Chapter 1:

1. Proustite $\chi^{(2)}_{yyy}$ has 1.3×10^{-7} cm/stat-volt in Gaussian Units. What is the value in MKS units?

$$1.3\times10^{-7}\frac{cm}{\text{stat-volt}} \cdot \frac{1\text{ stat-volt}}{299.792458\,V} \times \frac{1m}{100cm} = 4.34\times10^{-12}\frac{m}{V}$$

3. $\bar{x}=\lambda\bar{x}^{(1)}+\lambda^2\bar{x}^{(2)}+\lambda^3\bar{x}^{(3)}+\cdots\ \lambda^0 x^{(0)}$ is not included because the position must begin at zero distance and not 1.

5. $\ddot{\bar{x}}+2\gamma\dot{\bar{x}}+\omega_0^2\bar{x}+a\bar{x}^2=-e\bar{E}(t)/m$; Derive an expression for the third-order $\bar{x}^{(3)}$ and consequently, $\chi^{(3)}_{1111}(\omega_q,\omega_m,\omega_n,\omega_p)$

$$\bar{x}=\lambda\bar{x}^{(1)}+\lambda^2\bar{x}^{(2)}+\lambda^3x^{(3)}+\cdots$$

$$\ddot{x}^{(1)}+2\gamma\dot{x}^{(1)}+\omega_0^{(1)}x^{(1)}=-eE(t)/m \quad \text{"Lorentz Model"}$$

$$\ddot{x}^{(2)}+2\gamma\dot{x}^{(2)}+\omega_0^2x^{(2)}+a[x^{(1)}]^2=0$$

$$\boxed{\ddot{x}^{(3)}+2\gamma\dot{x}^{(3)}+\omega_0^2x^{(3)}+2ax^{(1)}x^{(2)}=0}$$

$$\bar{x}^{(1)}(\omega_j)=-\frac{e}{m}\frac{E_j}{D(\omega_j)};\ D(\omega_j)=\omega_0^2-\omega_j^2-2i\omega_j\gamma$$

$$\ddot{x}^{(2)}+2\gamma\dot{x}^{(2)}+\omega_0^2x^{(2)}=\frac{-a(eE_1/m)^2}{D^2(\omega_1)}e^{-2i\omega_1t}$$

$$x^{(2)}(t)=x^{(2)}(2\omega_1)e^{-2i\omega_1t}$$

$$x^{(2)}(2\omega_2),\ x^{(2)}(\omega_1+\omega_2),\ x^{(2)}(\omega_1-\omega_2),\ x^{(2)}(0)$$

Susceptibilities, $P^{(1)}(\omega_j)=\epsilon_0\chi^{(1)}(\omega_j)E(\omega_j)$

$$\chi^{(1)}(\omega_j)=\frac{Ne^2/(\epsilon_0m)}{D(\omega_j)}=\frac{Ne^2/(\epsilon_0m)}{\omega_0^2-\omega_j^2-2i\omega_j\gamma}$$

$$P^{(2)}(2\omega_1)=-Nex^{(2)}(2\omega_1)$$

$$\chi^{(2)}(2\omega_1,\omega_1,\omega_1)=\frac{N(e^3/m^2)a}{\epsilon_0D(2\omega_1)D^2(\omega_1)}$$

$$=\frac{\epsilon_0^2ma}{N^2e^3}\chi^{(1)}(2\omega_1)\left[\chi^{(1)}(\omega_1)\right]^2$$

$\chi^{(3)}(\omega_1,\omega_2,\omega_3,-\omega_1,-\omega_2);\ \chi^{(3)}(\omega_1,\omega_2,\omega_3,-\omega_2,-\omega_3)$

$\chi^{(3)}(\omega_1+\omega_2,\omega_1,-\omega_1,-\omega_2);\ \chi^{(3)}(\omega_1+\omega_2,\omega_1,\omega_2,-\omega_3)$

$\chi^{(3)}(\omega_1+\omega_2,\omega_1,-\omega_2,-\omega_3);\ \chi^{(3)}(\omega_1+\omega_2,-\omega_1,-\omega_2,\omega_3)$

$\chi^{(3)}(\omega_1+\omega_2,-\omega_1,\omega_2,\omega_3);\ \chi^{(3)}(\omega_1+\omega_2,-\omega_1,-\omega_2,\omega_3)$

$\chi^{(3)}(\omega_1+\omega_2,-\omega_1,-\omega_2,-\omega_3);\ \chi^{(3)}(\omega_1+\omega_2,-\omega_1,-\omega_2,-\omega_3)$

$\chi^{(3)}(\omega_1\ \cdots\ )$

7. Determine Symmetry of Third-Order Susceptibility $\chi^{(3)}$

Kleinman's Symmetry 1.1

Indices can be permutated as long as the frequencies are permutated.

$$P_i(\omega_n+\omega_m) = \epsilon_0 \sum \sum \chi^{(2)}_{ijk}(\omega_n+\omega_m, \omega_n, \omega_m) E_j(\omega_n) E_k(\omega_m)$$

$$P_i(r,t) = P_i(\omega_n+\omega_m) e^{-i(\omega_n+\omega_m)t} + P_i(-\omega_n-\omega_m) e^{i(\omega_n+\omega_m)t}$$

$$\chi^{(2)}_{ijk}(-\omega_n-\omega_m, -\omega_n, -\omega_m) = \chi^{(2)}_{ijk}(\omega_n+\omega_m, \omega_n, \omega_m)^*$$

$$\chi^{(2)}_{ijk}(\omega_n+\omega_m, \omega_n, \omega_m) = \chi^{(2)}_{ikj}(\omega_n+\omega_m, \omega_m, \omega_n)$$

$$\chi^{(2)}_{ijk}(\omega_3=\omega_1+\omega_2) = \chi^{(2)}_{jkl}(\omega_1=-\omega_2+\omega_3)$$

$$\chi^{(2)}_{ijk}(\omega_3=\omega_1+\omega_2) = \chi^{(2)}_{jkl}(\omega_2=\omega_3-\omega_1)$$

$$\chi^{(2)}_{ijk}=(\omega_3=\omega_1+\omega_2) = \chi^{(2)}_{jkl}(\omega_3=\omega_1+\omega_2) = \chi^{(2)}_{kij}(\omega_3=\omega_1+\omega_2)$$

$$= \chi^{(2)}_{ikj}(\omega_3=\omega_1+\omega_2) = \chi^{(2)}_{jik}(\omega_3=\omega_1+\omega_2)$$

$$= \chi^{(2)}_{kji}(\omega_3=\omega_1+\omega_2)$$

$$P_i(\omega_o+\omega_n+\omega_m) = \epsilon_0 \sum_{jkl} \sum_{mno} \chi^{(2)}_{ijkl}(\omega_o+\omega_n+\omega_m, \omega_o, \omega_n, \omega_m) \times E_j(\omega_o) E_k(\omega_n) E_l(\omega_m)$$

$$P_i(r,t) = P_i(\omega_o+\omega_n+\omega_m) e^{-i(\omega_n+\omega_n+\omega_m)t} + P_i(-\omega_o-\omega_n-\omega_m) e^{i(\omega_n+\omega_n+\omega_m)t}$$

$$\chi^{(3)}_{ijkl}(-\omega_o-\omega_n-\omega_m, -\omega_n, -\omega_m) = \chi^{(3)}_{ijkl}(\omega_o+\omega_n+\omega_m, \omega_n, \omega_m)^*$$

1.6.1 $$\chi^{(3)}_{ijkl}(\omega_o+\omega_n+\omega_n, \omega_m, \omega_n) = \chi^{(3)}_{ijkl}(\omega_o+\omega_n+\omega_m, \omega_n, \omega_m)$$

$$\chi^{(3)}_{ijkl}(\omega_4=\omega_1+\omega_2+\omega_3) = \chi^{(3)}_{ijlk}(\omega_1=\omega_4-\omega_2-\omega_1) = \chi^{(3)}_{ikjl}(\omega_2=\omega_4-\omega_3-\omega_1)$$

$$= \chi^{(3)}_{iklj}(\omega_3=\omega_4-\omega_2-\omega_1) = \chi^{(3)}_{ilkj}($$

$$= \chi_{iljk} \qquad \chi_{jikl}$$

χ_i

ijkl
ijlk
iljk
ilkj
iklj
ikjl

$$\frac{4!}{(3-0)!\,0!}$$

$4\cdot3\cdot2\cdot1$

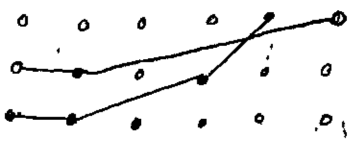
$$\boxed{{}^4_0C = \binom{4}{0}}$$

9. Crystal Class 432

Crystal System	Crystal Class	Nonvanishing Tensor Elements
Cubic	$432 = O$	$xyz = -xzy = yzx = -yxz = zxy = -zyx$
	$\bar{4}3m = T_d$	$xyz = xzy = yzx = yxz = zxy = zyx$
	$23 = T$	$xyz = yzx = zxy,\ xzy = yxz = zyx$
	$m3 = T_h,\ m3m = O_h$	Each Element Vanishes.

Isotropic Crystal Class : 432 (all elements vanish)

$$d_{il} = \begin{bmatrix} 0 & 0 & 0 & 0 & d_{31} & -d_{22} \\ -d_{22} & d_{22} & 0 & d_{31} & 0 & 0 \\ d_{31} & d_{31} & d_{33} & 0 & 0 & 0 \end{bmatrix} \quad \text{Class } 3m \ ; \ C_{3v} \quad d_{ijk} = \tfrac{1}{2}\chi^{(2)}_{ijk}$$



$$d_{il} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{Class } 432 \ ;$$

The compact format of class 432 represented as d_{il}, is an isotropic crystal class. As such, the material is described as a tensor with nonvanishing elements.

11. $$\chi^{(2)}(2w_1, w_1, w_1) = \frac{N(e^3/m^2)a}{\epsilon_0 D(2w_1) D^2(w_1)}$$

Kramer-Kronig Relationship: Conditions to relate real and imaginary parts of frequency-dependendent quantities, such as linear susceptibility.

$$\chi^{(1)}(w) \equiv \chi^{(1)}(w; w) = \int_0^{\infty} R^{(1)}(\tau)\, e^{iw\tau} \cdot d\tau \ ; \quad \chi^{(1)}(-w) = \chi^{(1)}(w)^*$$

$$w = Re(w) + i\,Im(w) \qquad = \int_0^{\infty} R^{(1)}(\tau)\, e^{i[Re(w)\tau]} \cdot e^{[-Im(w)\tau]}\, d\tau$$

In terms of susceptibility, $$Int = \int_{-\infty}^{\infty} \frac{\chi^{(1)}(w')dw'}{w'-w}$$

$$= \int_{-\infty}^{w-\delta} \frac{\chi^{(1)}(w')dw'}{w'-w} + \int_{w+\delta}^{\infty} \frac{\chi^{(1)}(w')dw'}{w'-w}$$

$$Re\,\chi^{(1)}(w) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{Im\,\chi^{(1)}(w')dw'}{w'-w}$$

$$Im\,\chi^{(1)}(w) = \frac{-1}{\pi} \int_{-\infty}^{\infty} \frac{Re\,\chi^{(1)}(w')dw'}{w'-w}$$

And hence,

$$Im\,\chi^{(1)}(w) = \frac{-2w}{\pi} \int_0^{\infty} \frac{Re\,\chi^{(1)}(w')}{w'^2 - w^2}\, dw$$

$$Re\,\chi^{(1)}(w) = \frac{-2w}{\pi} \int_0^{\infty} \frac{Im\,\chi^{(1)}(w')}{w'^2 - w^2}\, dw$$

$$\chi^{(2)}(2\omega_1,\omega_1,\omega_1) = \frac{N(e^3/m^2)a}{\epsilon_0 D(2\omega_1)D^2(\omega_1)} \; ; \; \chi^{(2)}(2\omega_1,\omega_1,\omega_1) = \frac{1}{i\pi}\int_{-\infty}^{\infty} \frac{\chi^{(2)}(2\omega';\omega';\omega')}{\omega'-\omega}d\omega'$$

1. $\omega_1 + \omega_2 = \omega_3$; L, d_{eff}, ΔK
$10\mu m$ $0.65\mu m$ $1cm$ $P=1W$

a) Output Power = (Quantum Efficiency) x I x $\frac{hc}{\lambda}$

b) The relationship for quantum efficiency

$$\boxed{I_3 = \frac{8 d_{eff}^2 \cdot \omega_3^2 \cdot I_1 \cdot I_2}{n_1 \cdot n_2 \cdot n_3 \cdot \epsilon \cdot c^2} L^2}$$

$$\boxed{Im\chi^{(2)}(\omega) = -\frac{4\omega i}{\pi}\int_0^{\infty}\frac{Re\chi^{(2)}(2\omega';\omega';\omega')}{\omega'^2-\omega^2}d\omega'}$$

$$\boxed{Re\chi^{(2)}(\omega) = \frac{4\omega i}{\pi}\int_0^{\infty}\frac{Im\chi^{(2)}(2\omega';\omega';\omega')}{\omega'^2-\omega^2}d\omega}$$

$$\chi^{(2)}(\omega_1;\omega_3,-\omega_2) = \frac{1}{i\pi}\int_{-\infty}^{\infty}\frac{\chi^{(2)}(\omega';\omega_3,-\omega_2)}{\omega'-\omega}d\omega'$$

$$Im\chi^{(2)}(\omega) = -\frac{i}{\pi}\int_{-\infty}^{0}\frac{Re\chi^{(2)}(\omega_1;\omega_3,-\omega_2)}{\omega'-\omega} - \frac{i}{\pi}\int_0^{\infty}\frac{Re\chi^{(2)}(\omega_1,\omega_3,-\omega_2)d\omega}{\omega'-\omega}$$

$$Re\chi^{(1)}(-\omega) = Re\chi^{(1)}(\omega) \; ; \; Im\chi^{(1)}(-\omega) = -Im\chi^{(1)}(\omega)$$

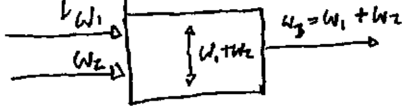
Thus,

$$\boxed{Im\chi^{(2)}(\omega) = \frac{2\omega i}{\pi}\int_0^{\infty}\frac{Re\chi^{(2)}(\omega_1;\omega_3,-\omega_2)d\omega}{\omega'^2-\omega^2}}$$

$$\boxed{Re\chi^{(2)}(\omega) = \frac{2\omega i}{\pi}\int_0^{\infty}\frac{Re\chi^{(2)}(\omega_1;\omega_3,-\omega_2)d\omega}{\omega'^2-\omega^2}}$$

3. Section 2.1: The wave Equation for Nonlinear Optical Media

- Sum Frequency Generation:



?

Maxwell's Equations: $\nabla\cdot\bar{D} = \rho$; $\nabla\cdot\bar{B} = 0$

$$\nabla\times\bar{E} = -\frac{\partial B}{\partial t} \; ; \; \nabla\times H = \frac{\partial D}{\partial t} + J \; ; \; \rho = 0 \; ; \; \bar{J} = 0 \; ; \; \bar{B} = \mu_0\bar{H} \; ; \; \bar{D} = \epsilon_0\bar{E} + \bar{P}$$

$$\nabla\times\nabla\times E + \mu_0\frac{\partial^2}{\partial t^2}D = 0 \; ; \; \nabla\times\nabla\times E + \frac{1}{c^2}\frac{\partial^2}{\partial t^2}E = -\frac{1}{\epsilon_0 c^2}\frac{\partial^2 P}{\partial t^2} \quad \left[\frac{kg\cdot m^2}{s^3}\left(\frac{m}{s}\right)^2 = \frac{kg}{s}\right]$$

$$\nabla\times\nabla\times\bar{E} = \nabla(\nabla\cdot E) - \nabla^2 E \; ; \; \nabla^2\bar{E} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\bar{E} = \frac{1}{\epsilon_0 c^2}\frac{\partial^2 P}{\partial t^2} \quad [kg/s]$$

$[kg/s^2]$

$$\nabla^2 E - \frac{1}{\epsilon_0 c^2}\frac{\partial^2}{\partial t^2}D = 0 \; ; \; P = P^{(1)} + P^{NL} \; ; \; \bar{D} = D^{(1)} + P^{NL} \; ; \; D^{(1)} = \epsilon_0\bar{E} + \bar{P}^{(1)}$$

$$\nabla^2 E - \frac{1}{\epsilon_0 c^2}\frac{\partial^2 D^{(1)}}{\partial t^2} = \frac{1}{\epsilon_0 c^2}\frac{\partial^2 P}{\partial t^2} \; ; \; D^{(1)} = \epsilon_0\epsilon^{(1)}\bar{E} \; ; \; D^{(1)} = \epsilon_0\epsilon^{(1)}\vec{E}$$

$$\nabla^2 E - \frac{\epsilon^{(1)}}{c^2}\frac{\partial^2 E}{\partial t^2} = \frac{1}{\epsilon_0 c^2}\frac{\partial^2 P^{NL}}{\partial t^2} \left[\frac{F/m}{(m/s)^2}\;\frac{kg\, m^2/s^2}{s^2} = \frac{s^4 A^2}{kg\, m^3}\;\frac{kg\, m^2}{m/s^2} = A^2 s^2\right]$$

$$E^{(1)}(r,t) = \sum E_n(r,t) \; ; \; D^{(1)}(r,t) = \sum D_n^{(1)}(r,t) \; ; \; P^{NL}(r,t) = \sum P_n^{NL}(r,t)$$

$$E_n(r,t) = E_n(r)e^{-i\omega_n t} + c.c. \; ; \; D_n(r,t) = D_n(r)e^{-i\omega_n t} + c.c. \; ; \; P_n^{NL}(r,t) = P_n^{NL}(r,t)e^{-i\omega_n t} + c.c.$$

$$D_n^{(1)}(r,t) = \epsilon_0\epsilon^{(1)}(\omega_n)\cdot\bar{E}_n(r,t) \; ; \; \nabla^2 E_n - \frac{\epsilon^{(1)}(\omega_n)}{c^2}\cdot\frac{\partial^2 E}{\partial t^2} = \frac{1}{\epsilon_0 c^2}\frac{\partial^2 P_n^{NL}}{\partial t^2} \left[\frac{kg\cdot m^2/s^2}{F/m\cdot m^2/s^2} = \frac{kg\, m}{F}\right]$$

$$D_n(r) = \epsilon_0\epsilon^{(1)}(\omega_n)\cdot E_n(r) \; ; \; \nabla^2 E_n(r) + \frac{\omega_n^2}{c^2}\epsilon^{(1)}(\omega_n)\cdot E_n(r) = \frac{-\omega_n^2}{\epsilon_0 c^2}P_n^{NL}(r)$$

Section 2.2: Coupled-Wave Equations for Sum-Frequency Generation:

$$E_3(z,t) = A_3 e^{i(k_3 z - \omega_3 t)} + c.c. \; ; \; k_3 = \frac{n_3\omega_3}{c} \; ; \; n_3^2 = \epsilon^{(1)}(\omega_3) \; [F/m]$$

$$P_3(z,t) = P_3 e^{-i\omega_3 t} + c.c. \; ; \; P_3 = 4\epsilon_0 d_{eff} E_1 E_2$$

Which represents the applied fields $i(=1,2)$

$$E_i(z,t) = E_i e^{-i\omega_i t} + c.c. \quad \text{where} \quad E_i = A_i e^{ik_i z}$$

Amplitude of Nonlinear Sum-Frequency Generation: $P_3 = 4\epsilon_0 d_{eff} A_1 A_2 e^{i(k_1+k_2)z} \equiv P_3 e^{i(k_1+k_2)z}$.

Nonlinear Energy of wave 3: $\left[\frac{\partial^2 A_3}{\partial z^2} + 2ik_3 \frac{\partial A_3}{dz} - k_3^2 A_3 + \frac{\epsilon^{(1)}(\omega_3)\omega_3^2 A_3}{c^2}\right] e^{i(k_3 z - \omega_3 t)} + c.c.$

$= \frac{-4 d_{eff}\omega_3^2}{c^2} A_1 A_2 e^{i[(k_1+k_2)z - \omega_3 t]} + c.c.$; $k_3^2 = \epsilon^{(1)}(\omega_3)\omega_3^2/c^2$

Phase-Matching Considerations:

$\Delta k = 0$;

$\left[\frac{kg}{s^2}\right]$ $\frac{d^2 A_3}{dz^2} + 2ik_3 \frac{dA_3}{dz} = \frac{-4 d_{eff}\omega_3^2}{c^2} A_1 A_2 e^{i(k_1+k_2-k_3)z}$; $\left|\frac{d^2 A_3}{dz^2}\right| \ll \left|k_3 \frac{dA_3}{dz}\right|$

Amplitude of Nonlinear Medium: is from integration of amplitude:

Required condition: $\frac{dA_3}{dz} = \frac{2id_{eff}\omega_3}{n_3 c} A_1 A_2 e^{i\Delta k z}$; $\Delta k = k_1 + k_2 - k_3$

$A_3(L) = \frac{2id_{eff} A_1 A_2}{n_3 c}\int_0^L e^{i\Delta k z} dz$

$= \frac{2id_{eff}\omega_3 A_1 A_2}{n_3 c}\left(\frac{e^{i\Delta k L} - 1}{i\Delta k}\right)$.

When analyzing the amplitude equations:

$\frac{dA_1}{dz} = \frac{2id_{eff}\omega_1}{n_1 c} A_3 A_2^* e^{-i\Delta k z}$

$\frac{dA_2}{dz} = \frac{2id_{eff}\omega_1}{n_1 c} A_3 A_1^* e^{-i\Delta k z}$

$\frac{dA_3}{dz} = \frac{i\omega_3}{2\epsilon_0 n_3 c} P_3 e^{i\Delta k z}$

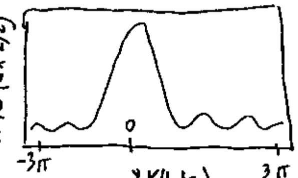
Intensity of ω_3 is the magnitude of a time-averaged Poynting vector;

$I_i = 2n_i \epsilon_0 c |A_i|^2$; $i = 1,2,3\ldots$ where $I_3 = \frac{8 n_3 \epsilon_0 d_{eff}^2 \omega_3^2 |A_1|^2 |A_2|^2}{n_3^2 \cdot c} \underbrace{\left|\frac{e^{i\Delta k L} - 1}{\Delta k}\right|^2}_{\text{squared modulus}}$

$\left|\frac{e^{i\Delta k L} - 1}{\Delta k}\right| = L^2\left(\frac{e^{i\Delta k L} - 1}{\Delta k L}\right)\left(\frac{e^{-i\Delta k L} - 1}{\Delta k L}\right) = 2L^2 \frac{(1 - \cos\Delta k L)}{(\Delta k L)^2} = L^2 \frac{\sin^2(\Delta k L/2)}{(\Delta k L/2)^2}$

$\boxed{= L^2 \operatorname{sinc}^2(\Delta k L/2)}$

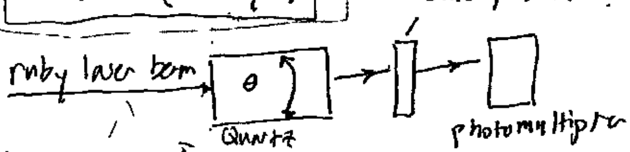
$L_{coh} = 2/\Delta k$; $\operatorname{sinc}^2(L/L_{coh})$

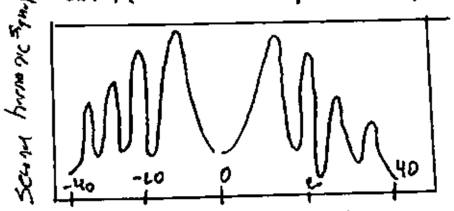


2.3 Phase Matching:

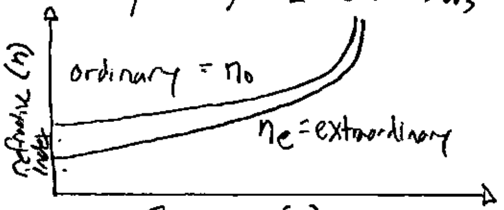
$\Delta k = k_1 + k_2 - k_3$

varies intensity according to $I_3 = I_3^{(max)}\left[\frac{\sin(\Delta k L/2)}{(\Delta k L/2)}\right]^2$





finally ; $n_3 - n_2 = (n_1 - n_2)\frac{\omega_1}{\omega_3}$



System	Linear Optical
Triclinic, monoclinic	Biaxial
Trigonal, tetragonal	Uniaxial
Cubic	Isotropic

	Positive Uniaxial ($n_e > n_o$)	Negative Uniaxial ($n_e < n_o$)
Type I	$n_3^o \omega_3 = n_1^e \omega_1 + n_2^e \omega_2$	$n_3^e \omega_3 = n_1^o \omega_1 + n_2^o \omega_2$
Type II	$n_3^o \omega_3 = n_1^o \omega_1 + n_2^e \omega_2$	$n_3^e \omega_3 = n_1^e \omega_1 + n_2^o \omega_2$

$\frac{n_1\omega_1}{c} + \frac{n_2\omega_2}{c} = \frac{n_3\omega_3}{c}$

$\omega_1 + \omega_2 = \omega_3$

In the case $\omega_1 = \omega_2$; $\omega_3 = 2\omega_1$

requires $n(\omega_1) = n(2\omega_1)$

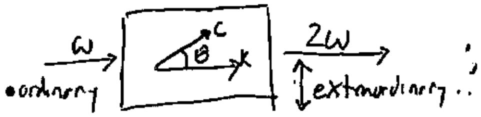
$n_3 = \frac{n_1\omega_1 + n_2\omega_2}{\omega_3}$

$n_3 - n_2 = \frac{n_1\omega_1 + n_2\omega_2 - n_2\omega_3}{\omega_3} = \frac{n\omega_1 - n_2\omega_2}{\omega_3}$

Angle Tuning:

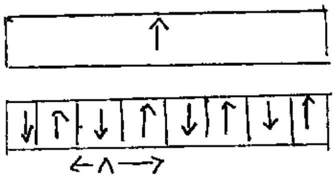
$\frac{1}{n_e(\theta)^2} = \frac{\sin^2\theta}{n_e^2} + \frac{\cos^2\theta}{n_o^2}$; n_e $\theta = 90°$; n_o @ $\theta = 0°$

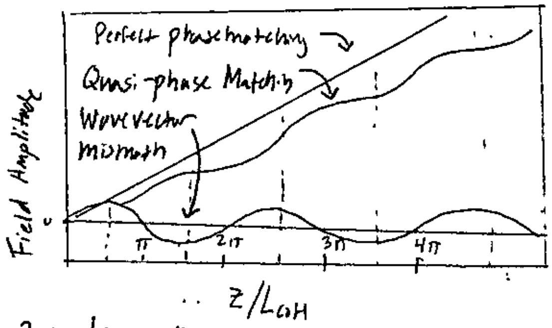
$n_e(2\omega, \theta) \doteq n_o(\omega)$ -or- $\frac{\sin^2\theta}{n_e(2\omega)^2} + \frac{\cos^2\theta}{n_o(2\omega)^2} = \frac{1}{n_o(\omega)^2}$



; $\sin^2\theta = \frac{\frac{1}{n_o(\omega)^2} - \frac{1}{n_o(2\omega)^2}}{\frac{1}{n_e(2\omega)^2} - \frac{1}{n_o(2\omega)^2}}$

2.4 Quasi-Phase Matching:





Mathematical Description:

$d(z) = \underbrace{d_{eff}\, \text{sign}[\cos(2\pi z/\Lambda)]}_{\text{"phase matching"}}$

$= d_{eff} \sum_{m=-\infty}^{\infty} G_m \exp(i k_m z)$ } "spatial variation" where $k_m = 2\pi m/\Lambda$; m^{th} fourier component

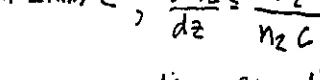
$G_m = (2/m\pi)\sin(m\pi/2)$; where $G_1 = 2/\pi$; Nonlinear Coupling Coefficient:

$\frac{dA_1}{dz} = \frac{2i\omega_1 d_m}{n_1 c} A_3 A_2^* e^{-i(\Delta K_m - 2K_m)z}$; $\frac{dA_2}{dz} = \frac{2i\omega_2 d_m}{n_2 c} A_3 A_1^* e^{-i(\Delta K_m - 2K_m)z}$.

where $d_m = d_{eff} \cdot G_m$ $\qquad \frac{dA_3}{dz} = \frac{2i\omega_3 d_m}{n_3 c} A_1 A_2 e^{i\Delta K_m z}$ $\qquad$ [N]

$\Delta K_m = K_1 + K_2 - K_3 + K_m$; $\Delta K_m = K_1 + K_2 - K_3 - 2\pi/\Lambda$; $d_m = (2/\pi) d_{eff}$; $\Lambda = 2L_{coh} = 2\pi/(K_1 + K_2 - K_3)$

2.5: The Manley-Rowe Relations:



$I_i = 2n_i \epsilon_0 c A_i A_i^*$ $[J^2 \cdot m/s]$

$I = I_1 + I_2 + I_3$

$\frac{dI}{dz} = 2n_i \epsilon_0 c \left(A_i^* \frac{\partial A}{\partial z} + A_i \frac{\partial A_i^*}{\partial z}\right)$ $[J^2/s]$

$\frac{dI}{dz} = \frac{dI_1}{dz} + \frac{dI_2}{dz} + \frac{dI_3}{dz}$

$= -8\epsilon_0 d_{eff}(\omega_1 + \omega_2 - \omega_3)\, \text{Im}(A_3 A_1^* A_2^* e^{-i\Delta K z}) = 0$ (J^2/s)

$\frac{dI_1}{dz} = 2n_i \epsilon_0 c \frac{2 d_{eff} \omega_1^2}{K_1 c}(i A_1^* A_3 A_2^* e^{-i\Delta K z} + c.c)$

$= 4\epsilon_0 d_{eff} \omega_1 (i A_3 A_1^* \cdot A_2^* e^{-i\Delta K z} + c.c)$ $[J^2/s]$

$= -8\epsilon_0 d_{eff} \omega_1\, \text{Im}(A_3 A_1^* A_2^* e^{-i\Delta K z} + c.c)$ $[J^2/s]$

$\frac{dI_2}{dz} = -8\epsilon_0 d_{eff} \omega_2\, \text{Im}(A_3 A_1^* A_2^* e^{-i\Delta K z})$

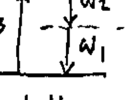
$\frac{dI_3}{dz} = -8\epsilon_0 d_{eff} \omega_3\, \text{Im}(A_3^* A_1 A_2 e^{i\Delta K z})$

$= 8\epsilon_0 d_{eff} \omega_3\, \text{Im}(A_3 A_1^* A_2^* e^{-i\Delta K z})$ $[J^2/s]$

$\underbrace{\frac{d}{dz}\left(\frac{I_1}{\omega_1}\right) = \frac{d}{dz}\left(\frac{I_2}{\omega_2}\right) = -\frac{d}{dz}\left(\frac{I_3}{\omega_3}\right)}_{\text{"Manley-Rowe Relations"}} : [N \cdot s]$.

$\frac{d}{dz}\left(\frac{I_2}{\omega_2} + \frac{I_3}{\omega_3}\right) = 0$; $\frac{d}{dz}\left(\frac{I_1}{\omega_1} + \frac{I_3}{\omega_3}\right) = 0$; $\frac{d}{dz}\left(\frac{I_1}{\omega_1} - \frac{I_2}{\omega_2}\right) = 0$

$M_1 = \frac{I_2}{\omega_2} + \frac{I_3}{\omega_3}$; $M_2 = \frac{I_1}{\omega_1} + \frac{I_3}{\omega_3}$, $M_3 = \frac{I_1}{\omega_1} - \frac{I_2}{\omega_2}$



5. E 2.7.29 / 2.7.2 $\quad \frac{du_2^2}{d\zeta} = \pm 2\left[(1-u_2^2)^2 u_2^2 - \Gamma^2\right]^{1/2}$ "Second-Harmonic Generation" - Twice the Frequencies.

The Jacobi-Elliptic Function is a periodic function for a given set of initial conditions. Being a coupled equation dependent upon u_2, u_1, θ, z, and ℓ, the final solution represents a hyperbolic relationship to intensity along with medium.

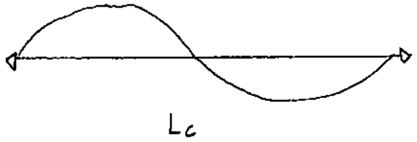
7. $\Delta\nu_c = \frac{1}{n^{(g)}} \frac{c}{2 \cdot L_c}$ where $n^{(g)} = n + \nu \frac{dn}{d\nu}$ and $n^{(g)}$ is the group index.

$\frac{m\lambda}{2} = L_c$; $\lambda = \frac{c}{n\nu}$; $n\nu = cm/2L_c$ $\quad \Delta(n\nu) = \Delta(cm/2L_c)$; $\Delta(n\nu) = n\Delta\nu + \nu\Delta n = n\Delta\nu + \nu\left(\frac{dn}{d\nu}\right)\Delta\nu$

$\Delta(cm/2L_c) = c/2L\Delta(m) = c/2L_c$

$\therefore \Delta\nu = \frac{c}{2L_c(n + \nu\, dn/d\nu)} = \frac{v_g}{2L_c} : \frac{c}{2L_c}$

$v_g = c/[n + \nu(dn/d\nu)]$; $n_g = n + \nu(dn/d\nu)$



$\frac{m\lambda}{2} = L_c$; i. $\lambda = \frac{c}{n \cdot \nu}$; $\frac{2L_c}{m} = \frac{c}{n\cdot\nu}$; $n\nu = \frac{cm}{2\cdot L_c}$; $(\Delta n\nu) = \Delta\left(\frac{cm}{2L_c}\right)$.

$(\Delta n\nu) = n\Delta\nu + \nu\Delta n = n\Delta\nu + \nu\left(\frac{dn}{d\nu}\right)\Delta\nu$

9. 2.4.1 to 2.4.6.

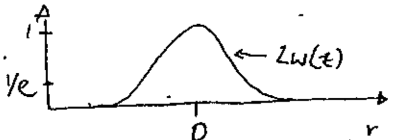
$\Delta(cm/2L_c) = \frac{c}{2L_c}(\Delta m)$; $\frac{c}{2\cdot L_c}(\Delta m) = n\cdot\Delta\nu + \nu\left(\frac{dn}{d\nu}\right)\Delta\nu = \left(n + \frac{\nu\, dn}{d\nu}\right)\cdot\Delta\nu$

$$\boxed{\Delta\nu = \frac{c}{2 \cdot L_c\left(n + \nu\frac{dn}{d\nu}\right)} = \frac{1}{2L_c}\left[\frac{c}{n + \nu\left(\frac{dn}{d\nu}\right)}\right] = \frac{v_g}{2L_c}}$$

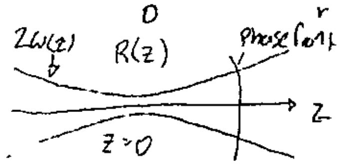
$v_g = c/\left[n + \nu\left(\frac{dn}{d\nu}\right)\right]$; $n_g = \underbrace{n + \nu(dn/d\nu)}_{\text{Group index}}$

Quasi-Phase Matching relates to the birefringence of a material and describes the relation to Nonlinear coupling. The period (Λ) aids with the spatially-dependence coupling coefficient. The coefficient depends on m-th Fourier components. In addition to, the change of amplitude per z. The wavevector mismatch is given by the difference of wavevectors.

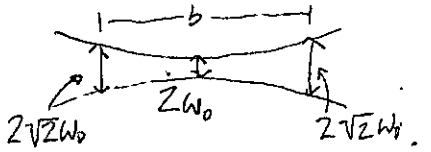
11. 2.10.9. $A_q(r,z) = \frac{A_q(z)}{1+i\xi} e^{-qr^2/w_0^2(1+i\xi)}$ "Solution to 2.10.7" "trial solution" "Derived from Paraxial Amp. Equation"



2.10.7 $2ik_q\frac{\partial A_q}{\partial z} + \nabla_T^2 A_q = -\frac{\omega_q^2}{c^2}\chi^{(q)}\cdot A_1^q e^{i\Delta kz}$; $\Delta k = qk_1 - k_q$



2.10.10 $\frac{dA_q}{dz} = \frac{iq\omega}{2n_q c}\chi^{(q)}\cdot A_1^q \frac{e^{i\Delta kz}}{(1+i\xi)^{q-1}}$ "Derivative of trial solution"



13. coupled Amp $\chi^{(3)}(3\omega;\omega,\omega,\omega)$ and $\chi^{(3)}(\omega;3\omega,-\omega,-\omega)$:

Paraxial Equation : $2ik_n\frac{\partial A_n}{\partial z} + \nabla_T^2 A_n = -\frac{\omega_n^2}{\epsilon_0 c^2} P_n e^{i\Delta kz}$

Intensity Gaussian Beam Eqn: $2ik_q\frac{\partial A_q}{\partial z} + \nabla_T^2 A_q = -\frac{\omega_q^2}{c^2}\underbrace{\chi^{(q)}\cdot A_q^q}_{\text{Polarization}} e^{i\Delta kz}$ $A_q(r,z) = \frac{A_q(z)}{1+i\xi}e^{-qr^2/w_0^2(1+i\xi)}$

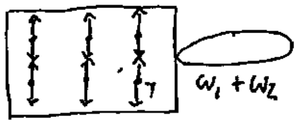
$2ik_3\frac{\partial A_3}{\partial z} + \nabla_T^2 A_3 = -\frac{\omega_3^2}{c^2}\chi^{(3)}\cdot A_1^3 \cdot e^{i\Delta kz}$; $A_3(r,z) = \frac{A_3(z)}{1+i\xi}e^{-3r^2/w_0^2(1+i\xi)}$

$$\boxed{2ik_3\left[\frac{3i\omega}{2n_3 c}\chi^{(3)}\cdot A_1^{(3)}\frac{e^{i\Delta kz}}{(1+i\xi)^2}\right] + \nabla^2\left[\frac{A_3(z)}{1+i\xi}e^{-3r^2/w_0^2(1+i\xi)}\right] = -\frac{\omega_3^2}{c^2}\chi^{(3)}\cdot A_1^3 e^{i\Delta kz}}$$

$+\omega,+\omega$; $k_3 = k + k_2 + k_3$
$-\omega,-\omega$; $k_3 = k - k_2 - k_3$

$\frac{\partial A_q}{\partial z} = \frac{iq\omega}{2n_q c}\chi^{(q)} A_1^q \frac{e^{i\Delta kz}}{(1+i\xi)^{q-1}}$

$\frac{\partial A_3}{\partial z} = \frac{i3\omega}{2n_3 c}\chi^{(3)} A_1^3 \frac{e^{i\Delta kz}}{(1+i\xi)^2}$

15. 

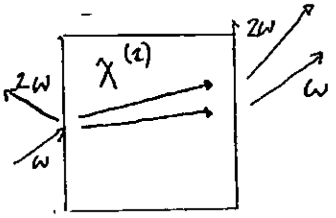
$\nabla\cdot E = \underset{\text{Free charge}}{\rho}/\epsilon_0$ $\nabla\times E = -\frac{\partial B}{\partial t}$

$\nabla\cdot B = 0$ $\nabla\times B = \mu_0 \underbrace{j}_{\text{current}} + \frac{1}{c^2}\frac{\partial E}{\partial t}$

Derive the coupled amplitude Equation for second-order harmonics.

Nonlinear Optics at an interface: Section 2.11

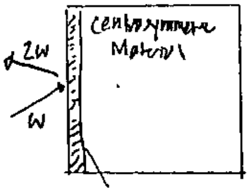
1/



Wave Fundamental Frequency Incident to surface

$$\bar{E}_i(r,t) = E_i(w_i)e^{-iwt} + c.c \quad \text{where} \quad E_i(w_i) = A_i(w_i)e^{ik_i(w_i)r}$$

Partial Reflectance and Transmittance: $\underbrace{E_T(W_i) = A_T(w_i)e^{ik_T(w_i)r}}_{\text{Determined by Fresnel Equations.}}$



Symmetry Broken at surface.

Transmitted wave will create a nonlinear polarization: $\tilde{P}(\bar{r},t) = \bar{P}e^{-iw_st} + c.c.$ Where $P = pe^{ik_s(w_s)\cdot r}$ and $p = \hat{p}\epsilon_0\chi^{(2)}_{eff} A_T^2(w_i)$

$$k_s(w_s) = 2k_T(w_i)$$

$$\nabla^2 E(w_s) + [\epsilon(w_s)w_s^2/c^2]E(w_s) = -(w_s^2/\epsilon_0 c^2)p_\perp e^{ik_s r}$$

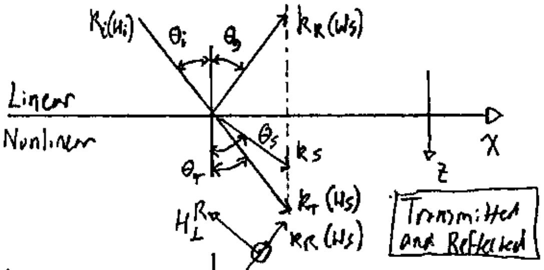
The Energy Transmitted from Fresnel Equations:

$$E_T(w_s) = \underbrace{A_T(w_s)e^{ik_T(w_s)r}}_{\text{Infinite plane wave}} + \underbrace{\frac{(w_s^2/\epsilon_0 c^2)}{|k_s|^2 - |k_T(w_s)|^2} p_\perp \cdot e^{ik_s\cdot r}}_{\text{Particular solution.}}$$

$k_s = \sqrt{\epsilon(w_s)}w_s/c$

$|k_T(w_s)|^2 = \epsilon_T(w_s)w_s^2/c^2$

Geometry of Transmitted and reflected:



$$E_R(w_s) = A_R(w_s)e^{ik_R(w_s)r} \; ; \; k_X^s = k_{R,x}(w_s) = k_{T,s}(w_s)$$

$k_s = \underbrace{\epsilon_s^{1/2}}_{\text{Fictitious Dielectric constant}} w_s/c$

$k_T(w_s) = \epsilon_T^{1/2}(w_s)w_s/c$

$k_R(w_s) = \epsilon_R^{1/2}(w_s)w_s/c$

$k_i(w_i) = \epsilon_R^{1/2}(w_s)w_i/c$

Where ϵ_R = Dielectric constant

ϵ_T = Linear Dielectric constant.

