Chapter 1: Newton's Laws of Motion:

1.1 $\vec{b} = \hat{x} + \hat{y}$; $\vec{c} = \hat{x} + \hat{z}$

$$\vec{b} + \vec{c} = 2\hat{x} + \hat{y} + \hat{z}$$

$$5\vec{b} + 2\vec{c} = 7\hat{x} + 5\hat{y} + 2\hat{z}$$

$$\vec{b} \circ \vec{c} = 1$$

$$\vec{b} \times \vec{c} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{vmatrix} = \hat{x} - \hat{y} + \hat{z}$$

Dot Product

$A = \langle a_1, b_1, c_1 \rangle$

$B = \langle a_2, b_2, c_2 \rangle$

$A \circ B = a_1 \cdot a_2 + b_1 \cdot b_2 + c_1 \cdot c_2$

Cross Product

$$A \times B = \begin{vmatrix} x & y & z \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

Angle between vectors

$A \circ B = |A||B| \cos\theta$

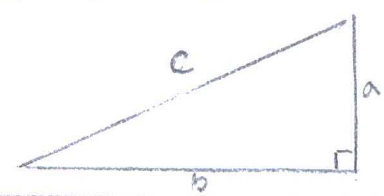
1.2 If $\vec{b} = (1,2,3)$ and $\vec{c} = (3,2,1)$

then $\vec{b} \circ \vec{c} = 1 \cdot 3 + 2 \cdot 2 + 3 \cdot 1$
$= 10$

$$\vec{b} \circ \vec{c} = |b||c| \cos\theta$$
$$= \sqrt{1^2 + 2^2 + 3^2} \sqrt{3^2 + 2^2 + 1^2} \cdot \cos\theta$$
$$\theta = 19°$$

Pythagoras Law

$a^2 + b^2 = c^2$



1.3 If $\vec{r} = x + y + z$

then $\vec{r} \circ \vec{r} = |r||r| \cos\theta$

$$= \sqrt{x^2 + y^2 + z^2} \sqrt{x^2 + y^2 + z^2} \cos 0°$$
$$= x^2 + y^2 + z^2$$

1.4 If $b = (1,2,4)$ and $c = (4,2,1)$

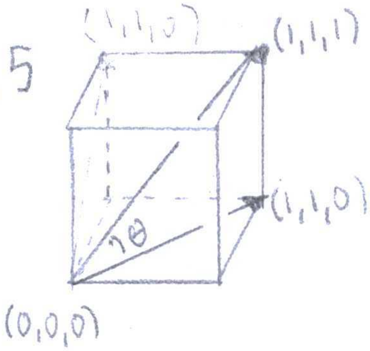
then $\vec{b} \circ \vec{c} = 1 \cdot 4 + 2 \cdot 2 + 4 \cdot 1$
$= 12$

$$\vec{b} \circ \vec{c} = |b||c| \cos\theta$$

$$= \sqrt{1^2+2^2+4^2}\sqrt{4^2+2^2+1^2}\cos\theta$$

$$\theta = 55°$$

1.5



If $\vec{r_1} = \langle 1,1,1\rangle$ and $\vec{r_2} = \langle 1,1,0\rangle$

then $\vec{r_1}\cdot\vec{r_2} = 1\cdot1 + 1\cdot1 + 1\cdot0$

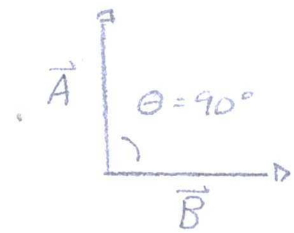
$$= 2$$

$$\vec{r_1}\cdot\vec{r_2} = \sqrt{1^2+1^2+1^2}\sqrt{1^2+1^2+0^2}\cdot\cos\theta$$

$$\theta = 35°$$

Orthoganol Vectors:

$$\vec{A}\cdot\vec{B} = 0$$



1.6 If $\vec{b} = \hat{x} + s\hat{y}$ and $\vec{c} = \hat{x} - s\hat{y}$

then $b\cdot c = 1\cdot1 - s\cdot s$

$$= 1 - s^2$$

$$b\cdot c = \sqrt{1+s^2}\sqrt{1+s^2}\cos\theta$$

$$s = 1$$

1.7 (Equation 1.6) "Scalar product"

$$\vec{r}\cdot\vec{s} = |r||s|\cos\theta$$

(Equation 1.7) "Dot Product"

$$\vec{r}\cdot\vec{s} = \ldots = \sum_{n=1}^{3} r_n s_n$$

Proof by induction:

Base case (1-dimension vector):

$$r \circ s = \sqrt{\sum_{n=1}^{1} n^2}\sqrt{\sum_{n=1}^{1} n^2}\cos\theta$$

$$= \sum_{n=1}^{1} n^2 \qquad \text{at } \theta = 0^\circ$$

Inductive Hypothesis (k-dimensional vector):

$$r \circ s = \sqrt{\sum_{n=1}^{k} n^2}\sqrt{\sum_{n=1}^{k} n^2}\cos\theta$$

$$= \sum_{n=1}^{k} n^2$$

Inductive Step ((k+1)-dimensional vector):

$$r \circ s = \sqrt{\sum_{n=1}^{k+1} n^2} \cdot \sqrt{\sum_{n=1}^{k+1} n^2} \cdot \cos\theta$$

$$= \sum_{n=1}^{k+1} n^2 \qquad \text{at } \theta = 0^\circ$$

1.8 a) (Equation 1.7) "Dot Product"

$$r \circ s = r_1 s_1 + r_2 s_2 + r_3 s_3 = \sum_{n=1}^{3} r_n \cdot s_n$$

$$\vec{r} \circ (\vec{u} + \vec{v}) = \vec{r} \circ \vec{s} \quad \text{when} \quad s = (u + v)$$

$$= r_1 s_1 + r_2 s_2 + r_3 s_3$$

$$= r_1(u_1 + v_1) + r_2(u_2 + v_2) + r_3(u_3 + v_3)$$

$$= r_1 u_1 + r_2 u_2 + r_3 u_3 + r_1 v_1 + r_2 v_2 + r_3 v_3$$

$$= \overline{r} \circ \overline{u} + \overline{r} \circ \overline{V}$$

b) $\frac{d}{dt}(\vec{r} \circ \vec{s}) = \frac{d}{dt} \sum_{i=1}^{n=3} r_i \cdot s_i$

$$= \frac{d}{dt}\left(r_1 s_1 + r_2 \circ s_2 + r_3 s_3\right)$$

$$= \frac{d\vec{r}}{dt}(s_1 + s_2 + s_3) + (r_1 + r_2 + r_3)\frac{d\vec{s}}{dt}$$

$$= \frac{d\vec{r}}{dt}\vec{s} + \vec{r}\frac{d\vec{s}}{dt}$$

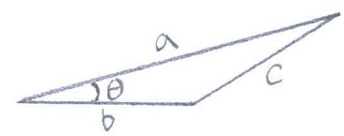
1.9 $(\vec{a}+\vec{b})^2 = \vec{a} \circ \vec{a} + 2\vec{a} \cdot \vec{b} + \vec{b} \circ \vec{b}$

$$= a^2 + b^2 + 2a \cdot b$$

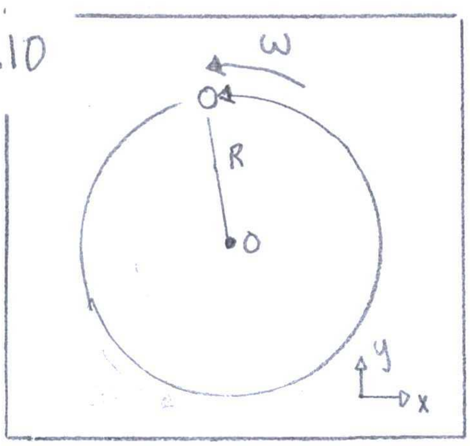
Law of cosines when $\theta = 0°$

Law of Cosines:

$$c^2 = a^2 + b^2 - 2ab\cos\theta$$



1.10



"Particle moves in a circle"

