[redacted]: Mechanical Equilibrium: $V_{L,R} = \frac{N_{L,R}}{N} \cdot V_{TOT}$; $N_{L,R}$ = # particles Left & Right

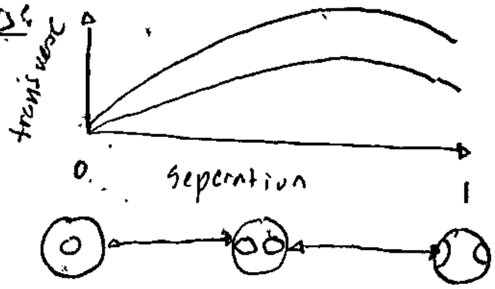
$N_L + N_R = N$; $V_L + V_R = V_{TOT}$

Work on Mechanical System: $\int_{\frac{1}{2}V_{TOT}}^{\frac{N/2+R}{N}V_{TOT}} P_L\,dV_L + \int_{\frac{1}{2}V_{TOT}}^{\frac{N/2-R}{N}V_{TOT}} P_L\,dV_L = \left(\frac{1}{2}\ln\left(1-\left(\frac{2R}{N}\right)^2\right)\right) + \frac{R}{N}\ln\frac{1+2R/N}{1-2R/N}$

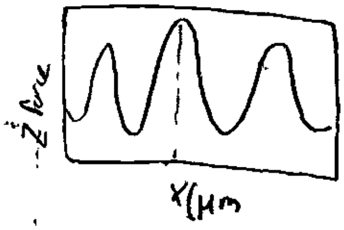
Ensemble Average: $\langle W \rangle = \sum_{R=-N/2}^{N/2} W(R)\,Pr(R)$; $Pr(R) = \left(\frac{1}{2}\right)^N \binom{N}{\frac{N}{2}+R}$

Binomially Expand $W(R) = Nk_BT\left(2\left(\frac{R}{N}\right)^2 + \underbrace{O\left(\left(\frac{R}{N}\right)^4\right)}_{=0 \,@\, N\to\infty}\right)$, $\lim_{N\to\infty}\langle W_{ideal}\rangle = \frac{2k_BT}{N}\langle R^2\rangle = \frac{1}{2}k_BT$

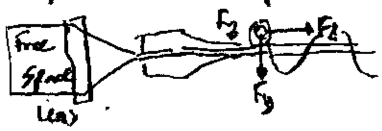
Torque of Optical Tweezers:



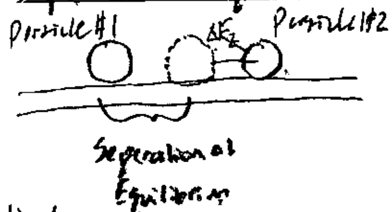
Forces on channels:

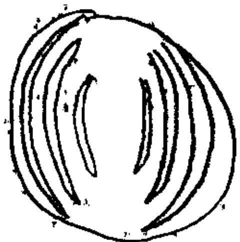


Experimental Setup:

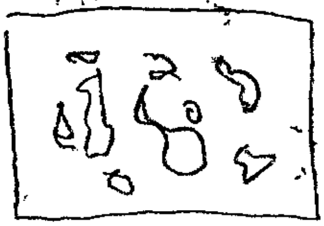


Seperation at Equilibrium:



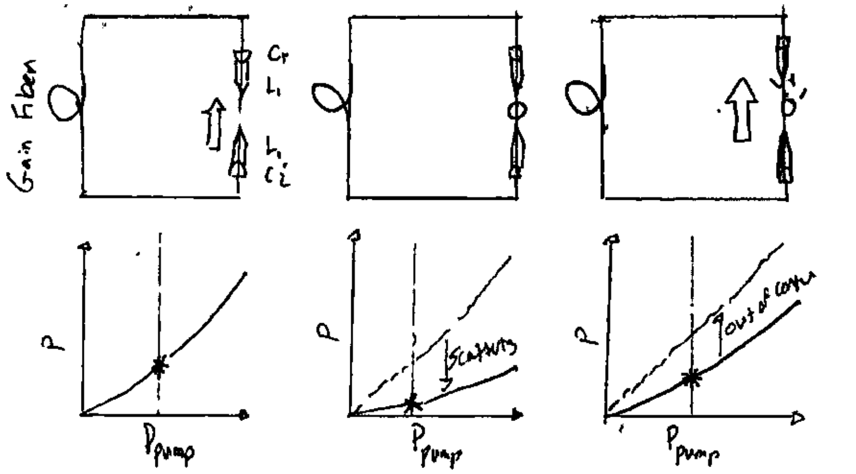


Speckle Patterns possible:



Working principle of intracavity optical trapping:

Steps to Trap or Principle to intracavity optical Trapping:

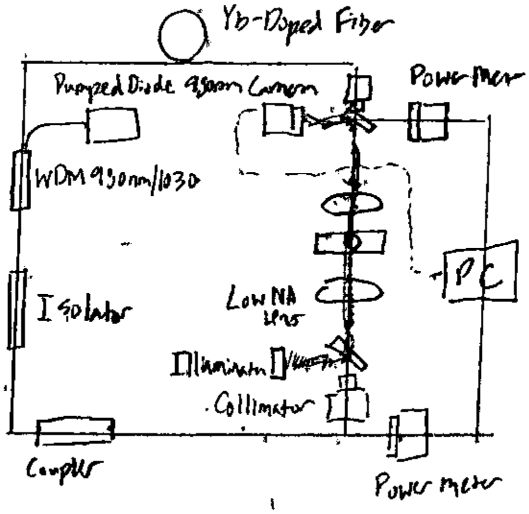


Above lasing Threshold

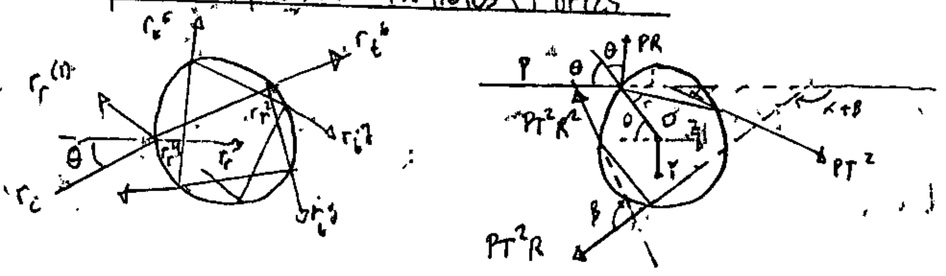
Particle is centered on trap

Laser power below reading because of scattering

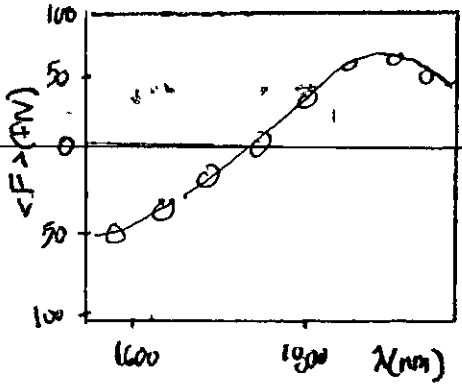
Setup of optical Trap:



Theory of Spherical Particles\ Forces



Force from parts of spin vs wavelength.



The force is negative when electric-dipole dominates, and positive when the magnetic dipole dominates

Mie Scattering [sphere]

$M = \nabla \times (r\psi)$ - Magnetic Harmonic

$N = \frac{\nabla \times M}{k}$ - Electric Harmonic

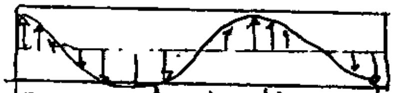
$$\frac{\int_0^{2\pi}\int_0^{\pi} E_0 \sum i^n \frac{2n+1}{n(n+1)} \left(M_{o1n}^{(1)}(k,r) - i N_{e1n}^{(1)}(k,r)\right) M_{mn} \sin\theta}{\iint |M_{mn}|^2 \sin\theta \, d\theta \, d\phi}$$

Belinfante-Rosenfeld Stress-energy Tensor : $M_{\mu\nu} = \int d^3x \, M^0_{\mu\nu} ; M^{\mu}_{\nu\lambda} = (x T^{\mu}_{\lambda} - x_\lambda T^{\mu}_{\nu}) + S^{\mu}_{\nu\lambda}$

Transverse Angular Momentum Spin : $s = \mathrm{Im}[\epsilon_0 (E^* \times E) + \mu_0 (H^* \times H)]/4\omega$ $\quad J = s + l$

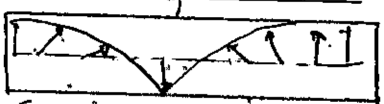
Longitudinal Angular Momentum

Angular Quantum # : $l = r \times \mathrm{Im}[\epsilon \cdot E^* \cdot (\nabla) E + \mu_0 H^* \cdot (\nabla) H]/4\omega$



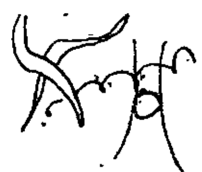
Electric Field Vector : $E_{\pm} = E_0 (1, \pm i, 0)$

Transverse Angular Momenta



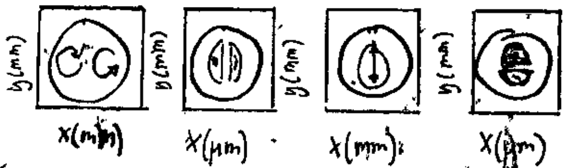
Non-Zero longitudinal Modes : $s \propto E_0^2 \cdot \hat{e}_z$

Enzyme Forces

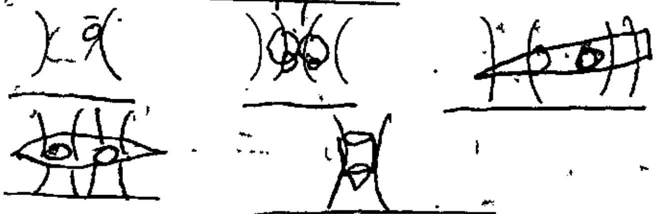


Optical Manipulation:

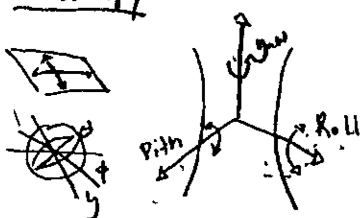
Field Distributions of Spin



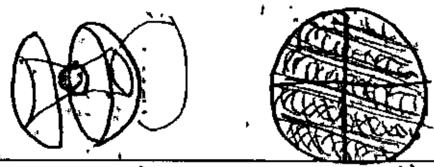
Photon Force Microscopy:



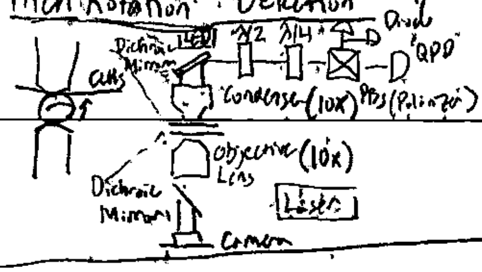
Anisotropy



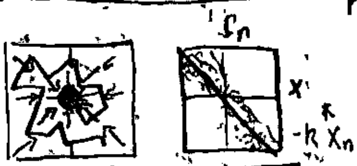
Trapping Principle and Interference:



Pitch Rotation Detection



Brownian Motion:



$$K^* = \sum_{n=1}^{N} \frac{x_n f_n}{x_n^2} ; D^* = \frac{\Delta t}{2\gamma^2} \sum_{n=1}^{N} (f_n - K^* x_n)^2$$

K^* = Stiffness ; f_n^* = Drag Force

$$\gamma \frac{dx(t)}{dt} = \underbrace{-k_x x(t)}_{\text{Force Spring}} + \underbrace{\sqrt{2D} W_x(t)}_{\text{Dirac delta correlated White noise}}$$

Friction Coefficient

Probability $P(\{x_n\}|\theta)$; $x_n = x(t_n)$

$O(\theta) = \int \{dx_n\} P(\{x_n\}|\theta) O(\{x_n\})$

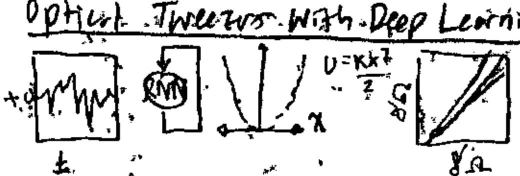
Where $P(\theta|\{x_n\}) = \frac{P(\{x_n\}|\theta) P(\theta)}{Z(\{x_n\})}$ — Baysian Inference

$P_{(k_x/\gamma, D)}(x, y|\{x_n\}_{n=1}^{N}) = N(x|k_n, \gamma_n y) \times \mathrm{InvGamma}(\gamma|\alpha_n, \beta_n)$ — Gauss-Inverse

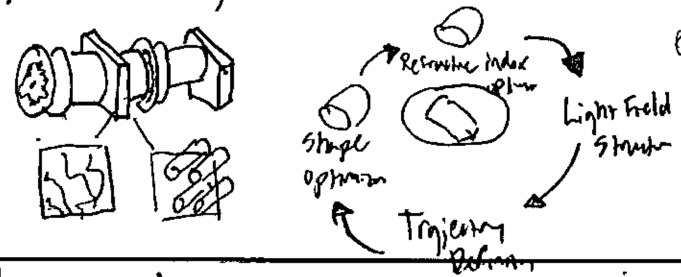
Initial Gauss inverse:

$P_{(k_x/\gamma, D)}(x, y|\emptyset) = N(x|k_0, \gamma_0 y) \times \mathrm{InvGam}(y|\alpha_0, \beta_0)$

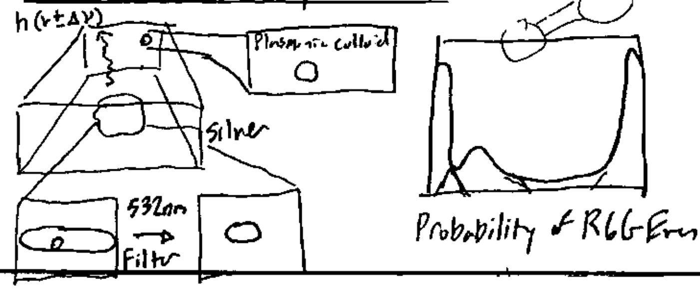
Optical Tweezers With Deep Learning



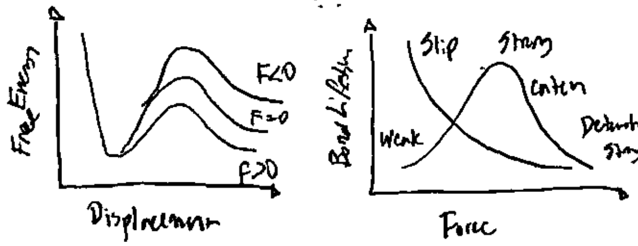
Light-assisted organization of active colloidal matter



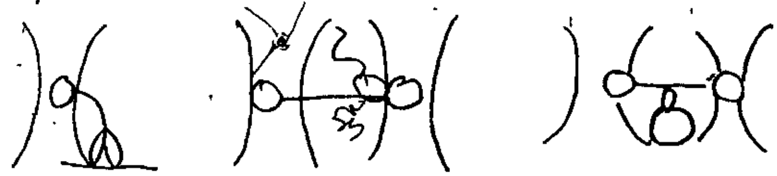
Combination with Spectroscopy



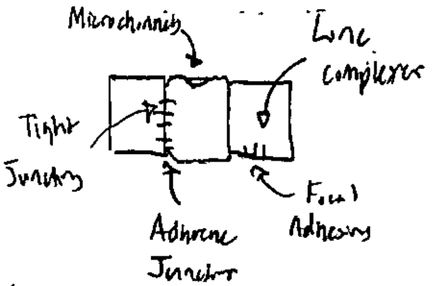
Biophysical Applications:

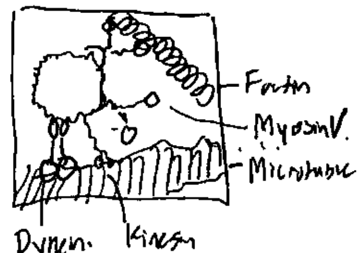


Single Bead Two Bead Three Bead

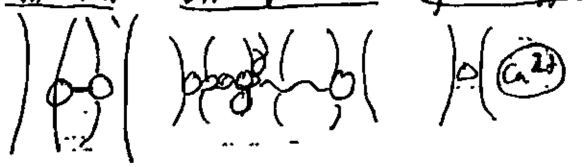


Cell Surface:





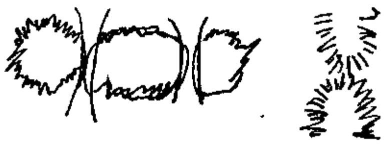
Dual Trap Dual Trap-confocal Optical Trapping



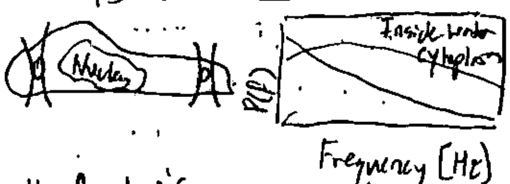
Heat Sensing:

Cellular Phase Imaging:

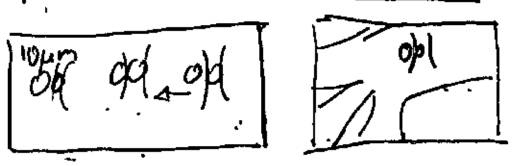
Membrane Tethering:



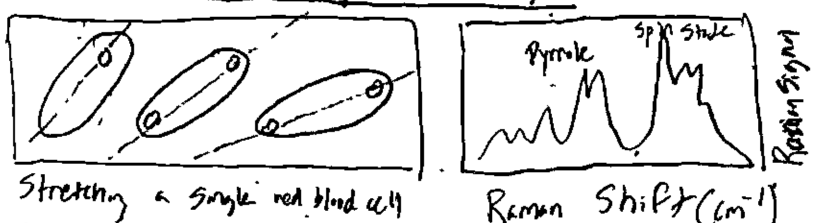
Quantifying Live Cell Material



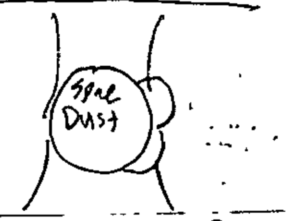
Optical Analysis of Single cells



Raman Tweezers for single-cell Analysis



Space Tweezers



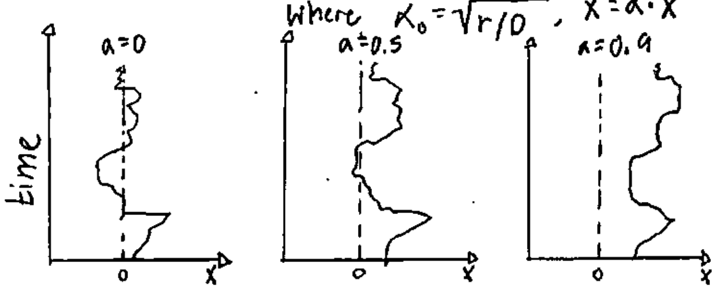
★ Raman Sorting, Id, Counting ★

Particle Resetting:

Normal Diffusion: $P(x,t) = \frac{1}{\sqrt{4\pi D t}} e^{-x^2/4Dt}$ "Gaussian"

Actual Diffusion: $P(x) = \frac{\alpha_0}{2} e^{-\alpha_0 |x|}$ "Laplace"

where $\alpha_0 = \sqrt{r/D}$, $x' = a \cdot x$



Particle resetting $x \longrightarrow ax$; Master Particle Resetting Equation

$$\frac{\partial P(x,t)}{\partial t} = D\frac{\partial^2 P(x,t)}{\partial x^2} - rP(x,t) + \frac{r}{a}P(x/a,t)$$

$[x, x+\delta]$; $[x/a, x/a + \delta/a]$

With steady state approximation:

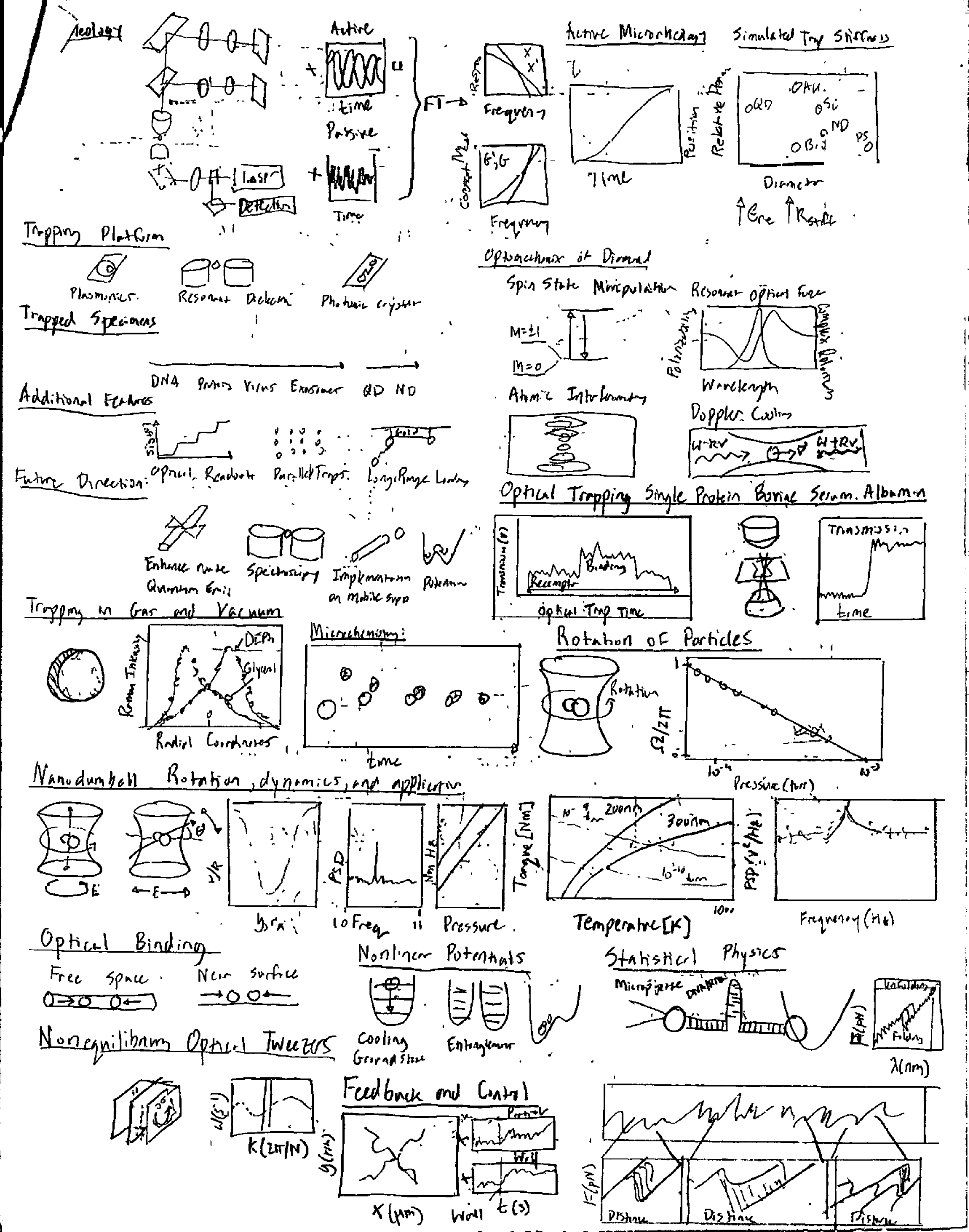
$$D\frac{d^2 P(x,t)}{dx^2} - rP(x) + \frac{r}{a}P(x/a) = 0$$

Fourier Transform to obtain:

$$-(r + Dk^2)\hat{P}(k) + r\hat{P}(ak) = 0$$

With a solution:

$$\hat{P}_{ss}(k) = \prod_{j=0}^{\infty} \frac{r}{r + Dk^2 a^{2j}}$$



Rheology
Active
time
Passive
Laser
Detection
Time
FT
Frequency
Active Microrheology
Time
Position
Simulated Trap Stiffness
QD
Au
Si
ND
Bi
PS
Diameter
Trapping Platform
Plasmonics
Resonant Dielectric
Photonic crystal
Trapped Specimens
DNA Protein Virus Exosome QD ND
Additional Features
Optical Readout
Parallel Traps
Long-Range Loading
Future Direction:
Enhance Quantum Emit.
Spectroscopy
Implementation on Mobile Supp.
Potential
Optomechanics of Diamond
Spin State Manipulation
M=±1
M=0
Resonant Optical Force
Wavelength
Atomic Interferometry
Doppler Cooling
Optical Trapping Single Protein Bovine Serum Albumin
Binding
Release
Optical Trap Time
Transmission
time
Trapping in Gas and Vacuum
Raman Intensity
DEPh
Glycerol
Radial Coordinates
Microrheology:
time
Rotation of Particles
Rotation
Ω/2π
Pressure (torr)
Nanodumbbell Rotation, dynamics, and application
E
PSD
Freq
Pressure
Torque [Nm]
200nm
300nm
Temperature [K]
Frequency (Hz)
Optical Binding
Free space
Near surface
Nonlinear Potentials
Cooling Ground State
Entanglement
Statistical Physics
Micropipette
DNA
Unfolding
Folding
F (pN)
λ (nm)
Nonequilibrium Optical Tweezers
K(2π/N)
Feedback and Control
Well
y (μm)
X (μm)
t (s)
F (pN)
Distance
Distance
Distance

Fourier Transform Identity: $F\left(\frac{df(x)}{dx}\right) = ik\hat{f}(k)$; $F\left(f\left(\frac{x}{a}\right)\right) = a\hat{f}(ak)$

How do you evaluate $F\left(D\frac{d^2P(x)}{dx^2}\right) = D(i(k))^2 f(k)$

Therefore, $-(r+Dk^2-ikv)\hat{P}(k) + rP(ak) = 0$; $P_{ss}(k) = \prod_{j=0}^{\infty} \frac{r}{r-iVka^j+DK^2a^{2j}}$, $\lim_{a\to 0} P_{ss} = \frac{r}{(r-iVK+DK^2)}$

$$P(x) = \frac{\sqrt{\frac{r}{D}}}{2\sqrt{1+\frac{v^2}{4Dr}}} \exp\left[-\sqrt{\frac{r}{D}}|x|\left(\sqrt{1+\frac{v^2}{4Dr}} - \frac{v\cdot \mathrm{sgn}(x)}{2\sqrt{Dr}}\right)\right]$$

$$= \frac{1}{\sqrt{\frac{4D}{r}+\frac{v^2}{r^2}}} \exp\left[-\sqrt{\frac{r}{D}+\frac{v^2}{4D^2}}|x| + \frac{v}{2D}x\right]$$

$P_{ss}(x) = \frac{\kappa_0}{2\sqrt{1+\lambda_0^2}} e^{-(\sqrt{1+\lambda_0^2}-\mathrm{sgn}(x)\lambda_0)\kappa_0|x|}$

Why?

$\kappa_0 = \sqrt{r/D}$; $\lambda_0 = V/(2\sqrt{Dr})$

"Shape of Fourier Transform: $C\cdot\exp(-a|x|+bx)$"

$\hat{P}_j(k) = \langle e^{-ikX_j}\rangle$

$= \frac{r}{(r-iV_jk+D_jk^2)}$; $V_j = Va^j$; $D_j = Da^{2j}$

$F(c\cdot\exp(-a|x|+bx)) = \frac{2ac}{a^2-(b+ik)^2}$

Substituting; $a = \sqrt{\frac{r}{D}+\frac{v^2}{4D^2}}$, $b = \frac{v}{2D}$, $c = \frac{1}{\sqrt{\frac{4D}{r}+\frac{v^2}{r^2}}}$

$P_{ss}(x) = 2\frac{\sqrt{\frac{r}{D}+\frac{v^2}{4D^2}}}{\sqrt{\frac{4D}{r}+\frac{v^2}{r^2}}} \frac{1}{\frac{r}{D}+\frac{v^2}{4D^2}-\left(\frac{v}{2D}+ik\right)^2}$

$= \frac{r}{r-ikV+Dk^2}$

"Full Stochastic Resetting"

"Steady-state Position Distribution of Drift Diffusion With Partial Stochastic Resetting"

$P_j(x) = \frac{\kappa_j}{2\sqrt{1+\lambda_j^2}} e^{-(\sqrt{1+\lambda_j^2}-\mathrm{sgn}(x)\lambda_j)\kappa_j|x|}$

$\kappa_j = \sqrt{r/D_j}$; $\lambda_j = V_j/(2\sqrt{D_j r})$

$\underbrace{\langle X_{ss}\rangle = \sum_{j=0}^{\infty}\langle X_j\rangle = \sum_{j=0}^{\infty} V_j/r = \frac{v}{r}\frac{1}{1-a}}_{\text{Mean}}$; $\underbrace{\sigma^2(X_{ss}) = \sum_{j=0}^{\infty}\sigma_j^2 = \sum_{0}^{\infty}\left(\frac{2D_j}{r}+\frac{V_j^2}{r^2}\right) = \left(\frac{2D}{r}+\frac{v^2}{r^2}\right)\frac{1}{1-a^2}}_{\text{Variance}}$

Standardized Position: $(X_{ss}-\langle X_{ss}\rangle)/\sigma(X_{ss})$; $\langle e^{-ik(X_{ss}-\langle X_{ss}\rangle)/\sigma(X_{ss})}\rangle = \prod_{j=0}^{\infty}\langle e^{-\frac{ik(X_j-\langle X_j\rangle)}{\sigma(X_{ss})}}\rangle$

Expanding the components to second order, and taking expectation yields:

$e^x = 1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots$; $e^{-x} = 1-\frac{x}{1!}+\frac{x^2}{2!}-\frac{x^3}{3!}+\cdots$; $\langle e^{-ik(X_j-X_j)/\sigma(X_{ss})}\rangle$

$\cong 1 - \underbrace{\frac{ik\sigma_j}{\sigma(X_{ss})}}_{\text{Not real}} - \frac{k^2\sigma_j^2}{\sigma^2(X_{ss})} \simeq e^{-\frac{k^2\sigma_j^2}{2\sigma^2(X_{ss})}}$

$\simeq \underbrace{e^{-k^2/2}}_{\text{Gaussian Random Variable.}}$

Sharp Partial Resetting:

Diffusion Time: $P(x,t) = \frac{1}{\sqrt{4\pi Dt}} e^{\frac{-x^2}{4Dt}}$

$0 \xrightarrow{D} \tau \xrightarrow{R} a^2\tau \xrightarrow{D} a^2\tau+\tau \xrightarrow{R} a(a^2\tau+\tau)\ldots$

if $x \to ax$; $t \to a^2\tau$

; Convergence to a Gaussian!

D=Diffusion, R=Reset ; $T_{ene} = \sum_{n=0}^{\infty} a^{2n}\cdot\tau = \frac{\tau}{1-a^2}$ Time Oscillation

$T_{eff} = \sum_{n=1}^{\infty} a^{2n}\cdot\tau = \frac{a^2\tau}{1-a^2}$

Time Dependent Solution: $T_{eff}(m) = t - a^{2m}[1-a^2]\sum_{n=1}^{m} t_n\cdot a^{-2n}$

How was this derived!

Remember, Fourier Transform Identities: $F\left(\frac{dP(x)}{dx}\right) = ik\hat{P}(k)$; $F\left(f\left(\frac{x}{a}\right)\right) = a\hat{F}(ak)$

Where did the solution come from: $\hat{P}_{ss}(k) = \frac{r}{r + Dk^2}$; $P_{ss}(x) = \frac{\alpha_0}{2}\exp(-\alpha_0|x|)$

· A stochastic representation: $X_{ss} = \sum_{j=0}^{\infty} X_j$; Where $\{X_0, X_1, X_2, X_3\}$ are Laplace Random Variables

The Fourier Transform of a Laplace Distribution with variance σ^2 ; $f(x) = \frac{1}{\sqrt{2}\sigma} e^{(-\sqrt{2}|x|/\sigma)}$

Stochastic Representation:

$X_{ss} = \sum_{j=0}^{\infty} X_j$

Moments of a Steady State Distribution

Variance, Kurtosis

$\langle X_{ss}^m \rangle = (-i)^m \frac{d^m \hat{P}_{ss}(k)}{dk^m}\Big|_{k=0}$

$\langle e^{-ikX_j} \rangle \equiv \hat{P}_j(k) = \frac{r}{r + Dk^2 a^{2j}}$

$$F[f(x)] = F\left[\frac{1}{\sqrt{2}\sigma} e^{-\sqrt{2}|x|/\sigma}\right] \cong F\left[\frac{1}{\sqrt{2}\sigma} e^{-k_0|x|}\right] \text{ ; where } k_0 = \sqrt{2}/\sigma$$

$$= \int_{-\infty}^{\infty} e^{-2\pi i k x} \cdot e^{2\pi x k_0} dx = \int_{-\infty}^{0} e^{-2\pi i k x} e^{2\pi x k_0} dx + \int_0^{\infty} e^{-2\pi i k x} e^{-2\pi x k_0} dx$$

$$= \int_{-\infty}^{0} [\cos(2\pi kx) - i\sin(2\pi kx)] e^{2\pi k_0 x} dx + \int_0^{\infty} [\cos(2\pi kx) - i\sin(2\pi kx)] e^{-2\pi k_0 x} dx$$

if $u = -x$; $du = -dx$; $F_x\left[e^{-k_0|x|}\right](k) = \int_0^{\infty} [\cos(2\pi ku) + i\sin(2\pi ku)] e^{-2\pi k_0 u} du$

$$+ \int_0^{\infty} [\cos(2\pi ku) - i\sin(2\pi ku)] e^{-2\pi k_0 u} du$$

$$= 2\int_0^{\infty} [\underbrace{\cos(2\pi ku)}_{\text{even}} - i\underbrace{\sin(2\pi ku)}_{\text{odd}}] e^{-2\pi k_0 u} du$$

$$= 2\int_0^{\infty} \cos(2\pi ku) e^{-2\pi k_0 u} du \text{ ;}$$

Damped cosine Integral

$$\int_0^{\infty} e^{-\omega T} \cos(\omega t) d\omega = \frac{T}{t^2 + T^2}$$

"Integration by parts"

$$F_x\left[e^{-2\pi k_0|x|}\right](k) = \frac{1}{\pi} \frac{k_0}{k^2 + k_0^2}$$

$$= \frac{1}{\pi\sqrt{2}\sigma}\left(\frac{\sqrt{2}/\sigma}{k^2 + \sqrt{2}/\sigma}\right)$$

$$= \frac{1}{\pi\sqrt{2}\sigma}(\sqrt{2}/\sigma) \frac{1}{\frac{\sigma^2 k^2}{2} + 1}$$

$$= \frac{\sqrt{\sigma}}{\pi}\left(\frac{1}{\frac{\sigma^2 k^2}{2} + 1}\right)$$

$$\frac{d\hat{P}_j(k)}{dk} = -\frac{2a^{2j} Drk}{(a^{2j} Dk^2 + r)^2}$$

$$\frac{d^2 P_j(k)}{dk^2} = \frac{2a^{2j} Dr(3a^{2j} Dk^2 - r)}{(a^{2j} \cdot Dk^2 + r)^3}$$

$$\frac{d^3 P_j(k)}{dk^3} = -\frac{24 a^{4j} D^2 rk(a^{2j} \cdot Dk^2 - r)}{(a^{2j} \cdot Dk^2 + r)^4}$$

$$\frac{d^4 P_j(k)}{dk^4} = \frac{24 a^{4j} D^2 r(5a^{4j} D^2 k^4 - 10a^{2j} Drk^2 + r^2)}{(a^{2j} \cdot Dk^2 + r)^5}$$

The Moments of Stochastic Steady State Distribution:

$\langle X_j \rangle = 0$

$\langle X_j^2 \rangle = \frac{2Da^{2j}}{r} = \sigma^2(X_j)$

$\langle X_j^3 \rangle = 0$

$\langle X_j^4 \rangle = \frac{24D^2 a^{4j}}{r^2}$

$$\sigma^2(X_{ss}) = \sum_{j=0}^{\infty} \frac{2D}{r} a^{2j} = \boxed{\frac{2D}{r} \frac{1}{1-a^2} \text{ ; } \sigma_j^2 = 2Da^{2j}/r}$$

$$\langle X_{ss}^4 \rangle = \langle (X_0 + X_1 + \cdots)^4 \rangle = \sum_{i=0}^{\infty} \langle X_i^4 \rangle + 3\sum_{i=0}^{\infty}\sum_{j=0}^{\infty} \langle X_i^2 \rangle \langle X_j^2 \rangle - 3\sum_{i=0}^{\infty} \langle X_i^2 \rangle^2$$

$$= \frac{24D^2}{r^2} \frac{1}{1-a^4} + 3\sum_{i=0}^{\infty} \langle X_i^2 \rangle \sum_{j=0}^{\infty} \langle X_j^2 \rangle - \frac{12D^2}{r^2} \frac{1}{1-a^4} = \frac{12D^2}{r^2} \frac{1}{1-a^4} + 3\left(\frac{2D}{r} \frac{1}{1-a^2}\right)^2$$

$$\text{Kurtosis}(X_{ss}) = \frac{\langle X_{ss}^4 \rangle}{\sigma^4(X_{ss})} = \frac{\frac{12D^2}{r^2} \frac{1}{1-a^4} + 3\left(\frac{2D}{r} \frac{1}{1-a^2}\right)^2}{\left(\frac{2D}{r} \frac{1}{1-a^2}\right)^2} = \frac{6}{1+a^2}$$

$$\langle e^{-ikX_{ss}/\sigma(X_{ss})} \rangle = \prod_{j=0}^{\infty} \frac{1}{1 + \frac{Dk^2 a^{2j}}{r\sigma^2(X_{ss})}} \text{ ; } \lim_{a \to 1} \langle e^{-ikX_{ss}/\sigma(X_{ss})} \rangle = \prod_{j=0}^{\infty} e^{-\frac{Dk^2 a^{2j}}{r\sigma^2(X_{ss})}} = e^{-k^2/2} \text{ ; when } \frac{Dk^2}{r\sigma^2(X_{ss})} \ll 1$$

Drift-Diffusion with Particle Resetting: $\frac{dP(x,t)}{dt} = D\frac{\partial^2 P(x,t)}{\partial x^2} - V\frac{\partial P(x,t)}{\partial x} - rP(x,t) + \frac{r}{a}P(x/a, t)$

V = Drift velocity ; $D\frac{d^2 P(x)}{dx^2} - V\frac{dP(x)}{dx} - rP(x) + \frac{r}{a}P(x/a) = 0$

$$F\left[D\frac{d^2 P(x)}{dx^2} - V\frac{dP(x)}{dx} - rP(x) + \frac{r}{a}P(x/a)\right] = 0$$

Laplace Transform: $\tilde{P}_0(k,s) = \frac{1}{r+Dk^2+s}$

$$\tilde{P}_1(k,s) = \frac{-r}{Dk^2[a^2-1]}\left[e^{[-(r+Dk^2)-Dk^2(a^2-1)]t} - e^{-(r+Dk^2)t}\right] \qquad \frac{1}{s-a} = e^{at}$$

$$= \frac{-r}{Dk^2[a^2-1]}\left[\frac{1}{s+(r+Dk^2)+Dk^2(a^2-1)} - \frac{1}{s+r+Dk^2}\right]$$

$$= \frac{-r}{Dk^2[a^2-1]}\left[\frac{1}{s+r+\cancel{Dk^2}+Dk^2a^2-\cancel{Dk^2}} - \frac{1}{s+r+Dk^2}\right]$$

$$= \frac{-r}{Dk^2[a^2-1]}\,\frac{(\cancel{s}+\cancel{r}+Dk^2)-(\cancel{s}+\cancel{r}+Dk^2a^2)}{(s+r+Dk^2a^2)(s+r+Dk^2)}$$

$$= \frac{-r}{Dk^2[a^2-1]}\,\frac{Dk^2[1-a^2]}{(s+r+Dk^2a^2)(s+r+Dk^2)} = \frac{r}{(r+a^2Dk^2+s)(r+Dk^2+s)}$$

Time-Dependent Variance: $\sigma^2(X(t)) = \frac{2D}{r(1-a^2)}\left(1-e^{-r(1-a^2)t}\right)$

$\langle X^2(s)\rangle = -\frac{\partial^2 P(k,s)}{\partial k^2}\Big|_{k=0}$; We define $F(j) = \frac{r}{r+Dk^2a^{2j}+s}$

$P(k,s) = \sum_{m=0}^{\infty}\tilde{P}_m(k,s) = \frac{1}{r}\sum_{m=0}^{\infty}\prod_{j=0}^{\infty}F(j)$; $\frac{d\tilde{P}(k,s)}{dk} = \frac{1}{r}\sum_{m=0}^{\infty}\left[\left(\prod_{j=0}^{m}F(j)\right)\left(\sum\frac{F'(j)}{F(j)}\right)\right]$; where $F'(j) = \frac{dF(j)}{dk}$

The second Derivative $\tilde{P}(k,s)$ can be written:

$$\frac{d^2P(k,s)}{dk^2} = \frac{1}{r}\sum_{m=0}^{\infty}\left[\left(\prod^{m}F(j)\right)\left(\sum\frac{F(j)}{F(j)}\right)^2 + \left(\prod_{j=0}^{m}F(j)\right)\left(\sum_{j=0}^{m}\frac{F''(j)\cdot F(j) - F'(j)^2}{F(j)^2}\right)\right]$$

The first and second Derivatives: $F'(j) = -\frac{2rDka^{2j}}{(r+Dk^2a^{2j}+s)^2}$

$$F''(j) = -\frac{2rDa^{2j}}{(r+Dk^2a^{2j}+s)^2} + \frac{2rDa^{2j}k[4(r+s)Da^{2j}k+4Da^{4j}k]}{(r+Dk^2a^{2j}+s)^4}$$

After substituting: $F'(j)|_{k=0} = 0$

$$F''(j)|_{k=0} = -\frac{2rDa^{2j}}{(r+s)^2}$$

Substituting, $\frac{\partial^2 P(k,s)}{dk^2} = \frac{1}{r}\sum\left[\prod_{j=0}^{m}\left(\frac{r}{r+s}\right)\right]\left(\sum_{j=0}^{m}\frac{2rDa^{2j}}{(r+s)^2}\cdot\frac{r+s}{r}\right)$

$$= -\frac{2D}{1-a^2}\left(\frac{1}{s(r+s)} - a^2\frac{1}{(r+s)(r(1-a^2)+s)}\right)$$

Which is inverted to find time dependence, $\sigma^2(X) = \langle X^2(t)\rangle = \mathcal{L}^{-1}\left(-\frac{d^2P(k,s)}{dk^2}\Big|_{k=0}\right) = \frac{2D}{r(1-a^2)}\left(1-e^{-r(1-a^2)t}\right)$

Assuming $\{t_1, \dots t_m\}$; $\{t_1, t_2 - t_1, \dots, t_m - t_{m-1}\}$: $0 \xrightarrow{D} t_1 \xrightarrow{R} a^2 t_1 \xrightarrow{D} a^2 t_1 + t_2 - t_1 \xrightarrow{R} a^2(a^2 t_1 + t_2 - t_1) \xrightarrow{D} a^2(a^2 t_1 + t_2 - t_1) + t_3 - t_2$

A rearranged question: $0 \to t_1 \to a^2 t_1 \to (a^2-1)t_1 + t_2 \to (a^4 - a^2)t_1 + a^2 t_2 \to (a^4-a^2)t_1 + (a^2-1)t_2 + t_3$

In, turn yields!

$$\to t + \sum_{n=1}^{m} \left(a^{2m+2-2n} - a^{2m-2n}\right) t_n ,$$

Therefore,

$$T_{eff}(m) = t + \sum_{n=1}^{m}\left(a^{2m+2-2n} - a^{2m-2n}\right) t_n$$
$$= t - a^{2m}\left[1 - a^2\right] \sum_{n=1}^{m} t_n a^{-2n}$$

$$P(x, t, T_{eff}(m)) = \frac{1}{\sqrt{4\pi D T_{eff}(m)}} e^{-\frac{x^2 n=1}{4 D T_{eff}(m)}} \; ; \; Pr(m, t) = \frac{(rt)^m e^{-rt}}{m!}$$ "Poisson Distribution" m = resetting events

Averaging the Distribution over reset: $\sum_{m=0}^{\infty} \frac{(rt)^m e^{-rt}}{m! \sqrt{4\pi D T_{eff}(m)}} e^{-\frac{x^2}{4 D T_{eff}(m)}}$

Which is Fourier Transformed to give

$$\sum_{m=0}^{\infty} \hat{P}_m(k, t, T_{eff}) \equiv \sum_{m=0}^{\infty} \frac{(rt)^m}{m!} e^{-rt - Dk^2 T_{eff}(m)}$$

$$\hat{P}_m(k,t) = \int_0^t dt_m \int_0^{t_m} dt_{m-1} \int_0^{t_2} \frac{m!}{t^m} P_m(k, t, T_{eff}(m)) dt_1$$

Calculating the integrals and Laplace Transforming $P_m(k, s) = \int_0^{\infty} P_m(k,t) e^{-st} dt$

Practice: $\tilde{P}_0(k, t) = \int_0^t \frac{0!}{t^0} P_0(k, t, T_{eff}(m)) dt = \int_0^t \hat{P}_0(k,t) = e^{-(r + Dk^2)t}$; where $T_{eff} = 1$

Particle Regardless of time: Fourier Transformation

$$P_1(k,t) = \int_0^{t_1} \frac{1}{t} P_1(k, t, T_{eff}(m)) dt = \int_0^{t} \frac{1!}{t} \frac{(rt)}{1!} e^{-rt - Dk^2(t + (a^2-1)t_1)} \cdot e^{-rt} dt$$
$$= r \int_0^t e^{-rt} \cdot e^{-rt - Dk^2(t + (a^2-1)t_1)} dt_1 = r \cdot e^{[rt - Dk^2 t]} \int_0^t e^{-Dk^2(a^2-1)t_1} dt_1$$
$$= \frac{-r \cdot e^{-(r + Dk^2)t}}{+Dk^2[a^2-1]} \left(e^{-Dk^2(a^2-1)t} - 1\right)$$

$$P_2(k,t) = \int_0^t dt_2 \int_0^{t_2} \frac{(rt)^2}{t^2} e^{-Dk^2(t + (a^2-1)(t_1 a^2 + t_2))} \cdot e^{-rt} dt_1$$
$$= r^2 e^{-(r+Dk^2)t} \int_0^t \int_0^{t_2} e^{-Dk^2(a^2-1)(t_1 a^2 + t_2)} dt_1$$
$$= r^2 e^{-(r+Dk^2)t} \int_0^t \left[\frac{e^{-Dk^2(a^2-1)(t_1 a^2 + t_2)}}{-Dk^2(a^2-1)a^2}\right]_0^{t_2} dt_2$$
$$= r^2 e^{-(r+Dk^2)t} \int_0^t \left[\frac{e^{-Dk^2(a^2-1)(t_2 a^2 + t_2)} - e^{-Dk^2(a^2-1)t_2}}{-Dk^2(a^4 - a^2)}\right] dt_2$$
$$= \frac{r^2 e^{-(r+Dk^2)t}}{(Dk^2)(a^4 - a^2)} \left[\frac{e^{-Dk^2(a^2-1)(ta^2+t)} - 1}{-Dk^2(a^4 - 1)} - \frac{e^{-Dk^2(a^2-1)t} - 1}{-Dk^2(a^2-1)}\right]$$

Energy Cost Per Passage: Stochastic Resetting and constant return times:

$$\langle T_r\rangle = \left(\frac{1}{r}+\tau_0\right)\left[e^{\sqrt{rL^2/D}}-1\right]$$

To compute energy cost, calculate return time:

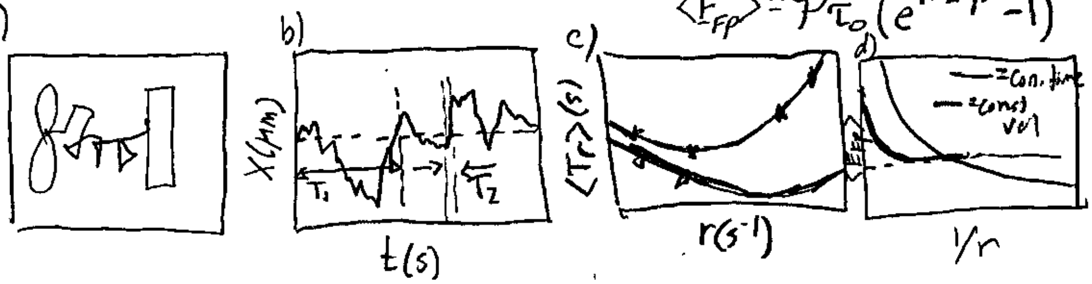
$$\Phi_{FP}(t) = \frac{1}{p}\int_{-\infty} d\vec{R}\,\delta[t-\tau(R)]\int_0^{\infty} dt' f(t')\,G_{abs}(\vec{R},t')$$

Probability of reset   Reset Free Propagator

Geometric Mean: $\frac{p}{1-p}$; setting $\tau(R)=\tau_0$

$$\langle E_{FP}\rangle = \mathcal{P}\tau_0\left(e^{\sqrt{rL^2/D}}-1\right)$$

Virtual Absorbing Wall a)



Realistic velocity Protocol: $\langle E_{FP}\rangle = \frac{\mathcal{P}L}{v}\left[\frac{2\sinh\kappa_0 L}{\kappa_0 L}-1\right]$; for $v<v_{max}$;

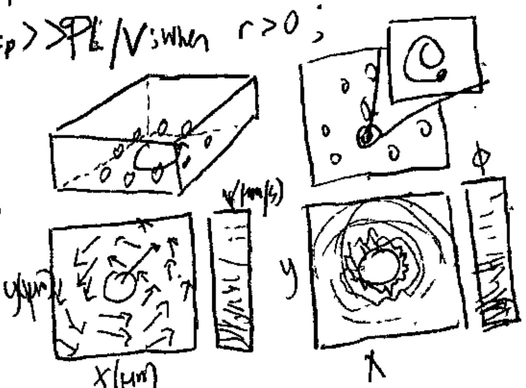
Spatial Crossover between Far from Equilibrium Dynamics:

when $\langle E_{FP}\rangle \equiv 0$; when $r=0$; at $v=v_{max}$ $\langle E_{FP}\rangle >> 2L\gamma/6$

$\langle E_{FP}\rangle > \mathcal{P}L/v$; when $r>0$;

Peclet Number (Pe)
- Low Pe = Strongly inhomogeneous and steady state profiles

High Pe [Far to Non Thermal Equilibrium]



Average Particle Flow Can be Described by hydrodynamic pair Shear Rate $\dot\gamma$; Brownian Relaxation a^2/D are characterized by Péclet Number $P_e=|\dot\gamma|a^2/D$. Measured by $\dot\gamma = r\partial_r(V_\theta/r)$; $P_e(r)\sim 1/r^5$.