$\boxed{\text{Angle of Incidence}}$

$$\epsilon_R^{1/2}(w_i)\sin\theta_i = \epsilon_R^{1/2}(w_s)\cdot\sin\theta_R$$
$$= \epsilon_T^{1/2}(w_s)\sin\theta_T$$
$$= \epsilon_s^{1/2}\sin\theta_s$$

$\boxed{\text{Magnetic and Electric Fields}}$

$\boxed{\text{Continuity of Tangential Components}}$

$$E_y: A_\perp^R(w_s) = A_\perp^T(w_s) + P_\perp/[\epsilon_0(\epsilon_s - \epsilon_T(w_s))]$$

$$H_x: -\epsilon_R^{1/2}(w_s)A_\perp^R(w_s)\cos\theta_R = \epsilon_T^{1/2}(w_s)A_\perp^T(w_s)\cos\theta_T + p_\perp\cos\theta_s\epsilon_s^{1/2}/[\epsilon_0(\epsilon_s - \epsilon_T(w_s))]$$

$$E_\perp^R(w_s) = \frac{-p_\perp e^{ik_R(w_s)r}}{\epsilon_0[\epsilon_T^{1/2}(w_s)\cos\theta_T + \epsilon_R^{1/2}(w_s)\cos\theta_R][\epsilon_T^{1/2}(w_s)\cos\theta_T + \epsilon_s^{1/2}\cos\theta_s]} \equiv A_\perp^R(w_s)e^{ik_R(w_s)\cdot r}$$

$$E_\perp^T(w_s) = \frac{-p_\perp}{\epsilon_0[\epsilon_T(w_s) - \epsilon_s]}\left[e^{ik_s\cdot r} - \frac{\epsilon_s^{1/2}\cos\theta_s + \epsilon_R^{1/2}\cos\theta_R}{\epsilon_T^{1/2}(w_s)\cos\theta_T + \epsilon_R^{1/2}(w_s)\cos\theta_R}\cdot e^{ik_T(w_s)r}\right]$$

k_T = Homogeneous Contribution

k_s = inhomogeneous Wave contribution.

$$= \left[A_\perp^R(w_s) + \frac{(w_s^2/\epsilon_0 c^2)p_\perp}{2k_T(w_s)}\left(\frac{e^{i\Delta k z} - 1}{\Delta k}\right)\right]e^{ik_T(w_s)r}$$

$$k_s - k_T = \Delta K\hat{z} = (w_s/c)\left[\epsilon_s^{1/2}\cos\theta_s - \epsilon_T^{1/2}(w_s)\cos\theta_T\right]\hat{z}$$

$$= A_\perp^T(w_s)e^{ik_T(w_s)r} \text{ "Plane Wave"}$$

$$A_\perp^T(w_s) = A_\perp^R(w_s) + \frac{(w/c)^2 p_\perp(iz)}{2\epsilon_0 k_T(w_s)} = A_\perp^R(w_s) + \frac{i(w/c)p_\perp z}{2\epsilon_0\epsilon^{1/2}(w_s)} \; ; \text{ -or- } A_\perp^R \simeq -\frac{p_\perp}{4\epsilon_0\epsilon}$$

$$A_\perp^T \simeq -\frac{\pi p_\perp}{4\epsilon_0\epsilon}[1 - 2ik_T(w_s)z]$$

$$t = \lambda/4\pi$$

17. Derive Manley-Rowe Relations: $\boxed{\begin{matrix}\omega_1 \longrightarrow \\ \omega_2 \longrightarrow \\ \omega_3 \longrightarrow\end{matrix}}$ "Three optical waves propagating and mutually interacting".

19. (2.7.12) $\Delta K = 2K_1 - K_2$

(2.7.11) $\frac{dA_2}{dz} = \frac{i\omega_2^2 d_{eff}}{K_2 c^2} A_1^2 e^{i\Delta K z}$

I, L, To solve the integral, then complex or modular should be considered.

$$\underbrace{A_1 = \left(\frac{I}{2n_1\epsilon_0 c}\right)^{1/2} u_1 e^{i\phi_1} ; A_2 = \left(\frac{I}{2n_2\epsilon_0 c}\right)^{1/2} u_2 e^{i\phi_2}}_{\boxed{\text{"Coupled-Equations"}}}$$

$I = I_1 + I_2$

$I_j = 2n_j \epsilon_0 c |A_j|^2$

$u_1(z)^2 + u_2(z)^2 = 1$ $\boxed{\text{"Spatially Invarient"}}$

$l = \left(\frac{n_1^2 \cdot n_2 \cdot \epsilon_0 \cdot c}{2I}\right)\frac{c}{\omega_1 d_{eff}}$; $\theta = 2\phi_1 - \phi_2 + \Delta K z$; $\xi = z/l$

Normalized phase-mismatch parameter:

$\Delta S = \Delta K l$; $\frac{du_1}{d\xi} = u_1 u_2 \sin\theta$

$\frac{du_2}{d\xi} = -u_1^2 \sin\theta$.

$\frac{d\theta}{d\xi} = \Delta S + \frac{\cos\theta}{\sin\theta}\frac{d}{d\xi}(\ln u_1^2 u_2)$

$= 0$, and the function is maximum.

$0 = \frac{d}{d\xi}(\ln u_1^2 u_2 \cdot \cos\theta) = \frac{d}{d\xi}\ln\Gamma$; $\boxed{\text{"independent value in coupled-equations"}}$

Also, $\frac{du_2}{d\xi} = \pm(1-u_2^2)(1-\cos^2\theta)^{1/2}$; $\frac{du_2}{d\xi} = \pm(1-u_2^2)\left(1-\frac{\Gamma^2}{u_1^4 u_2^2}\right)^{1/2} = \pm(1-u_2^2)\left(1-\frac{\Gamma^2}{(1-u_2^2)^2 u_2^2}\right)^{1/2}$

$u_2\frac{du_2}{d\xi} = \pm\left[(1-u_2^2)^2 u_2^2 - \Gamma^2\right]^{1/2}$; $\frac{du_2^2}{d\xi} = \pm 2\left[(1-u_2^2)^2 u_2^2 - \Gamma^2\right]^{1/2}$ "Jacobi Elliptic Function"

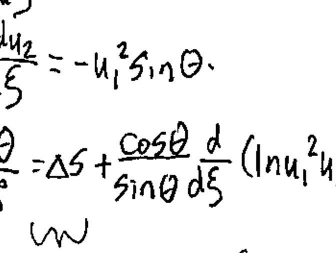
$\cos\theta = 0$; $\sin\theta = -1$; $\frac{du_1}{d\xi} = -u_1 u_2$; $\frac{du_2}{d\xi} = u_1^2 = 1 - u_2^2$; $u_2 = \tanh(\xi + \xi_0)$

$u_1(0) = 1$; $u_2(0) = 0$

$u_1 = \text{sech}(\xi)$

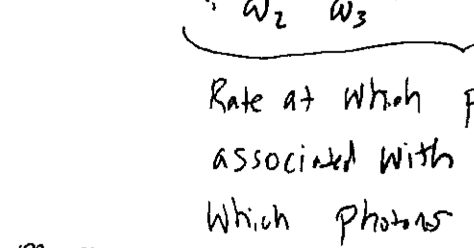
$l = \frac{(n_1 n_2)^{1/2} c}{2\omega_1 d_{eff}|A_1(0)|}$

u_2



$\xi = z/l$

Plot of u_1 and u_2



$\xi = z/L$

$I_i = 2n_i\epsilon_0 c A_i A_i^*$; $\frac{dI_i}{dz} = 2n_i\epsilon_0 c\left(A_i^*\frac{dA_i}{dz} + A_i\frac{dA_i^*}{dz}\right)$

Thus, $\frac{dI_1}{dz} = 2n_i\epsilon_0 c\frac{2d_{eff}\cdot\omega_1^2}{K_1 c^2}(iA_1^* A_3 A_2^* e^{-i\Delta K z} + c.c)$

$= 4\epsilon_0 d_{eff}\cdot\omega_1(iA_3 A_1^* A_2^* e^{-i\Delta K z} + c.c)$

$= -8\epsilon_0 d_{eff}\cdot\omega_1 \text{ Im}(A_3 A_1^* A_2^* e^{-i\Delta K z})$

$\frac{dI_2}{dz} = -8\epsilon_0 d_{eff}\cdot\omega_2 \text{ Im}(A_3 A_1^* A_2^* e^{-i\Delta K z})$

$\frac{dI_3}{dz} = -8\epsilon_0 d_{eff}\omega_3 \text{ Im}(A_3^* A_1 A_2 e^{i\Delta K z})$

$= 8\epsilon_0 d_{eff}\omega_3 \text{ Im}(A_3 A_1^* A_2^* e^{-i\Delta K z})$

$I = I_1 + I_2 + I_3$

$\frac{dI}{dz} = \frac{dI_1}{dz} + \frac{dI_2}{dz} + \frac{dI_3}{dz}$

$= -8\epsilon_0 d_{eff}(\omega_1 + \omega_2 - \omega_3)\text{Im}(A_3 A_1^* A_2^* e^{-i\Delta K z}) = 0$

$\frac{d}{dz}\left(\frac{I}{\omega_1}\right) = \frac{d}{dz}\left(\frac{I}{\omega_2}\right) = \frac{-d}{dz}\left(\frac{I}{\omega_3}\right)$; $\omega_3 = \omega_1 + \omega_2$

$$\underbrace{M_1 = \frac{I_2}{\omega_2} + \frac{I_3}{\omega_3},\quad M_2 = \frac{I_1}{\omega_1} + \frac{I_3}{\omega_3},\quad M_3 = \frac{I_1}{\omega_1} + \frac{I_2}{\omega_2}}$$

Rate at which photons are created is associated with the frequency at which photons are created: Manley-Rowe

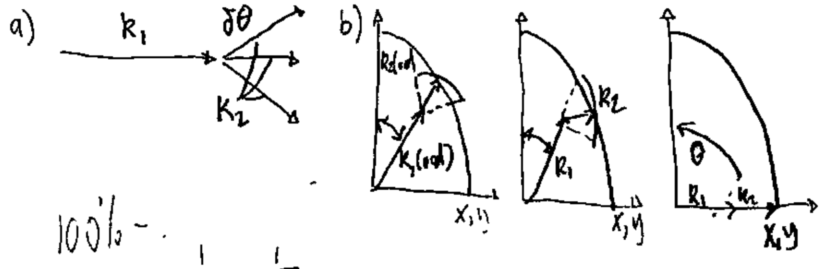
$I_1 = \frac{P}{\pi W_0^2} = 2\pi \epsilon_1 A_1^2$; W_0 = focal spot size ; $b \equiv \frac{2\pi W_0^2}{\lambda_1/n_1} = L$; $A_1 = \left(\frac{P}{\epsilon_0 c \lambda_1 L}\right)^{1/2}$; $S = \left(\frac{16\pi^2 d_{eff}^2 L \cdot P}{\epsilon_0 c n_1 n_2 \lambda_1^3}\right)^{1/2}$

21. $P_{2\omega} = K\left[\frac{128\pi^2 \omega_1^3 d_{eff}^2 \cdot L}{c^4 n_1 n_2}\right] P_\omega^2$; The power of second-harmonic generation is related to the square of incident power. The length of medium is a scalar to incident power... $\eta = \frac{u_2^2(L)}{u_1^2(0)}$

23. Maxwell's Equations: $\nabla \cdot D = \rho$; $\nabla \times \bar{E} = -\frac{\partial B}{\partial t}$
$\nabla \cdot B = 0$; $\nabla \times \bar{H} = \frac{\partial D}{\partial t} + J$

25. Prove power fraction of Gaussian laser is $\omega(z)$ given by $1 - 1/e^2 = 0.865$

Critical and Noncritical Phasematching



100% -

e $\quad e^{\frac{1}{2\cdot 2}} - e^{\frac{1}{2\cdot 1}}$

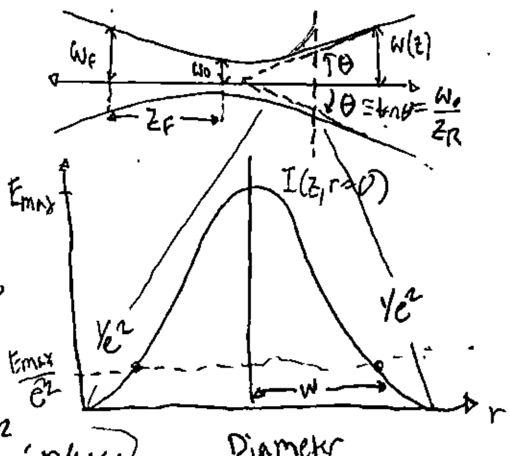
The nonlinear medium is highly dependent to the Electric field because of the theoretical demonstration of the paraxial equation: $D = D^{(1)} + P^{NL}$; $D^{(1)} = \epsilon_0 E + P^{(1)}$

$$\nabla^2 E - \frac{1}{\epsilon_0 c^2}\frac{\partial^2}{\partial t^2} E = \frac{1}{\epsilon_0 c^2}\frac{\partial^2 P}{\partial t^2}$$

$$\nabla^2 E - \frac{1}{\epsilon_0 c^2}\frac{\partial^2 D^{(1)}}{\partial t^2} = \frac{1}{\epsilon_0 c^2}\frac{\partial^2 P^{NL}}{\partial t^2}$$

In isotropic Medium; $D^{(1)} = \epsilon_0 \epsilon^{(1)} E$

$$\boxed{\nabla^2 E - \frac{\epsilon^{(1)}}{c^2}\frac{\partial^2 E}{\partial t^2} = \frac{1}{\epsilon_0 c^2} \cdot \frac{\partial^2 P^{NL}}{\partial t^2}}$$



$W(z) = W_0\sqrt{1 + (z/z_R)^2}$; $W_0 = 1$

$I = I_0 e^{-2r^2/W^2}$

Characteristics of a Gaussian Beam

Beam Radius : $W(z) = W_0\sqrt{1 + (z/z_R)^2}$

Radius of Curvature of Wavefront : $R(z) = z\left(1 + \left(\frac{z_R}{z}\right)^2\right)$

Transverse Phase : $\Phi_T(r, z) = \frac{kr^2}{2R(z)}$

Longitudinal Phase : $\Phi_L(z) = kz - \arctan\left(\frac{z}{z_R}\right)$

RayLeigh Length : $Z_R = \frac{\pi W_0^2}{\lambda}$

$$I = I_0 e^{-2(r/w)^2/w^2} \quad ; r/w = 1$$
$$= I_0 e^{-2} = I_0/e^2 \; ; \; \frac{I}{I_0} = \frac{1}{e^2}$$

$$= 1\frac{I_0}{I} - \frac{I}{I_0} = \boxed{1 - \frac{1}{e^2} = 0.865}$$

1. 3. 5. 20

$$\chi_{ij}^{(1)}(\omega_p) = \frac{N}{\epsilon_0 \hbar}\sum_n\left[\frac{\mu_{an}^i \cdot \mu_{na}^j}{(\omega_{na} - \omega_p) - i\gamma_{na}} + \frac{\mu_{na}^i \cdot \mu_{an}^j}{(\omega_{na} + \omega_p) + i\gamma_{na}}\right]$$

$N = 10^{17}\,cm^{-3}$, $\mu = 2.5 e a_0$, $\lambda = 0.6\mu m$

$(FWHM) = 10\,GHz$

Solution to the Paraxial Wave Equation:

$$\frac{E(r, z, t)}{E_0} = \frac{W_0}{W(z)}\exp\left[-\left(\frac{r}{W(z)}\right)^2\right]\exp\left[i(\omega t - \Phi_T - \Phi_L)\right]$$

Linear Susceptibility : $\langle \hat{p} \rangle = \langle \psi | \hat{\mu} | \psi \rangle$; $\chi_{ij}^{(1)}(\omega_p) = \frac{N}{\epsilon_0 \hbar} \frac{\mu_{an}^i \mu_{na}^j}{(\omega_{na}-\omega_p) - i\gamma_{na}} = \frac{N}{\epsilon_0 \hbar} \mu_{an}^i \mu_{na}^j \frac{(\omega_{na}-\omega_p) + i\gamma_{na}}{(\omega_{na}-\omega_p)^2 + \gamma_{na}^2}$

Electric Dipole Moment

Susceptibility is greatest at the transition wavelength

$n(\omega) = \sqrt{1 + \chi^{(1)}(\omega)}$

ω_{mg} = transition Frequency

ω_p = Wavelength of pump

$I = I_0 e^{-2r^2/w^2}$

Absorption crosssection

Absorption Coefficient

$\alpha = N \cdot \sigma = N \cdot \chi^{(1)''} \omega/c$
$= 2n'' \omega/c = \chi^{(1)''} \omega/c \approx \sum \frac{f_{na} N e^2}{2 m \epsilon_0 c \gamma_{na}} \left[\frac{\gamma_{na}^2}{(\omega_{na}-\omega)^2 + \gamma_{na}^2} \right]$

ω_{na} = transition Frequency

γ_n = transition rate or decay rate

3.5.20 So, $\chi_{ij}^{(1)}(\omega_p) = \frac{N}{\epsilon_0 \hbar} \sum \left[\frac{\mu_{an}^i \cdot \mu_{na}^j}{(\omega_{na}-\omega_p) - i\gamma_{na}} + \frac{\mu_{na}^i \cdot \mu_{an}^j}{(\omega_{na}+\omega_p) + i\gamma_{na}} \right]$

$= \frac{N}{\epsilon_0 \hbar} \mu_{an}^i \mu_{ng}^j \frac{(\omega_{na}-\omega_p)}{(\omega_{na}-\omega_p)^2 + \gamma_{na}^2} = \frac{10^{17} cm^{-3} (2.5 e \cdot a_0)}{(8.854\times10^{-12} \frac{F}{m}) \frac{6.626\times10^{-34} J\cdot s}{2\pi}} \frac{(3.14\times10^{15}\ 1/s)}{(3.14\times10^{15} \frac{1}{s})^2 + (10 GHz \times \frac{10^9 Hz}{1 GHz})^2}$ ★

$f_{na} = \frac{2 m \omega_{na} |\mu_{na}|^2}{3 \hbar e^2}$; $\sum_i f_{na} = 1$; $\gamma_{nm} = \frac{1}{2}(\Gamma_n + \Gamma_m) + \gamma_{nm}^{(col)}$; $\tau = 1/\Gamma_n$; FWHM $= 2\gamma_{na}$

★ $= \frac{10^{17} cm^{-3} \times \left(\frac{100 cm}{1m}\right)^3 (2.5 e a_0) \cdot 3.18\times10^{-16} \cancel{s}}{9.337\times10^{-46} \frac{F}{m} \cdot s}$

$2\pi f = \omega$

$\frac{hc}{\lambda} = h\nu$ $2\pi\nu = \omega$

$\frac{2\pi c}{\lambda} = \omega$

$= \frac{7.95\times10^{17} \cdot e \cdot a_0}{9.337\times10^{-46} \frac{F}{m}} = 8.51\times10^{52} \cdot e \cdot a_0 \cdot \frac{m}{F}$

$\frac{2 \cdot 3.14 \cdot 3\times10^{8} \frac{m}{s}}{0.6 \mu m \times \frac{1m}{10^6 \mu m}} = \omega$

$= 8.51\times10^{52} \cdot 1.60\times10^{-19} C \cdot 0.52917\times10^{-10} \frac{1}{m} \cdot \frac{m}{F}$ $= 3.14\times10^{15} \frac{1}{s}$

$\times \frac{1 Farad}{96500 C \cdot mol^{-1}} \times \frac{1 mol}{6.022\times10^{23}}$

$\boxed{= 1.2399\times10^{-5}}$

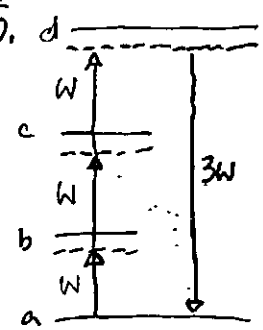
$\frac{c}{\omega} = \frac{\lambda}{2\pi}$

$\alpha = \chi^{(1)} \cdot \omega / c = 1.2399\times10^{-5} \cdot 3.14\times10^{15} \frac{1}{\cancel{s}} \frac{\cancel{s}}{2.998\times10^{8} m} = \boxed{129.86 \frac{1}{m}}$

$\frac{2\pi c}{\omega}$

$n(\omega) = \sqrt{1 + \chi^{(1)}(\omega)} = \sqrt{1 + 1.2399\times10^{-5}} = \boxed{1.000006199}$

3. Verify $\sigma_{max} = \frac{g_b}{g_a} \frac{\lambda^2}{2\pi}$; $g_b = 2J_b + 1$; $g_a = 2J_a + 1$; If $\gamma_{n'a} = \frac{1}{2}\Gamma_n$ goes into $\chi_{res}^{(1)} = \frac{i |\mu_{na}|^2}{\epsilon_0 \hbar \gamma_{n'a}}$

5. 

3rd Harmonic Gen.

$\rho^{(1)}(t) = \int_{-\infty}^{t} \frac{-i}{\hbar} [\hat{V}(t'), \hat{\rho}^{(1)}]_{nm} e^{(i\omega_{nm} + \gamma_{nm})(t'-t)} dt'$ where $\Gamma_n = \frac{\omega_{na}^3 |\mu_{na}|^2}{3\pi \epsilon_0 \hbar c^3}$; $\chi_{res}^{(1)} = \frac{i |\mu_{na}|^2}{\epsilon_0 \hbar} \frac{3\pi \epsilon_0 \hbar c^3}{2\omega_{na}^3 |\mu_{na}|^2}$

Electric Dipole Approx

$\alpha = \chi^{(1)'} \omega / c$
$= 6\pi i \left(\frac{\lambda}{2\pi}\right)^3 \cdot \frac{\omega}{c}$

$= \frac{3}{2} \pi i \left(\frac{\lambda}{2\pi}\right)^3$

$= 6\pi i \left(\frac{\lambda}{2\pi}\right)^3$

$\rho_{nm} = \rho_{nm}^{(0)} + \lambda \rho_{nm}^{(1)} + \lambda^2 \rho_{nm}^{(2)} + \ldots$

$\boxed{= 3 \frac{\lambda^2}{2\pi} = \frac{(2J_b+1)}{(2J_a+1)} \frac{\lambda^2}{2\pi}}$

$\hat{V}(t') = -\hat{\mu}\cdot E(t)$; $\rho_{nm}^{(0)} = 0$ for $n \neq m$; $\vec{E}(t) = \sum E(\omega_p) e^{-i\omega_p t}$

$$[\hat{V}(t), \hat{\rho}^{(0)}] = \sum [V(t)_{nv}\rho_{vm}^{(0)} - \rho_{nv}^{(0)}\cdot V(t)_{vm}] = -\sum[\mu_{nv}\cdot\rho_{vm}^{(0)} - \rho_{nv}^{(0)}\mu_{vm}]\cdot E(t)$$
$$= -(\rho_{mm}^{(0)} - \rho_{nn}^{(0)})\mu_{nm}\cdot\vec{E}(t)$$

$$\rho_{nm}^{(1)}(t) = \frac{i}{\hbar}(\rho_{mm}^{(0)} - \rho_{nn}^{(0)})\mu\cdot e^{-(i\omega_{nm}+\gamma)t} = \int_{-\infty}^{t} E(t') e^{(i\omega_{nm}+\gamma_{nm})t'}\,dt' = \frac{i}{\hbar}(\rho_{mm}^{(0)} - \rho_{nn}^{(0)})\mu_{nm}\cdot\sum E(\omega_p)\times e^{-(i\omega_{nm}+\gamma_{nm})t}\underbrace{\int_{-\infty}^{t} e^{[i(\omega_{nm}-\omega_p)+\gamma_{nm}]t'}\,dt'}$$

$$e^{-(i\omega_{nm}+\gamma_{nm})t}\left(\frac{e^{[i(\omega_{nm}-\omega_p)+\gamma_{nm}]t'}}{i(\omega_{nm}-\omega_p)+\gamma_{nm}}\right)\Bigg|_{-\infty}^{t}$$

$$\rho_{nm}^{(1)} = \hbar^{-1}(\rho_{mm}^{(0)} - \rho_{nn}^{(0)})\sum\frac{\mu_{nm}\cdot E(\omega_p)\cdot e^{-i\omega_p t}}{(\omega_{nm}-\omega_p) - i\gamma_{nm}}$$

Induced Dipole Moment:

$$\langle\mu(t)\rangle = \mathrm{Tr}(\rho^{(1)}\hat{\mu}) = \sum\rho_{nm}^{(1)}\mu_{mn} = \sum\hbar^{-1}(\rho_{mm}^{(0)} - \rho_{nn}^{(0)})\sum\frac{\mu_{mn}[\mu_{nm}\cdot E(\omega_p)]e^{-i\omega_p t}}{(\omega_{nm}-\omega_p) - i\gamma_{nm}} \;;\; \langle\mu(t)\rangle = \sum\langle\mu(\omega_p)\rangle e^{-i\omega_p t}$$

First order Polarization: $P(\omega_p) = N\langle\mu(\omega_p)\rangle = \epsilon_0\chi^{(1)}(\omega_p)\cdot\vec{E}(\omega_p)$

where $\chi^{(1)}(\omega_p) = \frac{N}{\epsilon_0\hbar}\sum_{nm}(\rho_{mm}^{(0)} - \rho_{nn}^{(0)})\frac{\mu_{nm}\mu_{mn}}{(\omega_{nm}-\omega_p) - i\gamma_{nm}}$

$-\omega_{nm} = \omega_{mn}$; $\gamma_{nm} = \gamma_{mn}$

$$\chi^{(1)}(\omega_p) = \frac{N}{\epsilon_0\hbar}\sum\rho_{mn}^{(0)}\left[\frac{\mu_{mn}\mu_{nm}}{(\omega_{nm}-\omega_p) - i\gamma_{nm}} + \frac{\mu_{nm}^{i}\mu_{mn}^{j}}{(\omega_{nm}+\omega_p) + i\gamma_{nm}}\right]$$

$\rho_{nn}^{(0)} = 1$; $\rho_{mm}^{(0)} = 0$ for $m \neq a$ ..

$$\chi_{ij}^{(1)}(\omega_p) = \frac{N}{\epsilon_0\hbar}\frac{\mu_{an}\mu_{na}}{(\omega_{na}-\omega_p) - i\gamma_{m}} = \frac{N}{\epsilon_0\hbar}\mu_{an}^{i}\mu_{na}^{j}\underbrace{\frac{(\omega_{na}-\omega_p) + i\gamma_{m}}{(\omega_{na}-\omega_p)^2 + \gamma_{nm}^2}}_{\text{Lorentzian Line shape}}$$

FWHM 2γ

<u>Density Matrix Calculation</u>

<u>Second-order susceptibility</u>

- $\rho_{nm}^{(2)} = e^{-(i\omega_{nm}+\gamma_{nm})t}\int_{-\infty}^{t}\frac{-i}{\hbar}[V,\hat{\rho}^{(1)}]_{nm}e^{(i\omega_{nm}+\gamma_{nm})t'}\,dt'$

where $[V,\rho^{(1)}]_{nm} = -\sum(\mu_{nv}\rho_{vm}^{(1)} - \rho_{nv}^{(1)}\mu_{vm})\cdot E(t)$

- $\rho_{vm}^{(1)} = \hbar^{-1}(\rho_{mm}^{(0)} - \rho_{vv}^{(0)})\sum\frac{\mu_{vm}\cdot E(\omega_p)}{(\omega_{vm}-\omega_p) - i\gamma_{vm}}e^{-i\omega_p t}$

$\rho_{vm}^{(1)} = \hbar^{-1}(\rho_{vv}^{(0)} - \rho_{nn}^{(0)})\sum\frac{\mu_{nv}\cdot E(\omega_p)(\omega_q)}{(\omega_{nv}-\omega_p) - i\gamma_{nv}}e^{-i\omega_p t}$

$E(t) = \sum E(\omega_q)e^{-i\omega_q t}$

- $[V,\rho^{(1)}]_{nm} = -\hbar^{-1}\sum(\rho_{mm}^{(0)} - \rho_{vv}^{(0)})\times\sum\frac{[\mu_{nv}\cdot E(\omega_q)][\mu_{vm}\cdot E(\omega_p)]}{(\omega_{vm}-\omega_q) - i\gamma_{vm}}e^{-i(\omega_p+\omega_q)t}$
$+ \hbar^{-1}\sum(\rho_{vv}^{(0)} - \rho_{nn}^{(0)})\times\sum\frac{[\mu_{nv}\cdot E(\omega_p)][\mu_{vm}\cdot E(\omega_q)]}{(\omega_{nv}-\omega_p) - i\gamma_{nv}}e^{-i(\omega_p+\omega_q)t}$

- $\rho_{nm}^{(2)} = \sum\sum e^{-i(\omega_p+\omega_q)t}$
$$\times\left\{\frac{\rho_{mm}^{(0)} - \rho_{vv}^{(0)}}{\hbar^2}\frac{[\mu_{nv}\cdot E(\omega_q)][\mu_{vm}\cdot E(\omega_p)]}{[(\omega_{nm}-\omega_p-\omega_q) - i\gamma_{nm}][(\omega_{vm}-\omega_p) - i\gamma_{nv}]} - \frac{\rho_{vv}^{(0)} - \rho_{nn}^{(0)}}{\hbar^2}\frac{[\mu_{nv}\cdot E(\omega_q)][\mu_{vm}\cdot E(\omega_q)]}{[(\omega_{nm}-\omega_p-i\gamma_{nm})][(\omega_{nv}-\omega_p) - i\gamma_{nv}]}\right\} = \boxed{\sum\sum K_{nm}\, e^{-i(\omega_p+\omega_q)t}}$$

- $\langle\bar{\mu}\rangle = \sum_{nm}\rho_{nm}\mu_{mn} = \sum\langle\mu(\omega_r)\rangle e^{-i\omega_r t}$

$\langle\mu(\omega_p+\omega_q)\rangle = \sum_{nmv}\sum_{(pq)}K_{nmv}\mu_{mn}$; $P^{(2)}(\omega_p+\omega_q) = \epsilon_0\sum_{jk}\sum_{(pq)}\chi_{ijk}^{(2)}(\omega_p+\omega_q,\omega_q,\omega_p)E(\omega_q)E_k(\omega_p)$

Third order Susceptibility: $\rho_{nm}^{(3)} = e^{-(i\omega_{nm}+\gamma_{nm})t} \cdot \int_{-\infty}^{t} \frac{-i}{\hbar}[\hat{V}, \hat{\rho}^{(2)}]_{nm} e^{(i\omega_{nm}+\gamma_{nm})t'} dt'$

$$[V, \rho^{(2)}]_{nm} = -\sum_{v} (\mu_{nv} \cdot \rho_{vm}^{(2)} - \rho_{nv}^{(2)} \mu_{vm}) \cdot \tilde{E}(t)$$

$$\rho_{vm}^{(2)} = \sum_{l} \sum_{pq} K_{vml} \cdot e^{-i(\omega_p+\omega_q)t} \quad ; \quad E(t) = \sum E(\omega_r) e^{-i\omega_r t}$$

$$[\hat{V}, \rho^{(2)}]_{nm} = -\sum_{vl} \sum_{pqr} [\mu_{nv} \cdot E(\omega_r)] K_{vml} e^{-i(\omega_p+\omega_q+\omega_r)t}$$

$$+ \sum_{vl} \sum_{pqr} [\mu_{vm} \cdot E(\omega_r)] K_{nvl} \cdot e^{-i(\omega_p+\omega_q+\omega_r)t}$$

$$\rho_{nm}^{(3)} = \frac{1}{\hbar} \sum \sum \left\{ \frac{[\mu_{nv} \cdot E(\omega_r)] K_{vml}}{(\omega_{nm}-\omega_p-\omega_q-\omega_r) - i\gamma_{nm}} - \frac{[\mu_{vm} \cdot E(\omega_r)] \cdot K_{nvl}}{(\omega_{nm}-\omega_p-\omega_q-\omega_r) - i\gamma_{nm}} \right\} e^{-i(\omega_p+\omega_q+\omega_r)t}$$

Polarization Oscillation frequency $(\omega_p+\omega_q+\omega_r)$: $P(\omega_p+\omega_q+\omega_r) = N\langle \mu(\omega_p+\omega_q+\omega_r)\rangle$

$$\langle \tilde{\mu} \rangle = \sum \rho_{nm} \mu_{mn} \equiv \sum \langle \mu(\omega_s) \rangle e^{-i\omega_s t}$$

↳ Expressed as a nonlinear Polarization term:

$$P_K(\omega_p+\omega_q+\omega_r) = \epsilon_0 \sum \sum \chi_{Kjih}^{(3)}(\omega_p+\omega_q+\omega_r, \omega_r, \omega_q, \omega_p) \times E_j(\omega_r) E_i(\omega_q) E_h(\omega_p)$$

$$\chi_{Kjih}^{(3)}(\omega_p+\omega_q+\omega_r, \omega_r, \omega_q, \omega_p) = \frac{N}{\epsilon \hbar^3} P_I \sum \Bigg\{ \frac{(\rho_{mm}^{(0)} - \rho_{ll}^{(0)}) \mu_{mn}^{K} \mu_{nv}^{j} \mu_{vl}^{i} \mu_{lm}^{h}}{[(\omega_{nm}-\omega_p-\omega_q-\omega_r) - i\gamma_{nm}][(\omega_{vm}-\omega_p-\omega_q) - i\gamma_{vm}][(\omega_{lm}-\omega_p) - i\gamma_{lm}]}$$

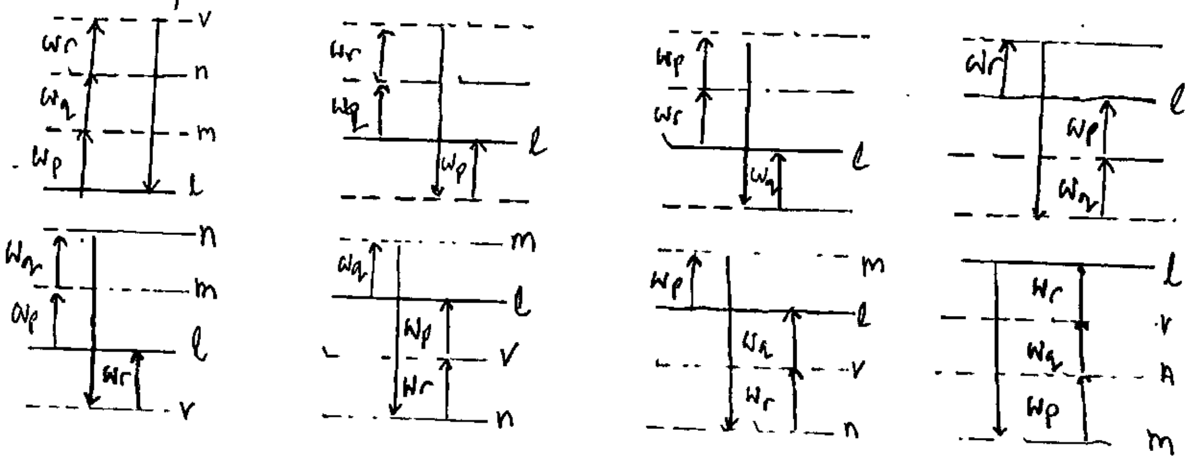
$$- \frac{(\rho_{ll}^{(0)} - \rho_{vv}^{(0)}) \mu_{mn}^{K} \cdot \mu_{nv}^{j} \mu_{lm}^{j} \mu_{vl}^{h}}{[(\omega_{nm}-\omega_p-\omega_q-\omega_r) - i\gamma_{nm}][(\omega_{nv}-\omega_p-\omega_q) - i\gamma_{nv}][(\omega_{lv}-\omega_p) - i\gamma_{lv}]}$$

$$- \frac{(\rho_{vv}^{(0)} - \rho_{ll}^{(0)}) \mu_{mn}^{K} \mu_{vm}^{j} \mu_{nl}^{i} \mu_{lv}^{h}}{[(\omega_{nm}-\omega_p-\omega_q-\omega_r) - i\gamma_{nm}][(\omega_{nv}-\omega_p-\omega_q) - i\gamma_{nv}][(\omega_{lv}-\omega_p) - i\gamma_{lv}]}$$

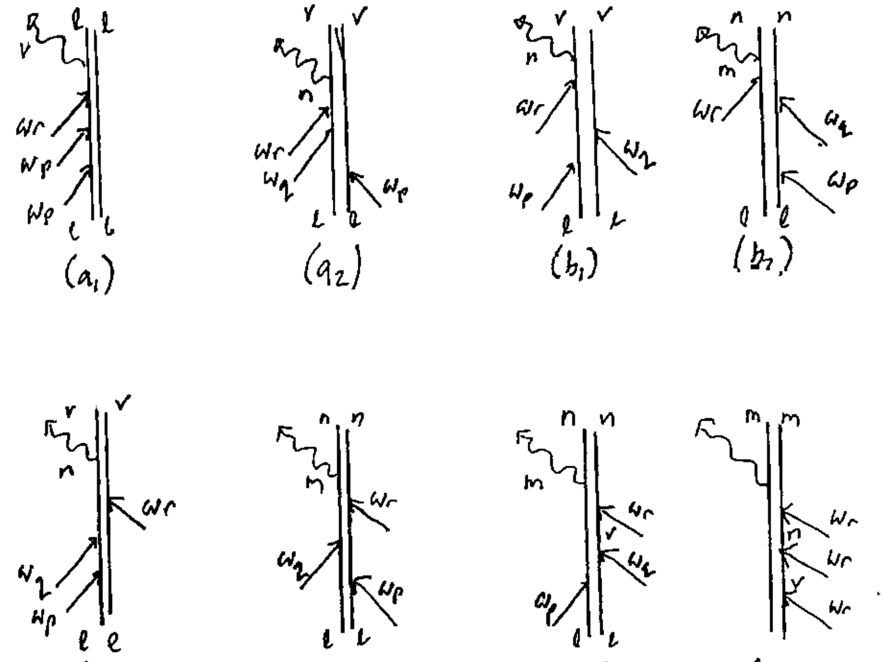
$$+ \frac{(\rho_{ll}^{(0)} - \rho_{nn}^{(0)}) \mu_{mn}^{K} \mu_{vm}^{j} \mu_{lv}^{i} \mu_{nl}^{K}}{[(\omega_{vn}+\omega_p+\omega_q+\omega_r) + i\gamma_{vn}][(\omega_{nl}-\omega_p-\omega_q) - i\gamma_{nl}][(\omega_{ml}-\omega_p) - i\gamma_{ml}]} \Bigg\}$$

All permutations of resonance structure:

Where P_l = Permutation Operator of l.



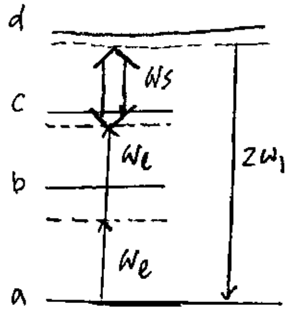
7.



(a₁) (a₂) (b₁) (b₂)

(c₁) (c₂) (d₁) (d₂)

9. $\chi^{(1)}(2\omega_1+\omega_s)$ & $\chi^{(3)}(\omega_{sum}=\omega_1+\omega_1+\omega_s)$, $I(\omega_s)\ll 1$



$\boxed{\text{Electromagnetically Induced Transparency}}$ a technique to render a material system transparent to resonant laser radiation.