Components:

$\vec{r} = A\hat{x} + B\hat{y}$

where $A = R\cos(\omega t)$

$B = R\sin(\omega t)$

Time-dependent vector:

$$\vec{r}(t) = R\cos(\omega t)\hat{x} + R\sin(\omega t)\hat{y}$$

Velocity:

$$\dot{\vec{r}}(t) = \frac{d}{dt}\left(R\cos(\omega t)\hat{x} + R\sin(\omega t)\hat{y}\right)$$

$$= -R \cdot \omega \sin(\omega t)\hat{x} + R\omega\cos(\omega t)\hat{y}$$

Acceleration:

$$\ddot{\vec{r}}(t) = \frac{d}{dt}\left(-R\omega\sin(\omega t)\hat{x} + R\omega\cos(\omega t)\hat{y}\right)$$

$$= -R\omega^2\cos(\omega t)\hat{x} - R\omega^2\sin(\omega t)\hat{y}$$

Magnitude:

$$|\ddot{\vec{r}}(t)| = \sqrt{(-R\omega^2\cos(\omega t))^2 + (-R\omega^2\sin(\omega t))^2}$$

$$= R\cdot\omega^2$$

Trigonometric Identity:
$\cos^2(\theta) + \sin^2(\theta) = 1$

Direction:

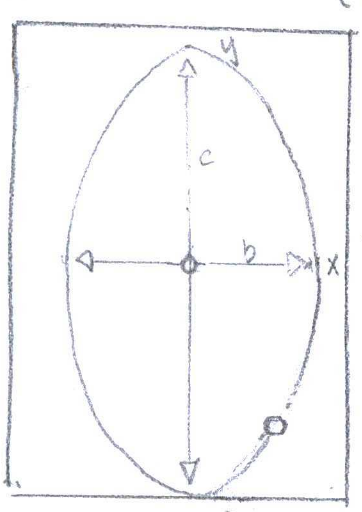
$$|\ddot{\vec{r}}(t)| = R\cdot\omega^2$$

(pg 29) "Centripetal acceleration"

$$r\omega^2$$

The direction around a circle derived an inward vector. The vector is centripetal acceleration.

1.11 (from problem) "Position of a particle"

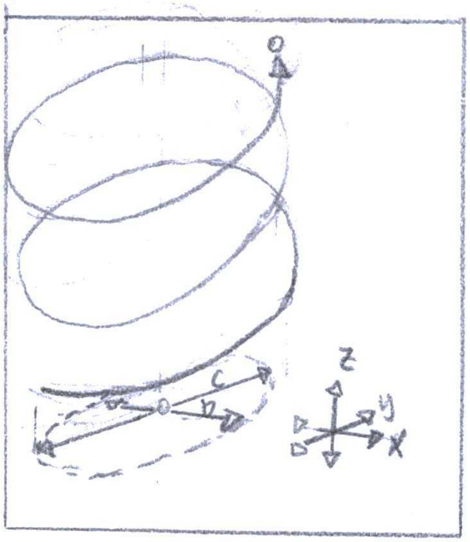


$$r(t) = b\cos(\omega t)\hat{x} + c\sin(\omega t)\hat{y}$$

The particle's orbit is elliptical. Coefficients to unit vectors, $\hat{x}$ and $\hat{y}$ have amplitudes b and c, respectively.

While ω describes the angular frequency around the orbit.

1.12



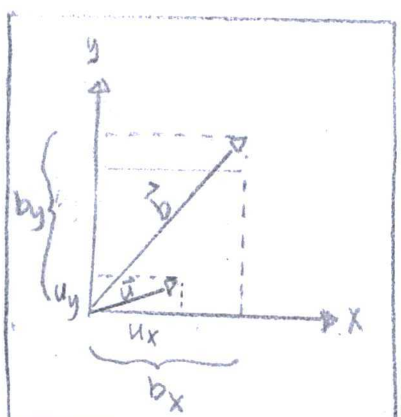
"Moving particle as a Function of time"

(from problem) "position of moving particle"

$$r(t) = b\cos(\omega t)\hat{x} + c\sin(\omega t)\hat{y} + V_0 t\hat{z}$$

The particle moves through space and time in an elliptical spiral. b and c are amplitudes. V_0 is a velocity. ω is the rotational frequency.

1.13



"Vectors"

If $\vec{u}$ has a magnitude about one, then $(u \circ b)^2 + (u \times b)^2 = (u_x b_x + u_y b_y)^2 + (u_x b_y - u_y b_x)^2$

$$= b_x^2(u_x^2 + u_y^2) + b_y^2(u_x^2 + u_y^2)$$

$$= \left(\sqrt{b_x^2 + b_y^2}\right)^2$$

$$= |b|^2$$

1.14

(from problem) "Triangle inequality"

Third length on a triangle

$$|\vec{a}+\vec{b}| \leq (|\vec{a}|+|\vec{b}|)$$

$$\rightarrow |\vec{c}| \leq |\vec{a}|+|\vec{b}|$$

or

$$|\vec{a}+\vec{b}| \leq (|\vec{a}|+|\vec{b}|)^2$$

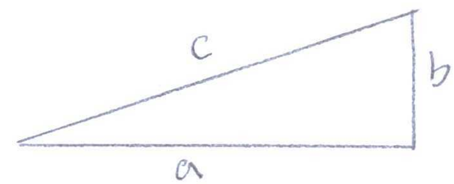
$$|\vec{a}+\vec{b}|^2 \leq (|\vec{a}|+|\vec{b}|)^2$$

$$a_x b_x + a_y b_y \leq \sqrt{(a_x^2+a_y^2)}\sqrt{(b_x^2+b_y^2)}$$

...with many positive solutions and a base case at unit length.

Triangle Inequality Theorem:

The sum of two sides around a triangle is greater than the third length.



$c < a+b$

$b < a+c$

$a < b+c$

1.15

(Equation 1.9) "cross product"

$$\vec{r} \times \vec{s} = \det\begin{bmatrix} \hat{x} & \hat{y} & \hat{z} \\ r_x & r_y & r_z \\ s_x & s_y & s_z \end{bmatrix}$$

If $\vec{r} = \langle 1,0,0 \rangle$ and $\vec{s} = \langle 1,1,0 \rangle$, then the output is $\vec{t} = \langle 0,0,1 \rangle$. The new vector is perpendicular to $\vec{r}$ and $\vec{s}$. Magnitude is one for $\vec{t}$, also $|r||s|\sin\theta$.

