (1-x)y(1-y)$$

b) Actual Equation:

$$4\pi\epsilon_0 \Phi(x,y) = \frac{16}{\pi^2} \sum_{m=0}^{\infty} \frac{\sin[(2m+1)\pi x]}{(2m+1)^3} \left\{ 1 - \frac{\cosh[(2m+1)\pi(y-1/2)]}{\cosh[(2m+1)\pi/2]} \right\}$$

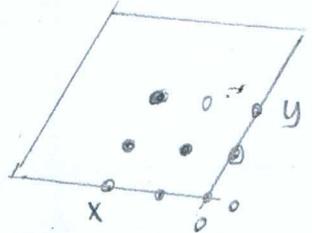
Variational Equation:

$$4\pi\epsilon_0 4f(x,y) = 5x(1-x)y(1-y)$$



"Graph represents potential gradient above a square"

1.24



Unit Square

Shape = Square

Dimension = Area [2D]

Charge = q

a) From Problem 1.21(b),

$$4\pi\epsilon_0 \Phi(x, y) = \frac{16}{\pi^2} \sum_{m=0}^n \frac{\sin[(2m+1)\pi x]}{(2m+1)^3} \left\{ \begin{array}{l} 1 - \frac{\cosh((2m+1)\pi(y-x))}{\cosh((2m+1)\pi/2)} \\ \end{array} \right.$$

Actual Function		Iteration (#)					
X	y	0	1	2	3	4	5
0.25	0.25	1.00	0.73	0.72	0.72	0.72	0.72
0.25	0.50	1.00	0.73	0.72	0.72	0.72	0.72
0.50	0.50	1.00	0.92	0.93	0.92	0.92	0.93

b)

Jacobi Method		Iteration (#)					
X	y	0	1	2	3	4	5
0.25	0.25	1.00	0.68	0.47	0.93	0.47	0.57
0.25	0.50	1.00	0.83	0.83	0.83	0.83	0.83
0.50	0.50	1.00	0.92	0.94	0.93	0.94	0.93

b)

Gauss-Siedel		Iteration (#)					
X	y	0	1	2	3	4	5
0.25	0.25	1.00	0.68	0.72	0.71	0.72	0.72
0.25	0.50	1.00	0.68	0.73	0.72	0.72	0.72
0.50	0.50	1.00	0.97	0.91	0.92	0.92	0.93

\times = Jacobi
 \circ = Gauss-Siedel
* = Actual
--- = Experimental