P_e $(0.1<P_e(r)<100)$

To quantify Flow of particles:

$V_\theta(r)$ = Azimuthal particle velocity

$V_i^2=\tilde{B}_{ij}^{12}(\vec{r}_{12}; h, a')V_j^1$; $\tilde{B}_{ij}^{12}\propto$ Mobility Tensor

Eigenvalues: $\tilde{B}_{ij}^{12}$; $1/r$ $B_{\parallel}^{12}\sim 1/r^3$ and $B_{\perp}^{2}\sim 1/r^5$

Measurements and characterization of the dynamics of Tracer Particles in an active Network Polystyrene $r=0.75\,\mu m$ and $a=0.25\,\mu m$

Results: ξ = Mesh Network size; a = radius of particles

Trajectories expanded over law cone

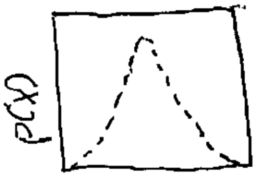
$$\langle \Delta r^2(t)\rangle = \frac{1}{N_t}\sum_{k=1}^{N}(r_k(t)-r_k(0))^2$$

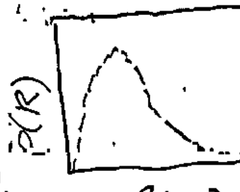
Mean Squared Displacement

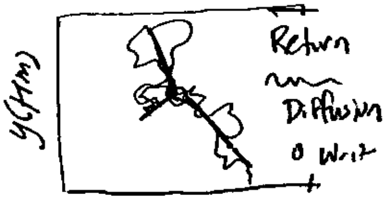
Experimental Realization of Diffusion with Stochastic Resetting

Radial velocity $v = \sqrt{V_x^2 + V_y^2}$

--- = Theoretical Prediction







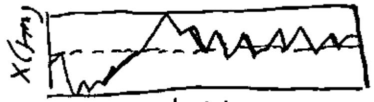
Non-Gaussian

$$\boxed{\rho(X) = \frac{\alpha_0}{2} e^{-\alpha_0 |X|} \quad \text{Where } \alpha_0 = \sqrt{r/D}}$$

"Time"

$$\boxed{\rho(R) = \alpha_0^2 R K_0(\alpha_0 R)}$$ "Radially"

$K_n(z)$ = Modified Bessel of the Second Kind



The radial steady state density is given by: $\rho(R) = p_0^{c.v} \rho_{Diff}(R) + (1 - p_0^{c.v}) \rho_{Ret}(R)$

Radial Steady-State Distribution:

$$\rho(R) = p_D^{c.b} \rho_{Diff}(R) + (1 - p_0^{c.b}) \rho_{Ret}(R)$$

"Radial steady-state Position"

Where $p_0 = (1 + r\tau_0)^{-1}$: Steady state probability

and $\rho_{Diff}(R) = \alpha_0^2 R K_0(\alpha_0 R)$ and $\rho_{Ret}(R)$

$$= \left[\frac{1}{\alpha_0} - R\left[K_0(\alpha_0 R) L_{-1}(\alpha_0 R) + K_1(\alpha_0 R) L_0(\alpha_0 R)\right]\right]$$

L_{-1} = Struve Function

"Non-homogeneous Bessel"

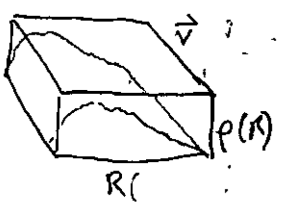
$$x^2 \frac{d^2 y}{dx^2} + x\frac{dy}{dx} + (x^2 - \alpha^2) y = \frac{4\left(\frac{x}{2}\right)^{\alpha+1}}{\sqrt{\pi}\,\Gamma(\alpha + \frac{1}{2})}$$

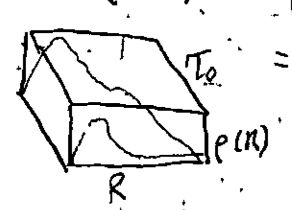
$$p_D^{c.v} = \left(1 + \frac{\pi r}{2\alpha_0 v}\right)^{-1}$$

$$\rho_{Diff}(R) = \alpha_0^2 R K_0(\alpha_0 R) \quad \text{Bessel}$$

$$\rho_{Ret}(R) = \frac{2\alpha_0^2}{\pi} R K_1(\alpha_0 R)$$

Because of rotational symmetry





Energy Cost Per Resetting

$E = \mathcal{P} \cdot \tau(R)$; Where $\mathcal{P}$ = Laser power fixed

$\tau(R)$ = time for the laser to trap at distance R

A Random cost to return an object *

Proportional to the return time whose probability density function is:

$$\phi(t) = \int dR\, \delta\left[t - \tau(R)\right] \int dt' f(t') G_0(\vec{R}, t)$$

; $\tau(R)$ = Return time = R/v

$G_0(\vec{R}, t)$ = Stochastic Dynamics

$f(t) = r \cdot e^{-rt}$

$G_0(R, t) = \frac{1}{4\pi D t} e^{-R^2/4Dt}$

Energy Spent per Resetting:

$$\mathcal{H}(E) = \frac{E}{E_0^2} K_0(E/E_0); \; E_0 = \alpha_0^{-1} v^{-1} \mathcal{P}$$

K_0 = K-Distribution "Hyperbolic or inverse Gaussian distribution i.e. variance-gamma"

$\langle E \rangle \propto v^{-1}$; "Stiffness $K = G\mathcal{P}$; where G = conversion

$\langle E \rangle / E_{min} = V_{max} / v$; Where $V_{max} \approx \frac{1}{2} \xi P / \gamma$

Energy Cost Per Passage: Stochastic Resetting and constant return times:

$$\langle T_r \rangle = \left(\frac{1}{r} + \tau_0\right)\left[e^{\sqrt{rL^2/D}} - 1\right]$$

To compute energy cost, calculate return time:

$$\Phi_{FP}(t) = \frac{1}{p}\int_{-\infty} d\vec{R}\, \delta[t - \tau(R)] \int_0^{\infty} dt' f(t')\, G_{abs}(\vec{R}, t')$$

Probability of reset — Reset Free Propagation

Geometric Mean: $\frac{p}{1-p}$; setting $\tau(R) = \tau_0$

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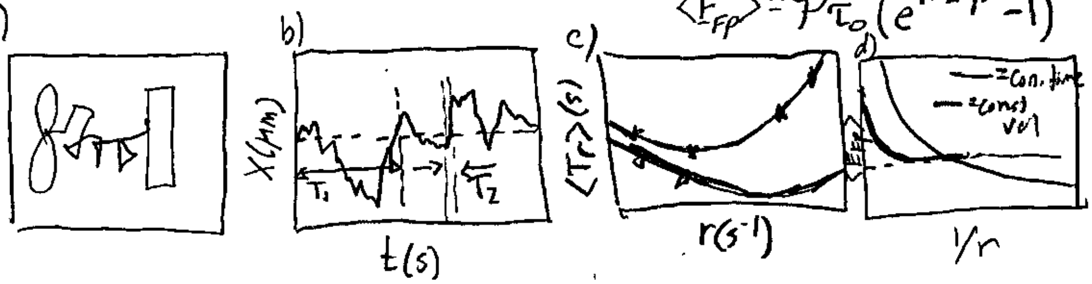


Realistic velocity Protocol: $\langle E_{FP} \rangle = \frac{\mathcal{P}L}{v}\left[\frac{2\sinh \kappa_0 L}{\kappa_0 L} - 1\right]$; for $v < v_{max}$; $\langle E$ ...

Spatial Crossover between Far from Equilibrium Dynamics:

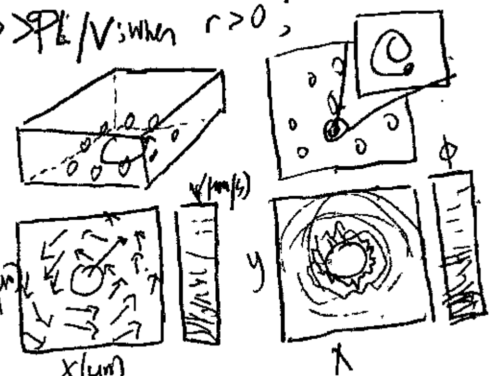
when $\langle E_{FP} \rangle \equiv 0$; when $r \approx 0$; at $v = v_{max}$ $\langle E_{FP} \rangle \gg 2L\gamma/\mathcal{C}$

$\langle E_{FP} \rangle \gg \mathcal{P}L/V$; when $r > 0$;

Peclet Number (P_e)

- Low P_e = Strongly inhomogeneous and Steady State profiles

High P_e [Far to Near Thermal Equilibrium]



Average Particle Flow Can be Described by hydrodynamic p.m

Shear Rate $\dot{\gamma}$; Brownian Relaxation a^2/D

are characterized by Péclet Number $P_e = |\dot{\gamma}| a^2/D$. Measured by $\dot{\gamma} = r\partial_r (V_\theta/r)$; $P_e(r) \sim 1/r^3$.

P_e $(0.1 < P_e(r) < 100)$

To quantify Flow of Particles:

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Trajectories expanded over low conc

$$\langle \Delta r^2(t) \rangle \approx \frac{1}{N_t}\sum_{k=1}^{N} (r_k(t) - r_k(0))^2$$

Mean Squared Displacement

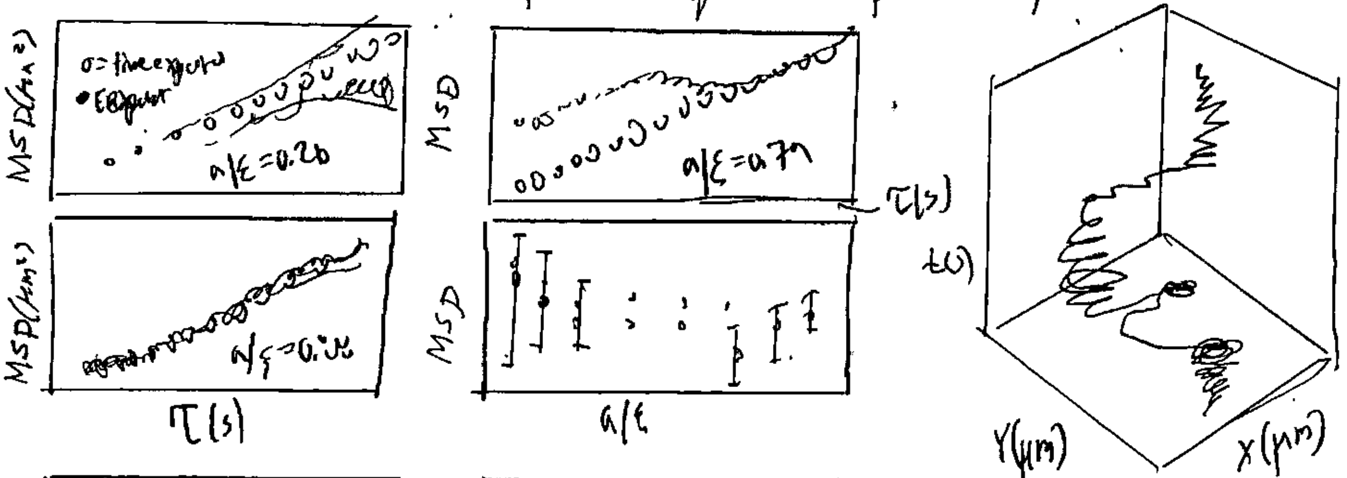
Ergodicity: comparative of Ensemble-average mean-square displacement to time averaged mean-square displacement.

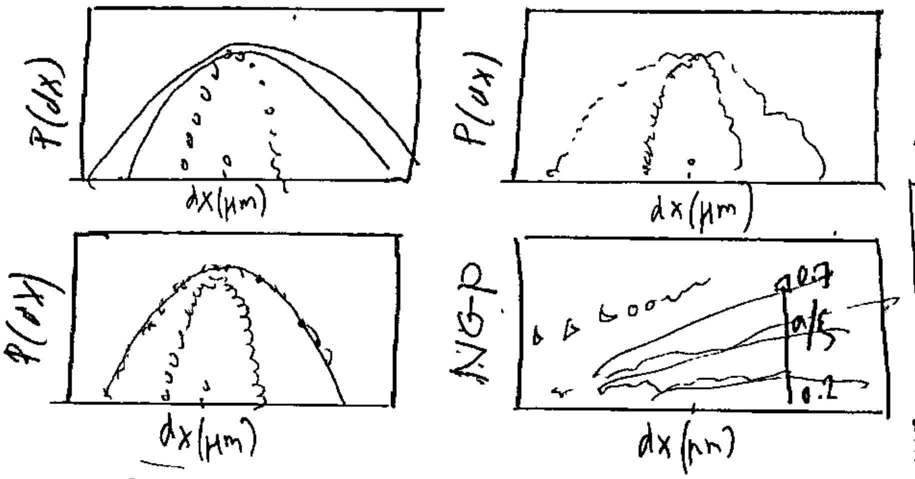
$$\overline{\delta^2_K}(\tau) = \frac{1}{T_K-\tau}\int_0^{T_K-\tau} \left(\vec r_K(t+\tau) - \vec r_K(t)\right)^2 dt$$; τ = lag time ; $\vec r_K(t)$ position of K^{th} particle, T_K = K^{th} trajectory ; t = integration variable for "temporal average"

$$\langle \overline{\delta^2_K}(\tau)\rangle = \frac{1}{N_\tau}\sum_{k=1}^{N} \delta_K^2(\tau)$$

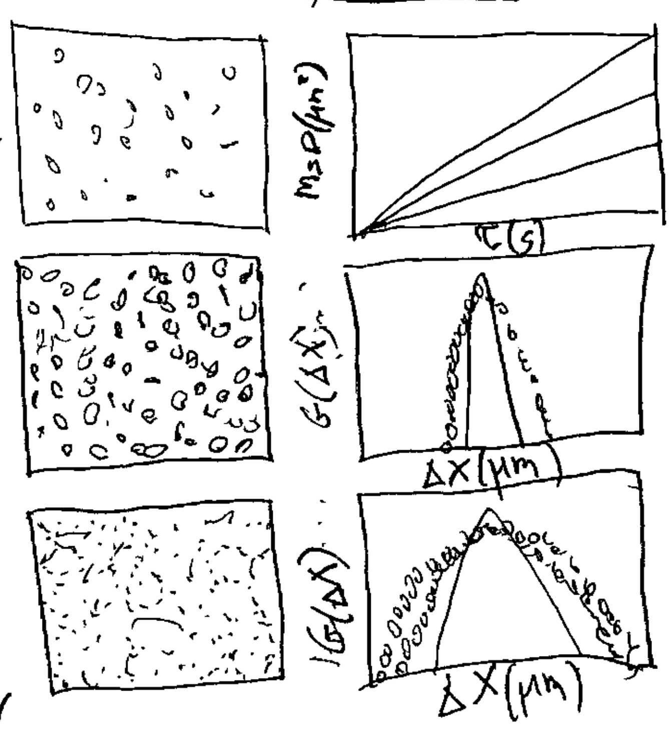
Probability Distribution Function: $$NGP(\tau) = \frac{\langle \Delta x(\tau)^4\rangle}{3\langle \Delta x(\tau)^2\rangle^2} - 1$$ "Non Gaussian parameter"

Relationship of the time expected average vs expected average



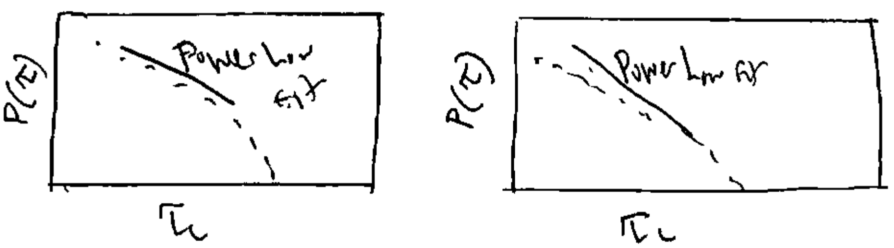


Disordered-induced Fickian, yet non-Gaussian Diffusion in Hetrogeneous media



PDF = Probability Density Function; NGP

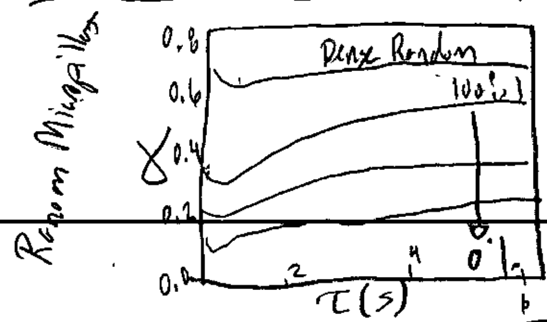
NGP = Non Gaussian Parameter



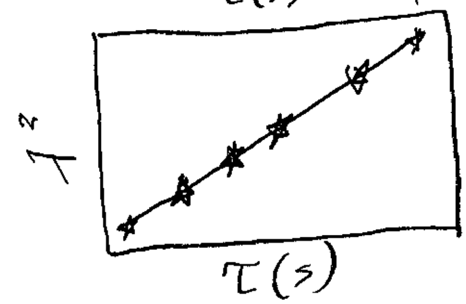
Probability of time. $P(\tau_c) \sim \tau_c^{-v}$

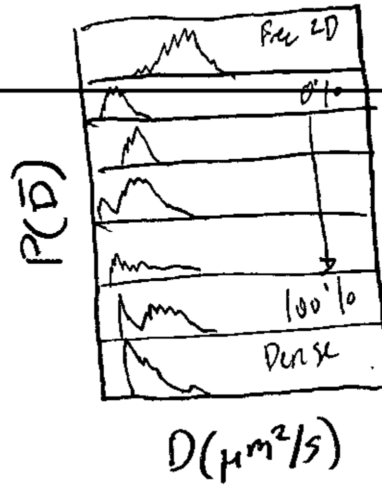
$MSD = 4D\tau^n$

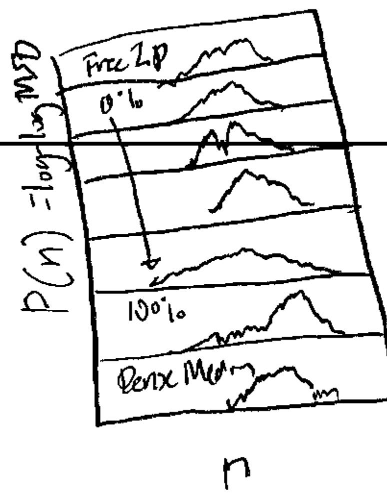
Measurement of Non-Gaussinity



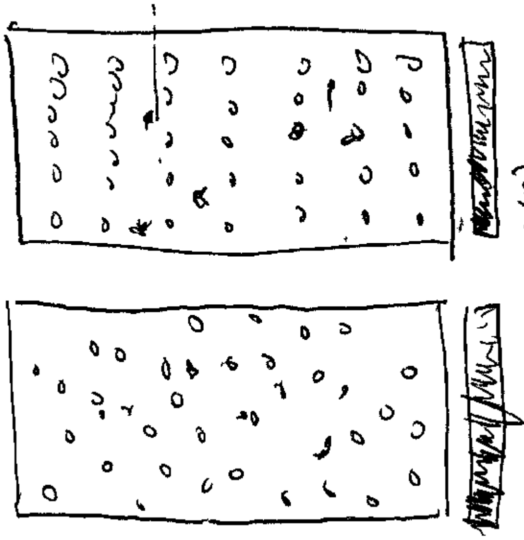
$$\kappa(t) = \frac{\langle \Delta x^4(t)\rangle}{3\langle \Delta x(t)\rangle^2} \; ; \; \Delta G(x) = A e^{-|x|/\lambda} \; ; \; \lambda^2 \sim \tau$$

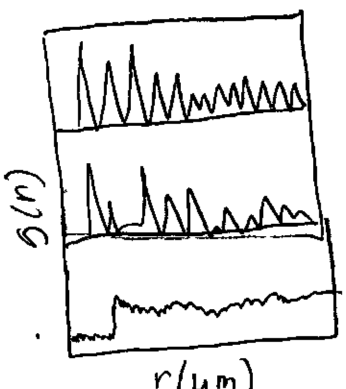






2-D color Map Diffusion:



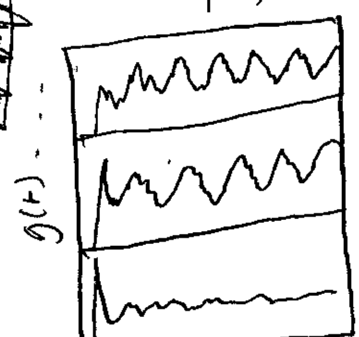


$g(r)$ = pair corelation function

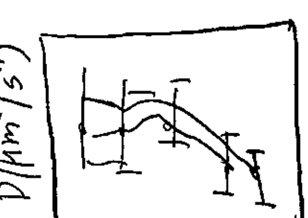
$$\Delta G(\Delta x, t) = \sum_i^N \frac{\omega_i}{\sqrt{4\pi D t}} e^{-x^2/4Dt}$$ "Displacement Probability"

$$P(D) = \frac{1}{\langle D\rangle} e^{-D/\langle D\rangle}$$

"Diffusive Constant Distribution"



Airy Distribution Tails:



Airy Distribution Experiment:

Area of Brownian Excursion:

$$A = \int_0^T x(t)\,dt$$

Probability Distribution:

$$P(A,T) = \frac{1}{\sqrt{D_0 T^3}} F\left(\frac{A}{\sqrt{D_0 T^3}}\right)$$

The Laplace Transform was inverted to provide a closed form:

$$F(\xi) = \frac{2\sqrt{6}}{\xi^{10/3}} \sum_{k=1}^{\infty} e^{-\beta_k/\xi^2} \beta_k^{2/3} U(-5/6, 4/3, \beta_k/\xi^2)$$

$U(\dots)$ = confluent hypergeometric series

$\beta = 2\alpha_k^2/27$, α = ordered absolute values of $Ai(\xi)$

$$-\ln P(A,T) \simeq \begin{cases} \frac{2\alpha_1^3}{27}\frac{D_0 T^3}{A^2} \; ; A \ll \sqrt{D_0 T^3} \;\; \varphi_{1,c} \\ \frac{6A^2}{D_0 T^3} \; ; A \gg \sqrt{D_0 T^3} \end{cases}$$

: Dimensionless Parameter $\tilde{A} \equiv A D_0^{-1/2} \cdot T^{-3/2}$

Particle's Mean Square Displacement: $D_0 = \langle \Delta x^2(t)\rangle / 2t$

Long Time: $-\ln P(A \ll \sqrt{D_0 T}) \simeq T I(a)$

Optimal Fluctuation Method: $S[x(t)] = \frac{1}{2}\int_0^T dt\, \dot{x}^2(t)$

With Lagrange Multiplier: $L(x,\dot{x}) = \dot{x}^2/2 - \lambda x$ where $x(0) \equiv x(T) = 0$; $\lambda = 12A/T^3$

The optimal Trajectory: $x_A^*(t) = (6At/T^2)(1-t/T)$

Scaling the Distribution from $P(A,T) = \frac{1}{\sqrt{D_0 T^3}} f\left(\frac{A}{\sqrt{D_0 T^3}}\right)$: $P(X,t) = \frac{T}{A}\bar{p}_A\left(\frac{XT}{A}, \frac{t}{T}\right)$

This scaled distribution is known as a Fermi-Spohn Distribution.

$$\bar{p}_A\left(z, \frac{t}{T}\right) \simeq \frac{2\alpha_1}{3}\frac{Ai^2\left(\frac{2\alpha_1}{3}z - \alpha_1\right)}{[Ai'(-\alpha_1)]^2} \; ; \; \tilde{A} \ll 1$$

For a parabolic $x_w(t)$: $x_w(t) = Ct(1-t/T)$ Where $Ai'(z) \equiv (d/dz)Ai(z)$

The trajectory: $x(t') = X + (6a - 4X)\left|1 - \frac{2t'}{T}\right| - (6a-3X)\left(1 - \frac{2t'}{T}\right)^2$

The solutions contour

$$x(t') = \begin{cases} \frac{4X^3}{9A^2}\left(\frac{3A}{2X} - \left|\frac{T}{2} - t'\right|\right)^2 & \left|\frac{T}{2} - t'\right| \le \frac{3A}{2X} \\ 0 & \left|\frac{T}{2} - t'\right| \ge \frac{3A}{2X} \end{cases}$$

The conditional Distribution ΔS:

$$-\ln \bar{p}_A\left(z, \frac{1}{2}\right) \simeq \frac{\Delta S}{D_0} = \frac{A^2}{D_0 T^3} g(z) \quad \tilde{A} \gg 1$$

Large-Deviations:

$$g(z) = \left.\begin{cases} 8(z-3/2)^2, & 0 < z \le 3 \\ (8/9)z^3 - 6, & z \ge 3 \end{cases}\right\}$$ Third Derivative $z = 3$

The subcritical regime $X \le X_{c1}(t)$;

$$-\ln \bar{p}_A\left(z, \frac{t}{T}\right) \simeq \frac{A^2}{D_0 T^3}\frac{[z - z(t)]^2}{2\sigma^2(t/T)}$$

where $z(t) = T x_A^*(t)/A$;

$$\sigma^2(s) = s(1-s)(3s^2 - 3s + 1)$$

Experiment, Simulation, and Vervaat Transform

$X_{Br}(t) = X_{Bm}(t) - \frac{t}{T}X_{Bm}(t)$ "Bridge"

$\downarrow$ "Vervaat Transform"

$X_{ex}(t) = X_{Br}(t + \tau \bmod T) - X_{Br}(t)$ "Excursion"

The DV Formalism and the Small-A Tail

$-\ln p(a \ll \sqrt{D_0 T}) \simeq T I(a)$ ← Rate Function

"Legendre-Fenchel Trans" $I(a) = \max[ka - \tilde{I}(k)]$

"Scaled Cumulant Generating Function" $\tilde{I}(k) = \lim_{T\to\infty} \frac{1}{T}\ln\langle e^{Tka}\rangle$

Averaging against Any Distribution

$$P(A,T) = \frac{1}{\sqrt{D_0 T^3}} f\left(\frac{A}{\sqrt{D_0 T^3}}\right)$$

→ According to the DV Method: $I(k) = \xi_{max}\left(\hat{L}^{(k)}\right)$

where ξ_{max} = maximal eigenvalue of $L^{(k)} \equiv \hat{L} + kx$

which is a tilted version of Fokker-Planck

$$\frac{\partial P(x,t)}{\partial t} = \hat{L}P(x,t)$$

$$I(k) = -E_{min}\left(\hat{H}^{(k)}\right)$$

$$-\frac{D_0}{2}\frac{\partial^2 \psi^{(k)}}{\partial x^2} - kx\psi^{(k)} = E^{(k)}\psi^{(k)}$$

$$E_n^{(k)} = (D_0/2)^{1/3} k^{2/3} \alpha_n \; ; \; n = 1, 2, 3$$

"Quantum Mechanical Solution"

Where the wavefunction is:

$\psi_n^{(n)}(x) \propto Ai[(-2k/D_0)^{1/3} x - \alpha_n]$, and the SCGF is given by

$I(k) = -(D_0/2)^{1/3} \cdot k^{2/3} \cdot \alpha_1$

Plugging into $I(a) = \max[ka - I(k)]$

then $k(a) = -4D_0(\alpha_1/3a)^3$

Mapping Ferrari-Spohn Model:

$X_w(t) = Ct^{1/2}(T-t)^{1/2}$; $\Delta X = x(t') - X_w(t')$

$$P(\Delta X) = \frac{l\,Ai^2(l\Delta X - \alpha_1)}{Ai'(-\alpha_1)^2}$$

$$l = [-2\ddot{X}_w(t')/D_0^2]^{1/3}$$

where Path Integral is

$\int Dx(t)\, e^{-\sigma[x(t)]/D_0}$; $S[x(t)]$ = Weiner

$$\int Dy(t)\, e^{-\sigma[y(t)]/D_0}$$

$S[y(t)] = S[X_w(t) + y(t)] = S_0 + \frac{1}{2}\int_0^T (\dot{y}^2 + 2\dot{x}\dot{y})\,dt$

$= S_0 + \frac{1}{2}\int_0^T (\dot{y}^2 - 2\ddot{X}y)\,dt$ where integration by parts is used.

$S_0 = \frac{1}{2}\int_0^T \dot{X}_w^2\,dt$; $X_w(t) = C_0 t(1 - t/T)$

$\ddot{X}_w(t) = -2C/T$

$\bar{S}[y(t)] = S[y(t)] + \frac{2C}{T}\int_0^t y(t)\,dt + S_0$

Distribution of ΔX is given by:

$$P_c(\Delta X) = \frac{\int Dy(t)\, e^{-\bar{\sigma}[y(t)]/D_0} \cdot \delta(y(t') - \Delta X)}{\int Dy(t)\, e^{-\bar{\sigma}[y(t)]/D_0}}$$

$CT \gg \sqrt{D_0 T}$; $l = \left(\frac{4C}{D_0^2 T}\right)^{1/3}$

The canonical Ensemble:

$$P(X;\mu) = \frac{\int Dx(t)\, e^{-S_\mu[x(t)]/D_0}\,\delta(x(t') - X)}{\int Dx(t)\, e^{-S_\mu[x(t)]/D_0}}$$

$$S_\mu[x(t)] = S[x(t)] + \mu D_0 \int_0^T x(t)\,dt$$

$$P_c(\Delta X) = P(\Delta X;\mu = \tfrac{2C}{D_0 T})$$

Joint Probability Density: $P(X,A)$

$P(X,A) = P(A)\,p(X|A)$

Where $P(A)$ is the Airy Distribution.

The joint probability of the canonical ensemble would be $p(X,A) \propto e^{-\mu A} P(X,A)$

$p(X;\mu) = \frac{F(X,\mu)}{N(\mu)}$ where;

$F(X,\mu) = \int_0^\infty e^{-\mu A} P(X,A)\,dA$ and

$N(\mu) = \int_0^\infty e^{-\mu A} P(A)\,dA$. enforces the normalization condition $\int_0^\infty p(X;\mu)\,dX = 1$.

Note $N(\mu)$ is the Laplace Transform of $p(X,\mu)$.

$$p(X|A) = \frac{P(X,A)}{P(A)} \approx \frac{e^{\mu^*} \cdot F(X,\mu^*)}{e^{\mu^* A} N(\mu^*)} = \frac{F(X,\mu^*)}{N(\mu^*)}$$

Where $\mu^* = \mu^*(A) = -k = 4D_0\left(\frac{\alpha T}{3A}\right)^3$.

Stopped

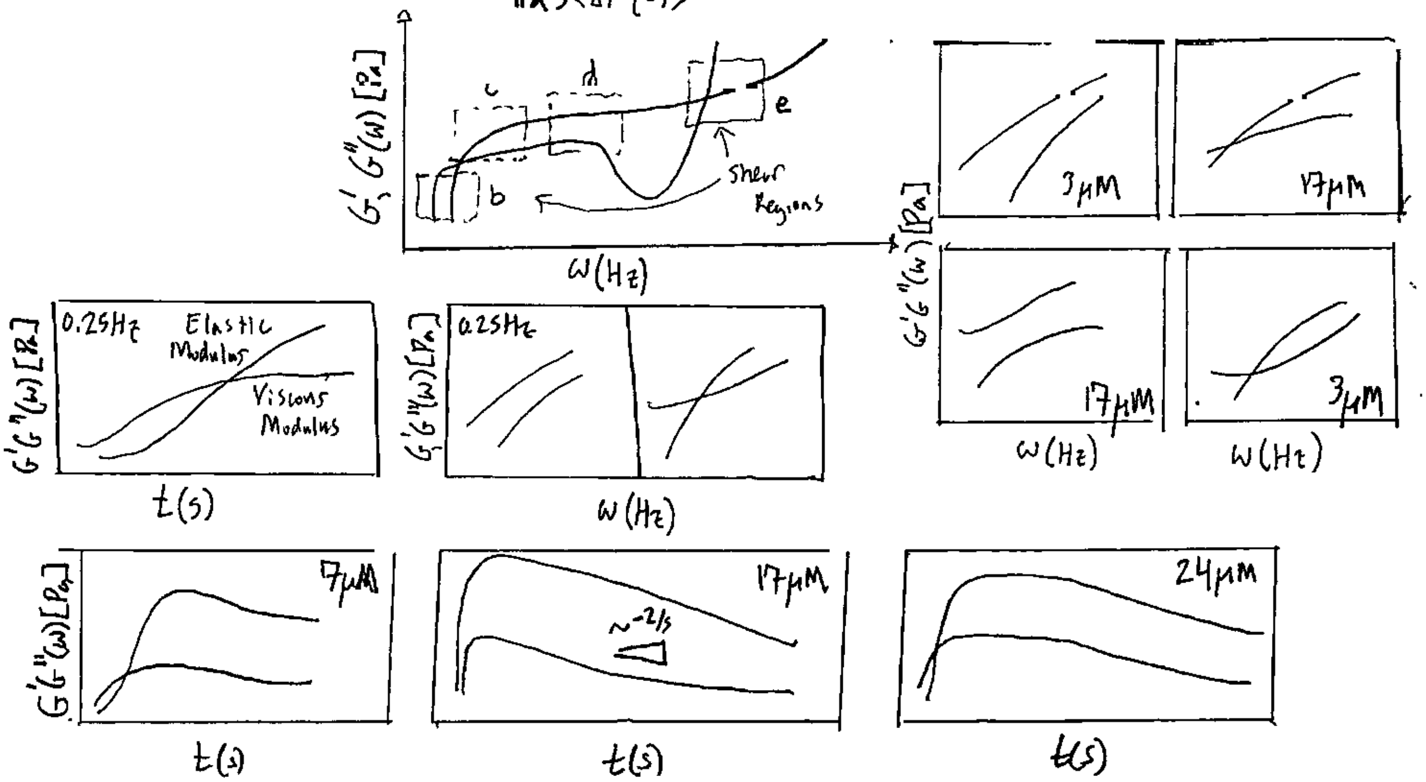
Kinetics of Actin Networks formation Measured by time resolved particle tracking metrology; $\xi = \mu m$

$\xi = 0.3/\sqrt{C_a}$; C_a = Monomer Concentration

Mean Square Displacement: $\langle \Delta r^2(\tau) \rangle = \langle |\vec{r}(t+\tau) - \vec{r}(t)|^2 \rangle$

Laplace-transformed complex shear modulus $G(s)$ and Laplace of MSD $\langle \Delta r^2(s) \rangle$

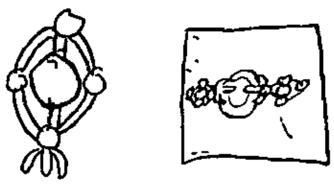
$$G(s) = \frac{k_B T}{\pi a s \langle \Delta r^2(s) \rangle} \; ; \; \tilde{G}(\omega) = G'(\omega) + i G''(\omega)$$

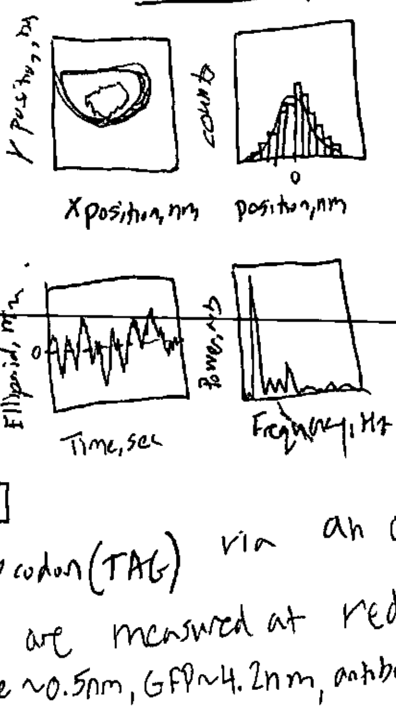


Average Filament Length: $L \sim D \sqrt{\frac{\eta_{eff}/\eta_0 - 1}{A}}$; $\tau = \frac{1}{\omega_e} \cong \frac{\eta_0 \, l^4}{k_B T}$

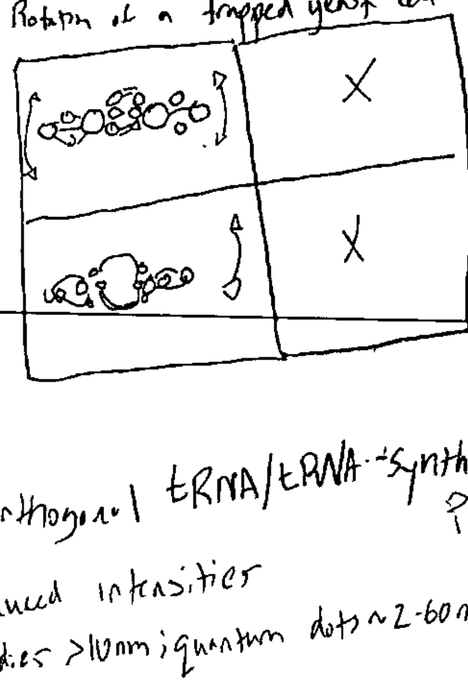
~~Auxillary Optomechanical Tools for 3D~~ Cell Manipulation

Cell Clamp:





Rotation of a trapped yeast cell.



Live cell Molecule Tracking, and Localization Microscopy

Genetic Code expansion

↳ bioorthogonal labels

GCE-based labeling [Non-Canonical amino acid]

↳ Protein sequence → in-frame Stop codon (TAG) via an orthogonal tRNA/tRNA-synthase pair

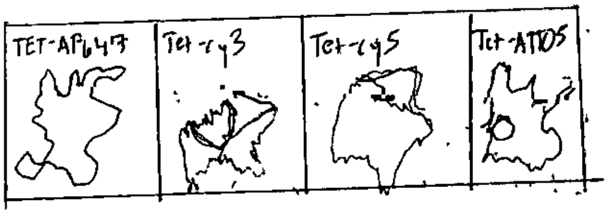
- FL-dyes are photostable, and are measured at reduced intensities
- The label is small (FL-dye ~0.5nm, GFP~4.2nm, antibodies >10nm; quantum dots ~2-60nm)
- Labeling does not require amplification
- Flexibility in modifying a dye.