Described in (1990), Harris et Al.

$$\psi(r,t)=C_a'(t)\cdot u_a(r)e^{-i\omega_a t}+C_d'(t)\cdot u_d(r)e^{-i\omega_d t}+C_c'(t)u_c(r)e^{-i\omega_c t}$$

$$\hookrightarrow = C_a(t)\cdot u_a(r)+C_d(t)u_d(r)e^{-i\omega_4 t}+C_c(t)u_c(r)e^{-i(\omega_4-\omega_s)t}$$

Schrödinger Equation: $i\hbar\frac{\partial\psi}{\partial t}=H\psi$; where $\hat{H}=H_0+\hat{V}$

$$\hat{V}=-\hat{\mu}\left(E_4e^{-i\omega_4 t}+E_s^*e^{i\omega_s t}\right)$$

$$i\hbar\left[\dot{C}_a u_a+\dot{C}_d u_d e^{-i\omega_4 t}-i\omega_4 C_d u_d e^{-i\omega_4 t}+\dot{C}_c\cdot u_c\cdot e^{-i(\omega_4-\omega_s)t}-i(\omega_4-\omega_s)C_c u_c e^{-i(\omega_4-\omega_s)t}\right]$$

$$=C_a\hbar\omega_a u_a+C_d\hbar\omega_d u_d e^{-i\omega_4 t}+C_c\hbar\omega u_c e^{-i(\omega_4-\omega_s)t}$$

$$+\hat{V}\left[C_a u_a+C_d u_d e^{-i\omega_4 t}+C_c u_c e^{-i(\omega_4-\omega_s)t}\right]$$

$$i\hbar\dot{C}_a=\hbar\omega_a\cdot C_a+V_{ad}\cdot C_d e^{-i\omega_4 t}$$

$$i\hbar\left[\dot{C}_d e^{-i\omega_4 t}-i\omega_4 C_d e^{-i\omega_4 t}\right]=\hbar\omega_d C_d e^{-i\omega_4 t}+V_{da}\cdot C_a + V_{dc}C_c e^{-i(\omega_4-\omega_s)t}$$

$$i\hbar\left[\dot{C}_c e^{-i(\omega_4-\omega_s)t}-i(\omega_4-\omega_s)C_c e^{-i(\omega_4-\omega_s)t}\right]=\hbar\omega_c C_c e^{-i(\omega_4-\omega_s)t}+V_{cd}\cdot C_d e^{-i\omega_4 t}$$

$V_{ad}^*=V_{da}=-\mu_{da}\cdot E_4 e^{-i\omega_4 t}$; $\hbar\omega_a\to\hbar\omega_{aa}=0$; $\hbar\omega_d\to\hbar\omega_{da}$; $\hbar\omega_c\to\hbar\omega_{ca}$

$V_{dc}^*=V_{cd}=-\mu_{cd}E_s^*e^{i\omega_s t}$;

$\Omega=\mu_{da}\cdot E_4/\hbar$ and $\Omega_s^*=\mu_{cd}\cdot E_s^*/\hbar$; $\dot{C}_a=iC_d\Omega^*$

$$\dot{C}_d-i\delta C_d=iC_a\Omega+iC_c\Omega_s$$

$$\dot{C}_c-i(\delta-\Delta)C_c=iC_d\Omega_s^*$$

$$\delta\equiv\omega_4-\omega_{da}\ ;\ \Delta=\omega_s-\omega_{dc}$$

$$C_j=C_j^{(0)}+\lambda C_j^{(1)}+\lambda^2C_j^{(2)}+\cdots$$

$$\dot{C}_a^{(0)} + \lambda \dot{C}_a^{(1)} = i C_d^{(0)} \cdot \lambda \Omega^* + i C_d^{(1)} \lambda^2 \Omega^*$$

$$(\dot{C}_d^{(0)} - i\delta C_d^{(0)}) + \lambda(\dot{C}_d^{(1)} - i\delta C_d^{(1)}) = i C_\alpha^{(0)} \Omega \lambda + i C_\alpha^{(1)} \Omega \lambda^2 + i C_c^{(0)} \Omega_i \lambda + i C_c^{(1)} \Omega_s \lambda^2$$

$$[\dot{C}_c^{(0)} - i(\delta-\Delta) C_c^{(0)}] + \lambda[\dot{C}_c^{(1)} - i(\delta-\Delta) C_c^{(1)}] = i C_d^{(0)} \lambda \Omega_s^* + i C_d^{(1)} \lambda^2 \Omega_s^*$$

$$\dot{C}_a^{(0)} = 0 ; \quad \dot{C}_d^{(0)} - i C_d^{(0)} = i C_a^{(0)} \Omega + i C_c^{(0)} \Omega d ; \quad \dot{C}_c^{(0)} - i(\delta-\Delta) C_c^{(0)} = i C_d^{(0)} \Omega_s .$$

$$C_a^{(0)} = 1 ; \quad C_d^{(0)} = C_c^{(0)} = 0$$

$$C_a^{(1)} = 0 ; \quad \dot{C}_d^{(1)} - i\delta C_d^{(1)} = i\Omega + i C_c^{(0)} \Omega_s ; \quad \dot{C}_c^{(1)} - i(\delta-\Delta) C_c^{(1)} = i C_A^{(0)} \Omega_s^*$$

$$\dot{C}_d - i\delta C_d = i\Omega + i\Omega_s C_c ; \quad \dot{C}_c - i(\delta-\Delta) C_c = i\Omega_s^* C_d .$$

$$0 = \Omega + \delta C_d + \Omega_s C_c$$

$$0 = \Omega_s^* \cdot C_d + (\delta-\Delta) C_c \quad ; \quad C_d = \frac{\Omega(\delta-\Delta)}{|\Omega_s|^2 - \delta(\delta-\Delta)}$$

Induced Dipole Moment: $\tilde{p} = \langle \psi | \mu | \psi \rangle = \langle \psi^{(0)} | \hat{\mu} | \psi^{(1)} \rangle + \langle \psi^{(1)} | \hat{\mu} | \psi^{(0)} \rangle$

$$= \langle a | \hat{\mu} | d \rangle C_d e^{-i\omega_4 t} + c.c. = \mu_{ad} C_d e^{-i\omega_4 t} + c.c.$$

$$= \frac{\mu_{ad} \Omega(\delta+\Delta)}{|\Omega_s|^2 - (\delta+\Delta)\delta} \quad \text{"Dipole Moment Amplitude"}$$

$$\chi_{eff}^{(1)} = \frac{N |\mu_{da}|^2}{\epsilon_0 \hbar} \frac{(\delta+\Delta)}{|\Omega_s|^2 - (\delta+\Delta)\delta}$$

Polarization: $P = N p \equiv \epsilon_0 \chi_{eff}^{(1)} E$

Induced Dipole Moment

susceptibility

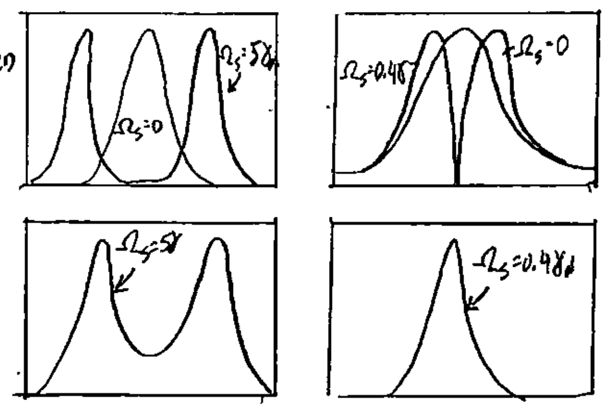
$$p = \frac{\mu_{ad} \Omega (\delta + \Delta + i\gamma_c)}{|\Omega_s|^2 - (\delta + i\gamma_d)(\delta + \Delta + i\gamma_c)}$$

$$\chi_{eff}^{(1)} = \frac{N}{\epsilon_0 \hbar} \frac{|\mu_{da}|^2 (\delta + \Delta + i\gamma_c)}{|\Omega_s|^2 - (\delta + i\gamma_d)(\delta + \Delta + i\gamma_c)}$$

On resonance, $\chi_{eff}^{(1)} = \frac{N}{\epsilon_0 \hbar} \frac{|\mu_{da}|^2 i\gamma_c}{|\Omega|^2 + \gamma_c \gamma_d}$

$\mathrm{Im}\,\chi^{(1)}$

$[\chi^{(3)}]$



$\delta/\gamma_d = (\omega_4 - \omega_{da})/\gamma_d \qquad \delta/\gamma_d = (\omega_4 - \omega_{da})/\gamma_d$

$$\dot{C}_a = i C_b \Omega_{ba}^*$$

$$\dot{C}_b - i C_b \delta_1 = i C_a \Omega_{ba} + i C_c \Omega_{cb}^*$$

$$\dot{C}_c - i C_c \delta_2 = i C_b \Omega_{cb} + i C_d \Omega_{dc}^*$$

$$\dot{C}_d - i C_d (\delta_2 + \Delta) = i C_c \Omega_{dc}$$

$$C_b = -\Omega_{ba}/\delta_1$$

Sum-Frequency Generation:

$$\psi(r,t) = C_a(t) u_a(r) + C_b(t) u_b(r) e^{-i\omega t} + C_c(t) u_c(r) e^{-i2\omega t} + C_d(t) u_d(r) e^{-i(2\omega+\omega)t}$$

$$i\hbar[\dot{C}_a u_a + \dot{C}_b u_b e^{-i\omega t} - i\omega C_b u_b e^{-i\omega t} + \dot{C}_c u_c e^{-i2\omega t} - 2i\omega C_c u_c e^{-i2\omega t}$$
$$+ \dot{C}_a u_d e^{-i(2\omega+\omega_c)t} - i(2\omega+\omega_c) C_d u_d e^{-i(2\omega+\omega_c)t}]$$

$$= \hbar\omega_{ba} C_b u_b e^{-i\omega t} + \hbar\omega_{ca} C_c u_c e^{-i2\omega t} + \hbar\omega C_d u_d e^{-i(2\omega+\omega_c)t}$$
$$+ \hat{V}[C_a u_a + C_b u_b e^{-i\omega t} + C_c u_c e^{-i2\omega t} + C_d u_d e^{-i(2\omega+\omega)t}]$$

$$i\hbar \dot{C}_a = V_{ab} C_b e^{-i\omega t}$$

$$i\hbar(\dot{C}_c - i2\omega C_c) e^{-i2\omega t} = \hbar\omega_{ca} C_c e^{-i2\omega t} + V_{cb} C_b e^{-i\omega t} + V_{cd} C_d e^{-i(2\omega+\omega)t}$$

$$i\hbar[\dot{C}_d - i(2\omega+\omega_c) C_d] e^{-(2\omega+\omega_c)t} = \hbar\omega_{da} C_d e^{-(2\omega+\omega_c)t} + V_{dc} C e^{-i2\omega t}$$

$$V_{ba} = V_{ab}^* = -\mu_{ba} \cdot E e^{-i\omega t} = -\hbar\Omega_{ba} e^{-i\omega t}$$

$$V_{cb} = V_{bc}^* = -\mu_{cb} \cdot E e^{-i\omega t} = -\hbar\Omega_{cb} e^{-i\omega t}$$

$$V_{dc} = V_{cd}^* = -\mu_{dc} E e^{-i\omega t} = -\hbar\Omega_{dc} e^{-i\omega_s t}$$

$$-C_c = \frac{C_b \Omega_{cb}}{\delta_2} + \frac{C_d \Omega_{dc}^*}{\delta_2} \; ; \; C_d = \frac{-C_c \Omega_{dc}}{(\delta_2+\Delta)} = \frac{-\Omega_{ba}\Omega_{cb}\Omega_{dc}}{\delta_1\delta_2(\delta_2+\Delta)} + C_d \frac{|\Omega_{dc}|^2}{(\delta_2+\Delta)\delta_2}$$

$$= \frac{\Omega_{dc}\Omega_{cb}\Omega_{ba}}{\delta_1\delta_2(\delta_2+\Delta)}\left[1 - \frac{|\Omega_{dc}|^2}{\delta_2(\delta_2+\Delta)}\right]^{-1} = \frac{\Omega_{dc}\Omega_{cb}\Omega_{ba}}{\delta_1[\delta_2(\delta_2+\Delta)-|\Omega_{dc}|^2]}$$

$$\tilde{p} = \langle \psi|\hat{\mu}|\psi\rangle = \langle u_a|\hat{\mu}|C_d u_d\rangle + c.c. = \mu_{ad}\cdot C_d + c.c.$$ $\boxed{\text{Induced Electric Dipole}}$

$$p = \frac{-\mu_{ad}\Omega_{dc}\Omega_{cb}\Omega_{ba}}{\delta_1[\delta_2(\delta_2+\Delta)-|\Omega_{dc}|^2]} = \frac{-\mu_{ad}\mu_{dc}\mu_{cb}\mu_{ba}E^2\cdot E_S}{\hbar^3\cdot\delta_1[\delta_2(\delta_2+\Delta)-|\Omega_{dc}|^2]} \equiv \frac{3\epsilon_0\chi^{(3)}E^2 E_c}{N}$$

$$\chi^{(3)} = \frac{-N\mu_{ad}\mu_{dc}\mu_{cb}\mu_{ba}}{3\epsilon_0\hbar\delta_1[\delta_2(\delta_2+\Delta)-|\Omega_{dc}|^2]} \doteq \frac{-N\mu_{ad}\mu_{dc}\mu_{cb}\mu_{ba}}{3\epsilon_0\hbar\delta_1[(\delta_1+i\gamma_c)(\delta_2+\Delta+i\gamma_d)-|\Omega_{dc}|^2]}$$

1: Derive 4.1.19 for n to be a complex quantity $\tilde{n}_0$.

$$n = \underbrace{n_0}_{\text{"Usual" "Weak"}} + \underbrace{\bar{n}_2\langle\tilde{E}^2\rangle}_{\text{"Second order refractive index" with time average}} \; ; \; \tilde{E}(t) = E(\omega)e^{-i\omega t} + c.c. \; ; \; \langle\tilde{E}(t)^2\rangle = 2E(\omega)E(\omega)^* = 2|E(\omega)|^2$$

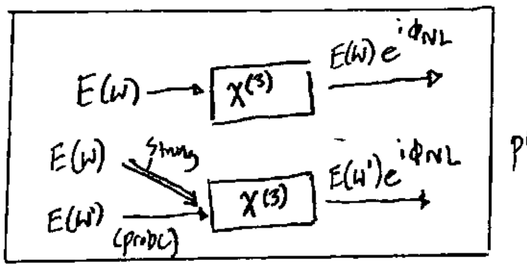
$$n = n_0 + 2\bar{n}_2|E(\omega)|^2$$

$$P^{NL}(\omega) = 3\epsilon_0\chi^{(3)}(\omega=\omega+\omega-\omega)|E(\omega)|^2E(\omega)$$

$\boxed{\text{Optical Kerr Effect}}$ $\longrightarrow$ $P^{TOT}(\omega) = \epsilon_0\chi^{(1)}E(\omega) + 3\epsilon_0\chi^{(3)}|E(\omega)|^2E(\omega) \equiv \epsilon_0\chi_{eff}E(\omega)$

$$\chi_{eff} = \chi^{(1)} + 3\chi^{(3)}|E(\omega)|^2 \; ; \; \text{Generally } n^2 = 1+\chi_{eff}$$

$$[n_0 + 2\bar{n}_2|E(\omega)|^2]^2 \doteq 1+\chi^{(1)}+3\chi^{(3)}|E(\omega)|^2$$



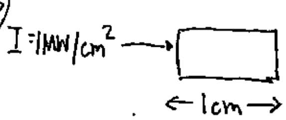
$$n_0 = \sqrt{1+\chi_i^{(1)}} \; ; \; \bar{n}_2 = \frac{3\chi^{(3)}}{4n_0}$$

$$P^{NL}(\omega') = 6\epsilon_0\chi^{(3)}(\omega'=\omega'+\omega-\omega)|E(\omega)|^2E(\omega')$$

$$n = n_0 + 2\bar{n}_2^{(cross)}|E(\omega)|^2$$

$$n_2^{(cross)} = \frac{3\chi^{(3)}}{2n_0}$$

Hch.p. (3)



$$(-1\le\sigma\le1) \; ; \; \hat{\epsilon} = \frac{\hat{x}+i\sigma\hat{y}}{(1+\sigma^2)^{1/2}}$$

$$I = 2n_0'\cdot\epsilon_0\cdot c|E(\omega)|^2$$

$n = n_0 + n_2 I$ $\boxed{\text{Intensity Dependent Refractive Index}}$

$$I = 2n_0'\epsilon_0 c|E(\omega)|^2$$

$$n = n' + in''$$

$$= 2(n_0' + in_0'')\epsilon_0 c|E(\omega)|^2$$

$$2\bar{n}_2|E(\omega)|^2 = n_2 I \; ; \; n_2 = \frac{\bar{n}_2}{(n_0'+in_0'')\epsilon_0 c}$$

$$= \frac{3}{4n_0(n_0'+in_0'')\epsilon_0 c}\chi^{(3)}$$

Propagation through Isotropic Nonlinear Mediums:

$$E = E_+\hat{\sigma}_+ + E_-\hat{\sigma}_- \; ; \; \hat{\sigma}_\pm = \frac{\hat{x}\pm i\hat{y}}{\sqrt{2}} \; ; \; \hat{\sigma}_+ = \text{Left hand circular}$$

Decomposition Identities: $\hat{\sigma}_- =$ Right hand circular polarization

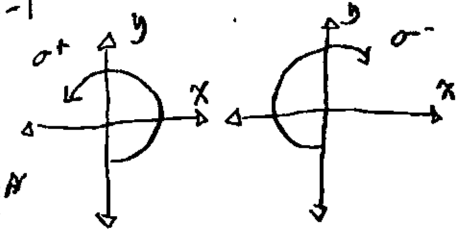
$\hat{\sigma}_\pm^* = \hat{\sigma}_\mp$, $\hat{\sigma}_\pm\cdot\hat{\sigma}_\pm = 0$, $\hat{\sigma}_\pm\cdot\hat{\sigma}_\mp = 1$; Products of $\boxed{E^*\cdot E}$

$$= (E_+^*\sigma_+^* + E_-^*\sigma_-^*)(E_+\sigma_+ + E_-\sigma_-)$$

$$= E_+^*E_+ + E_-^*E_- = |E_+|^2 + |E_-|^2$$

$$E\cdot E = (E_+\sigma_+ + E_-\sigma_-)^2$$

$$= 2E_+E_-$$



$$P^{NL} = \epsilon_0 A(|E_+|^2+|E_-|^2)\vec{E} + \epsilon_0 B(E_+E_-)E^*$$

$P^{NL} = P_+\hat{\sigma}_+ + P_-\hat{\sigma}_-$; $P_+ = \epsilon_0 A(|E_+|^2+|E_-|^2)E_+ + \epsilon_0 B(E_+E_-)E_-^* = \epsilon A|E_+|^2E_+ + \epsilon_0(A+B)|E_-|^2E_+$

$P_- = \epsilon_0 A|E_-|^2E_- + \epsilon_0(A+B)|E_+|^2E_-$

$$\nabla^2 E(z,t) = \frac{\epsilon^{(1)}}{c^2}\frac{\partial^2 E(z,t)}{\partial t^2} + \frac{1}{\epsilon_0 c^2}\frac{\partial^2}{\partial t^2}P^{NL} \quad \boxed{\text{Wave Equation}}$$

$P_\pm = \epsilon_0 \chi^{NL}_\pm E_\pm$

$\chi^{NL}_\pm = A|E_\pm|^2 + (A+B)|E_\mp|^2$

$E(z,t) = E\exp(-i\omega t) + c.c.$; $P(z,t) = P\exp(-i\omega t) + c.c.$

To determine angle of rotation...

$\nabla^2 E_\pm(z,t) = \frac{\epsilon_\pm^{(eff)}}{c^2}\frac{\partial^2 E_\pm(z,t)}{\partial t^2}$; $\epsilon_\pm^{(eff)} = \epsilon^{(1)} + \chi^{NL}_\pm$

$E(z) = E_+\hat{\sigma}_+ + E_-\hat{\sigma}_- = A_+e^{in_+\omega z/c}\hat{\sigma}_+ + A_-e^{in_-\omega z/c}\hat{\sigma}_-$

$= \left(A_+e^{i(1/2)\Delta n\omega z/c}\hat{\sigma}_+ + A_-e^{-(1/2)\Delta n\omega z/c}\hat{\sigma}_-\right)e^{i(1/2)(n_++n_-)\omega z/c}$

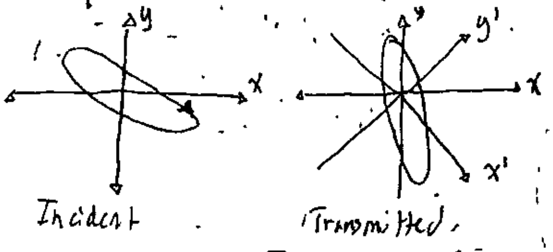
$\theta = \frac{1}{2}\Delta n\frac{\omega}{c}z$; $k_m = \frac{1}{2}(n_++n_-)\omega/c$

$E(z) = (A_+\hat{\sigma}e^{i\theta} + A_-\sigma_-e^{-i\theta})e^{ik_m z}$

$n_\pm^2 = n_0^2 + \chi^{NL}_\pm = n_0^2 + [A|E_\pm|^2 + (A+B)|E_\mp|^2]$

$= n_0^2\left(1 + \frac{1}{n_0^2}[A|E_\pm|^2 + (A+B)|E_\mp|^2]\right)$

$\Delta n \equiv n_+ - n_- = \frac{B}{2n_0}(|E_-|^2 - |E_+|^2)$ "Difference of refractive index"



$\hat{\sigma}_\pm e^{\pm i\theta} = \frac{\hat{x}' \pm i\hat{y}'}{\sqrt{2}}$; $x' = x\cos\theta - y\sin\theta$

$y' = x\sin\theta + y\cos\theta$

$\delta n_{circ} = \frac{1}{2n_0}A|E|^2$

$\delta n_{lin} = \frac{1}{2n_0}(A + \frac{1}{2}B)|E|^2$

$\frac{\delta n_{linear}}{\delta n_{circular}} = \begin{cases} 4 & : \text{For Molecular orientation} \\ \frac{3}{2} & : \text{Nonresonant electronic Nonlinearities} \\ 1 & : \text{Electrostriction} \end{cases}$

$x' = x\cos\theta - y\sin\theta$; $y' = x\sin\theta + y\cos\theta$

$x\cos\theta - y\sin\theta + ix\sin\theta + iy\cos\theta$

$e^{x\theta} - e^{-x\theta}$

5. Molecular orientation Effect : $\langle\alpha_{ij}\rangle = \alpha\delta_{ij} + \gamma_{ij}$; to 4.4.30

$\alpha = \frac{1}{3}(a+b+c)$; $\gamma_{ij} = C\sum(3\delta_{ik}\delta_{jl} - \delta_{ij}\delta_{kl})\overline{E_k^{loc}(t)E_l^{loc}(t)}$

$C = \frac{(a-b)^2+(b-c)^2+(a-c)^2}{90kT}$

$\langle\alpha_{ij}\rangle = \frac{1}{3}(a+b+c)\delta_{ij} + \left[\frac{(a-b)^2+(b-c)^2+(a-c)^2}{9kT}\right]\sum(3\delta_{ik}\delta_{jl} - \delta_{ij}\delta_{kl})\underbrace{\overline{E_k^l(t)E_l^l(t)}}_{\text{Lorentz Local Field}}$

7. $F_M = (-e/c)\vec{v}\times\vec{B}$

a) $E = \hat{x}(E_0e^{i(kz-\omega t)} + c.c.)$

$F_{EM} = -e(E_0e^{-i\omega t} + c.c.)\left[\hat{x}\left(1 - \frac{\dot{z}}{c}\right) + \hat{z}\left(\frac{\dot{x}}{c}\right)\right]$

$k = k\hat{k} = k_x\hat{x} + k_y\hat{y} + k_z\hat{z}$

$\nabla\times E = -\frac{\partial B}{\partial t}$; $B = -\int\nabla\times E\,dt$ .... $\frac{\sqrt{\epsilon}\hat{k}\times E}{\sqrt{\mu}}$, $\frac{k\times E}{n}$; $\nabla\times E_0e$ .... $i\hat{k}\times E$

(1) $\vec{B} = \frac{c}{i\omega}\vec{B}$

$= -\int\nabla\times\hat{x}(E_0e^{i(kz-\omega t)} + c.c.)dt = -\frac{\nabla\times E}{-i\omega}$ "Magnetic field of a plane"

$B = -\nabla\times E\,dt$

$= -\nabla\times\int\hat{x}(E_0e^{i(kz-\omega t)} + c.c.)dt$

$= -\frac{\nabla\times E}{-i\omega} = \frac{E_0e^{i(kz-\omega t)} + \hat{x}ikE_0e^{i(kz-\omega t)}}{i\omega}$

$= \frac{1}{i\omega}\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)\hat{x}\left(E_0e^{i(k_x x + k_y y + k_z z)}e^{-i\omega t}\right)$

$= \frac{1}{i\omega}\left[E_0e^{i(kz-\omega t)} + \hat{x}ikE_0e^{i(kz-\omega t)}\right]$

$= \frac{1}{i\omega}[1 + \hat{x}ik]E_0e^{i(kz-\omega t)}$

$= -\left(\frac{e}{c}\right)[1 + \hat{x}ik]\nabla\times\frac{E}{i\omega} = -eE_0e^{-i\omega t}[1 + \hat{x}ik]e^{ikz} = -(eE_0e^{-i\omega t})\left[\frac{1}{c} + \ldots\right]$

$= \frac{1}{i\omega}\left[1 + \hat{x} i k_x + i k_y + i k_z\right] E_0 e^{-i(k}$

(b) $\chi^{(2)}(2\omega)$

9. Beam Waist Radius: $w(z) = W_0\sqrt{1+(z/z_R)^2}$; Gaussian Beam: $E = E_0 e^{-x^2/w_0^2}$

Phase Change of an Optical Field: $\phi = \tilde{n} k_0 L = (n + n_2|E(x)|^2) k_0 L$

$= (n + n_2|E_0 e^{-x^2/w_0^2}|) k_0 L$

$L = 1\text{cm}$, $I = 10\text{GW/cm}^2$

~~$\phi = (n_0 + n_2 w_0^2 \sqrt{\pi}\, 10\text{GW/cm}^2) k_0\, 1\text{cm}$~~

$\phi = \int_{-\infty}^{\infty} (n_0 + n_2 |E_0 e^{-x^2/w_0^2}|) k_0 L\, dx$

$\boxed{= (n_0 + n_2 w_0^2 \sqrt{\pi} E_0 e^{-x^2/w_0^2}) k_0 L}$

Fused Silica:

$n_0 = 1.47$, $n_2 (m^2/W) = 3.67 \times 10^{-20}$

$\boxed{\phi = 1.47 + 3.67\times10^{-20} \cdot 10\text{GW/cm}^2 \cdot 1\text{cm} \cdot k_0}$

The accuracy of the measurement relies on n_0, and n_2 previous measurements.

Note: Dispersion Relation

$E(r,t) = E_0 e^{-i(\omega t - kx)}$

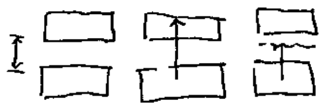
$\frac{\omega}{k} = \frac{c}{n} = \frac{c}{1 - \delta + i\beta}$

$k = \frac{\omega}{c}(1 - \delta + i\beta)$

$E(r,t) = E_0 e^{-i(\omega t - x(\frac{\omega}{c})(1-\delta+i\beta))}$

$= \underbrace{E_0 e^{-i\omega(t - r/c)}}_{\text{vacuum}} \cdot \underbrace{e^{-i(2\pi\delta/\lambda) r}}_{\phi\text{-shift}} \cdot \underbrace{e^{-(2\pi\beta/\lambda) r}}_{\text{decay}}$

11. $\frac{4d N \delta}{dt} = \frac{\kappa I}{\hbar\omega} = \frac{(N_c - N_c^{(0)})}{\tau_R}$



$N_c = N_c^{(0)} + \frac{\kappa I \tau_R}{\hbar\omega}$ Response time

Free-Electron Response:

$\epsilon(\omega) = \epsilon_b - \frac{\omega_p^2}{\omega(\omega + i/\tau)}$ ← Plasma frequency $\cong 1 + N\alpha(\omega) = 1 - \frac{Ne^2}{\epsilon_0 m \omega^2}$

Dielectric constant

1. $\bar{P} = \epsilon_0 \chi^{(1)} \bar{E} + \epsilon_0 \chi^{(2)} E^2 + \epsilon_0 \chi^{(3)} E^3 + \ldots$

$W = -\int_0^{\hat{E}} P(E')\,dE' = -\frac{1}{2}\chi^{(1)}E^2 - \frac{1}{3}\chi^{(2)}E^3 - \frac{1}{4}\chi^{(3)}E^3$

$n_0^2 = \epsilon_b - \frac{N_c(0)e^2}{\epsilon_0 m\omega(\omega + i/\tau)}$; $\omega_p^2 = N_c e^2/\epsilon_0 m$ ∝

$\underbrace{= W^{(2)} + W^{(3)} + W^{(4)} + \ldots}_{\text{Energy Stored in polarizing medium.}}$

$n = \sqrt{\epsilon_r \mu_r}$

$\mu_r = \mu/\mu_0$ "Relative Magnetic Permeability"

$\chi^{(n-1)} = -\frac{n W^{(n)}}{\epsilon_0 E^n}$

$= \frac{-1}{\epsilon_0 (n-1)!} \frac{\partial^n W}{\partial E^n}\Big|_{E=0}$

$n_2 = \frac{-N_c(0)e^2}{\epsilon_0 m\omega(\omega + i/\tau)}$

For the Hydrogen Atom: $\boxed{\frac{W}{2R} = -\frac{1}{2} - \frac{9}{4}\left(\frac{E}{E_{at}}\right)^2 - \frac{3555}{64}\left(\frac{E}{E_a}\right)^4}$

$n = n_0 + n_2 I$;

$\boxed{n_2 = \frac{\kappa \tau e^2}{2\epsilon_0 n_0 m \hbar \omega^3}}$

$\omega = \frac{-\frac{1}{2}E^2}{2R} - \frac{\frac{1}{3}\frac{-1}{\epsilon_0 n!}\frac{\partial^2 W}{\partial E}E^3}{2R} - \frac{\frac{1}{4}\frac{-1}{\epsilon_0(2)!}\frac{\partial^3 W}{\partial^3 E}E^4}{2R}$

$\frac{-\frac{1}{2} e^{6} \hbar^{2} (4\pi\epsilon_0)^3 \hbar^{4}}{\frac{2 \cdot \hbar e^4}{32\pi^2\epsilon_0^2 \hbar}} = \frac{-16 e m}{(4\pi\epsilon_0)^3} \quad ?\pi^2\epsilon_0^2 = \frac{-11 e m}{4\pi\epsilon_0 \hbar^2}$

3. $V=\frac{1}{2}(k_a x^2+k_b y^2+K_c z^2)+Axyz$; $E(t)=E_1 e^{-i\omega_1 t}+E_2 e^{-i\omega_2 t}+c.c.$; $\hat{V}=-\mu E=-c\hat{x}\hat{E}$

Oscillator Strength: $f_{na}=\frac{2m\omega_{na}|\mu_{na}|^2}{3\hbar e^2}$; Oscillator Strength Sum Rule: $\sum_n f_{na}=1$

Linear Susceptibility in terms of Oscillator Strength:

$$\chi^{(1)}(\omega)=\sum_n \frac{N f_{na} e^2}{2\epsilon_0 m\omega_{na}}\left[\frac{1}{(\omega_{na}-\omega)-i\gamma_{na}}+\frac{1}{(\omega_{na}+\omega)+i\gamma_{na}}\right]\sim\sum_n f_{na}\left[\frac{Ne^2/\epsilon_0 m}{\omega_{na}^2-\omega^2-2i\omega\gamma_{na}}\right]$$

Index or Refraction to Dielectric Constant: $n(\omega)=\sqrt{\epsilon^{(1)}(\omega)}=\sqrt{1+\chi^{(1)}(\omega)}$

Absorption Coefficient: $\alpha=2n''\omega/c$; $I(z)=I_0 e^{-\kappa z}$

Polarizabilities: $\alpha=\frac{8L^3}{3a_0\pi^2 N}$; $\gamma=\frac{256L^3}{45a_0^3 e^2\pi^6 N^5}$ Response: $\tau=2\pi a_0/v$

$a_0=0.5\times10^{-10}$ m

$$\chi^{(2)}=N\beta;\ \beta=\beta_{xxx}=\frac{3e^3}{\hbar^2}\sum_{s,t\neq g}\frac{x_{gt}x_{ts}x_{sg}}{\omega_{tg}\omega_{sg}}=\boxed{\frac{3e^3}{\hbar^2}\frac{x_{gt}\cdot x_{ts}\cdot x_{sg}}{\omega_1\cdot\omega_2}}$$

Symmetric optical components: $S_{ijk}=\frac{1}{2}(\chi^{(2)}_{ijk}+\chi^{(2)}_{ikj})$; Antisymmetric optical components: $A_{ijk}=\frac{1}{2}(\chi^{(2)}_{ijk}-\chi^{(2)}_{ikj})$

$\boxed{\text{The oscillator possesses only symmetric components } (S_{ijk}) \text{ and not antisymmetric } (A_{ijk}).}$

Note: This term vanishes with second-harmonic generation, or whenever the Kleinmin symmetry condition is valid.

Polarization Dependence on Nonlinear symmetry:

$$P_i(\omega_\sigma)=2\epsilon_0\sum_{jk}\chi^{(2)}_{ijk}(\omega_\sigma=\omega_1+\omega_2)E_jF_k$$

$$=\epsilon_0\sum_{jk}S_{ijk}(E_jF_k+E_kF_j)+\epsilon_0\sum A_{ijk}(E_jF_k-E_kF_j)\ ;\ A_{123}=A_{231}=A_{312}$$

When Isotropic $E_jF_k=\pm E_kF_j$, the material is considered an isotropic, chiral medium. This would imply $\boxed{P_i(\omega_\sigma)\propto\epsilon_0 A_{123}\,E\times F}$,

$H|\psi\rangle=E|\psi\rangle$; $\left[\frac{1}{2}mv^2+\frac{1}{2}(k_a x^2+k_b y^2+k_c z^2)+Axyz\right]|\psi\rangle=(E_1 e^{-i\omega_1 t}+E_2 e^{-i\omega_2 t}+c.c.)|\psi\rangle$

$\boxed{v=\sqrt{(E_1 e^{-i\omega_1 t}+E_2 e^{-i\omega_2 t}-V)\frac{2}{m}}}$; $\boxed{\tau=2\pi a_0\Big/\sqrt{(E_1 e^{-i\omega_1 t}+E_2 e^{-i\omega_2 t}-V)\frac{2}{m}}}$

$$\chi^{(2)}_{ijk}(\omega_p+\omega_q,\omega_q,\omega_p)=\frac{N}{\epsilon_0\hbar^2}\sum\Big\{(\rho^{(0)}_{mm}-\rho^{(0)}_{vv})\frac{\mu^i_{mn}\mu^j_{nv}\mu^k_{vm}}{[(\omega_{nm}-\omega_p-\omega_q)-i\gamma_{nm}][(\omega_{vm}-\omega_p)-i\gamma_{nm}]}$$

$$-(\rho^{(0)}_{vv}-\rho^{(0)}_{nn})\frac{\mu^i_{mn}\mu^j_{vm}\mu^k_{nv}}{[(\omega_{nm}-\omega_p-\omega_q)-i\gamma_{nm}][(\omega_{nv}-\omega_p)-i\gamma_{nm}]}\Big]$$

$$[\hat{V},\rho^{(1)}]=-\sum(\mu_{nv}\rho^{(1)}_{vm}-\rho^{(1)}_{nv}\mu_{vm})\vec{E}(t)$$

$$=-\hbar^{-1}\sum(\rho^{(0)}_{mm}-\rho^{(0)}_{vv})\times\sum_{pq}\frac{[\mu_{nv}\cdot E(\omega_q)][\mu_{vm}\cdot E(\omega_p)]}{(\omega_{vm}-\omega_q)-i\gamma_{vm}}e^{-i(\omega_p+\omega_q)t}$$

$$+\hbar^{-1}\sum(\rho^{(0)}_{vv}-\rho^{(0)}_{nn})\times\sum\frac{[\mu_{nv}\cdot E(\omega_p)][\mu_{vm}\cdot E(\omega_q)]}{(\omega_{nv}-\omega_q)-i\gamma_{nv}}e^{-i(\omega_p+\omega_q)t}$$

Chapter 6:

2. $\chi^{(3)}$; n_0 Eq 6.3.36b or 6.3.36b

$$6.3.36b \quad \chi^{(1)} = \left[N(\rho_{bb}-\rho_{aa})^{(eq)}|\mu_{ba}|^2\frac{T_2}{\epsilon_0\hbar}\right]\frac{\Delta T_2 - i}{1+\Delta^2T_2^2}$$ n_0 num

6.3.34a, 6.3.34b

$$6.3.38b \quad \chi^{(3)} = -\frac{4}{3}N(\rho_{bb}-\rho_{aa})^{(eq)}|\mu_{ba}|^4\frac{T_1T_2^2}{\epsilon_0\hbar^3}\frac{\Delta T_2 - 1}{(1+\Delta^2T_2^2)^2}$$

6.3.22b

$$6.3.33b \quad \chi^{(3)} = \frac{\alpha_0(0)}{3\omega_{ba}/c}\left[\frac{\Delta T_2 - i}{(1+\Delta^2T_2^2)^2}\right]\frac{1}{|E_s^0|^2}$$

$$6.3.34a \quad \chi^{(3)} = \frac{\alpha_0(\Delta)}{3\omega_{ba}/c}\left[\frac{\Delta T_2 - i}{(1+\Delta^2T_2^2)^2}\right]\frac{2\epsilon_0 c}{I_s^0}$$ n_0 num

$$6.3.34b \quad \chi^{(3)} = \frac{\alpha_0(\Delta)(\Delta T_2 - i)}{3\omega_{ba}/c}\frac{2\epsilon_0 c}{I_s^\Delta}$$

$$6.3.30 \quad I_s^0 = 2\epsilon_0 c|E_s^0|^2$$ $\times n_0$

$$6.3.31 \quad I_s^\Delta = 2\epsilon_0 c|E_s^\Delta|^2 = I_s^0(1+\Delta^2T_2^2)$$

$$6.3.22b \quad \alpha_0(0) = -\frac{\omega_{ba}}{c}\left[N(\rho_{bb}-\rho_{aa})^{(eq)}|\mu_{ba}|^2\frac{T_2}{\epsilon_0\hbar}\right]$$ n_0 down

4. Bloch Equation: $\dot{p} = (i\Delta - \frac{1}{T_2})p - \frac{i}{\hbar}|\mu|^2 E w$; $\dot{w} = -\frac{w+1}{T_1} - \frac{2i}{\hbar}(pE^* - p^*E)$

$E(t) = Ee^{-i\omega t} + c.c.$

at $t=0$; $p(0)=0$; $w(0)=-1$

$$\frac{dp}{dt} = (i\Delta - \frac{1}{T_2})p - \frac{i}{\hbar}|\mu|^2 E w = (i\Delta - \frac{1}{T_2})p - \frac{i}{\hbar}|\mu|^2(Ee^{-i\omega t} + c.c)w$$

$$\dot{p} - (i\Delta - \frac{1}{T_2})p = -\frac{i}{\hbar}|\mu|^2(E\cdot e^{-i\omega t} + c.c)w$$

"Bernouli" $\quad y' + p(x)y = Q(x)$; $I = e^{\int p(x)dt}$; $y = \frac{1}{I(x)}\left[\int I(x)Q(x)dx + C\right]$

$$I(x) = e^{\int -(i\Delta - \frac{1}{T_2})dt} = e^{-(i\Delta - \frac{1}{T_2})t} \; ; \; y = p = \frac{1}{e^{-(i\Delta - \frac{1}{T_2})t}}\int e^{-(i\Delta - \frac{1}{T_2})}\left(-\frac{i}{\hbar}|\mu|^2(Ee^{-i\omega t} + c.c)\right)w\,dt$$

$$= \frac{1}{e^{-(i\Delta - \frac{1}{T_2})t}}\,\frac{-(+i\Delta - \frac{1}{T_2})\left(\frac{\cancel{i}}{\hbar}|\mu|^2\cdot E e^{-i\omega t}\cdots e\right)\cancel{w}}{\cancel{i}\cancel{w}} =$$

$$p = -\frac{(i\Delta - \frac{1}{T_2})}{\hbar}|\mu|^2\cdot E e^{-(i\omega t + (i\Delta - \frac{1}{T_2})t)} + c = -\frac{(i\Delta - \frac{1}{T_2})}{\hbar}|\mu|^2 E\cdot e^{-(i(\omega+\Delta) - \frac{1}{T_2})t} + C$$

"Bernouli" $\quad y' + p(x)y = Q(x)$; $I = e^{\int p(x)dt}$; $y = \frac{1}{I(x)}\left[\int I(x)Q(x)dx\right]$

$p(0) = 0$ @ $\quad C = +\frac{(i\Delta - \frac{1}{T_2})}{\hbar}|\mu|^2 E$

$$= \left(1 - e^{-(i(\omega+\Delta) - \frac{1}{T_2})t}\right)\frac{(i\Delta - \frac{1}{T_2})}{\hbar}|\mu|^2 E$$

$$\dot{w} + \frac{w+1}{T_1} = -\frac{2i}{\hbar}(pE^* - p^*E)$$

$$I = e^{\int\frac{w+1}{T_1}dt} = e^{\frac{(w+1)}{T_1}t} \; ; \; y = \frac{1}{e^{\frac{(w+1)}{T_1}t}}\int e^{\frac{(w+1)t}{T_1}}\cdot\left(-\frac{2i}{\hbar}(pE^* - p^*E)\right)$$

Burnd

$$= \frac{-2T_1\cdot i}{\hbar}\frac{(pE^* - p^*E)}{(w+1)} \;\cdots\; w(t) = w_0 - (1+w_0)e^{-t/T_2}\left[\cos\Omega' t + \frac{1}{\Omega' T_2}\sin\Omega' t\right]$$

$$w_0 = \frac{-(1+\Delta^2T_2^2)}{1+\Delta^2T_2^2 + \Omega^2T_1T_2}$$

6. Verify $\int_{-\infty}^{\infty} \frac{-\frac{1}{2}\Omega^2(\delta-\Delta+i/T_2)(\delta+2i/T_2)(\Delta-i/T_2)^{-1}}{(\delta+i/T_1)(\delta-\Delta+i/T_2)(\Delta+\delta+i/T_2)-\Omega^2(\delta+i/T_2)}\,d\delta=0$; Quick method: Cauchy Principle Value. "symmetric function"

$$\int_{-\infty}^{\infty}\frac{x^2}{x^5-x}dx=0$$

Chapter 7 ① Verify 7.1.19 through 7.1.21 satisfy 7.1.18

$$A(x,z)=A_0\operatorname{sech}(x/x_0)e^{i\gamma z}$$

$$2ik_0\frac{\partial}{\partial z}\left[A_0\operatorname{sech}(x/x_0)e^{i\gamma z}\right]+\frac{\partial^2}{\partial x^2}\left[A_0\operatorname{sech}(x/x_0)e^{i\gamma z}\right]=-3\chi^{(3)}\frac{\omega^2}{c^2}|A|^2A$$

$$2ik_0(i\gamma)A_0\operatorname{sech}(x/x_0)e^{i\gamma z}+\frac{A_0e^{i\gamma z}}{x_0}\left[-\operatorname{sech}(x/x_0)\tanh\left(\frac{x}{x_0}\right)-\operatorname{sech}(x/x_0)\tanh(x/x_0)\right]=-3\chi^{(3)}\frac{\omega^2}{c^2}|A|^2A$$

$$2ik_0(i\gamma)A_0\operatorname{sech}(x/x_0)e^{i\gamma z}+\frac{A_0e^{i\gamma z}}{x_0^2}\left[\tanh^2\left(\frac{x}{x_0}\right)\operatorname{sech}\left(\frac{x}{x_0}\right)-\operatorname{sech}^3\left(\frac{x}{x_0}\right)\right]=-3\chi^{(3)}\frac{\omega^2}{c^2}|A|^2A$$

$$-A_0\operatorname{sech}(x/x_0)e^{i\gamma z}\left[\frac{\tanh^2(x/x_0)-\operatorname{sech}^2(x/x_0)}{x_0^2}-2k_0\gamma\right]=-3\chi^{(3)}\frac{\omega^2}{c^2}A_0^3\operatorname{sech}^3\left(\frac{x}{x_0}\right)e^{3i\gamma z}$$

$$-\left(\frac{1}{x_0^2}+2k_0\gamma\right)=-3\chi^{(3)}\frac{\omega^2}{c^2}A_0^2\operatorname{sech}^2(x/x_0)e^{2i\gamma z}$$

$$x_0=\frac{1}{k_0}\sqrt{\frac{n_0}{2n_2|A_0|^2}}=\frac{1}{k_0}\sqrt{\frac{n_0}{n_2I}}\ ;\ \gamma=k_0\bar{n}_2|A_0|^2/n_0=k_0n_2I/(2n_0)$$

$$-\left(k_0^2\left(\frac{2n_2|A_0|^2}{n_0}\right)+2k_0^2\frac{\bar{n}_2|A_0|^2}{n_0}\right)=-3\chi^{(3)}\frac{\omega^2}{c^2}A_0^2\operatorname{sech}^2\left(\frac{x}{x_0}\right)e^{2i\gamma z}$$

$$-3k_0^2\left(\frac{2n_2|A_0|^2}{n_0}\right)=-3\chi^{(3)}A_0^2\operatorname{sech}^2\left(\frac{x}{x_0}\right)e^{2i\gamma z}\frac{\omega^2}{c^2}$$

$$-3\cdot 2\frac{n_2}{n_0}=\chi^{(3)}\operatorname{sech}^2\left(\frac{x}{x_0}\right)e^{2i\gamma z}\ ;\ n_2=3\chi^{(3)}/4n_0\ ;$$

$$\frac{2\cdot 3\cancel{\chi^{(3)}}}{4n_0^2}=\cancel{\chi^{(3)}}\operatorname{sech}^2\left(\frac{x}{x_0}\right)e^{2i\gamma z}\ ;\quad \frac{3}{2}\frac{1}{n_0^2}=\operatorname{sech}^2\left(\frac{x}{x_0}\right)e^{2i\gamma z}$$

$$=\operatorname{sech}^2\left(\frac{x}{x_0}\right)\cosh i\gamma z+\sinh(i\gamma z)$$

3. Derive $\Delta I/|I|$ relative to sample position z and position z_0 $\quad w(z)=\sqrt{1+(z/z_0)^2}$

Also contains n_2, and L. Confocal parameter b. Nonlinear phase shift $\int$

Gaussian Laser beam: $I=I_0e^{-2r^2/w_0^2 w^2}$ $\quad R$

$\Phi_{NL}=n_2(\omega/c)\int_0^L I(z)dz$

$=n_2(\omega/c)\cdot I_0\cdot L_{eff}$

Curvature: $R(z)=z(1+(\frac{z_R}{z})^2)$

For field as a function of $z-z_0$.