1.16 a) (Equation 1.7) "Dot Product"

$$r \circ s = r_1 s_1 + r_2 s_2 + r_3 s_3$$

$$= \sum_{n=1}^{3} r_n \cdot s_n$$

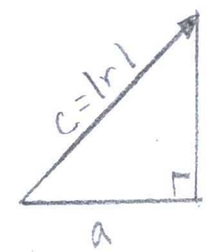
If $r = \langle a, b \rangle$, then $r \circ r = a^2 + b^2$

$$= c^2$$

$$= |r|^2$$

$$|r| = \sqrt{r \circ r}$$

Pythagoreas' Law:



$a^2 + b^2 = c^2$ For 90°-triangles

b) If $\vec{r}$ is orthoganol to $\vec{s}$, then possibly

$r = \langle \hat{x}, 0 \rangle$ and $s = \langle 0, \hat{y} \rangle$ where

$$\vec{r} \circ \vec{s} = \sum_{n=1}^{2} r_n \cdot s_n$$

$$= x + y$$

$$= |\vec{r} \circ \vec{s}|^2$$

$$|r \circ s| = \sqrt{x + y}$$

1.17 a) (Equation 1.9) "Vector Product"

$$P_x = r_y s_z - r_z s_y$$

$$P_y = r_z s_z - r_x s_z$$

$$P_z = r_x s_y - r_y s_x$$

Equivalently,

$$r \times s = \det \begin{bmatrix} \hat{x} & \hat{y} & \hat{z} \\ r_x & r_y & r_z \\ s_x & s_y & s_z \end{bmatrix}$$

$$\vec{r} \times (\vec{u} + \vec{v}) = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ r_x & r_y & r_z \\ u_x + v_x & u_y + v_y & u_z + v_z \end{vmatrix}$$

$$= \vec{r} \times \vec{u} + \vec{r} \times \vec{v}$$

b) $$\frac{d}{dt}(\vec{r} \times \vec{s}) = \frac{d}{dt} \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ r_x & r_y & r_z \\ s_x & s_y & s_z \end{vmatrix}$$

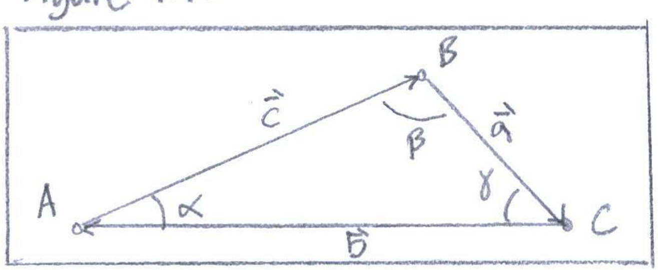
$$= \frac{d}{dt}(r_y s_z - r_z s_y)\hat{x} - \frac{d}{dt}(r_x s_z - r_z s_x)\hat{y} + \frac{d}{dt}(r_x s_z - r_z s_x)\hat{z}$$

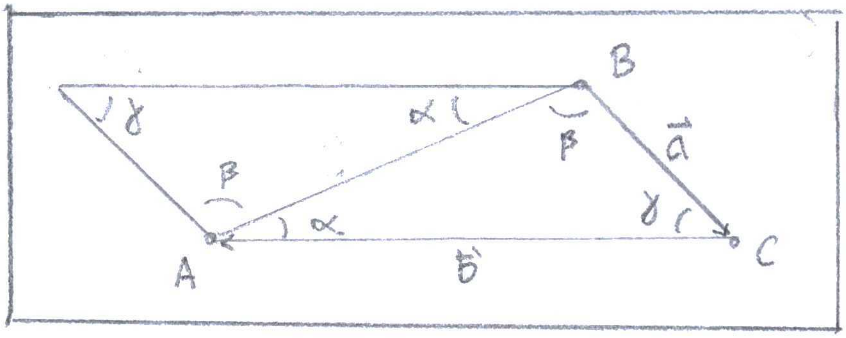
$$= \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ r_x & r_y & r_z \\ \frac{ds_x}{dt} & \frac{ds_y}{dt} & \frac{ds_z}{dt} \end{vmatrix} + \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{dr_x}{dt} & \frac{dr_y}{dt} & \frac{dr_z}{dt} \\ s_x & s_y & s_z \end{vmatrix}$$

$$= \vec{r} \times \frac{d\vec{s}}{dt} + \frac{d\vec{r}}{dt} \times \vec{s}$$

"Figure 1.15"

1.18





"Parallelogram"

a)

Area Parallelogram

$= |\vec{a} \times \vec{b}|$

$= 2 \cdot (\text{Area Triangle})$

$$\text{Area Triangle} = \frac{1}{2}|\vec{a} \times \vec{b}|$$
$$= \frac{1}{2}|\vec{b} \times \vec{c}|$$
$$= \frac{1}{2}|\vec{c} \times \vec{a}|$$

b) $\vec{c} = \vec{b} \sin\alpha$

$= \vec{a} \sin\beta$

So, $\dfrac{\vec{b}}{\sin\beta} = \dfrac{\vec{a}}{\sin\alpha}$

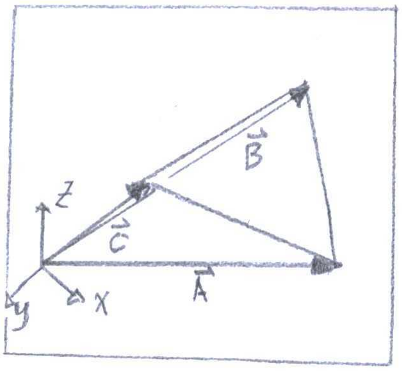
The relationship between vectors works for each side, and the "law of sines."

$$\frac{\vec{a}}{\sin\alpha} = \frac{\vec{b}}{\sin\beta} = \frac{\vec{c}}{\sin\gamma}$$

1.19

$$\frac{d}{dt}\left[\vec{a} \cdot (\vec{v} \times \vec{r})\right] = \frac{d}{dt}\sum_{i=1}^{n} a_i \cdot (\vec{v} \times \vec{r})_i$$
$$= \sum_{i=1}^{n} \mathring{a}_i \cdot (\vec{v} \times \vec{r})_i$$
$$= \mathring{\vec{a}} \cdot (\vec{v} \times \vec{r})$$

1.20



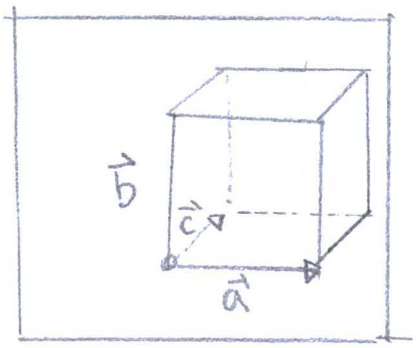
"Three vectors... three corners of triangle"

$$\text{Area of Triangle} = \text{Area}_{AB} + \text{Area}_{BC} + \text{Area}_{AC}$$

$$= \tfrac{1}{2}|\vec{A}\times\vec{B}| + \tfrac{1}{2}|\vec{B}\times\vec{C}| + \tfrac{1}{2}|\vec{C}\times\vec{A}|$$

$$= \tfrac{1}{2}|\vec{B}\times\vec{C} + \vec{C}\times\vec{A} + \vec{A}\times\vec{B}|$$

1.21



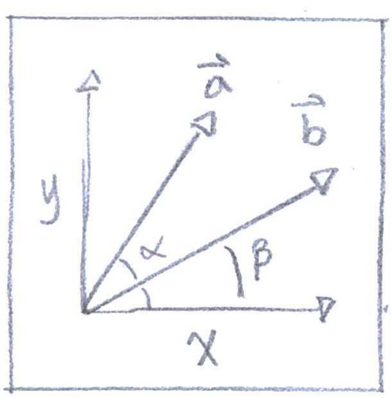
"Parallelpiped"

$$\text{Volume} = \text{length} \times \text{width} \times \text{height}$$

$$= a \cdot b \times b \cdot c$$

$$= a \cdot (b \times c)$$

1.22



"Vectors in... xy-plane"

a)