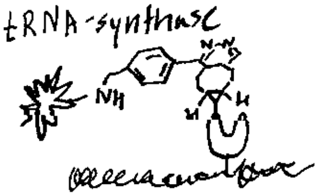
Calibrating Conditions:

Epidermal Growth Factor Receptor [EGFP] corner TAG codon into a single expression receptor to encode cognate pair of tRNA : tRNA-synthase

Signal-to-Noise Ratio

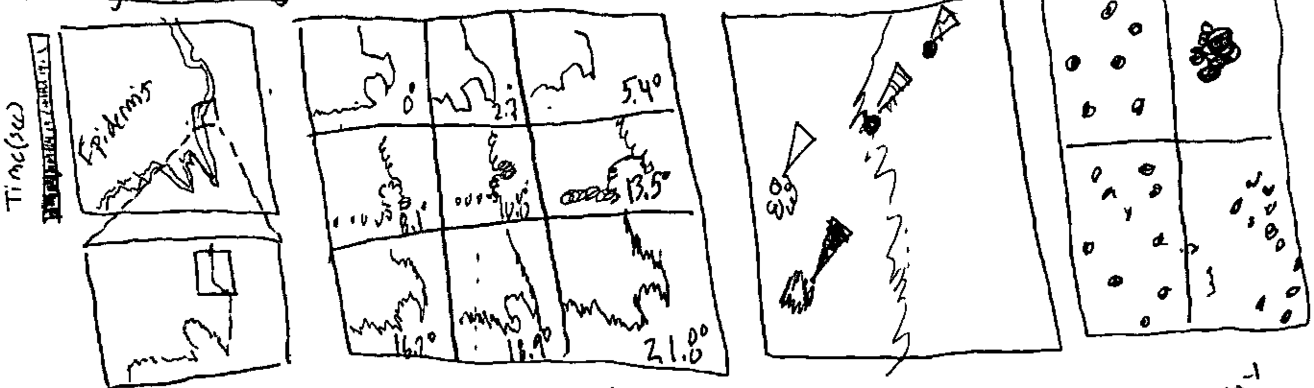
Total Internal Reflection Mode (TIRF)



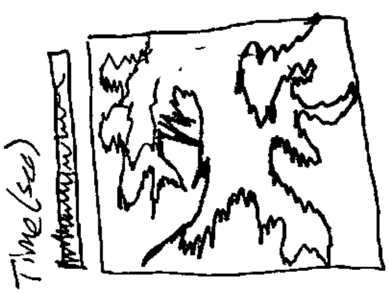


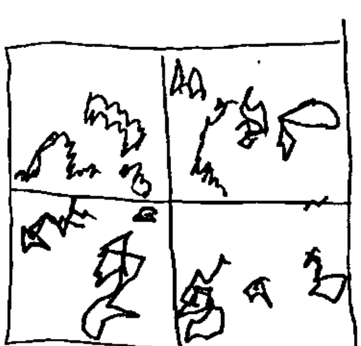
Shaker B V345 BCNK

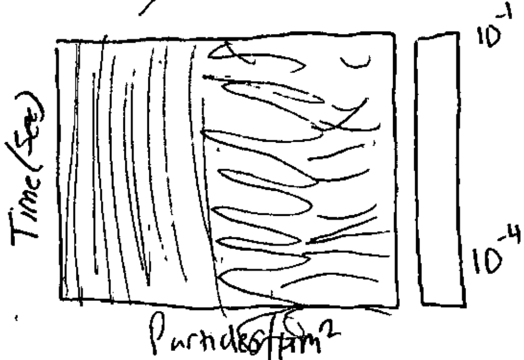
Biorthogonal Labeling:

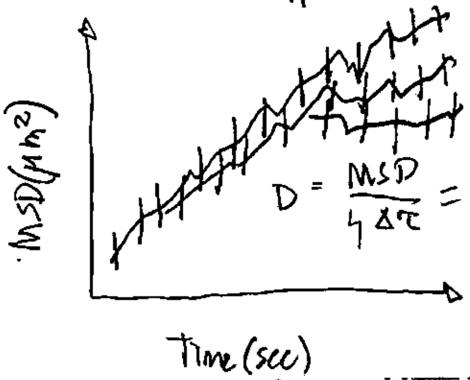


Biorthogonal labeling enables super resolution of Shaker B live cells.









$$D = \frac{MSD}{4\Delta\tau} = \frac{a^2}{4t_{1/2}}$$

$$MSD = \frac{1}{N}\sum (r - \bar{r})^2$$

$$MSD\text{-Distribution}(\Delta\tau) = \frac{1}{T-\Delta\tau}\sum_{t=0}^{T-\Delta\tau}(x(t+\Delta t)-x(t))^2$$

Multiple Peaks in the Displacement Distribution of active random Walkers

Markovian random walks have no correlations at different times.

Spatial Correlations - correlations between motion at subsequent points of time.

Temporal Correlations - relationship to time intervals between direction direction changes. "Non-Poisson, Levy Walker Levy Flight"

Probability Distribution Functions Von Hove Distribution is gaussian.

Homogeneous System - in istropic and flat energy landscape with no temporal correlations.

$$P_r(\vec{r},\tau) = \underbrace{\int P_d(\vec{r},\tau)}_{\text{Prob. [Discrete] Displacement}} \cdot P_c(\vec{r}-r,\tau)\,dr \quad \text{[continuous]}$$

$$P_c(\vec{r},\tau) = K_d \exp\left(-\frac{r^2}{2\alpha^2(r)}\right); \quad K_d = \frac{1}{2^{d/2-1}\cdot\alpha^d\cdot\Gamma\left(\frac{d}{2}\right)\Omega_d}$$

Ω_d = surface area of d-sphere.

PDF of the total displacement

For general discrete process

$\boxed{\text{continuous}}$ "Thermal contact"

$$P_d(\vec{r},\tau) = \sum_{n=0}^{\infty} P_n(\vec{r},\tau) = \sum_{n=0}^{\infty} \underbrace{q_n(\tau)}_{\text{"prob of n hops"}}\cdot \underbrace{p_n(\vec{r})}_{\text{"Prob of } r\text{"}}$$

$\boxed{\text{Discrete}}$ "n hops"

$\alpha^2(\tau) = \langle r^2 \langle \tau \rangle\rangle$ "Mean squared Displacement"

$$P_r(\vec{r},\tau) = \frac{1}{2^{d/2-1}\alpha^d\Gamma\left(\frac{d}{2}\right)\Omega_d}\int P_d(\vec{r},\tau)\exp\left(-\frac{(\vec{r}-r)^2}{2\alpha^2}\right)dr$$

$$= \frac{1}{2^{d/2-1}\cdot\alpha^{d-2}\cdot\Gamma\left(\frac{d}{2}\right)}\int P_d(\vec{r},\tau)\cdot\frac{r^{d-2}}{r}\cdot\exp\left(-\frac{(r^2+r'^2)}{2\alpha^2}\right)\sinh\left(\frac{rr'}{\alpha^2}\right)dr'$$

Moment	Uncentered	Centered
1^{st}	$E(X)$	
2^{nd}	$E(X^2)$	$E[(X-\mu)^2]$
3^{rd}	$E(X^3)$	$E[(X-\mu)^3]$
4^{th}	$E(X^4)$	$E[(X-\mu)^4]$

We now define a Fourier Transform of $P_d(\vec{r},\tau)$:

$\tilde{P}_d(\vec{K},\tau) = \int d\vec{r}\, e^{ikr} P_d(\vec{r},\tau)$; Inverting the Fourier of $P_d(\vec{r},\tau)$

$$e^{-ik\vec{r}}\tilde{P}_d(\vec{k},\tau) = \int d\vec{r}\cdot P_d(\vec{r},\tau); \quad P_d(\vec{r},\tau) = 2\pi\int e^{-ik\vec{r}}\,\tilde{P}_d(\vec{k},\tau)\,dk$$

$$= \frac{2\pi}{-ir}P_d(\vec{k},\tau)\cdot \underline{e^{-ikr} - e^{+ikr}} = -\frac{4\pi}{r}\cdot P_d(\vec{k},\tau)\cdot\frac{e^{-ikr}-e^{kr}}{2i}$$

$$-4\pi P_d(\vec{k},\tau)\frac{\sin(kr)}{r} = -4\pi P_d(\vec{k},\tau)\cdot \mathrm{sinc}(kr) \quad \boxed{\text{"Unsure"}}$$

Surface Area $[\Omega]$ "Hypersphere"

$$\Omega = 2\pi^{n/2}/\Gamma\left(\tfrac{1}{2}n\right)$$

Volume $[V]$ "Hypersphere"

$$V = \frac{\pi^{n/2}}{\Gamma\left(\frac{n}{2}+1\right)}$$

$$P_d(\vec{r},\tau) = \frac{\Omega_d}{(2\pi)^d}\int dk\,k^{d-1}\,\mathrm{sinc}(kr)\,\tilde{P}_d(\vec{k},\tau)$$

$$P_r(\vec{r},\tau) = \frac{\Omega_d}{2^{3d/2-1}\cdot\pi^d\cdot\alpha^{d-2}\cdot r\cdot\Gamma\left(\frac{d}{2}\right)}\times\int\tilde{P}_d(\vec{k},\tau)\cdot k^{d-2}\cdot r'^{d-3}\cdot\sin(kr')\exp\left(\frac{-(r^2+r'^2)}{2\alpha^2}\right)\sinh\left(\frac{rr'}{\alpha^2}\right)dr'dk$$

Integrating over r' yields:

$$P_r(\vec{r},\tau) = \frac{\Omega(d)\exp\left(\frac{-r^2}{2\alpha^2}\right)}{2^{d+1}\pi^d(d-2)ir}\times\int_0^{\infty}dk\,P_d(\vec{k},\tau)\,k^{d-2}\left\{{}_1F_1\left[\frac{d}{2}-1,\frac{1}{2},-\frac{(\alpha^2k-ir)^2}{2\alpha^2}\right]-{}_1F_1\left[\frac{d}{2}-1,\frac{1}{2},-\frac{(\alpha^2k+ir)^2}{2\alpha^2}\right]\right\}$$

$d=3$: $$P_r(\vec{r},\tau) = \frac{1}{2\pi^2 r}\int_0^\infty dk\, \tilde{P}_d(\vec{k},\tau)\cdot k\sin(kr)\exp\left(-\frac{\kappa^2 k^2}{2}\right)$$

$d=2$: $$P_r(\vec{r},\tau) = \frac{1}{2\pi\kappa}\exp\left(-\frac{r^2}{2\kappa^2}\right)\int_0^\infty dk\, P_d(\vec{k},\tau)\int_0^\infty dx\, e^{-x^2/2}\cdot I_0\left(\frac{xr}{\kappa}\right)J_0(\kappa k x)$$

$d=1$: $$P_r(r,\tau) = \frac{1}{\pi}\int_0^\infty e^{-\kappa^2 k^2/2}\cdot\cos(kr)\,\tilde{P}_d(k,\tau)\,dk$$

Fourier Transform of the model Special Discrete Process

Step size distribution $s(l)$

Temporal correlation $g_n(\tau)$

Probability Density $p_{n,l}(\vec{r})$

$$p_n(\vec{r}) = \int d\vec{l}\, p_{n,l}(\vec{r})$$

$$p_{n+1,\vec{l}}(\vec{r}) = \gamma_0 \frac{s(l)}{\Omega_d}\underbrace{\int d\vec{l}\, p_{n,l}(\vec{r}-\vec{l})}_{\text{Random Direction of d-dimensions}} + \underbrace{\gamma_f p_{n,\vec{l}}(\vec{r}-\vec{l})}_{\text{Identical Direction of length to hop n}} + \underbrace{\gamma_b p_{n,-\vec{l}}(\vec{r}-\vec{l})}_{\text{Exact opposite of hop n}}$$

Prob-Uncorrelated Hop

Azimuthal Prob Density

$$p_n(\vec{r}) = \frac{1}{(2\pi)^d}\int d\vec{k}\, e^{-i\vec{k}\vec{r}}\cdot\tilde{p}_n(\vec{k}) = \frac{\Omega_d}{(2\pi)^d}\int dk\, \tilde{p}_n(\vec{k})\, k^{d-1}\operatorname{sinc}(kr)$$

$$n=0,1;\quad p_0(\vec{r}) = \frac{\delta(r)}{\Omega_d r^{d-1}};\quad p_1(\vec{r}) = \frac{s(r)}{\Omega_d r^{d-1}}$$

Explicit Derivation of $\tilde{p}_n(k)$ $\quad \tilde{P}_{n,\vec{l}}(\vec{k}) = \int d\vec{r}\, e^{ikr} p_{n,\vec{l}}(\vec{r})\quad;\quad p_{n,l}(\vec{r}) = \frac{1}{(2\pi)^d}\int dk\, e^{-ikr}\cdot\tilde{p}_{n,l}(\vec{k})$

$$\tilde{p}_{n+1,\vec{l}}(\vec{k}) = \int d\vec{r}\, e^{i\vec{k}\cdot\vec{r}}\Big[\gamma_0\frac{s(l)}{\Omega_d}\cdot\frac{1}{(2\pi)^d}\int dl\, dk'\, e^{-ik'(r-l)}\cdot\tilde{p}_{n,l}(\vec{k}') + \gamma_b\frac{1}{(2\pi)^d}\int d\vec{k}'\, e^{-i\vec{k}'(r-l)}\cdot p_{n,-\vec{l}}(\vec{k}')$$
$$+\gamma_f\frac{1}{(2\pi)^d}\int d\vec{k}'\cdot e^{-i\vec{k}'\cdot(r-l)}\cdot\tilde{p}_{n,\vec{l}}(\vec{k}')\Big]$$

$$= \gamma\frac{s(l)}{\Omega d}\int d\vec{l}\cdot d\vec{k}'\, e^{ik'l} p_{n,l}(k')\,\delta(\vec{k}-\vec{k}') + \gamma_f\int dk\, e^{ikl} p_{n,l}(\vec{k})\,\delta(\vec{k}-k')$$
$$+\gamma_B\int d\vec{k}\, e^{ikl}\cdot p_{n,-l}(\vec{k}')\,\delta(\vec{k}-k')$$

$$= \gamma_0\frac{s(l)}{\Omega d}\int d\vec{l}'\, e^{ikl} p_{n,l'}(\vec{k}) + \gamma_f e^{ik\cdot l} p_{n,l}(\vec{k}) + \gamma_b e^{ikl} p_{n-1}(\vec{k})$$

This may be written as: $|\tilde{p}_{n+1}(\vec{k})\rangle = (\gamma_0 M_0 + \gamma_f M_f + \gamma M_b)|\tilde{p}_n(\vec{k})\rangle$

$$[M_0]_{l,l'} = \frac{s(l)}{\Omega_d}e^{ikl}$$

$$[M_f]_{l,l} = e^{ikl}\delta(\vec{l}-\vec{l})$$

$$[M_b]_{ll} = e^{i\vec{k}\vec{l}}\cdot\delta(\vec{l}+\vec{l})$$

Note: $\tilde{p}_n(\vec{k}) = \langle 1|\tilde{p}_n(\vec{k})\rangle$

Thus the $|\tilde{p}_n(\vec{k})\rangle$ are related to $\tilde{p}_n(\vec{k})$ by a Fourier Transform of equation: $\tilde{p}_n(\vec{k}) = \langle 1|\tilde{p}_n(\vec{k})\rangle$ with $|\tilde{p}_n(\vec{k})\rangle = (\gamma_o M_o + \gamma_f M_f + \gamma_b M_b)^{n-1} |\tilde{p}_1(\vec{k})\rangle$

The real distribution $|\tilde{p}_1(\vec{k})$ after the first hop is:

$$\tilde{p}_{1,\ell}(\vec{k}) = \int d\vec{r}\, e^{i\vec{k}\vec{r}} p_{1\ell}(\vec{r}) = \frac{s(\ell)}{\Omega_d}\int d\vec{r}\, e^{i\vec{k}\vec{r}}\,\delta(\vec{r}-\vec{\ell}) = \frac{s(\ell)}{\Omega_d} e^{i\vec{k}\vec{\ell}}$$

Integrating over all possible vectors $\vec{\ell}$ of hop n, we obtain

$$\tilde{p}_1(\vec{k}) = \int d\vec{\ell}\, p_{1,\ell}(\vec{k}) = \int d\ell \frac{s(\ell)}{\Omega_d} e^{i\vec{k}\vec{\ell}} = \int d\ell\, s(\ell)\,\mathrm{sinc}(k\ell) \equiv f(k)$$

The function is symmetric $f(k) = f(-k)$

$$M_o|\tilde{p}_1(m\vec{k})\rangle = f(mk)|\tilde{p}_1(\vec{k})\rangle$$

$$M_f|\tilde{p}_1(m\vec{k})\rangle = |\tilde{p}_1[(1+m)\vec{k}]\rangle$$

$$M_b|\tilde{p}_1(m\vec{k})\rangle = |\tilde{p}_1[(1-m)\vec{k}]\rangle$$

$$\langle 1|\tilde{p}_1(m\vec{k})\rangle = f(mk)$$

$$\tilde{p}_n(\vec{k}) = \int d\vec{\ell}\, p_{n,\ell}(\vec{k}) = \langle 1|(\gamma_o M_o + \gamma_f M_f + \gamma_b M_b)^{n-1}|\tilde{p}_1(\vec{k})\rangle$$

Is this wrong?

$$M_b^2 = 1$$

$$M_f M_b M_f = M_b;\ M_o M_b M_o = M_o$$

$$M_o M_b M_f = M_o;\ M_o M_b M_o = M_o$$

The steps can be written as a linear combination of $\tilde{p}_n(\vec{k})$

$$\tilde{p}_n(\vec{k},\gamma_b) = \sum_{m=0}^{[\frac{n}{2}]} A_{m,n}(\gamma_b)\cdot \tilde{p}_{n-2m}(\vec{k},0)$$

$$P_d(\vec{r},\tau,\gamma_b) = \sum \tilde{q}_n(\tau)\, p_n(\vec{r},\gamma_b=0)$$

$$\tilde{p}_n(\vec{k}) = \langle 1|[\gamma_o M_o + \gamma_f M_f]^{n-1}|\tilde{p}_1(\vec{k})\rangle$$

$$= \sum_{M=1}^{n} \gamma_o^{M-1}\gamma_f^{N-M} \sum_{m_1=1}^{n}\cdots\sum_{m_M=1}^{n} \delta_{n,\sum_i^M m_i} \prod f(m_k)$$

Steps to evaluate PDF Displacement

1) Calculate the function $f(k)$ using $\tilde{p}_1(\vec{k}) = \int d\ell\, p_{1,\ell}(\vec{k}) = \int d\vec{\ell}\frac{s(\ell)}{\Omega_d} e^{ik\ell} = \int d\ell\, s(\ell)\,\mathrm{sinc}(k\ell) \equiv f(k)$

2) Calculate the Fourier Transform of the discrete distribution given n steps $\tilde{p}_n(\vec{k})$

$$\tilde{p}_n(\vec{k}) = \int d\vec{\ell}\, p_{n,\ell}(\vec{k}) = \langle 1|(\gamma_o M_o + \gamma_f M_f + \gamma_b M_b)^{n-1}|\tilde{p}_1(k)\rangle$$

3) Evaluate the Fourier Transform of discrete distribution $P_d(\vec{k})$ using the Fourier

$$P_d(\vec{r},\tau) = \sum_{n=0}^{\infty} \underset{\text{prob of } n \text{ steps}}{q_n(\tau)} \cdot \underset{\text{prob. of } n \text{ step distance}}{p_n(\vec{r})}$$

4) Calculate the convolution of the discrete and continuous processes using:

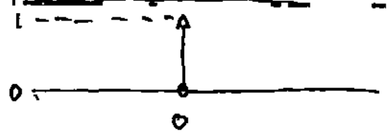
$$P_r(\vec{r},\tau) = \frac{\Omega(d)\exp\left(-\frac{r^2}{2\kappa^2}\right)}{2^{d+1}\pi^d(d-2)ir} \times \int_0^{\infty} dk\, P_d(\vec{k},\tau)\, k^{d-2}\left\{{}_1F_1\left[\frac{d}{2}-1,\frac{1}{2},-\frac{(\kappa^2 k - ir)^2}{2\kappa^2}\right] - {}_1F_1\left[\frac{d}{2}-1,\frac{1}{2},-\frac{(\kappa^2 k + ir)^2}{2\kappa^2}\right]\right\}$$

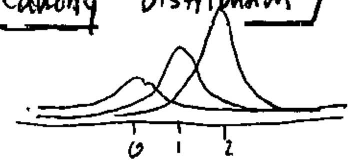
Where ${}_1F_1$ = Confluent Hypergeometric function.

Specific Distributions: "How P_r behaves for step size distributions"

Three types of step size distributions are considered: $s(\ell)$
&
Three types of temporal correlations are considered: q_n

Distributions: $\boxed{\text{Dirac Delta Distribution}}$ $\boxed{\text{Modified Gaussian Distribution}}$ $\boxed{\text{Cauchy Distribution}}$





$$s_D(\ell) = \delta(\ell - a)$$

$$s_g = \left(\frac{\ell}{a}\right)^{2\nu} e^{-\nu\ell^2/a^2} \frac{2\nu^{\nu+1/2}}{a\Gamma(\nu+\frac{1}{2})}$$

$$s_c(\ell) = \frac{4a\ell^2}{\pi(\ell^2+a^2)^2}$$

Most Probable:

$$\langle \ell \rangle_D = \int_0^\infty \ell s_D(\ell)\, d\ell = a$$

$$\langle \ell \rangle_G = \int_0^\infty \ell s_G(\ell)\, d\ell = a\frac{\sqrt{\nu}\,\Gamma(\nu)}{\Gamma(\nu+1/2)}$$

$$\langle \ell \rangle_C = \int_0^\infty \ell s_c(\ell)\, d\ell = \infty$$

Variance:

$$\sigma_D^2 = \langle \ell^2 \rangle_D - \langle \ell \rangle_D^2 = 0$$

$$\sigma_G^2 = \langle \ell^2 \rangle_G - \langle \ell \rangle_G^2 = a^2\left(1 + \frac{1}{2\nu} - \frac{\nu\Gamma^2(\nu)}{\Gamma^2(\nu+1/2)}\right)$$

"Does have asymptotic $\nu \to \infty$"

$$\sigma_G^2 = \langle \ell^2 \rangle - \langle \ell \rangle_G^2 = \infty$$

$f(k)$:

$$f_D(k) = \mathrm{sinc}(ak)$$

$$f_G(k) = {}_1F_1\left(\nu+\frac{1}{2}, \frac{3}{2}, -\frac{a^2k^2}{4\nu}\right) = \frac{(-1)^{\nu-1}(\nu-1)!\sqrt{\nu}\, e^{-a^2k^2/4\nu} \cdot H_{2\nu-1}\left(\frac{ak}{2\sqrt{\nu}}\right)}{ak(2\nu-1)!}$$

$$f_c(k) = e^{-ak}$$

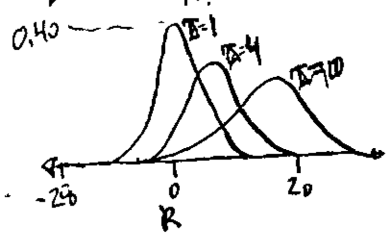
Temporal Correlations: $\boxed{\text{Poissonian}}$ $\boxed{\text{Binomial}}$ $\boxed{\text{Geometric}}$

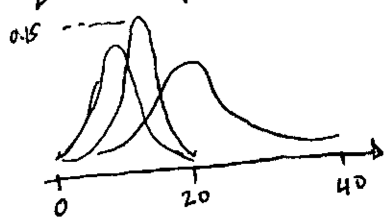
$$q_n^{Pois}(\tau) = \frac{\tau^n e^{-\tau}}{n!}$$

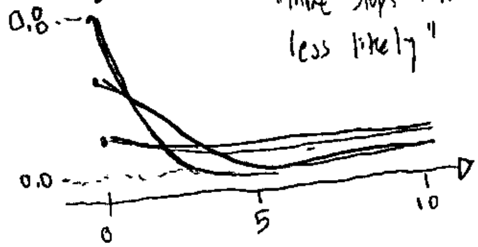
$$q_n^{bin}(\tau) = \binom{\tau}{n} p^n (1-p)^{\tau-n}$$

$$q_n(\tau) = \tau^{-1}(1-\tau^{-1})^n$$

"More steps always less likely"







$$\langle n \rangle = \sum n q_n = 3$$

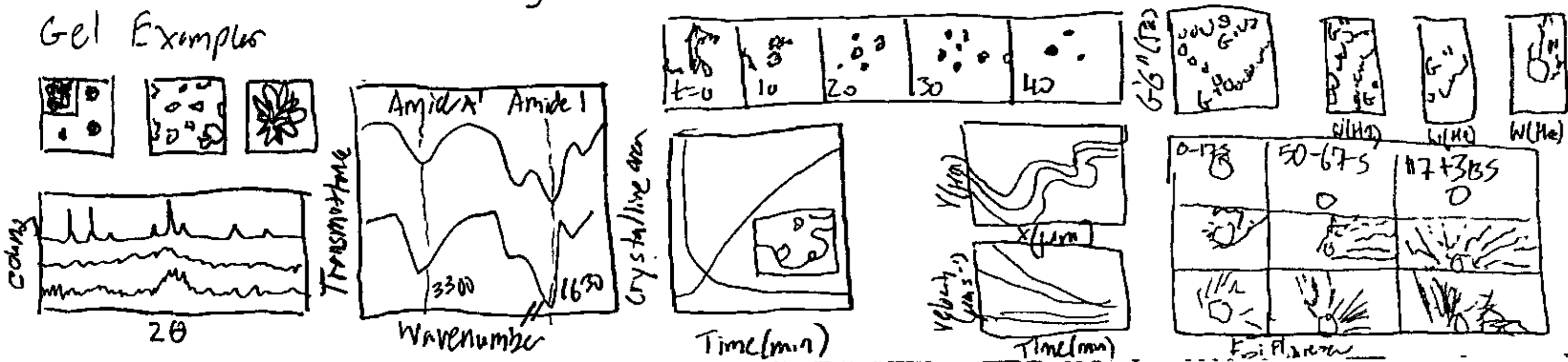
Most Probable

$$\langle n \rangle = \sum_{n=0}^\infty n q_n = 3$$

$$\langle n \rangle = \sum n q_n = 3$$

Real-Time In-Situ Monitoring of a Tunable Pentapeptide Gel-Crystal Transition

Gel Examples



Time

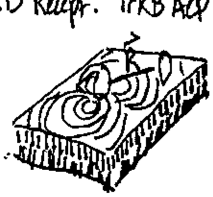
Temp

Conc

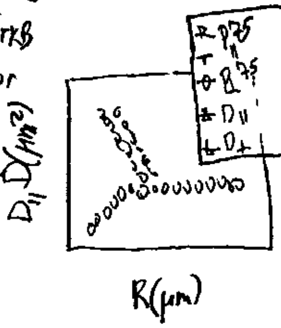
Flow Arrest in Plasma membrane: Neuron Plasma Membrane is a highly dynamic area on the outside of the cell.

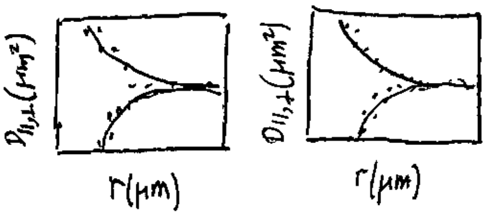
- Flow or at investigation with surface cell, G-proteins.

TrkB Recepr. TrkB Act Single TrkB Receptor

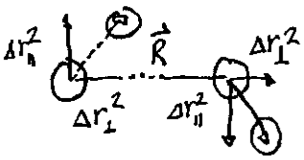


Correlated Displacement: $D_{\parallel,\perp}(R,\tau) = \left\langle \Delta r^{\alpha}_{\parallel,\perp}(\tau)\,\Delta r^{\beta}_{\parallel,\perp}(\tau)\,\delta(|r^{-\alpha\beta}|-R)\right\rangle_{\alpha\neq\beta}$



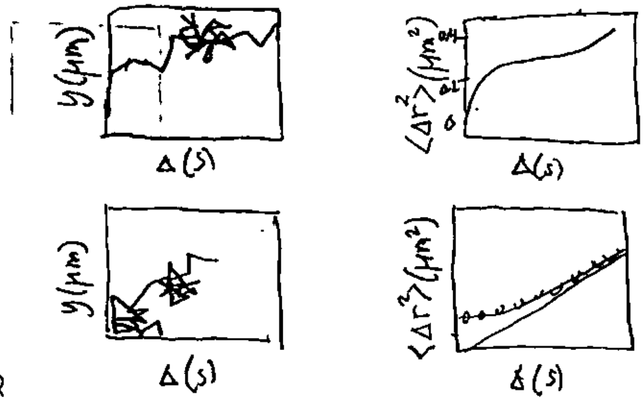


"correlated Diffusion"



Characterization of department within the plasma membrane

Transient confinement of Motion:



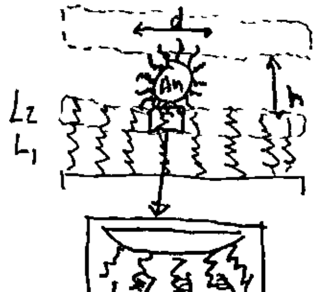
Mean square Deviation: $\langle \Delta r^2\rangle = \langle r_D^2\rangle[1+A_1\exp(-\Delta/t_0)]$

Modified mean-square displacement:

$\langle \Delta r^2\rangle = \langle \Delta r_D^2\rangle\left[1+A_1\exp\left(\frac{-\Delta}{t_1}\right)\right]+4D_0\Delta$

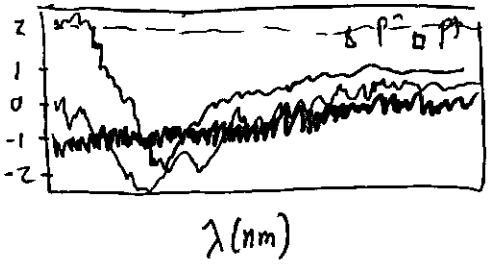
Nanoparticle Mobility over a surface as a probe for weak transient disordered peptide interaction

- A way to measure single amino acid mutations on a transient peptide peptide interaction.



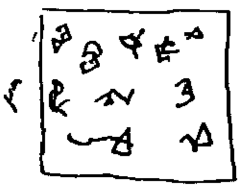
P−: C-E-E-G-A-S-R-H-W-E-E-E

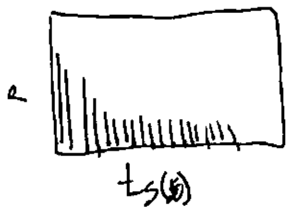
P+: C-E-E-G-A-S-R-H-W-E-K-A-K



Diffusion: $\langle \Delta \chi^2(\tau)\rangle = 4Dt^{\alpha}$ Diffusion Exponent.

Transport coefficient $[D\ \text{"}\mu m^2/s^{\alpha}\text{"}]$





Eqns

Optical Resonance of Dielectrics (sphere): $n\dfrac{j_\ell'(nkr)}{j_\ell(nkr)} = \dfrac{y_\ell'(kr)}{y_\ell(kr)}$; r = radius ; n = index of refraction ; k = wave vector ; $j_\ell(z)$ = Bessel 1st ; $y_\ell(z)$ = Bessel 2nd

Bessel Functions: $x^2\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} + (x^2-\alpha^2)y = 0$

1st: $J_\alpha(x) = \sum_{m=0}^{\infty} \dfrac{(-1)^m}{m!\,\Gamma(m+\alpha+1)}\left(\dfrac{x}{2}\right)^{2m+\alpha}$

spherical: $j_n(x) = (-x)^n\left(\dfrac{1}{x}\dfrac{d}{dx}\right)^n\dfrac{\sin x}{x}$; $y_n(x) = -(-x)^n\left(\dfrac{1}{x}\dfrac{d}{dx}\right)^n\dfrac{\cos x}{x}$

Eigenfrequencies: $\nu_{\ell m} = \nu_\ell^{(0)} - \frac{1}{3}\varepsilon\eta\nu_\ell^{(0)}\left(1 - 3\dfrac{m^2}{\ell(\ell+1)}\right)$; $\nu_\ell^{(0)}$ = unperturbed freq. m = Quantum # ; η = shape deformation ; ℓ = number of wavelengths ; m = Intensity Maxima

Power Transfer Ratio: $T = \dfrac{\pi^4}{4}\operatorname{sinc}^2\left[\frac{1}{2}\sqrt{1+\left(\dfrac{\Delta\beta L_0}{\pi}\right)^2}\right]$

clockwise -or- counterclockwise

$\Delta\beta$ = phase mismatch ; L_0 = ?

Refractive index: $n_\pm = n[1 \pm r_2\Omega(n^{-2}-1)/c]$; n = refractive index ; r_2 = radius of resonator ; c = speed of light ; $\lambda = c/\omega c$; Optical wavelength ; $\dfrac{\lambda}{n}\left(\dfrac{dn}{d\lambda}\right)$ = Lorentz correction to Fresnel-Fizeau drag coefficient

Sagnac Effect: $t_1 = \dfrac{2\pi R + \Delta L}{c}$; $\Delta L = R\omega t_1$

$= \dfrac{2\pi R}{c \pm R\omega}$; $\Delta t = \dfrac{4\pi R^2\omega}{c^2 - R^2\omega^2}$

Light Propagation: $\omega_c \to \omega_c \pm |\Delta_{sag}|$; $\Delta_{sag} = \pm\Omega\dfrac{n r_2\omega_c}{c}\left(1 - \dfrac{1}{n^2} - \dfrac{\lambda}{n}\dfrac{dn}{d\lambda}\right)$

Time Propagation optical pulse: $\tau = L\left(\dfrac{n}{c} + \dfrac{\omega}{c}\dfrac{dn}{d\omega}\right) = L\left(\dfrac{n}{c} - \dfrac{\lambda}{c}\dfrac{dn}{d\lambda}\right)$; Group Velocity ; $D = \dfrac{d}{d\lambda}\left(\dfrac{\tau}{L}\right) = -\dfrac{1}{c\lambda}\left(\lambda^2\dfrac{d^2n}{d\lambda^2}\right)$

$v_1 = \dfrac{\omega_1}{k_1}$; $v_2 = \dfrac{\omega_2}{k_2}$; $v_g = \dfrac{\omega_2-\omega_1}{k_2-k_1}$

Optical Resonator Hamiltonian $H = H_0 + H_{int} + H_{dr}$

"Optical Modes" $H_0 = \Delta_L a_1^\dagger a_1 - (\Delta_L + \Delta_{sag})a_2^\dagger a_2 + \omega_m b^\dagger b$

"Optical Modes" coupling (Optical + Mech) + (optical resonator) $H_{int} = -\zeta x_0 a_1^\dagger a_1(b^\dagger + b) + J(a_1^\dagger a_2 + a_2^\dagger a_1)$

$H_{dr} = i(\varepsilon_L a_1^\dagger - \varepsilon_L^* a_1)$

$\Delta_L = \omega_L - \omega_c$; a_1, a_2 = optical annihilation operators ; J = optical strength ; b = mechanical annihilation operator ; $\zeta = \omega_c/r$ = coupling strength ; r = radius

$\dot{a}_1 = (i\Delta_L - \gamma_1)a_1 + i\zeta x_0(b + b^\dagger)a_1 - iJa_2 + \varepsilon_L$; $\varepsilon_L = \sqrt{2\gamma_1 P_{in}/\hbar\omega_L}$; Driving P_{in} = input power

$\dot{a}_2 = [i(\Delta_L + \Delta_{sag}) - \gamma_2]a_2 - iJa_1$; $\gamma_{1,2} = \dfrac{\omega_c}{Q_{0,1,2}}$

$\dot{b} = -(i\omega_m + \gamma_m)b + i\zeta x_0 a_1^\dagger a_1$; $x_0 = \sqrt{\hbar/2m\omega_m}$; γ_m = Dampening rate

Steady-state $a_1, a_2, b = 0$; Balancing [illegible]

$[\omega_m^2 + \gamma_m^2]x_s = \hbar\zeta|a_1|_s^2$

$|a_1|_s^2$ = optical Density

x_s = displacement

[illegible] = $\dfrac{x_s(\Delta_{sag} \neq 0) - x_s(\Omega = 0)}{x_s(\Omega = 0)}$

[illegible]

$a\,|n\rangle = \sqrt{n}\,|n-1\rangle$; $a^\dagger|n\rangle = \sqrt{n+1}\,|n+1\rangle$

$[N, a] = Na - aN = (a^\dagger a)a - a(a^\dagger a) = a(aa^\dagger - a^\dagger a)$ $= a^\dagger[a, a^\dagger] = a^\dagger$

$H = \hbar\omega N + \dfrac{\hbar\omega}{2}$

$[H, a^\dagger] = \hbar\omega a^\dagger$; $[H, a] = -\hbar\omega a$

$H(a|\psi\rangle) = aH|\psi\rangle - \hbar\omega a|\psi\rangle = aE|\psi\rangle - \hbar\omega a|\psi\rangle$

$H|\psi\rangle = E|\psi\rangle$; $H(a|\psi\rangle) = (E - \hbar\omega)(a|\psi\rangle)$

$H(a^\dagger|\psi\rangle) = (E + \hbar\omega)(a^\dagger|\psi\rangle)$

Supermodes: $a_\pm = (a_1 \pm a_2)/\sqrt{2}$

$H_0 = \omega_+ a_+^\dagger a_+ + \omega_- a_-^\dagger a_- + \omega_m b^\dagger b$; $\Delta\omega_\pm = -\Delta_L - \frac{1}{2}\Delta_{sag} \pm J$

$H_{dr} = \dfrac{i}{\sqrt{2}}\left[\varepsilon_L(a_+^\dagger + a_-^\dagger) - H.c.\right]$; $H_{int} = -\dfrac{\zeta x_0}{2}(a_+^\dagger a_- b + b^\dagger a_-^\dagger a_+) + \dfrac{\Delta(\omega)}{2}(a_+^\dagger a_- + a_-^\dagger a_+)$

$\underbrace{p = a_-^\dagger a_+}_{\text{Ladder operator}}$; $\delta n = \underbrace{a_+^\dagger a_+ - a_-^\dagger a_-}$

Population Inversion operator of optical supermodes

$\dot{b} = -(\gamma_m + i\omega_m) b + \frac{i\zeta x_0}{2} p$

$\dot{p} = -2(\gamma + iJ)p + \frac{i}{2}(\Delta_{sag} - \zeta x_0 b)\delta n + \frac{1}{\sqrt{2}}(\epsilon_L^* a_+ + \epsilon_L a_-^\dagger)$; $\gamma = (\gamma_1 + \gamma_2)/2$

Mechanical Gain : $G = G_0 + G$

$G_0 = \frac{(\zeta x_0)^2 \gamma \delta n}{2(2J - \omega_m)^2 + 8\gamma^2}$; $G = \frac{|\epsilon_L|^2 (\zeta x_0)^2 (\omega_m - 2J)(\Delta_{sag} + 2\Delta_L)\gamma}{4[\beta^2 + (2\Delta_L + \Delta_{sag})^2\gamma^2][(2J - \omega_m)^2 + 4\gamma^2]}$

Population Inversions

$\delta n \approx \frac{2J|\epsilon_L|^2}{\beta_0^2 + 4\gamma^2\Delta_L^2}(\Delta_L + \Delta_{sag})$; $\beta_0 = \beta(\Delta_{sag} = 0)$

$\beta = J^2 + \gamma^2 - \Delta_L^2 + \frac{(\zeta x_0)^2 n_b}{4} - \Delta_{sag} \cdot \Delta_L$

Stimulated Emitted Phonon Number:

$N_b = \exp[2(G - \gamma_m)/\gamma_m]$

Threshold Condition:

$P_{th} \approx \frac{2\hbar\gamma\gamma_m\omega_e[M + \gamma^2(2\Delta_L + \Delta_{sag})^2]}{\gamma_1 J (\zeta x_0)^2 (\Delta_L + \Delta_{sag})}$

$M = (J^2 + \gamma^2 - \Delta_L^2 - \Delta_{sag} \cdot \Delta_L)^2$

Optical Spinning Speed

$R = 10\log_{10}\frac{N_b(\Omega > 0)}{N_b(\Omega < 0)}$

Acoustic Nonreciprocity : $N_b(\Delta_{sag} > 0) \neq N_b(\Delta_{sag} < 0)$

$H_0' = \omega_c a_1^\dagger a_1 + (\omega_c - \Delta_{sag}) a_2^\dagger a_2 + \omega_m b^\dagger b$; $H_{int}' = -\zeta x_0 a_1^\dagger a_1 (b^\dagger + b) + J(a_1^\dagger a_2 + a_2^\dagger a_1)$; $\zeta = \omega_c / r_1$

$H_{dr}' = i(\epsilon_L e^{-i\omega_L t} a_1^\dagger - \epsilon_L^* e^{i\omega_L t} a_1)$

$U = e^{-i\omega_L t(a_1^\dagger a_1 + a_2^\dagger a_2)}$. Rotating Frame: $H = U^\dagger H U - iU^\dagger \frac{\partial U}{\partial t}$ "Product Rule"

$H = H_0 + H_{int} + H_{dr}$; $H_0 = -\Delta_L a_1^\dagger a_1 - (\Delta_L + \Delta_{sag}) a_2^\dagger a_2 + \omega_m b^\dagger b$; $H_{int} = -\zeta x_0 a_1^\dagger a_1 (b^\dagger + b) + J(a_1^\dagger a_2 + a_2^\dagger a_1)$

Supermode Operators: $a_\pm = (a_1 \pm a_2)/\sqrt{2}$ $\qquad H_{dr} = i(\epsilon_L a_1^\dagger - \epsilon_L^* a_1)$

$[a_+, a_+^\dagger] = [a_-, a_-^\dagger] = 1$; $[a_\pm, a_\mp^\dagger] = 0$

Where $H_0 = \omega_+ a_+^\dagger a_+ + \omega_- a_-^\dagger a_- + \omega_m b^\dagger b$

$H_{dr} = \frac{i}{\sqrt{2}}[\epsilon_L(a_+^\dagger + a_-^\dagger) - H.c.]$; $\omega_\pm = -\Delta_L - \frac{1}{2}\Delta_{sag} \pm J$

$H_{int} = H_{int}^0 + H_{int}'$

$H_{int} = H_{int}^0 + H_{int}' = -\frac{\zeta x_0}{2}[(a_+^\dagger a_+ + a_-^\dagger a_-) + (a_+^\dagger a_- + a_-^\dagger a_+)](b^\dagger + b) + \frac{\Delta_{sag}}{2}(a_+^\dagger a_- + a_-^\dagger a_+)$

Rotating Frame: $H_{int}^0 = -\frac{\zeta x_0}{2}[a_+^\dagger a_- b e^{i(2J - \omega_m)t} + a_+ a_-^\dagger b^\dagger e^{-i(2J - \omega)t}$
$+ a_+^\dagger a_- b^\dagger e^{i(2J + \omega_m)t} + a_+ a_-^\dagger b e^{-i(2J + \omega_m)t}$
$+ (a_+^\dagger a_+ + a_-^\dagger a_-)(b^\dagger e^{i\omega_m t} + b e^{-i\omega_m t})]$

Under the approximation: $2J + \omega_m, \omega_m \gg |2J - \omega_m|$

$n_\pm = n[1 \pm R\Omega(n^{-2} - 1)/c]$ $\qquad H_{eff} = (\Delta_+ - i\gamma)a_{cw}^\dagger a_{cw} + (\Delta_- - i\gamma)a_{ccw}^\dagger a_{ccw} + (J - i\gamma_c)a_{cw}^\dagger a_{ccw}$

$\Delta_{sag} = \frac{nR\Omega}{c}\left(1 - \frac{1}{n^2} - \frac{\lambda}{n}\frac{dn}{d\lambda}\right)$; $\qquad + (J - i\gamma_c)a_{ccw}^\dagger a_{cw} + i\sqrt{\gamma_{ex}}\, a_{in}(a_{cw}^\dagger - a_{cw})$

$\Delta_\pm = \Delta_L + J \pm \Delta_{sag}$; $\gamma = (\gamma_a + \gamma_{ex})/2 + \gamma_c$

$\Delta_a = \omega_a - \omega_l$; $\omega_{1/2} = \omega_a + J \pm \omega' - i(\gamma \pm \gamma')$; $2\omega' = \left[2(D^2+4J^2\gamma_c^2)^{1/2} + 2D\right]^{1/2}$

$2\gamma' = \left[2(D^2+4J^2\gamma_c^2)^{1/2} - 2D\right]^{1/2}$; $D = \Delta_{sag}^2 + J^2 - \gamma_c^2$