$\Delta\Phi_{NL}^{(max)}=-n_2\cdot\frac{\omega_0}{c}I_0\cdot L$

$$E=\left[\frac{2}{\pi w^2(z)}\right]^{0.5}\exp\left[\frac{-r^2}{w^2(z)}-jkz-\frac{j\pi r^2}{\lambda R(z)}+j\phi_0(z)\right]$$

$$W(z)=W_0\left[1+\left(\frac{\lambda z}{\pi W_0^2}\right)^2\right]^{0.5};\ R(z)=z+\frac{(\pi W_0^2/\lambda)^2}{z};\ \phi_0(z)=\tan^{-1}\left(\frac{\lambda z}{\pi W_0^2}\right)$$

Far field Divergence Angle: $\theta_0=\frac{\lambda}{\pi W_0}$; Derivation: $U(r,z)=A(z)\exp\left[\frac{-jkr^2}{2q(z)}\right]$

$q(z)=q(z_0)+(z-z_0)$

Result: $\frac{\Delta I}{I}=\frac{4\Phi_{max}\cdot x}{(x^2+1)(x^2+9)}$; $x=2(z-z_0)/b$ $\quad \frac{1}{q}=\left(\frac{1}{q}\right)-j\left(\frac{1}{q}\right)$;

Phase Delay $\phi(r) = \frac{\pi r^2}{\lambda R} = \frac{kr^2}{2R}$; $\left(\frac{1}{2}\right) = \frac{1}{R}$

$f(r) = f(0)\exp\left[-\left(\frac{r}{r_0}\right)^2\right]$; $\left(\frac{1}{2}\right) = \frac{2}{R\omega^2(z)} = \frac{\lambda}{\pi\omega^2}$; $\frac{1}{q} = \frac{1}{R} - \frac{j\lambda}{\pi\omega^2}$ $\boxed{E = \left(\frac{\omega_0}{\omega}\right)\exp\left(\frac{-r^2}{\omega^2} - jkz - \frac{j\pi r^2}{\lambda R} + j\phi_0\right)}$

(5) $\frac{dA_3}{dz} = -\alpha_3 A_3 - i k_4^* A_4^* e^{i\Delta k z}$; where $\Delta K = (k_1 + k_2 - k_3 - k_4)$

?

$\frac{dA_4}{\partial z} = \alpha_4 A_4 + i k_3^* A_3^* e^{-i\Delta k z}$ Paraxial Equation: $ik_3 A_3' e^{ik_3 z} + \frac{dA_3'}{dz} e^{ik_3 z} = ik_3 A_3 e^{ik_3 z} + ikA_4^* e^{ik_3 z}$

General Solution: $A_3 = A_3' e^{ik_3 z}$; $A_4 = A_4' e^{-ik_3 z}$; ; $A_3 \frac{A_4}{A_4'} = A_3'$

(7) $\begin{bmatrix} P_x \\ P_y \end{bmatrix} = 6\epsilon_0 \begin{bmatrix} \chi_{1111} B_x F_x + \chi_{1221} B_y F_y & \chi_{1122}(B_x F_y + B_y F_x) \\ \chi_{1122}(B_y F_x + B_x F_y) & \chi_{1111} B_y F_y + \chi_{1221} B_x F_x \end{bmatrix} \begin{bmatrix} S_x^* \\ S_y^* \end{bmatrix}$

Mch

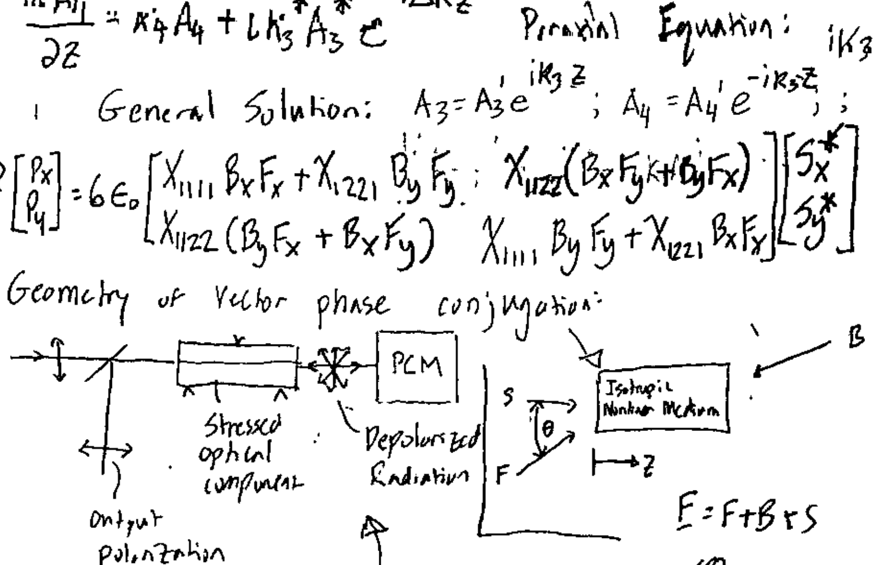
$\frac{dA_3'}{dz} \cancel{e^{ik_3 z}} = ikA_4^* \cancel{e^{ik_3 z}}$

$ikA_3 e^{-ik_3 z} = ikA_4^*$

$\boxed{A_3' = A_4' e^{-2ik_3 z} A_3 \text{ and } A_4^* = A_3 e^{-ikz}}$

$\alpha_3 = ik = \alpha_4$ A;

Geometry of vector phase conjugation:



$E = F + B + S$

$\chi_{ijkl}^{(3)} = \chi_{1122}(\delta_{ij}\delta_{kl} + \delta_{ik}\delta_{jl}) + \chi_{1221}\delta_{il}\delta_{jk}$

Polarization Properties of phase conjugation:

$P = 6\epsilon_0 \chi_{1122}(E \cdot E^*)E + 3\epsilon_0 \chi_{1221}(E \cdot E)E^*$

$= \epsilon_0 A(E \cdot E^*)E + \frac{1}{2}\epsilon_0 B(E \cdot E)^*$

$P = 6\epsilon_0 \chi_{1122}(F+B+S)(F+B+S)^*(F+B+S) + 3\epsilon_0 \chi_{1221}(F+B+S)^2(F+B+S)^*$

$= 6\epsilon_0\left[\chi_{1122}(F+B+S)^2(F+B+S) + \chi_{1221}(F+B+S)^2(F+B+S)/2\right]$

$= 6\epsilon_0\left[\chi_{1122}\cancel{(F_x F_x^* + F_y F_y^* + B_x B_x^* + B_y B_y^*)(F+B+S) +} \frac{\chi_{1221}}{2}(F_x F_x + F_y F_y + B_x B_x + B_y B_y)(F+B+S)^*\right]$

$= 6\epsilon_0\left[\chi_{1122}(F_x F_x^* F_x + F_x F_x^* F_y + F_y F_y^* F_x + F_y F_y^* F_y + B_x B_x^* B_x + B_x B_x^* B_y + B_y B_y^* B_x + B_y B_y^* B_y)\right.$

$+ (F_x F_x^* + F_y F_y^* + B_x B_x^* + B_y B_y^*)(S_x + S_y) + (F_x F_x^* B_x + F_y F_y^* B_x + F_x F_x^* B_y + F_y F_y^* B_y$

$+ B_x B_x^* F_x + B_y B_y^* F_x + B_x B_x^* F_y + B_y B_y^* F_y)\cancel{(S_x + S_y)}$

$= 6\epsilon_0 \chi_{1221}(B_x F_x + B_y F_y)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} S_x^* \\ S_y^* \end{bmatrix}$ Note: Due to no off-diagonal terms and and cross-product evaluation.

9. $\alpha = \frac{\alpha_0}{1 + I/I_s}$ becomes $\alpha = \frac{\alpha_0}{1 + I/I_s} + \alpha_1$

Optical: Bistability and Optical Switching:

R = Reflectance; $R = |\rho|^2$ and $T = |\tau|^2$.

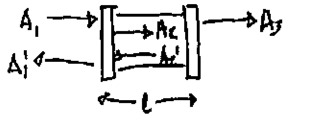
T = Transmittance. With $R + T = 1$

The incident fields are related: $A_2' = \rho A_2 e^{2ikl - \alpha l}$

$A_2 = \tau A_1 + \rho A_2'$

Optical Bistability. Two different output intensities are possible for a given input intensity

Optical Multistability. A circumstance when two or more stable outputs are possible.



$A_2 = \frac{\tau A_1}{1-\rho^2 e^{2ikl-\kappa l}}$ "Airy's Function" - Fabry Perot

Absorptive Bistability: Assume $\rho^2 = R$ and $e^{2ikl-\kappa l} = 1-\kappa l$ "Expansion" $A_2 = \frac{\tau A_1}{1-R(1-\kappa l)}$

α_1 cannot be larger than $0 << 2k_i l - (\kappa_0 + \kappa_1) l$

$\boxed{\alpha_1 << 2k_i - \alpha_0}$

or $\alpha_1 << \alpha_0$

Analogously: $I_2 = \frac{T I_1}{[1-R(1-\kappa l)]^2}$

When simplified $C = \frac{R\kappa l}{1-R}$ becomes

$1+C = (1-R+R\kappa l)/(1-R)$

$= [1-R(1-\kappa l)]/T$

$I_2 = \frac{I_1}{T(1+C)^2}$

The derivation would change for intensity by overall intensity being dependent upon two absorptive constants in the denominator of Intensity.

11. How are Fabry-Perot intensity requirements different for switching modified by the inclusion of loss, and still allow absorption? How large can absorption be?

Optical Switching: $r = i\sqrt{R}$; $t = \sqrt{T}$; $R+T=1$

The field at output port 1 is by:

$E_1 = E_s(rt + rte^{i\phi_{NL}})$

Where $\phi_{NL} = n_2(\omega/c) I L = n_2(\omega/c)|t|^2(2n_0\epsilon_0 c)|E_s|^2 L$

Intensity at output port 1 is then:

$|E_1|^2 = |E_s|^2|r|^2|t|^2(1+e^{i\phi_{NL}})(1+e^{-i\phi_{NL}})$

$= 2|E_s|^2 RT(1+\cos\phi_{NL})$

Output at port 2 is given by

$E_2 = E_s(r^2 + t^2 e^{i\phi_{NL}})$

Intensity at port 2 is:

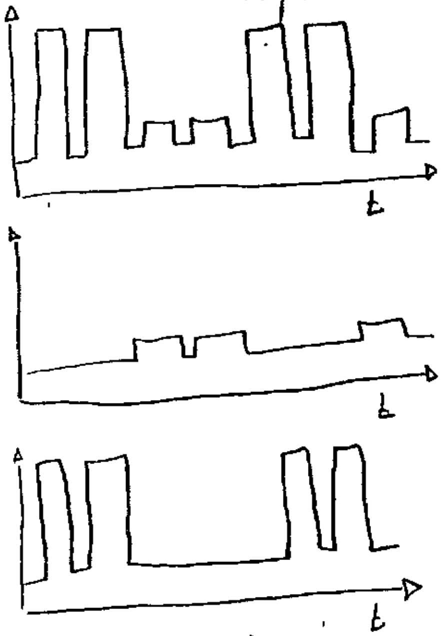
$|E_2|^2 = |E_s|^2[R^2 + T^2 - 2RT\cos\phi_{NL}]$

Note: $|E_1|^2 + |E_2|^2 = |E_s|^2$

Input field

output 1

output 2

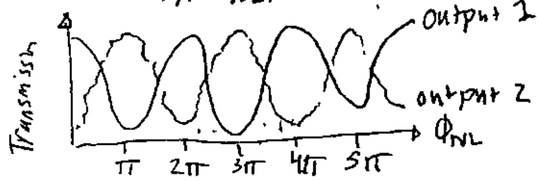


The nonlinear phase shift is: $\phi_{NL} = n_2(\omega/c)\int_0^L I(z)\,dz$

$I(z) = I_0 e^{-\alpha z}$

$= n_2(\omega/c) I_0 L_{eff}$

$L_{eff} = \frac{1-e^{-\alpha L}}{\alpha}$



The intensity requirements for switching are different than bistability by the fact switching depends on absorption coefficient, while bistability is an Airy Function dependent upon Reflectance and Absorption. The absorption can be as large as $(1-e^{-\alpha L})/\alpha$.

13. $\frac{dI_1}{dz} = \beta I_1 I_2$

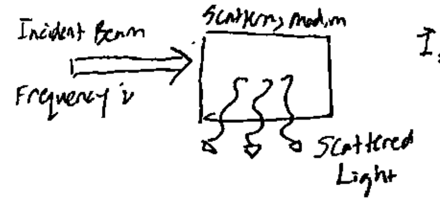
$\frac{dI_2}{dz} = -\beta I_1 I_2$

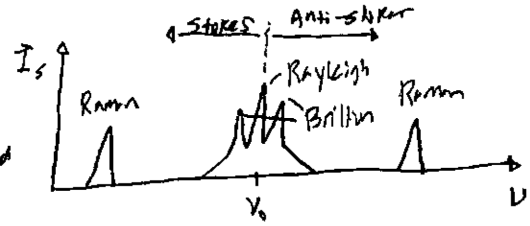
15

17

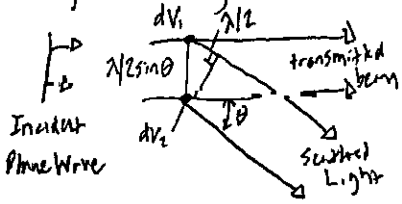
19

Chapter 8: Spontaneous Light Scattering:





Light Scattering Model:



Process	Shift (cm^{-1})	Linewidth	Relaxation	Gain
Raman	1000	5	10^{-12}	5×10^{-5}
Brillouin	0.1	5×10^{-3}	10^{-9}	10^{-4}
Rayleigh	0	5×10^{-4}	10^{-8}	10^{-6}
Rayleigh-Wing	0	5	10^{-12}	10^{-5}

$\epsilon = \bar{\epsilon}\delta_{ik} + \Delta\epsilon_{ik}$ ← varying spatial or temporal dielectric tensor

Dielectric tensor; Dielectric constant

$\Delta\epsilon_{ik} = \underbrace{\Delta\epsilon\,\delta_{ik}}_{\text{Scalar contribution}} + \underbrace{\Delta\epsilon_{ik}^{(t)}}_{\text{"traceless tensor contribution"}}$

→ Brillouin Scattering, Rayleigh Scattering

→ Light Scattering Tensor [traceless $\sum_i \epsilon_{ii}^{(t)} = 0$]

$\Delta\epsilon_{ik}^{(t)} = \underbrace{\Delta\epsilon_{ik}^{(s)}}_{\text{"symmetric"}} + \underbrace{\Delta\epsilon_{ik}^{(a)}}_{\text{"antisymmetric"}}$

→ Rayleigh Wing Scattering; → Raman Scattering

$\boxed{\Delta\epsilon, \Delta\epsilon_{ik}^{(s)}, \Delta\epsilon_{ik}^{(a)} \text{ are statistically independent}}$

Scattering Coefficient:

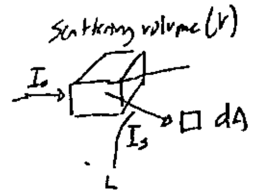
$$I_s = \frac{I_0 R \cdot V}{L^2}$$

R ← scattering coefficient; V ← volume; L ← Length

Power on Detector: $dP = I_s dA$

Scattering Region: $d\Omega = dA/L^2$

$$\frac{dP}{d\Omega} = I_0 R V$$



Scattering Cross Section: $P = \sigma I_0\ \sigma\ [W/m^2]$

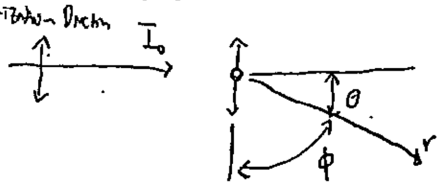
σ ← Total scattering cross-section

$$\frac{dP}{d\Omega} = I_0\frac{d\sigma}{d\Omega}; \quad \sigma = \int_{4\pi}\frac{d\sigma}{d\Omega}d\Omega; \quad \text{Scattering Coefficient}: R = \frac{N}{V}\frac{d\sigma}{d\Omega}$$

Microscopic Theory of Light Scattering:

$E = E_0 e^{-i\omega t} + c.c$; $I_0 = (2nc\epsilon_0)|E_0|^2$

$\tilde{p} = \epsilon_0\alpha(\omega)E_0 e^{-i\omega t} + c.c.$;



Geometry of light scattered

$$I_s = \frac{n\langle\ddot{\tilde{p}}^2\rangle}{16\pi^2\epsilon_0 c^3 L^2}\sin^2\phi = \frac{n\omega^4\epsilon_0|\alpha(\omega)|^2|E_0|^2}{8\pi^2 c^3 L^2}\sin^2\phi$$

$$\frac{d\sigma}{d\Omega} = \frac{1}{16\pi^2}\frac{\omega^4}{c^4}|\alpha(\omega)|^2\sin^2\phi$$

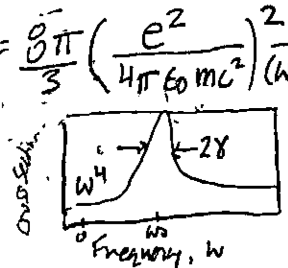
$$\sigma = \int_{4\pi} d\Omega\frac{d\sigma}{d\Omega} = \frac{8\pi}{3}\frac{1}{16\pi^2}\frac{\omega^4}{c^4}|\alpha(\omega)|^2 = \frac{1}{6\pi}\frac{\omega^4}{c^4}|\alpha(\omega)|^2$$

Cross section is dependent upon angle, polarization.

Lorentz model of a Harmonic oscillator

$$\alpha(\omega) = \frac{e^2/m\epsilon_0}{\omega_0^2 - \omega^2 - 2i\omega\gamma}$$; ω_0 = resonance frequency; γ = Dipole Dampening Rate

$$\sigma = \frac{8\pi}{3}\left(\frac{e^2}{4\pi\epsilon_0 mc^2}\right)^2\frac{\omega^4}{(\omega_0^2-\omega^2)^2 + 4\omega^2\gamma^2}$$



* Chart changes when above or below resonance frequency *

$$\frac{dA_1}{dx} = iKA_2\, e^{-i\Delta k x} \; ; \; \frac{dA_2}{dx} = iK^* A_1 e^{i\Delta k x} \; ; \; K = \frac{\omega^2 \Delta\epsilon^{\vartheta}}{2k_x c^2}$$

$$\frac{dA_1}{dx} = iKA_2 \; ; \; \frac{dA_2}{dx} = iK^* A_1 \; ; \; A_1(x) = A_1(0)\cos(|K|x) \; ; \; A_2(x) = \frac{iK^*}{|K|} A_1(0)\sin(|K|x)$$

$$|A_1(x)|^2 + |A_2(x)|^2 = |A_1(0)|^2$$

Diffraction Efficiency:

$$\eta = \frac{|A_2(L)|^2}{|A_1(0)|^2} = \sin^2(|K|L)$$

$$I = Kv\frac{\langle \Delta\tilde{\rho}^2\rangle}{\rho_0^2} = 2Kv\left|\frac{\Delta\rho}{\rho_0}\right|^2$$

$$|K| = \frac{\omega \gamma_e}{2nc\cos\theta}\left(\frac{I}{2Kv}\right)^{1/2}$$

; $n = 1.33$, $\gamma_e = 0.92$, $v = 1.5\times10^3$ m/sec

$$K = 2.19\times10^{11}\ \mathrm{N\,m^{-2}}$$

Starting from 8.4.2.5 ; $\delta K = -\frac{1}{2} q\,\delta\theta$

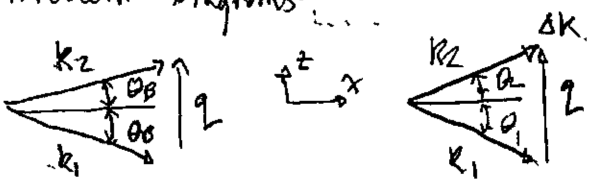
$$I = Kv\frac{\langle \Delta\tilde{\rho}^2\rangle}{\rho_0^2} = 2Kv\left|\frac{\Delta\rho}{\rho_0}\right|^2$$

$$\boxed{\delta\theta = \frac{-2}{q}\Delta K = \frac{-2}{q}\left[\sqrt{2}|K|\left[1 - \frac{|K|L\cos|K|L}{\sin|K|L}\right]^{1/2}\right]}$$

@ $|K|L = \pi/2$ $\quad \delta\theta = 2\sqrt{2}|K|/q$; $2\delta\theta = 0.22^\circ$

$\Delta K = 0$ is 100%

Wavevector Diagrams:



$$k\cos\theta_1 - k\cos\theta_2 = \Delta K \; ; \; k_1 = k_2 = k$$

$$k\sin\theta_1 + k\sin\theta_2 = q \; ; \; \theta_B = \sin^{-1}\frac{q}{2k} = \sin^{-1}\frac{\lambda}{2\Lambda}$$

$$\theta_1 = \theta_B + \Delta\theta$$

$$\theta_2 = \theta_B - \Delta\theta$$

$$\cos(\theta_B \pm \Delta\theta) = \cos\theta_B \mp (\sin\theta_B)\Delta\theta$$

$$(2k\sin\theta_B)\Delta\theta = \Delta K = -\Delta\theta q$$

$$A_1(x) = e^{-i(1/2)\Delta K x} A_1(0)\left(\cos sx + i\frac{\Delta K}{2s}\sin sx\right)$$

$$A_2(x) = ie^{i(1/2)\Delta K x} A_1(0)\frac{K^*}{s}\sin sx$$

$$s^2 = |K|^2 + (\tfrac{1}{2}\Delta K)^2$$

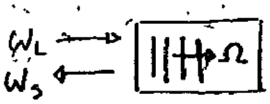
$$\eta(\Delta K) = \frac{|A_2(L)|^2}{|A_1(0)|^2} = \frac{|K|^2}{|K|^2 + (\frac{1}{2}\Delta K)^2}\sin^2\left[\left[|K|^2 + (\tfrac{1}{2}\Delta K)^2\right]^{1/2} L\right]$$

$$\eta(\Delta K) = \eta(0) + \Delta K\left.\frac{d\eta}{d(\Delta K)}\right|_{\Delta K=0} + \frac{1}{2}(\Delta K)^2\left.\frac{d^2\eta}{d(\Delta K)^2}\right|_{\Delta K=0} + \cdots$$

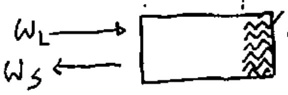
$$\eta(\Delta K) = \eta(0)\left[1 - \frac{(\Delta K)^2}{4|K|^2}\underbrace{\left(1 - \frac{|K|K\cos(|K|L)}{\sin(|K|L)}\right)}\right]$$

Chapter 9: Stimulated Brillouin and Stimulated Rayleigh Scattering

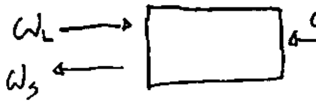
1.



"Brillouin Scattering"
$\omega_S = \omega_L - \Omega$

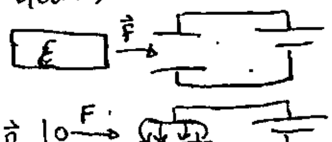


Thermal noise from photons
"SBS Generator"
"Stimulated" Brillouin



Weak Stokes Seed
"SBS Amplifier"

Electrostriction:



Dielectric Slab between capacitor

Fringing Field of a Capacitor generates Dipole of molecule.

Potential Inside an Electric Field,

$$u = \tfrac{1}{2}\epsilon\epsilon_0 E^2 \; ; \; \text{Dipole Moment } \vec{p} = \epsilon_0 \alpha E$$

Energy Stored:

$$U = -\int_0^E p\cdot dE' = -\int_0^E \epsilon_0\alpha E' dE' = -\tfrac{1}{2}\epsilon_0\alpha E\cdot E \equiv -\tfrac{1}{2}\epsilon_0\alpha E^2$$

$$F = -\nabla U = \tfrac{1}{2}\epsilon_0\alpha\nabla(E^2)$$

Capacitor in a Dielectric Liquid:

$$\Delta\epsilon = \left(\frac{\partial\epsilon}{\partial\rho}\right)\Delta\rho$$

$$\Delta u = \frac{1}{2}\epsilon_0 E^2 \Delta\epsilon = \frac{1}{2}\epsilon_0 E^2 \left(\frac{\partial\epsilon}{\partial\rho}\right)\Delta\rho$$

Work per unit volume: $\Delta W = p_{st}\frac{\Delta V}{V} = -p_{st}\frac{\Delta\rho}{\rho}$

Electrostrictive pressure: $p_{st} = -\frac{1}{2}\epsilon_0 \rho\left(\frac{\partial\epsilon}{\partial\rho}\right)E^2 \equiv -\frac{1}{2}\epsilon_0\gamma_e E^2$

Where: $\gamma_e = \rho(\partial\epsilon/\partial\rho)$ "Electrostrictive constant"

Charge Density:

$\Delta\rho = -(\partial\rho/\partial p)\Delta p$

$= -\rho\left(\frac{1}{\rho}\frac{\partial\rho}{\partial p}\right)p_{st} \equiv -\rho C p_{st}$

Where $C = \rho^{-1}(\partial\rho/\partial p)$ "Compressibility"

$\equiv \frac{1}{2}\epsilon_0 \rho C \gamma_e E^2$ "Density change in an electric field"

$\equiv \frac{1}{2}\epsilon_0 \rho C \gamma_e \langle \underbrace{\tilde{E}\cdot\tilde{E}} \rangle$

"Static and hypersonic components"

"time-average"

Consider a Monochromatic field:

$\tilde{E}(t) = Ee^{-i\omega t} + c.c.$

If SBS, then $\langle\tilde{E}\cdot\tilde{E}\rangle = 2E\cdot E^*$

$\Delta\chi = \frac{1}{2}\epsilon_0 C\gamma_e^2\langle\tilde{E}\cdot\tilde{E}\rangle$

$\vec{P} = \epsilon_0 C_T \cdot \gamma_e^2 \underbrace{|E|^2 E}$ because $\vec{P} = \Delta\chi E$

Represented as "third"-order

$\vec{P} = 3\epsilon_0\chi^{(3)}(\omega = \omega+\omega-\omega)|E|^2 E$; $\chi^{(3)} = \frac{1}{3}C_T\gamma_e^2$

$\gamma_e = \rho(\partial\epsilon/\partial\rho)$; $\gamma_e = n^2 - 1$

Lorentz-Lorenz Law: $\frac{\epsilon^{(1)}-1}{\epsilon^{(1)}+2} = \frac{1}{3}N\gamma^{(1)}$

$\gamma_e = \gamma_{ex}\chi_1^{(1)}$
$= \frac{(n^2-1)(\epsilon+2)}{3}$
$= \frac{(n^2-1)(n^2+2)}{3}$

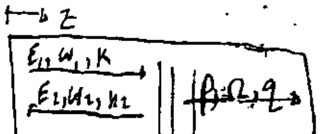
$\boxed{\gamma_e = (n^2-1)(n^2+2)/3}$ Isothermal compressibility

$C_T = \rho^{-1}(\partial\rho/\partial P)$

$\chi^{(3)} = \frac{\epsilon_0\gamma}{3v^2\rho}\left[\frac{(n^2-1)(n^2+2)}{3}\right]^2$ as a $\boxed{\frac{NKT}{V\rho^2}}$ $\boxed{n^2 = \epsilon}$

result: $n_2 = (3/4n_0^2\epsilon_0 c)\chi^{(3)}$

Stimulated Brillouin Scattering (Induced by Electrostriction)



$\omega_2 = \omega_1 - \Omega_B$;

$\Omega_B = |q_B|v$

$q_B = k_1 - k_2$

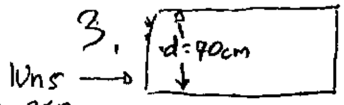
$\Omega = \omega_1 - \omega_2$

$E(z,t) = E_1(z,t) + E_2(z,t)$

$E_1(z,t) = A_1(z,t)e^{i(k_1 z - \omega_1 t)}$

$E_2(z,t) = A_2(z,t)e^{i(-k_2 z - \omega_2 t)}$

3.



What is the minimum pulse energy?

$\rho(z,t) = \rho_0 + [\rho(z,t)e^{i(qz-\Omega t)} + c.c.]$

$\frac{\partial^2\rho}{\partial t^2} - T'\nabla^2\frac{\partial\rho}{\partial t} - v^2\nabla^2\rho = \nabla\cdot f$

v = velocity of sound;

T' = Dampening parameter.

$f = \nabla p_{st}$; $p_{st} = -\frac{1}{2}\epsilon_0\gamma_e\langle E^2\rangle$

$\nabla\cdot f = \epsilon_0\gamma_e q^2[A_1A_2^* e^{i(qz-\Omega t)} + c.c.]$

$-2i\Omega\frac{\partial\rho}{\partial t} + (\Omega_B^2 - \Omega^2 - i\Omega T_B)\rho - 2iqv^2\frac{\partial\rho}{\partial z}$
$= \epsilon_0\gamma_e q^2 A_1 A_2^*$

"Brillouin Linewidth" $T_B = q^2 T'$

$\rho(z,t) = \epsilon_0\gamma_e q^2 \frac{A_1A_2^*}{\Omega_B^2 - \Omega^2 - i\Omega T_B}$

Spatial Evolution of optical fields

$\frac{\partial^2 E_i}{\partial z^2} - \frac{1}{(c/n)^2}\frac{\partial^2 E_i}{\partial t^2} = \frac{1}{\epsilon_0 c^2}\frac{\partial^2 P_i}{\partial t^2}$ $i = 1, 2$

$\tilde{P} = \epsilon_0\Delta\chi\tilde{E} = \epsilon_0\Delta\epsilon\tilde{E} = \epsilon_0\rho_0^{-1}\gamma_e\tilde{\rho}E$

Phase-Matched

$P_1 = p_1 e^{i(k_1 z - \omega t)}$ $P_2 = p_2 e^{i(-k_2 z - \omega_2 t)}$

$p_1 = \epsilon_0\gamma_e\rho_0^{-1}\rho A_2$ $p_2 = \epsilon_0\gamma_e\rho_0^{-1}\rho^* A_1$

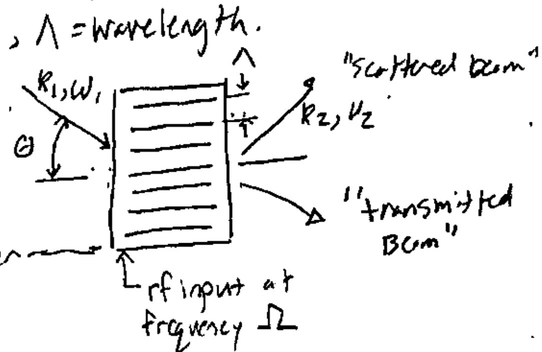
5 Acoustoptics: Bragg scattering: Occurs for long-distance interaction lengths which are phase-matched. Leads to diffracted beam. An analogy to X-ray scattering. Appreciable (>50%)

0.46

Raman-Nath Scattering: Scattering in short interaction lengths. Phase-matching is not important. Orders are present.

Bragg Scattering: $\Lambda = 2\pi v/\Omega$; v = velocity, Ω = frequency, Λ = wavelength.

Constructive interference - $\lambda = 2\Lambda \sin\theta$



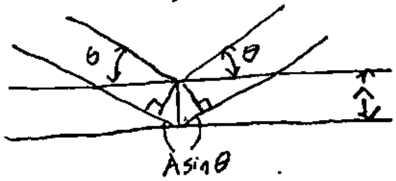
Alternatively: $k_2 = k_1 + q$

$\omega_2 = \omega_1 + \Omega$

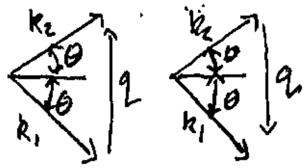
Opposite Direction:

$k_2 = k_1 - q$;

$\omega_2 = \omega_1 - \Omega$



As example:



Assumption: $\Delta\tilde{\epsilon} \simeq \Delta\rho$; $\Delta\tilde{\epsilon} = \frac{\partial\epsilon}{\partial\rho}\Delta\tilde{\rho} = \gamma_e \frac{\Delta\rho}{\rho_0}$ — Electrostrictive constant; Acoustic Density

$[\Delta(\epsilon^{-1})]_{ij} = \sum p_{ijkl} S_{kl}$: $S_{kl} = \frac{1}{2}\left(\frac{\partial d_k}{\partial x_l} + \frac{\partial d_l}{\partial x_k}\right)$ "Strain tensor"

strain-optic tensor

d_k = displacement of a particle.

7. $\theta_1, \Omega \updownarrow$

$\lambda = 2\Lambda \sin\theta$

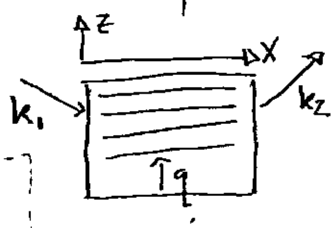
$\Lambda = 2\pi v/\Omega$

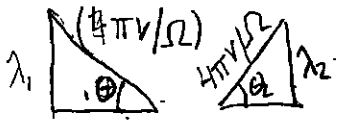
$\lambda = 2(2\pi v/\Omega)\sin\theta$

$(\Delta\epsilon)_{il} = -\sum_{jk} \epsilon_{ij} [\Delta(\epsilon^{-1})]_{jk} \epsilon_{kl}$

$\ldots = -\sum \epsilon_{ij} [\sum p_{ijkl} S_{kl}] \epsilon_{kl}$

$= (\epsilon_{il} - \epsilon_{il})$





$\frac{\lambda_1}{\sin\theta_1} = \frac{\lambda_2}{\sin\theta_2}$; $\frac{\lambda_1}{\lambda_2} = \frac{\sin\theta_1}{\sin\theta_2}$; $2\lambda_1 = \lambda_2$; $\boxed{\frac{1}{2} = \frac{\sin\theta_1}{\sin\theta_2}}$ $\boxed{\sin\theta_2 = 2\sin\theta_1}$

$|k|L = \pi/2$, $L = 1.1\,cm$, $\Lambda = 30\,\mu m$.

$\nabla^2 E - \frac{n^2 + \Delta\epsilon}{c^2}\frac{\partial^2 E}{\partial t^2} = 0$

$\omega_2 = \omega_1 + \Omega$

$E_1 = A_1 e^{i(k_1 r - \omega_1 t)} + c.c$

$E_2 = A_2 e^{i(k_2 r - \omega_2 t)} + c.c.$

$k_2 \simeq k_1 + q$

$\Delta\tilde{\epsilon} = \Delta\epsilon\, e^{i(q\cdot r - \Omega t)} + c.c.$

$\Delta\epsilon = \gamma_e \Delta\rho/\rho_0$

$\frac{\partial^2 A_1}{\partial x^2} + \frac{\partial^2 A_1}{\partial z^2} + 2ik_{1x}\frac{\partial A_1}{\partial x} + 2ik_{1z}\frac{\partial A_1}{\partial z} - (k_{1x}^2 + k_{1z}^2)A_1$

$+ \frac{n^2\omega_1^2}{c^2}A_1 + \frac{\omega_2^2}{c^2}A_2 \Delta\epsilon^* e^{i(k_2 - k_1 - q)r} = 0$

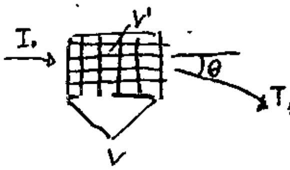
$2ik_{1x}\frac{\partial A_1}{\partial x} = -\frac{\omega_2^2}{c^2}A_2\Delta\epsilon^* e^{i(k_2 - k_1 - q)r}$

$(k_2 - k_1 - q)\cdot r = -\Delta k x$: $\frac{dA_1}{dx} = \frac{i\omega_2^2\Delta\epsilon^*}{2k_{1x}c^2}A_2 e^{-i\Delta k x}$

$\frac{dA_2}{dx} = \frac{i\omega_2^2\Delta\epsilon}{2k_{2x}c^2}A_1 e^{i\Delta k x}$

Polarizability of a Dielectric sphere: $\alpha = 4\pi\epsilon \frac{\epsilon_1 - \epsilon}{\epsilon_1 + 2\epsilon} a^3$; $\sigma = \frac{8\pi\omega^4}{3 \cdot c^4} a^6 \epsilon^2 \left(\frac{\epsilon_1 - \epsilon}{\epsilon_1 + 2\epsilon}\right)^2$

$I_{v'} = v^2 I_{mol}$
describes



Mean-Square Displacement:

$\overline{\Delta v^2} = \overline{(v-\bar{v})^2} = \overline{v^2} - \bar{v}^2$

$I_v = I_{mol} \overline{\Delta v^2} \frac{V}{v'} = \tilde{N} I_{mol}$

$\tilde{N} = NV$

$R = N \frac{d\sigma}{d\Omega}$; $R = \frac{N}{16\pi^2} \frac{\omega^4}{c^4} |\alpha(\omega)|^2 \sin^2\phi$

$n = 1 + \frac{1}{2} N\alpha(\omega)$

$R = \frac{\omega^4}{c^4} \frac{|n-1|^2}{4\pi^2 N} \sin^2\phi$

$\boxed{1.}$ $\lambda = 500\,nm$

Derivation · Lorentz model

$m\ddot{r} + \Gamma\dot{r} + \omega_0^2 r = -m\omega_0^2 r - e[E(t) + \frac{v}{c} B(t)]$

$\vec{r}(t) = Re(\vec{r}_e e^{-i\omega t})$; $= -\frac{e}{m} E_0 e^{i\phi}$

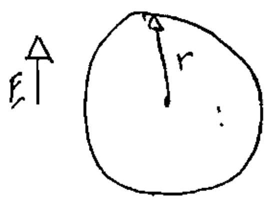
$r_e = \left[\frac{-e/m}{\omega_0^2 - \omega^2 - i\omega\Gamma}\right] E_0 e^{i\phi}$

1. $\sigma = \frac{8\pi}{3}\left(\frac{e^2}{4\pi\epsilon_0 m_e c^2}\right)\left[\frac{2\pi c}{\lambda}\right]^4$

$= \frac{8\pi}{3}\left(\frac{(1.602\times10^{-19} C)^2}{4\pi\, 8.85\times10^{-12} \frac{C^2 s^2}{kg\, m^2} (9.109\times10^{-31})^2 kg^2}\right)\left[\frac{2\pi\, 2.998\times10^{8} m/s}{500\,nm \frac{1\,m}{10^9\,nm}}\right]^4 = 4.693\times10^{95} \frac{1}{\frac{kg\cdot m^3}{s^4}}$ $\boxed{~~7.74\times10^{91}~~ \frac{mol}{N\cdot m}}$

$R = \frac{\left(\frac{2\pi c}{\lambda}\right)^4}{c^4} \frac{|1.0003-1|^2}{4\pi^2\, 2.686\times10^{19} cm^{-3}} \sin^2 90° = 0.53\times10^{39} \frac{1}{cm^2}$; $I = I_0 e^{-\frac{(x-\bar{x})^2}{2\sigma}}$ $\boxed{\text{Attenuating Distance}}$

3. Verify 9.2.12



$\Phi_1 = \sum\left(A_n r^n + \frac{B_n}{r^{n+1}}\right) P_n \cos\theta$

$\Phi_2 = \sum\left(C_n r^n + \frac{D_n}{r^{n+1}}\right) P_n \cos\theta$

Boundary Conditions:

1. $(\Phi_1)_{r\to\infty} = -E_0 z = -E_0 r\cos\theta$
2. $(\Phi_1)_{r=a} = (\Phi_2)_{r=a}$ since ϕ is continuous.
3. $\epsilon_1 \left(\frac{d\Phi_1}{dr}\right)_{r=a} = \epsilon_2 \left(\frac{d\Phi_2}{dr}\right)$
4. At $(r=0)$, Φ_2 must not have singularity.