(1.6) "Scalar Product"

$$\vec{r}\cdot\vec{s} = rs\cos\theta$$

(1.7) "Dot Product"

$$\vec{r}\cdot\vec{s} = r_1 s_1 + r_2 s_2 + r_3 s_3 = \sum_{n=1}^{3} r_n s_n$$

$$\vec{a}\cdot\vec{b} = a_x \cdot b_x + a_y \cdot b_y$$

$$= |a||b|\cos(\alpha-\beta)$$

$$= |a|\cos(\alpha)\cdot|b|\cos(\beta) + |a|\sin(\alpha)|b|\sin(\beta)$$

$$\cos(\alpha-\beta) = \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)$$

b)

$$\vec{a}\times\vec{b} = |a||b|\sin(\alpha-\beta)$$

$$= |a|\sin(\alpha)|b|\sin(\beta) - |a|\cos(\alpha)|b|\cos(\beta)$$

$$\sin(\alpha-\beta) = \sin(\alpha)\sin(\beta) - \cos(\alpha)\cos(\beta)$$

1.23 (Back Cover) "Vector Identities"

$$\vec{A}\circ(\vec{B}\times\vec{C}) = \vec{B}\cdot(\vec{C}\times\vec{A}) = \vec{C}\circ(\vec{A}\times\vec{B})$$

$$\vec{A}\times(\vec{B}\times\vec{C}) = \vec{B}(\vec{A}\cdot\vec{C}) - \vec{C}(\vec{A}\circ\vec{B})$$

(from problem) "Unknown Vector Properties"

$\vec{b}\circ\vec{v} = \lambda$ and $\vec{b}\times\vec{v} = \vec{c}$

<u>V as λ, $\vec{b}$, and $\vec{c}$:</u>

① $\vec{b}\times(\vec{b}\circ\vec{v} + \vec{v}\times\vec{b}) = \vec{b}\times(\vec{b}\circ\vec{v}) + \vec{b}\times(\vec{v}\times\vec{b})$

$$= \vec{b}\times(\vec{b}\cdot\vec{v}) + \vec{v}(\vec{b}\circ\vec{b}) - \vec{b}\times(\vec{b}\circ\vec{v})$$

$$= \vec{v}(\vec{b}\cdot\vec{b})$$

② $\vec{b}\times(\vec{b}\circ\vec{v} + \vec{v}\times\vec{b}) = \vec{b}\lambda + \vec{b}\times\vec{c}$

① = ② $\quad \vec{v}(\vec{b}\circ\vec{b}) = \vec{b}\lambda + \vec{b}\times\vec{c}$

$$\vec{V} = \frac{\vec{b}\lambda + \vec{b} \times \vec{c}}{\vec{b} \cdot \vec{b}}$$

1.24 (from author) "First order equation"

$$\frac{dF}{dt} = F$$

$$\int \frac{dF}{F} = \int dt$$

$$F = e^{t+c}$$

$$= Ce^{t}$$ ...one arbitrary constant for a 1^{st} order equation.

1.25 (by author) "Differential Equation"

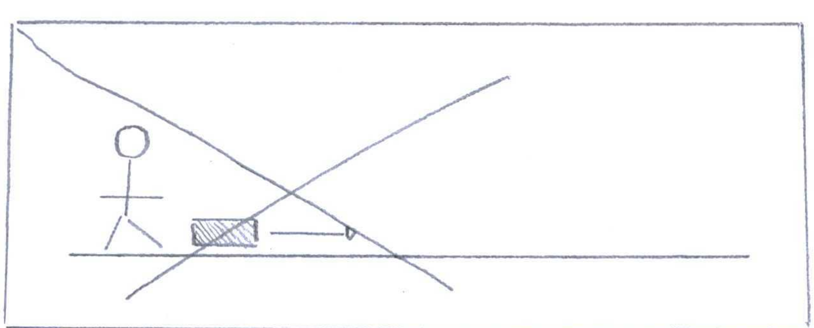
$$\frac{dF}{dt} = -3F$$

$$\int \frac{dF}{F} = -3 \int dt$$

$$F = -3e^{t+c}$$

$$= -3C \cdot e^{t}$$

1.26

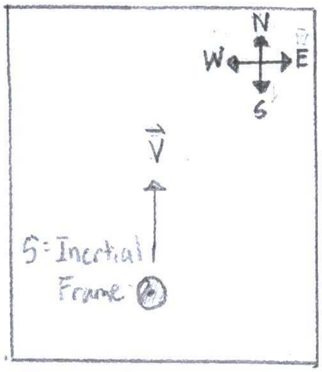


1.26

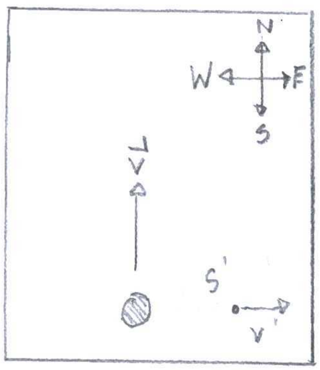
a) Position:

$$X = V \cdot t + S_{x,0}$$

$$y = V \cdot t + S_{y,0}$$



"Kick a frictionless puck due north"

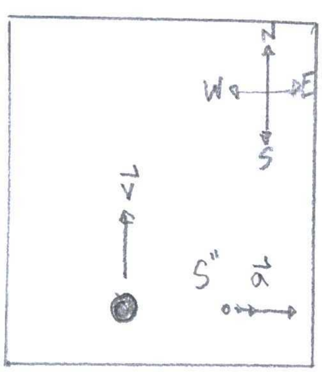


Position relative to S':

$$X = V' \cdot t + S'_{x,0}$$

$$y = v \cdot t + S'_{y,0}$$

"Observer S' travels with a constant velocity east"



Position relative to S'':

$$X = \frac{1}{2} a t^2 + V' t + S''_{x,0}$$

$$y = V \cdot t + S''_{x,0}$$

"Observer S'' travels with constant acceleration east"

Roughly defined on pages 14-15

$$F = m \cdot \ddot{X}(t)$$

$$\ddot{X}(t) = \frac{F}{m}$$

$$\dot{X}(t) = \int_0^t \frac{F}{m} dt$$

$$= \frac{F}{m} t + V_0$$

$$X(t) = \int \dot{X}(t) dt$$

$$= \int \frac{F}{m} t \, dt$$

$$= \frac{F}{2m} t^2 + V_0 t + X_0$$

with conditions $\dot{X}(0) = V_0$, $X(0) = X_0$

b) Position from S':

$$x' = -v' \cdot t = S'_{x,0}$$

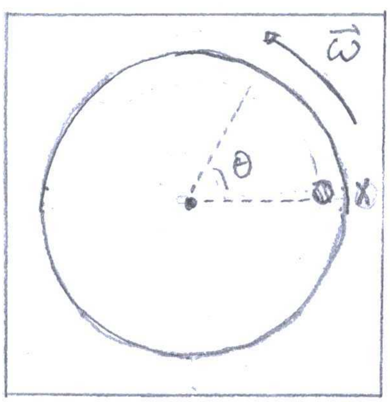
$$y'' = v \cdot t - S'_{y,0}$$

c) Position from S'':

$$x'' = -\frac{1}{2}at^2 - v' \cdot t - S''_{x,0}$$

$$y'' = v \cdot t + S''_{y,0}$$

1.27



"Shove a puck... across a turn-table... through center"

Description from someone at rest:

Angular frequency (ω) about a turn-table proposes circular motion by an angular velocity ($\dot{\phi}$). The rotational velocity (Equation 1.43) is $\vec{v} \equiv \dot{\vec{r}} = \dot{r}\hat{r} + r\dot{\phi}\hat{\phi}$, where in polar components. The components are $v_r = \dot{r}$ and $v_\phi = r\dot{\phi} = r\omega$. From the observer, $x \equiv r\cos\phi$ and $y = r\sin\phi$, both sinusoidal relations (Equation 1.37).