$\eta = \left|\frac{\Delta\omega(\Omega \neq 0)}{\Delta\omega(\Omega = 0)}\right| = \left[\frac{D^2+4J^2\gamma_c^2}{(J^2+\gamma_c^2)^2}\right]^{1/4} > 1$

$\Delta\omega = \omega_1 - \omega_2 = 2\omega' - 2\omega$

"Enhancement Factor" | "Enhancement of Transmission" Steady-State: $\bar{a}_{cw} = \frac{\sqrt{\gamma_{ex}}\, a_{in}(\gamma + i\Delta_-)}{(\gamma+i\Delta_+)(\gamma+i\Delta_-)+(J-i\gamma_c)^2}$

Input-output relation, $a_{out} = a_{in} - \sqrt{\gamma_{ex}}\,\bar{a}_{cw}$

$\bar{a}_{ccw} = \frac{\sqrt{\gamma_{ex}}\, a_{in}(iJ+\gamma_c)}{(\gamma+i\Delta_+)(\gamma+i\Delta_-)+(J-i\gamma_c)^2}$

Optical Transmission rate: $T = \left|\frac{a_{out}}{a_{in}}\right|^2 = \left|1 - \frac{\gamma_{ex}(\gamma+i\Delta_-)}{(\gamma+i\Delta_+)(\gamma+i\Delta_-)+(J-i\gamma_c)^2}\right|^2$

Rotation Rate: $\Delta T(\Delta_a = 0) \approx \frac{J^2+\gamma_c^2+2J\Delta_{sag}}{(\gamma+\gamma_c)^2+4J^2}$

Where $\beta = \frac{|\bar{a}_{cw}|^2}{|\bar{a}_{ccw}|^2} = \frac{\gamma^2+(\Delta_a+J-\Delta_{sag})^2}{J^2+\gamma_c^2}$

Multiple particle Detection: $H_{eff} = (\Delta'_+ - i\gamma')a^\dagger_{cw}a_{cw} + (\Delta'_- - i\gamma')a^\dagger_{ccw}a_{ccw} + C_1 a^\dagger_{cw}a_{ccw} + C_2 a^\dagger_{ccw}a_{cw} + i\sqrt{\gamma_{ex}}\,a_{in}(a^\dagger_{cw} - a_{cw})$

$\Delta_a + J - \Delta_{sag}$

Where $\Delta'_\pm = \Delta_a + \sum_{i=1}^{N} J_i \pm \Delta_{sag}$; $\gamma' = (\gamma_0 + \gamma_{ex})/2 + \sum_{i=1}^{N}\gamma_{c,i}$;

$C_{1,2} = \sum_{i=1}^{N}(J_i - i\gamma_{ci})e^{(\mp i2m\beta_j)}$

Air-Liquid Interface

Optical Force: $F_o = \frac{nP}{c}$

P = input optical power

c = speed of light, n = refractive index

Shear-flow Equation: $F_s = \mu A \frac{u}{y}$; μ = Dynamic viscosity ; u = speed of liquid at the air-liquid interface

y = distance of stem ; A = area

Speed at liquid-phase Boundary: $u = \frac{yn}{\mu A c}P$

Cross section area for scattering: $\sigma_{sca} = \frac{2\lambda^2}{3\pi}\alpha^6\left|\frac{m^2-1}{m^2+2}\right|$; $\alpha = \frac{2\pi r}{\lambda}$; $m = \frac{n}{n_1}$ — sphere / silicone

Intensity within a mode volume: $I = \frac{F \cdot P_i}{A_m}$; $F_{scat} = \frac{2\sigma_{sca} \cdot I \cdot n_1}{c}$; $F_d = 6\pi\mu_1 r v$

Tension of Taper:

$T = 6.19\mu R^{5/2}\Omega \int_{-B/2}^{n}\left(h - \sqrt{r^2-x^2} + r\right)dx$; μ = viscosity, r = radius of taper, R = Radius of sphere

Index of refraction (Whispering Gallery Modes): $n_\pm = n[1 \pm nv(n^{-2}-1)/c]$; $v = R\Omega$

$\Delta\omega_F = 2\omega\frac{nR\Omega}{c}\left(1 - \frac{1}{n^2} - \frac{\lambda}{n}\frac{dn}{d\lambda}\right) = \eta\Omega$

Rate Equations for Amplitude: $\dot{a}_\Theta + \left[\frac{t^2}{T} + i(\omega - \omega_{res} + \eta\Omega)\right]a_\Theta = i\frac{t}{T}a_{in}$

Optical Transmission

$\vec{T} = \left|\frac{\vec{a}_{out}}{a_{in}}\right|^2 = 1 - \frac{-t^4}{t^4 + T^2(\omega - \omega_{res} \pm \eta\Omega)^2}$

$\vec{a}_{out} = \left(1 + \frac{t^2}{2}\right)\vec{a}_{in} + i t a_\Theta$

$\dot{a}_G + \left[\frac{t^2}{T} + i(\omega - \omega_{res} - \eta\Omega)\right]a_G = i\frac{t}{T}\overleftarrow{a}_{in}$

$\overleftarrow{T} = \left|\frac{\overleftarrow{a}_{out}}{a_{in}}\right|^2 = 1 - \frac{t^4}{t^4 + T^2(\omega - \omega_{res} - \eta\Omega)^2}$

$\overleftarrow{a}_{out} = \left(1 - \frac{t^2}{2}\right)\overleftarrow{a}_{in} + i t a_G$

Pump optical [illegible] $2\pi R = m\lambda/n_{eff}$; R: Radius [illegible]

Doppler Shift: $f_s = [illegible]$; Propagation β [illegible]

Approximate WGM Wavenumber: $\beta_{mq} \approx \frac{1}{R}\left[m + \alpha_q\left(\frac{m}{2}\right)^{1/3} - \frac{n}{\sqrt{n^2-1}}\right]$

$\kappa \cong \left(\frac{m}{2}\right)^{1/3}(\alpha_{q,s} - \alpha_q)$

n = refractive index ; α_q : qth zero of Airy $Ai(-\alpha_q)$ [illegible]

WGM Resonators: Resonant Wavelengths: $m\lambda = 2\pi R n_{eff}$ Sensitivity of Relative Humidity: $S = \frac{1}{\lambda}\frac{\delta\lambda}{\delta RH}$

Frequency of water thread vibrations: Total Sensitivity: $S = S_{rad} + S_{refraction\ index} = \frac{1}{\lambda}\left(\frac{1}{R}\frac{\delta R}{\delta RH} + \frac{1}{n_{eff}}\frac{\delta n_{eff}}{\delta RH}\right)$

$f_i = \frac{i}{2L}\sqrt{\frac{T}{\rho'}}$ || ρ' = mass density per length L; T = Tension of water bridge; i = order of mode

Change of relative Humidity: $\Delta RH_{min} = \frac{\Delta\lambda_{min}}{\frac{d\lambda}{dRH}}$

Tension of Liquid: Quality-Factors: $Q = \lambda / \Delta\lambda_{min}$

$T = a\left[\frac{\pi D^2}{8}\right]\cdot(\epsilon_0(\epsilon-1)E^2) + b\gamma\rho D$ || Diffusion Model: $X_p = \sqrt{2Dt}$

Diffusion constant

ϵ_0 = Vacuum permittivity; D = Bridge Diameter

E = Electric field; ϵ = relative permittivity

γ = Surface tension; a & b = ratio between

γ = Surface Tension

R = Droplet Radius

Eigenfrequencies of Capillary Shape

$f_i^2 = \frac{\gamma i(i+1)(i+2)}{4\pi^2 R^3[(i+2)\rho_o + (i+1)\rho_w]} = \frac{k}{m}$

Electromotive-Forces and Interfacial tension

$Q = \frac{1}{\kappa\lambda}$; $\kappa = \frac{8\mu\pi^2}{\rho V_g \Lambda^2}$; $V_g = \sqrt{\frac{4.5\pi\gamma}{\rho\Lambda}}$ Λ = capillary wavelength; K = Absorption coefficient

V_g = group velocity, μ = liquid velocity, ρ = mass density

ρ_o = mass density of octane

ρ_w = mass density of water; i = mode order

m = effective mass; k = spring constant

r = changes of water droplet radius

$m\ddot{r}(t) + b\dot{r}(t) + kr(t) = 2\pi[A(t)]^2/c$

b: Dampening coefficient, c = speed of light

$|A(t)|^2$ = optical power circulating the droplet

$r(t)$ = cavity inflation

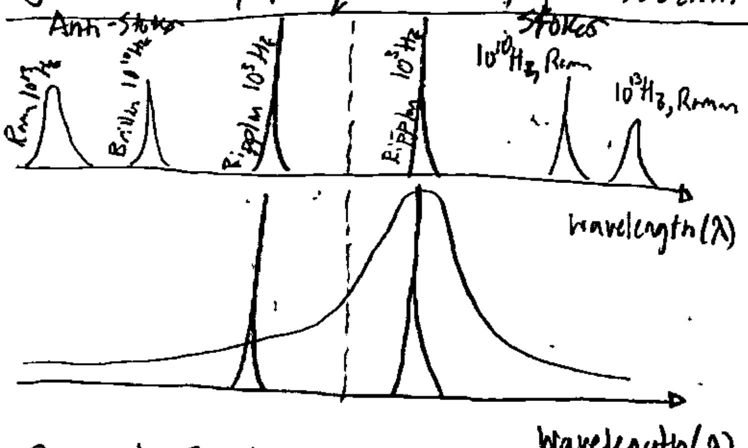
Optical resonance Frequency $\Delta W(t) = \Delta W_0 - \frac{2\pi w r(t)}{\lambda N}$

λ = optical wavelength of resonator

N = the integer describing the circumference over resonant wavelength.

ΔW_0 = Detuning between the frequency of pump and cavity

W = Angular Frequency



Optical Field Inside Resonator

$\dot{A}(t) + A(t)[\kappa c - I\Delta W(t)] = IB\sqrt{\frac{\kappa c}{T_o}}$

κ = optical pump loss; B = input pump field

$I = \sqrt{-1}$; T_o = photon round trip time

Power Threshold: $P = \frac{5R^2 \omega^2 \sqrt{mk}}{32}\frac{1}{Q_{opt}^3 \cdot Q_{cap}}$

Relative Humidity: $S = \frac{1}{\lambda}\frac{\delta\lambda}{\delta RH}$; $S_{tot} = S_{Rad} + S_{Refractive\ Index} = \frac{1}{\lambda}\left[\frac{1}{R}\frac{\delta R}{\delta RH} + \frac{1}{n_{eff}}\frac{\delta n_{eff}}{\delta RH}\right]$

Shape of Droplet:

$\frac{1}{r_1} + \frac{1}{r_2} = \frac{\Delta P}{\gamma}$; r_1, r_2 = radii

γ = surface tension ΔP = pressure difference

$\Delta RH_{min} = \frac{\Delta\lambda_{min}}{\frac{d\lambda}{dRH}}$; $X_p = \sqrt{2Dt}$

Elliptical Integrals: $F(\phi,k) = \int_0^{\theta'}\frac{d\theta}{\sqrt{1-k^2\sin^2\theta}}$

Cross-section of cylindrical Droplets: $Z = \pm[adF(\phi,k) + DE(\phi,k)]$ $E(k,\theta') = \int_0^{\theta'}\sqrt{1-k^2\sin^2\theta}\,d\theta$

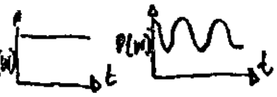
Spheroidal Approximation:

$n_p k_{l,m,n} \cdot R_L = m - \alpha_n\left(\frac{m}{2}\right)^{1/2} + \frac{(2l-2m+1)R_1}{2R_2} - \frac{Pn_r}{\sqrt{n_r^2 - n_0^2}} + \frac{3\alpha_n^2}{20}\left(\frac{m}{2}\right)^{-1/3} - \frac{\alpha_n}{12}\left[\frac{(2l-2m+1)R_1^3}{R_2^2} + \frac{2n_r^3 P(2P^2-3)}{(n_r^2-n_0^2)^{3/2}}\right]\left(\frac{m}{2}\right)^{-2/3}$

$a = \frac{D\cos\theta - d}{D - d\cos\theta}$; d = radius; D = radius; a = contact angle

Position: $X^2 = D^2(1 - k^2\sin^2\phi)$

$Z = \pm\frac{ad + D}{D}\sqrt{D^2 - X^2}$

Water Microdroplet:  $\Delta T = \frac{Q}{r}\Delta r$; ΔT = Transmission ; Q = Quality Factor ; Δr = radius ; r = radius

QM of our Water Droplet: $Q_m = \pi/(\kappa\lambda)$

λ = acoustical wavelength

κ = acoustical coefficient

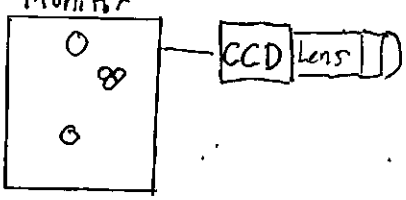
Eigen Frequency: $f = \sqrt{2\gamma/(\pi^2\rho r^3)} \approx \sqrt{\kappa/(\frac{2}{3}\pi r^3\rho)}$

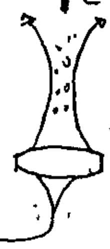
Thermal Cycling Fluctuation: $k_B T = \kappa\langle\Delta r^2\rangle$

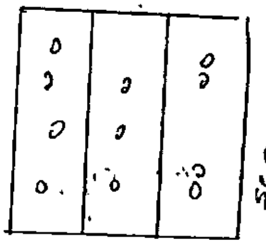
η = Dynamic viscosity

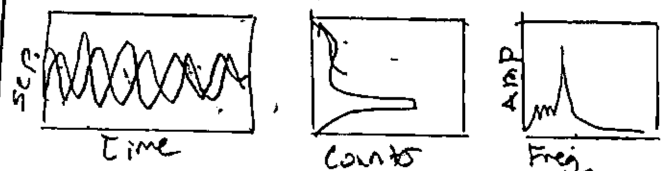
Acoustical Attenuation: $\alpha = \frac{\omega^2}{2\rho c^3}\left(\frac{4}{3}\eta + \zeta\right)$; c = speed of Light ; ζ = Volume viscosity ; ω = acoustic Frequency

Monitor

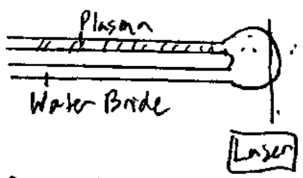






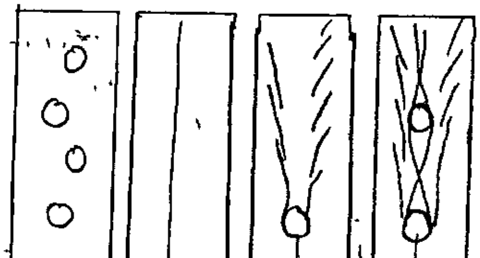


Droplet Setup:



Setup:

- Silica Spheres [0.2dB/km]
- GRIN Lens
- 8.4μm x 9.8μm



Coolable, almost free of damping, or mechanical energy

Site Note: Levitating Microsphere:

Electrodynamic Calculation of Forces on a dielectric sphere

$E_{in} = \hat{y}E_0 \cos k(x-x_0)\cos\omega t$; $B_{in} = \hat{z}(E_0/c)\sin k(x-x_0)\sin\omega t$

Where $k = \omega/c$. Electrodynamic Plane wave scattering:

$$\nabla^2 E(r) + k^2\epsilon(r)E(r) = 0$$

$$\nabla^2 B(r) + k^2\epsilon(r)B(r) = 0$$

Natural Dimensionless Length Scales: $k|\sqrt{\epsilon}|r$

When $k|\sqrt{\epsilon}|r \ll 1$; $|\sqrt{\epsilon}| > 1$:

Under perturbation theory, lowest order, $\nabla^2 E(r) = 0$

With an optical wavelength $\lambda = 2\pi/k = 1\mu m$, $\epsilon = 2$.

The electrostatic solution should be $r \lesssim 110nm$

Polarizability from electrostatic theory

$\alpha_{ind} = 3\epsilon_0 V\frac{\epsilon-1}{\epsilon+2}$; The optical potential is therefore: $U_{opt} = -(1/4)(\mathrm{Re}\,\alpha_{ind})E_0^2\cos^2 k(x-x_0)$

For spheres larger than $r \gtrsim 1/k|\sqrt{\epsilon}|$; Forces become large.

The Exact Force is determined by integrating the Maxwell Stress Tensor T_{ij} : $F_x = \epsilon_0\oint_S dA\sum_{j=x,y,z} T_{xj}\hat{n}_j$

* $k\sqrt{\epsilon}r \lesssim 1$ = Normal ; $k\sqrt{\epsilon}r \gtrsim 1$: Unusual

Considering a standing wave: $\omega_m = \left(\frac{6k^2 I_0}{\rho c}\mathrm{Re}\frac{\epsilon-1}{\epsilon+2}\right)^{1/2}$

I_0 = Field Intensity ; ρ = Mass Density

Total Trap Depth: $U_0 = (3 I_0 V/c)\,\mathrm{Re}\frac{\epsilon-1}{\epsilon+2}$

Adsorbed Power $P_{abs} = 12\pi\frac{I_0}{\lambda}V\,\mathrm{Im}\frac{\epsilon-1}{\epsilon+2}$; $\alpha_{ind} = 3\epsilon_0 V\frac{\epsilon-1}{\epsilon+2}$

* Material Properties: Precise, $\rho, \epsilon \propto \lambda \propto \frac{\Delta V}{2\pi} \propto$ Intensity (I_0), Temp,

Note propagation loss: 10dB/km ; $I_0 \sim 10\ W/\mu m^2$

Cooling Rate $\frac{dE}{dt} = -\alpha_g\sqrt{\frac{2}{3\pi}}(\pi r^2)P\frac{1}{v_{rms}}\frac{\gamma_{sh}+1}{\gamma_{sh}-1}\left(\frac{T_{int}}{T} - 1\right)$

- Where P, v_{rms}, T are background properties.
- Phenomenological accomodation factor α_g
- γ_{SH} = Specific Heat Ratio Diatomic gas

$\frac{dE}{dt} = \sum(\hbar ck)R_{abs,k}$; Black Body Radiation.

$R_{abs,k} = 3ck(V/V_k)\,n_k\,\mathrm{Im}\frac{\epsilon(\omega_k)-1}{\epsilon(\omega_k)+1}$; Temp, Permittivity constant

V = Volume. $n_k = (e^{\hbar ck/k_BT} - 1)^{-1}$

Sphere Blackbody Radiation: $\frac{dE}{dt} = \frac{72\zeta(5)}{\pi^2} \frac{V}{c^3\hbar^4} \mathrm{Im}\frac{\epsilon_{bb}-1}{\epsilon_{bb}+2}(k_B T)^5$

Note: Temperature as a function of Pressure and Intensity:

Mean Free Path: $\frac{d\rho}{dt} = -\gamma_g P/2 \mp -(8/\pi)(P/\bar{v}r\rho)\rho$; P and v are the background gas and pressure and mean speed

Fluctuation-Dissipation: $\frac{dE}{dt} = -\gamma_g(E-k_BT)$; characteristic phonon time: $\tau_g = \hbar\omega_m/\gamma_g k_B T$

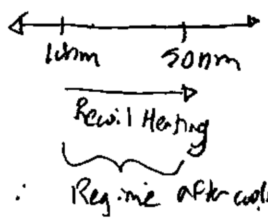
Collisions of Molecules: $R_{coll} \approx \pi P v r^2/k_B T$; Mechanical Quality Factor $Q_g = \omega_m/\gamma_g$

$\boxed{N_{osc}^{(g)} \equiv \omega_m \tau_g/2\pi}$; $r = 50nm$, $\omega_m/2\pi = 0.5MHz$; Density Matrix Describing Scattering $\hat{\rho} \to \int dk P(k) e^{ik\hat{r}} \cdot \rho \cdot e^{-ik\hat{r}}$

Motion in the x-direction: "Atom" $\gamma_{sc} = \left(\frac{2}{5}\right)\left(\frac{\omega_r}{\omega_m}\right)R_{sc}$; $\omega_r = \frac{\hbar k^2}{2\rho V}$

A dimensionless parameter to characterize photon recoil heating: $\phi \equiv \gamma_{sc}/\omega_m = \frac{4\pi^2}{5}\frac{\epsilon-1}{\epsilon+2}(V/\lambda^3)$; $R_{sc} = 48\pi^3 \frac{I_0 V^2}{\lambda^4 \hbar\omega}\left(\frac{\epsilon-1}{\epsilon+2}\right)^2$

Coherent Oscillations $N_{osc}^{(sc)} = \frac{1}{2\pi\phi} \propto \lambda^3/V$; $r = 50nm, \epsilon = 2, \lambda = 1\mu m$ $N_{osc} \sim 150$



$\boxed{; K_{sc} = 12\pi^2\omega(r^2/\lambda^3 \epsilon)\left(\frac{\epsilon-1}{\epsilon+2}\right)^2}$

Optical Cooling: $E_1 \propto \cos k_1 x$; $E_2 \propto \cos(k_2 x - \pi/4)$

Derivation of Optomechanical Coupling Strength:

$\frac{\delta\omega}{\omega} = -\frac{1}{2}\frac{\int d^3r\, \delta P(r)\cdot E(r)}{\int d^3r\, \epsilon_0 E^2(r)}$; $E(r)$ = Bare Cavity Mode Profile

$\delta P(r)$ = Variation in Permittivity

One readily finds: $P(r') \approx \alpha_{ind} E(r)\delta(r-r')$; $r = cm$.

$\delta\omega = -\frac{3V}{4V}\frac{\epsilon-1}{\epsilon+2}\cos(2kx - 2\phi)\omega$

$H_{om} = \hbar\delta\omega \hat{a}^\dagger\hat{a}$; coupling Strength: $g = \frac{3V}{4V_c}\frac{\epsilon-1}{\epsilon+2}\omega$

Optical Self-cooling Eqns:

$\frac{d}{dt}\hat{a}_1 = (i\delta_1 - \kappa/2)\hat{a}_1 - \frac{i\Omega}{2} + \sqrt{\kappa}\hat{a}_{1,in}$; $\frac{d}{dt}\hat{a}_2 = (i(\delta_2 + 2gkz) - \kappa/2)\hat{a}_2 - \frac{i\Omega}{2}\sqrt{25'} + \sqrt{\kappa}\hat{a}_2$

The Rotating Frame Hamiltonian:

$H = \underbrace{-\hbar\delta_1\hat{a}_1^\dagger\hat{a}_1}_{\text{GS-Detuning}} \underbrace{- \hbar g_1(\cos 2k_1\hat{x} - 1)\hat{a}_1^\dagger\hat{a}_1}_{\text{Ground State-Function}} \underbrace{- \hbar\delta_2\hat{a}_2^\dagger\hat{a}_2}_{\text{ES Detuning}} \underbrace{- \hbar g_2\cos[2(k_2\hat{x} - \pi/4)]\hat{a}_2^\dagger\hat{a}_2}_{\text{ES Function}} + \underbrace{\frac{\hat{p}^2}{2m}}_{KE}$

$+ \frac{\hbar\Omega}{2}[\underbrace{(\hat{a}_1 + \hat{a}_1^\dagger)}_{\text{Mode 1 Drive}} + \underbrace{\sqrt{25'}(\hat{a}_2 + \hat{a}_2^\dagger)}_{\text{Mode 2 Drive}}]$

Ω = Driving Amplitude Mode 1 ; $\Omega\sqrt{25'}$ = Driving Amplitude Mode 2

δ_i = Detuning between Modes

The optomechanical coupling strength: $g_i = \frac{3V}{4V}\frac{\epsilon-1}{\epsilon+2}\omega_i$

Expanding around the opto-mechanical coupling term of mode 2 @ $x=0$

$\hbar g\cos(2[kx-\pi/4])\hat{a}_2^\dagger\hat{a}_2 \approx \underbrace{2\hbar gk\hat{x}\hat{a}_2^\dagger\hat{a}_2}$, To calculate cooling rate:

Ratio of Intracavity Intensity: | Radiation Pressure of a Fabry-Perot | $a_i \to a_i + \alpha_i$; $\hat{x} \to \hat{x} + x_0$

Mode 1 ($\delta_1 = 0$) "Responsible Cooling" Where, α_i and x_0 are mean values

Mode 2

$\xi = |\alpha_1/\alpha_2| = \left|\left(\frac{-i\Omega}{K}\right) \Big/ \left(\frac{-i\Omega}{2}\frac{\sqrt{2\xi}}{(K/2)-i\delta_2'}\right)\right|$; where $\delta_2' = \delta_2 + 2gkx_0$

Net cooling Rate

Small ξ ; $\delta_2' < 0$

$= \xi' K^2/(K^2 + 4\delta_2'^2)$

$$\Gamma \equiv R_{opt,-} - R_{opt,+} \simeq K\Omega_m^2\Big[\underbrace{((\delta_2+\omega_m)^2+(K/2)^2)^{-1}}_{\text{anti-Stokes "cooling"}} - \underbrace{((\delta_2-\omega_m)^2+(K/2)^2)^{-1}}_{\text{"Stokes" Heating}}\Big] ; \Omega_m \equiv 2gkx_m|\alpha_1|\sqrt{2\xi}$$

$x_m \equiv \sqrt{\hbar/2m\omega_m}$

$\Omega_m \lesssim K, \omega_m$ $\xi \leq 1$

Steady State phonon Number: $\langle n_p\rangle \approx \delta_{pf} + \gamma_{sc}/\Gamma$

$\langle n_f\rangle \approx R_{opt,+}/\Gamma$ is the fundamental cooling of laser

Minimized when $\delta_2 = -(1/2)\sqrt{K^2+4\omega_m^2}$:

Considering Maximum cooling Rate $\Gamma \sim K$; $\langle n_f\rangle \approx \frac{K^2}{16\omega_m^2} + \phi\frac{\omega_m}{K}$ ← photon recoil heating

Minimized for $K/\omega_m = 2\phi^{1/3}$; $\langle n_p\rangle_{min} = 3\phi^{2/3}/4 \propto (r/\lambda)^2 \ll 1$

Cavity design: $r = 50nm$; $\omega_m/2\pi = 0.5MHz$; $L = 1cm$; $w = 25\mu m$; $(V_c = (\pi/4)Lw^2)$

$F = \pi c/KL$ = "Cavity Finesse" ; Assuming Negligible Collisions ; $2\xi, \Omega_m/K, \Omega_m/\omega_m < 1/2$

Normal Finesse $(\mathcal{F}) = \frac{\Delta\lambda}{\delta\lambda} = \frac{\pi}{2\arcsin(1/\sqrt{F})}$; $F = \frac{4R}{(1-R)^2}$

Note: Transverse motion introduces errors of order $(\Delta y/w)^2$ @ $T = 300K$

$\Delta y/w \approx \sqrt{\phi\omega_r t}$

Photon Shot Noise: $\omega_m^2(t) = \omega_{m,0}^2\left(1 + \frac{\delta N(t)}{N_0}\right)$; Mechanical oscillator Frequency (ω_m) : N_0 = Mean Photon Number, $\omega_{m,0}$ = Mean Frequency

Phonon Number $n \to n \pm 2$ at rate R proportional to spectral density. $2\omega_{m,0}$.

$$R_{n\to n+2} = \frac{\pi\omega_{m,0}^2}{16}S(2\omega_{m,0})(n+2)(n+1)$$

$$R_{n\to n-2} = \frac{\pi\omega_{m,0}^2}{16}S(2\omega_{m,0})n(n-1)$$

Where $S(\omega) = \frac{2}{\pi N_0^2}\int_0^\infty dt\cos\omega t\langle\delta N(t)\delta N(0)\rangle$

Which is evaluated to $S(\omega) = \frac{1}{\pi N_0} \frac{4\pi}{K^2 + 4\omega^2}$

Number of oscillations due to shot noise: $N_{osc}^{(SN)} = \frac{\omega_{m,0}}{2\pi R_{rec}} = \frac{\epsilon+2}{\epsilon-1} \frac{V_c \rho}{3\pi c \hbar k^3} \frac{\omega_{m,0}}{K}(K^2 + 16\omega^2)$

Blackbody Radiation: Each absorption event provides a momentum Kick: $\hbar k_x$. $k = 2\pi/\lambda$; $V_c = (\pi/4) L w^2$

$\boxed{\epsilon(\omega) \approx \epsilon_{bb}}$

$$\gamma_{bb} = \frac{2\pi^4}{63} \frac{(k_B T)^6}{c^2 \hbar^5 \rho \omega_m} \operatorname{Im} \frac{\epsilon_{bb} - 1}{\epsilon_{bb} + 2} ; R_{n \to n \pm 1} = \gamma_{BB}(n + 1/2 \pm 1/2)$$

Anisotropy of sphere: $a/b \approx 1$; deviation from an ideal sphere is small.

$$\alpha_{ind} \approx \alpha_{ind,0}\left(1 \pm \frac{9}{20} \frac{\epsilon-1}{\epsilon+2}\left[(a/b)^{4/3} - 1\right]\right) ; \alpha_{ind,0} \approx 3\epsilon_0 V \frac{\epsilon-1}{\epsilon+2}$$

"Polarizability of Sphere" "Degrees of Freedom"

$$\delta_\omega = \delta\omega_0 + \delta\omega_0 \cos 2\theta$$

"Associated with the center of mass" $\delta\omega_0 = \frac{27}{80} \frac{V}{V_c} \left(\frac{\epsilon-1}{\epsilon+2}\right)^2 \left[(a/b)^{4/3} - 1\right] \omega \cos(2kx - 2\phi)$

$$\frac{da_1}{dt} = -\left(i\delta\omega_0 \cos 2\theta + \frac{K}{2}\right)a_1 + \frac{i\Omega}{2} ; \frac{dp_\theta}{dt} = 2\hbar\delta\omega_0 |a_1|^2 \sin\theta - \gamma_\theta p_\theta + F_\theta(t)$$

$\gamma_\theta = 5\sqrt{3/(2\pi)}\, a_\theta P/(V_{rms} r \cdot \rho)$; Since $a \approx r \approx b$.

a_θ is a phenomenological accomodation coefficient

$$\underbrace{\langle F(t) F(t')\rangle = 2D\delta(t - t')}_{\text{"Noise Free Correlation"}} ; D = \gamma_\theta \cdot k_B T / I_\theta$$

$$\frac{d\theta}{dt} = \frac{p_\theta}{I_\theta}$$

Where θ, I_θ are related to Moment of Inertia

Where γ_θ, F_θ are damping coefficients.

Fluctuations of Trap Frequency: $\delta\omega_m(t) = \epsilon_\theta \omega_{m,0} \cos 2\theta(t)$

$$R_{0 \to 2} = \int_0^{\infty} dt \cos 2\omega_{m,0} t \langle \delta\omega_m(0) \delta\omega_m(t)\rangle$$

Where $\epsilon_\theta = \frac{9}{40} \frac{\epsilon-1}{\epsilon+2}\left((a/b)^{4/3} - 1\right)$

Denoting $\delta\theta(t) = \theta(t) - \theta(0)$

$$R_{0 \to 2} = \frac{1}{2} \int_0^{\infty} dt \cos 2\omega t \,(\epsilon_\theta \omega_{m,0}^2) \langle \cos 2\delta\theta(t)\rangle$$; Where $\langle e^{2i\delta\theta(t)}\rangle \approx \exp(-\langle \delta\theta^2(t)\rangle/2)$

One finally finds, $\frac{R_{0 \to 2}}{\omega_{m,0}} = \epsilon_\theta^2 \frac{\sqrt{2\pi}\, \omega_{m,0}}{8\sqrt{\langle \omega_r^2\rangle}} \exp\left(-\frac{\omega_{m,0}^2}{2\langle \omega_r^2\rangle}\right)$; $\omega_r = \frac{d\theta}{dt}$; $\sqrt{\langle \omega_r^2\rangle}$

At maximum, $R_{0 \to 2}/\omega_{m,0} \sim 0.2\epsilon_0^2$

Entanglement Transfer: EPR correlations:

$$X_{+,in}^{(j)} = (\hat{a}_{in}^{(j)} + \hat{a}_{in}^{(j)\dagger}), \quad X_{-,in}^{(j)} = (\hat{a}_{in}^{(j)} - \hat{a}_{in}^{(j)\dagger})/i$$

Optical Levitation by Radiation Pressure:

★Ashkin★

100-500mW of 5145Å laser in TEM with 15-25μ Diameter & 5cm lens

Beam waist ($2w_0 \cong 25\mu$) ; $n = 1.65$

mass $= 10^{-4}$ g

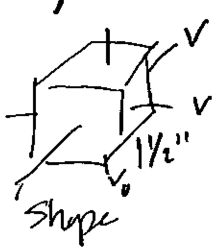
Thermal Force: $F \propto 1/p$ to 10 torr

Maximum Transverse Trapping: $1/7$ g

Transverse stability 40mW

Containment of charged particles

★Langmuir★



$$V = 5.15 \frac{V_0}{a^2}\left[z^2 - \tfrac{1}{2}(x^2+y^2)\right]$$

The three phase potential distribution:

$$V = A\left[z^2\cos\Omega t + y^2\cos(\Omega t + 2\pi/3) + x^2\cos(\Omega t + 4\pi/3)\right]$$

The equations of motion for a charged particle: Mathieu's Equation:

$$\frac{d^2u}{d\xi^2} + (a - 2q\cos 2\xi)u = 0$$

$u = x, y,$ or z and $2\xi = \Omega t$

The values of a and q are found to be

$$q_x = q_y = q_z = 4(e/m)\frac{5.15 V_{ac}}{a^2\Omega^2}$$

$$a_z = -\frac{8}{3\Omega^2}\frac{5.15}{a^2}(e/m)\cdot(V_x + V_z)$$

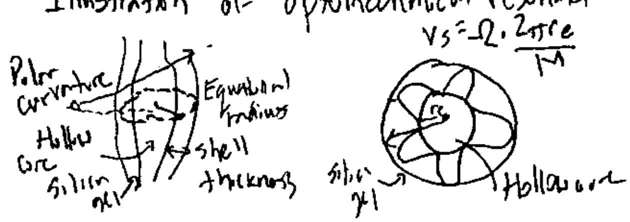
$$a_y = \frac{16}{3\Omega^2}\frac{5.1}{a^2}(e/m)(V_x - \tfrac{1}{2}V_y)$$

$$a_x = \frac{16}{3\Omega^2}\frac{5.15}{a^2}(e/m)(V_y - \tfrac{1}{2}V_x)$$

$$a_x + a_y + a_z = 0$$

Acoustic Whispering-gallery Modes in Optomechanical Shells

Illustration of optomechanical resonator:



Rayleigh-Lamb Modes: : Rayleigh waves (large R) and Lamb waves, Radial displacement

Longitudinal Modes: Compression waves - trajectory azimuthal

Transverse (Love) Modes: Polar waves, or Shear waves.

Of particular interest $\langle (X^{(A)}_{\pm,in}(\omega) \pm X^{(B)}_{\pm,in}(\omega))^2 \rangle / 2 = e^{-2R} < 1$

"The form created by a non-degenerate optical parametric amplifier" (NOPA)

Hamiltonian of the cavity mode: $H = i\hbar(\beta/2)(c^{(A)}c^{B} - c^{(A)\dagger}c^{(B)\dagger})$

Heisenberg Equation of Motion: $\frac{d}{dt}c^{(j)} = -\frac{\kappa_c}{2}c^{j} - \frac{\beta}{2}c^{j\dagger} + \underbrace{\sqrt{\kappa_c}\,c_{in}}_{\text{cavity input field}}$

$$c^{j}_{out} = \sqrt{\kappa_c}\,c^{j} - c^{j}_{in}$$

$$c^{(j)}(t) = (1/\sqrt{2\pi}) \cdot \int d\omega\, e^{-i\omega t}\, c^{(j)}(\omega)$$

Quadrature Operators: $X^{(j)}_{+} = c^{(j)} + c^{(j)\dagger}$ and $X^{(j)}_{1} = (c^{j} - c^{j\dagger})/i$

One can show that, $X^{(A)}_{\pm out} \pm X^{(B)}_{\pm out} = \frac{\kappa_c - \beta + 2i\omega}{\kappa_c + \beta - 2i\omega}(X^{(A)}_{\pm in} \pm X^{(B)}_{\pm in})$
when $\beta < \kappa_c$

$$= e^{-R}(X^{(A)}_{\pm in}(\omega) \pm X^{(B)}_{\pm in}(\omega))$$

Joint Variance:

$$\Delta_{ERR} \equiv \langle (X^{(A)}_{\pm m}(t) \mp X^{(B)}_{\pm m}(t))^2 \rangle / 2$$

$$= e^{-2R} + \underbrace{\frac{\kappa^2}{16\omega_m^2}(3e^{2R} + 2\sinh 2R)}_{\text{Heating}} + \underbrace{\frac{4\phi\omega_m}{\kappa}}_{\text{"cooling"}} \quad \text{where } e^{-R} = \frac{\kappa_c - \beta}{\kappa_c + \beta}$$

$$\frac{d\Delta_{ERR}}{d(\kappa/\omega)} = e^{-2R} + 3(\phi/2)^{2/3} \cdot (3e^{2R} + 2\sinh 2R)^{1/3}$$

<u>Squeezed Light Generation:</u> $H_m = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega_m^2\hat{x}^2(1 + 2\epsilon_m \sin 2\omega_m t)$

"Motion" $= \hbar\omega b^\dagger b - i\frac{\hbar\beta}{2}(b^2 e^{2i\omega_m t} - b^{\dagger 2} \cdot e^{-2i\omega_m t}) + 2\{\hbar\beta b^\dagger b \sin 2\omega_m t\}$

$$\hat{x} = \sqrt{\frac{\hbar}{2m\omega_m}}(\hat{b} + b^\dagger) \; ; \; \hat{p} = i\sqrt{\frac{\hbar m\omega_m}{2}}(b^\dagger - \hat{b})$$

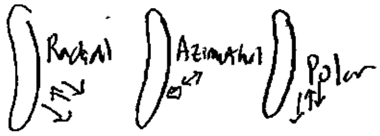
"External" $H_e = -i\frac{\hbar\beta}{2}(b^2 e^{2i\omega_m t} - b^{\dagger 2}e^{-2i\omega_m t})$ $\quad \beta = \epsilon\omega_m/2$

$$+ 2\hbar\beta b^\dagger b \sin 2\omega_m t$$

Heisenberg Equation of Motion:

$$\frac{d}{dt}a_2 = -\frac{\kappa}{2}a_2 - i\Omega(\hat{b} + b_2^\dagger e^{2i\omega_m t}) + \sqrt{\kappa}\,a_{2,in}$$

$$\frac{d}{dt}b = -i\Omega(\hat{a}_2 + \hat{a}_2^\dagger e^{2i\omega_m t}) + iF(t)e^{i\omega_m t} + \beta b^\dagger - 2i\beta b \sin 2\omega_m t$$



Optomechanical Coupling:

$$g_0 = \left(\frac{\pi \gamma c}{2 \epsilon r}\right) \cdot M \cdot \frac{\omega \cdot \sqrt{\kappa}}{L \cdot \sqrt{2 \cdot m_{eff} \cdot \Omega}}$$

L = circumference

γ = Electrostrictive coefficient

M = Momentum Parameter

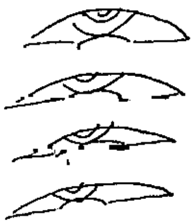
Ω = Acoustic Frequency

☆ May Measure mass in liquid ☆

Acoustic Velocity ; $t \approx \frac{2\pi r_c}{M}$; $R \approx \frac{2\pi}{M}$

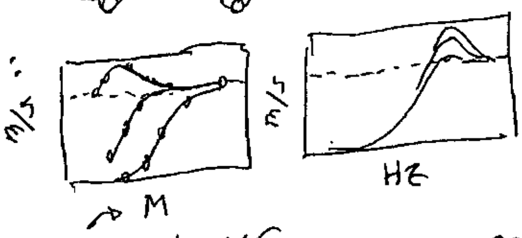
Rayleigh-Lamb Modes:

Polar cross-section: R=0.6, R=0.4, R=0.2, R=0.02

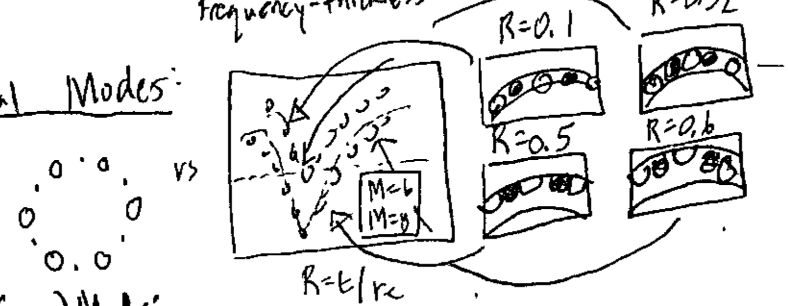


Equatorial cross-section: R=0.6 R=0.4 R=0.2 R=0.1

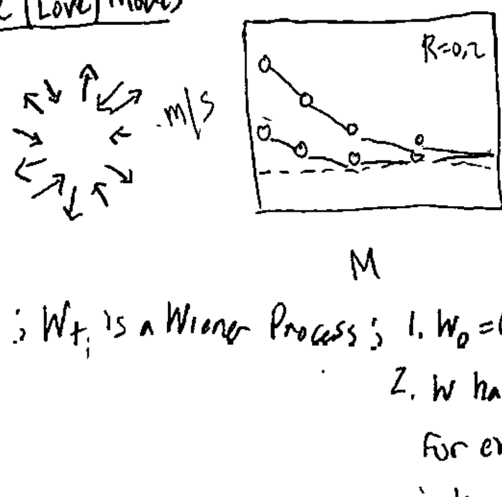
Velocity Dispersion:



Longitudinal Modes:



Transverse (Love) Modes:



Ornstein-Uhlenbeck Process:

Stochastic Differential Equation

$dx_t = -\theta x_t dt + \sigma dW$; $\theta > 0$; $\sigma > 0$; W_t is a Wiener Process ; 1. $W_0 = 0$

$\dot{x}_t = -\theta x(t) + F(t)$

$$\underbrace{\ddot{x}(t)}_{\text{accelon}} = -\underbrace{\gamma_m \dot{x}(t)}_{\text{Velocity}} - \underbrace{f_m^2 x(t)}_{\text{Position}} + \underbrace{F(t)}_{\text{White Noise external}}$$

2. W has independent increments for every $t > 0$; $W_{t+u} - W_t$, $u \geq 0$ independendent of past W_s, $s \leq t$.

3. W has Gaussian increments $W_{t+u} - W_t$ is normally distributed with mean 0 and variance u; $W_{t+u} - W_t \sim N(0, u)$

4 W is a continuous path, W_t

Solving (Gröblacher 2012, Kalmykov 2012, Coffey)

$$S(f) = \frac{a}{(f_m^2 - f^2)^2 + f^2 \gamma_m^2} \propto \quad |H(if)|^2 = \frac{a}{(f^2 - f_0^2) + f^2 (f_0/Q)^2}$$

γ = FWHM $\qquad F_{res} = f_m \sqrt{1 - \frac{\gamma_m^2}{2 f_m^2}}$

$\boxed{\text{Fokker-Planck Equation}}$

$F_{sat} = (h/\lambda)(1/2\tau_N)[p/(1+p)]$ "Saturation lifetime" $= \tau$; p = Saturation parameter.

$= \frac{I}{I_s}\left[\frac{\gamma^2}{4(\Delta\nu^2+\gamma^2/4)}\right]$

"Related to the dipole force of the Gaussian beam"

<u>Potential</u> : $U = (h\Delta\nu/2)\ln(1+p)$

I = Intensity

$\Delta\nu$ = Detuning from resonance

$\gamma = \frac{1}{2\pi\tau_N}$ "Natural linewidth"

$m\langle v_i^2\rangle = k_B T = \frac{\hbar\Gamma}{4}\left(\frac{\Gamma}{2\delta} - \frac{2\delta}{\Gamma}\right)$ ☆

Doppler Cooling i.e. average kinetic energy

δ = Angular Frequency, v_i = Velocity. ☆ Valid low intensity

$\hbar^2 K^2/2m \ll \hbar\Gamma$; "Recoil Energy"

Γ = Rate of spontaneous emission.

Minimum Temperature

$k_B T_{Dopp} = \frac{\hbar\Gamma}{2}$ ☆

<u>Diffusion</u>: $\langle x^2\rangle = 2Dt$

$D = k_B T/\alpha$

Where α = Viscous Damping

$F = -\alpha \cdot v$

Change of Heating

: N = Number of photons.