First Boundary Condition: $\Phi_1 = -E_0 r\cos\theta = \sum\left(\underbrace{\cancel{A_n r^n}}_{0 \text{ as } r\to\infty\ (1)} + \frac{B_n}{r^{n+1}}\right) P_n \cos\theta - E_0 r\cos\theta$

$\Phi_2 = \sum\left(C_n r^n + \underbrace{\cancel{\frac{D_n}{r^{n+1}}}}_{0\ @r=0,\text{ must not be zero.}}\right) P_n \cos\theta$; $\frac{B_n}{a^{n+1}} = C_n a^n$

Thus,

$-\epsilon_1 (n+1) \frac{B_n}{a^{n+2}} = \epsilon_2 n C_n a^{n-1}$

When $n=1$; $\frac{B_1}{a^2} - E_0 a = C_1 a$

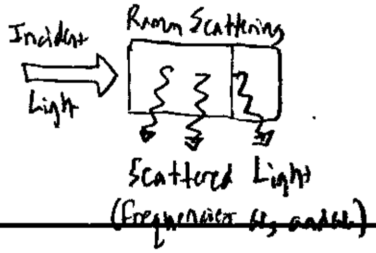
$\epsilon_1\left(\frac{2B_1}{a^3} + E_0\right) = -\epsilon_2 C_1$; $B_1 = \frac{\epsilon_2 - \epsilon_1}{\epsilon_2 + 2\epsilon_1} a^3 E_0$

$C_1 = \frac{3\epsilon_1}{2\epsilon_1 + \epsilon_2} E_0$

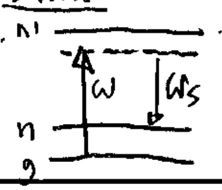
Signifying, $\Phi_1 = \left(\frac{\epsilon_2 - \epsilon_1}{\epsilon_2 + 2\epsilon_1} \frac{a^3}{r^3} - 1\right) E_0 z$; $\Phi' = \frac{\epsilon_2 - \epsilon_1}{\epsilon_2 + 2\epsilon_1} \frac{a^3}{r^3} E_0 z$ $\boxed{m = \frac{\epsilon_2 - \epsilon_1}{\epsilon_2 + 2\epsilon_1} a^3 E_0}$

$\Phi_2 = -\frac{3\epsilon_1}{2\epsilon_1 + \epsilon_2} E_0 z$; $\Phi_2' = \frac{\epsilon_2 - \epsilon_1}{\epsilon_2 + 2\epsilon_1} E_0 z$; $E_2 = \frac{3\epsilon_1}{2\epsilon_1 + \epsilon_2} E_0$

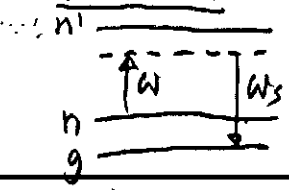
Chapter 10: Stimulated Raman Scattering and Stimulated Rayleigh-Wing Scattering



Stokes:



Anti-Stokes



$P_s = D m_L (m_s + 1)$

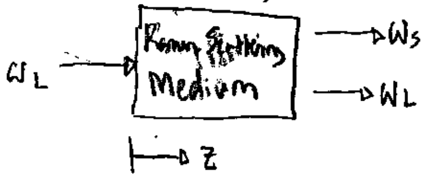
Probability; Proportionality; mean number per mode; mean number of photons per Stokes mode

$$\underbrace{\frac{dm_s}{dt}}_{\text{"Rate of mean photon occupation"}} = D m_L (m_s+1) \; ; \; \underbrace{\frac{dm_s}{dz}}_{\text{"Spatial Growth Rate"}} = \frac{1}{c/n}\frac{dm_s}{dt} = \frac{1}{c/n} D m_L (m_s+1)$$

Note: Sometimes Stokes ($m_s \ll 1$); $\frac{dm_s}{dz} = \frac{1}{c/n}\frac{dm_s}{dt} = \frac{1}{c/n} D m_L$

$$m_s(z) = m_s(0) + \frac{1}{c/n} D m_L \cdot z$$

Raman Scattering Medium:



Opposite Emitting Case:

$$\frac{dm_s}{dz} = \frac{1}{c/n} D m_L m_s \quad (m_s \gg 1)$$

$$m_s(z) = m_s(0) e^{Gz} \quad (m_s \gg 1)$$

where $G = \frac{D m_L}{c/n}$

Total Rate of Laser Loss:

$$\frac{dm_L}{dz} = -Mb\frac{dm_s}{dz} = \frac{-D m_L M b}{c/n}$$

Number of modes the system can radiate. Geometric factor

Direction of a particular Stokes mode:

$$b = \frac{\int |F(\theta,\phi)|^2 d\Omega/4\pi}{|F(\theta_s,\phi_s)|^2}$$

Total Number of Stokes Modes:

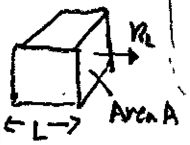
$$M = \frac{V\omega_s^2 \Delta\omega}{\pi^2 (c/n)^3}$$

$$\underbrace{\frac{dm_L}{dz} = -N\sigma m_L}_{\text{"Rate of loss of laser photon by cross section"}}$$

Parameter D in terms of cross section σ:

$$D = \frac{N\sigma(c/n)}{Mb}$$

Geometry of the region which laser and Stokes are defined:



Raman Gain Coefficient:

$$G = \frac{N\sigma\pi^2 c^3 m_L}{V\omega_s^2 \Delta\omega b n^3} = \frac{N\pi^2 c^3 m_L}{V\omega_s^2 b n^3}\left(\frac{\partial\sigma}{\partial\omega}\right)$$

Raman Gain Coefficient (no volume):

$$G = \frac{N\pi^2 c^2}{\omega_s^2 b n^2 \hbar\omega_L}\left(\frac{\partial\sigma}{\partial\omega}\right)_0 I_L$$

cross-section line-center: $\sigma \doteq \left(\frac{\partial\sigma}{\partial\omega}\right)_0 \Delta\omega$ Laser Intensity: $I_L = \frac{m_L\hbar\omega_L}{A(nL/c)} = \frac{m_L\hbar\omega_L c}{Vn}$

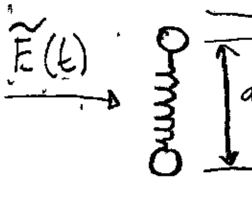
Differential Cross-section:

$$\left(\frac{\partial\sigma}{\partial\omega}\right)_0 = 4\pi b\left(\frac{\partial^2\sigma}{\partial\omega\partial\Omega}\right)$$

so,

$$G = \frac{4\pi^3 N c^2}{\omega_s^2\hbar\omega_L n_s^2}\left(\frac{\partial^2\sigma}{\partial\omega\partial\Omega}\right) I_L$$

Stimulated Raman Scattering Described by Nonlinear Polarization



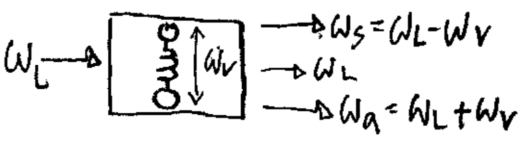
$$q_0 + \tilde{q}(t) \; ; \; \frac{d^2\tilde{q}}{dt^2} + 2\gamma\frac{d\tilde{q}}{dt} + \omega_v^2 \tilde{q} = \frac{F(t)}{m}$$

$$\tilde{\alpha}(t) = \alpha_0 + \left(\frac{\partial\alpha}{\partial q}\right)_0 \tilde{q}(t) \; ; \; \tilde{n}(t) = \sqrt{\epsilon(t)} = [1 + N\tilde{\alpha}(t)]^{1/2}$$

Dipole Moment: $\tilde{p}(z,t) = \epsilon_0 \alpha \tilde{E}(z,t)$; $W = \frac{1}{2}\langle \tilde{p}(z,t)\cdot\tilde{E}(z,t)\rangle$

$$= \frac{1}{2}\epsilon_0 \alpha \langle \tilde{E}^2(z,t)\rangle \; ; \; F = \frac{\partial W}{\partial q} = \frac{\epsilon_0}{2}\left(\frac{d\alpha}{dq}\right)_0 \langle \tilde{E}^2(z,t)\rangle$$

Common Scattering:



Driving Frequency:

$$\tilde{I}(t) = I_0 + I_1 \cos[(\omega_L - \omega_s)t + \phi]$$

$$\frac{dA_1}{\partial z} + \frac{1}{c/n}\frac{\partial A_1}{\partial t} = \frac{i\omega\gamma_e}{2nc\rho_0}\rho A_2 \; ; \quad \frac{dA_1}{dz} = \frac{i\epsilon_0 \omega q^2 \gamma_e^2}{2nc\rho_0}\frac{|A_2|^2 A_1}{\Omega_B^2 - \Omega^2 - i\Omega\Gamma_B} \; ; \quad \frac{dI_2}{dz} = -g I_1 I_2$$

$$-\frac{\partial A_2}{\partial z} + \frac{1}{c/n}\frac{\partial A_2}{\partial t} = \frac{i\omega\gamma_e}{2nc\rho_0}\rho^* A_1 \; ; \quad \frac{dA_2}{dz} = \frac{-i\epsilon_0 \omega q^2 \gamma_e^2}{2nc\rho_0}\frac{|A_1|^2 A_2}{\Omega_B^2 - \Omega^2 + i\Omega\Gamma_B} \; ; \quad \frac{dI_2}{dz} = -g I_1 I_2$$

To solve the coupled dependent equation, assume a constant:

Assume I_1 = constant ; $I_2(z) = I_2(L) e^{g I_1 (L-z)}$

-or-

I_2 = constant ; $I_1(z) = I_1(0) e^{-g I_2 z}$

Gain Factor:

$$g = g_0 \frac{(\Gamma_B/2)^2}{(\Omega_B - \Omega)^2 + (\Gamma_B/2)^2}$$

$$g_0 = \frac{\gamma_e^2 \omega^2}{n v c^3 \rho_0 \Gamma_B}$$

For fused-silica, SiO_2 [$\Omega_B/2\pi = 25{,}800$ MHz, $\Gamma_B/2\pi = 78$ MHz,

Assume fused silica $n = 1.0001$ $\qquad g_0 = 0.045$ m/GW].

$\lambda = \frac{c}{\nu} = \frac{2\pi c}{\omega}$

$$\gamma_e = \frac{(n^2-1)(n^2+2)}{3} = \frac{(1.001^2-1)(1.001^2+2)}{3} = 2.002 \times 10^{-4}$$

$2\pi\nu = \omega$

$$\omega = \frac{2\pi c}{\lambda} = \frac{2\pi(2.998\times10^8 \text{ m/s})}{350 \text{ nm} \times \frac{1 \text{ m}}{10^9 \text{ nm}}} = 5.38 \times 10^{15} \text{ 1/s}$$

v = "velocity of sound" = 1.1×10^5 m/sec

$\rho_0 = 2.202$ g/cm³

$T_p = \Gamma_B^{-1} = 10$ ns ; $\Gamma_B = 1 \times 10^8 \, s^{-1}$

$$g_0 = \frac{1.78 \times 10^{-23} \, \cancel{s}}{\frac{m^4}{s^2}\cdot\frac{g}{cm^3}\cdot\frac{1}{s}\times\left(\frac{100 \text{ cm}}{1 \text{ m}}\right)^3 \cdot \frac{1 \text{ kg}}{1000 \text{ g}}} = 1.77 \times 10^{-26} \frac{s^2}{m \cdot kg}$$

"Much smaller than the book.

$\frac{s}{m^2 \, kg}$

$$g > 0.045 \text{ m/GW} \frac{(1\times10^8 s^{-1}/2)^2}{(25{,}800 \text{ MHz}\cdot 2\pi - \Omega) + (1\times10^8 s^{-1}/2)^2}$$

$(25{,}800 \text{ MHz} - 2\pi) + (\Omega) > 0.045 \text{ m/GW} (1\times10^8 s^{-1}/2)^2 - (1\times10^8 s^{-1}/2)^2$

$(1 \frac{m}{GW}) \; 25{,}800 \text{ MHz} \cdot 2\pi - 0.045 \text{ m/GW} (1\times10^8 s^{-1}/2)^2 + (1\times10^8 s^{-1}/2)^2 > \Omega = \omega_1 - \omega_2$

(1 m/GW)

$\boxed{(\Omega_B - \Omega)^2 = \Gamma_B/2 \, ?}$

5. Explain why pulse duration is low in duration than the excitation radiation?

$\Delta E \Delta t \geq \hbar/2 \quad \Delta E = (\text{Excitation} - SBS) \gg 1 \; ; \; \Delta t \geq \hbar/2\Delta E \ll 1$

$\frac{dI_1}{dz} = g I_1 I_2$; $\frac{dI_2}{dz} = -g I_1 I_2$; Assume I_1 is constant. $I_2(z) = I_2(L) e^{g I_1 (L-z)}$

$E_2(z) = \epsilon_0 E_2(L) e^{g \epsilon E (L-z)}$

When $L = 0$, and $E \gg$

Chapter 10 Intensity of Raman Scattering: $\tilde{I}(t) = I_0 + I_1 \cos[(\omega_L-\omega_s)t+\phi]$

Total Optical Field: $\tilde{E}(z,t) = A_L e^{i(k_L z-\omega_L t)} + A_s e^{i(k_s z-\omega_s t)} + c.c.$ — where $K = k_L - k_s$, and $\Omega = \omega_L - \omega_s$

Time varying Force: $\tilde{F}(z,t) = \epsilon_0 \left(\frac{\partial\alpha}{\partial q}\right)[A_L A_s^* e^{i(Kz-\Omega t)} + c.c.]$

The solution to $\frac{d^2\tilde{q}}{dt^2} + 2\gamma\frac{d\tilde{q}}{dt} + \omega_v^2\tilde{q} = \frac{\tilde{F}(t)}{m}$ "Exponential, sine, or cosine"

$$\tilde{q}(t) = q(\Omega)e^{i(Kz-\Omega t)} + c.c.$$ Trial

$$-\Omega^2 q(\Omega) - 2i\Omega\gamma q(\Omega) + \omega_v^2 q(\Omega) = \frac{\epsilon_0}{m}\left(\frac{\partial\alpha}{\partial q}\right)_0 A_L A_s^*$$ Derived from time-varying energy

Thus we find, $$q(\Omega) = \frac{(\epsilon_0/m)(\partial\alpha/\partial q)_0 A_L A_s^*}{\omega_v^2 - \Omega^2 - 2i\Omega\gamma}$$

The polarization would then be defined as:

$$\tilde{P}(z,t) = N\tilde{p}(z,t) = \epsilon_0 N\tilde{\alpha}(z,t)\tilde{E}(z,t) = \epsilon_0 N\left[\alpha_0 + \left(\frac{\partial\alpha}{\partial q}\right)_0 \tilde{q}(z,t)\right]\tilde{E}(z,t)$$

and consequently,

$$\tilde{P}^{NL}(z,t) = \epsilon_0 N\left(\frac{\partial\alpha}{\partial q}\right)_0 [q(\Omega)e^{i(Kz-\Omega t)} + c.c.] \times [A_L e^{i(k_L z-\omega_L t)} + \underbrace{A_s e^{i(k_s z-\omega_s t)}}_{\text{"Stokes polarization"}}]$$

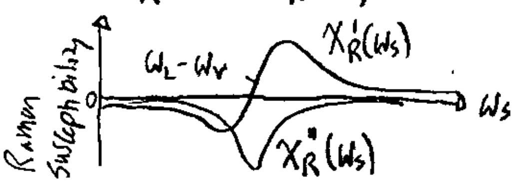
Stokes polarization continued: $\tilde{P}_s^{NL}(z,t) = P(\omega_s)e^{-i\omega_s t} + c.c.$

Complex amplitude: $P(\omega_s) = N\epsilon_0\left(\frac{\partial\alpha}{\partial q}\right)_0 q^*(\Omega)A_L e^{ik_s z}$ plugging in $q(\Omega)$

$$= \frac{(\epsilon_0^2 N/m)(\partial\alpha/\partial q)_0^2 |A_L|^2 A_s}{\omega_v^2 - \Omega^2 + 2i\Omega\gamma} e^{ik_s z}$$

$$= 6\epsilon_0 \chi_R(\omega_s)|A_L|^2 A_s e^{ik_s z}$$

The real and Imaginary components of $\chi_R(\omega_s)$ $= \chi_R'(\omega_s) + i\chi_R''(\omega_s)$



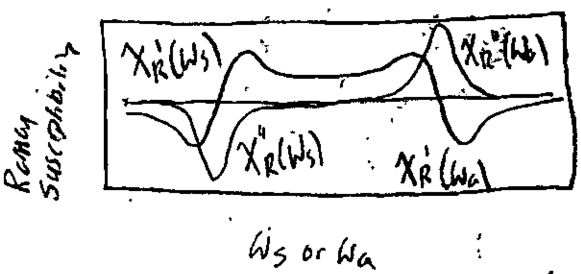
Where $$\chi_R(\omega_s) = \frac{\epsilon_0(N/6m)(\partial\alpha/\partial q)_0^2}{\omega_v^2 - (\omega_L-\omega_s)^2 + 2i(\omega_L-\omega_s)\gamma}$$

Near Resonance: $$\chi_R(\omega_s) = \frac{(\epsilon_0 N/12m\omega_v)(\partial\alpha/\partial q)_0^2}{[\omega_s - (\omega_L-\omega_v)] + i\gamma}$$ "Resonance Raman"

The field amplitude: $\frac{dA_s}{dz} = -\kappa_s A_s$; $\kappa_s = -3i\frac{\omega_s}{n_s c}\chi_R(\omega_s)|A_L|^2$

Anti-Stokes: $$\chi_R(\omega_a) = \frac{\epsilon_0(N/6m)(\partial\alpha/\partial q)_0^2}{\omega_v^2 - (\omega_L-\omega_a)^2 + 2i(\omega_L-\omega_a)\gamma}$$; since $\omega_L - \omega_s = -(\omega_L - \omega_a)$, $\chi_R(\omega_a) = \chi_R(\omega_s)^*$

Approximated as: $$\chi_R(\omega_a) = \frac{-(\epsilon_0 N/12m\omega_v)(\partial\alpha/\partial q)_0^2}{[\omega_a - (\omega_L+\omega_v)] + i\gamma}$$



Amplitude $\frac{dA}{dz} = -\kappa_a A_a$;

$$\kappa_a = -3i\frac{\omega_a}{n_a c}\chi_R(\omega_a)|A_L|^2$$

N: The "gain" for stokes and anti-stokes is $$g(\Omega) = \frac{(\epsilon_0/m)(\partial\alpha/\partial q)_0(A_L A_s^* + A_a^* A_L)}{\omega_v^2 - \Omega^2 - 2i\Omega\gamma}$$

An amplitude would be best represented by 10^{12} proportionality.

(Note: Coherent Anti-stokes Raman Spectroscopy (CARS)
Coherent Stokes Raman Spectroscopy (CSRS)

Chapter 11: Electrooptic and Photorefractive Effects

$P_i(\omega) = 2\epsilon_0 \sum_{jk} \chi^{(2)}_{ijk}(\omega = \omega + 0) E_j(\omega) E_k(0)$

Kerr electrooptic effect: $P_i(\omega) = 3\epsilon_0 \sum_{jkl} \chi^{(3)}_{ijkl}(\omega = \omega + 0 + 0) E_j(\omega) E_k(0) E_l(0)$ "Nonlinear Polarization"

Linear Electrooptic Effect: Anisotropic material with field vector $\vec{D}$ and $\vec{E}$:

$$D_i = \epsilon_0 \sum \epsilon_{ij} E_j \quad \text{or explicitly,} \quad \begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix} = \epsilon_0 \begin{bmatrix} \epsilon_{xx} & \epsilon_{xy} & \epsilon_{xz} \\ \epsilon_{yx} & \epsilon_{yy} & \epsilon_{yz} \\ \epsilon_{zx} & \epsilon_{zy} & \epsilon_{zz} \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix}$$

If we assume the dielectric constant is symmetric about $i \neq j$.
Then a real symmetric matrix would be: $\begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix} = \epsilon_0 \begin{bmatrix} \epsilon_{xx} & 0 & 0 \\ 0 & \epsilon_{yy} & 0 \\ 0 & 0 & \epsilon_{zz} \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix}$

Energy transfer density per unit volume: $U = \frac{1}{2}\vec{D}\cdot\vec{E} = \frac{1}{2}\epsilon_0 \sum_{ij} \epsilon_{ij} E_i E_j$

The shape of the ellipsoid:

$$= \frac{1}{2\epsilon_0}\left[\frac{D_x^2}{\epsilon_{xx}} + \frac{D_y^2}{\epsilon_{yy}} + \frac{D_z^2}{\epsilon_{zz}}\right]$$ shape of an ellipsoid.

$$X = \left(\frac{1}{2\epsilon_0 U}\right)^{1/2} D_x,\; Y = \left(\frac{1}{2\epsilon_0 U}\right)^{1/2} D_y,\; Z = \left(\frac{1}{2\epsilon_0}\right)^{1/2} D_z$$

and $\dfrac{X^2}{\epsilon_{xx}} + \dfrac{Y^2}{\epsilon_{yy}} + \dfrac{Z^2}{\epsilon_{zz}} = 1$ "optical Indicatrix" "index ellipsoid"

In other coordinate systems,

$$\left(\frac{1}{n^2}\right)_1 x^2 + \left(\frac{1}{n^2}\right)_2 y^2 + \left(\frac{1}{n^2}\right)_3 z^2 + 2\left(\frac{1}{n^2}\right)_4 yz + 2\left(\frac{1}{n^2}\right)_5 xz + 2\left(\frac{1}{n^2}\right) xy = 1$$

Notes: Used to describe the optical properties of an anisotropic material
Semi-major and minor axes of a crystal

Impermeability Tensor (η_{ij}): $E_i = \frac{1}{\epsilon_0}\sum_j \eta_{ij} D_j$ "Inverse of D-field"

$$\text{So, } \eta_{ij} = (\epsilon^{-1})_{ij};\; U = (1/2\epsilon_0)\sum_{ij} \eta_{ij} D_i D_j$$

$$1 = \eta_{11}x^2 + \eta_{22}y^2 + \eta_{33}z^2 + 2\eta_{12}xy + 2\eta_{23}yz + 2\eta_{13}xz$$

By comparison,

$$\left(\frac{1}{n_2}\right) = \eta_{11};\; \left(\frac{1}{n_2}\right) = \eta_{22};\; \left(\frac{1}{n_2}\right) = \eta_{33};\; \left(\frac{1}{n_2}\right) = \eta_{12};\; \left(\frac{1}{n_2}\right) = \eta_{23};\; \left(\frac{1}{n_2}\right) = \eta_{13}$$

Next assume a power series, $\eta_{ij} = \eta^{(1)}_{ij} + \sum_k r_{ijk} E_k + \sum_{kl} s_{ijkl} \cdot E_k \cdot E_l + \ldots$

A simplified r_{ij}, r_{ijk} is presented:

$$h = \begin{cases} 1 & \text{for } ij = 11 \\ 2 & \text{for } ij = 22 \\ 3 & \text{for } ij = 33 \\ 4 & \text{for } ij = 23, 32 \\ 5 & \text{for } ij = 13, 31 \\ 6 & \text{for } ij = 12, 21 \end{cases}$$

$\Delta\left(\frac{1}{n^2}\right)_i = \sum r_{ij} E_j$; This relationship is a 2D-matrix:

$$\begin{bmatrix} \Delta(1/n^2)_1 \\ \Delta(1/n^2)_2 \\ \Delta(1/n^2)_3 \\ \Delta(1/n^2)_4 \\ \Delta(1/n^2)_5 \\ \Delta(1/n^2)_6 \end{bmatrix} = \underbrace{\begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \\ r_{41} & r_{42} & r_{43} \\ r_{51} & r_{52} & r_{53} \\ r_{61} & r_{62} & r_{63} \end{bmatrix}}_{\text{"Electroptic Coefficients"}} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix}$$

$\propto\left(\frac{1}{n^2}\right) \propto$ Rate $\cdot E$

As example; Potassium Dihydrogen Phosphate [KDP] and ADP have point group symmetry $\bar{4}2m$.

$$r_{ij} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ r_{41} & 0 & 0 \\ 0 & r_{41} & 0 \\ 0 & 0 & r_{63} \end{bmatrix}$$ (for class $\bar{4}2m$) "two independent elements"

Barium Titanate

$$r_{ij} = \begin{bmatrix} 0 & 0 & r_{13} \\ 0 & 0 & r_{13} \\ 0 & 0 & r_{33} \\ 0 & r_{42} & 0 \\ r_{42} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$ (for class 4mm).

See table 11.2.1 for Electroptic coefficients and refractive index

Electrooptic Modulators

How to construct an electrooptic modulator using KDP.

KDP is uniaxial, with no applied electric field the index ellipsoid in the standard crystallographic coordinate system by the equation.

$\frac{X^2}{n_o^2} + \frac{Y^2}{n_o^2} + \frac{Z^2}{n_e^2} = 1$; If an electric field is applied to the crystal, then the index ellipsoid takes the form:

$\frac{X^2}{n_o^2} + \frac{Y^2}{n_o^2} + \frac{Z^2}{n_e^2} + 2r_{41} E_X YZ + 2r_{41} E_Y XZ + 2r_{63} E_Z XY = 1$ "No-longer uniaxial"

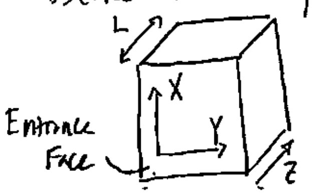
Sometimes electric field is only in the z-direction $[E_z]$

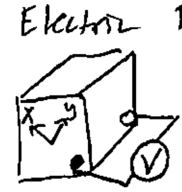
$\frac{X^2}{n_o^2} + \frac{Y^2}{n_o^2} + \frac{Z^2}{n_e^2} + 2r_{63} \cdot E_z \cdot X \cdot Y = 1$; $X = \frac{x-y}{\sqrt{2}}$; $Y = \frac{x+y}{\sqrt{2}}$; $Z = z$

To simplify. $\left(\frac{1}{n_o^2} + r_{63} \cdot E_z\right)x^2 + \left(\frac{1}{n_o^2} - r_{63} E_z\right)y^2 + \frac{z^2}{n_e^2} = 1$

Alternatively, $\frac{x^2}{n_x^2} + \frac{y^2}{n_y^2} + \frac{z^2}{n_e^2} = 1$; where $r_{63} E_z \ll 1$, the new principal axes are
$n_x = n_o - \frac{1}{2} n_o^3 r_{63} E_z$; $n_y = n_o + \frac{1}{2} n_o^3 r_{63} \cdot E$

As an image: Absence of an applied field   Principle Axes in the presence of an Electric Field.

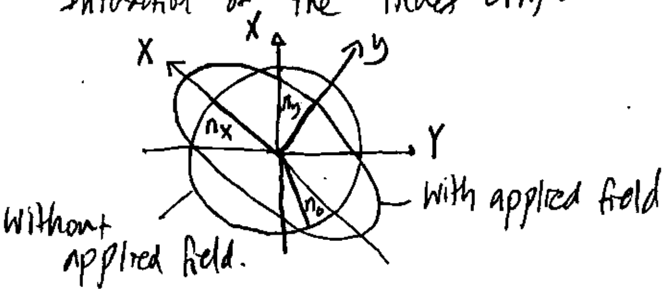




When a beam of light enters the medium, then the x-and-y phase difference change.

$\Gamma = (n_y - n_x)\frac{\omega L}{c}$ "retardation"

Intersection of the index ellipsoid:



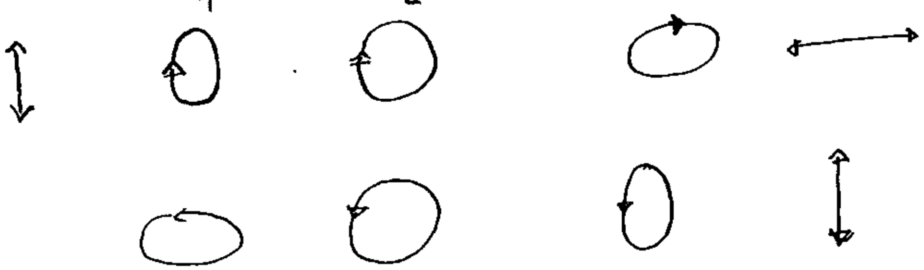
From earlier equations ; $\Gamma = \dfrac{n_o^3 r_{63} \cdot E_z \omega L}{c}$; when $E_z = V/L$ "longitudinal" electric light modulator"

$= \dfrac{n_o^3 r_{63}\, \omega V}{c}$; $V_{\lambda/2} = \dfrac{\pi c}{\omega n_o^3 r_{63}}$

$\boxed{V = \text{Voltage !!!}}$

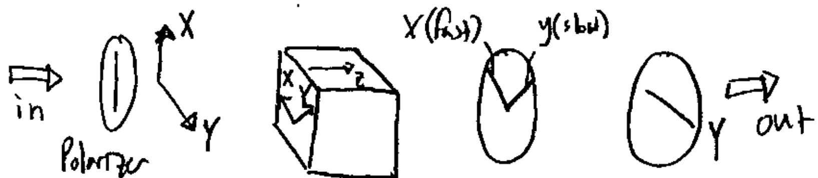
"Half-wave Voltage" $\Gamma = \pi \dfrac{V}{V_{\lambda/2}}$
~10KeV = Visible

$\Gamma = 0$ $\quad \frac{\pi}{4}$ $\quad \frac{\pi}{2}$ $\quad \frac{3\pi}{4}$ $\quad \pi$



$\frac{5\pi}{4}$ $\quad \frac{3\pi}{2}$ $\quad \frac{7\pi}{4}$ $\quad 2\pi$

Construction of a voltage-controllable intensity modulator



Enters Polarizer → X orientation → Different propagation velocities → Change fast and slow axis →

$\dfrac{dx}{dt} = -\dfrac{\partial\phi}{k\partial t}\dfrac{\partial x}{\partial\phi} C = \dfrac{\omega}{k}$ $\qquad \Gamma_B = \dfrac{\pi}{2}$

$\phi = kx - \omega t$ $\qquad \Gamma = \pi\dfrac{V}{V_{\lambda/2}} + \dfrac{\pi}{2}$

Group velocity $\qquad$ y-shifts phase.

$v_g = \dfrac{\partial\omega}{\partial k} = \dfrac{c}{n + \omega\frac{\partial n}{\partial\omega}}$

$\tilde{E} = E_{in} e^{-i\omega t} + c.c$; $E_{in} = E_{in}\hat{x} = \dfrac{E_{in}}{\sqrt{2}}(\hat{x} + \hat{y})$; $E = \dfrac{E_{in}}{\sqrt{2}}(\hat{x} + e^{i\Gamma} \cdot \hat{y})$

$\hat{Y} = (-\hat{x} + \hat{y})/\sqrt{2}$ "transmitted"

$E_{out} = \dfrac{E_{in}}{2}(-1 + e^{i\Gamma})\hat{Y}$; $T = \dfrac{|E_{out}|^2}{|E_{in}|^2} = \sin^2(\Gamma/2)$

$V(t) = V_m \sin \omega_m t$

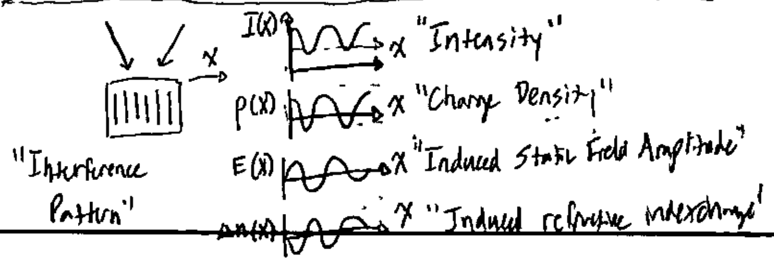
Thus, the retardation is given by $\Gamma = \dfrac{\pi}{2} + \dfrac{\pi V_m}{V_{\lambda/2}} \sin \omega_m t$

$$T = \sin^2\left(\frac{\pi}{4} + \frac{\pi V_m}{2V_{\lambda/2}} \sin \omega_m t\right) = \frac{1}{2}\left[1 + \sin\left(\frac{\pi V_m}{V_{\lambda/2}} \sin \omega_m t\right)\right] \simeq \frac{1}{2}\left(1 + \frac{\pi V_m}{V_{\lambda/2}} \sin \omega_m t\right)$$

Best described by $\phi = (n_x - n_o)\dfrac{\omega L}{c} = -\dfrac{n_o^3 r_{63} E_z \omega L}{2c} = \dfrac{n_o^3 r_{63} V \omega}{2c}$.

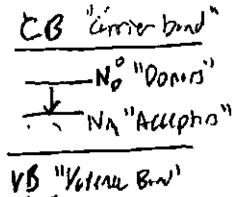
Introduction to the Photorefractive Effect

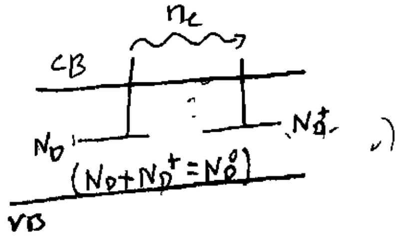
$$\phi = (n_x - n_y)\frac{\omega L}{c} \propto \frac{n^5 r_{33} E^{\omega L}}{2c} \propto \frac{n_{23} V \omega}{2c}$$



Note: The change of index of refraction which results from optically induced electrons and holes.
Cannot be described as nonlinear susceptibility.