1.28. (Equation 1.25) "Net Force"

$$(\text{net force on particle } \alpha) = \vec{F}_\alpha = \sum_{\beta \neq \alpha} \vec{F}_{\alpha\beta} + \vec{F}_\alpha^{ext}$$

(Equation 1.26) "Newtons 2nd law"

$$\dot{P}_\alpha = \sum_{\beta \neq \alpha} F_{\alpha\beta} + F_\alpha^{ext}$$

(Equation 1.27) "Total Force"

$$\dot{P} = \sum_\alpha \sum_{\beta \neq \alpha} F_{\alpha\beta} + \sum_\alpha F_\alpha^{ext}$$

(Equation 1.28) "Double sum"

$$\sum_\alpha \sum_{\beta \neq \alpha} F_{\alpha\beta} = \sum_\alpha \sum_{\beta > \alpha} (F_{\alpha\beta} + F_{\beta\alpha})$$

(Equation 1.29) "Net external Force"

$$\dot{P} = \sum_\alpha F_\alpha^{ext} = F^{ext}$$

If $N = 3$, then

$$\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_1^{ext}$$

$$\vec{F}_2 = \vec{F}_{21} + \vec{F}_{23} + \vec{F}_2^{ext}$$

$$\vec{F}_3 = \vec{F}_{31} + \vec{F}_{32} + \vec{F}_3^{ext}$$

$$\dot{\vec{P}}_1 = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_1^{ext}$$

$$\dot{\vec{P}}_2 = \vec{F}_{21} + \vec{F}_{23} + \vec{F}_2^{ext}$$

$$\dot{\vec{P}}_3 = \vec{F}_{31} + \vec{F}_{32} + \vec{F}_3^{ext}$$

$$\dot{\vec{P}} = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{21} + \vec{F}_{23} + \vec{F}_{31} + \vec{F}_{32} + \vec{F}_1^{ext} + \vec{F}_2^{ext} + \vec{F}_3^{ext}$$

$$\sum_{\alpha=1}^{3} \sum_{\beta \neq \alpha}^{3} \vec{F}_{\alpha\beta} = \vec{F}_{12} + \vec{F}_{21} + \vec{F}_{13} + \vec{F}_{31} + \vec{F}_{23} + \vec{F}_{32}$$

$$\vec{F}^{ext} = \vec{F}_1^{ext} + \vec{F}_2^{ext} + \vec{F}_3^{ext}$$

1.29 Net Force:

$$\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14}$$

$$\vec{F}_2 = \vec{F}_{21} + \vec{F}_{23} + \vec{F}_{24}$$

$$\vec{F}_3 = \vec{F}_{31} + \vec{F}_{32} + \vec{F}_{34}$$

$$\vec{F}_4 = \vec{F}_{41} + \vec{F}_{42} + \vec{F}_{43}$$

Inertia:

$$\dot{\vec{P}}_1 = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} + \vec{F}_1^{ext}$$

$$\dot{\vec{P}}_2 = \vec{F}_{21} + \vec{F}_{23} + \vec{F}_{24} + \vec{F}_2^{ext}$$
$$\dot{\vec{P}}_3 = \vec{F}_{31} + \vec{F}_{32} + \vec{F}_{34} + \vec{F}_3^{ext}$$
$$\dot{\vec{P}}_4 = \vec{F}_{41} + \vec{F}_{42} + \vec{F}_{43} + \vec{F}_4^{ext}$$

Total Force:

$$\dot{\vec{P}} = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} + \vec{F}_{21} + \vec{F}_{23} + \vec{F}_{24} + \vec{F}_{31} + \vec{F}_{32} + \vec{F}_{34} + \vec{F}_{41} + \vec{F}_{42} + \vec{F}_{43}$$
$$+ \vec{F}_1^{ext} + \vec{F}_2^{ext} + \vec{F}_3^{ext} + \vec{F}_4^{ext}$$

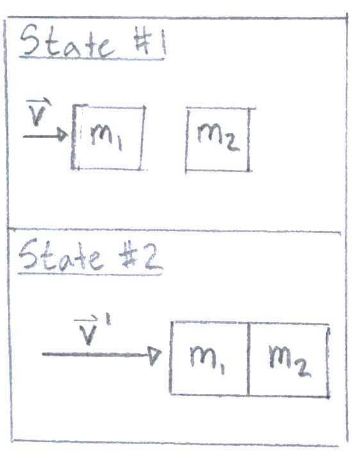
Double Sum:

$$\sum_{\alpha=1}^{4} \sum_{\beta \neq \alpha}^{4} \vec{F}_{\alpha\beta} = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} + \vec{F}_{21} + \vec{F}_{23} + \vec{F}_{24} + \vec{F}_{31} + \vec{F}_{32} + \vec{F}_{34} + \vec{F}_{41} + \vec{F}_{42} + \vec{F}_{43}$$

Net external Force:

$$\vec{F}^{ext} = \vec{F}_1^{ext} + \vec{F}_2^{ext} + \vec{F}_3^{ext} + \vec{F}_4^{ext}$$

1.30



"Object one collides with velocity ($\vec{v}$)... collides with stationary object 2."

"Two objects stick together and move off with common velocity v'"

(Page 83) "Principle of Conservation of Momentum"

$\vec{P} = \sum m_\alpha \vec{v}_\alpha$ ... where $\alpha = 1, 2, 3 \ldots N$

Conservation of Momentum:

State #1: $\vec{P} = m_1 \vec{v}_1 + \cancel{m_2 \vec{v}_2} \to 0$

$= m_1 \vec{v}_1$

State #2: $\vec{P} = (m_1 + m_2)\vec{v}'$

Momentums equate because conservation:

$$m_1 \vec{v} = (m_1 + m_2)\vec{v}'$$

$$\vec{v}' = \frac{m_1}{m_1 + m_2} \cdot \vec{v}$$

1.31 (Equation 1.20) "Newton's Third Law"

$\vec{F}_{12} = -\vec{F}_{21}$ ... object 1 exerts a force on object 2, then object 2 exerts a force on object 1.