$\frac{dW_{Heating}}{dt} = \frac{d}{dt}\left(\frac{p^2}{2m}\right) = \frac{N(p_r)^2}{2M}$; p_r = momentum recoil

Room Temp Displacement:

$\langle[\Delta X(t)]^2\rangle = \frac{2k_B T}{m\omega_0^2}\left[1 - e^{-t/2\tau_p}\left(\cos\omega_1 t + \frac{\sin\omega_1 t}{2\omega_1\tau_p}\right)\right]$

Where $\tau_p = \frac{m}{\gamma}$ "Momentum Relaxation"

γ = Stokes friction coefficient

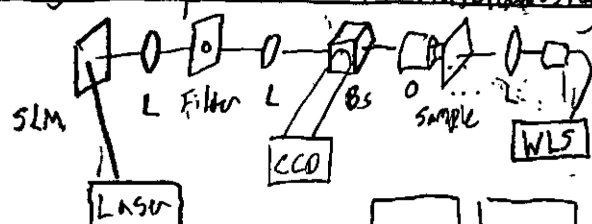
$\omega_1 = \sqrt{\omega_0^2 - 1/(2\tau_p)^2}$

Mean velocity:

$\bar{v} = \frac{\sqrt{\langle[\Delta x(t)]^2\rangle}}{t} = \frac{\sqrt{2D}}{\sqrt{t}}$

Trapping and Guiding Microparticles with Autofocusing Airy Beams.

Experimental Setup: "Airy"

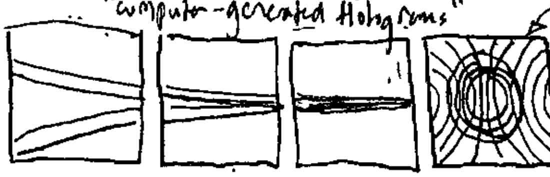


"White Light Source"

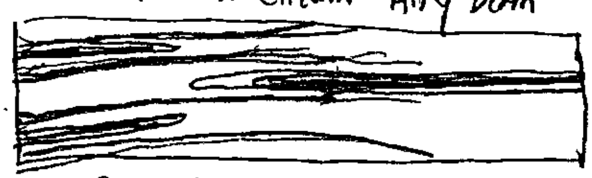
Generation of Autofocusing Beam:

"Generation of Circular Airy Beam"

"computer-generated Holograms"



"Transverse Intensity Patterns"



Beam bends outward and not inward.

To understand odd behavior of beam to become a Bessel Beam, we consider Fresnel-Huygens Diffraction. <u>Optical Field Envelope</u>:

$$E(\rho,z) = -\frac{ik}{z}\exp\left(\frac{ik\rho^2}{2z}\right)\int_0^\infty r\,E(r,0)\,J_0\left(\frac{k\rho r}{z}\right)dr$$

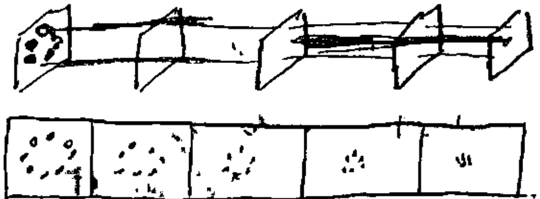
Where $E(r,0)$ represents the initial Airy profile:

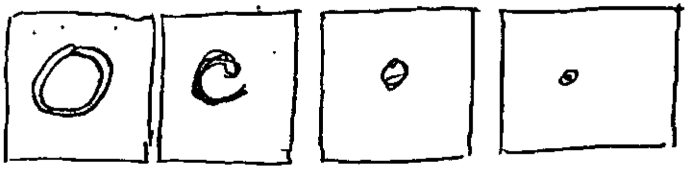
$$E(\rho,z) = -\frac{ik}{z}\exp\left(\frac{ik\rho^2}{2z}\right) r_0 A_0 J_0\left(\frac{k\rho r_0}{z}\right)$$

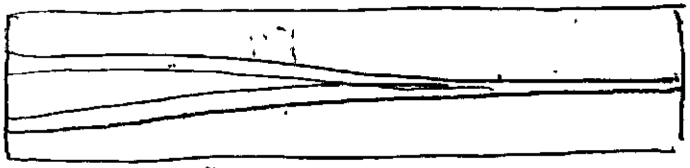
$$E(r,0) = A_0\left[\frac{1}{\cdot}\left(\frac{r_0-r}{w}\right)\right]\exp\left[\pm\alpha\left(\frac{r_0-r}{w}\right)\right]$$

$$A_0 = w\exp\left(\frac{\alpha^3}{3}\right)\left(1-\frac{w\alpha^2}{r_0}\right)$$

<u>Experimental Demonstration of Particle Guidance</u>:



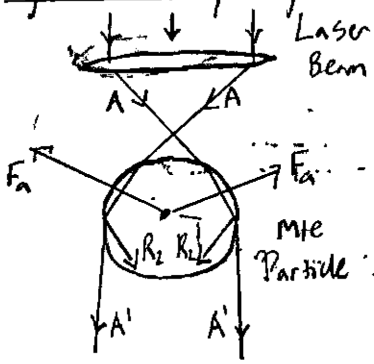




"Bessel"

<u>Observation of a Single-beam Gradient force optical trap for dielectric particles</u>:

<u>Diagram of ray optics</u>:



$$F_{scat} = \frac{I_0}{c}\frac{128\pi^5 r^6}{3\lambda^4}\left(\frac{m^2-1}{m^2+2}\right)n_b \quad ; 2R \ll \lambda \text{ "Rayleigh Scattering"}$$

$$= n_b P_{scat}/c$$

$$F_{grad} = -\frac{n_b}{2}\alpha\nabla E^2 = -\frac{n_b^3 r^3}{2}\left(\frac{m^2-1}{m^2-2}\right)\nabla E^2 \quad ; m = \text{index of refraction.}$$

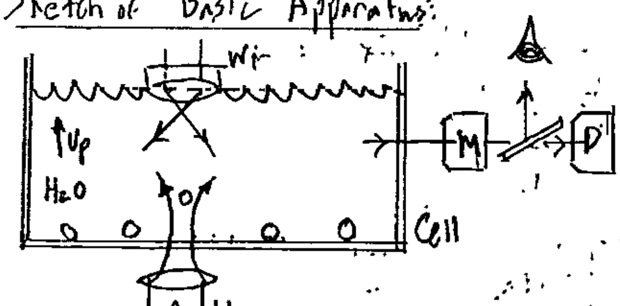
α = polarizability

"Ratio of Backward" Axial Gradient

$$R = \frac{F_{grad}}{F_{scat}} = \frac{3\sqrt{3}}{64\pi^5}\frac{n_b^2}{\left(\frac{m^2-1}{m^2+2}\right)}\frac{\lambda^5}{r^3 w_0^2} \geq 1$$

Boltzmann Factor: $\exp(-U/kT) \ll 1$; where $U = n_b\cdot\alpha\cdot E^2/2$

<u>Sketch of Basic Apparatus</u>:



<u>Derivation of Ornstein-Uhlenbeck with Gravity</u>:

$$m\dot{v} + \gamma v + mg + \nabla U = \xi ; \langle\xi(t)\xi(t')\rangle = 2k_B T_\gamma \delta(t-t')$$

$$m\dot{v} + \gamma v + mg + \nabla\tfrac{1}{2}kx^2 = \xi$$

$$m\dot{v} + \gamma v + mg + k\cdot x = \xi \quad ; \langle\xi(t)\xi(t')\rangle$$

$$m\dot{v} + \gamma v + mg + kx = \sqrt{2k_B T_\gamma \delta(t-t')} \quad = 2k_B T_\gamma \delta(t-t')$$

$$m\dot{v} + \gamma v + kx = -mg + \sqrt{2k_B T}\,\xi(t)$$

$$\boxed{\dot{x} = -\frac{k}{\gamma}x + \frac{m}{\gamma}g + \sqrt{\frac{2k_B T}{\gamma}}\,\xi(t)}$$

Stochastic Differential

$$dx = \theta(\mu - x)\,dt + \sigma\,dW$$

$$\mu = \frac{m}{k}g ; \theta = \frac{k}{\gamma} ; \sigma^2 = 2D$$

<u>Vasicek Model</u> $dr = a(b-r)dt + \sigma dW$; b = "long term mean level" "future"

"Mean Reversion"

a = "Speed of reversion" "velocity at trajectory"

σ = "instantaneous volatility" "randomness"

$$r = r_0 e^{-at} + b(1-e^{-at}) + \sigma e^{-at}\int_0^t e^{as}\,dW$$

$$E[r] = r_0 e^{-at} + b(1-e^{-at}) \; ; \; Var[r_t] = \frac{\sigma^2}{2a}(1-e^{-2at})$$

$$\boxed{E\left[\int_0^{t'} \sigma e^{a(t-t')}\,dW \int_0^{t''} \sigma e^{a(t-t'')}\,dW\right]}$$

Fourier Transform of $\langle r(t) r(t') \rangle$ Correlation Function

<u>Very High Dampening Regime:</u> • Effect of the Brownian Forces on velocity is larger than that of the external force.

Maxwell Distribution $[p]$: $\rho(x,p,t) \approx f(x,t)\, e^{-p^2/2mkT}$

$\lambda \xi = 8\pi a^3 \eta$ "Drag Coefficient" ; $\tau = \beta^{-1}$ "Diffusion Time"

$$\frac{\partial f}{\partial t} = -\frac{\partial}{\partial x}\left[\frac{K}{\beta} f - \frac{kT}{\beta}\frac{\partial f}{\partial x}\right]$$

$2\pi \rho \bar{\omega}$

Starting from $\boxed{\text{Klein-Kramers Equation}}$

$$\frac{\partial \rho}{\partial t} + \frac{p}{m}\frac{\partial \rho}{\partial x} - \frac{\partial V}{\partial x}\frac{\partial \rho}{\partial p} = \frac{\xi}{m}\frac{\partial}{\partial p}\left(\rho p + mkT\frac{\partial \rho}{\partial p}\right)$$

$$\frac{\partial \rho}{\partial t} = \beta\left(\frac{\partial}{\partial p} - \frac{1}{\beta}\frac{\partial}{\partial x}\right)\left(\rho p + kT\frac{\partial \rho}{\partial p} - \frac{K}{\beta}\rho + \frac{kT}{\beta}\frac{\partial \rho}{\partial x}\right) - \frac{\partial}{\partial x}\left(\frac{K}{\beta}\rho - \frac{kT}{\beta}\frac{\partial \rho}{\partial x}\right)$$

$x + p/\beta = x_0$ (constant)

$$\frac{\partial}{\partial x} = \frac{\partial x_0}{\partial x}\frac{\partial}{\partial x_0} = \frac{\partial}{\partial x_0} \; ; \; \frac{\partial}{\partial p} = \frac{\partial x_0}{\partial p}\frac{\partial}{\partial x_0} = \frac{1}{\beta}\frac{\partial}{\partial x_0}$$

$$\frac{\partial f}{\partial t} = -\int \frac{\partial}{\partial x}\left(\frac{K}{\beta}\rho - \frac{kT}{\beta}\frac{\partial \rho}{\partial x}\right)dp$$

...

$$\frac{d}{dt} f(x_0,t) = -\frac{\partial}{\partial x_0}\left(\frac{K(x_0)}{\beta} f(x_0,t) - \frac{kT}{\beta}\frac{\partial f(x_0,t)}{\partial x_0}\right)$$

$\boxed{\text{"Smoluchowski Equation"}}$

$|p| \leq \sqrt{kT}$

<u>Low Damping Regime:</u> Small Viscosity means that Brownian Forces cause only a tiny perturbation in the undamped energy..

Klein-Kramers Equation with canonical variables (x,p) as a Diffusion Equation in Energy (E) and phase (W).

- Accepted when energy is slow varying and phase fast-varying.
- Average over fast phase variable, and get slow energy variable.

Time Average along a Trajectory to a saddle point of potential energy:

$$\bar{\rho}(E,t) \equiv \frac{1}{T}\int_0^{T'} \rho(E,W,t)\,dW$$

T' = time to execute one periodic motion. $dt = dW$

$$I(E) = \oint_{E=\text{const}} p\,dq \quad \text{"Action"}$$

- Always Librational ie. closed trajectories.
- Harmonic Oscillator

$$E = \frac{p^2}{2m} + \frac{m\omega^2 x^2}{2}$$

Trajectories in phase space are (closed) ellipses

Now the Klein-Kramers Equation:
$$\frac{\partial \rho}{\partial t} = \frac{\partial V}{\partial x}\frac{\partial \rho}{\partial p} - \frac{p}{m}\frac{\partial \rho}{\partial x} + \underbrace{\beta\frac{\partial}{\partial p}\left(\rho p + mkT\frac{\partial \rho}{\partial p}\right)}$$

When zero becomes Liouville's Theorem,

$$\frac{\partial \rho}{\partial t} = \frac{\partial V}{\partial x}\frac{\partial \rho}{\partial p} - \frac{p}{m}\frac{\partial \rho}{\partial x}$$ Dissipation of Energy

$E = \frac{p^2}{2m} + V(x)$; we have:

$$\dot{x} = \pm\sqrt{2[E - V(x)]/m}$$

$$\int_{x_1}^{x} \frac{dx'}{\sqrt{2[E-V]/m}} = t + w$$ where the constant of integration w defines phase.

$\dot{W} = 1$; $\dot{E} = 0$; $\dot{p} = -\frac{dV}{dx}$

We have by the chain rule:

$$\frac{\partial \rho}{\partial x} = \frac{\partial \rho}{\partial E}\frac{\partial E}{\partial x} + \frac{\partial \rho}{\partial W}\frac{\partial W}{\partial x},\quad \frac{\partial \rho}{\partial p} = \frac{\partial \rho}{\partial E}\frac{\partial E}{\partial p} + \frac{\partial \rho}{\partial W}\frac{\partial W}{\partial p}$$

$$\frac{\partial E}{\partial x} = \frac{\partial V}{\partial x},\ \frac{\partial W}{\partial x} = \frac{1}{\dot{x}} = \frac{m}{p};\ \frac{\partial E}{\partial p} = \frac{p}{m},\ \frac{\partial W}{\partial p} = 0$$

$$\frac{\partial}{\partial x} = \frac{\partial V}{\partial x}\frac{\partial}{\partial E} + \frac{m}{p}\frac{\partial}{\partial W},\ \frac{\partial}{\partial p} = \frac{p}{m}\frac{\partial}{\partial E}$$

$$\frac{\partial \rho}{\partial t} = -\frac{\partial \rho}{\partial W} + \frac{\beta p}{m}\frac{\partial}{\partial E}\left(\rho p + pkT\frac{\partial \rho}{\partial E}\right)$$

$$\bar{\rho} = \frac{1}{T'}\int_0^{T'} \rho(E,W,t)\,dW$$

$$\overline{\frac{\partial \rho}{\partial t}} = \overline{\frac{\partial \rho}{\partial W}} + \frac{\beta p}{m}\frac{\partial}{\partial E}\left(\rho p + pkT\frac{\partial \rho}{\partial E}\right)$$

Ending with

$$\frac{\partial \rho}{\partial t} = \beta\frac{\partial}{\partial I}\left(I\rho + kT\frac{2\pi I}{\omega}\frac{\partial \rho}{\partial I}\right)$$

Casmir : Markus Klute : Introduction to Nuclear and Particle Physics

Polarized Particles: $M \sim [\bar{u}_1 \cdot T^{\mu} \cdot v_2] \epsilon_{3\mu}$
- Explicit Wavefunction
- Known Polarization

Spin-Averaged Amplitude: $\langle |M|^2 \rangle$
- Average over polarizations of initial-state particles
- Sum over polarizations of final state particles in squared-amplitude.

Spin Sums: $M \sim [\bar{u}_1 T u_2]$; $|M|^2 \sim [\bar{u}_1 T u_2][\bar{u}_1 T u_2]^*$

$$\sim [\bar{u}_1 T u_2][u_1^\dagger \gamma^0 T u_2]^\dagger$$

$$\sim [\bar{u}_1 T u_2][u_2^\dagger T^\dagger \cdot \gamma^{0\dagger} \cdot u_1]$$

$$\sim [\bar{u}_1 T u_2][u_2^\dagger \cdot \gamma^0 \gamma^0 T^\dagger \gamma^0 u_1]$$

$$\sim [\bar{u}_1 T u_2][\bar{u}_2 T u_1]$$

$$|M|^2 \sim [\bar{u}_1 T u_2][\bar{u}_2 \bar{T} u_1]$$

Applying the completeness relation:

$$\sum_{s_i = 1,2} u_i^{s_i} \cdot \bar{u}_i^{s_i} = (\not{p}_i + m_i)$$

$$\sum_{s_2} |M|^2 \sim [\bar{u}_1 T (\not{p}_2 + m_2) \bar{T} \cdot u_1]$$

$$\sim [\bar{u}_1 Q u_1]$$

$$[\bar{u}_1 Q u_1] = (\bar{u}_1)_i Q_{ij} (u_1)_j$$

$$= Q_{ij} (u_1 \bar{u}_1)_{ji}$$

$$= [Q(u_1 \bar{u}_1)]_{ii}$$

$$= Tr[Q(u_1 \bar{u}_1)]$$

$$\sum_{s_1} |M|^2 \sim Tr[Q(\not{p}_1 + m_1)]$$

Summary: $M \sim [\bar{u}_1 T u_2]$; $\langle |M|^2 \rangle \sim \frac{1}{2} Tr[T(\not{p}_2 + m_2)\bar{T}(\not{p}_1 + m_1)]$

Casimir's Trick: $\sum [\bar{u}_a T u_b][\bar{u}_a T u_b]^* = Tr[T_1(\not{p}_b + m_b)\bar{T}_2(\not{p}_a + m_a)]$

$$\sum_{1,2} v_i^{s_i} \bar{v}_i^{s_i} = (\not{p}_i - m_i)$$

Traces $Tr(A+B) = Tr(A) + Tr(B)$
$Tr(\alpha A) = \alpha Tr(A)$
$Tr(AB) = Tr(BA)$
$Tr(ABC) = Tr(CAB) + Tr(BCA)$

Gamma Matrices $g_{\mu\nu} \cdot g^{\mu\nu} = 4$
$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}(\times I)$
$\gamma_\mu \cdot \gamma^\nu \gamma^\mu = \gamma_\mu (2g^{\mu\nu} - \gamma^\mu \gamma^\nu)$
$= 2\gamma^\nu - \gamma_\mu \gamma^\mu \gamma^\nu$
$= 2\gamma^\nu - 4\gamma^\nu$
$= -2\gamma^\nu$

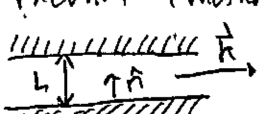
Trace Identities: $Tr(\gamma^\mu \gamma^\nu) = Tr(\gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu)/2$
$= Tr(2g^{\mu\nu})/2$
$= g^{\mu\nu} Tr(1)$
$= 4g^{\mu\nu}$

$Tr(\gamma^\mu \gamma^\nu \gamma^\lambda \gamma^\sigma) = 4(g^{\mu\nu} g^{\lambda\sigma} - g^{\mu\lambda} g^{\nu\sigma} + g^{\mu\sigma} g^{\nu\lambda})$

Trace with γ^5
$Tr(\gamma^5) = 0 \; ; \; \gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$
$Tr(\gamma^5 \gamma^\mu) = 0 \; ; \; Tr(\gamma^5 \gamma^\mu \gamma^\nu \gamma^\lambda) = 0$
$Tr(\gamma^5 \gamma^\mu \gamma^\nu) = 0$
$Tr(\gamma^5 \gamma^\mu \gamma^\nu \gamma^\lambda \gamma^\sigma) = 4i\epsilon^{\mu\nu\lambda\sigma}$

$$\epsilon^{\mu\nu\lambda\sigma} \equiv \begin{cases} -1 & \text{: even permutation} \\ +1 & \text{: odd permutation} \\ 0 & \text{: 2 indices are the same} \end{cases}$$

Casimer Effect: "Attractive force between conducting plates due to vacuum fluctuations"



Cross-section $[a^2]$
$a \gg L$

Boundary-condition: $z=0$ and $z=L$
$\sin L\ell = 0$
$\ell = m\pi/L \quad m = 0, 1, 2 \ldots$
$\omega^2 = k^2 c^2 + m^2\pi^2 c^2 / L^2$

Transverse Electric Modes:
$$\vec{A}^{E}_{\vec{k},m} = \sin\left(\frac{m\pi z}{L}\right)(\hat{k}\times\hat{n}) e^{ik\hat{x}\vec{s}} + c.c. \quad m \geq 1$$

Transverse Magnetic Modes:
$$\vec{A}^{M}_{\vec{k},m} = \left\{\left[\frac{ck}{\omega}\cos\left(\frac{m\pi z}{L}\right)\hat{n} - \frac{i m\pi c}{L\omega}\sin\left(\frac{m\pi z}{L}\right)\hat{k}\right] e^{ik\hat{k}\vec{s}}\right\} + c.c. \quad m \geq 0$$

Density of States: - Number of modes with given m & k between k and $k+dk$.

$$\frac{dk_x}{\left(\frac{2\pi}{a}\right)} \frac{dk_y}{\left(\frac{2\pi}{a}\right)} = \frac{a^2 \cdot k\,dk}{2\pi} = 4\frac{a^2}{\pi} d(k^2) = \frac{a^2}{c^2}\frac{\omega\, d\omega}{2\pi}$$

- For a given ω: $m = 0, \ldots, \mathrm{Int}\left(\frac{\omega L}{\pi c}\right)$

$$\delta(\omega)d\omega = \frac{a^2}{2\pi c^2}\left[1 + 2\,\mathrm{Int}\left(\frac{\omega L}{\pi c}\right)\right]\omega\, d\omega$$
$$= \frac{a^2}{2\pi c^2}\left[1 + 2\sum_{m=1}^{\infty}\Theta\left(\omega - \frac{m\pi c}{L}\right)\right]\omega\, d\omega$$

Zero-point energy in the value $a^2 \times L$; $W(L) = \int_0^\infty \frac{\hbar\omega}{2}\sigma(\omega)\,d\omega$

$$= \frac{a^2\hbar}{4\pi c^2}\left[\int_0^\infty \omega^3 d\omega + 2\sum_{n=1}^{\infty}\int_{\frac{n\pi c}{L}}^{\infty} \omega^2 d\omega\right]$$

add convergence term $e^{-\lambda\omega/c}$; ~~with cutoff~~ at high frequencies $\omega \gg \lambda/c$

$\therefore$ $W(L) = \text{const} + \frac{a^2\hbar}{2\pi c^2}\sum_1 I_m$; where $I_m = \int_{\frac{m\pi c}{L}}^{\infty} \omega^2 e^{-\lambda\omega/c}\cdot d\omega$

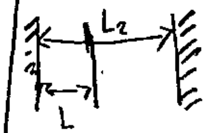
$$= c^2\frac{\partial^2}{\partial\lambda^2}\int_{\frac{m\pi c}{L}}^{\infty} e^{-\lambda\omega/c}\cdot d\omega = c^3\frac{\partial^2}{\partial\lambda^2}\left[-\frac{e^{-m\pi\lambda/L}}{\lambda}\right]$$

$$\sum_m I_m = \frac{c^3\pi}{L}\frac{\partial^2}{\partial\lambda^2}\left[\frac{1}{\left(\frac{\pi\lambda}{L}\right)}\cdot\frac{1}{e^{(\pi\lambda/L)}-1}\right]$$

When expanded at $x<0$ ($x = \pi\lambda/L$ $\lambda \to 0$)

$$\frac{1}{x(e^x-1)} = \frac{1}{x^2} - \frac{1}{2x} + \frac{1}{12} - \frac{x^2}{720} + \cdots$$

$$W(L) = W_0 + \frac{a^2\hbar c}{2}\left[\frac{6L}{\pi^2\lambda^4} - \frac{1}{\pi\lambda^3} - \frac{2\pi^2}{720L^3} + \cdots\right]$$



$$W_T = W(L) + W(L_0 - L) = \frac{a^2\hbar c}{2}\left[\frac{6L}{\pi^2\lambda^4} + \frac{6\cdot(L_2-L)}{\pi^2\lambda^4} - \frac{2}{\pi\lambda^3} - \frac{2\pi^2}{720}\left(\frac{1}{L^3} + \frac{1}{(L_0-L)^3}\right) + \cdots\right]$$

$$\boxed{\text{Casimir Energy}: U(L) = \frac{-\pi^2\hbar c}{720}a^2\cdot\frac{1}{L^3}}$$

Cavity Quantum Electrodynamics

Electric Field Amplitude

$E_{vac} = [\hbar\omega/(2\epsilon_0 V)]^{1/2}$

Elementary Frequency: $\Omega_{ef} = D_{eg}E_{vac}/\hbar$

Mode Density is given by:

$\rho_0(\omega) = \omega^2 V/\pi^2 c^3$

assuming volume $\gg \lambda^3$ or $(2\pi c/\omega)^3$

Fermi Golden Rule

$$T_0 = 2\pi\Omega_{ef}^2\frac{\rho_0(\omega)}{3} = \frac{\omega^3}{3\pi\hbar c^3}\frac{|D_{eg}|^2}{\epsilon_0}$$

i.e.:

"Naturally Excited"

Casimir Effect and Quantum Vacuum

Energy Density of vacuum: $\langle T_{\mu\nu}\rangle = -\mathcal{E}g_{\mu\nu}$; Einstein Equations: $R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -8\pi G(T_{\mu\nu} - \mathcal{E}g_{\mu\nu})$

Excitation in vacuum

(1997) Lamoreaux: Casimir between parallel conducting plates:

$\mathcal{F} = -\frac{\hbar c\pi^2}{240d^4}$ @ 1% Precision *Can be calculated many ways*

$\lambda = (2.14 \pm 0.13 \times 10^{-3}\,eV)$

Non-Interacting Quantum Field: $\mathcal{E} = \pm\frac{1}{2}\sum\hbar\omega_0$

(1) Casimer Effect is a fluctuation of a fine structure constant and vanishes at $\alpha \to 0$; i.e. (relativistic, retarded) Van der Waals Force between metal,

(2) Casimer effects are calculated as S-matrix elements, i.e. Feynman diagrams with external lines, without reference to vacuum. The usual calculations are based on $\frac{1}{2}\sum_i \hbar\omega$; $\boxed{\text{Energy of smooth, charge Distribution}}$

$$= \frac{1}{2}\int dx\,dy\,[\rho(x)\cdot\rho(y)/[x-y]]$$

-or- natively, $\frac{1}{8\pi}\int dx\,[E(x)]^2$ $\boxed{\text{"Stored"}}$

Dependence of the Casimer Effect on the fine structure constant

Frequencies which dominate the Casimer Force:

$$\underbrace{\omega_{pe}^2 = \frac{4\pi e^2 n}{m}}_{\text{"Plasma Freq."}}\;;\; \underbrace{\delta^{-2} = \frac{2\pi\omega|\sigma|}{c^2}}_{\text{"Skin Depth"}}\;;\; \sigma = \frac{ne^2}{m(\gamma_0 - i\omega)}$$

n = total electrons/volume

γ_0 = Damping Drude oscillator

$\alpha \gg \frac{mc}{4\pi\hbar n d^2}$

Casimer Effect without the vacuum

* Original Goal: Van der Waals between polarizable molecules at separations so large that relativistic (retardation)

$$\Delta E = -\frac{23\hbar c}{4\pi R^7}a_1 a_2\;;\; a_j = \text{static polarizability}$$

$$\vec{P} = a\vec{E}.$$

Polarizable Molecules opposite at a conducting plate

$$\Delta E = -3\hbar c a/8\pi R^4$$

Expressed as Green's Function for fluctuating field

$$\epsilon = \frac{\hbar}{2\pi}\,\mathrm{Im}\int d\omega\,\omega\,\mathrm{Tr}\int d^3x\,[G(x,x,\omega+i\epsilon) - G_0(x,x,\omega+i\epsilon)]$$

G = the Full Greens Function

G_0 = Free Greens Function.

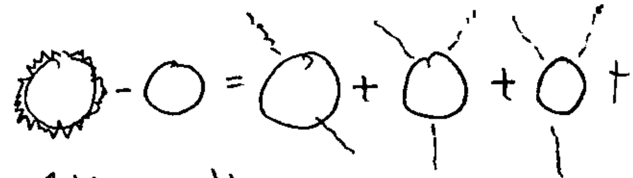
$$\frac{1}{\pi}\mathrm{Im}\int[G(x,x,\omega+i\epsilon) - G_0(x,x,\omega+i\epsilon)] = \frac{d\Delta N}{d\omega}$$

$$F = -\hbar c\pi/24a^2$$

$$\mathcal{L}_{int} = \frac{1}{2} g\,\sigma(x)\Phi^2(x) \;;\; \sigma(x) = \delta(x - a/2) + \delta(x + a/2)$$

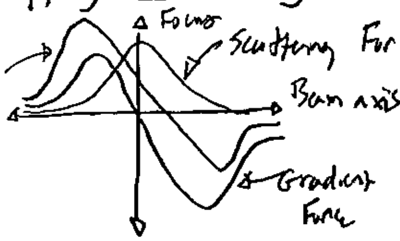
$$F(a,g,m) = -\frac{g^2}{\pi}\int_m^\infty \frac{t^2\,dt}{\sqrt{t^2-m^2}} \times \frac{e^{-2at}}{4t^2+4gt+g^2(1-e^{-2at})}$$

$$\lim_{g\to\infty} F(a,g,m) = -\int_m^\infty \frac{dt}{\pi}\,\frac{t^2}{\sqrt{t^2-m^2}\,(e^{2ta}-1)}$$



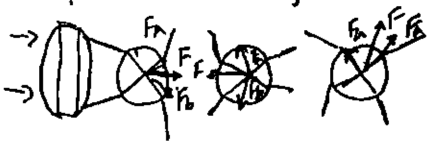
Multiple Optical Trapping and Binding: New routes to Self-assembly

Theoretical Basis:



Lorentz Force: $F = \underbrace{(p\cdot\nabla)}_{\text{Induced Dipole}}E + \underbrace{\dot{p}}_{\text{Time Derivative}}\times B$

Ray Optics Regime:



Total Force $\sum_i PR$

and infinite PT^2R^m: Fresnel Reflection

$$|F| = |Q|\frac{n_m P}{c}$$

Total Optical Force:

$$F = -\frac{n_m}{2}\alpha\nabla(E^2) + \frac{16\pi^4\alpha^2|E|^2}{3\varepsilon_0 n_m \lambda^4}\hat{k}$$

Optical Frequency Effective Polarizability

Radiation wave-vector

Permittivity

Wavelength

$$\alpha = 4\pi\varepsilon_0 n_m^2 \cdot r^3\left(\frac{m^2-1}{m^2+2}\right)$$

Mie and Lorenz-Mie Regime:

Maxwell Stress Tensor

$$\langle F\rangle = \int\langle T(r,t)\rangle\cdot n(r)\,da$$

$$= \varepsilon_m \cdot E_i E_j + \mu_m^{-1} B_i B_j - \frac{1}{2}\left(\varepsilon_m E^2 + \mu_m^{-1} B^2\right)\delta_{ij}$$

Threshold Stability for Gaussian Beam:

$$m = \frac{n_p}{n_m}$$

$$\text{Ratio} = \frac{3\sqrt{3}}{64\pi^5}\,\frac{n_m^2}{\left(\frac{m^2-1}{m^2+2}\right)}\,\frac{\lambda^5}{r^3 w_0^2} \geq 1$$

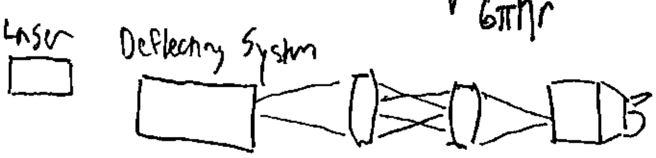
$$\text{"}\frac{F_{Grad}}{F_{Scr}}\text{"}$$

where $z = \pi w_0^2/\sqrt{3}\lambda$

Counter-Propagating Dual Beam Trap:

"Fiber Trap"

Diffusion Distance: $d = \sqrt{2\frac{k_B T}{6\pi\eta r}t}$



Typical Parameters

Device	[KHz]	[°/o]	[mRad]	[μRad]
Galvo-mirror	1	✫	500	10
Piezo-mirror	1	✫	50	0.1
AOD	10-50	50	30	<1nm
EOD	>100	80	1	<1nm

$$\Delta E_{ABC} = \left(\frac{I f_{ep}^2 \alpha_0^2}{4\pi \epsilon_0^2 c A_p R^3}\right)\left\{\cos^2\phi\left(\cos kR + kR\sin kR - k^2R^2\cos kR\right) - 2\sin^2\phi\left(\cos kR + kR\sin kR\right)\right\}\cos(\ell \delta \varphi)$$

$$\Delta E^0_{ABC} = \left[\frac{I f_{ep}^2 \alpha_0^2 (1 - 3\sin^2\phi)}{8\sqrt{2\pi}\,\epsilon_0^2 r^3 c A_p}\right]\frac{\cos(\ell \Delta\varphi)}{(\eta - \cos\varphi)^{3/2}}$$

Collapsing Optically Bound Structures:

$$\Delta E = \left(\frac{k^3 \alpha_0^A \alpha_0^B}{16\pi^2 c \epsilon_0^2}\right)\left(\left[I_s D_1 + 2 I_p C_1\right]\frac{\cos kR}{kR} + \left[I_s D_3 + 2 I_p C_3\right]\left(\frac{\cos kR}{k^3R^3} + \frac{\sin kR}{k^2R^2}\right)\right)$$

$$= \left[\frac{(2I_p - I_s)\alpha_0^A \alpha_0^B}{16\pi^2 c \epsilon_0^2}\right]\frac{(1 - 3\cos^2\theta)}{R^3}$$

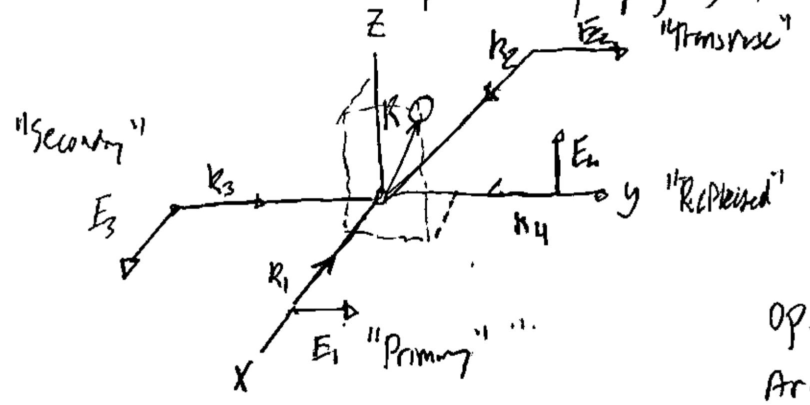
where $C = (1 - n\cos^2\theta_1)\cos(kR\sin\theta_1\cos\theta_2)$
$D = (2 - n\sin^2\theta_1)\cos(kR\cos\theta_2)$ } Angles

I_p and I_s = Irradiance.
primary, secondary

Large = Array of Particles

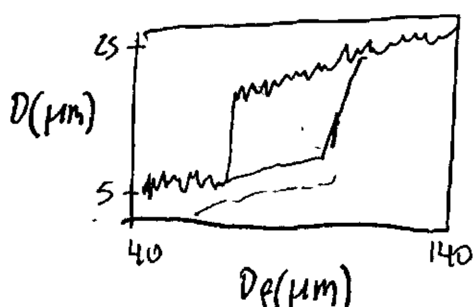
Small = Casimir-Polder Forces

Pair of Particles by counter propagating Beam:

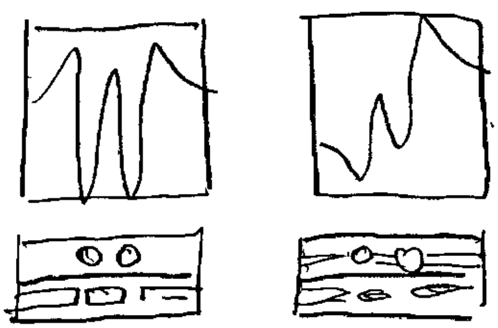


Experiments of Optical Binding:

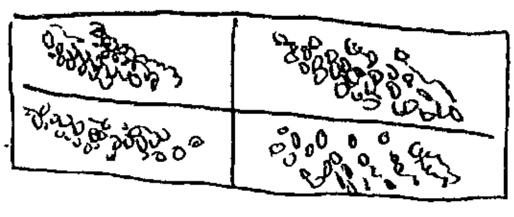
Longitudinal Binding Structures:



Optically Bound Arrays:

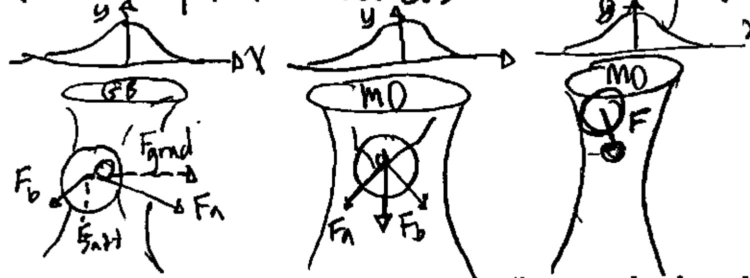


Self-organization:



Optical Trapping of Structured Light:

Principal of Optical Tweezers: Three regimes exist: Rayleigh, intermediate, Ray optics

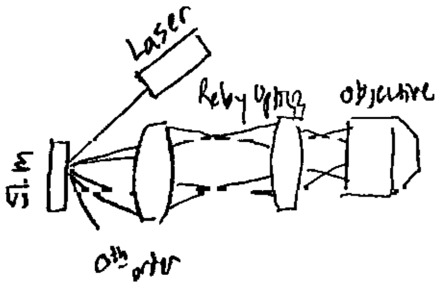


$\xi = k_m a$; $k_m = 2\pi n/\lambda_0$; a = radius

$\xi \gg 1$: ray optics

$\xi \ll 1$ and $\xi \cdot n_p/n_m$: Rayleigh Regime

Holographic Optical Trapping:



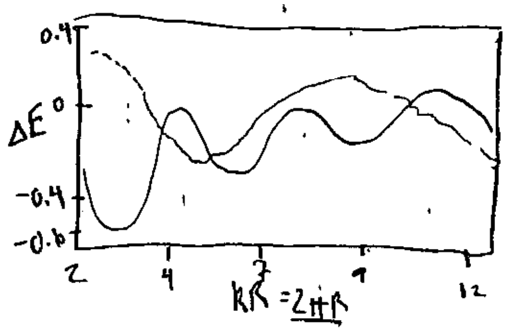
Major Advantages as compared to time-sharing:

- Organization in three dimensions
- Independent shape of traps
- Remove abberations
- Arbitrary power.

Gerchberg-Saxton used for light pattern

Fresnel ⟷ Fourier Regime e^{ikR}

Electric-Field: $E = k^2 (R \times \mu) \times R \dfrac{e^{ikR}}{4\pi \varepsilon_0 R} + \left[3\hat{R}(\hat{R}\cdot\mu) - \mu\right]\left(\dfrac{1}{4\pi\varepsilon_0 R^3} - \dfrac{ik}{4\pi\varepsilon_0 R^2}\right)$

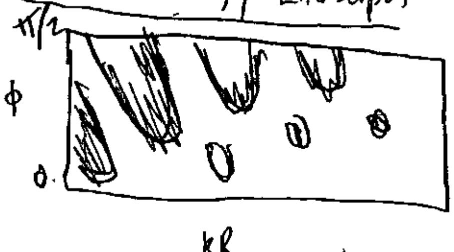


Quantum Electrodynamics Derivation:

$$\Delta E_{ind}(k, R) = \left(\frac{2I}{\varepsilon_0 c}\right) e_i e_k \,\mathrm{Re}\left[\alpha_{ij}^{A}(k)\, V_{jk}(k, R)\, \alpha_{kl}^{B}(k)\right] \cos(k \cdot R)$$

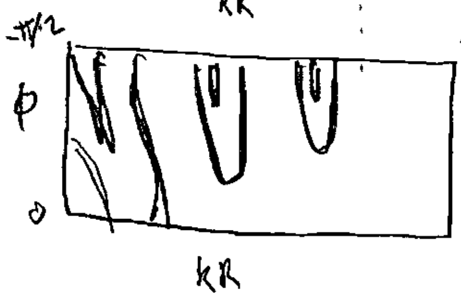
$$V(k, R) = \frac{\exp(ikR)}{4\pi\varepsilon_0 R^3}\underbrace{\left[(1 - ikR)(\delta - 3\hat{R}_i \hat{R}_j)(\delta - R_{jj})\right]}_{\text{Dipole-Dipole Interaction Tensor}}$$

Potential Energy Landscapes



$$\Delta E = \left(\frac{I(\omega)}{2\pi c \varepsilon_0^2 R^3}\right) \mathrm{Re}\left[\alpha^2(-k, k) \cos(kR \sin\phi \cos\xi)\right.$$
$$\left.\times \left\{(1 - ikR)(1 - 3\cos^2\phi) - k^2 R^2 \sin^2\phi\right\} \exp(ikR)\right\}$$

...continued for aggregates...



Broadband Irradiation!

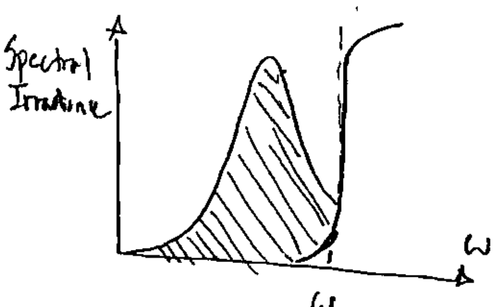
$$S(\omega) = \frac{K}{(\omega_0 - \omega)^2 + \gamma^2}$$

"Lorentzian Form"

Particle Susceptibility:

$$\chi^{(A)}(\omega) = \chi^{(B)}(\omega) = \frac{2N}{3\hbar}\frac{\omega_{n0}(\mu^{n0})^2}{\varepsilon(\omega_n^2 - \omega^2)}$$

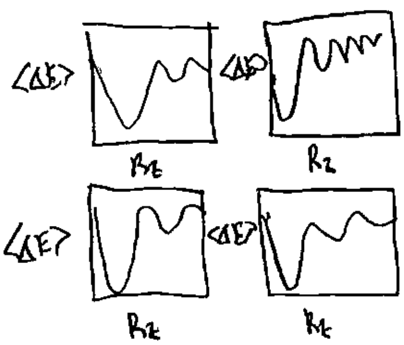
Where $\mu^{n0} = \langle n|\mu|0\rangle$

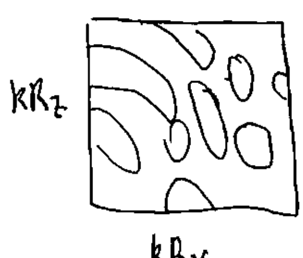


$$\langle \Delta E_{avg} \rangle = \mathrm{Re}\left[\frac{4KN^2|\mu|^4 \omega_n^3}{9c\hbar^2\varepsilon_0}\int_0^{\omega}\frac{1}{(\omega_0^2 - \omega^2)^2}\frac{V_{jj}(\omega|\varsigma R)}{(\omega_0 - \omega)^2 + \gamma^2}\cos\left(\frac{\omega(R_{00})}{c}\right)d\omega\right]$$

Multiple Particle Optical Binding:

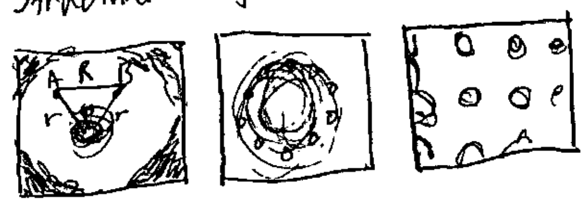
* at integral values Three or more particles.