Photorefractive Equations: Assumption





Assume variational population levels described by the rate equations:

$$\frac{\partial N_D^+}{\partial t} = (sI + \beta)(N_D^0 - N_D^+) - \gamma n_e N_D^+ \quad ; \quad \frac{\partial n_e}{\partial t} = \frac{\partial N_D^+}{\partial t} + \frac{1}{e}(\nabla \cdot j)$$

"Photoionization cross-section of donor" "Thermal Generation Rate" Recombination Effects "charge of electron^{-1}" "Current Density"

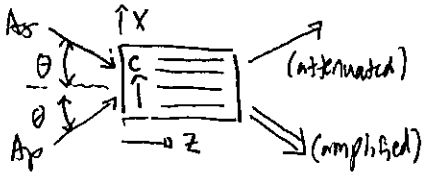
Current flow described as: $\vec{j} = n_e e \mu E + eD\nabla n_e + j_{PH}$

"Electron mobility" "Diffusion constant" "photogalvanic contribution"

$\epsilon_{dc} \nabla \cdot \vec{E} = -e(n_e + N_A - N_D^+)$ "Maxwell Equation" $\Delta\epsilon = -\epsilon^2 r_{eff} |E|$ "change of dielectric constant"

The optical field obeys the wave equation: $\underbrace{\nabla^2 E_{opt} + \frac{1}{c^2}\frac{\partial^2}{\partial t^2}(\epsilon + \Delta\epsilon)\tilde{E}_{opt} = 0}_{\text{"Photorefractive Phenomena"}}$

Two-beam Coupling of Photorefractive Materials



$$\tilde{E}_{opt}(r,t) = [\underbrace{A_p(z)e^{ik_p r} + A_s(z)e^{ik_s r}}_{\text{"slow-contributions"}}]e^{-i\omega t} + c.c.$$

$I = n_0 \epsilon_0 c \langle \tilde{E}_{opt}^2 \rangle$ or $I = I_0 + (I_1 e^{iqx} + c.c)$

Where $I_0 = 2n_0\epsilon_0 c(|A_p|^2 + |A_s|^2)$; $I_1 = 2n_0\epsilon_0 c(A_p A_s^*)(\hat{e}_p \cdot \hat{e}_s)$

$$\underbrace{q = q\hat{x} = k_p - k_s}_{\text{"Grating Wavevector"}}$$

$I = I_0[1 + m\cos(qx + \phi)]$; $m = 2|I_1|/I_0$

Modulation Index (m) ; where $\phi = \tan^{-1}(\mathrm{Im}\,I_1/\mathrm{Re}\,I_1)$

Steady State Solutions: $E = E_0 + (E_1 e^{iqx} + c.c)$ $\quad j = j_0 + (j_1 e^{iqx} + c.c)$

$n_e = n_{e0} + (n_{e1} e^{iqx} + c.c)$ $\quad N_D^+ = N_{D0}^+ + (N_{D1}^+ e^{iqx} + c.c)$

$(sI_0 + \beta)(N_D^0 - N_{D0}^+) = \gamma n_{e0} N_{D0}^+$: $j_0 = \text{constant}$ $\quad E = E\hat{x}$; $j = j\hat{x}$

$j_0 = n_{e0} e \mu E_0 + j_{ph,0}$; $N_{D0}^+ = n_{e0} + N_A$

$N_{D0}^+ = N_A$; $n_{e0} = \dfrac{(sI_0 + \beta)(N_D^0 - N_D)}{\gamma N_A}$ "Number density"

First-order quantities with spatial dependence e^{iqx} ($E_0 = 0$):

$sI_1(N_D^0 - N_A) - (sI_0 + \beta)N_{D1}^+ = \gamma n_{e0} N_{D1}^+ + \gamma n_{e1} N_A$, $j_1 = 0$; $-n_{e0} e E_1 = iq k_B T n_{e1}$

$iq \epsilon_0 \epsilon_{dc} E_1 = -e(n_{e1} - N_{D1}^+)$

To find the amplitude of the spatially varying part:

$$\underbrace{E_1 = -i\left(\frac{sI_1}{sI_0 + \beta}\right)\left(\frac{E_D}{1 + E_D/E_q}\right)}_{\text{Varying part of electric field.}}$$

Where $E_D = \dfrac{q k_B T}{e}$; $E_q = \dfrac{e}{\epsilon_0 \epsilon_{dc} q} N_{eff}$

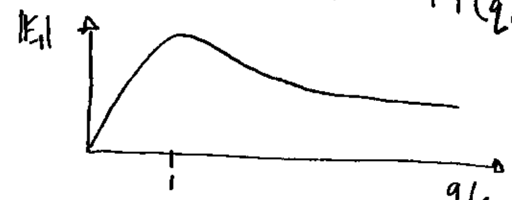
$N_{eff} = N_A(N_D^0 - N_A)/N_D^0$

"Effective Trap Density"

Total Dependence of Electricity on q: "Debye Huckel Screening"

$$E_1 = -i\left(\frac{sI_1}{sI_0 + \beta}\right) E_{opt} \frac{2(q/q_{opt})}{1 + (q/q_{opt})^2} \;;\; q_{opt} = \left(\frac{N_{eff} e^2}{k_B T \epsilon_0 \epsilon_{dc}}\right)$$

$$E_{opt} = \left(\frac{N_{eff} k_B T}{4 \epsilon_0 \epsilon_{dc}}\right)^{1/2}$$



$q = 2n \frac{\omega}{c} \sin\theta$; $E_1 = -i \dfrac{A_p A_s^*}{|A_s|^2 + |A_p|^2} (\hat{e}_p \cdot \hat{e}_s) E_m$ where $E_m = \dfrac{E_D}{1 + E_D/E_q}$

"Spatial Growth rate of Signal"

$\boxed{\text{"ordinary"}}$

"For a geometry" $\epsilon^2 r_{eff} = \sum_{ijklm} r_{ijk} (\epsilon_{il} \hat{e}_l^s)(\epsilon_{jm} \hat{e}_m^p) \hat{q}_k$ $\quad r_{eff} = r_{13} \sin\left(\dfrac{\alpha_s + \alpha_p}{2}\right)$

Change of Dielectric constant: $\Delta\epsilon = -\epsilon^2 r_{eff} E_1$

$$r_{eff} = n^{-4}\left[n_o^4 r_{13} \cos\alpha_s \cos\alpha_p + 2n_e^2 n_o^2 r_{42} \cos\tfrac{1}{2}(\alpha_s + \alpha_p) + n_e^4 r_{33} \sin\alpha_s \sin\alpha_p\right] \sin\tfrac{1}{2}(\alpha_s + \alpha_p)\Big)$$

$$P^{NL} = (\Delta\epsilon e^{iqr} + c.c)(A_s e^{ik_s r} + A_p e^{ik_p r})$$

$q = k_p - k_s$

α_s and α_p ; propagation vector of signal and pump.

$$\underbrace{P_s^{NL} = \Delta\epsilon^* A_p e^{ik_s r} = -i \epsilon^2 r_{eff} E_m \frac{|A_p|^2 A_s}{|A_p|^2 + |A_s|^2} e^{ik_s r}}_{\text{Nonlinear polarization}}$$

$$P_p^{NL} = \Delta\epsilon A_s e^{ik_p r} = -i \epsilon^2 r_{eff} E_m \frac{-|A_s|^2 A_p}{|A_p|^2 + |A_s|^2} e^{ik_p r}$$

$$2ik\frac{dA_s}{dz_s}e^{ik_s n} = -\frac{w^2}{c^2}P_s^{NL} \; ; \; \frac{dA_s}{dz_s} = \frac{\omega}{2c}n^3 r_{eff} E_m \frac{|A_p|^2 A_s}{|A_p|^2+|A_s|^2} \; ; \; I_s = 2n\epsilon_0 c|A_s|^2$$

$$\frac{dI_s}{dz_s} = (n\epsilon_0 c(A_s^* dA_s/dz_s + c.c.) = \Gamma\frac{I_s I_p}{I_s+I_p}$$

Where, $\Gamma = \frac{\omega}{c}n^3 r_{eff} E_m$

$$\frac{dI_p}{dz_p} = -\Gamma\frac{I_s I_p}{I_s+I_p}$$

Problem 1: Estimate $E_D, E_q, E_{opt}, E_1, r_{eff}, \Delta\epsilon$, and Γ

Assume $N_{eff} = 10^{12}\,cm^{-3}$, $n = 10^{0}$ and $\theta_s = \theta_p = 5^\circ$

$$\underbrace{E_D = \frac{qk_B T}{e} \; ; \; E_q = \frac{e}{\epsilon_0 \epsilon_{dc}}N_{eff}}_{\text{"characteristic fields"}}$$

$$\underbrace{E_1 = -i\left(\frac{sI_1}{sI_0+\beta}\right)\left(\frac{E_0}{1+E_D/E_q}\right)}_{\text{"Static electric field"}} \; ; \; \underbrace{E_{opt} = \left(\frac{N_{eff}k_B T}{4\epsilon_0 \epsilon_{dc}}\right)^{1/2}}_{\text{"Energy of Huckel screening wave number"}}$$

$$\underbrace{\Delta\epsilon = -\epsilon^2 r_{eff}|E|}_{\text{"optical-frequency Dielectric constant change"}}$$

$$\underbrace{r_{eff} = r_{13}\sin\left(\frac{\alpha_s+\alpha_p}{2}\right)}_{\text{"ordinary waves"}} \; ; \; \underbrace{r_{eff} = n^{-4}[n_0^4 r_{13}\cos\alpha_s\cos\alpha_p + 2n_e^2 n_0^2 r_{42}\cos\tfrac{1}{2}(\alpha_s+\alpha_p) + n_e^4 r_{33}\sin\alpha_s\sin\alpha_p]\sin\tfrac{1}{2}(\alpha_s+\alpha_p)}_{\text{"Extraordinary"}}$$

$$\underbrace{\Gamma = \frac{\omega}{c}n^3 r_{eff} E_m}_{\text{"Photorefractive Gain"}}$$

Problem 3: $\underbrace{\chi_{ijk}^{(2)} \text{ relationship to } r_{ijk}}_{\text{Linear electrooptic effect}} \; ; \; \underbrace{\chi_{ijkl} \text{ relationship to } s_{ijkl}}_{\text{Quadratic electrooptic effect}}$

$$\eta_{ij} = \eta_{ij}^{(0)} + \sum r_{ijk}E_k + \sum s_{ijkl}E_k E_l + \dots \; ; \; D = \epsilon_0(E+P) = \epsilon_0 E + [\epsilon_0 E\chi_e + \epsilon_0\chi^{(2)}E^2 + \epsilon_0\chi^{(3)}E^3]$$

$$\epsilon E = \epsilon_0 E + \epsilon_0 \chi E + \epsilon_0\chi^{(2)}E^2 + \epsilon_0\chi^{(3)}E^{3}$$

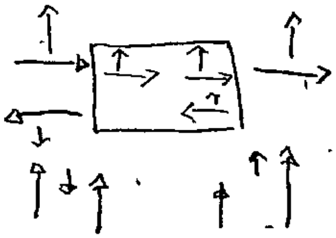
$$\eta_{ij} = (\epsilon)_{ij}^{-1} \; ; \; \epsilon_{ij} = \frac{1}{\eta_{ij}} \qquad \epsilon = \epsilon_0(1+\chi^{(1)}+\chi^{(2)}E+\chi^{(3)}E^2)$$

$$\boxed{\epsilon = (1+\chi^{(1)}+\chi^{(2)}E+\chi^{(3)}E^2) \quad \eta_{ij} = (\eta_{ij}^0+\sum r_{ij}E+\sum s_{ijk}E^2)}$$

Chapter 12: Optically Induced Damage and Multiphoton Absorption

Physical Mechanism:
- Linear Absorption, leading to localized heating and cracking of the optical material. ($\geq 1\mu sec$).
- Avalanche breakdown, dominant for ($\leq 1\mu sec$) and high-intensity
- Multiphoton ionization or multiphoton dissociation
- Direct Field Ionization

Note: Tends to occur at the exiting surface



Avalanche-Breakdown Model

$Q = e\bar{E}d$ where $d = \frac{1}{2}at^2 = \frac{1}{2}(e\bar{E}/m)t^2$

$= e^2E^2t^2/2m$ for $t \leq \tau$

$= e^2E^2t\tau/2m$

time, time interval

Rate at which an electron gains energy $P = \frac{\partial Q}{\partial t} = e^2E^2\tau/2m$

$\frac{dN}{dt} = \frac{fNP}{W}$ ← Absorbed power,

Ionization Threshold

$N(t) = N_0 e^{gt}$ where $g = \frac{fe^2E^2\tau}{2Wm}$

The occurence of laser damage is thus expressed as: $\frac{fe^2\bar{E}^2\tau T_p}{2mW} > \ln(N^{th}/N_0)$

The threshold intensity is thus,

$I_{th} = n\epsilon_0 c\langle \bar{E}^2\rangle = 2n\epsilon_0 c \frac{Wm}{fe^2\tau T_p}\ln(N^{th}/N_0)$

Influence of Laser Pulse Duration: states: the fluence (energy per unit area) required to produce laser damage decreases with pulse duration $T_p^{-1/2}$.

Heat Transport Equation:

$$(\rho C)\frac{\partial \tilde{T}}{\partial t} - K\nabla^2\tilde{T} = N(1-f)\hat{P}$$

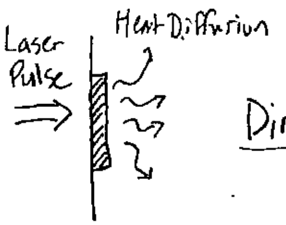
Heat capacity per unit volume. Thermal Conducting. Temp Distribution. Free Electrons. Fraction of Absorbed Power. Power absorbed.

Temperature rise per time relates to temperature per length

Which becomes $(\rho C)\frac{\Delta T}{T_p} = K\frac{\Delta T}{L^2}$

$L = (DT_p)^{1/2}$; $D = K/\rho C$ "Diffusion Constant"

ie How fast does temperature diffuse across material.



Direct Photoionization: When the laser strength is above the atomic field strength. $E_{at} = e/4\pi\epsilon_0 a_0^2$

$I_{at} = \frac{1}{2}n\epsilon_0 c E_{at}^2 \approx W/m^2$

Multiphoton Absorption and Multiphoton Excitation

1. Multiphoton Absorption is used to study high-lying electronic states not accessible by selection rules
2. Two-photon Microscopy has been used to eliminate background of materials by focusing beam only at position.
3. Multiphoton Absorption and multiphoton ionization lead to laser damage of optical materials and is used to write permanent refractive index.
4. Multiphoton absorption constitutes a nonlinear loss mechanism that can limit the efficiency of nonlinear optical devices and switches.

Theory of Single- and Multiphoton Absorption and Fermi's Golden Rule.

Atomic wavefunction ∘ $i\hbar \frac{\partial \psi(r,t)}{dt} = \hat{H}\psi(r,t)$; $\hat{H} = \hat{H}_0 + \hat{V}(t)$; $\underbrace{V(t) = -\hat{\mu}\tilde{E}(t)}$; $\hat{\mu} = -e\hat{r}$

Interaction energy with applied optical field.

Assuming monochromatic waves:

$$\tilde{E}(t) = Ee^{-i\omega t} + c.c.$$

Wavefunction associated with eigenstates: $\psi_n(r,t) = u_n(r)e^{-i\omega_n t}$

Where $\omega_n = E_n/\hbar$

Then, $\hat{H}_0 u_n(r) = E_n u_n(r)$

∘ $i\hbar \frac{\partial \psi(r,t)}{dt} = (\hat{H}_0 + V(t))\psi(r,t)$, Also expressed with a $\boxed{\text{linear combination}}$ of eigenstates: $\psi(r,t) = \sum a_l(t) u_l(r) e^{-i\omega_l t}$

Suddenly, $i\hbar \sum \frac{da_l}{dt} u(r) e^{-i\omega_l t} + i\hbar \sum (-i\omega_l) a_l(t) u_l(r) e^{-i\omega t}$
$= \sum a_l(t) E_l u_l(r) e^{-i\omega t} + \sum a_l(t) \hat{V} u_l(r) e^{-i\omega t}$

∘ Following orthonormality, $\int u_m^* u_l(r)\, d^3r = \delta_{ml}$

we obtain $i\hbar \frac{da_m}{dt} = \sum a_l(t) V_{ml} e^{-i\omega_{lm} t}$, where $\omega_{lm} = \omega_l - \omega_m$ $\boxed{\text{Matrix}}$

$$V_{ml} = \int u_m^*(r)\hat{V} u_l(r)\, d^3r$$

∘ Powers of interaction: $a_m(t) = a_m^{(0)}(t) + \lambda a_m^{(1)}(t) + \lambda^2 a_m^{(2)}(t) + \dots$

Linear One Photon Absorption ; N=1, first-order in the field.

$$\frac{da_m^{(N)}}{dt} = (i\hbar)^{-1} \sum a_l^{(N-1)} V_{ml} e^{-i\omega_{m} t} \quad N=1,2,3$$

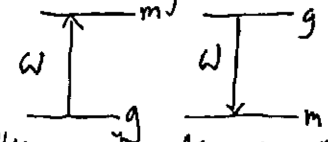
$\boxed{\text{Perturbation}}$

$a_g^{(0)}(t) = 1$; $a_l^{(0)}(t) = 0$ for $l \neq g$. $V_{mg} = -\mu_{mg}(Ee^{-i\omega t} + E^* e^{i(\omega t)})$

The perturbation then becomes $\frac{da_m^{(1)}}{dt} = -(i\hbar)^{-1}\mu_{mg}\left[Ee^{i(\omega_{mg}-\omega)t} + E^* e^{i(\omega_{mg}+\omega)t}\right]$

This equation is integrated to give: $a_m^{(1)}(t) = -(i\hbar)^{-1}\mu_{mg}\int_0^t dt' \left[Ee^{i(\omega_{mg}-\omega)t'} + E^* e^{i(\omega_{mg}+\omega)t'}\right]$

$$= \frac{\mu_{mg} E}{\hbar(\omega_{mg}-\omega)}\left[e^{i(\omega_{g}-\omega)t} - 1\right] + \frac{\mu_{mg} E^*}{\hbar(\omega_{mg}-\omega)}\left[e^{i(\omega_{mg}+\omega)t} - 1\right]$$



"Absorption" "Stimulated" Emission

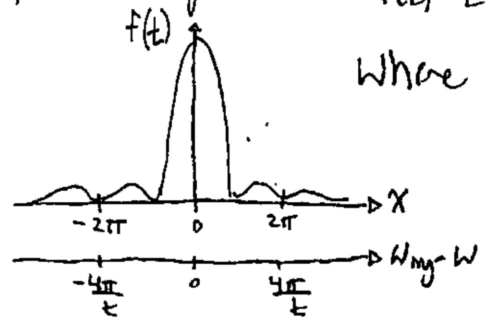
Probability of transition amplitude: $p_m^{(1)}(t) = |a_m^{(1)}(t)|^2 = \frac{|\mu_{mg} \cdot E|^2}{\hbar^2}\left|\frac{e^{i(\omega_{mg}-\omega)t} - 1}{\omega_{mg} - \omega}\right|^2$

$$= \frac{|\mu_{mg}E|^2}{\hbar^2}\frac{4\sin^2[(\omega_{mg}-\omega)t/2]}{(\omega_{mg}-\omega)^2} \equiv \frac{|\mu_{mg}E|^2}{\hbar^2} f(t)$$

Time Dependence: $f(t) = t^2\left(\frac{\sin^2 x}{x^2}\right)$

Where $x = (\omega_{mg}-\omega)t/2$

where $f(t) = \frac{4\sin^2[(\omega_{mg}-\omega)t/2]}{(\omega_{mg}-\omega)^2}$



$\lim_{t\to\infty} f(t) = 2\pi t\underbrace{\delta(\omega_{mg}-\omega)}_{\text{Dirac Delta Function}}$

Thus, $p_m^{(1)}(t) = \frac{|\mu_{mg}E|^2 t}{\hbar^2} 2\pi\delta(\omega_{mg}-\omega)$

- The final state $\rho_f(\omega_{mg})$ is defined such that $\rho_f(\omega_{mg})d\omega_{mg}$ is the probability that the transition frequency lies between $\omega_{mg}+d\omega_{mg}$.
- Atomic physics $\rho_f(\omega_{mg})$ – Atomic Lineshape Function. $\int_0^\infty \rho_f(\omega_{mg})d\omega_{mg}=1$
- A well known example of a density of final states is the Lorentzian lineshape function: $\rho_f(\omega_{mg}) = \frac{1}{\pi}\frac{\Gamma/2}{(\underbrace{\bar{\omega}_{mg}}_{\text{Line center freq.}}-\omega_{mg})^2+(\Gamma/2)^2}$ Full-width at Half-max. in Angular Frequency.
- Probability Density averaged over all possible values of transition Frequency:

$$p_m^{(1)}(t) = \frac{|\mu_{mg}E|^2 t}{\hbar^2}\int_0^\infty \rho_f(\omega_{mg})2\pi\delta(\omega_{mg}-\omega)d\omega_{mg}$$

$$= \frac{2\pi|\mu_{mg}E|^2 t}{\hbar}\rho_f(\omega_{mg}=\omega)$$

Transition Rate for Linear Absorption:

$$R_{mg}^{(1)} = \frac{p_m^{(1)}(t)}{t} = \frac{2\pi|\mu_{mg}E|^2}{\hbar^2}\rho_f(\omega_{mg}=\omega)$$

$\boxed{\text{Fermis Golden Rule}}$ $R_{mg}^{(1)} = \sigma_{mg}^{(1)}(\omega)I$ "Absorption" cross section

where $I = 2n\epsilon_0 c|E|^2$

$$\sigma_{mg}^{(1)}(\omega) = \frac{\pi}{n\epsilon_0 c}\frac{|\mu_{mg}|^2}{\hbar^2}\rho_f(\omega_{mg}=\omega)$$

Two-photon Absorption: $N=1$ and $N=2$

$$V_{nm} = -\mu_{nm}(Ee^{-i\omega t}+E^*e^{i\omega t}) \simeq -\mu_{nm}Ee^{-i\omega t}$$

$$\frac{d}{dt}a_n^{(2)}(t) = (i\hbar)^{-1}\sum a_m^{(1)}(t)V_{nm}e^{-i\omega_{mn}t}$$

$$= -(i\hbar)^{-1}\sum \frac{\mu_{nm}\mu_{mg}E^2}{\hbar(\omega_{mg}-\omega)}\left[e^{i(\omega_{ng}-2\omega)t} - e^{i(\omega_{nm}-\omega)t}\right]$$

Amplitude $a_n^{(2)}(t) = \sum_m \frac{\mu_{nm}\mu_{mg}E^2}{\hbar^2(\omega_{mg}-\omega)}\left[\frac{e^{i(\omega_{ng}-2\omega)t}-1}{\omega_{ng}-2\omega}\right]$

Prob of Amplitude $p_n^{(2)}(t) = |a_n^{(2)}(t)|^2 = \left|\sum \frac{\mu_{nm}\mu_{mg}E^2}{\hbar^2(\omega_{mg}-\omega)}\right|^2\left|\frac{e^{i(\omega_{ng}-2\omega)t}-1}{\omega_{ng}-2\omega}\right|^2$

$$\simeq \left|\sum \frac{\mu_{nm}\mu_{mg}E^2}{\hbar^2(\omega_{mg}-\omega)}\right|^2 2\pi t\delta(\omega_{ng}-2\omega)$$

$$= \left|\sum_m \frac{\mu_{nm}\mu_{mg}E^2}{\hbar^2(\omega_{mg}-\omega)}\right|^2 2\pi\rho_f(\omega_{mg}=2\omega)$$

$\boxed{\text{Rate}}$ $R_{ng}^{(2)} = \frac{p_n^{(2)}(t)}{t} = \sigma_{ng}^{(2)}(\omega)I^2$;

"Population Decay Rate"

$$\sigma_{ng}^{(2)}(\omega) = \frac{1}{4n^2\epsilon_0^2c^2}\left|\sum \frac{\mu_{nm}\mu_{mg}}{\hbar^2(\omega_{mg}-\omega)}\right|^2 2\pi\rho_f(\omega_{ng}=2\omega)$$

"cross-section units photons/area-s

Göppert-Mayer (GM) GM

$R_{ng}^{(2)} = \tilde{\sigma}_{ng}^{(2)}(\omega)\bar{I}^2$ where $\bar{I} = \frac{2n\epsilon_0 c}{\hbar\omega}|E|^2$

where $\tilde{\sigma}_{ng}^{(2)}(\omega) = \frac{\omega^2}{4n^2\epsilon_0^2c^2}\left|\sum \frac{\mu_{nm}\mu_{mg}}{\hbar(\omega_{mg}-\omega)}\right|^2 2\pi\rho_f$

When tuned, $\rho_f(\omega_{ng}=2\omega) \approx (2\pi T_n)^{-1}$

$$\tilde{\sigma}_{ng}^{(2)} = \frac{|\mu_{nm}\mu_{mg}|^2}{4\epsilon_0^2\hbar^2c^2T_{n1}}$$

Multiphoton Absorption:

$$R_{mg}^{(1)} = \left|\frac{\mu_{mg}E}{\hbar}\right|^2 2\pi\rho_f(\omega_{mg}-\omega)$$

$$R_{ng}^{(2)} = \left|\sum_m \frac{\mu_{nm}\mu_{mg}E^2}{\hbar^2(\omega_{mg}-\omega)}\right|^2 2\pi\rho_f(\omega_{ng}-2\omega)$$

$$R_{og}^{(3)} = \left|\sum_{mn} \frac{\mu_{on}\mu_{nm}\mu_{mg}E^3}{\hbar^3(\omega_{ng}-2\omega)(\omega_{mg}-\omega)}\right|^2 2\pi\rho_f(\omega_{og}-3\omega)$$

$$R_{pg}^{(4)} = \left|\sum_{onm} \frac{\mu_{po}\mu_{on}\mu_{nm}\mu_{mg}E^4}{\hbar^4(\omega_{og}-3\omega)(\omega_{ng}-2\omega)(\omega_{mg}-\omega)}\right|^2 2\pi\rho_f(\omega_{pg}-4\omega)$$

1. Derive an expression relating two-photon cross-section $\sigma^{(2)}$ to $\chi^{(3)}$

$$\chi^{(3)}(\omega_\sigma;\omega_1,\omega_2,\omega_3)=\frac{\int_0^\infty d\tau_1\int_0^\infty d\tau_2\int_0^\infty d\tau_3 \times R^{(3)}(\tau_1,\tau_2,\tau_3)e^{i(\omega_1\tau_1+\omega_2\tau_2+\omega_3\tau_3)}}{\int_0^\infty d\tau_1\int_0^\infty d\tau_2\int_0^\infty d\tau_3 \times \sigma_{ng} I^2 e^{i(\omega_1\tau_1+\omega_2\tau_2+\omega_3\tau_3)}}$$

Chapter 13: Ultrashort-Pulse Propagation Equation:

1) Wave Equation in the time domain $\nabla^2 E(r,t)-\frac{1}{\epsilon_0 c^2}\frac{\partial^2 D^{(1)}(r,t)}{\partial t^2}=\frac{1}{\epsilon_0 c^2}\frac{\partial^2 P(r,t)}{\partial t^2}$

2) Fourier Transforms: $\tilde{E}(r,t)=\int E(r,\omega)e^{-i\omega t}d\omega/2\pi$

$$\tilde{D}(r,t)=\int D^{(1)}(r,\omega)e^{-i\omega t}d\omega/2\pi$$

$$\tilde{P}(r,t)=\int P(r,\omega)e^{-i\omega t}d\omega/2\pi$$

3) Assume $D^{(1)}(r,\omega)$ and $E(r,\omega)$ are related through a dispersion equation

$$D^{(1)}(r,\omega)=\epsilon_0\epsilon^{(1)}(\omega)E(r,\omega)$$

4) Wave equation in the frequency domain $\nabla^2 E(r,\omega)+\epsilon^{(1)}(\omega)(\omega^2/c^2)E(r,\omega)=-(\omega^2/\epsilon_0 c^2)P(r,\omega)$

5) Derive a wave equation for slowly varying field amplitude $\tilde{A}(r,t)$

$$\tilde{E}(r,t)=\tilde{A}(r,t)e^{i(k_0 z-\omega_0 t)}+c.c.\ ;\ \underbrace{k_0=[\epsilon^{(1)}(\omega_0)]^{1/2}\omega_0/c}_{\text{"linear proportion of wavevector"}}$$

6) Spectral Content: $\tilde{A}(r,t)=\int A(r,\omega)e^{-i\omega t}d\omega/2\pi$

7) Slow varying field Amplitude in the frequency domain:

$$\left[\nabla_\perp^2+\frac{\partial^2}{\partial z^2}+2ik_0\frac{\partial}{\partial z}+[k^2(\omega)-k_0^2]\right]A(r,\omega)=\frac{-\omega^2}{\epsilon_0 c^2}P(r,\omega)e^{-ik_0 z}$$

where $k^2(\omega)=\epsilon^{(1)}(\omega)(\omega^2/c^2)$

as a power series $k(\omega)=k_0+(\omega-\omega_0)k_1(\omega_0)+D(\omega)$

and that $D(\omega)=\sum_{n=2}^{\infty}\frac{1}{n!}(\omega-\omega_0)^n k_n(\omega_0)$

8) N^{th}-derivative of the wave-vector $k_n(\omega_0)=d^n k(\omega)/d\omega^n$

$$k^2(\omega)=k_0^2+2(\omega-\omega_0)k_1k_0+\underbrace{2k_0D(\omega)+2(\omega-\omega_0)k_1D(\omega)+(\omega-\omega_0)^2k_1^2+D^2(\omega)}_{\text{"Higher-order Dispersion"}}$$

9) The wave-equation becomes

$$\left[\nabla_\perp^2+\frac{\partial^2}{\partial z^2}+2ik_0\frac{\partial}{\partial z}+2(\omega-\omega_0)k_0k_1+2k_0D+2(\omega-\omega_0)k_1D+(\omega-\omega_0)^2k_1^2\right]A(r,\omega)=(\omega_0^2/\epsilon_0c^2)P(z,\omega)e^{-ik_0z}$$

multiplied by $\exp[-i(\omega-\omega_0)t]$;

$$\left[\nabla_\perp^2+\frac{\partial^2}{\partial z^2}+2ik_0\left(\frac{\partial}{\partial z}+k_1\frac{\partial}{\partial t}\right)+2ik_1D\frac{\partial}{\partial t}+2k_0D-k_1^2\frac{\partial^2}{\partial t^2}\right]A(r,t)=-\frac{1}{\epsilon_0c^2}\frac{\partial^2 p}{\partial t^2}e^{-i(k_0z-\omega_0t)}$$

$$\hat{D}=\sum_{n=2}^{\infty}\frac{1}{n!}k_n\left(i\frac{\partial}{\partial t}\right)^n=-\frac{1}{2}k_2\frac{\partial^2}{\partial t^2}-\frac{i}{6}k_3\frac{\partial^3}{\partial t^3}+\ldots$$

Differential operator

10) Slow varying amplitude of Polarization: $\tilde{P}(r,t) = \tilde{p}(r,t)\, e^{-i(k_0 z - w_0 t)} + c.c.$

$$\frac{\partial P}{\partial t} = -i w_0 \left[\left(1 + \frac{i}{w_0}\frac{\partial}{\partial t}\right)\tilde{p}\right] e^{i(k_0 z - w_0 t)} + c.c$$

$$\frac{\partial^2 P}{\partial^2 t} = -w_0^2 \left[\left(1 + \frac{i}{w_0}\frac{\partial}{\partial t}\right)^2 \tilde{p}\right] e^{i(k_0 z - w_0 t)} + c.c.$$

$$\left[\nabla_\perp^2 + \frac{\partial^2}{\partial z^2} + 2ik_0\left(\frac{\partial}{\partial z} + k_1\frac{\partial}{\partial t}\right) + 2k_0\tilde{D} + 2ik_1\tilde{D}\frac{\partial}{\partial t} - k_1^2\frac{\partial^2}{\partial t^2}\right] A(r,t) = -\frac{w_0^2}{\epsilon_0 c^2}\left(1 + \frac{i}{w_0}\frac{\partial}{\partial t}\right)^2 \tilde{p}(r,t)$$

11) Convert the wave equation to a retarded time frame

$$z' = z; \quad \tau = t - \frac{1}{v_g} z = t - k_1 z$$

with $\frac{d}{dz} = \frac{\partial}{\partial z'} - k_1\frac{\partial}{\partial \tau}$ and $\frac{\partial}{\partial t} = \frac{\partial}{\partial \tau}$

12) New reference frame becomes:

$$\left[\nabla_\perp^2 + \underbrace{\frac{\partial^2}{\partial z'^2}}_{\text{"small"}} - 2k_1\frac{\partial}{\partial z'}\frac{\partial}{\partial \tau} + k_1^2\frac{\partial^2}{\partial \tau^2} + 2ik_0\left(\frac{\partial}{\partial z'} - k_1\frac{\partial}{\partial \tau} + k_1\frac{\partial}{\partial \tau}\right) + 2k_0\tilde{D} + 2ik_1 D\frac{\partial}{\partial \tau} - k_1^2\frac{\partial^2}{\partial \tau^2}\right] A(r,t)$$
$$= -\frac{w_0^2}{\epsilon_0 c^2}\left(1 + \frac{i}{w_0}\frac{\partial}{\partial \tau}\right)^2 \tilde{p}(r,t).$$

Simplifying, $\left[\nabla_\perp^2 - 2k_1\frac{\partial}{\partial z'}\frac{\partial}{\partial \tau} + 2ik_0\frac{\partial}{\partial z'} + 2k_0\tilde{D} + 2ik_1\tilde{D}\frac{\partial}{\partial \tau}\right] A(r,t) = -\frac{w_0^2}{\epsilon_0 c^2}\left(1 + \frac{i}{w_0}\frac{\partial}{\partial \tau}\right)^2 \tilde{p}(r,t)$

Alternatively, $\left[\nabla_\perp^2 + 2ik_0\frac{\partial}{\partial z'}\left(1 + \frac{ik_1}{k_0}\frac{\partial}{\partial \tau}\right) + 2k_0\tilde{D}\left(1 + \frac{ik_1}{k_0}\frac{\partial}{\partial \tau}\right)\right]\tilde{A}(r,t) = \frac{-w_0^2}{\epsilon_0 c^2}\left(1 + \frac{i}{w_0}\frac{\partial}{\partial \tau}\right)^2 \tilde{p}(r,t)$

$$k_1/k_0 = v_g^{-1}/(n_0 w_0/c) = n_g/(n_0 w_0); \quad n_g = c/v_g$$

Envelope Equation (1997)

$$\left[\left(1 + \frac{i}{w_0}\frac{\partial}{\partial \tau}\right)^{-1}\nabla_\perp^2 + 2ik_0\frac{\partial}{\partial z'} + 2k_0 D\right]\tilde{A}(r,t) = -\frac{w_0^2}{\epsilon_0 c^2}\left(1 + \frac{i}{w_0}\frac{\partial}{\partial \tau}\right) p(r,t)$$

Brabec-Krausz Equation Related to the Non-linear Schrodinger Equation.

Contains, space-time coupling, self-steepening, and High-order dispersion.

$\chi^{(3)}(w) = \chi^{(3)}(w_0) + (w - w_0)\frac{d\chi^{(3)}}{dw}$ where the derivative to be evaluated at w_0

$$p(w) = 3\epsilon_0\left[\chi^{(3)}(w_0) + (w - w_0)\frac{d\chi^{(3)}}{dw}\right]|A(w)|^2 A(w)$$
$$\tilde{p}(r,\tau) = 3\epsilon_0\left[\chi^{(3)}(w_0) + \frac{d\chi^{(3)}}{dw}\, i\frac{d}{d\tau}\right]|\tilde{A}(r,t)|^2\tilde{A}(r,t)$$

FT

Noting: $\left(1 + \frac{i}{w_0}\frac{\partial}{\partial \tau}\right)^2 = \left(1 + \frac{2i}{w_0}\frac{\partial}{\partial \tau} - \frac{1}{w_0^2}\frac{\partial^2}{\partial \tau^2}\right) \approx \left(1 + \frac{2i}{w_0}\frac{\partial}{\partial \tau}\right)$

Ultrashort-pulse Propagation Equation

$$\left[\nabla_\perp^2 + 2ik_0\frac{\partial}{\partial z'}\left(1 + \frac{i}{w_0}\frac{\partial}{\partial \tau}\right) + 2k_0\tilde{D}\left(1 + \frac{i}{w_0}\frac{\partial}{\partial \tau}\right)\right]\tilde{A}(r,\tau)$$
$$= (-3/c^2) w_0^2 \chi^{(3)}(w_0)\left[1 + \left(2 + \frac{w_0}{\chi^{(3)}(w_0)}\frac{d\chi^{(3)}}{dw}\right)\frac{i}{w_0}\frac{\partial}{\partial \tau}\right]|\tilde{A}(r,\tau)|^2\tilde{A}(r,\tau)$$

Interpretation of the Ultrashort-Pulse Propagation Equation:

$$\frac{\partial A(r,\tau)}{dz'} = \Big[\underbrace{\frac{i}{2k_0}\nabla_\perp^2}_{\text{"Spreading from diffraction" [Spatial]}} - \underbrace{\frac{i}{2}k_2\frac{\partial^2}{\partial\tau^2}}_{\text{"spreading from group velocity" [Temporal]}} + \underbrace{\frac{3i\omega_0}{2n_0 c}\chi^{(3)}(\omega_0)|A(r,\tau)|^2}_{\text{"Nonlinear Acquisition of phase"}}\Big]\tilde{A}(r,\tau)$$

$L_{dif} = \frac{1}{2}k_0 w_0^2$ [Diffraction Length]

$L_{dis} = T^2/|k_2|$ [Dispersion Length]

Dependendent.

$$L_{NL} = \frac{2n_0 c}{3\omega_0\chi^{(3)}|A|^2} = \frac{1}{(\omega_0/c)n_2 I} \text{ [Nonlinear Length]}$$

Self-Steepening: $k_1\partial A/\partial t = (1/v_g)\partial A/\partial t = (n_0^{(g)}/c)\partial A/\partial t$

$$\frac{\partial A}{\partial z} + \frac{n_0^{(g)}}{c}\frac{\partial A}{\partial t} = \frac{i}{2k_0}\nabla_\perp^2\tilde{A} - \frac{i}{2}k_2\frac{\partial^2 A}{\partial t^2} + \frac{i3\omega_0}{2n_0 c}\chi^{(3)}(\omega_0)|\tilde{A}|^2\tilde{A}$$
$$+ \frac{i3\omega_0}{2n_0 c}\chi^{(3)}(\omega_0)\left(2 + \frac{\omega_0}{\chi^{(3)}(\omega_0)}\frac{d\chi^{(3)}}{d\omega}\right)\frac{i}{\omega_0}\frac{\partial}{\partial t}|\tilde{A}|^2\tilde{A}$$

Non-linear coefficients: $\gamma_1 = \frac{3\omega_0}{2n_0 c}\chi^{(3)}(\omega_0)$; $\gamma_2 = \frac{3\omega_0}{2n_0 c}\chi^{(3)}(\omega_0)\left(1 + \frac{1}{2}\frac{\omega_0}{\chi^{(3)}}\frac{\partial\chi^{(3)}}{\partial\omega}\right)$

$$\frac{dA}{dz} + \frac{n_0^{(g)}}{c}\frac{\partial A}{\partial t} = \frac{i}{2k_0}\nabla_\perp^2\tilde{A} - \frac{i}{2}k_2\frac{\partial^2\tilde{A}}{\partial t^2} + i\gamma_1|A|^2A - 2\gamma_2\frac{1}{\omega_0}\frac{\partial}{\partial t}(|A|^2A)$$

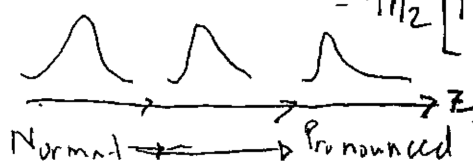
$$\frac{\partial}{\partial t}(|A|^2A) = \frac{\partial}{\partial t}(A^2A^*) = \tilde{A}^2\frac{\partial A^*}{\partial t} + 2\tilde{A}^*A\frac{\partial A}{\partial t} = 2|\tilde{A}|^2\frac{\partial A}{\partial t} + \tilde{A}^2\frac{\partial A^*}{\partial t}$$

$$\underbrace{\frac{dA}{dz}}_{\text{"Intensity Dependendent contribution"}} + \frac{n_{eff}^{(g)}}{c}\frac{\partial A}{\partial t} = \frac{i}{2k_0}\nabla_\perp^2\tilde{A} - \frac{i}{2}k_2\frac{\partial^2 A}{\partial t^2} + i\gamma_1|A|^2A - \underbrace{\frac{2\gamma_2}{\omega_0}\tilde{A}^2\frac{\partial A^*}{\partial t}}_{\text{"Dispersive Four wave Mixing"}}$$

Where $n_{eff}^{(g)} = n_0^{(g)} + \frac{4\gamma_2 c}{\omega_0}|A|^2 = n_0^{(g)} + n_2^{(g)}I$

$$n_2^{(g)} = \frac{3}{n_0^2\epsilon_0 c}\chi^{(3)}(\omega_0)\left[1 + \frac{1}{2}\frac{\omega_0}{\chi^{(3)}(\omega_0)}\frac{d\chi^{(3)}}{d\omega}\right]$$
$$= 4n_2\left[1 + \frac{1}{2}\frac{\omega_0}{\chi^{(3)}(\omega_0)}\frac{d\chi^{(3)}}{d\omega}\right]$$

Self-Steepening:



; $L_{ss} = \frac{cT}{n_2^{(g)}I}$

Space-Time Coupling:

A dispersionless, linear material: $\underbrace{\left(1+\frac{i}{\omega_0}\frac{\partial}{\partial\tau}\right)^{-1}}_{\text{"space-time coupling"}}\nabla_\perp^2\tilde{A}(r,\tau) + 2ik_0\frac{\partial}{\partial z'}\tilde{A}(r,\tau) = 0$

Which becomes $\nabla_\perp^2A(r,\tau) + \left(1+\frac{i}{\omega_0}\frac{\partial}{\partial\tau}\right)2ik_0\frac{\partial}{\partial z'}A(r,\tau) = 0$; if $A(r,\tau) = a(r)e^{-i\delta\omega t}$: then

$\nabla_\perp^2 a(r) + 2i(k_0+\delta k)\frac{\partial}{\partial z'}a(r) = 0$; Where $\delta k = k_0(\delta\omega/\omega_0)$; $[1 + (i/\omega_0)\partial/\partial\tau]$ "Different Frequency Components".

Supercontinuum Generation: A process to generate white light

Intense-Field Nonlinear Optics: $\tilde{P}(t) = \epsilon_0 \chi^{(1)} E(t) + \epsilon_0 \chi^{(2)} E(t)^2 + \epsilon_0 \chi^{(3)} E(t)^3 + \ldots$

Rabi Frequency $\Omega = \mu_0 a E/\hbar$; When $E > E_{atom}$ issues $E_{at} = \frac{e}{4\pi\epsilon_0 a_0^2} = \frac{e}{4\pi\epsilon_0 (4\pi\epsilon_0 \hbar^2/mc^2)^2} = 6\times10^{11}\ V/m$

Motion of a Free Electron in a Laser Field: $I_{at} = \frac{1}{2}\epsilon_0 c E_{at}^2 = 4\times10^{16} W/cm^2$

$E(t) = E e^{-i\omega t} + c.c$; $m\ddot{x} = -e\tilde{E}(t) = \cancel{-eE e^{-i\omega t} + c.c.}$

$x(t) = x e^{-i\omega t} + c.c.$; $x = eE/m\omega^2$; Time-averaged kinetic Energy $K = \frac{1}{2}m\langle \dot{x}(t)^2\rangle$

-or- $\dot{x}(t) = (-i\omega x)e^{-i\omega t} + c.c$; $K = \frac{e^2E^2}{m\omega^2} = \frac{e^2E_0^2}{4m\omega^2}$

Equation of Motion: $m\ddot{z} = \underbrace{\left[\left(-\frac{ieE}{m\omega}\right)e^{-i\omega t} + c.c\right]\left[Be^{-i\omega t} + c.c\right]}_{\text{"Nonlinearity to magnetic field"}}$

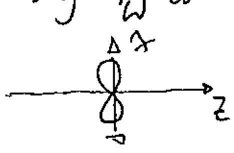
i.e nonlinear drift.

In the case of linear polarization in the x-direction ; $E(t) = E_0 \cos(\omega t - \omega z/c)$

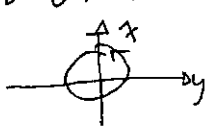
$x = \frac{\beta c}{\omega}\cos\eta$; $y = 0$; $z = \frac{\beta^2 c}{8\omega}\sin 2\eta$ where $\eta = \omega(t - z/c)$; $\beta = eE_0/\gamma'\omega$; $\gamma'^2 = m^2c^2 + e^2E_0^2/2\omega^2$

In the case of circularly polarized light ; $E_x = E_0\cos(\omega t - \omega z/c)$, $E_y = E_0 \sin(\omega t - \omega z/c)$

$x = \frac{\beta c}{\omega}\sin\omega t$; $y = \frac{\beta c}{\omega}\cos\omega t$; $z = 0$. Where $\gamma'^2 = m^2c^2 + e^2E_0^2/\omega^2$



Liner polarized electron



Circularly polarized field

Lorentz Invariant: $a^2 = \frac{K}{mc^2} = \frac{e^2E^2}{m^2c^2\omega^2}$

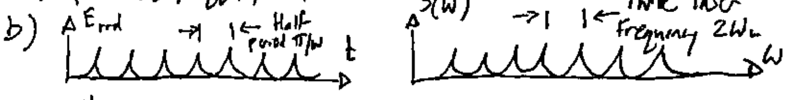
The relation could be expressed: $a^2 = \frac{1}{2\pi}\frac{I r_0 \lambda^2}{mc^3}$

Where $r_0 = e^2/4\pi\epsilon_0 mc^2$

High-Harmonic Generation: When an intense laser illuminates a medium and all odd harmonics at laser frequency are emitted in the forward direction.

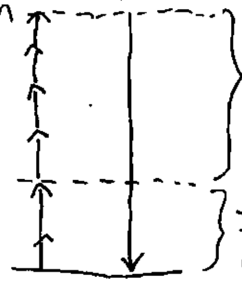
i.e When an electron within light is above a threshold frequency than ionization.