(Page 83) "Conservation of Momentum"

$\vec{P} = \sum m_\alpha \vec{v}_\alpha$ ... where $\alpha = 1, 2, 3 \ldots N$

Relationship between Newton's Third Law and Conservation of Momentum:

$$F_{12} = -F_{21}$$

$$\underbrace{F_{21} + F_{12} = \frac{d}{dt} \sum m_\alpha V_\alpha}_{\text{"Conservation of Momentum"}}$$

1.32

(from problem) "Electric Field"

$$\vec{E}(r_1) = \frac{1}{4\pi\varepsilon_0} \frac{q_2}{s^2} \hat{s}^2$$

$$= \frac{\vec{F}^{el}_{12}}{q_1}$$

(from problem) "Magnetic Field"

$$\vec{B}(r_1) = \frac{\mu_0}{4\pi} \frac{q_2}{s^2} \vec{V}_2 \times \hat{s}$$

$$= \frac{\vec{F}^{mag}_{21}}{q_1}$$

$$\vec{F}^{mag}_{12} = \frac{\mu_0}{4\pi} \frac{q_1 q_2}{s^2} \vec{V}_2 \times \hat{s} \leq \frac{V_1 V_2}{c^2} \frac{1}{4\pi\varepsilon_0} \frac{q_2}{s^2} \hat{s}^2$$

$$\leq \frac{V_1 V_2}{c^2} \vec{F}^{el}_{12}$$

1.33

1.34 (from problem) "Total Angular Momentum"

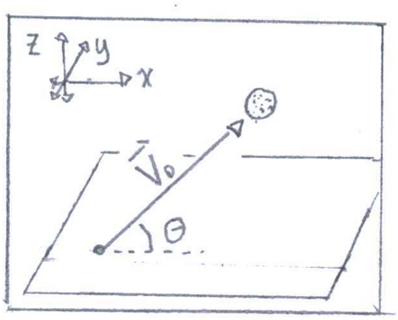
$$\vec{L} = \sum_\alpha \vec{r}_\alpha \times \vec{p}_\alpha$$

$$= \sum_\alpha \vec{r}_\alpha \times \left(\frac{d}{dt}\left(\sum_{\beta \neq \alpha} \vec{F}_{\alpha\beta} + \vec{F}_\alpha^{ext}\right)\right)$$

$$= \frac{dL}{dt} \sum_\alpha r \times \left(\sum_{\beta \neq \alpha} \vec{F}_{\alpha\beta} + \vec{F}_\alpha^{ext}\right)$$

...with Newton's 2^{nd} law in a conserved system.

1.35



"A golf ball is hit from the ground"

(Page 15) "Particle Position"

$$x(t) = \int \dot{x}(t)dt = x_0 + v_0 t + \frac{F_0}{2m}t^2$$

$$= x_0 + v_0 t + \frac{1}{2}at^2$$

<u>Coordinates as a Function of time:</u>

$$x(t) = x_0 + V_{x,0} t + \frac{1}{2}at^2$$

$$= \vec{V}_x \cdot t$$

$$= V_0 \cdot \cos\theta \cdot t$$

$$y(t) = y_0 + V_{y,0} t + \frac{1}{2}at^2$$

$$= 0$$

$$z(t) = z_0 + v_{z,0}\, t - \frac{1}{2} a_t t^2$$

$$\doteq -\frac{1}{2}(9.806\, m/s^2) t^2 + v_0 \cdot \sin\theta \cdot t$$

$\boxed{\begin{array}{l}\text{Cite: "The NIST Reference on Constants,} \\ \text{Units, and Uncertainty."} \\ \text{Standard acceleration of gravity} \\ g_n = 9.80665\, m\, s^{-2}\end{array}}$

At $z=0$, $$t_{1,2} = \frac{-v_0 \sin\theta \pm \sqrt{(v_0 \sin\theta)^2 - 4(-\frac{1}{2} 9.806 \frac{m}{s^2})(}{-2\, \frac{1}{2}(9.806\, \frac{m}{s^2})}$$

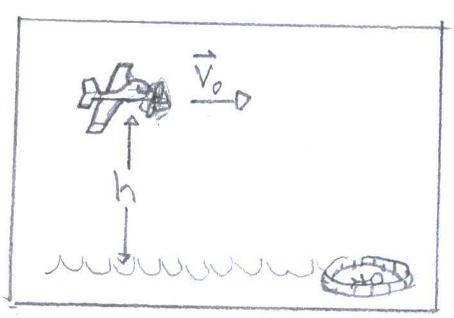
$$= \frac{2 v_0 \sin\theta}{9.806\, \frac{m}{s^2}},\ 0$$

"Quadratic formula",
said by a Babylonian.

$$x\left(t = \frac{2 v_0 \sin\theta}{9.806\, \frac{m}{s^2}}\right) = v_0 \cos\theta \cdot \frac{2 v_0 \sin\theta}{9.806\, m/s^2}$$

$$= \frac{2 v_0^2 \cos\theta \sin\theta}{9.806\, m/s^2}$$

1.36



"A plane, which is flying horizontally ... must drop a bundle of supplies to a ... raft"

a) Newton's 2^{nd} Law:

X-direction: $F_x = ma_x = 0$

y-direction: $F_y = ma_y = ma_g$

Position:

$$x(t) = \frac{1}{2}a_x t^2 + v_x t + x_0$$

$$= v_x \cdot t$$

$$y(t) = -\frac{1}{2}a_y t^2 + v_y t + y_0$$

$$= -\frac{1}{2}gt^2 + h$$

b) General solution: distance dropping before raft:

$$x(t) = d = -v_0 t$$

$$y(t) = -\frac{1}{2}gt^2 + h \rightarrow d = -v_0\sqrt{\frac{2h}{g}}$$

Exact solution: distance dropping before raft

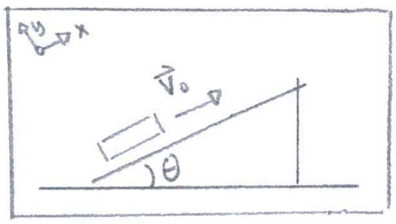
$v_0 = 50\ m/s$; $h = 100\ m$; $g \cong 10\ m/s^2$

$$d = -v_0\sqrt{\frac{2h}{g}} = -223\ m$$

c) Time interval within ± 10 m:

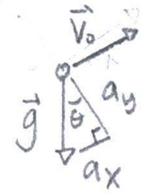
$$\Delta t = \frac{d \pm 10}{v_0} = \pm 0.4 \text{ seconds}$$

1.37



"A frictionless puck... slides up a plane"

Free-body Diagram:



a) Newton's 2^{nd} Law:

X-coordinate: $X(t) = X_0 + V_x t + \frac{1}{2} a_x t^2$

$$= \vec{V}_0 t - \frac{1}{2} g \sin\theta \, t^2$$

y-coordinate: $y(t) = y_0 + V_y t + \frac{1}{2} a_y t^2$

$$= -\frac{1}{2} g \cos\theta \, t^2$$

b) Time till a return to bottom:

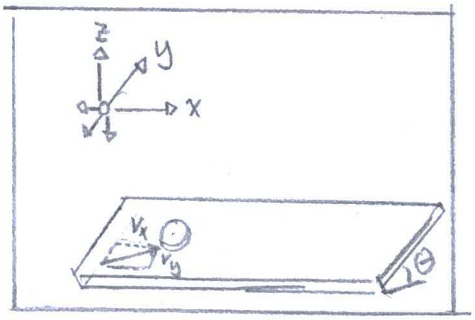
$$X(t) = V_0 t - \frac{1}{2} g \sin\theta \, t^2$$

$$= 0$$

$$t_{1,2} = \frac{-V_0 \pm \sqrt{V_0^2 - 4(-\frac{1}{2} g \sin\theta) \cdot 0}}{2 \cdot (-\frac{1}{2} g \sin\theta)}$$

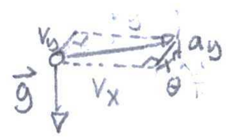
$$= \frac{2V_0}{g \sin\theta}, 0$$

1.38



"frictionless puck...
slides across board"

Free-body diagram:



a) Newton's 2nd Law:

X-component:

$$x(t) = x_0 + V_{0,x} t + \frac{1}{2} a_x t^2$$

$$= V_{0,x} \cdot t$$

y-component:

$$y(t) = y_0 + V_{0,y} t + \frac{1}{2} a_y t^2$$

$$= V_{0,y} t - \frac{1}{2} g \sin\theta \cdot t^2$$

Z-component:

$$z(t) = z_0 + V_{0,z} t + \frac{1}{2} a_z t^2$$

$$= -\frac{1}{2} g \cos\theta t^2$$

Distance travelled after return:

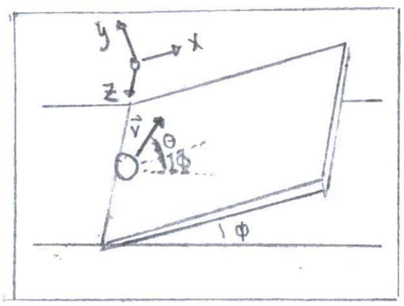
$$z(t) = \frac{-1}{2} g \cos\theta t^2$$

$$= 0$$

$$t = \sqrt{\frac{2}{g\cos\theta}}$$

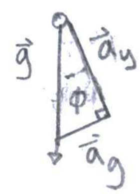
$$x\left(t = \sqrt{\frac{2}{g\cos\theta}}\right) = V_{0,x} \cdot \sqrt{\frac{2}{g\cos\theta}}$$

1.39



"A ball is thrown ...up an inclined plane...at an angle θ above the plane."

Free-body Diagram:



Note: Problem had exact answers only if, "θ above horizontal."

Position:

x-coordinate: $F = m\vec{a}_x$

$$x(t) = x_0 + V_{0,x} \circ t + \frac{1}{2} a_x \circ t^2$$

$$= v\cos(\theta)t - \frac{1}{2}a_g\sin(\phi)t^2$$

y-coordinate: $F = m\vec{a}_y$

$$y(t) = y_0 + V_{0,y}t + \frac{1}{2}a_y \circ t^2$$

$$= v\sin(\theta)t - \frac{1}{2}a_g\cos(\phi)t^2$$

z-coordinate: $F = m \cdot a_z = 0$

$$z(t) = z_0 + V_{0,z} \circ t - \frac{1}{2}a_z \circ t^2$$

$= 0$

Distance before landing:

Time till return:

$$y(t) = V_0 \sin(\theta)t - \frac{1}{2} g \cos(\phi) t^2$$

$$= 0$$

$$t = \frac{2V_0 \sin(\theta)}{g\cos(\phi)}$$

Distance:

$$X\left(t = \frac{2v\sin(\theta)}{g\cos(\phi)}\right) = V_0\cos(\theta)\cdot\frac{2V_0\sin(\theta)}{g\cos(\phi)}$$

$$-\frac{1}{2} g\cdot\sin(\phi)\left(\frac{2V_0\sin(\theta)}{g\cos(\phi)}\right)^2$$

$$= \frac{2V_0^2\sin\theta(\cos\theta\cos\phi - \sin\phi\sin\theta)}{g\cos^2\phi}$$

$$= \frac{2V_0^2\sin\theta\cos(\theta+\phi)}{g\cos^2\phi}$$

Sum difference Identity:

$\cos(\theta \pm \phi) = \cos\theta\cos\phi \mp \sin\theta\sin\phi$

Maximum possible range:

Critical angle:

$$\frac{dX\left(t = \frac{2v\sin\theta}{g\cos\phi}\right)}{d\theta} = \frac{d}{d\theta}\left[\frac{2V_0^2\sin\theta\cos(\theta+\phi)}{g\cos^2\phi}\right]$$

$$= \frac{2v_0^2(\cos\theta\cos(\theta+\phi) - \sin\theta\sin(\theta+\phi))}{g\cos^2\phi}$$

$$= 0$$

$$2v_0^2\cos(2\theta+\phi) = 0$$

$$\theta_{max} = \frac{\pi}{2} - \frac{\phi}{2}$$

Maximum Distance:

$$X(\theta_{max}) = \frac{2v_0^2\sin(\frac{\pi}{2} - \frac{\phi}{2})\cos(\frac{\pi}{2} - \frac{\phi}{2} + \phi)}{g\cos^2\phi}$$

$$= \frac{v_0^2(1-\sin\phi)}{g\cos^2\phi}$$

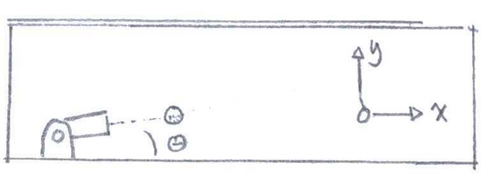
$$= \frac{v_0^2}{g(1+\sin\phi)}$$

Sum difference Identity:

$\sin(\theta+\phi) = \sin\theta\cos\phi \pm \cos\theta\sin\phi$

Double angle Identity:

$\sin(2\theta) = 2\sin\theta\cos\theta$

1.40 

"A cannon shoots a ball at an angle... above ground"

a) Position:

X-coordinate: $X(t) = X_0 + V_{0,x} \cdot t + \frac{1}{2} a_x \cdot t^2$

$= V_0 \cos\theta \cdot t$

y-coordinate: $y(t) = y_0 + V_{0,y} \cdot t - \frac{1}{2} a_y \cdot t^2$

$= V_0 \sin\theta \cdot t - \frac{1}{2} g \cdot t^2$

b) Maximum r:

If $r = \sqrt{x^2 + y^2}$

$= \sqrt{(V_0 \cos\theta\, t)^2 + (V_0 \sin\theta \cdot t - \frac{1}{2} g \cdot t^2)^2}$

$= \sqrt{\frac{1}{4} g^2 t^4 - g V_0 \sin\theta \cdot t^3 + V_0^2 t^2}$

then $\frac{dr}{dt} = \frac{d}{dt} \sqrt{\frac{1}{4} g^2 t^4 - g V_0 \sin\theta\, t^3 + V_0^2 t^2}$

$= \dfrac{g t^3 - 3 g t^2 V_0 \sin\theta + 2 V_0^2 t}{2 \sqrt{\frac{1}{4} g^2 t^4 - g V_0 \sin\theta\, t^3 + V_0^2 t^2}}$

$= 0$

$t = \dfrac{3 g V_0 \sin\theta \pm \sqrt{(-3 g V_0 \sin\theta)^2 - 4(g)(2V_0)^2}}{2g}$

When discriminant is zero for the parabola, then minimum:

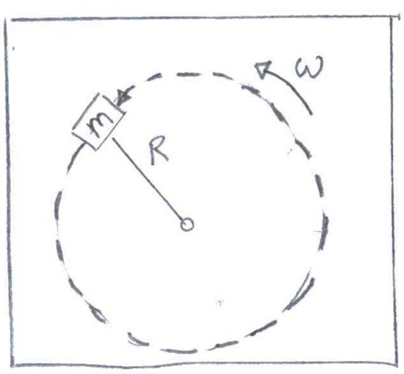
$$(-3gv_0\sin\theta)^2 - 4(g)(2v_0)^2 = 0$$

$$\sin^2\theta = \frac{8}{9}$$

$$\theta = \sin^{-1}\left(\sqrt{\frac{8}{9}}\right)$$

$$= 70.5^\circ$$

1.41.