$kR_x = 0$

Structured Light:



Gaussian Laguerre Beam

Clusters in LG Beam

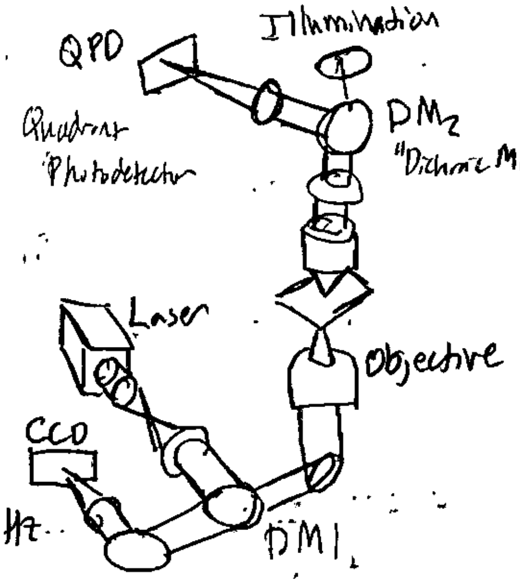
ΔE against Δ4

Rayleigh Regime: $\langle F\rangle = \frac{1}{4}\mathrm{Re}(\alpha_p)\nabla|E|^2 + \frac{\sigma(\alpha_p)}{2c}\mathrm{Re}(E\times H^*) + \sigma(\alpha_p)c\nabla\times\left(\frac{\epsilon_0}{4\omega i}E\times E^*\right)$

$F_{grad} = \frac{-2\pi n_m r^3}{c}\left(\frac{n^2-1}{n^2+2}\right)\nabla I(r)$ Polarizability: $\alpha_p = \frac{\alpha_0}{1-i\alpha_0 k^3/6\pi\epsilon_m}$

$\alpha_0 = 4\pi n_m^2 r^3 \epsilon_0\left(\frac{n^2-1}{n^2+2}\right)$

$F_{scat} = \frac{8\pi n_m k^4 r^6}{3c}\left(\frac{n^2-1}{n^2+2}\right)I(r)\hat{z}$



Finite-Time Difference Domain:

• Numerical simulation.

Langevin Equation: $\Gamma_0\dot{x} = -k_0 x + F_{th}$

Stokes — Drag coefficient $= 6\pi\eta a$

Trap Stiffness $k_0 = m\Omega_0^2$

Driving Motion

Resonant Frequency: $\nu \le 1/2\pi\sqrt{k_0/m} \le 10^4$ Hz

Trap Stiffness: $0.5 \le k_0 \le 50$ pN/μm

Power spectral Density: $S_x(f) = \frac{4\Gamma_0 k_B T/k_0^2}{1+f^2/f_c^2}$ where $f_c = k_0/(2\pi\Gamma_0)$

Mean Square Displacement: $\int_0^\infty S_x(f)\,df = \langle x^2\rangle = \frac{k_B T}{k_0}$

"Measured on CCD or QPD"

Optical Torques for Rotation:

$\boxed{\text{Spin Angular Momentum (SAM)}}$

↳ Derived from Poynting vector.

↳ Polarized Light

$\boxed{\text{Orbital Angular Momentum (OAM)}}$

↳ Azimuthal Phase $[\exp(i\ell\phi)]$

ℓ = integer value, topical charge

ϕ = Azimuthal Angle.

Overview of Structured Light beams

Electromagnetic waves, λ, A, ϕ, t

Laguerre-Gaussian Beams A, ℓ

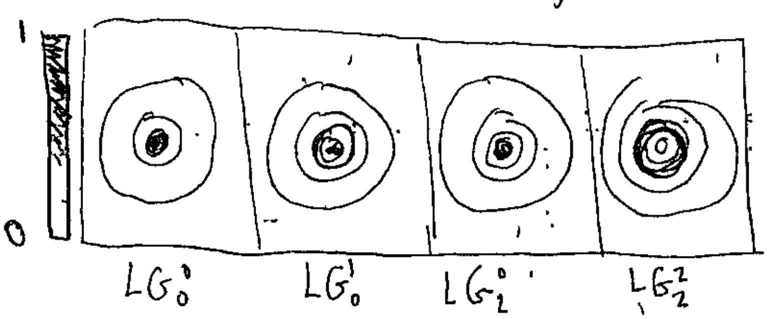
$LG_p^\ell(\rho,\phi,z) = \frac{\omega_0}{\omega(z)}\sqrt{\frac{2p!}{\pi(|\ell|+p)!}}\left(\frac{\sqrt{2}\rho}{\omega(z)}\right)^{|\ell|} L_p^{|\ell|}\left[2\left(\frac{\rho}{\omega(z)}\right)^2\right]$

Beam waist — Laguerre Polynomial

$\boxed{\text{OAM}}$ $\times\exp\left[i(2p+|\ell|+1)\zeta(z)\right]$ Longitudinal phase

$\times\exp\left[-\left(\frac{\rho}{\omega(z)}\right)^2\right]\exp\left[-\frac{ik\rho^2}{2R(z)}\right]\exp(i\ell\phi)$

Curvature of Beam Wavefront



Bessel Beam: $B(\rho,\phi,z) = E_0 \underbrace{J_\ell(k_t \rho)}_{\substack{\text{Bessel} \\ \text{Polynomial} \\ \text{"Transverse"}}} \underbrace{\exp(ik_z z)}_{\text{"Longitudinal"}} \exp(\ell\phi)$

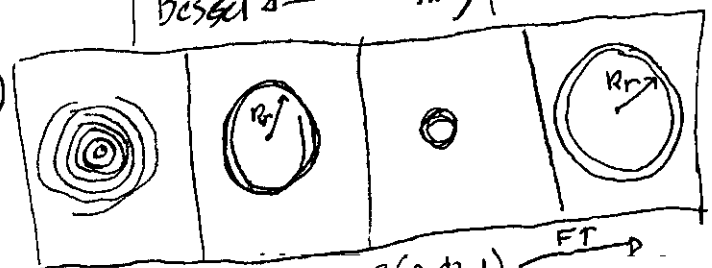
$\boxed{\text{OAM}}$

$|k| = k = 2\pi/\lambda$

$|k|^2 = k_1^2 + k_z^2$

$$B(\rho,\phi,0) = J_\ell(k_\ell \rho)\exp\left[-\left(\frac{\rho}{w_0}\right)^2\right]\exp(-i\ell\phi)$$

Bessel $\xrightarrow{FT}$ Ring



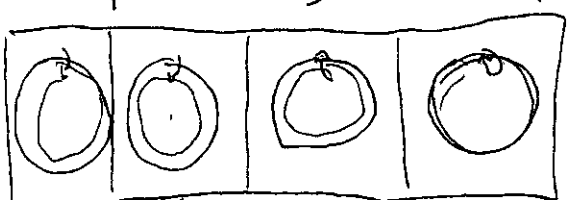
$B(\rho,\phi,0) \xrightarrow{FT}$ Rings $\quad B(\rho,\phi,1) \xrightarrow{FT}$

Perfect Vortex Beams

radius

$$V_\ell(\rho,\theta) = \exp\left(-\frac{\rho-\rho_0}{\Delta\rho^2}\right)e^{i\ell\theta}$$

$\Delta\rho$ = width of ring



$\ell=1 \quad \ell=5 \quad \ell=10 \quad \ell=15$

Mathieu-Gauss Beams

$x = f\cosh\xi\cos\eta \;;\; y = f\sinh\xi\sin\eta \;;\; z = z$

radial coordinate $= \xi$; angular coordinate $= \eta$

$$\left[\frac{\partial^2}{\partial\xi^2} + \frac{\partial^2}{\partial\eta^2} + \frac{f^2k_t^2}{2}(\cosh 2\xi - \cos 2\eta)\right]u_T(\xi,\eta) = 0$$

f = semi-focus distance ; $f^2 = a^2 - b^2$ a and b

$e = f/a$

When Hemholtz Eqn split into radial and angular

k_T = Transverse Component.

Mathieu:

$$M_m^e(\xi,\eta,z:q) = \underbrace{C_m}_{\text{Normalization}} \underbrace{Je_m(\xi,q)ce(\eta:q)}_{\text{Mathieu Functions}}\exp(ik_z z)$$

$$M_m^o(\xi,\eta,z:q) = S_m\, Jo_m(\xi,q)\, se_m(\eta:q)\exp(ik_z z)$$

Normalization

Mathieu Functions

$$\frac{d^2y}{dx^2} + (a - 2q\cos(2x))y = 0$$

Mathieu-Gauss Modes are described

$$MG_m^e(\xi,\eta,z:q) = \exp\left(-\frac{ik_t^2}{2k}\frac{z}{\mu}\right)M_m^e(\xi,\eta,z,q) \times \exp\left(\frac{-r^2}{\mu w_0^2}\right)\frac{\exp(ikz)}{\mu}$$

$\boxed{\text{"Even"}}$

$$MG_m^o(\xi,\eta,z:q) = \exp\left(-\frac{ik_t^2}{2k}\frac{z}{\mu}\right)M_m^o(\xi,\eta,z,q) \times \exp\left(\frac{-r^2}{\mu w_0^2}\right)\frac{\exp(ikz)}{\mu}$$

$\boxed{\text{"Odd"}}$

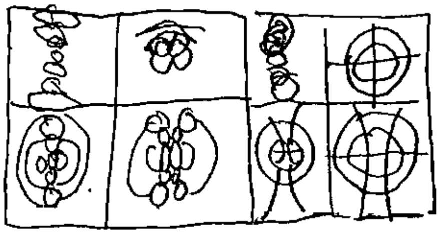
With definitions $x = f_0(1 + iz/z_R)\cosh\xi\cos\eta$

$y = f_0(1 + iz/z_R)\sinh\xi\sin\eta$

Rayleigh Range : $z_R = kw_0^2/2$

Beam Waist

$\mu = 1 + iz/z_R$



Mathieu Beams

$$A_0(s_x,s_y,\xi)=Ai\left[s_x-\left(\frac{\xi}{2}\right)^2+ib\xi\right]Ai\left[s_y-\left(\frac{\xi}{2}\right)^2+ib\xi\right]$$

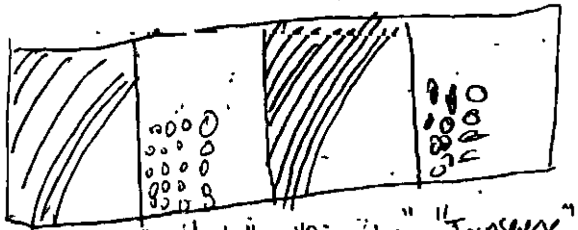
$$\times\exp\left[b(s_x+s_y)-b\xi^2+ib\xi^2-\frac{\xi^3}{6}+\frac{i\xi(s_x+s_y)}{2}\right]$$

Airy-Gaussian Beams:

$$A(s_x,s_y,\xi)=Ai\left[s_x-\left(\frac{\xi}{2}\right)^2\right]Ai\left[s_y-\left(\frac{\xi}{2}\right)^2\right]\times\exp\left[\frac{i\xi}{2}\left(s_x+s_y-\frac{\xi^3}{3}\right)\right]$$

$s_x=x/x_0$; $s_y=y/y_0$

$\xi=z/(kx_0^2)$



$$F^{-1}\{A_0(s_x,s_y)\}\propto\exp\left[-b(k_x^2+k_y^2)\right]\exp\left[\frac{i(k_x^3+k_y^3)}{3}\right]$$ k_x and k_y are transverse phases

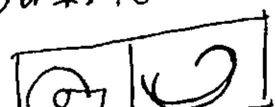
Ince-Gaussian Beams: "orthogonal solutions to the paraxial Equation"

"Even" $$IG^e_{p,m}(r,\epsilon)=\frac{Cw_0}{w(z)}C^m_p(i\xi,\epsilon)C^m_p(\eta,\epsilon)\exp\left(\frac{-r^2}{w^2(z)}\right)\times\exp\left\{i\left[kz+\frac{kr^2}{2R(z)}-(p+1)\zeta(z)\right]\right\}$$

"odd" $$IG^o_{p,m}(r,\epsilon)=\frac{Sw_0}{w(z)}S^m_p(i\xi,\epsilon)S^m_p(\eta,\epsilon)\exp\left(\frac{-r^2}{w^2(z)}\right)\times\exp\left\{i\left[kz+\frac{kr^2}{2R(z)}-(p+1)\zeta(z)\right]\right\}$$

Ince Equation: $w''+\xi\sin(2z)w'+(\eta-p\xi\cos(2z))w=0$; $\epsilon=2f_0^2/w_0^2$

Helico-Conical Beams: $\psi(r,\phi)=l\phi(K-r/r_0)$; l = topical charge, K is 0 or 1. r_0



Vector Light Fields: "Solutions to the Helmholtz Equation"

$$U(r)=u_1(r)e^{i\delta_1}\hat{e}_R+u_2(r)e^{i\delta_2}\hat{e}_L$$

Phases

"Right-hand" "Left Hand"

Orthogonal Functions

As LG^l_p basis,

$$U(r)=\frac{1}{\sqrt{2}}\left(LG^l_pe^{i\delta_1}\hat{e}_R+LG^l_pe^{i\delta_2}e_L\right)$$

$$U(r)=\frac{1}{\sqrt{2}}\left(LG^l_0\hat{e}_R+LG^{-l}_0e^{i\delta}e_L\right)$$

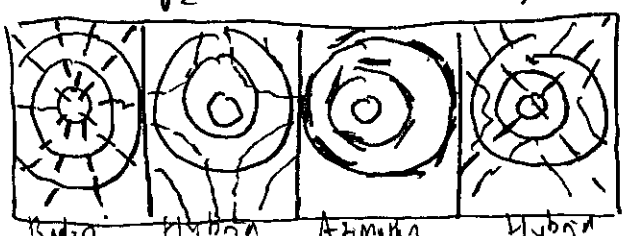
Orthogonal vectors:

$$U_{TE}(r)=\frac{1}{\sqrt{2}}\left(LG^1_0\hat{e}_R+LG^{-1}_0e_L\right)$$

$$U_{TM}(r)=\frac{1}{\sqrt{2}}\left(LG^1_0\hat{e}_R-LG^{-1}_0\hat{e}_L\right)$$

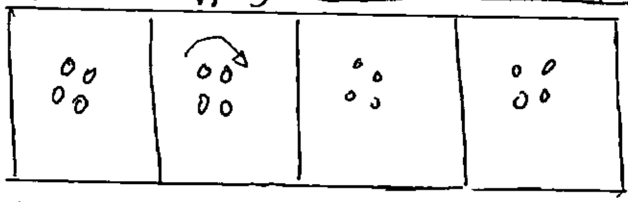
$$U_{He}(r)=\frac{1}{\sqrt{2}}\left(LG^1_0e_L+LG^{-1}_0e_R\right)$$

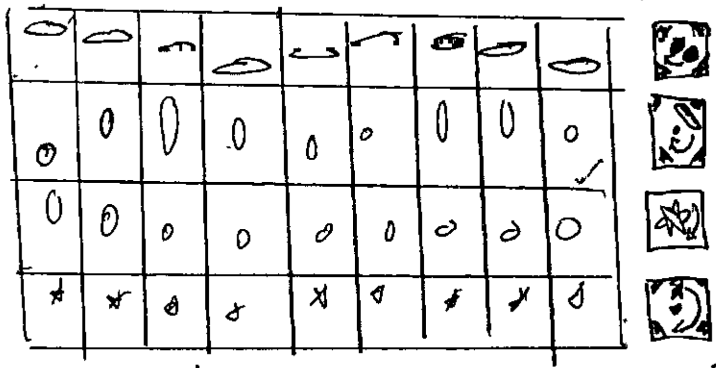
$$U_{He}(r)=\frac{1}{\sqrt{2}}\left(LG^1_0\hat{e}_L-LG^{-1}_0\hat{e}_R\right)$$



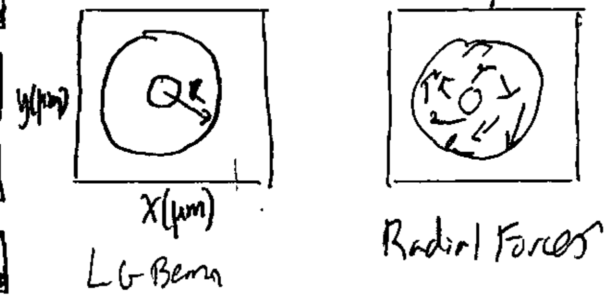
Optical Trapping with Structured Light

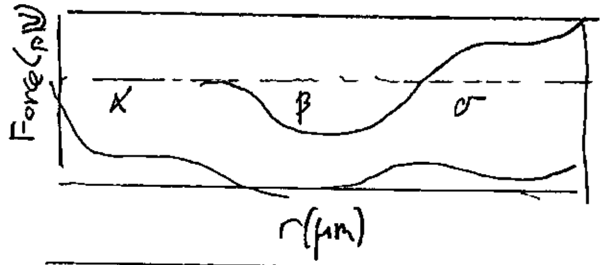
"Spatial Light Modulators" SLMs
"Digital Micromirror Devices" DMDS
"Holographic optical Tweezers" HOTS



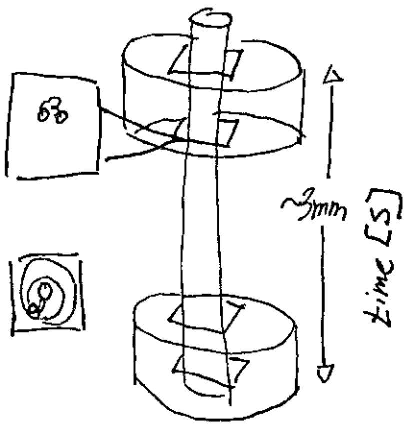


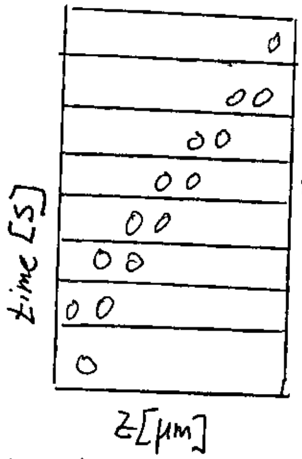
Anomalous Motion of particle in high order LG Beam

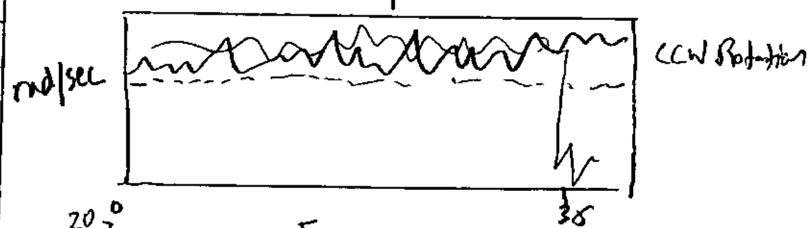


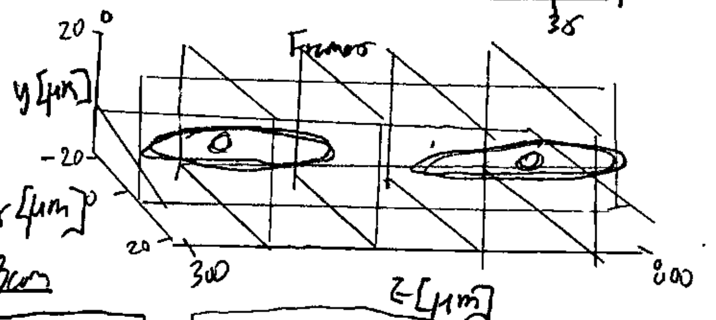


Different Optical Planes:

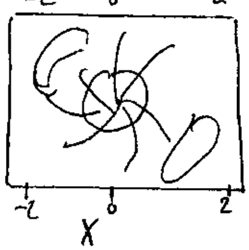








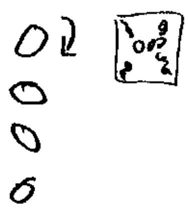
Optical Trapping in Pure Vortex



Airy Beam

$\boxed{\text{Parallel planes}}$

Mathieu Beam

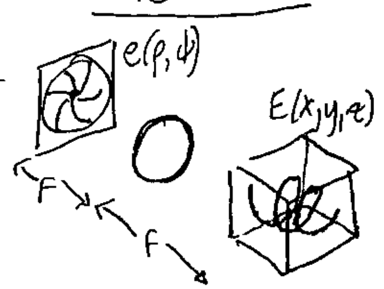


Optical Trapping ~~with Holographic~~ Traps

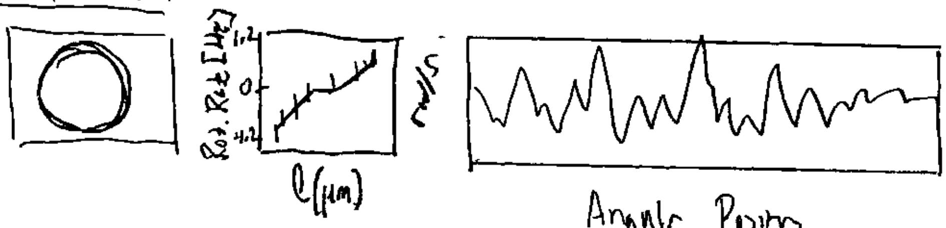
- SLMs
- Solutions to the wave equations
- Diffractive optical Elements

↳ Diffraction.

HC Beams



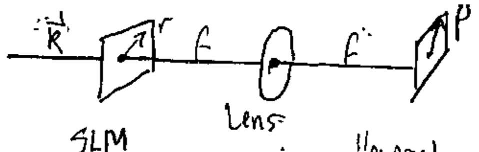
Perfect Vortex



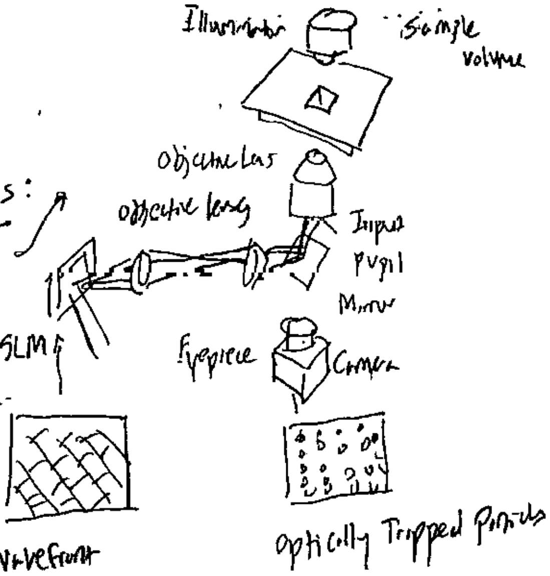
Optical Trapping with Modulated Beams

$$R(\theta) = a\frac{\lambda}{NA}\left[1+\frac{1}{l_0}\frac{\partial\phi(\theta)}{\partial\theta}\right]\ ;\ \phi(\theta)=l[\theta+\alpha\sin(m\theta+\beta)]$$

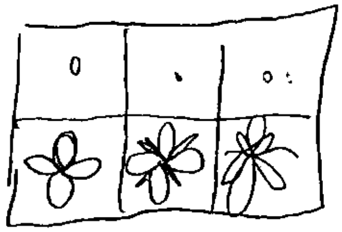
Optical Tweezers Using Computer Generated Holograms:



Experimental setup:



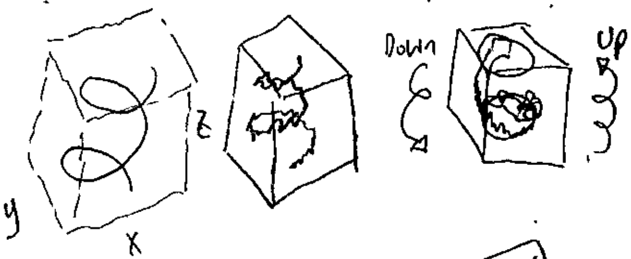
Complex Optical Pattern



Optical Trapping with Optical Solenoids

$$u_{l,\ell}(r,z)=\sum_{m=|l-\gamma R|}^{|l|}\frac{l-m}{\gamma^2}J_m(q_mR)\exp\left[\frac{i(l-m)}{\gamma}z\right]\exp(im\theta)J_m(q_m r)$$

where $q_m^2 = k^2-(l-m)^2/\gamma^2$ and $[l-\gamma R]$ is an int.



Experimental Theoretical

Optical Trapping and Transporting along 3D Trajectories

$$R_0(s)=R\left(\cos\left(\frac{s}{R}\right)\cos\beta,\ \sin\left(\frac{s}{R}\right),\ \cos\left(\frac{s}{R}\right)\sin\beta\right)$$

Radius angle

Radius of curvature

Rotated angle $-\beta$ about y-axis.

Note: $S(t)=\int_0^t |c'(\tau)|d\tau$

$$|c'(t)|=\sqrt{R(t)^2+R'(t)^2+u_z'(t)^2} = \frac{dc}{dt}$$

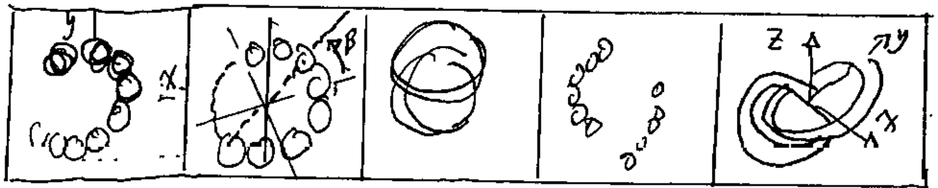
$$E(r)=\frac{1}{T}\int_0^T \Phi(r,t)\Phi(r,t)\,|c_2'(t)|\,dt$$

$$c_2(t)=(x_0(t),y_0(t),z_0(t))$$

$$L=\int_0^t|c_2'(t)|\,dt$$

$$|c_2'(t)|=[x_0'(t)^2+y_0'(t)^2]^{1/2}$$

$$\Phi(r,t)=\exp\left\{\frac{i}{\rho^2}[yx_0(t)-xy_0(t)]\right\}\exp\left[\frac{i2\pi m}{S(T)}S(t)\right]$$



Polymorphic Beam

$$E(r_0)=\int_0^T g(t)\exp\left[-\frac{ik}{2f^2}u_z(t)r_0^2\right]\exp\left[\frac{ik}{f}r_0R(t)\right]dt$$

$R(t)=[R(t)\cos t,\ R(t)\sin t]$

Complex Amplitude Distribution

$$E(r,z=u_z(t))=-\frac{i\lambda\exp[iku_z(t)]}{f}\int_0^T g(t)\,\delta\left[\frac{1}{f}(R(t)-r)\right]dt$$

$\boxed{\text{Phase}}$ $g(t)=|g(t)|\exp[i\Phi(t)]$

Variation of phase

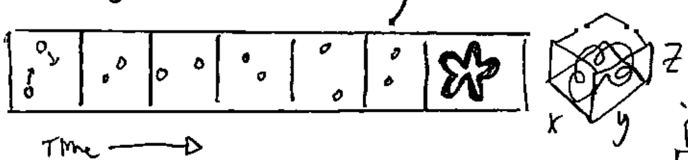
$$\Psi(t)=\frac{2\pi l}{S(T)}S(t)=\Phi(t)+ku_z(t)$$

The 3D-curve has $S(t) = \int_0^t [x_0 y_0' - y x'] \, d\tau$

With a spline:

$$b_n(\tau) = \underbrace{(1-\tau)^3 P_s^{(n)} + 3\tau(1-\tau)^2 T_s^{(n)}}_{\text{Starting}} + \underbrace{3\tau^2(1-\tau)^2 T_e^{(n)} + \tau^3 P_e^{(n)}}_{\text{Ending}}$$

"Bezier spline"

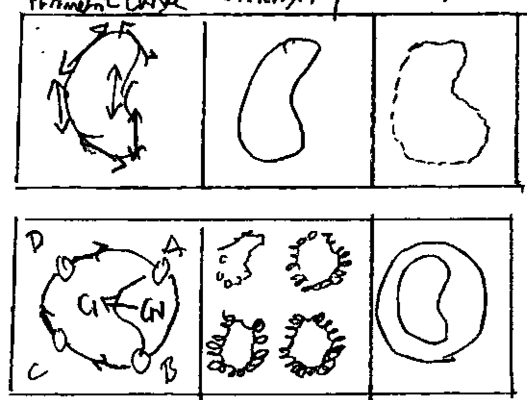


Quadratic Phase:

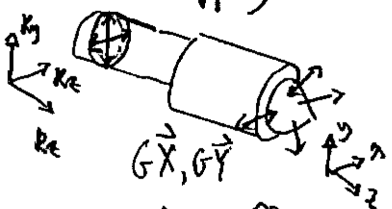
$$\phi(r, t) = \exp\left\{ i\pi \frac{[x - x_0(t)]^2 + [y - y_0(t)]^2}{\lambda f^2} z_0(t) \right\}$$

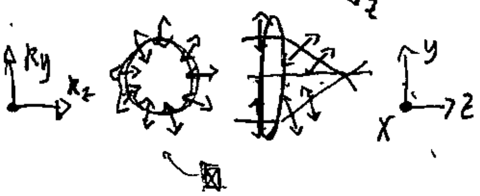
Defocusing Parameter $z_0(t)$, along $\vec{C}_2(t)$

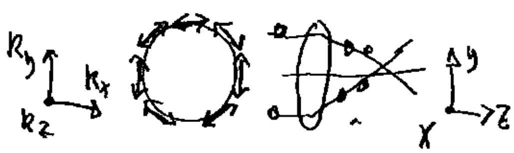
$C(t) = \{ b_1(\tau), b_2(\tau) \ldots b_m(\tau) \}$



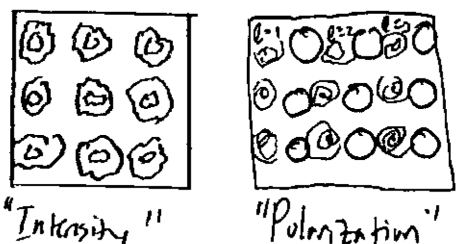
Optical Trapping With Vector Beams

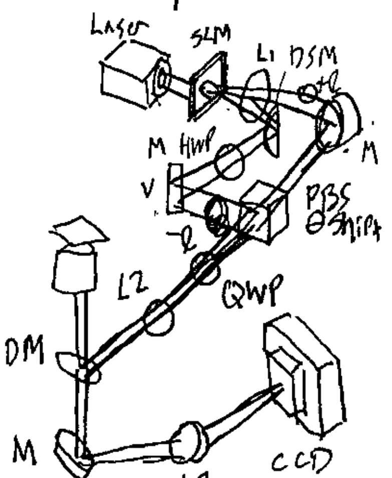






Schematic





CCD = Charge Coupled Device

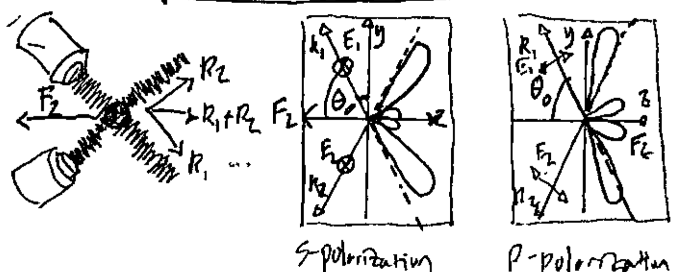
PBS = Plate Beam Splitter

HFP = Half Wave Plate

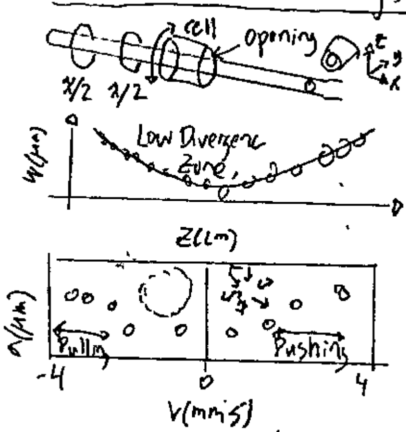
SLM = Spatial Light Modulator

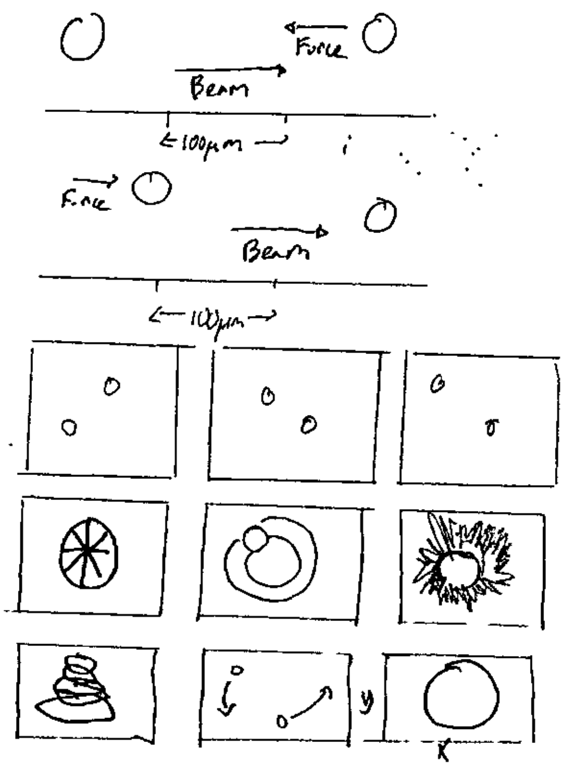
DSM = Dichroic Silver mirror

Optical Traps with Tractor Beams

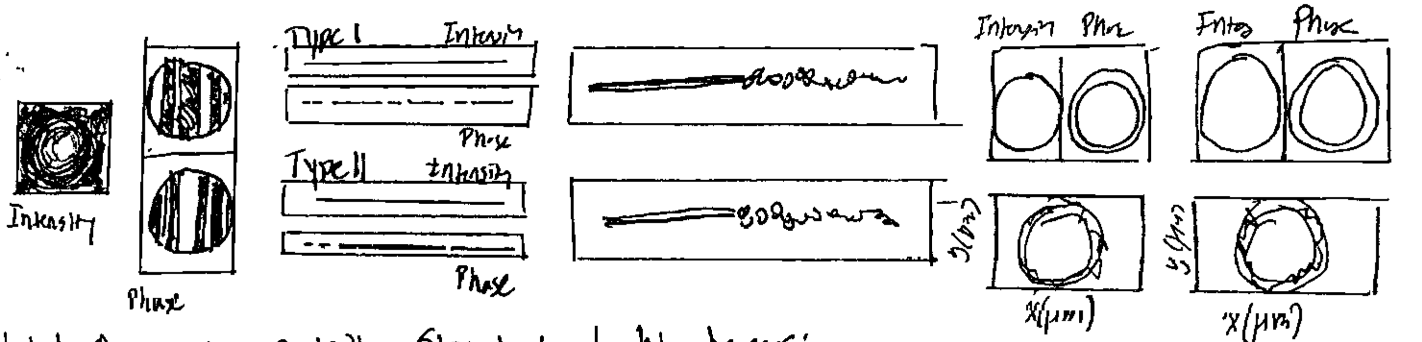


Demonstration of optical Tractor Beams:

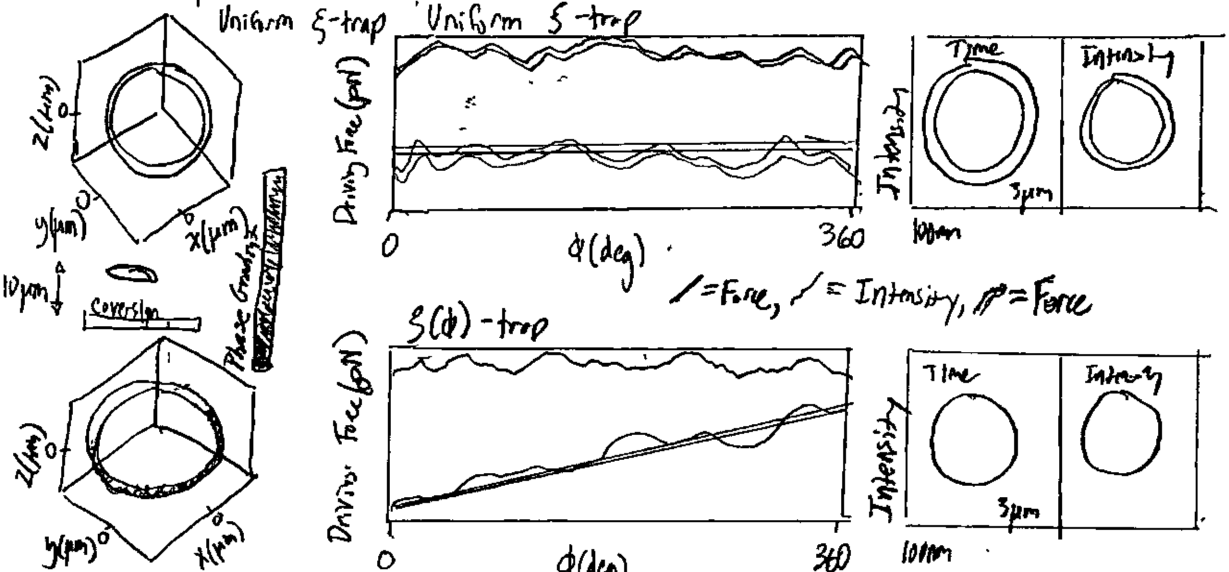




$\boxed{\text{Phase Gradient}}$



Levitated Optomechanics with Structured Light beams:



$m\ddot{x} = -\Gamma_0\dot{x} - k_0 x + F_{th}$; $k_0 = m\Omega_0^2$; $\Omega_0/(2\pi)$; $S_x(\omega) = \frac{k_B T}{\pi m}\frac{\Gamma_0}{(\omega^2-\Omega_0^2)^2+\omega^2\Gamma_0^2}$ $\leftarrow \boxed{\text{Power Spectral Density}}$

$\boxed{\text{Friction coefficient}}$ $\boxed{\text{stiffness}}$ $\boxed{\text{Trap Frequency}}$ "Lorentzian"

Determined by $\sqrt{k_0/m}$

Regimes: $\Gamma_0 (\gg 2\Omega_0)$ @ ATM

$\Gamma_0 = 2\Omega_0$: Critically Dampening.

$\Gamma \ll \Omega_0$ @ <10 mbar

$Q = \Omega_0/\Gamma_0$ $\leftarrow \boxed{\text{Dampening coefficient changed by pressure.}}$

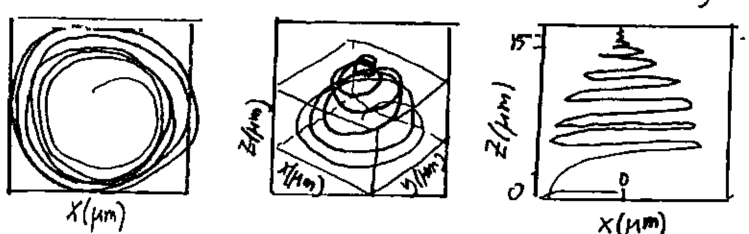
$\boxed{\text{Quality Factor}}$

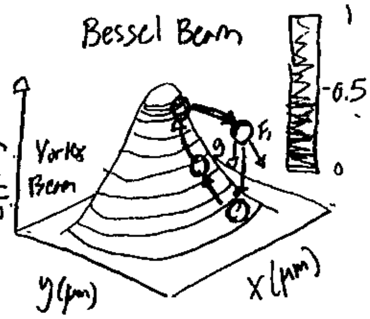
$\boxed{\text{Many Cites}}$

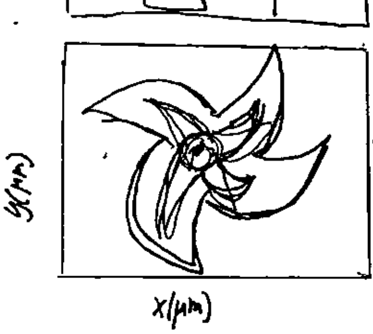
$\int_0^\omega S_x(\omega)\,d\omega = \langle x^2 \rangle = \frac{k_B T_{cm}}{m\Omega_0^2}$ "Minimizing is macroscopic ground state cooling"

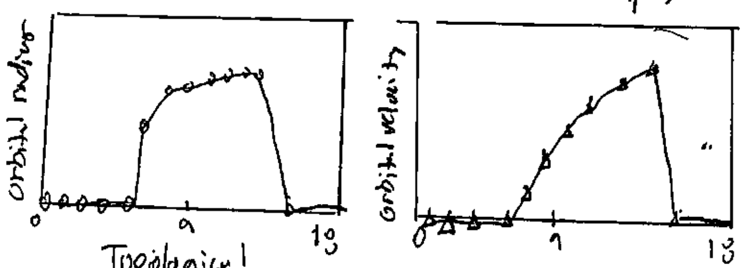
"Bessel Beam" $\boxed{\text{Vortex Beam}}$

(1) Trapped in rotation at Bessel Beam
(2) Horizontally launched into free space by vortex beam
(3) Driven by scattering and gradient forces.







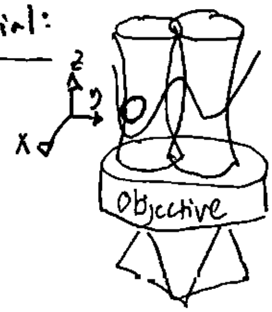


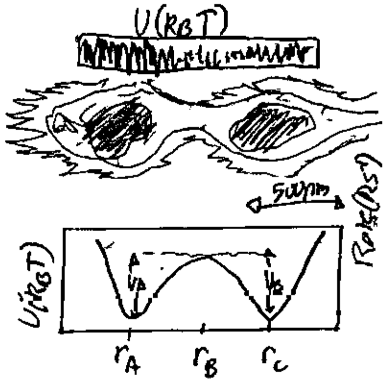
$\boxed{\text{OAM}}$

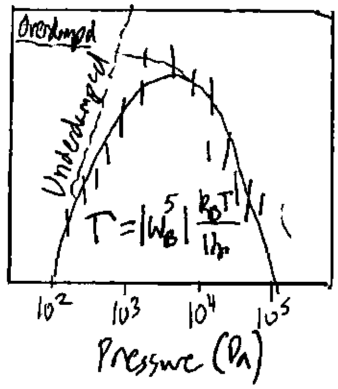
Note: Orbital Motion [l] is related to radius and velocity.

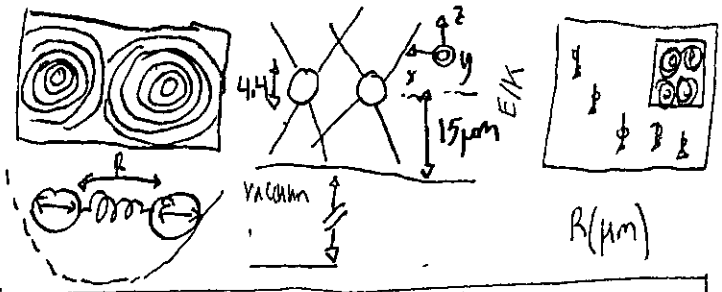
Double-Well Optical Potential:

Biomedical Application of Optical Trapping With Structured Light









Acting Gradient Force: $\langle F_{grad} \rangle = \frac{\alpha'}{2} \langle \nabla E^2 \rangle$

E = Electric field,

Approx: $\langle F_{grad,q} \rangle = -k_q q$

Optomechanics or Levitated particle

Optically Trapped particles: the basics

$\omega_L = 2\pi c/\lambda$: complex polarizability $(\alpha[\omega_L])$:

$$\alpha(\omega_L) = 4\pi\epsilon_0 R^3 \frac{\epsilon_R(\omega_L) - 1}{\epsilon_R(\omega_L) + 2} ; \epsilon_R(\omega_L) = n(\omega_L)^2$$

Absorption cross-section: $\sigma_{Abs} = \omega_L \alpha''/(c\epsilon_0)$

Equations of Motion:

$$M\ddot{q}(t) = -M\Gamma_{cm}\dot{q}(t) - M\omega_q^2 q(t) + \sqrt{2\pi S_{ff}}\,\eta(t)$$

Total Momentum Dampening Rate.

$\sqrt{k/M}$

White noise $\langle \eta(t) \rangle = 0$, $\langle \eta(t)\eta(t') \rangle = \delta(t-t')$

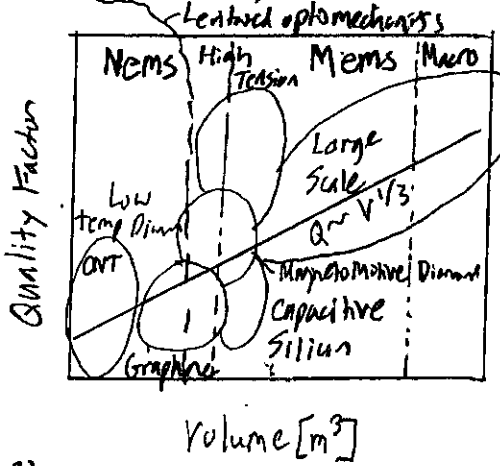
S_{ff} = Force Spectral Density

Spring Constants

$$k_q = \frac{4\alpha' P_{opt}(t)}{\pi\epsilon\epsilon_0 W_x W_y W_q^2}$$

W_q = Beam waist

P_{opt} = Power of optical beam



Momentum Dampening Rate

$2\pi S_{ff} = 2Mk_B \cdot T_{env} \cdot \Gamma_{cm}$

$$\Gamma_{gas} = \frac{6\pi\eta_{gas} \cdot R}{m} \frac{0.619}{0.619 + Kn}(1 + C_K) \approx 0.31Kn/(0.785 + 1.152Kn + Kn^2)$$

Dynamic Viscosity

Knudsen Number (mean free path/R)

@Low-pressures: $\Gamma_{gas}^{Kn\gg1} = \frac{8}{3}\sqrt{\frac{2m_{gas}}{\pi k_B T_{im}}} R^2 P_{gas}$

@Discrete photon nature of light

$$\Gamma_{rad} = \frac{C_{dp} \cdot P_{scat}}{Mc^2}$$

$C_{dp} = \frac{2}{5}$ "Parallel"

$= \frac{4}{5}$ "perpendicular"

$P_{sat} = |\alpha|^2 k_L^4 \cdot I_{opt}/6\pi\epsilon_0^2$

Temp & Pressure Generalizations

Autocorrelation function, power spectral density, C.O.M.