Corckum's Model of High-Harmonic Generation in linearly polarized laser field. An electron emits radiation when colliding with core



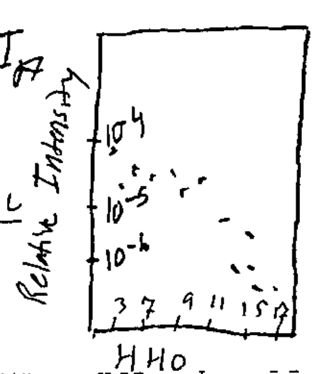
The spectrum of the emitted radiation is determined by the Fourier Transform of the pulse train

Schematic Representation of Empirical Relation



$\hbar\omega_{max} = 3.17K + I_p$

Experimental Data of High-harmonic Generation



$\epsilon = 1 - \frac{\omega_p^2}{\omega^2}$ where $\omega_p^2 = \frac{Ne^2}{\epsilon_0 m}$ "Plasma Frequency"

- When N is small $\omega_p^2 < \omega^2$ (~~undodense~~ underdense plasma), dielectric constant is (+)-positive $n = \sqrt{\epsilon}$ (real)
  N is large $\omega_p^2 > \omega^2$ (overdense plasma), dielectric constant is (−)-negative $n = \sqrt{\epsilon}$ (imag.)
- Linear Polarizability: $\chi^{(1)}(\omega) = N\epsilon_0 \alpha(\omega)$, $\alpha(\omega) = \frac{e^2/m\epsilon_0}{\omega_0^2 - \omega^2 - 2i\omega\gamma}$

which in a nonresonant limit reduces to $\alpha_{bound} = \frac{e^2}{\epsilon_0 m \omega_0^2}$

Common effects in Plasma

1) Ponderomotive Effects - When electrons are expelled from regions of high strength.
2) Relativistic Effects - Intense laser beams and electrons are accelerated to near light speed.

$$E_{rel} = \frac{2mc^2}{\lambda e}\ ;\ \text{where } \lambda = 2\pi c/\omega\ ;\ I_{rel} = \frac{1}{2}\epsilon_0 c E_{rel}^2$$

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}\ ;\ \text{Refractive index of plasma: } n^2 = 1 - \frac{\omega_p^2}{\gamma\omega^2}$$

$$\omega_p^2 = Ne^2/\epsilon_0 m$$

Strongly Relativistic Limit: $\gamma^2 = 1 + \frac{e^2 E_0^2}{m^2\omega^2 c^2}$; Calculating the nonlinear coefficient

$$\gamma = 1 + \frac{1}{2}\frac{e^2 E_0^2}{m^2\omega^2 c^2} \equiv 1 + X$$

$$n^2 = 1 - \frac{\omega_p^2}{\omega^2(1+X)} \simeq 1 - \frac{\omega_p^2}{\omega^2}(1 - X) = n_0^2 + \frac{\omega_p^2}{\omega^2}X$$

$$n \simeq n_0 + \frac{1}{2n_0}\frac{\omega_p^2}{\omega^2}X = n_0 + n_2 I$$

Setting I equal to $\frac{1}{2}n_0\epsilon_0 c E_0^2$; $n_2 = \frac{\omega_p^2 e^2}{2n_0^2 m^2 c^3 \omega^4}$

$$n_2 = \frac{1}{2\pi n_0^2}\left(\frac{\omega_p}{\omega}\right)^2\left[\frac{\lambda^2}{(mc^2)/(r_0/c)}\right]\ ;\ \lambda = \frac{2\pi c}{\omega}$$

$$r_0 = \frac{e^2}{4\pi\epsilon_0 mc^2}$$

Fundamental Power of Plasma:

$$P_{rel} = \frac{mc^2}{(r_0/c)} \simeq 9.2\times10^9\,W.$$

$$P_{critical\ self\text{-}focusing} \simeq \frac{\lambda^2}{8 n_0 n_2} = 6.7\left(\frac{\omega}{\omega_p}\right)^2 GW = \underbrace{\frac{\pi}{4}n_0\left(\frac{\omega}{\omega_p}\right)^2\left(\frac{mc^2}{r_0/c}\right)}_{\text{"relativistic unit of optical power"}}$$

Nonlinear Quantum Electrodynamics

$E_{rel} = 2mc^2/e\lambda$ "Characteristic field strength"

$$E_{QED} = \frac{m^2c^2}{e\hbar} = \frac{mc^2}{e\lambda_c}\ ;\ \lambda_c = \frac{h}{mc}$$ "Compton Wavelength": Maximum localized Wavelength.

$= 1.32\times10^{18}\,V/m$: Schwinger Limit. : $I_{QED} = \frac{1}{2}\epsilon_0 c E_{QED}^2 = 4\times10^{29}\,W/cm^2 = 4\times10^{33}\,W/m^2$

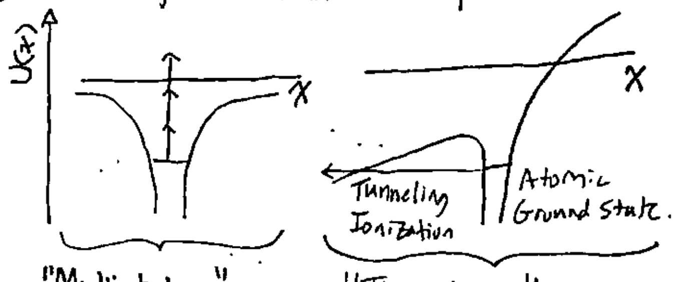
$$\epsilon_{ik} = \delta_{ik} + \left(\frac{e^2}{4\pi\epsilon_0}\right)^2\frac{\hbar}{45\pi m^4 c^7}\left[2(E^2 - B^2c^2)\delta_{ik} + 7B_iB_kc^2\right]$$

In a plane wave, $\epsilon_{ik} = \delta_{ik} + \left(\frac{e^2}{4\pi\epsilon_0}\right)^2\frac{7\hbar}{45\pi m^4 c^7}B_iB_kc^2$; $\epsilon_{ik} = \delta_{ik} + \frac{7}{45\pi}\left(\frac{e^2}{4\pi\epsilon_0\hbar c}\right)\frac{B_iB_kc^2}{E_{cr}^2}$

Timeline [1994 - Electron Ejection, 1988 High-Harmonic Generation 33rd order
1993 - Theoretical model of HHG, 1994 - QM model of HHG
1997 - 221st order detected, 1998, 1999 Gas-filled capillary waveguide phase match,
2014 - Theoretical Result of electron collision rate]

Tunnel Isolation and the Keldysh Model

1) Multiphoton ionization, or absorption. leads to photoionization.
2) Tunnel Ionization



"Multiphoton" "Tunneling" ≈ Atomic Binding.

Keldysh (1965) proved a quantity known as the Keldysh Parameter γ_K.
Specifically, $\gamma_K > 1$ = Multiphoton Ionization, and Tunneling.
The constant is represented as: $\gamma_K = \frac{eE}{\omega_L \sqrt{2m I_p}} = \sqrt{I_p/2K}$
"Ponderomotive Energy"
$= \omega \tau = (\omega_L/|eE|)\sqrt{m_r I_p}$

Nonlinear Optics of Plasmas and Relativistic Nonlinear Optics

Plasma: a partially or fully ionized gas.
1) Multiphoton ionization creates plasma, which creates a linear response to optical properties.
2) A plasma can respond in an intrinsically nonlinear manner to an applied optical field.

Description of Plasma Formation: N_e = Number of free electrons per unit volume

$$\frac{dN_e}{dt} = \frac{dN_i}{dt} = (N_T - N_i)\sigma^{(N)} I^N - r N_e N_i$$

N_i = Number of positive ions.
N_T = Total number of atoms present

Where $\sigma^{(N)}$ denotes N-photon cross-section.
r denotes electron-ion recombination rate. Note: Electron-density increases monotonically during laser pulse.

Electric Field: $\tilde{E}(t) = E e^{-i\omega t} + c.c.$

Position of an Electron: $x(t) = x e^{-i\omega t} + c.c.$ Where $x = eE/m\omega^2$

Dipole Moment associated with response: $\tilde{p}(t) = p e^{-i\omega t} + c.c. = -e\tilde{x}(t)$

Polarizability $\alpha(\omega)$ is defined by $p = \epsilon_0 \alpha(\omega) E$.
Where $\alpha(\omega) = -\frac{e^2}{\epsilon_0 m \omega^2}$

Dielectric Constant: $\epsilon = 1 + N\alpha(\omega) = 1 - \frac{Ne^2}{\epsilon_0 m \omega^2}$

$$\Delta t = \frac{7}{45\pi}\left(\frac{e^2}{4\pi\varepsilon_0\hbar c}\right)\frac{1}{E_{QED}^2} \;;\; \chi^{(3)} = \frac{7}{45\pi}\left(\frac{e^2}{4\pi\varepsilon_0\hbar c}\right)\frac{1}{E_{QED}^2} = \frac{7}{45\pi}\frac{1}{137}\frac{1}{E_{QED}^2}$$

$$\chi^{(3)}\ \text{lim} : 2.1\times10^{-39}\,m^2/V^2 \;;\; n_2 = 5.9\times10^{-37}\,m^2/W \;;\; P_{cr} = \frac{\lambda^2}{8 n_0 n_2} \;;\; P_{cr} = 2.1\times10^{23}\,W$$

Problem 1: $\tilde{E}(z,t) = E_0\cos\omega t\,\hat{x}$; $\tilde{B}(z,t) = B_0\cos\omega t\,\hat{y}$

with $B_0 c - E_0$. $(x,y,t) = (0,0,0)$

a) $m\ddot{x} = -e\tilde{E}(t) = E_0\cos\omega t\,\hat{x}$

$m\frac{dv}{dt} = eE_0\cos\omega t$

$m\,dV = E_0\cos\omega t\,dt$

$m\frac{dx}{dt} = E_0\,\omega\sin\omega t$

$\boxed{V_x(t) = \frac{E_0}{m}\omega\sin\omega t}$

$\boxed{x(t) = -\frac{E_0\omega^2}{m}\cos\omega t}$

$m\ddot{y} = -eE(t) = -e\,\mathcal{E}\cdot B(t) = -ec\,B_0\cos\omega t$

$\boxed{V_y(t) = +\frac{e}{m}E_0\,\omega\sin\omega t}$

$\boxed{y(t) = -\frac{eE_0\,\omega^2\cos\omega t}{m}}$

$mF = -eV\times B$

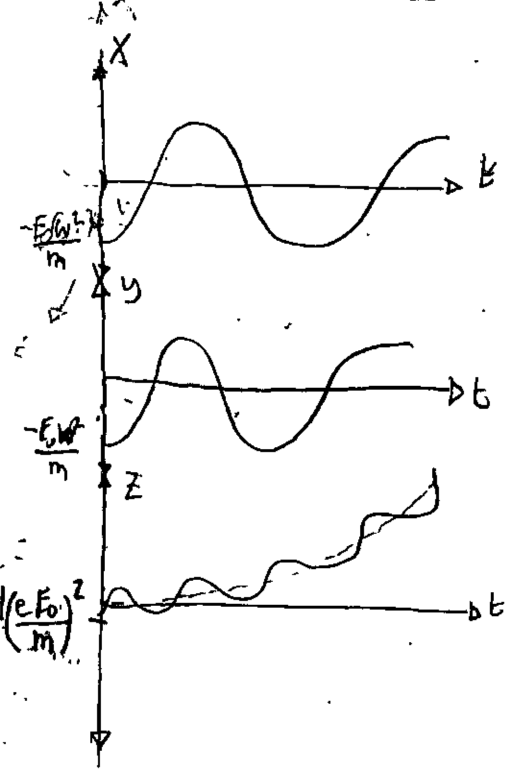
$m\ddot{z} = \left[+\frac{eE_0}{m}\omega\sin\omega t\right]\left[+e\frac{E_0\omega}{m}\sin\omega t\right]$

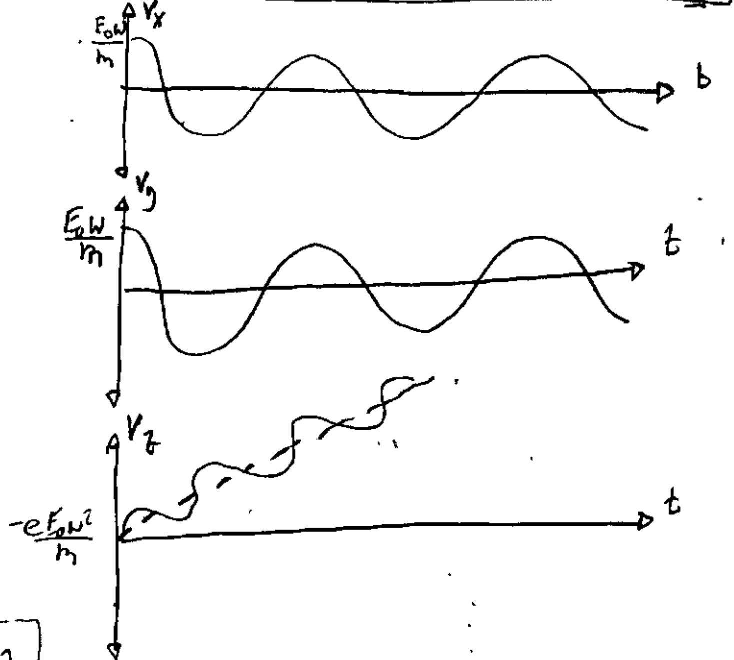
$= -\left(\frac{eE_0\omega}{m}\right)^2\sin^2\omega t$

$V_z(t) = -\left(\frac{eE_0\omega}{m}\right)^2\int(1-2\cos\omega t)\,dt'$

$\boxed{V_z(t) = -\left(\frac{eE_0\omega}{m}\right)^2\left[t - \frac{2}{\omega}\sin\omega t\right]}$ $1-2\cos t$

$$\boxed{\begin{aligned} z(t) &= -\left(\frac{eE_0\omega}{m}\right)^2\left[\frac{t^2}{2} + \frac{2}{\omega^2}\cos\omega t + \frac{2}{\omega^2}\right] \\ &= -\left(\frac{eE_0\omega}{m}\right)^2\left[\frac{t^3}{2} + \frac{2}{\omega^2}[\cos\omega t + 1]\right]\end{aligned}}$$





Forgot to incorporate the initial conditions. Would shift the function to $x=0, y=0, z=0$ at $t=0$.

b. $\langle KE \rangle = \langle \frac{1}{2} m v^2 \rangle = \frac{1}{2} m \langle v^2 \rangle = \frac{1}{2} m \langle v_x^2 + v_y^2 + v_z^2 \rangle$

Peak Energy would be E_0

c. $x = \frac{\beta c}{\omega} \sin \omega t$; $y = \frac{\beta c}{\omega} \cos \omega t$; $z = 0$

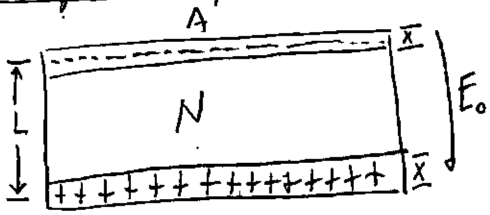
Chapter 14: Nonlinear Optics of Plasmonic Systems:

Plasmonics: Property of metals

Simple Derivation of Plasma Frequency

~~A static electric field E_0 applied~~ to a slab of metallic material of free charge density

Plasma Frequency is key to plasmonics.



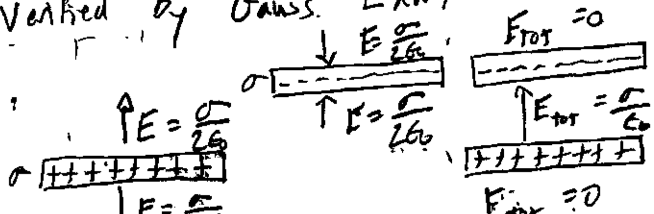
Total Electron charge: $Q = -NeAx$

At bottom of slab: $NeAx$

Surface charge Density: $\sigma_{top} = -Nex \equiv -\sigma$

$\sigma_{bottom} = Nex$

Total Field in the region: $E_{tot} = \frac{\sigma}{\epsilon_0} = \frac{Nex}{\epsilon_0}$ Verified by Gauss' Law,



The forces generated on the slab are:

Force is given by $F = Q E_{tot}$

Therefore, $m\ddot{x} = -\frac{NeAx\, Nex}{\epsilon_0}$; $\cancel{N} m \cancel{A} x \ddot{x} = -\frac{N e^2 x \cancel{A}}{\epsilon_0}$

$\ddot{x} = -\frac{Ne^2}{\epsilon_0 m}$; The solution is $x(t) = X_0 e^{-i\omega_p t} + c.c.$

Where $\underbrace{\omega_p = \sqrt{\frac{Ne^2}{\epsilon_0 m}}}_{\text{"Plasma Frequency"}}$

"The frequency at which a collection of free electrons oscillate"

The Drude Model

- A description of the optical properties of a metal or plasmonic system.
- The model treats the metal as a gas of free electrons.
- Related to the Lorentz model, but restoring force and resonant frequency ω_0 vanish.
- The section can be obtained by the limit of $\omega_0 \to 0$ of Lorentz Model.

Laser field: $\tilde{E}(t) = E_0 e^{-i\omega t} + c.c.$;

Force of an Electron: $F(t) = -e\tilde{E}(t) - 2m\gamma\dot{x}$

Equation of Motion: $\ddot{x} + 2\gamma\dot{x} = -eE(t)/m$

Solutions: $x(t) = x_0 e^{-i\omega t} + c.c.$

Where $x_0 = \frac{eE_0/m}{\omega^2 + 2i\omega\gamma}$

Polarization Amplitude: $P(\omega) = -eX_0$

Linear Susceptibility: $P = \epsilon_0 \chi^{(1)} E_0$

Susceptibility:

$\chi^{(1)} = \frac{Ne^2/m}{\omega^2 + 2i\omega\gamma} = -\frac{\omega_p^2}{\omega^2 + 2i\omega\gamma}$

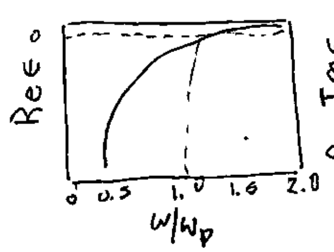
Dielectric $\epsilon^{(1)}(\omega) = 1 + \chi^{(1)}(\omega)$

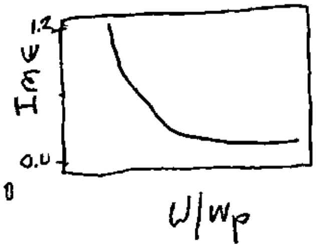
$= 1 - \frac{\omega_p^2}{\omega^2 + 2i\omega\gamma}$

Drude Equation:

$\epsilon^{(1)}(\omega) = \left[1 - \frac{\omega_p^2}{\omega^2 + 4\gamma^2}\right] - i\left[\frac{2\omega_p^2 \gamma}{\omega(\omega^2 + 4\gamma^2)}\right]$

Frequency Dependence of the Drude Model





$$\lim_{\gamma \to 0} k^2c^2 = \omega^2 - \omega_p^2$$

The standard Drude result is generalized as:

$$\epsilon^{(1)}(\omega) = \left[1 - \frac{\omega_p^2}{\omega^2 + 4\gamma^2}\right] - i\left[\frac{2\omega_p^2\gamma}{\omega(\omega^2+4\gamma^2)}\right]$$

$$= \underbrace{\epsilon_\infty}_{\text{"Nonresonant Contribution"}} - \underbrace{\frac{\omega_p^2}{\omega^2 - 2i\omega\gamma}}_{\text{"Free Electrons"}} \quad \text{-or-} \quad = \underbrace{\epsilon_\infty}_{\text{"Nonresonant Contributions"}} - \underbrace{\frac{\omega_p^2}{\omega^2 - 2i\omega\gamma}}_{\text{"Free Electrons"}} + \underbrace{\frac{N_b e^2/(\epsilon_0 m)}{\omega_0^2 - \omega^2 - 2i\omega\gamma_b}}_{\text{"Bound Electrons"}}$$

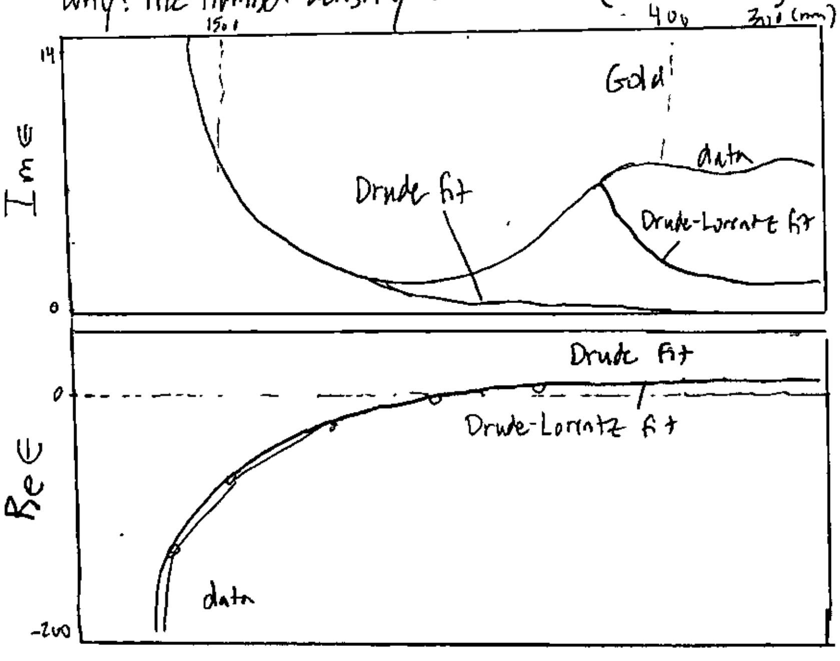
Hydrodynamic Model: $\underset{\text{"Effective electron mass"}}{m} N\left[\frac{\partial v}{\partial t} + (v\cdot\nabla)v + \underbrace{\frac{v}{\tau}}_{\text{"Dampening time"}}\right] = -eN(\vec{E} + \vec{V}\times\vec{B}) - \underbrace{\nabla p}_{\text{Quantum Pressure}}$

14.4: Optical Properties of Gold

: Gold is a Noble metal, used for plasmonics, and chemically resistant. Why? The number density of electrons ($5.90\times10^{28}\,m^{-3}$) is large.

Mechanisms for "free" electrons:

1) Conduction band [Particle-in-a-box]
2) Interband Transitions between conduction and valence.
3) Hot-electron contribution [Fermi-smearing]



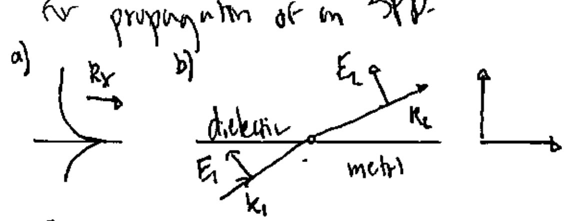
Angular Frequency (ω)

14.5: Surface Plasmon Polariton

Surface Plasmon Polaritons (SPP)
↳ Travels on surface of two materials, i.e metal and dielectric.

Plasma Oscillation:

a) Symbolic Representation of metal and a dielectric
b) Geometry of dispersion relation for propagation of an SPP.



Electric Field of Light: $E_m(r,t) = (A_{m,x}\hat{x} + A_{m,z}\hat{z})e^{i(k_x x - \omega t)}e^{ik_m z}$
for $z<0$ of the form: $\tilde{E}_d(r,t) = (A_{dx}\hat{x} + A_{dz}\hat{z})e^{i(k_x x - \omega t)}e^{ik_d z}$
for $z>0$ of the form:

Examination How four field amplitudes A_{mx}, A_{mz}, A_{dx}, and A_{dz} are related.

$$k_x A_{mx} + k_{mz} A_{mz} = 0$$
$$k_x A_{dx} + k_{dz} A_{dz} = 0$$
$$A_{mx} - A_{dx} = 0$$

$\epsilon_m A_{mx} - \epsilon_d A_{dx} = 0$ ... Each constitute a set of four homogeneous equations

$k_{dz}\epsilon_m - k_{mz}\epsilon_d = 0$; The wave equation $\nabla^2 E - (\epsilon/c^2)\cdot(\partial^2 E/\partial t^2) = 0$

leads to $k_x^2 + k_m^2 = \epsilon_m(\omega^2/c^2)$ and $k_x^2 + k_d^2 = \epsilon_d(\omega^2/c^2)$

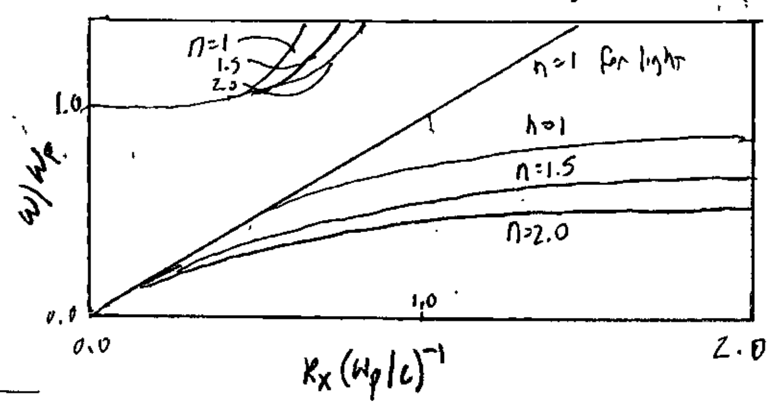
$$k_x^2 = \frac{\omega^2}{c^2}\frac{\epsilon_m\epsilon_d}{\epsilon_m+\epsilon_d} \quad \text{and} \quad k_x = \frac{\omega}{c}\sqrt{\frac{\epsilon_m\epsilon_d}{\epsilon_m+\epsilon_d}}$$

The other two wavevectors perpendicular to the interface

$$k_{m,z} = k\sqrt{\frac{\epsilon_m^2}{\epsilon_m+\epsilon_d}} \quad \text{and} \quad k_{dz} = k\sqrt{\frac{\epsilon_d^2}{\epsilon_m+\epsilon_d}}$$

"Longitudinal component of the wavevector does relate to frequency"

<u>Plot of the dispersion relation:</u>



The pronounced nonlinear effects appear by the propagation constant:

$\Delta K_x = \gamma_{sp} S_{sp}$ ← Power unit length.

<u>Electric Field Enhancement in Plasmonic Systems.</u>

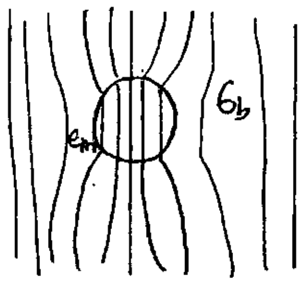
The lightning rod effect: an enhancement of the local electric field at a sharp point or tip.

Practiced with a metallic sphere of radius a with dielectric constant ϵ_m placed in a background material of dielectric constant ϵ_b.

With electric field of a strength E_0. Assume wavelength λ of size $a << \lambda$.

The result is $E_{in} = \frac{3\epsilon_b}{\epsilon_m + 2\epsilon_b} E_0$; Dipole moment expressed as: $p = \epsilon_0\epsilon_m \alpha E_0$.

where the polarizability of the sphere is: $\alpha = 4\pi a^3 \frac{3\epsilon_b}{\epsilon_m + 2\epsilon_b}$.



Fröhlich condition

Problem 1: For s-polarized light, $\tilde{E}_m(r,t) = r_p E_m(r,t)$ for $z < 0$ and $z < 0$.
i r

No, because the homogeneous equations would approach zero at the interface region.

Chapter 1:

2. $P = 1W$, Area $= 30\mu m$, $n = 2$, $\chi^{(2)} = 4\times10^{-11} m/V$; Calculate $P(2\omega)$, $P(2\omega) = \epsilon_0 \chi^{(2)} E_1^{\,2}$

$P = E_0 e^{-i\omega t}$; $n = \sqrt{1+\epsilon_0}$; $P(2\omega) = (n^2-1)\chi^{(2)} \cdot E_0^2 \cdot e^{-\cancel{...}\,t} = 3.4\times10^{-11}\frac{m}{V} E_0^2 e^{-4\pi i/(6.626\times10^{-34})}$

$\boxed{= 1.2\times10^{-10}\frac{m}{V} E_0^2 e^{-4\pi i E/G}}$

$$\nabla^2 E - \frac{n^2}{c^2}\frac{\partial^2 E}{\partial t^2} = \frac{1}{\epsilon_0 c^2}\frac{\partial^2 P^{NL}}{\partial t^2} = \mu_0 \frac{\partial^2 P^{NL}}{\partial t^2}$$

$\mu(2\omega)$;

$$\emptyset + \frac{n^2}{c^2}\omega^2 E_0 e^{-i\omega t} = \mu_0 (4\pi\nu)^2 E_0^2 e^{-4\pi i \nu t} \;;\; \mu(2\omega) = \boxed{\mu_0 = \frac{n^2}{c^2 E_0}}$$

$P(2\omega)$; $n^2 = \frac{\epsilon}{\epsilon_0}$; $P(2\omega) = n^2 \epsilon_0 \chi^{(2)} E_{at}^2$; $E_{at} = e/(4\pi\epsilon_0 a_0^2)$

$= 4 \cdot 8.85 \frac{C}{N\cdot m^2} \cdot 4\times10^{-11}\frac{m}{V}(5.14\times10^{11}\frac{V}{m})$; $= 5.14\times10^{11} V/m$

Intensity $= \frac{\text{Power}}{\text{Area}} = \frac{\text{Power}}{30\mu m} = \frac{1}{2}\epsilon c E^2 = \frac{1}{2} n_0 \epsilon_0 c E_0^2$; $E_0 = \sqrt{\frac{2\,\text{Power}}{\pi(\frac{D}{2})^2 n_0 \epsilon_0 c}}$; $P(2\omega) = \epsilon_0 \chi^{(2)} \frac{E_0^2}{4}$

Assuming $N = 10^{28}$ atoms/m^3

$= \sqrt{\frac{2(1W)}{\pi\left(\frac{30\mu m\times10^{-6}}{2}\right)^2 2 \cdot 8.85\frac{C}{N\cdot m^2}\cdot 2.998\times10^{8}\frac{m}{s}}}$

$\mu(2\omega) = 4.7\times10^{-11}\frac{C}{m^2}\cdot\frac{m^3}{10^{28}\text{atoms}}$

$\boxed{= 4.7\times10^{-39} C/\text{atom}}$

$\boxed{P(2\omega) = 8.85\frac{V}{m}\cdot 4\times10^{-11}\frac{m}{V} = \frac{0.26}{4}\frac{C}{m^3}}$

$ea_0 = 8.5\times10^{-30}$ C m

$\boxed{\mu(2\omega) = 5.56\times10^{-10}\, ea_0}$

$P(\omega) = \boxed{\epsilon_0 \chi^{(1)} E_0}$; $\boxed{\mu(\omega) = P(\omega)\cdot N}$; $\boxed{\mu(2\omega)/\mu(\omega)}$

4. $$\chi^{(3)}_{ijkl}(\omega_q, \omega_m, \omega_n, \omega_p) = \frac{Nbe^4[\delta_{ij}\delta_{kl} + \delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}]}{3\epsilon_0 m^3 \cdot D(\omega_q)D(\omega_m)D(\omega_n)D(\omega_p)}$$

Prove the properties $\chi_{1122} = \chi_{1212} = \chi_{1221} = \chi_{1133} = \chi_{1313} = \chi_{1331} = \chi_{2233} = \chi_{2323}$

For $\delta_{ij} = 1, i=j$ and $\delta_{ij} = 0, i\neq j$

$= \chi_{2332} = \chi_{2211} = \chi_{2121} = \chi_{2112} = \chi_{3311} = \chi_{3131} = \chi_{3113}$

$\chi^{(3)}_{ijkl}(\omega_q,\omega_m,\omega_n,\omega_p) = \frac{Nbe^4}{3\epsilon_0 m^3 D(\omega_q)D(\omega_m)D(\omega_n)D(\omega_p)} \times[\delta_{ij}\delta_{kl} + \delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}]$

$= \chi_{3322} = \chi_{3232} = \chi_{3223} = \frac{1}{3}\chi_{1111} = \frac{1}{3}\chi_{2222} = \frac{1}{3}\chi_{3333}$

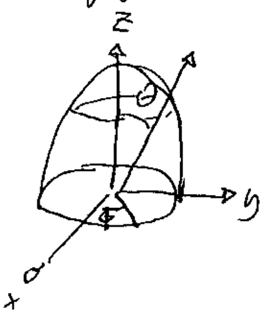
		i	j	k	l
		1 2 3	1 2 3	1 2 3	1 2 3
i	1		1 0 0	1 0 0	1 0 0
	2	X	0 1 0	0 1 0	0 1 0
	3		0 0 1	0 0 1	0 0 1
j	1	1 0 0		1 0 0	1 0 0
	2	0 1 0	X	0 1 0	0 1 0
	3	0 0 1		0 0 1	0 0 1
k	1	1 0 0	1 0 0		1 0 0
	2	0 1 0	0 1 0	X	0 1 0
	3	0 0 1	0 0 1		0 0 1
l	1	1 0 0	1 0 0	1 0 0	
	2	0 1 0	0 1 0	0 1 0	X
	3	0 0 1	0 0 1	0 0 1	

Each $= 1\cdot1 + 0\cdot$ equal to :

$[\delta_{ij}\delta_{kl} + \delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}] = 1$, except $\chi_{1111} = \chi_{2222} = \chi_{3333} = 3$.

6. Verify $d_{eff} = d_{31}\sin\theta - d_{22}\cos\theta\sin 3\phi$ and $d_{eff} = d_{22}\cos^2\theta\cos 3\phi$

$P = 4\epsilon_0 d_{eff} E(\omega_1)E(\omega_2)$



3m : Negative Uniaxial Crystal : C_{3v}, M_q, M_5, 6 no. elements, X_3

$$\boxed{\begin{array}{l} F_2(\theta,\phi,d) \text{ memo } \boxed{\text{Type 1:}} \\ f_1 \equiv o \text{ ray} \quad f_2 \equiv o \text{ ray} \quad f_3 \equiv e \text{ ray} \\ P(\omega_3) = F_2(\theta,\phi,d)E(\omega_1)E(\omega_2) \\ n_3^{ext} = \frac{f_1}{f_3} n_1^{ord} + \frac{f_2}{f_3} n_2^{ord} \end{array}}$$

Purposed to define crystal polarization and Type I/II

$$P_{oo}^{e}(\omega_3) = b_i d_{ijk}(\omega_3,\omega_2,\omega_1)a_j a_k E_j(\omega_2)E_k(\omega_1)$$

Klienmans Symmetries

$$M_q = \begin{pmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$ threefold rotation about X_3 axis

$$M_5 = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$ X_2X_3 plane of reflection

General Conditions:

$$\begin{pmatrix} 0 & -d_{11} & 0 & 0 & d_{15} & -d_{22} \\ -d_{22} & d_{22} & 0 & d_{15} & 0 & 0 \\ d_{31} & d_{31} & d_{33} & 0 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & 0 & 0 & d_{15} & -d_{22} \\ -d_{22} & d_{22} & 0 & d_{15} & 0 & 0 \\ d_{15} & d_{15} & d_{33} & 0 & 0 & 0 \end{pmatrix}$$

Approximated by Klienmans Diagrams

$$\begin{pmatrix} P_1 \\ P_2 \\ P_3 \end{pmatrix} = \begin{pmatrix} d_{11} & d_{12} & d_{13} & \cdots & d_{16} \\ d_{21} & d_{22} & \cdot & \cdots & d_{26} \\ d_{31} & \cdot & \cdot & \cdots & d_{36} \end{pmatrix} \begin{pmatrix} L_{11} \\ L_{22} \\ L_{33} \\ L_{23}+L_{32} \\ L_{31}+L_{13} \\ L_{12}+L_{21} \end{pmatrix}$$

2<4

3, 3.1

3<4

Rewritten as...

$$P_i = d_{im}\epsilon_m$$

..., with

$$\epsilon_m^{ee} = \begin{pmatrix} \cos^2\theta\cos^2\phi \\ \cos^2\theta\sin^2\phi \\ \sin^2\theta \\ -2\sin\theta\sin\phi \\ -2\sin\theta\sin\phi \\ \cos^2\theta\sin 2\phi \end{pmatrix} \times E^e(\omega_2)E^e(\omega_1)$$

$$e_m^{oo} = \begin{pmatrix} \sin^2\phi \\ \cos^2\phi \\ 0 \\ 0 \\ 0 \\ -\sin 2\phi \end{pmatrix} \times E^o(\omega_2)E^o(\omega_1)$$

$$(e_m^{eo}) = (e_m^{oe}) = \begin{pmatrix} -\frac{1}{2}\cos\theta\sin 2\phi \\ +\frac{1}{2}\cos\theta\sin 2\phi \\ 0 \\ -\sin\theta\cos\phi \\ \sin\theta\sin\phi \\ \cos\theta\cos 2\phi \end{pmatrix} \times E^o(\omega_2)E^e(\omega_1)$$

Effective Susceptibility (d_{eff}):

$$\begin{pmatrix} P_x(2w) \\ P_y(2w) \\ P_z(2w) \end{pmatrix} = 2\varepsilon_0 \begin{pmatrix} d_{11} & d_{12} & d_{13} & d_{14} & d_{15} & d_{16} \\ d_{21} & d_{22} & d_{23} & d_{24} & d_{25} & d_{26} \\ d_{31} & d_{32} & d_{33} & d_{34} & d_{35} & d_{36} \end{pmatrix} \begin{pmatrix} E_x^2(w) \\ E_y^2(w) \\ E_z^2(w) \\ 2E_y(w)E_z(w) \\ 2E_x(w)E_z(w) \\ 2E_x(w)E_y(w) \end{pmatrix}$$

3m
$$\begin{pmatrix} 0 & -d_{11} & 0 & 0 & d_{15} & -d_{22} \\ -d_{22} & d_{22} & 0 & d_{15} & 0 & 0 \\ d_{31} & d_{31} & d_{33} & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & d_{15} & -d_{22} \\ -d_{22} & d_{22} & 0 & d_{15} & 0 & 0 \\ d_{15} & d_{15} & d_{33} & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} P_x(2w) \\ P_y(2w) \\ P_z(2w) \end{pmatrix} = 2\varepsilon_0 \begin{pmatrix} 0 & -d_{11} & 0 & 0 & d_{15} & -d_{22} \\ -d_{22} & d_{22} & 0 & d_{15} & 0 & 0 \\ d_{31} & d_{31} & d_{33} & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} E_x^2(w) \\ E_y^2(w) \\ E_z^2(w) \\ 2E_y(w)E_z(w) \\ 2E_x(w)E_z(w) \\ 2E_x(w)E_y(w) \end{pmatrix}$$

$$P_x(2w) = 2\varepsilon_0\left(-d_{11}E_y^2(w) + 2d_{15}E_x(w)E_z(w) - 2d_{22}E_x(w)E_y(w)\right)$$

$$P_y(2w) = 2\varepsilon_0\left(-d_{22}E_x^2(w) + d_{22}E_y^2(w) + 2d_{15}E_y(w)E_z(w)\right)$$

$$P_z(2w) = 2\varepsilon_0\left(d_{31}\left[E_x^2(w) + E_y^2(w)\right] + d_{33}E_z^2(w)\right)$$

Assuming $\vec{E}(w) = E(w)\begin{pmatrix} -\sin\phi \\ \cos\phi \\ 0 \end{pmatrix}$; $\vec{E}(2w) = E(2w)\begin{pmatrix} \cos\theta\cos\phi \\ \cos\theta\sin\phi \\ -\sin\phi \end{pmatrix}$

$$P_x(2w) = 2\varepsilon_0\left(-d_{11}\cos^2\phi - 2d_{22}\sin 2\phi\right)E^2(w)$$

$$P_y(2w) = 2\varepsilon_0\left(-d_{22}\sin^2\phi + d_{22}\cos^2\phi\right)E^2(w)$$

$$= 2\varepsilon_0 d_{22}\cos(2\phi)E^2(w)$$

$$P_z(2w) = 2\varepsilon_0\left(d_{31}\right)E^2(w)$$

$$P_c(2w) = 2\varepsilon_0 E^2(w)\Big[-d_{11}\cos^3\phi\cos\theta - 2d_{22}\sin 2\phi\cos\theta\cos\phi$$

$$+ d_{22}\cos 2\phi\cos\theta\sin\phi$$

$$- d_{31}\sin\phi\Big]$$

3m General Conditions:

$$\begin{pmatrix} 0 & 0 & 0 & 0 & d_{24} & -d_{22} \\ -d_{22} & d_{22} & 0 & d_{24} & 0 & 0 \\ d_{31} & d_{31} & d_{33} & 0 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & 0 & 0 & d_{31} & -d_{22} \\ -d_{22} & d_{22} & 0 & d_{31} & 0 & 0 \\ d_{31} & d_{31} & d_{33} & 0 & 0 & 0 \end{pmatrix}$$

$$P_x(2\omega) = 2\epsilon_0 d_{eff} \cdot \vec{E}(\omega)\vec{E}(\omega) = 2\epsilon_0 d_{eff} \begin{pmatrix} E_x^2 \\ E_y^2 \\ E_z^2 \\ 2E_yE_z \\ 2E_xE_z \\ 2E_xE_y \end{pmatrix} E^2(\omega)$$

$$= 2\epsilon_0 (d_{31} \cdot 2E_xE_z + d_{22}E_xE_y) E^2(\omega)$$

$$P_y(2\omega) = 2\epsilon_0 (-d_{22}E_x^2 + d_{22}E_y^2 + d_{31} \cdot 2E_yE_z) E^2(\omega)$$

$$P_z(2\omega) = 2\epsilon_0 (d_{31}E_x^2 + d_{31}E_y^2 + d_{33}E_z^2) E^2(\omega)$$

$P(2\omega) = 4\epsilon_0$ Remember, $\vec{E}(\omega) = E(\omega)\begin{pmatrix} -\sin\phi \\ \cos\phi \\ 0 \end{pmatrix}$; $\vec{E}(2\omega) = E(2\omega)\begin{pmatrix} \cos\theta\cos\phi \\ \cos\theta\sin\phi \\ -\sin\theta \end{pmatrix}$

$$P_x(2\omega) = 2\epsilon_0 (+d_{22}\sin\phi\cos\phi) E^2(\omega) = 2\epsilon_0 (d_{22}\sin 2\phi$$

$$P_y(2\omega) = 2\epsilon_0 (d_{22}\cos^2\phi + d_{22}\sin^2\phi) E^2(\omega) = 2\epsilon_0 d_{22} E^2(\omega)\cos 2\phi$$

$$P_z(2\omega) = 2\epsilon_0 (d_{31}\sin^2\phi + d_{31}\cos^2\phi) E^2(\omega) = 2\epsilon_0 d_{31} E^2(\omega)$$

$$P_e(2\omega) = 2\epsilon_0 E^2(\omega)[d_{22}\sin 2\phi\cos\theta\cos\phi + d_{22}\cos 2\phi\cos\theta\sin\phi - d_{31}\sin\theta] = \boxed{2\epsilon_0 E^2(\omega)\underbrace{[d_{22}\sin 3\phi\cos\theta - d_{31}\sin\theta]}_{d_{eff}}}$$

$$P(2\omega) = 2\epsilon_0 \begin{pmatrix} 0 & 0 & 0 & 0 & d_{24} & -d_{22} \\ -d_{22} & d_{22} & 0 & d_{24} & 0 & 0 \\ d_{31} & d_{31} & d_{33} & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} E_x^2(\omega) \\ E_y^2(\omega) \\ E_z^2(\omega) \\ 2E_y(\omega)E_z(\omega) \\ 2E_x(\omega)E_z(\omega) \\ 2E_x(\omega)E_y(\omega) \end{pmatrix}$$

$$P_x(2\omega) = 4\epsilon_0 (d_{22}E_x(\omega)E_z(\omega) - d_{22}E_x(\omega)E_y(\omega))$$

$$P_y(2\omega) = 2\epsilon_0 (-d_{22}E_x^2(\omega) + d_{22}E_y^2(\omega) + d_{24}E_y(\omega)E_z(\omega))$$

$$P_z(2\omega) = 2\epsilon_0 (d_{31}E_x^2(\omega) + d_{31}E_y^2(\omega) + d_{33}E_z^2(\omega))$$

Note, $\vec{E}(\omega) = E(\omega)\begin{pmatrix} -\sin\phi \\ \cos\phi \\ 0 \end{pmatrix}$; $\vec{E}(2\omega) = E(2\omega)\begin{pmatrix} \cos\theta\cos\phi \\ \cos\theta\sin\phi \\ -\sin\theta \end{pmatrix}$

$$P_x(2\omega) \cdot 4\epsilon_0 (+d_{22}\sin\phi\cos\phi)\vec{E}_x(2\omega) = 4\epsilon_0 d_{22}\sin\phi\cos^2\phi\cos\theta$$

$$P_y(2\omega) = 2\epsilon_0 (-d_{22}\sin^2\phi + d_{22}\cos^2\phi)\vec{E}_y(2\omega) = 2\epsilon_0 d_{22}\cos 2\phi$$

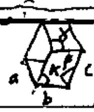
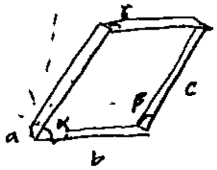
$$P_z(2\omega) = 2\epsilon_0 (d_{31}\sin^2\phi + d_{31}\cos^2\phi)\vec{E}_z(2\omega) = 2\epsilon_0 d_{31}(-\sin\theta)$$

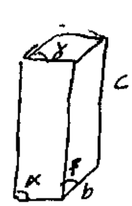
$$\vec{P}(2\omega) = 2\epsilon_0 \left[2d_{22}\sin\phi\cos^2\phi\cos\theta + d_{22}\cos 2\phi + d_{31}\sin\theta\right] E(2\omega)$$

$$\therefore\ d_{eff} = d_{22}\cos^2\theta\cos 3\phi$$

8. Verify ~~Tables~~ 1.5.2 and 1.5.4 with 1.5.3

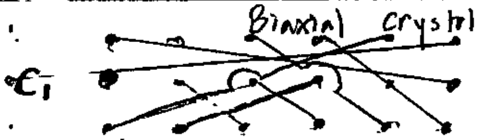
Table 1.5.2: Second-order susceptibility tensor for 32 crystal classes.