"An astronaut... twirling a mass"

Position:

R-component: $\vec{F}_R = m\cdot\vec{a}_R$

$$= -mR\omega^2$$

ϕ-component: $\vec{F}_\phi = m\vec{a}_\phi$

$$= 2mr\dot{\phi}$$

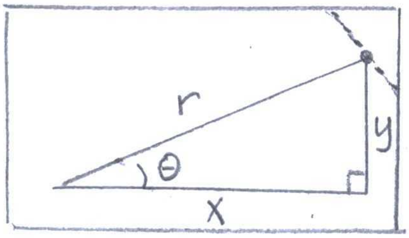
(1.48) "Newtons second law in Polar"

$$F = ma \longleftrightarrow \begin{cases} F_r = m(\ddot{r} - r\dot{\phi}^2) \\ F_\phi = m(r\ddot{\phi} + 2r\dot{\phi}) \end{cases}$$

Tension:

$$\vec{F}_T = \vec{F}_R = -mR\omega^2$$

1.42



Rectangular Coordinates

$x = r\cos\phi$

$y = r\sin\phi$

Polar coordinates

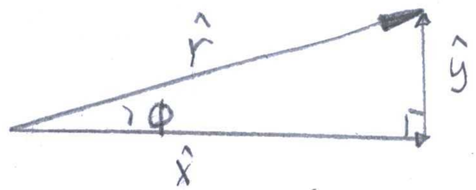
$r = \sqrt{x^2 + y^2}$

$\theta = \arctan(y/x)$

1.43

a) (1.59) "r unit vector"

$$\hat{r} = \hat{x}\cos\phi + \hat{y}\sin\phi$$



"φ unit vector"

$$\hat{\phi} = \arctan\left(\frac{y}{x}\right)$$

b) (1.42) "Radial rate of change"

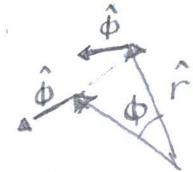
$\frac{d\hat{r}}{dt} = \dot{\phi}\,\hat{\phi}$ derives from a constant velocity.

$$\mathbf{r} = r\hat{r}$$

$$\dot{\mathbf{r}} = \underbrace{\dot{r}\hat{r}}_{=0} + r\frac{d\hat{r}}{dt}$$

$$= \dot{\phi}\,\hat{\phi}$$

(1.46) "Angular rate of change"

$\frac{d\hat{\phi}}{dt} = -\dot{\phi}\hat{r}$ derives from arc distance  or Newton's second law in Polar coordinates.

1.44 (1.56) "a solution"

$$\phi(t) = A\sin(\omega t) + B\cos(\omega t)$$

(1.55) "Angular acceleration"

$$\ddot{\phi} = -\omega^2 \phi$$

A first derivative,

$$\dot{\phi}(t) = \omega A\cos(\omega t) - \omega B\sin(\omega t)$$

Then second derivative,

$$\ddot{\phi}(t) = -\omega^2 A\sin(\omega t) - \omega^2 B\cos(\omega t)$$

$$= -\omega^2 \phi$$

1.45 If $v(t) = 4$, then $\frac{d}{dt}\left[v(t) \cdot v(t)\right]$ = "orthoganol vectors" $= 0$

1.46

a) Position:

r-coordinate: $r = \sqrt{(x-R)^2 + (y-R)^2}$

ϕ-coordinate: $\phi = \arctan\left(\frac{y}{x}\right)$

b) Position change in time:

r-coordinate: $\dot{r} = \dot{r}\hat{r} + r\dot{\phi}\hat{\phi}$

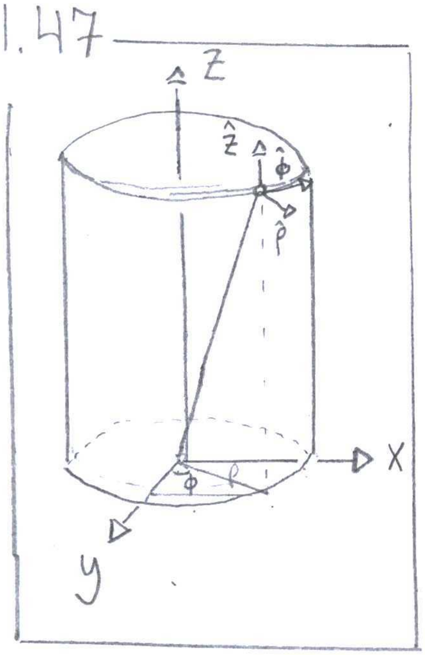
$$= \frac{(x-R)+(y-R)}{\sqrt{(x-R)^2+(y-R)^2}}\hat{r} + \sqrt{(x-R)^2+(y-R)^2}\,\hat{\phi}$$

ϕ-coordinate: $\dot{\phi} = -\dot{\phi}\hat{r}$

$$= \frac{-(x-R)(y-R)-(x-R)(y-R)}{(x-R)^2+(y-R)^2}\hat{r}$$

Without friction the puck spirals to an observer on the turntable. The frame is not inertial because independent of mass, gravity, and force in the equation. Inertial versions produce cardoids.

1.47



"Cylindrical Polar Coordinates"

a) The sketch is left.

"ρ" is the distance from the z-axis in both cylindrical and cartesian coordinates.

b) The $\hat{\rho}$ unit vector points away from the z-axis by projection on x-and-y axis. The $\hat{\phi}$ unit vector describes change in angle. While, the $\hat{z}$ unit vector points upward from vector.

c) When $r = (\rho, \phi, z)$

$$= \left(\sqrt{x^2+y^2}, \arctan\left(\frac{y}{x}\right), z\right)$$

$$\ddot{r} = r_\rho \hat{\rho} + r_\phi \hat{\phi} + r_z \hat{z}$$

$$= (\ddot{\rho} - \rho\dot{\phi}^2)\hat{\rho} + (\rho\ddot{\phi} + 2\dot{\rho}\dot{\phi})\hat{\phi} + \ddot{z}\hat{z}$$

1.48

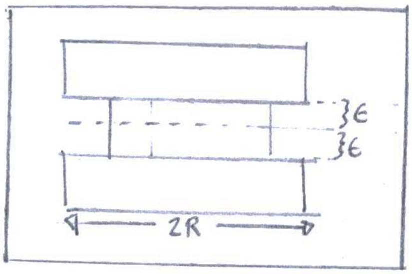
Cylindrical Coordinates:

$\rho = \sqrt{x^2+y^2}$; $\phi = \arctan\left(\frac{y}{x}\right)$; $z = \hat{z}$

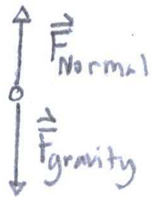
$$\dot{\vec{\rho}} = \dot{\rho}\hat{\rho} + \rho\frac{d\hat{\rho}}{dt} \;;\; \dot{\vec{\Phi}} = \dot{\phi}\hat{\Phi} + \phi\frac{d\hat{\Phi}}{dt} \;;\; \dot{\vec{z}} = \dot{z}\hat{z}$$

$$= \dot{\rho}\hat{\rho} + \rho\dot{\phi}\hat{\phi} \qquad = \dot{\phi}\hat{\phi} - \phi\dot{\phi}\hat{\rho}$$

1.49.



Free-body Diagram:



Newton's Second Law:

(pg 34) "Newton's Second in Cylindrical"

$$F_r = m(\ddot{\rho} - \rho\dot{\phi}^2)$$

$$F_\phi = m(\rho\ddot{\phi} + 2\dot{\rho}\dot{\phi})$$

$$F_z = m\ddot{z}$$

$$F_{TOT} = m \cdot a_g$$

$$= F_\rho\hat{\rho} + F_\phi\hat{\phi} + F_z\hat{z}$$

$$= m(\ddot{\rho} - \rho\dot{\phi}^2)\hat{\rho} + m(\rho\ddot{\phi} + 2\dot{\rho}\dot{\phi})\hat{\phi} + m\ddot{z}\hat{z}$$