Autocorrelation Function (ACF) $\langle q(t)q(0) \rangle = \frac{k_B T_{cm}}{M\omega_q^2} - \frac{1}{2}\sigma_q^2(t)$

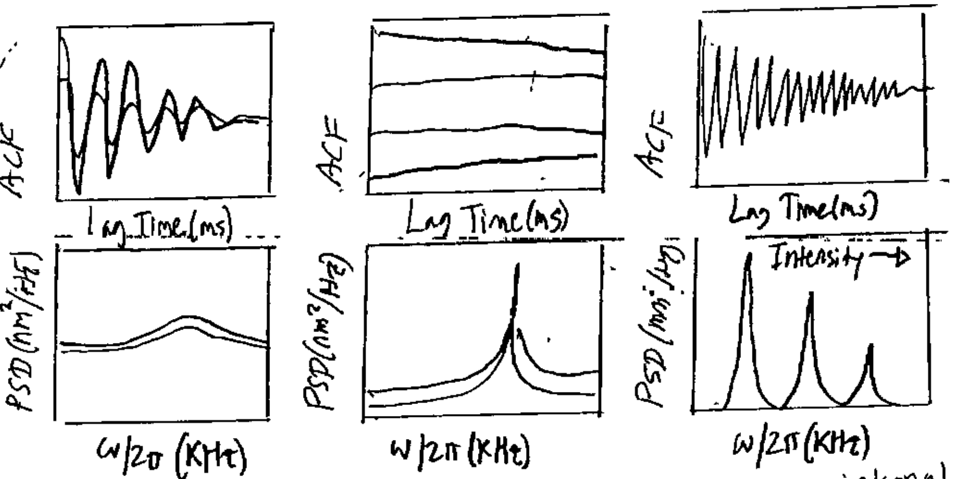
FT

Positional Variance: $\sigma_q^2(t) = \frac{2k_B T_{cm}}{M\omega_q^2}\left[1 - e^{-\frac{1}{2}\Gamma_{cm} t}\left(\cos(\omega_q t) + \frac{\Gamma_{cm}}{2\omega_q}\sin(\omega_q t)\right)\right]$

$$S_{qq}(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \langle q(t)q(0) \rangle e^{-i\omega t} dt$$; Power Spectral Densities:

$$S_{qq}(\omega) = \frac{\Gamma_{cm} k_B T_{cm}/\pi M}{(\omega^2 - \omega_q^2)^2 + \Gamma_{cm}^2 \cdot \omega^2}$$

$$\langle q(0)q(0) \rangle = \int_{-\infty}^{\infty} S_{qq}(\omega) d\omega = k_B T_{cm}/M\omega_q^2$$



Langavin Equation:

$$M\ddot{q}_i + M\Gamma_{gas}\dot{q} + M\omega_i^2\left[q_i + \left(\sum \xi q_i^2\right)q_i\right] = \sqrt{2\pi S_{ff}}$$

$$\xi_i = -1/\omega_i^2 \; ; \; \Delta\omega_i = \frac{3}{8}\omega_i \sum \xi a_j^2$$

$\boxed{\text{"Stochastic Equation"}}$

$$T_{cm}^{(q)} = M\omega_q^2 \langle q^2 \rangle$$

Quick Models: Thermally activated escape, heat engines, and internal temperature.

surface exchanges thermal energy is called $\alpha_c = \frac{T_{em} - T_{im}}{T_{int} - T_{im}}$; T_{im} = Temp impinging gas; T_{em} = Temp gas molecules

Absorption and Emission of Bulk;

$$T_{cm} = \frac{T_{im}\Gamma_{im} + T_{em}\Gamma_{em}}{\Gamma_{tot}} \; ; \; \Gamma_{tot} = \Gamma_{im} + \Gamma_{em}$$

Heating: Optical Absorption of trapped light & Blackbody

Cooling: Blackbody Emission & Energy Exchange of gas.

$$\Gamma_{em} = \frac{\pi}{8}\Gamma_{im}\sqrt{T_{em}/T_{im}}$$

Rates:

$$\dot{E}_{bb,abs} = \frac{24\zeta(5)}{\pi^2 \epsilon_0 c^3 \hbar^4}\alpha_{bb}^n (k_B T_{env})^5 \; ; \; \zeta(5) \approx 1.04 = \text{"Reimann Zeta Fn"}$$

$$\alpha_{bb}^n = 4\pi\epsilon_0 R^3 \times 0.1$$

$\approx \times 1$

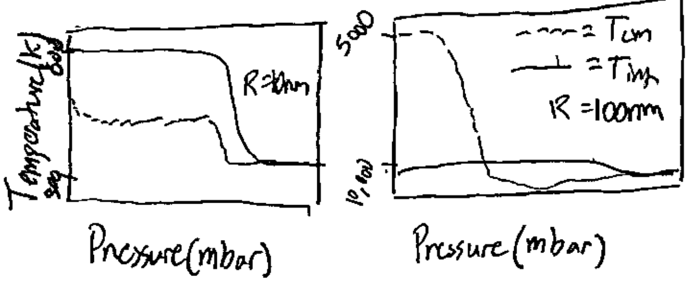
$$\dot{E}_{bb,emis} = \frac{-24\zeta(5)}{\pi^2 \epsilon_0 c^3 \hbar^4}\alpha_{bb}^n (k_B \cdot T_{int})^5$$

; v^{th}: mean thermal velocity

$\gamma = 7/5$: specific heat ratio.

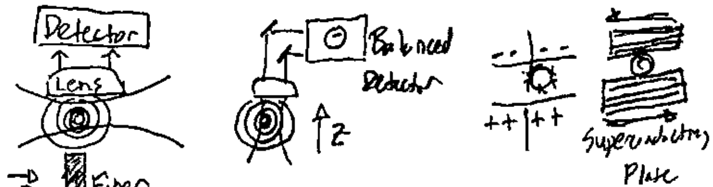
Cooling:

$$\dot{E}_{gas} = -\alpha_c\sqrt{\frac{2}{3\pi}}(\pi R^2) P_{gas} v^{th}\frac{\gamma_{sh}+1}{\gamma_{sh}-1}\left(\frac{T_{int}}{T_{im}} - 1\right) \; ; \; M c_{shc}\frac{dT_{int}}{dt} = I_{opt}\sigma_{abs} + \dot{E}_{gas}(T_{int}) + \dot{E}_{bb,abs} + \dot{E}_{bb,emis}(T_{int})$$



Detection and Calibration:

$$P_{scat} = |\alpha|^2 k_L^4 \cdot I_{opt}/6\pi\epsilon_0^2$$



Feedback cooling: Langavin Equation - $\ddot{q}(t) + \Gamma_{cm}\dot{q}(t) + \omega_q^2 q(t) = \frac{\sqrt{2\pi S_{ff}}}{m}\eta(t) + u_{fb}(t)$

$$u_{fb}(t) = G_q q(t) + G\dot{q}(t)$$

Limits of Feedback cooling: $T_{eff} = T_0 \frac{\Gamma_{cm}}{\Gamma_{cm} + \delta\Gamma}$; Equation of motion for oscillator phonon occupation:

$$\langle \dot{n}_m \rangle = B\langle n_m\rangle^2 - C\langle n_m \rangle + A \; ; \; A = \Gamma_{rad} - 6\Gamma_{ne} - \Gamma_{lin}$$

Damping Rate; Nonlinear Rate; Linear Rate

$$B = -24\Gamma_{ne}$$

$$C = 24\Gamma_{ne} + \Gamma_{lin}$$

$\boxed{\text{Sensing}}$:

$$F_{min} = \sqrt{\frac{4k_q k_B \cdot T_{cm} \cdot b}{\omega_q \cdot Q_m}}$$

b = Measurement bandwidth ; k_q = Spring constant.

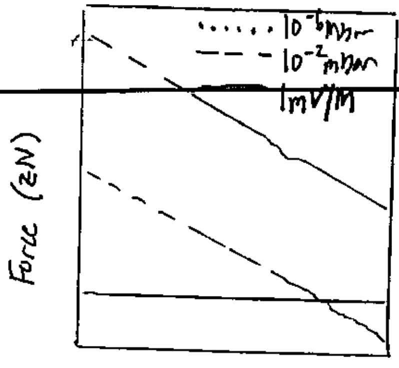
T_{cm} = Center of mass $\quad F_{min} = \sqrt{4k_B T_{cm} \cdot M\Gamma_{cm} \cdot b}$

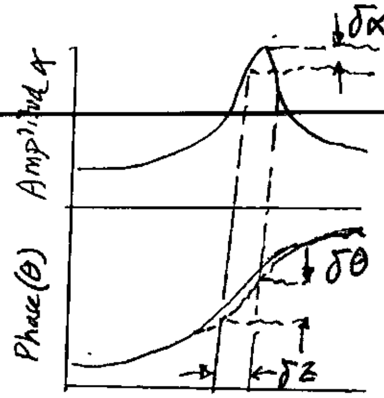
$$T_{eff} \cdot \Gamma_{eff} = T_{cm} \cdot \Gamma_{cm}$$

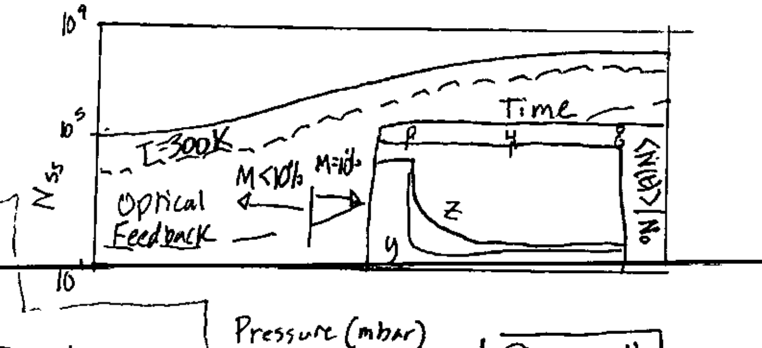
Detection of Surface Forces: Short-Range

Sensing with levitated cavity optomechanics:

$10^{-15} \, N \, Hz^{-1/2}$







$$|\delta\omega_{cav}/\omega_{cav}| = \frac{|\delta F/\delta z|}{2k_z}$$

Sensing via orientation:
$$N_\theta^{min} = \sqrt{\frac{4k_B T_\theta I \omega_\theta b}{Q_\theta}}$$

$\boxed{\text{Casimir Torques}}$

Gravitational wave sensor

T_θ = Temperature

ω_θ = Frequency

Q_θ = Quality factor librational motion

Levitated cavity optomechanics:

Quantum limited-detection "Ligo, Virgo, Fabry-Perot Cavity"

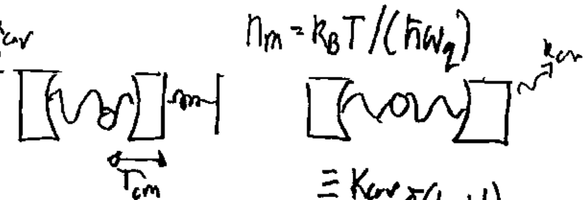
Optical Cooling: "Oscillators"

Levitated cavity optomechanics:

$$\frac{\hat{H}}{\hbar} = \underbrace{\frac{\omega_{opt}}{2}}_{\text{optical Frequency}}\big(\underbrace{\hat{p}_{opt}^2 + \hat{q}_{opt}^2}_{\text{conjugate variables}}\big) + \underbrace{\frac{\omega_q}{2}}_{\text{Mechanical Freq.}}\big(\underbrace{\hat{p}^2 + \hat{q}^2}_{\text{conjugate variables}}\big) + \underbrace{V_{int}(\hat{q}_{opt}, \hat{q})}_{\text{Interaction between optics and mechanics}}$$

Thermal Noise: $\langle \xi(t')\xi(t)\rangle \simeq \Gamma_{gas}(n_m + \tfrac{1}{2})\delta(t-t')$

$n_m = k_B T/(\hbar\omega_q)$



$$\equiv \frac{\kappa_{cav}}{2}\delta(t-t')$$

$$T_{cm} = \frac{\Gamma_{gas} \cdot T_{env} + \Gamma_{opt} \cdot T_{opt}}{\Gamma_{gas} + \Gamma_{opt}} \simeq \frac{\Gamma_{gas}}{\Gamma_{opt}} T_{env}$$

$\hat{Q} = \sqrt{\hbar/(M\omega_q)}\,\hat{q}$; $\hat{P} = \sqrt{\hbar\omega_q M}\,\hat{p}$ relates $\frac{P^2}{2m} + \frac{1}{2}m\omega^2 Q^2$

$\hat{q}_{opt} = \frac{1}{\sqrt{2}}(\hat{a}^\dagger + \hat{a})$, such that $\frac{\hat{H}}{\hbar} = \Delta\hat{a}^\dagger\hat{a} + \omega_q \cdot 2(\hat{p}^2 + \hat{q}^2) + g(a^\dagger + a)q$

Quantum BackAction regime: $C_{QM} = \frac{4g^2}{\kappa_{cav}\Gamma_{CM} \cdot n_m} \geq 1$

1) Maximizing ω_q to minimize mechanical occupation n_m for cooling rate Γ_{opt}

2) Maximizing the optomechanical coupling g.

3) Minimizing Γ_{rad}, the optical scattering contribution κ_{cav}

4) Stable trapping at high vacuum

Levitated cavity optomechanics: Four Chall.

Maximizing Mechanical Frequency:

$$\frac{\hat{H}}{\hbar} = \Delta a^\dagger a + \frac{P^2}{2M} + U_{class}(\hat{Q}) + U_{int}(a^\dagger a, \hat{Q})$$

Self-Trapping: $U_{class} \equiv U_{class}(n_{opt}, \hat{Q})$; $n_{opt} = \langle a^\dagger a\rangle$; $\hat{Q} \equiv z$; $U_d^{cav} = -A\hbar\hat{a}^\dagger\hat{a}\cos^2 k_L z\,\xi(\hat{x}, \hat{y})$

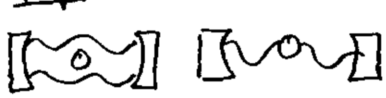
External Trapping "optical tweezer" $U_{class} \equiv V_d^{cav}(n_{opt}, \hat{z}) + \hat{U}_d(I_{opt}, z)$

$\boxed{\text{Dipole-Force Potentials}}$

Maximize Optomechanical Heating

Minimize Optomechanical Cooling

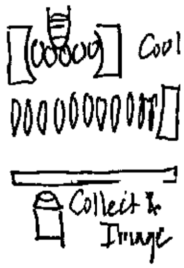
Trap

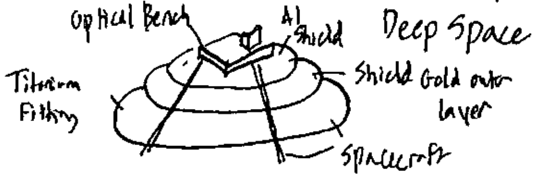


$\boxed{\text{Cavity Parameters}}$

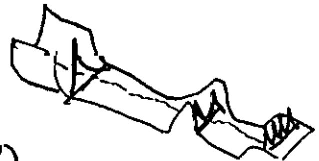
Interferometry: $\sigma_c = 2L\hat{v}_{DB}/n$; $\lambda_{DB} = h/p$; Harmonic Potential, $a_{eff} \approx \sqrt{k_B T_x / M \omega_x^2}$

High Mass Interferometry





Matterwave Interferometer



Magnetomechanical Doubleslit experiment

Decoherence: $\langle x|\rho(t)|x'\rangle \propto e^{-\bar{\Gamma} t}\langle x|\rho(0)|x'\rangle$

The decoherence rate is defined by $\bar{\Gamma}(|x-x'|) = \bar{\gamma}\left(1-e^{-\frac{|x-x'|^2}{4\tilde{a}^2}}\right)$; $2\tilde{a} = \lambda_{th}$

"Scaling Process"

$$= \begin{cases} \bar{\Lambda}|x-x'|^2, & |x-x'| \ll 2\tilde{a} \\ \bar{\gamma}, & |x-x'| \gg 2\tilde{a} \end{cases}$$

Interactions: Absorption:

$$\lambda_{th}^{bb} = \frac{\pi^{2/3} hc}{k_B T_{env}}$$

where $\Lambda \equiv \gamma/(4\tilde{a}^2)$. "Localization Param"

$$\langle x|\rho(t)|x'\rangle \propto e^{-\Lambda|x-x'|^2 t}\langle x|\rho(0)|x'\rangle$$

Wavefunction Collapse Models

"Stochastic & Nonlinear" wiener

$$d\psi(t) = \left[\frac{-i}{\hbar}H dt + \sqrt{\lambda}(q-\langle q\rangle)dW(t) - \frac{\lambda}{2}(q-\langle q\rangle)^2 dt\right]\psi(t)$$

Emission:

$$\Lambda^{bb}_{ab(emis)} = \frac{4\pi^4 c}{189\,\epsilon_0}\left[\frac{k_B T_{env}(int)}{\hbar c}\right]^6 \alpha''_{bb}$$

$$\lambda^{air}_{th} = \frac{2\pi\hbar}{\sqrt{2\pi m_{gas} \cdot k_B \cdot T_{env}}}$$

Scattering:

$$\Lambda^{bb}_{Scat} = \frac{8!\,\zeta(9)c}{9\pi^2\epsilon_0^2}\left[\frac{k_B T_{env}}{\hbar c}\right]^9 (\alpha'_{bb})^2$$

$$\tilde{\Lambda}_{air} = \frac{8\sqrt{2\pi}\, m_{gas} v_{th} P_{gas} R^2}{3\sqrt{3}\,\hbar^2}$$

$\downarrow \Lambda^{TW}_{Scat}$ $\quad \searrow \Lambda^{car}_{Scat}$

$\langle q\rangle = \langle\psi(t)|q|\psi(t)\rangle$

"Position expectation"

Force Density: $S_{FF}^{CSL} = \gamma_0^{CSL}\left(\frac{\hbar}{r_{CSL}}\right)^2 \alpha_{CSL}$

Mean phonon Number: $\langle \hat{n}(t)\rangle = \frac{\hbar\,\gamma_0^{CSL}\,\alpha_{CSL}}{M\omega_q r^2_{CSL}}$

Collapse rate: $\gamma_{CSL} = \frac{n_a}{n(r_{CSL})}\left(\frac{m_a n(r_{CSL})}{m_0}\right)^2 \gamma_0^{CSL}$

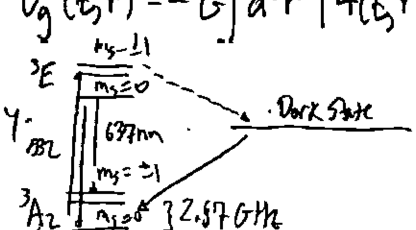
Continuous Spontaneous Localization.

Models: $i\hbar\, d\psi(t,r) = [H dt + U_g(\psi)]\,\psi(t,r)$ "Quantum Superposition"

Gravitational Collapse

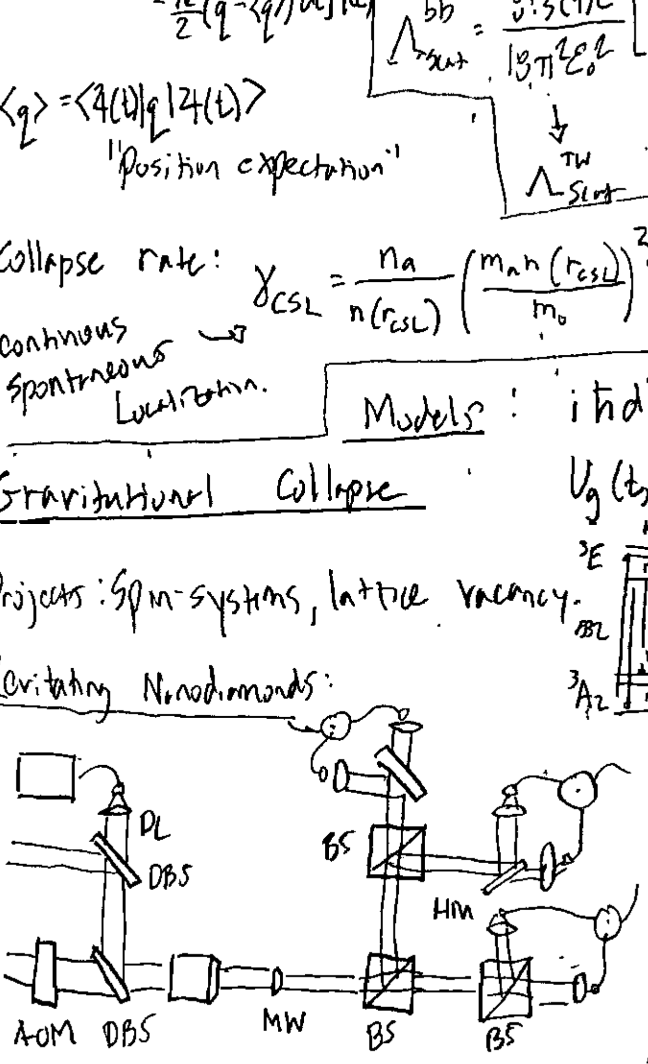
$U_g(t,r) = -G\int d^3r'\,|\psi(t,r')|^2 I_p(r-r')$ "Gravitational Interaction"

Projects: Spin-systems, lattice vacancy.



$\gamma_{DP} = \frac{GM^2}{2R\hbar}$ "Macroscopic Superposition"

Levitating Nanodiamonds:



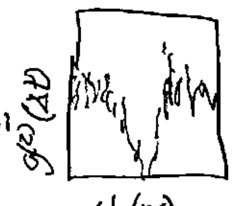
AOM = Acoustic optical Modulator

DBS = Dichroic Beam Splitter

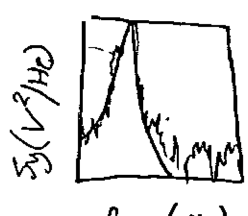
PL = Photoluminescence

BS = Beamsplitter

HM = Half Mirror



Photon Auto Correlation



Liberational Optomechanics

$\omega_q = \sqrt{K_q/M}$ "Vibrational Frequency"; $\omega_\theta = \sqrt{K_\theta/I}$ "torsional Freq"

"Mass" "moment of inertia"

$\omega_{x,y} = \sqrt{\frac{8 P_{opt} \chi_{11}}{\pi \rho c w_0^4}}$; $\omega_z = \sqrt{\frac{4 P_{opt} \chi_{11} k_L^2}{\pi \rho c w_0^2}}$

$g_q = \frac{\hbar}{2 M \omega_q} \frac{\partial W_{cav}}{\partial q}$ "Vibrational Coupling"; $g_\theta = \frac{\hbar}{I \omega_\theta} \frac{\partial W_{cav}}{\partial \theta}$ "torsional coupling"

$\omega_\phi = \sqrt{\frac{4 P_{opt} \chi_{11}}{\pi \rho c w_0^2 \ell^2} \left(\frac{\Delta\chi}{\chi_{11}} + \frac{(k_L \ell)^2}{12} \right)}$

$\omega_\alpha = \sqrt{\frac{4 P_{opt} \cdot \Delta\chi}{\pi \rho c w_0^2 \ell^2}}$

$U_{opt}^{cyl} = U_0^{cyl} f(r) \left[\frac{\chi_\perp}{\chi_{11}} + \frac{\Delta\chi}{\chi_{11}} (\vec{m} \cdot \vec{e}_x)^2 \right]$; $U_0^{cyl} = -\alpha_{cyl} E_0^2/4$

"optical potential"

$\alpha_{cyl} = \epsilon_0 V_{cyl} \chi_{11}$ "polarizability"

$\Delta\chi = \chi_{11} - \chi_\perp$

$\chi_{11} = \epsilon_r - 1$

$\chi_\perp = 2(\epsilon_r - 1)/(\epsilon_r + 1)$

"cylinder rotation"

Rotational optomechanics

sphere : Dampening

cylinder : Dampening, Torque, sinc.

Novel cooling Mechanism

"Whispering Gallery Modes"

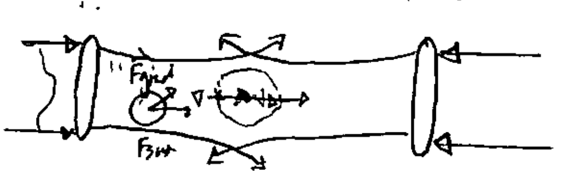
"Droplet Suspension"

Structure: Structured Light for Advanced optical Manipulation

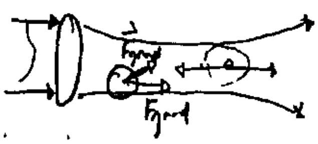
Optical Trapping gets

Optical Trapping:
- Conventional Optical Tweezers
- Holographic, optical Tweezers
- Tailored Scalar Trapping Potentials
- Energy flow & Optical Angular Momentum
- Customized Vectorial Light
- Focal Field Customization By Polarization

Principal of optical trapping

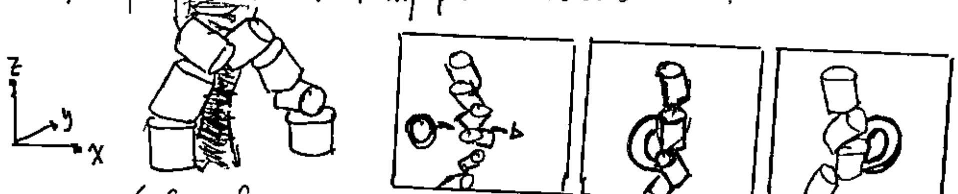


Principal of optical Tweezers

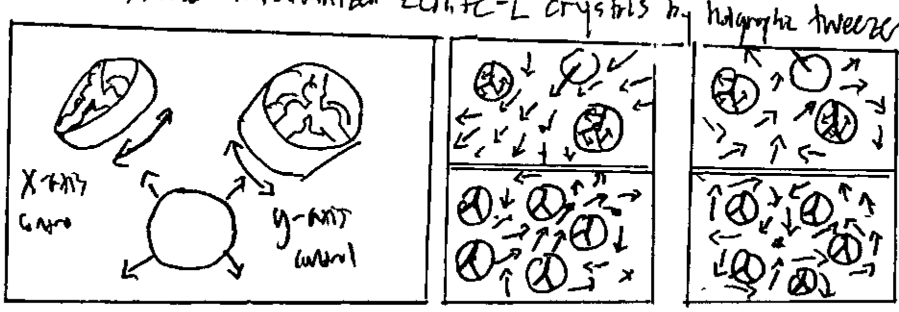


Tailored Scalar Light as advanced Trapping Potentials

Exemplary Application of Holographic Tweezers:



Surface-functionalized Zeolite-L crystals by holographic tweezer.



Holographic optical Tweezers

(HOTs) formed by digital Hologram on computer.

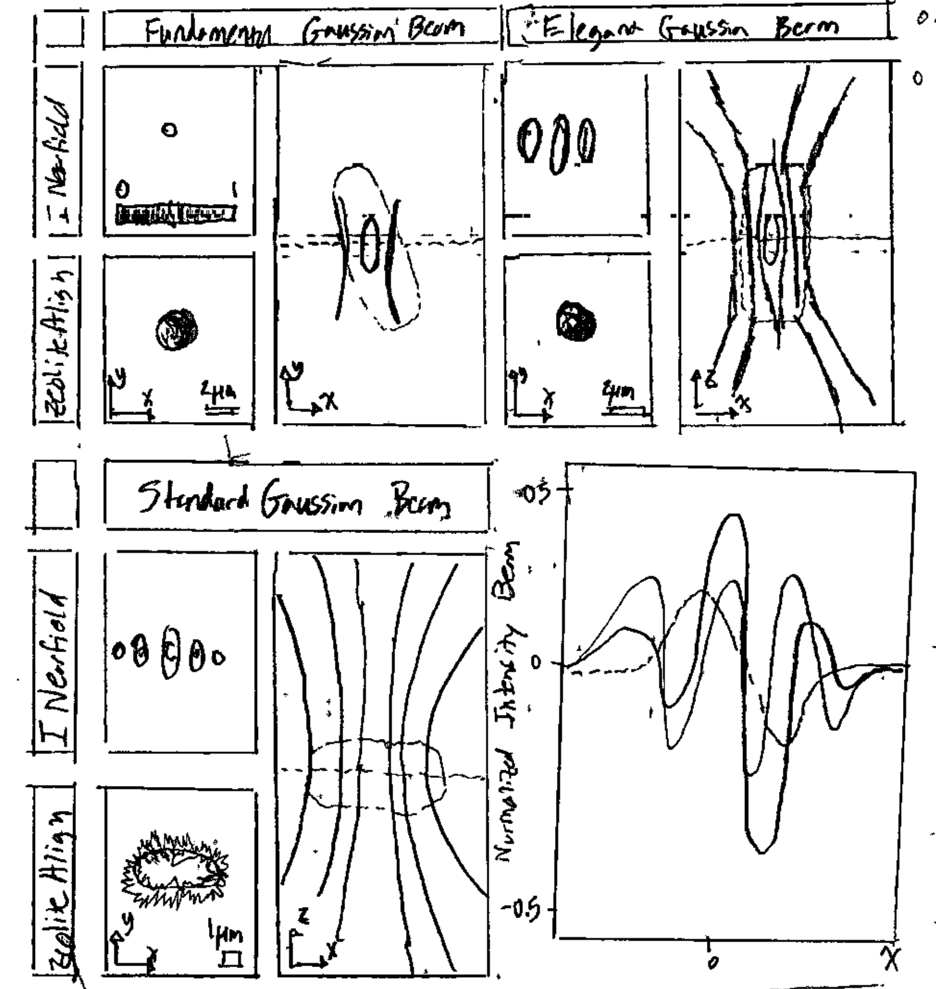
- formed by spatial light modulator
  -or-
- Digital mirror devices.

Experiments: Bio-microrobotics

Bacteria movements.

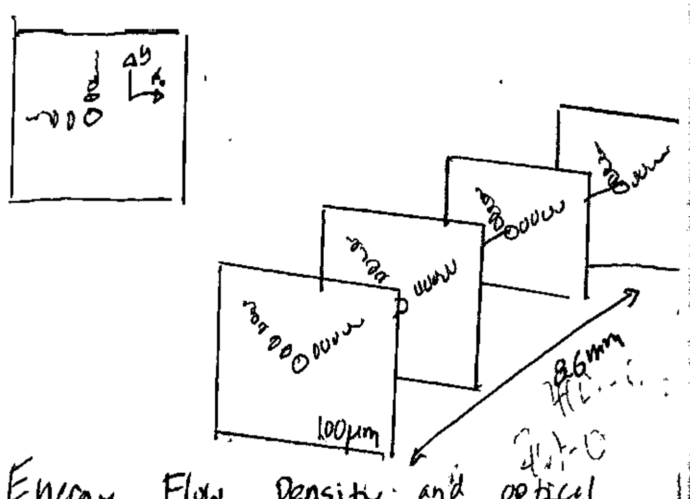
Hydrodynamics

Furne-Taylr Series : $\Phi_h = A' \mathrm{mod}[\Phi + \Phi_0 - A'\pi, 2\pi]$



Types of Structured Light Fields:

- Standard Hermite (HG)
- Laguerre (LG), ○ Ince-Gaussian (IG)



Energy Flow Density and optical Angular Momentum

Momentum Density : $\vec{p} = \vec{S}_p / c^2$

Where $\vec{S}_p \propto \vec{E} \times \vec{H}$: Electric and Magnetic Field

Flow : $\vec{p} = \vec{p}_0 + \vec{p}_s$

orbital and spin orbital contributions

$\frac{1}{2} mv^2$

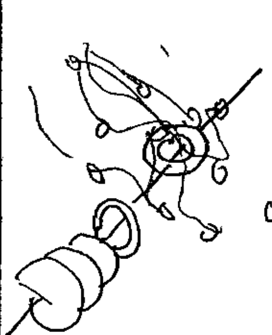
Spin Flow : $\vec{P}_0 = \frac{1}{K}(I_x \nabla \phi_x + I_y \nabla \phi_y)$ $\lambda = \frac{h}{mv}$

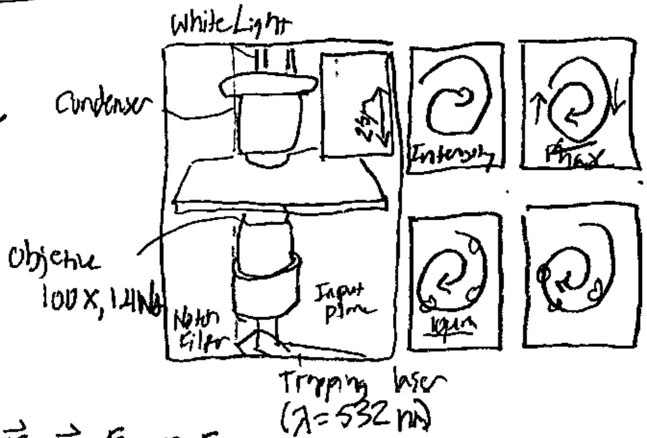
"Transverse"

Orbital Flow : $\vec{P}_s = \frac{-1}{2K}(\vec{e}_z \times \nabla S_3)$ "Longitudinal"

$\nabla = [\partial/\partial x, \partial/\partial y, \partial/\partial z]$

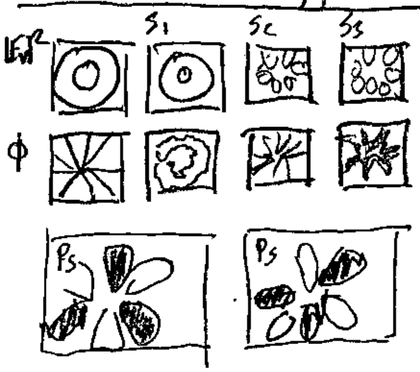
$I = |E|^2$, $\vec{e}_x = [0, 0, \ldots]$





Scalar Field : $\vec{E} = \vec{e}_u E_u + \vec{e}_v E_v$

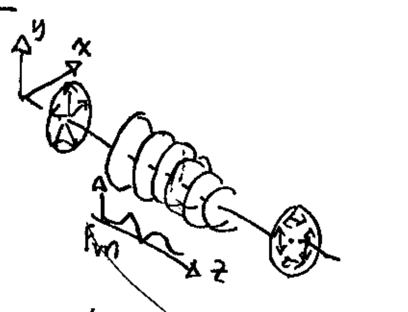
Interference Approach:

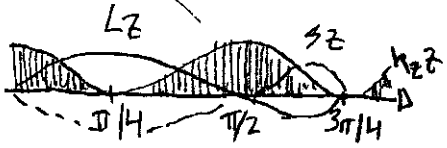


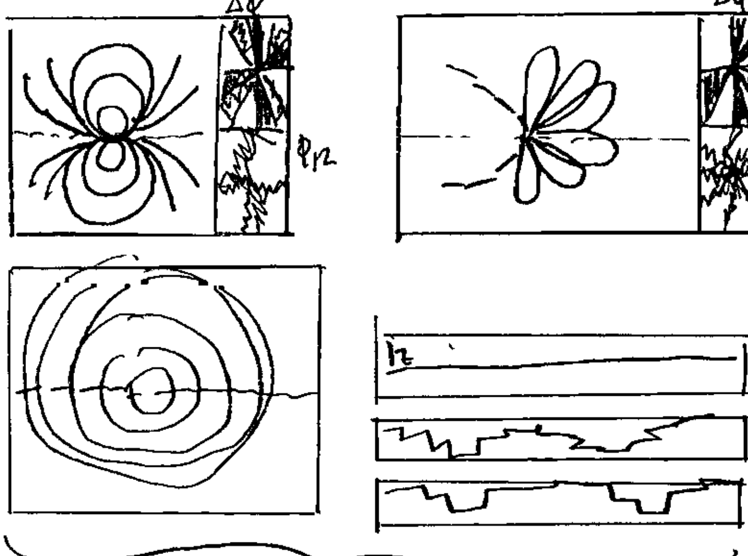
Orbital Angular Momentum : $\vec{L}_z = \int \vec{r} \times \vec{p}_0 \, d^2 r$,

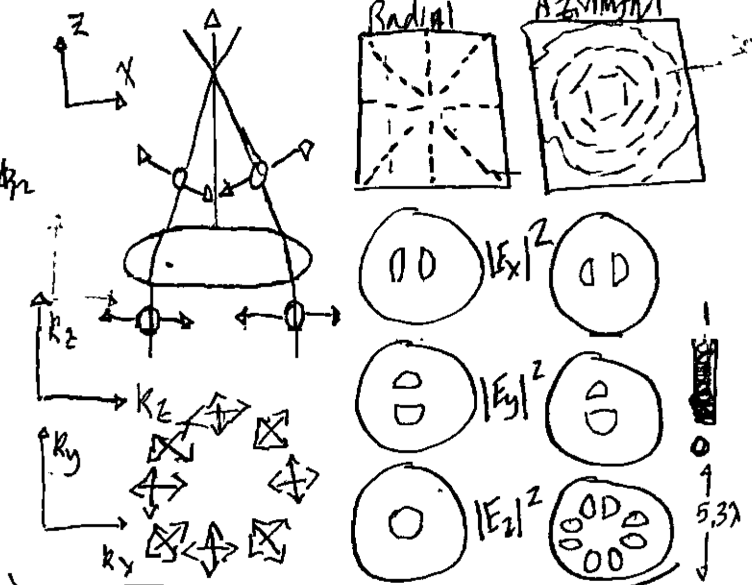
$S_z = \int r \times p_s \, d^2 r$

Spin Angular Momentum : $J = \vec{L} + \vec{S}$



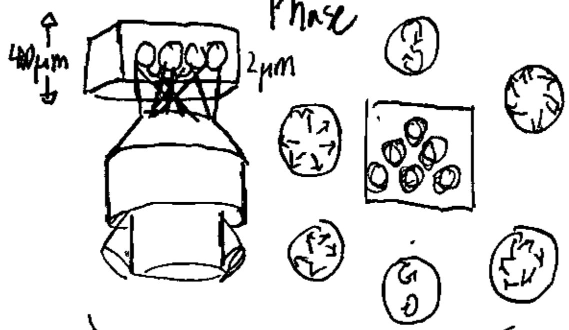






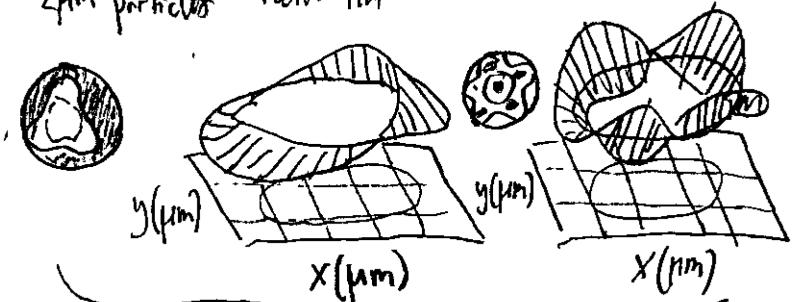
"Phase"

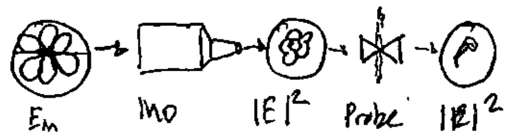
"Polarization"



2μm particles "Vector Hot"

Phase vortices.





"Polarization"

Recent Progress in engineering Casimers effect - application to nanophotonics, nanomechanics, and chemistry.

Parallel plates: $F = -\frac{\pi^2 \hbar c}{240 d^4}$; d = distance

Timeline:

Dielectric Response — Dissimilar Metals (2003)

Sphere plate (1957)

Force Electrostatic (1978)

AFM-based Measurement (1998)

Force

H-Switchable Mirrors (2004)

(2008) Metals in Fluid

Repulsive Force (2009)

Ge Plates (2009)

Transparent Conductor (2009)

Phase-change Materials (2000)

Graphene Magnetic Materials (2013)

Torque with optically anisotropic Materials (2018)

Casimer Force (1948)

(1958) Parallel Plate

(1997) Precise Casimer Force Measurement

(2001) Mems-based Measurement

(2002) Metal Plates

(2002) Gratings and Lateral Effect

(2008) Nanotrench

(2013) Metallic Nanostructure

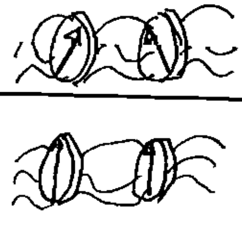
(2017) Si Nanostructure

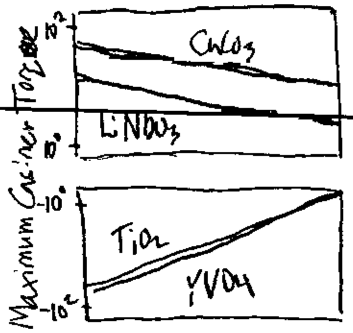
(2018) Twisphere

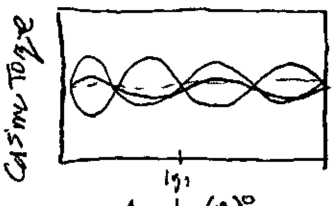
Geometry

Effect of Geometry: sphere-plate configuration: $F_c = \dfrac{-\pi^3 \hbar c R}{360 d^3}$ $(R \gg d)$

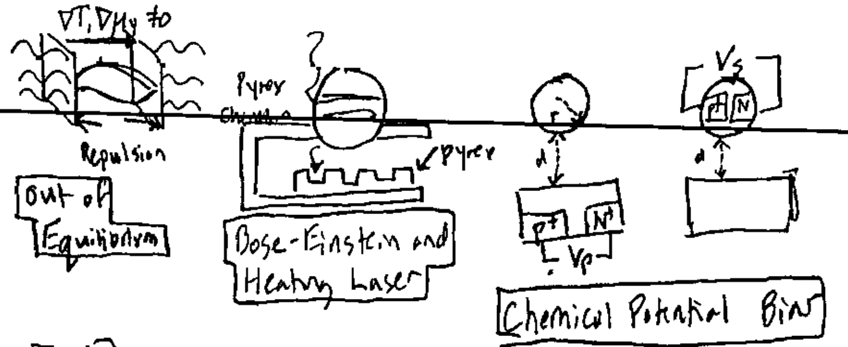
Optical Anisotropy and Casimir Torque:

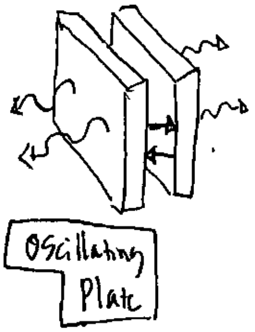


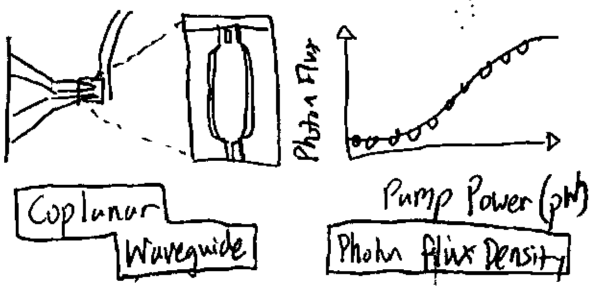




Nonequilibrium Casimir Forces:







Generation of photons from vacuum

Emitted spectrum of oscillating mirrors

$\left(\frac{a}{c}\right)^2 \omega (\Omega - \omega)$; Ω = frequency of oscillation

a = amplitude

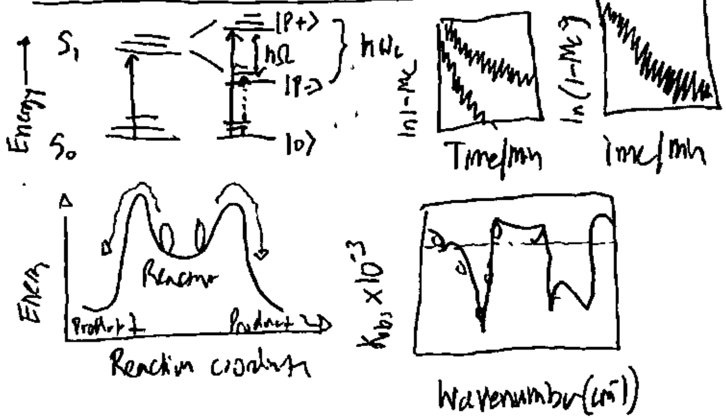
Maximum velocity: $\dfrac{\Omega}{3\pi}\left(\dfrac{v}{c}\right)^2$

Number of photons: $N = \sinh^2\left(\Omega t \frac{v}{c}\right)$

$\Omega = 2\omega_m$ $\chi^{(2)}$ and $\chi^{(3)}$ observed.

Dynamic

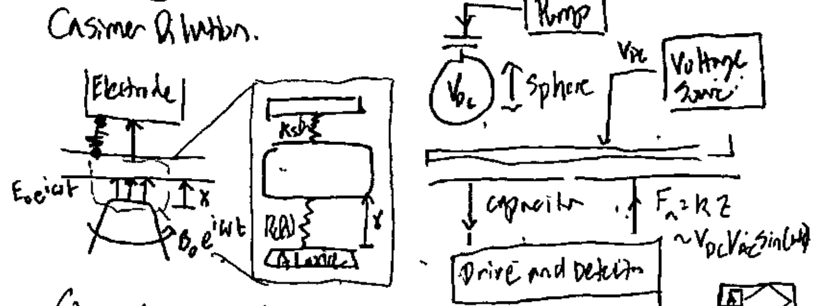
Vacuum Fluctuation Chemistry



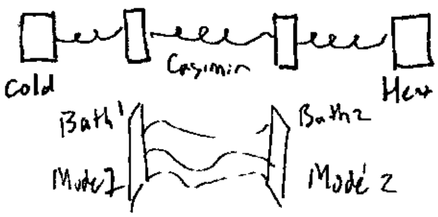
Applications to Nanotechnology:

Commercial Mems

Casimir Oscillator.



Quantum Fluctuations for Heat Transfer

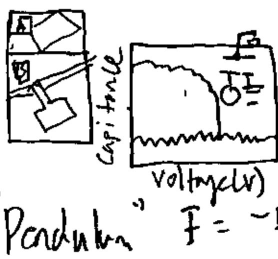


"Sphere and Plate"

$\theta = a\Delta C$; $bF = k\theta$; $F = \alpha\theta$

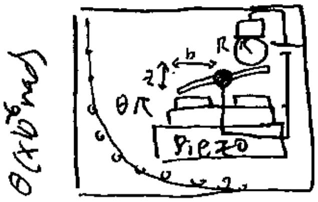
$F_e = -\epsilon_0 \pi R \dfrac{(V - V_0)^2}{z + z_0}$

Quantum Mechanical Actuation of Microelectromechanical Systems by Casimir Force



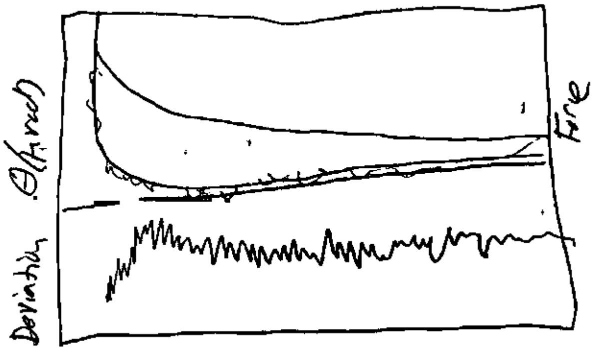
"Plates" $F_c = \dfrac{-\pi \hbar c}{240} \dfrac{1}{z^4}$ "Pendulum" $F = \dfrac{-\pi^3 \hbar c}{360} R \dfrac{1}{z^3}$

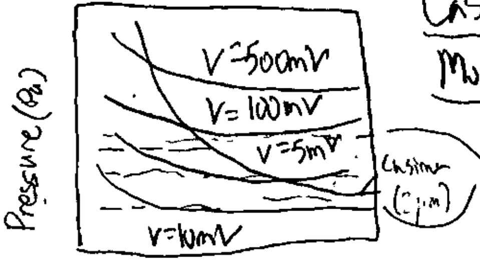
$$F_i(z) = \frac{\hbar}{2\pi c^2} R \int_0^\infty \int_1^\infty x p \, \xi^2 \left\{ \log\left[1 - \frac{(s-p)^2}{(s+p)^2} e^{-2pz\xi J_c}\right] + \log\left[1 - \frac{(s-p\epsilon)}{(s+p\epsilon)^2} e^{-2\xi kc}\right] \right\} dp \, d\xi$$



Anharmonic Casimir Oscillator (ACO) - The Casimir Effect in a Model MEM System

(1995)





Gravitational Pressure

Note: Plane geometry, parallel plates or Van der Waals pressure. (<20nm)

(>20nm)

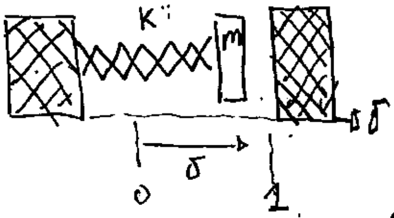
($d \ll hc/kT$)

Pressure

$P_c = \eta \frac{R}{d^4}$; $R \equiv \frac{\hbar c \pi^2}{240}$

$0 < \eta < 1$

"Stroke permitting"

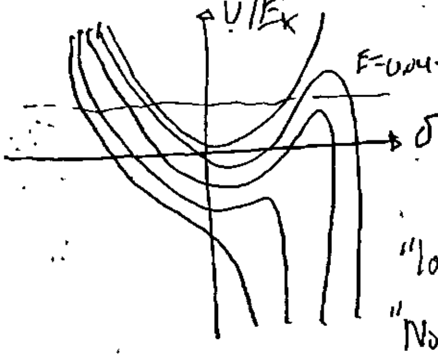


Stationary Phase: $\delta = \frac{\omega}{\omega_0}$

Restoring Pressure:

$$P(\delta) = P_K(\delta) + P_c'(\delta) = -K\omega_0\delta + \frac{R}{\omega_0^4(1-\delta)^4}$$

Potential-Energy per unit Area



Normalized potential Energy per unit area

"Simple harmonic" $C=0$

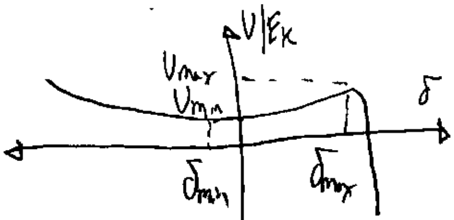
"local Extrema, stable and unstable" ($C < C_r$)

"No local extrema" ($C > C_r$)

Potential of Movable Plane

$$U(\delta) = U_K(\delta) + U_C(\delta) = E_K\left(\frac{1}{2}\delta^2 - \frac{1}{3}\frac{C}{(1-\delta)^3}\right)$$

Where $E_K = k\omega^2$; $C = \frac{R}{k\omega_0^5}$

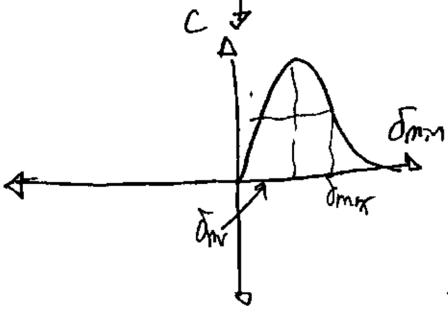


$\boxed{C < C_s}$

Unstable Equilibrium: $\frac{\partial U}{\partial \delta}\Big|_{\delta_{min}} = 0$

$0 < C < C_r$

$\delta_{min}(1-\delta_{min})^4 = C$

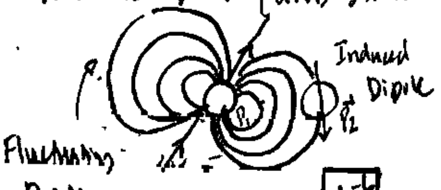


$\boxed{C < C_r}$

The Casimir Effect in Microstructured Geometries:

Note: 1948 Hendrik Casimir predicted a generalized version of Van der Waals

Van der Waals (Quasi-static Field)



$\boxed{d^{-6}}$

A fluctuating dipole p_1 induces a fluctuating electromagnetic dipole.

Classical Photonics

$E \sim e^{i\omega|x|/c - i\omega t}; \; p \sim e^{-i\omega t}$

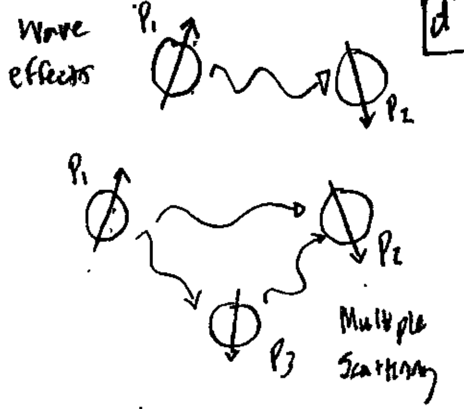
Casimir physics

$E \sim e^{-K|x|/c + Kt}; \; p \sim e^{Kt}$

"No oscillations"

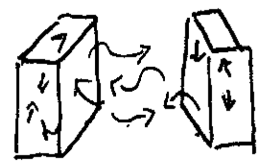
Casimir-Polder (waves/retardation)

$\boxed{d^{-7}}$



Non-additive field interactions lead to a breakdown of pairwise forces.

Casimir Effect (macroscopic bodies)

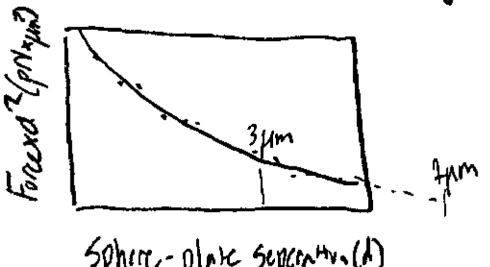


Many fluctuating dipole moments that all interact with each other

$$P_c = -\frac{\hbar c \pi^2}{240 d^4} = -\frac{1.3\times10^{-27}\,Nm^2}{d^4}$$

$$F_c = -\frac{\hbar c \pi^3 R}{360 d^3}$$

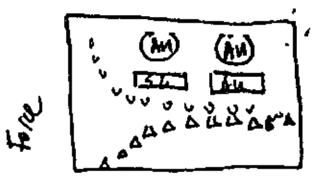
Lamoreaux's Force Data (1997)



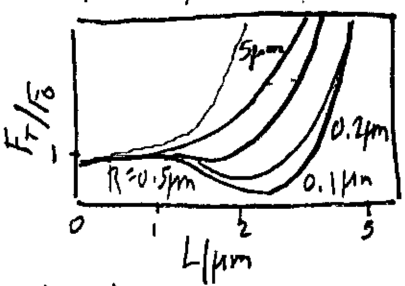
$\boxed{\text{Range of } d \sim 1\mu m \text{ and Force } \sim pN}$

$\boxed{\text{Thermal Fluctuations } k_B T R/d^2}$ $\boxed{R = 15.6\,\mu m}$

Repulsive and Attractive Forces



Sphere-plate at room temperature

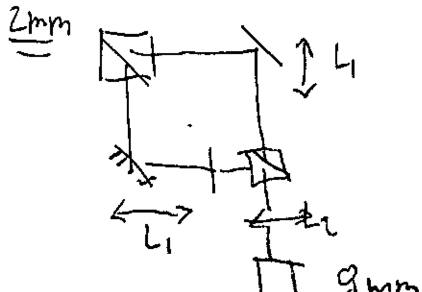


Dipole Relation

$p_2 \sim (\epsilon_2 - \epsilon_3)/(\epsilon_2 + 2\epsilon_3) E_1$

Evaluated with imaginary frequencies

$M = 4 = \frac{L_2}{L_1}$



Coupled Spherical Cavity:

Equipment: Immersion Oil [56822-50ML, $n = 1.516$]

Diameter [$75 \pm 1\,\mu m$]

Nikon CFI, Apochromat NIR 60X, W

Gimbal Mounted Mirror (GMM) x 2

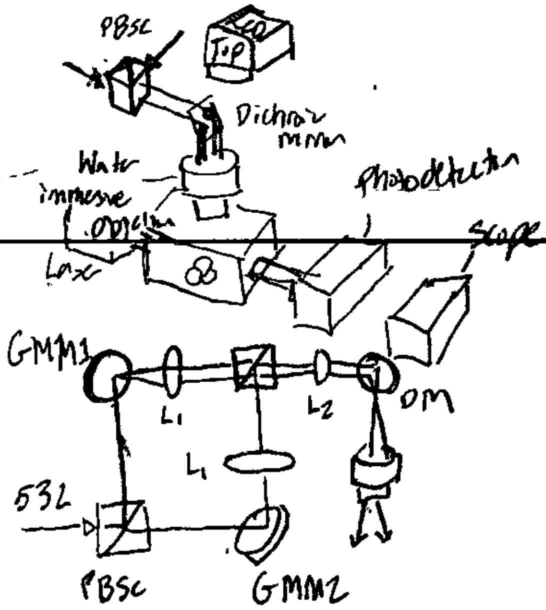
Polarizing Beam Splitters (PBSC)

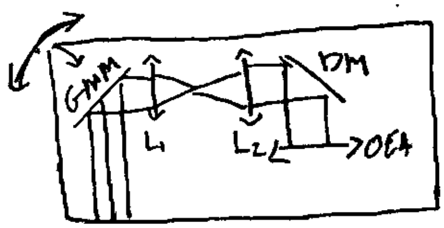
Objective Entrance Aperture (OEA)

4-F System

0.4W, 532 nm (0.2W)

Dichroic Mirror (DM)





Measurement of Casimir Force Between Two Spheres

$d = 400\,nm$ (~1% sphere radius)

Follows Lifshitz theory.

- Electrostatic and Hydrodynamic forces.

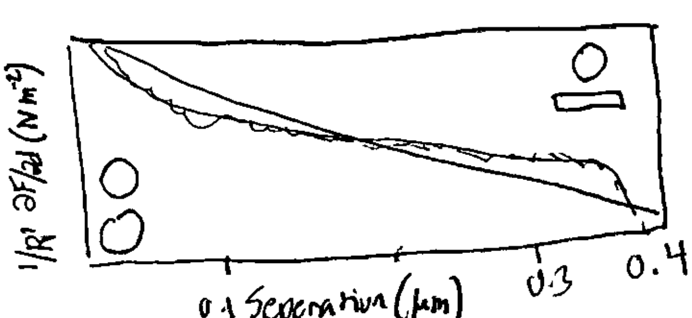
Errors: 1) Absolute Separation has uncertainty

2) Effective sensitivity of calibration

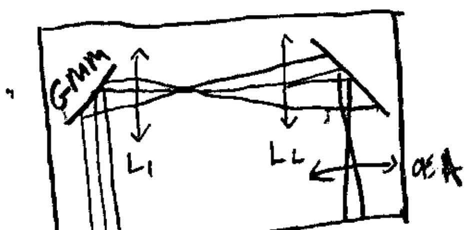
3) Discrepancy of piezo accuracy.

Conditions: $303.15 \pm 0.05\,K$, $15 \pm 9\%$ relative Humidity.

Mathematics: $F_c' = [(\partial F_c / \partial d)]$



Calibrated against spring constant.



xyz, rotx, Gms??

3 bends, buttons.

(1) Spectra Physics Millennia Prime

$^{-1}$ $\boxed{2.33mm\ Output}$

M Plan Apo 50x

$\boxed{4.00mm\ Focal\ Length}$

$\frac{1}{f} = \frac{1}{o} + \frac{1}{i}$ $\quad M = \frac{L_2}{L_1} \approx 1.75$

- Flip Mirror
- Mirror.
- Laser
- Matlab.
- Objective
- Sirf

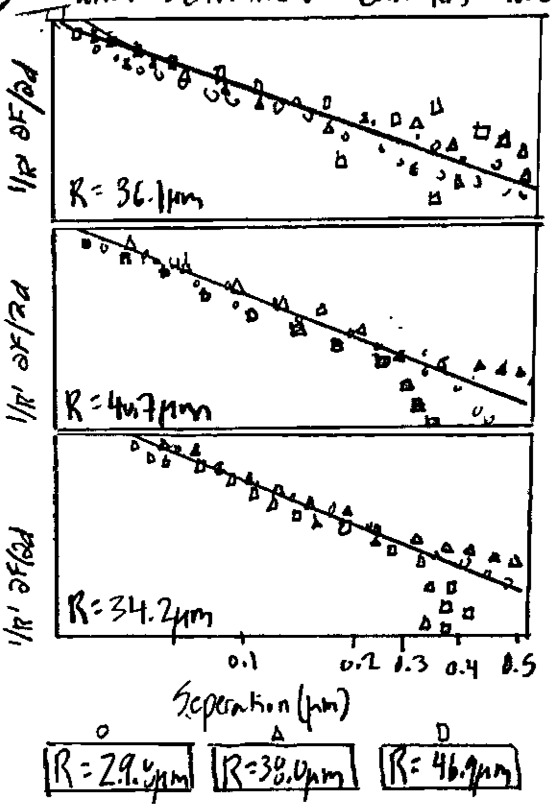
On the attraction between two perfectly conducting plates — H.B.G. Casimir

- Interaction between atom 1 and the dielectric is given by:
  → $\Delta E = -\frac{23\hbar c}{4\pi} K_1 K_2 n_2 \int_0^\infty dz \int_0^\infty \frac{2\pi \rho \, d\rho}{[(d+z)^2+\rho^2]^{7/2}}$ leads to $\Delta E = -\frac{23\hbar c}{40} \frac{K_1 K_2 n_2}{d^4}$ "Lifschitz" $\boxed{-1955}$

  "Van der Waals" $\boxed{-1937}$ — or — $\Delta E = -\frac{23}{4\pi} \hbar c \frac{K_1 K_2}{d^7}$ "Casimir" $\boxed{-1949}$

- Vibrations of a cavity
  → Cubic cavity $[L^3]$; Energy of Resonant Frequencies $[\frac{1}{2}\sum \hbar\omega]$
  → Possible vibrations of the cavity $0 \le x \le L$, $0 \le y \le L$, $0 \le z \le a$;
  with wave numbers $K_x = \frac{\pi}{L} n_x$, $K_y = \frac{\pi}{L} n_y$, $K_z = \frac{\pi}{a} n_z$
  where $K = \sqrt{K_x^2 + K_y^2 + K_z^2} = \sqrt{\chi^2 + K_z^2}$.
  → A standing wave per K_x, K_y, K_z
  $$\frac{1}{2}\sum \hbar\omega = \hbar c \frac{L^2}{\pi^2} \int_0^\infty\int_0^\infty \left[\frac{1}{2}\sqrt{K_x^2+K_y^2} + \sum_{n=1}^\infty \sqrt{n^2\frac{\pi^2}{a^2} + K_x^2 + K_y^2}\right] dK_x\, dK_y$$
  With polar coordinates:
  $$\frac{1}{2}\sum \hbar\omega = \hbar c \frac{L^2}{\pi^2} \cdot \frac{\pi}{2} \sum_{(0)1}^\infty \int_0^\infty \sqrt{\left(n^2\frac{\pi^2}{a^2} + \chi^2\right)}\, \chi\, d\chi$$
  → Interaction energy becomes:
  $$\delta E = \hbar c \frac{L^2}{\pi^2} \cdot \frac{\pi}{2} \left\{ \sum_{(0)1}^\infty \int_0^\infty \sqrt{\left(n^2\frac{\pi^2}{a^2} + \chi^2\right)}\, \chi\, d\chi - \int_0^\infty\int_0^\infty \sqrt{(K_z^2+\chi^2)}\, \chi\, d\chi \left(\frac{a}{\pi} dK_z\right) \right\}$$
  ☒ Introducing a substituted variable: $u = a^2\chi^2/\pi^2$
  $$\delta E = L^2 \hbar c \frac{\pi^2}{4a^3} \left\{ \sum_{(0)1}^\infty \int_0^\infty \sqrt{n^2+u}\, f(\pi\sqrt{n^2+u}/aK_m)\, du - \int_0^\infty\int_0^\infty \sqrt{n^2+u}\, f(\pi\sqrt{n^2+u}/aK_m)\, du\, dn \right\}$$
  ☒ Euler-Maclaurin Formula:
  $$\sum_{(0)1}^\infty F(n) - \int_0^\infty F(n)\, dn = -\frac{1}{12} F'(0) + \frac{1}{24\times 30} F'''(0) + \cdots$$
  ☒ Introducing $w = u + n^2$, we have
  $$F(n) = \int_{n^2}^\infty w^{1/2} f(w\pi/aK_m)\, dw, \text{ whence}$$
  $$F'(n) = -2n^2 f(n^2\pi/aK_m)$$
  $$F'(0) = 0 \; ; \; F'''(0) = -4$$
  ☒ The second-order term of Expansion becomes
  $$\delta E / L^2 = -\hbar c \frac{\pi^2}{24\times 30} \frac{1}{a^3} ;$$
  $$F = \hbar c \frac{\pi^2}{240} \frac{1}{a^4} = 0.013 \frac{1}{a_\mu^4} \text{ dyne/cm}^2$$

Spatial Derivative of Casimers Force



The plots show $\frac{1}{R'}\frac{\partial F}{\partial d} = 2\pi F_{pp}\left(1 + \frac{\beta' d}{R'} + \cdots\right)$

$\approx (2\pi F_{pp} \beta' d)\left(\frac{1}{R'}\right) + 2\pi F_{pp}$

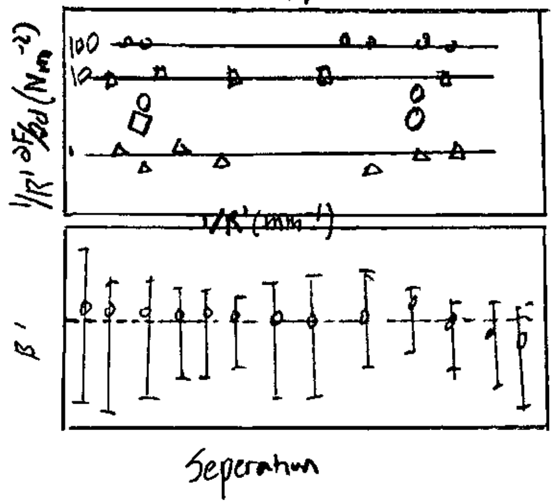
$= m\left(\frac{1}{R'}\right) + b$

Where $m = 2\pi F_{pp} \beta' d$

$b = 2\pi F_{pp}$

$\beta' = \frac{m}{bd} \cong 0$

Noted as:
- critical Casimir
- Hydrodynamic
- magnetic, etc



Demonstration of the Casimir Force in the 0.6 to 6µm Range

Attractive Force per unit area [Plates]

$F(a)/A = \frac{\pi^2}{240}\frac{\hbar c}{a^4} = 0.016 \frac{1}{a^4}\, dyn(\mu m^4)/cm^2$

where a = plate separation.

"Derived from Lifshitz theory as ϵ approaches infinity for Conductor"

Important!!!
- Geometry-dependent
- Dielectric-response
- Force relationship.

Proximity Force

$F = 2\pi R E$

where R = radius

E = potential

sphere-and-flat surface

$$F_c(a) = 2\pi R\left(\frac{1}{3}\frac{\pi^2}{240}\frac{\hbar c}{a^3}\right)$$

Corrections to Casimirs Force

- Finite Temperature $T \approx 300K$
  similar to Van der Waals constants temp-dependence

$$F_c^T(a) = F_c(a)\left(1 + \frac{720}{\pi^2} f(\xi)\right) \;;\; \xi = KTa/\hbar c$$

$$f(\xi) \approx \begin{cases} (\xi^3/2\pi)\zeta(3) - (\xi^4\pi^2/45) & \xi \le 1/2 \\ (\xi/8\pi)\zeta(3) - (\pi^2/720) & \xi > 1/2 \end{cases}$$

$\zeta(3) = 1.202\ldots$

- Finite Conductivity
  Where one plate is spherical

$$F_c'(a) = F_c(a)\left(1 + \frac{4c}{a\omega_p}\right) \;;\; \omega_p = \text{plasma frequency}$$

$\epsilon(\omega) = 1 - (\omega_p/\omega)^2$

Experiment left Residual Force on the order of 10^{-4} dynes

$\boxed{\text{distance} \cong 0.6\mu m}$

Stability of radiation-pressure particle traps: an optical Earnshaw theorem Ashkin (1950)

Earnshaw Theorem: A dielectric particle cannot be trapped by using only the scattering force of optical radiation pressure.

Corollary: Gradient or dipole force is necessary to any successful optical trap.

Principles: 1) Scattering of light momentum arises forces upon particles.

2) A Mie particle, or regime ($d \gtrsim \lambda$), is symmetric when the scattering force is parallel to the Poynting vector. Non-symmetric when transverse components to the Poynting vector.

3) Scattering Force $\propto$ cross-section of particle $\propto$ polarizability parallel to the Poynting Force

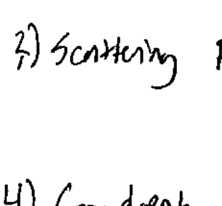
4) Gradient Force $\propto$ in-phase component of a particle's polarizability. i.e. directs (+)-polarization toward high-electric field strength. Δ Electrostrictive Force, for $d < \lambda$ = Rayleigh Force.

5) Minimum Kinetic Energy $\propto \hbar \gamma_N$; where $\gamma_N/2$ Natural line width.

6) Experiments show amount of beams $\mathrm{div}(E) = 0$, i.e., four or five beams could not be symmetrized.

7) Farfield ($z >> \pi w_0^2/\lambda$), $F_{scatt} \propto P(w)$; $F_z = K w^{-2} \exp(-2r^2/w^2)$

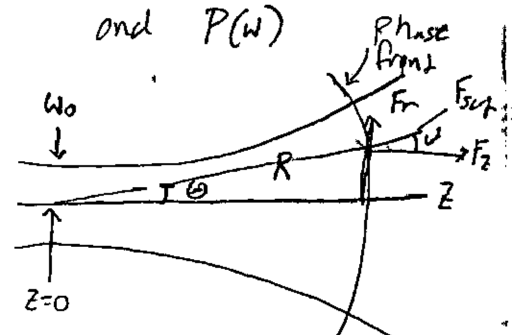
8) Four-beam Tetrahedral Arrangement. $F_r = F_z(r/z)$; $w = \lambda z / \pi w_0$



$\boxed{\text{Earnshaw's Theorem}}$: An electrostatic trap with a tetrahedral arrangement is unstable.

$F_z = F_1 + F_2 + \ldots$

K $\propto$ cross-section and $P(w)$



9) Six beams F_z generates F_x forces.

10) Lorentz representation for an electric-dipole

11) $F_g = (1/4)\, \mathrm{grad}(E^* \cdot \chi' \cdot \vec{E})$

$F_s = (1/2)\, \mathrm{Im}(\hat{x}_j E^* \cdot \chi'' \cdot \partial E/\partial x_j)$ where χ is Polarizability.

$\mu = \chi \cdot E = (\chi' + i\chi'')E$

$F_c = (1/2)\, \mathrm{Re}[\hat{x}_j \mu^* \cdot \partial E/\partial x_j]$

Dipole Moment, $E = \exp(-i\omega t)$

$\simeq F_s + F_{grad}$.

Light Traps using Spontaneous Force: Pritchard, Raab, and Bagnato. (1986)

Earnshaw theorem does not always apply to atoms.

Spontaneous light does confine atoms, at depths of Kelvin and volumes of cubic centimeters

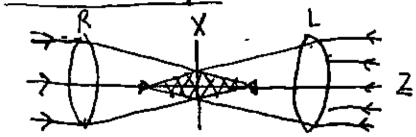
1) Gradient Force - interaction of induced dipole moment with field intensity.

~~2) Scattering Force - transfer of momentum from photon to particles.~~

Note: Dampens and cools.

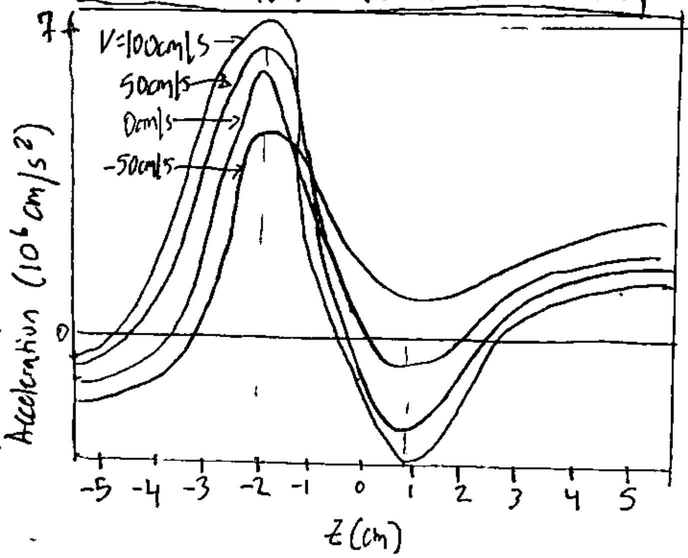
Internal Degrees of Freedom do change for an atom proportional to force and Poynting vector in a position-dependendent way

Basic Trap:

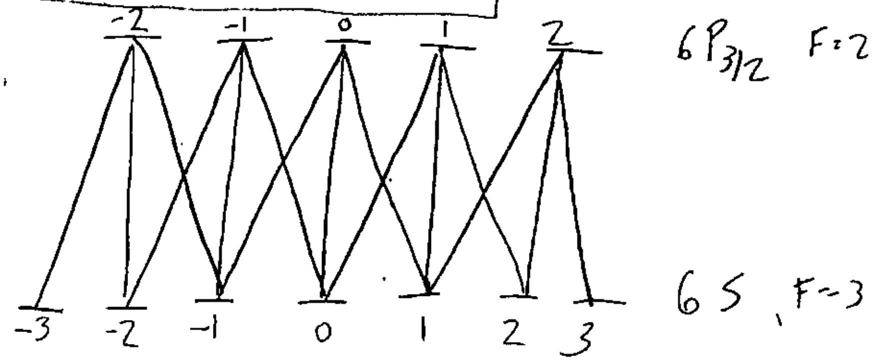


Polarization produces restoring force when aligned perpendicular on $\hat{x}$-and-$\hat{y}$

Acceleration felt by Sodium Atoms with various velocities in trap.

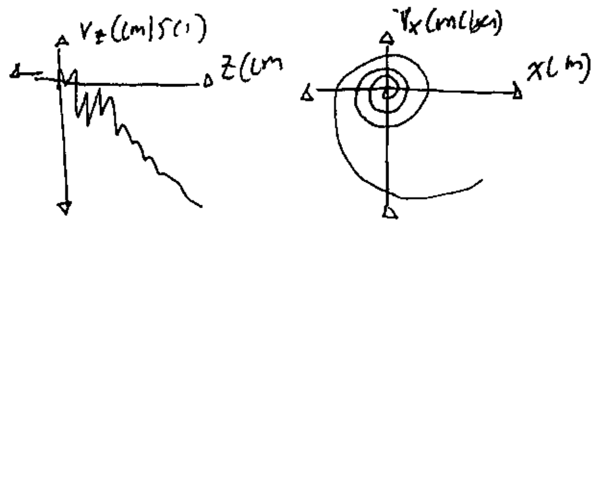


Laser Beams with opposite angular momentum generate longitudinal forces, even with transition.



Transition Probabilities of Cesium

Fact: Damping occurs for two-state system because acceleration <0 @ Z=0. i.e "left-and-right push-pull"



Retarded, or Casimir long-range Potentials · Larry Spruch (1986)

Newton Law $(1/r^2)$ ~ 18th Century

Van der Waals $(1/r^6)$ ~ 1930's

· Static uniform electric field $\vec{E}$

· Atoms dipole moment $\mu = \alpha E$

Electron-Electron

$V_{ee}(r) = e^2/r$

Electron-Atom

$V_{ea}(r) = \mu \cdot E$
$= \alpha E^2$
$= \alpha e^2/r^4$

"Static"

$V_{ea}^{dyn}(r) = \left(\frac{e^2/a_0}{\Delta E}\right)\left(\frac{a_0}{r}\right)^5 \frac{e^2}{r}$

Atom-Atom

$V_{aa}(r) = \dfrac{[\mu \cdot \mu'/r^3]^2}{\Delta E}$

$= [(ea_0)^2/r^3]^2 (a_0/e^2)$

$= (e^2/a_0)(a_0/r)^6$

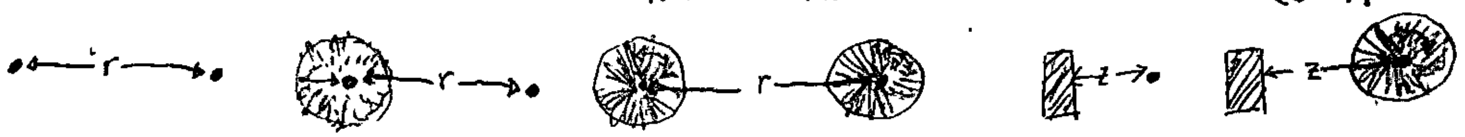
"Van der Waals"

Electron-Wall

$V_{ewall}(z) = e^2/z$

Atom-Wall

$V_{awall}(z) = |\mu|^2/z^3$
$= (ea_0)^2/z^3$

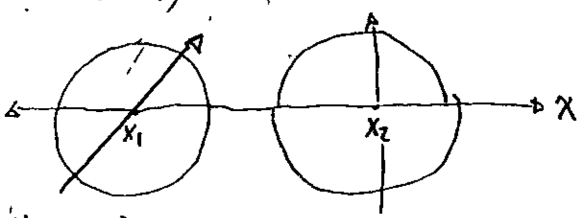


Classically, $U = KE + PE = \frac{1}{2}k(r_1^2 + r_2^2) - e^2(r_1 \cdot r_2 - 3z_1 z_2)r^3$ for atom-atom

Retardation of potential, (1941-1948); $\vec{E} = E_0(\omega, x)\sin\omega t$

The field $E_{2\to 1}$ generated by an oscillating dipole at X_2 is proportional to:

$$\frac{\mu_2 - 3\mu_2\hat{z}}{r^3} + \frac{\dot{\mu}_2 - 3\dot{\mu}_2\hat{z}}{cr^2} + \underbrace{\frac{\ddot{\mu}_2 - \ddot{\mu}_2\hat{z}}{c^2 r}}_{\text{only term important to the integrand.}}$$



$\mu_1(\omega,t) = \alpha_1(\omega)E_0(\omega,X_1)\sin\omega t$ $\quad$ $\mu_2(\omega,t) = \alpha_2(\omega)E_0(\omega,X_2)\sin\omega t$

$\longleftarrow |X_2 - X_1| \longrightarrow$

$N(\omega)d\omega = 2d^3\omega V/c^3 = 2\omega^2 d\omega \sin\theta\, d\theta\, d\phi\, V/c^3$

$V(X_1, X_2, t) = \int_0^\infty \mu_1(\omega,t) E_{2\to 1}(\omega,t) N(\omega)d\omega$

Classical

$= \dfrac{V}{c^5 r}\int_0^{c/r} \alpha_1(\omega)\alpha_2(\omega)\omega^2 \underbrace{|E_0(\omega, X_1)|^2}_{\omega}\omega^2 d\omega$

Quantum

$= \left(\dfrac{\hbar}{c^5 r}\right)\int_0^{c/r} \alpha_1(\omega)\cdot\alpha_2(\omega)\cdot\omega^5 d\omega$

Electron-Electron

$V_{ee}(r) = \dfrac{\hbar e^4}{c^3 m^2 r^3}$

Electron-Atom

$V_{ea}(r) = \dfrac{\hbar e^2 \alpha}{mcr^5}$

Atom-Atom

$V_{aa}(r) = \dfrac{\hbar c \alpha_1 \alpha_2}{r^7}$

Electron-Wall

$V_{ewall}(z) = (e^2\hbar/mcz^2)$

Atom-wall

$V_{awall}(z) = \alpha\hbar c/z^4$

$\boxed{137 \ll r}$

Cavity
Resonant Fr

Casimir Effect and the quantum vacuum   R.L. Jaffe (2005)

Stress tensor, $\langle T_{\mu\nu}\rangle \equiv -\mathcal{E} g_{\mu\nu}$ is in the Einstein Equations. $R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -8\pi G(\underbrace{T_{\mu\nu}}_{\text{"Excitations above vacuum"}} - \mathcal{E} g$

(1997) Lamoreaux measured Casimir's force per unit area.
between parallel plates: $F = -\frac{\hbar c \pi^2}{240 d^4}$ Universe Acceleration: $\lambda = 8\pi G\mathcal{E}$ $= (2.14 \pm 0.13 \times 10^{-3} eV)$

$\boxed{\text{Relationship to forces between metal plates}}$

1) Casimir effect is a function of the fine structure constant (α), $\alpha \to 0$, Casimir →
The (relativistic, retarded) van der Waals force between metal plates

2) Calculated as S-matrix elements i.e. Feynman diagrams

$$\frac{1}{2}\sum \hbar\omega = \frac{1}{2}\int dx\,dy\,[\rho(x)\rho(y)/|x-y|] = \frac{1}{8\pi}\int dx\,|\vec{E}(x)|^2,$$ where $\rho(x)$ = smooth charge distribution.

QED Graphs contributing to zero-point energy: 

$\boxed{\text{Dependence of Casimir Effect on fine structure constant}}$

Lifshitz ~ Casimir ~ Drude model of metals

ω_{pl} = $\underbrace{\text{plasma frequency}}_{\text{"Threshold for which conductivity goes to zero"}}$, $\underbrace{\delta = \text{skin depth}}_{\text{"Distance of penetration"}}$

$\boxed{\text{Drude Model}}$: $\omega_{pl}^2 = \frac{4\pi e^2 n}{m}$, $\delta^{-2} = \frac{2\pi\omega|\sigma|}{c^2}$

n = amount of conductive electrons $\sigma = \frac{ne^2}{m(\gamma_0 - i\omega)}$

m = effective mass

γ_0 = Damping Parameter ; $\delta \approx c/\sqrt{2}\,\omega_{pl}$

Perfect Drude Model: $\alpha \gg \frac{mc}{4\pi\hbar n d^2}$; $\boxed{\text{Generally } 0.5\mu m}$ Calculate α!!!

Note: QED interrelates energies at distances of $0.5\mu m \longrightarrow 0.52\,\text{Å}$

$\boxed{\text{Casimir Effect without a vacuum}}$ $\Delta E = -\frac{23\hbar c}{4\pi R^7} a_1 a_2$ Polarizability

Expressed in terms of the Green's Function for fluctuating fields

$$\mathcal{E} = \frac{\hbar}{2\pi}\,\text{Im}\int d\omega\,\omega\,\text{Tr}\int d^3x\,[\underbrace{G(x,x,\omega+i\epsilon)}_{\text{"Fluctuating field"}} - \underbrace{G_0(x,x,\omega+i\epsilon)}_{\text{"Trace over spin"}}]$$

$$\propto \frac{1}{\pi}\,\text{Im}\int[G(x,x,\omega+i\epsilon) - G_0(x,x,\omega+i\epsilon)] = \frac{d\Delta N}{d\omega}$$

Note: Expressed as Feynman Diagrams, S-matrix, Green's Function, Lippman-Schwinger Eqn

Founded by a Scalar field, $\phi = 0$ at $x = \pm a/2$ boundary conditions

Calculation of the force between $x = \pm a$, in the presence of $\sigma(x)$.

$\mathcal{L}_{int} = \frac{1}{2} g\sigma(x)\phi^2(x)$, where $\sigma(x) = \delta(x - a/2) + \delta(x + a/2)$; $\phi(\pm a/2) = 0$.

The effective energy is the sum of all one-loop Feynman diagrams with ...

$$F(a,g,m) = \frac{-g^2}{\pi}\int_m^{\infty}\frac{t^2\,db}{\sqrt{t^2-m^2}} \times \frac{e^{-2at}}{4t^2+4gt+g^2(1-e^{-2at})} \quad \text{as } g \to 0 \text{ ; } F=0$$

$$\lim_{g\to\infty} F(a,g,m) = -\int_m^{\infty}\frac{db}{\pi}\frac{t^2}{\sqrt{t^2-m^2}\,(e^{2ta}-1)}$$ then becomes $-\pi/24a^2$ at the $\lim_{m\to 0}$.

Cavity Quantum Electrodynamics

$\boxed{\text{Emission in Freespace: } \hbar\omega = E_f - E_i}$; $\boxed{\text{Zero-point energy} - \hbar\omega/2}$: $\boxed{\text{Root Mean Squared : } E_{vac} \text{ in a mode : } E_{vac} = \sqrt{\hbar\omega/(2\epsilon_0 V)}}$

Coupling of the atom to each mode is described by the frequency: $\boxed{\Omega_{ef} = D_{ef}E_{vac}/\hbar}$

where D_{ef} is the matrix element of the electric dipole.

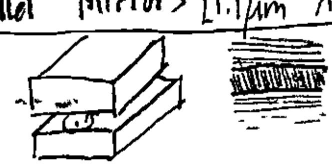
Probability of Photon Emission $[T_0]$ is the $\boxed{\text{Einstein A-Coefficient } \propto \Omega_{ef} \text{ and } \rho_0(\omega)}$ per unit time, $\boxed{\text{where } \rho_0(\omega) = \omega^2 V/\pi^2 c^3}$

$\boxed{\text{Fermi's "Golden Rule": } T_0 = 2\pi\Omega_{ef}^2\frac{\rho_0(\omega)}{3} = \frac{\omega^3}{3\pi\hbar c^3}\frac{|D_{ef}|^2}{\epsilon_0}}$

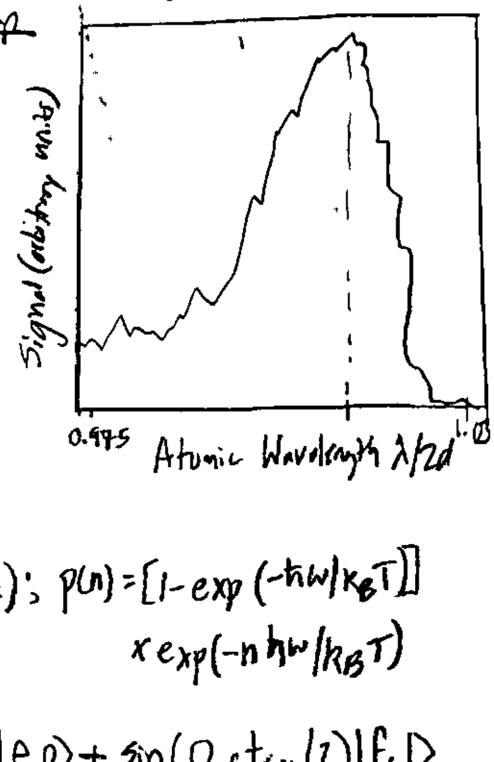
$\boxed{\text{Probability of Excitation Decay } P_e(t) = \exp(-T_0 t)}$

Boundary conditions for quantum electrodynamics was confirmed about 1970's by Karl Drexhage

Parallel Mirrors [1.1μm Apart @ Yale]



Survival of Excited Rydberg Atoms moving between a gap



Q-Factors were enhanced for the boundary conditions with Fabry-Perot resonators, and Spherical mirrors.

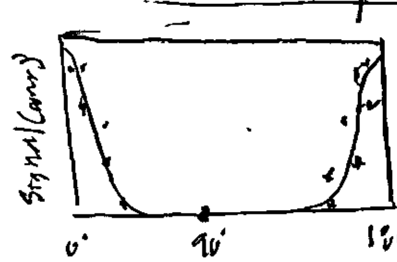
Low-Q

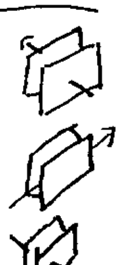
$$T_{cav} \cong T_0\,\frac{Q\lambda^3}{V}$$

High-Q

$$P_e(t) = \sum p(n)\cos^2(1/2\,\Omega_{ef}\sqrt{n+1}\,t); \quad p(n) = [1-\exp(-\hbar\omega/k_BT)] \times \exp(-n\hbar\omega/k_BT)$$

Orientation of Magnetic Field:





One-atom maser:

$$|\psi\rangle = \cos(\Omega_{ef}t_{int}/2)|e,0\rangle + \sin(\Omega_{ef}t_{int}/2)|f,1\rangle$$

$$(0 \leftarrow \Omega_{ef}t \rightarrow 1)$$

$$|\psi_{N+1}\rangle = \cos(\Omega_{ef}\sqrt{2}\,t_{int}/2)|e,1\rangle + \sin(\Omega_{ef}\sqrt{2}\,t_{int}/2)|f,2\rangle$$