Crystal System	Crystal Class	Nonvanishing Tensor Elements	Verification
Triclinic	$1 = C_1$ $\bar{1} = S_2$	Each element is independent and nonzero Each element vanishes	$a \neq c$; $\beta \neq 90°$ 
Monoclinic	$2 = C_2$ $m = C_{1h}$ $2/m = C_{2h}$	xyz, xzy, xxy, xyx, yxx, yyy, yzz, yzx, yxz, zyz, zzy, zxy, zyx (two-fold axis parallel to $\hat{y}$) xxx, xyy, xzz, xzx, xxz, yyz, yzy, yxy, yyx, zxx, zyy, zzz, zzx, zxz (mirror plane $\perp \hat{y}$) Each element vanishes	$a \neq b \neq c$; $\alpha = \gamma = 90° \neq \beta$ 
Orthorhombic	$222 = D_2$ $mm2 = C_{2v}$ $mmm = 2D_{2h}$	xyz, xzy, yzx, yxz, zxy, zyx xzx, xxz, yyz, yzy, zxx, zyy, zzz Each element vanishes.	$a \neq b \neq c$; $\alpha = \beta = \gamma = 90°$ 
Tetragonal	$4 = C_4$ $\bar{4} = S_4$ $422 = D_4$ $4mm = C_{4v}$ $\bar{4}2m = D_{2d}$ $4/m = C_{4h}$ $4/mmm = D_{4h}$	xyz = −yxz, xzy = −yzx, xzx = yzy, xxz = yyz, zxx = zyy, zzz, zxy = −zyx xyz = yxz, xzy = yzx, xzx = −yzy, xxz = −yyz, zxx = −zyy, zxy = zyx xyz = yxz, xzy = yzx, xzx = −yzy, xxz = −yyz, zxx = −zyy. zxy = zyx xzx = yzy, xxz = yyz, zxx = zyy, zzz xyz = yxz, xzy = yzx, zxy = zyx Each Element vanishes Each element vanishes.	$a = b \neq c$; $\alpha = \beta = \gamma = 90°$ 
Cubic	$432 = O$ $\bar{4}3m = T_d$ $23 = T$ $m3 = T_h$, $m3m = O_h$	xyz = −xzy = yzx = −yxz = zxy = −zyx xyz = xzy = yzx = yxz = zxy = zyx xyz = yzx = zxy, xzy = yxz = zyx Each element vanishes	$a = b = c$; $\alpha = \beta = \gamma = 90°$ 
Trigonal	$3 = C_3$ $32 = D_3$ $3m = C_{3v}$ $\bar{3} = S_6$, $\bar{3}m = D_{3d}$	xxx = −xyy = −yyz = −yxy, xyz = −yxz, xzy = −yzx, xzx = yzy, xxz = yyz, yyy = −yxx = −xxy = −xyx, zxx = zyy, zzz, zxy = −zyx xxx = −xyy = −yyx = −yxy, xyz = −yxz, xzy = −yzx, zxy = −zyx xzx = yzy, xxz = yyz, zxx = zyy, zzz, yyy = −yxx = −xxy = −xyx (mirror $\perp \hat{x}$) Each element vanishing	$a = b = c$; $\alpha = \beta = \gamma \neq 90°$ 

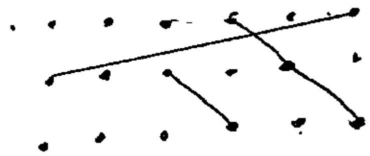
Hexagonal	$6 = C_6$ $xyz = -yxz$, $xzy = -yzx$, $xzx = yzy$, $xxz = yyz$, $zxx = zyy$, zzz, $zxy = -zyx$ $\bar{6} = C_{3h}$ $xxx = -xyy = -yxy = -yyx$, $yyy = -yxx = -xyx = -xxy$ $622 = D_6$ $xyz = -yxz$, $xzy = -yxz$, $zxy = -zyx$ $6mm = C_{6v}$ $xzx = yzy$, $xxz = yyz$, $zxx = zyy$, zzz $\bar{6}m2 = D_{3h}$ $yyy = -yxx = -xxy = -xyx$ $6/m = C_{6h}$ Each element vanishes $6/mmm = D_{6h}$ Each element vanishes.	 $a = b \neq c$ $\alpha = \beta = 90°$; $\gamma = 120°$

Biaxial crystal classes

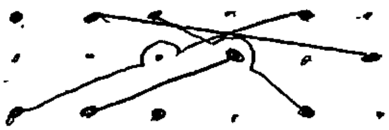
Class 1 C_1



class 2 C_2

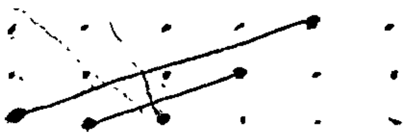


Class m C_{1h}



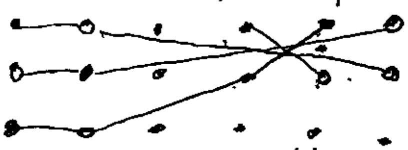
class 222 D_2 rot, ref

class mm2 C_{2v}



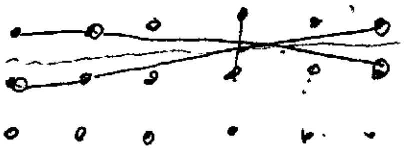
Uniaxial crystal classes.

Class 3 C_3



class 3m C_{3v}

Class $\bar{6}$ C_{3h}



Class $\bar{6}m2$ D_{3h}

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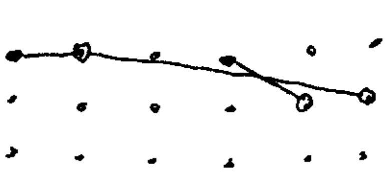
class C_6
6 and 4 C_4

class C_{6v}
6mm 4mm C_{4v}

class D_6
622 422 D_4

Class S_4
$\bar{4}$

Class 32 D_3



Class D_{2d}
$\bar{4}2m$

Class $\bar{4}3m$ T_d
23 T

class 432 O

Material	Point Group	d_{il} (pm/V)
Ag_3AsS_3 (proustite)	$3m = C_{3v}$	$d_{22} = 18$ $d_{15} = 11$
$AgGaSe_2$	$\bar{4}2m = D_{2d}$	$d_{36} = 33$
$AgSbS_3$ (pyragyrite)	$3m = C_{3v}$	$d_{15} = 8$ $d_{32} = 9$
beta-BaB_2O_4 (BBO) (beta barium borate)	$3m = C_{3v}$	$d_{22} = 2.2$
$CdGeAs_2$	$\bar{4}2m = D_{2d}$	$d_{36} = 235$
CdS	$6mm = C_{6v}$	$d_{33} = 78$ $d_{31} = -40$
GaAs	$\bar{4}3m$	$d_{36} = 370$
KH_2PO_4 (KDP)	$\bar{4}2m$	$d_{36} = 0.43$
KD_2PO_4 (KD*P)	$\bar{4}2m$	$d_{36} = 0.42$
$LiIO_3$	$6 = C_6$	$d_{15} = -5.5$ $d_{31} = -7$
$LiNbO_3$	$3m = C_{3v}$	$d_{32} = -30$ $d_{31} = -5.9$
Quartz	$32 = D_3$	$d_{11} = 0.3$ $d_{14} = 0.008$

10. Eqn 6.3.33 $\chi^{(1)} = \frac{-\kappa_0(0)}{\omega_{ba}/c}\frac{\Delta T_2 - i}{1+\Delta^2 T_2^2}$; $\chi^{(3)} = \frac{\kappa_0(0)}{3\omega_{ba}/c}\left[\frac{\Delta T_2 - i}{(1+\Delta^2T_2^2)^2}\right]\frac{1}{|E_s^0|^2}$

Rabi Freq | Field Strength

Eqn 6.3.23

$$\chi = -\frac{\kappa_0(0)}{\omega_{ba}/c}\frac{\Delta T_2 - i}{1+\Delta^2 T_2^2 + \Omega^2 T_1 T_2} \quad ; \quad \Delta = \omega - \omega_{ba} ; |E_s^0|^2 = \frac{\hbar^2}{4|\mu_{ba}|^2 T_1 T_2}$$

$$\kappa_0(0) = -\frac{\omega_{ba}}{c}\left[N(\rho_{bb}-\rho_{aa})^{(eq)}|\mu_{ba}|^2\frac{T_2}{\epsilon_0\hbar}\right]$$

Unsaturated, line center absorption coefficient

Kramer-Kronig Linear Optics

$$\chi^{(1)}(\omega) = \chi^{(1)}(\omega;\omega) = \int_0^\infty \underbrace{R^{(1)}(\tau)}_{\text{Linear Response Function}} e^{i\omega\tau}\,d\tau \ ; \chi^{(1)}(-\omega) = \chi^{(1)}(\omega)$$

Establishing $\text{Int} = \int_{-\infty}^{\infty}\frac{\chi^{(1)}(\omega')d\omega'}{\omega'-\omega}$ becomes

$$\int_{-\infty}^{\infty}\frac{\chi^{(1)}(\omega')d\omega'}{\omega'-\omega} = \lim_{\delta\to 0}\left[\int_{-\infty}^{\omega-\delta}\frac{\chi^{(1)}(\omega')d\omega'}{\omega'-\omega} + \int_{\omega+\delta}^{\infty}\frac{\chi^{(1)}(\omega')d\omega'}{\omega'-\omega}\right]$$

Int = Int(A) - Int(B) - Int(C) ; Int(A) = 0 because has no poles,
Int(C) = $-\pi i\chi(\omega)$ is from "Residue Theory".

$$\chi^{(1)}(\omega) = \frac{-i}{\pi}\int_{-\infty}^{\infty}\frac{\chi^{(1)}(\omega')d\omega'}{\omega'-\omega}$$

From here $\operatorname{Re}\chi^{(1)}(\omega) = \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{\operatorname{Im}\chi^{(1)}(\omega')d\omega'}{\omega'-\omega}$

$$\operatorname{Im}\chi^{(1)}(\omega) = \frac{-1}{\pi}\int_{-\infty}^{\infty}\frac{\operatorname{Re}\chi^{(1)}(\omega')d\omega'}{\omega'-\omega}$$

Kramer-Kronig Nonlinear Optics

$$\chi^{(3)}(\omega;\omega,\omega_1,-\omega_1) = \frac{1}{i\pi}\int_{-\infty}^{\infty}\frac{\chi^{(3)}(\omega';\omega',\omega_1,-\omega_1)d\omega'}{\omega'-\omega}$$

1. $\chi^{(1)} = \frac{-\kappa_0(0)}{\omega_{ba}/c}\frac{\Delta T_2 - i}{1+\Delta^2T_2^2}$

2. $\Omega^2 T_1 T_2 \Rightarrow \frac{|E|^2}{|E_s^0|^2}$

$$\chi = +\frac{\omega_{ba}}{c}\left[N(\rho_{bb}-\rho_{aa})|\mu_{ba}|^2\frac{T_2}{\epsilon_0\hbar}\right]\frac{\Delta T_2 - i}{1+\Delta^2T_2^2+\Omega^2T_1T_2}$$

If the denominator does not approach zero

1. $\chi^{(3)} = \mp\frac{\kappa_0(0)}{3\omega_{ba}/c}\left[\frac{\Delta T_2 - i}{(1+\Delta^2T_2^2)^2}\right]\frac{1}{|E_s^0|^2}$

The denominator does not approach zero

* Therefore, the first order relation contributes the most to two-level

12. $P^{(3)}(\omega) = 3\epsilon_0 \chi^{(3)} |E(\omega)|^2 E(\omega)$; $\underbrace{\vec{k}_1}_{\text{Nonlinear}}$ and $\underbrace{\vec{k}_2}_{\text{Linear}}$; $E_i(\omega) = E_0 e^{-i[k_1 x + \omega t]} + E_0 e^{-i(k_2 x - \omega t)}$

$P^{(3)}(\omega) = 3\epsilon_0 \chi^{(3)} \left| E_0^2 \left(e^{-2i(k_1 x - \omega t)} + 2e^{-i([k_1+k_2]x - 2\omega t)} + e^{-2i(k_2 x - \omega t)} \right) \right| E(\omega)$

$= 3\epsilon_0 \chi^{(3)} E_0^3 \left| \underbrace{e^{-3i(k_1 x - \omega t)}}_{\text{Linear term}} + \underbrace{3\left(e^{-i([2k_1+k_2]x - 3\omega t)} + e^{-i([k_1+2k_2]x - 3\omega t)}\right)}_{\text{Mixing term}} + \underbrace{e^{-3i(k_2 x - \omega t)}}_{\text{Nonlinear Term}} \right|$

Chapter 2:

1. $\omega_3 = \omega_1 + \omega_2$; length $\doteq L$; nonlinear coefficient d_{eff} phase mismatch ΔK

Estimate the quantum efficiency of upconversion of 10-μm infrared radiation using 1-cm-long proustite crystal, 1W laser @ 0.65μm.

Efficiency : $\eta = \frac{u_2^2(L)}{u_1^2(0)}$; $\underbrace{u_2(\zeta) = \tanh(\zeta)}_{\text{"Second-harmonic"}}$; $\underbrace{u_1(\zeta) = \operatorname{sech}(\zeta)}_{\text{"Fundamental"}}$; $u_1(0) = 1$; $\zeta = \left(\frac{16\pi^2 d_{eff}^2 \cdot L \cdot P}{\epsilon_0 c n_1 n_2 \lambda_1^2}\right)^{1/2}$

$$\eta = \frac{u_2^2(L)}{1} = \tanh^2\left(\frac{16\pi^2 d_{eff}^2 \cdot L \cdot P}{\epsilon_0 c \cdot n_1 \cdot n_2 \cdot \lambda_1^2}\right)^{1/2} = \tanh^2\left(\frac{16\pi^2 (4\times10^{-12}\,m/V)^2 \, 0.01m \cdot 1\,\frac{J}{s}}{8.85\times10^{-12}\,C\cdot V^{-1}\cdot m^{-1} \cdot 2.998\times10^8 \frac{m}{s} \cdot 1 \cdot 2 \cdot \left(0.65\mu m \times \frac{1m}{10^6 \mu m}\right)^2}\right)^{1/2}$$

$$= \tanh^2\left[\left(\frac{2.53\times10^{-23}}{1.46\times10^{-21}}\right)^{1/2}\right] = \tanh^2(0.13) = 0.017$$

$$\boxed{= 1.7\% \text{ Efficiency}}$$

2. Eq 2.2.10 $\frac{dA_3}{dz} = \frac{2i d_{eff} \omega_3}{n_3 c} A_1 A_2 e^{i\Delta k z}$

Eq 2.2.12b $\frac{dA_2}{dz} = \frac{2i d_{eff} \omega_2}{n_2 c} A_3 A_1^* e^{-i\Delta k z}$; $\Delta k = 0 = k_1 + k_2 - k_3$ $\boxed{\text{Perfect phase Matching}}$

Eq 2.2.12a $\frac{dA_1}{dz} = \frac{2i d_{eff} \omega_1}{n_1 c} A_3 A_2^* e^{-i\Delta k z}$.

Starting with $\frac{dA_3}{dz} = \frac{2i d_{eff} \cdot \omega_3}{n_3 c} A_1 A_2 e^{i(0)\cdot z}$; $A_3 = \frac{2i d_{eff} \cdot \omega_3}{n_3 c} \int_0^L e^{i\Delta k \cdot z} dz$

$= \frac{2i d_{eff} \omega_3 \cdot A_1 A_2}{n_3} \left[\frac{e^{i\Delta k \cdot L} - 1}{i\Delta k}\right]$

@ $\Delta k = 0$

$\frac{dA_1}{dz} = \frac{2i d_{eff} \omega_1}{n_1 c} A_3 A_2^* e^{-i\Delta k z}$

$A_1 = \frac{2i d_{eff} \omega_1}{n_1 c} A_3 A_2^* \int_0^L e^{-i\Delta k z} dz$

$= \frac{2i d_{eff} \cdot \omega_1}{n_1 c} A_3 A_2^* \left[\frac{e^{-i\Delta k L} - 1}{-i\Delta k}\right]$

$\frac{dA_2}{dz} = \frac{2i d_{eff} \omega_2}{n_2 c} A_3 A_1^* e^{-i\Delta k z}$; $A_2 = \frac{2i d_{eff} \omega_2}{n_2 c} A_3 A_1^* \left[\frac{e^{-i\Delta k L} - 1}{-i\Delta k}\right]$

$\boxed{\lim_{k\to0} A_3 = \frac{2i d_{eff} \omega_3 A_1 A_2}{n_3} \cdot L \; ; \; \lim_{k\to0} A_2 = \frac{2i d_{eff} \omega_2}{n_2 c} A_3 A_1^* \cdot L \; ; \; \lim_{k\to0} A_1 = \frac{2i d_{eff} \omega_1}{n_1 c} A_3 A_2^* \cdot L}$

9. Eqn: 2.9.2a $A_1(z) = \left[A_1(0)\left(\cosh gz - \frac{i\Delta k}{2g}\sinh gz\right) + \frac{\kappa_1}{g} A_2^*(0) \sinh gz\right] e^{i\Delta k z/2}$

$A_2(z) = \left[A_2(0)\left(\cosh gz - \frac{i\Delta k}{2g}\sinh gz\right) + \frac{\kappa_2}{g} A_1^*(0) \sinh gz\right] e^{i\Delta k z/2}$

where $g = [\kappa_1 \kappa_2^* - (\Delta k/2)^2]^{1/2}$; $\kappa_1 = \frac{2i\omega_1^2 d_{eff} A_3}{k_1 c^2}$; $\Delta k = k_3 - k_1 - k_2$.

$\omega_1 + \omega_2 = \omega_3$. A_3 of ω_3 is constant

$$\frac{dA_1}{dz} = \frac{2i\omega_1^2 d_{eff}}{K_1 c^2} A_3 A_2^* e^{i\Delta k z}$$; ① Differentiate the second equations: $$\frac{\partial^2 A_2}{\partial^2 z} = \frac{-2\Delta k \omega_2^2 d_{eff}}{K_2 c^2} A_3 A_1^* e^{i\Delta k z}$$

$$\frac{dA_2}{dz} = \frac{2i\omega_2^2 d_{eff}}{K_2 c^2} A_3 A_1^* e^{i\Delta k z}$$ ② Complex conjugate of first equation: $$\frac{dA_1^*}{dz} = \frac{-2i\omega_1^2 d_{eff}}{K_1 c^2} A_3^* A_2 e^{-i\Delta k z}$$

③ Solve for $A_1^* = \frac{2\omega_1^2 d_{eff}}{K_1 c^2} A_3^* A_2 \frac{e^{-i\Delta k z}}{\Delta k}$

④ Plug into equation [illegible] $\frac{\partial^2 A_2}{\partial^2 z} = \frac{4\omega_1^2\omega_2^2 d_{eff}^2}{K_1 K_2 c^4} A_3 A_3^* A_2 \equiv K^2 A_2$

⑤ Generate real coupling constant K given by: $K^2 = \frac{4\omega_1^2\omega_2^2 d_{eff}^2}{K_1 K_2 c^4}|A_3|^2$

⑥ Utilize general solution to partial diff eqn

$A_2(z) = C\sinh Kz + D\cosh Kz$

With initial conditions $A_2(0) = 0$ & $A_1(0) = $ arbitrary

$A_2(0) = [C(0) + iD] [illegible] = 0$ ∴ $D = 0$; $A_2(z) = C\sinh Kz$; $C = A_1^*(0)$

$\frac{4\omega_1^2\omega_2^2 d_{eff}}{[illegible]\, n_1^2 \omega_2^2\, c^2} = \frac{4\omega_1\omega_2 d_{eff}}{n_1 \cdot n_2 \cdot c^2}$; $A_1(0) \ne 0$ [illegible] $D \cdot 1 = $ constant

$D = $ constant $= A_1(0)$

$$\boxed{A_1(z) = A_1(0)\cosh(Kz)}$$
$$\boxed{A_2(z) = i\left(\frac{n_1\omega_2}{n_2\omega_1}\right)^{1/2}\frac{A_3}{|A_3|}A_1^*(0)\sinh Kz}$$

8. $A_2(0) = 0$; $A_1(0) = $ const

$\Delta K = $ Arbitrary.

Try again from Here

$$\frac{dA_1}{dz} = \frac{2i\omega_1^2 d_{eff}}{K_1 c^2} A_3 A_2^* e^{i\Delta k z}$$

$$\frac{dA_2}{dz} = \frac{2i\omega_2^2 d_{eff}}{K_2 c^2} A_3 A_1^* e^{i\Delta k z}$$

① Differentiate the first or second equation.

$$\frac{d^2 A_1}{d^2 z} = \frac{2\omega_1^2 d_{eff}}{\Delta K K_1 c^2} A_3 A_2^* e^{i\Delta k z}$$

② Complex conjugate of the first or second equation.

$$\frac{dA_2^*}{dz} = \frac{-2i\omega_2^2 d_{eff}}{K_2 c^2} A_3^* A_1 e^{-i\Delta k z}$$

③ Solve for the conjugate: $A_2^* = \frac{+2\omega_2^2 d_{eff}}{\Delta K K_2 c^2} A_3^* A_1 e^{-i\Delta k z}$

④ Plug the solution for conjugate amplitude into the second-order derivative:

$$\frac{d^2 A_1}{d^2 z} = \frac{4\omega_1^2\omega_2^2 d_{eff}^2}{\Delta K^2 K_1 K_2 c^4} A_3 A_3^* A_1 e^{-i\Delta k z} = K^2 A_1 \text{ ; where } K^2 = K_1 K_2 = \frac{4\omega_1^2\omega_2^2 d_{eff}^2}{\Delta K^2 K_1 K_2 c^4}|A_3|^2.$$

⑤ The general solution is $A_1(z) = B\sinh(Kz) + C\cosh(Kz)$.

$$A_2(z) = \frac{-BK}{K_1}\sinh(Kz) + \frac{CK}{K_1}\cosh(Kz)$$

⑥ Boundary conditions of $A_1(0) = $ constant, $A_3(0) = 0$; then.

$$A_1(z) = A_1\cosh(Kz) \text{ ; } A_2(z) = -A_1(0)\frac{[illegible]\,\omega_3 d_{eff}|A_3|}{(K_1 K_3)^{1/2} c^2}\frac{K_1 c^2}{2i\omega_1^2 d_{eff} A_3^*}\sinh Kz$$

$$= -A_1(0)\left(-i\left(\frac{n_1\omega_3}{n_3\omega_1}\right)^{1/2}\frac{|A_2|}{A_2^*}\sinh Kz\right)$$

⑦ Simplify $\frac{|A_2|}{A_2^*} = \frac{A_2}{A_2}\frac{|A_2|}{A_2^*} = \frac{A_2|A_2|}{|A_2|^2} = \frac{A_2}{|A_2|} = e^{i\phi_2}$

⑧ For $A_3(z) = i\left(\frac{n_1\omega_3}{n_3\omega_1}\right)^{1/2} A_1(0)\sinh(Kz)e^{i\phi_2}$

⑨ General solution for wavevector mismatch: $A_1(z) = (Fe^{igz} + Ge^{-igz})e^{-i\Delta kz/2}$

$$A_2(z) = (Ce^{igz} + De^{-igz})e^{i\Delta kz/2}$$

(10) Substituting the solution into the first equation:

$$(igFe^{igz} - igGe^{-igz})e^{-(1/2)i\Delta Kz} - \frac{1}{2}i\Delta K(Fe^{igz}+Ge^{-igz})e^{-(1/2)i\Delta Kz}$$
$$= [(F[ig-\tfrac{1}{2}i\Delta K]) + (G[ig+\tfrac{1}{2}i\Delta K])]e^{-\frac{1}{2}i\Delta Kz}$$
$$= [K_1 Ce^{igz} + K_1 De^{-igz}]e^{-\frac{1}{2}i\Delta Kz}$$

(11) Substituting the solution into the second equation:

$$(igCe^{igz} - igDe^{-igz})e^{(1/2)i\Delta Kz} + \frac{1}{2}i\Delta K(Ce^{igz}+De^{-igz})e^{(1/2)i\Delta K z}$$
$$= [C(ig+\tfrac{1}{2}i\Delta K) + D(ig-\tfrac{1}{2}i\Delta K)]e^{\frac{1}{2}i\Delta Kz}$$
$$= (K_2Fe^{igz} + K_2Ge^{-igz})e^{(\frac{1}{2})i\Delta Kz}$$

(12) Matrix form:

$$\begin{bmatrix} i(g-\frac{1}{2}\Delta K) & -K_1 \\ -K_2 & i(g+\frac{1}{2}\Delta K)\end{bmatrix}\begin{bmatrix} F \\ C\end{bmatrix} = 0 \; ; \; g^2 = -K_1K_2 + \frac{1}{4}\Delta K^2$$
$$g = \sqrt{K^2 + \frac{1}{4}\Delta K^2}$$

(13) We find that $A_1(0) = F+G$ and $A_2(0) = C+D$; $A_1(0) = \text{const}$; $A_2(0) = 0$

$$A_1(z) = F(ig - \tfrac{1}{2}i\Delta K) = K_1C \; ; \; -G(ig+\tfrac{1}{2}i\Delta K) = K_1D$$

(14) The boundary conditions lead to the third solution $A_1(z) = [Fe^{igz} + Ge^{igz}]e^{-\frac{1}{2}\Delta Kz}$

$A_2(z) = [Ce^{igz} + De^{-igz}]e^{\frac{1}{2}\Delta Kz}$

becoming

$$\boxed{\begin{aligned} A_1(z) &= \left[A_1(0)\cosh gz + \left(\frac{K_1}{g}A_2(0) + \frac{i\Delta K}{2g}A_1(0)\right)\sinh gz\right]e^{-\frac{1}{2}i\Delta Kz} \\ A_2(z) &= \left[A_2(0)\cosh gz + \left(\frac{-i\Delta K}{2g}A_2(0) + \frac{K_2}{g}A_1(0)\right)\sinh gz\right]e^{\frac{1}{2}i\Delta Kz}\end{aligned}}$$

(15) Sketch representative cases: $\Delta K = 0$; $A_1(z) = [A_1(0)\cosh gz + \frac{K_1}{g}A_2(0)\sinh gz](1-I_1)$

$A_2(z) = [A_2(0)\cosh gz + \frac{K_2}{g}A_1(0)\sinh gz](1-I_2)$

6. Second Harmonic Generation:

$z=0$: A_1

$$A_1 = \left(\frac{I}{2n_1\epsilon_0 c}\right)^{1/2} u_1 e^{i\phi_1}$$

$$A_2 = \left(\frac{I}{2n_2\epsilon_0 c}\right)^{1/2} u_2 e^{i\phi_2} \quad \text{where } I = I_1 + I_2 \; ; \; I_j = 2n_j\epsilon_0 c|A_j|^2$$

Normalized field amplitudes $u_1(z)^2 + u_2(z)^2 = 1$; $\zeta = z/F$

$$u_1 = I_1/e^{i\phi_1} = I_1/e^{i\left[\frac{\theta+\phi-\Delta K z}{2}\right]} \quad f = \left(\frac{n_1^2 n_2 \epsilon_0 c}{2I}\right)^{1/2}\frac{c}{\omega_1 d_{eff}}$$

A wave $E_3 = \mathrm{Re}[\hat{a}_3 A_3(z)e^{i(k_3z+\omega_3t)}]$

$= \hat{a}_3\rho(z)\cos[k_3z - \omega_3t + \phi_3(z)]$

becomes

$$\frac{dA_1^*}{dz} = i(2w^2K/K_1\cos^2\alpha_1)A_2^*A_1e^{i(2k_1-k_2)z}$$
$$\frac{dA_2}{dt} = -i(4w^2K/K_2\cos^2\alpha_2)A_1^*e^{i(2k_1-k_2)z}$$

$$= \left[1^{st}+3^{rd}+5^{th}\right] e^{-r^2/w(z)^2} \cdot e^{ikr^2/2R(z)} \cdot e^{i\phi(z)} + \left[1^{st}+3^{rd}+5^{th}\right]\left[\frac{2r^2}{w(z)^3}\frac{w_0}{z}\left(\frac{w_0}{w(z)}\right) e^{-r^2/w(z)^2} e^{ikr^2/2R(z)} e^{i\phi(z)}\right.$$

$$+ e^{-r^2/w(z)^2}\left[\frac{2ikr^2}{2R(z)^2}\left(\left[1+\left(\pi w_0^2/\lambda z\right)^2\right] - z \cdot 2\left(\frac{\pi w_0^2}{\lambda z}\right)\left(\frac{\pi w_0^2}{\lambda z^2}\right)\right) e^{ikr^2/2R(z)} \cdot e^{i\phi(z)}\right.$$

$$\left.\left. + \frac{i}{1+\left(\frac{\lambda z}{\pi w_0^2}\right)^2}\left(\frac{\lambda}{\pi w_0^2}\right) e^{ikr^2/2R(z)} \cdot e^{i\phi(z)}\right]\right]$$

$$= \left[\left[1^{st}+3^{rd}+5^{th}\right]' + \left[1^{st}+3^{rd}+5^{th}\right]\left(\frac{-w_0^2 r^2}{w(z)^4} + \frac{ikr^2}{R(z)^2}\left(\frac{R(z)}{z}\right) - 2\left(\frac{\pi w_0^2}{\lambda z}\right)\right)\right.$$

$$\left. - \frac{i}{1+\left(\frac{\lambda z}{\pi w_0^2}\right)^2}\left(\frac{\lambda}{\pi w_0^2}\right)\right] e^{-r^2/w(z)^2} e^{ikr^2/2R(z)} e^{i\phi(z)}$$

$$= \left[\frac{3}{4}\left(\frac{\pi w_0^2}{\lambda}\right)\left(\frac{w_0}{w(z)}\right) + \left[\frac{2}{z} + \left(\frac{\lambda}{\pi w_0^2}\right)^2\right]\left(\frac{w_0}{w(z)}\right)^3 + \left[2\left(\frac{\pi w_0^2}{\lambda z}\right)^2 - \frac{ikr^2}{R(z)^3}\frac{w(z)^2}{w_0} - 2\left(\frac{\lambda}{\pi w_0^2}\right) + \left[\left(\frac{\lambda}{\pi w_0^2}\right)^2 z w_0^2 - i\left(\frac{\lambda}{\pi w_0^2}\right) w_0^2\right] 3\left(\frac{\lambda}{\pi w_0^2}\right)^2 z\right] \times\right.$$

$$\left(\frac{w_0^2}{w(z)^5}\right) + \left[2\left(\frac{\lambda}{\pi w_0^2}\right)^2 z + \frac{ikr^2}{2R(z)^2} w_0\right] \cdot 5z\left(\frac{\lambda}{\pi w_0^2}\right)^2\left(\frac{w_0^5}{w(z)^7}\right) + \left[1^{st}+3^{rd}+5^{th}\right]\left[e^{-r^2/w(z)^2} e^{ikr^2/2R(z)} \cdot e^{i\phi(z)}\right]$$

$$= \left[\frac{3}{4}\left(\frac{b}{2}\right)\left(\frac{w_0}{w(z)}\right) + \left[\frac{2}{z} + \left(\frac{2}{b}\right)^2\right]\left(\frac{w_0}{w(z)}\right)^3 + \left[\frac{b^2}{2z^2} - \frac{ikr^2}{R(z)^3}\frac{w(z)^2}{w_0} - \left(\frac{5}{z}\right) - \left[\left(\frac{5^2}{z}\right) w_0^2 - i\left(\frac{5}{z}\right) w_0^2\right] 3\left(\frac{5^2}{z}\right)\right]\left(\frac{w_0^3}{w(z)^5}\right)\right.$$

$$+ \left[2\left(\frac{5^2}{z}\right) + \frac{ikr^2}{2R(z)} w_0\right] 5\left(\frac{5^2}{z}\right)\left(\frac{w_0^5}{w(z)^7}\right)\Bigg]' + \left[\frac{-2}{z^2}\left(\frac{b}{2}\right)^2\left(\frac{w_0}{w(z)}\right) + \left[\left(\frac{5}{z}\right)^2 - i\left(\frac{5}{z}\right)^2\right]\left(\frac{w_0}{w(z)}\right)^3 - \left[2\left(\frac{5}{z}\right)^2 + \frac{ikr^2}{2R(z)^2} w_0\right]\left(\frac{w_0^3}{w(z)^5}\right)\right] \times$$

$$\boxed{\left[-r^2\left(\frac{w_0^2}{w(z)^4}\right) + \frac{ikr^2}{R(z)^2 z} + \frac{5^2 b^2}{2z^2} - i\left(\frac{5}{z}\right)\left(\frac{w_0}{w(z)}\right)\right]} e^{-r^2/w(z)^2} e^{ikr^2/2R(z)} \cdot e^{i\phi(z)}$$

$$= \left[1^{st}+3^{rd}+5^{th}\right] e^{-r^2/w(z)^2} \cdot e^{ikr^2/2R(z)} \cdot e^{i\phi(z)} + \left[\frac{2r^2}{z^2}\left(\frac{b}{2}\right)^2\left(\frac{w_0^3 r}{w(z)^6}\right) + \frac{ikr^2}{R(z) \cdot z^2}\left(\frac{b}{2}\right)^2\left(\frac{w_0}{w(z)}\right) + \frac{4}{z^2}\left(\frac{b}{2}\right)^4\left(\frac{w_0}{w(z)}\right)\right.$$

$$- \frac{2i}{z^2}\left(\frac{5}{z}\right)\left(\frac{b}{2}\right)^2\left(\frac{w_0}{w(z)}\right)^2 - r^2\left[\left(\frac{5^2}{z}\right) - i\left(\frac{5}{z}\right)\right]\left(\frac{w_0^5}{w(z)^7}\right) + \frac{ikr^2}{R(z) \cdot z}\left[\left(\frac{5^2}{z}\right) - i\left(\frac{5}{z}\right)^2\right]\left(\frac{w_0}{w(z)}\right)^3 - \frac{2}{z^2}\left(\frac{b}{2}\right)^2\left[\left(\frac{5^2}{z}\right) - i\left(\frac{5^2}{z}\right)\right]\left(\frac{w_0}{w(z)}\right)^3$$

$$- i\left[\left(\frac{1}{z^2}\right) - i\left(\frac{1}{z^2}\right)\right] 5^3\left(\frac{w_0}{w(z)}\right)^4 + r^2\left[2\left(\frac{5^2}{z}\right) + \frac{ikr^2}{2R(z)^2} w_0\right]\left(\frac{w_0^5}{w(z)^9}\right) - \frac{ikr^2}{R(z) \cdot z}\left[2\left(\frac{5^2}{z}\right) + \frac{ikr^2}{2R(z)^2} w_0\right]\left(\frac{w_0^3}{w(z)^5}\right)$$

$$\left. + \frac{2}{z^2}\left(\frac{b}{2}\right)^2\left[2\left(\frac{5^2}{z}\right) + \frac{ikr^2}{2R(z)^2} w_0\right]\left(\frac{w_0^3}{w(z)^5}\right) + i\left(\frac{5}{z}\right)\left[2\left(\frac{5^2}{z}\right) + \frac{ikr^2}{2R(z)^2} w_0\right]\left(\frac{w_0^4}{w(z)^6}\right)\right] e^{-r^2/w(z)^2} \cdot e^{ikr^2/2R(z)} \cdot e^{i\phi(z)}$$

$$= 2ik\eta\left[\left[\frac{5}{4}\left(\frac{b}{2}\right) - \frac{2ikr^2}{z^3 R(z)}\left(\frac{b}{2}\right)^2 + \frac{4}{z^2}\left(\frac{b}{2}\right)^4\right]\boxed{\left(\frac{w_0}{w(z)}\right)} - \frac{2i}{z^2}\left(\frac{5}{z}\right)\left(\frac{b}{2}\right)^2\boxed{\left(\frac{w_0}{w(z)}\right)^2} + \left[\frac{2}{z} + \left(\frac{5}{z}\right)^2 + \frac{ikr^2}{R(z) \cdot z}\left[\left(\frac{5^2}{z}\right) - i\left(\frac{5}{z}\right)^2\right] - \frac{2}{z^2}\left(\frac{b}{2}\right)^2\left[\left(\frac{5^2}{z}\right) - i\left(\frac{5}{z}\right)^2\right]\right]\boxed{\left(\frac{w_0}{w(z)}\right)^3}\right.$$

$$- i\left[\left(\frac{1}{z^2}\right) - i\left(\frac{1}{z^2}\right)\right] 5^3\boxed{\left(\frac{w_0}{w(z)}\right)^4} + \left[\frac{2r^2}{z^2}\left(\frac{5}{2}\right)^2 + \frac{2}{z^2}\left(\frac{b}{2}\right)^2\left[2\left(\frac{5^2}{z}\right) + \frac{ikr^2}{2R(z)^2} w_0\right] - \frac{ikr^2}{R(z) \cdot z}\left[2\left(\frac{5^2}{z}\right) + \frac{ikr^2}{2R(z)^2} w_0\right]\right.$$

$$\left. + \left(\frac{2}{z}\right)\left(\frac{b}{2}\right)^2 + \frac{ikr^2}{R(z)^3} w_0\left(\frac{z^2}{b^2}\right) 4\left(\frac{b}{z}\right)^2 - \left[\frac{5^2}{z} w_0^2 - i\left(\frac{5}{z}\right) w_0^2\right] 3\left(\frac{5^2}{z}\right)\right]\left(\frac{1}{w_0}\right)\boxed{\left(\frac{w_0^3}{w(z)^5}\right)}$$

$$+ i\left(\frac{5}{z}\right)\left[2\left(\frac{5^2}{z}\right) + \frac{ikr^2}{2R(z)^2} w_0\right]\boxed{\left(\frac{w_0^4}{w(z)^6}\right)} + \left[r^2\left[\left(\frac{5}{z}\right)^2 - i\left(\frac{5}{z}\right)^2\right] + \left[2\left(\frac{5^2}{z}\right) + \frac{ikr^2}{2R(z)} w_0\right] 5\left(\frac{5^2}{z}\right)\right]\boxed{\left(\frac{w_0^5}{w(z)^7}\right)}$$

$$+ r^2\left[2\left(\frac{5}{z}\right) + \frac{ikr^2}{2R(z)^2} w_0\right]\left(\frac{w_0^5}{w(z)^9}\right) + \rightarrow \text{Forgot}\left(\frac{w_0}{w(z)}\right)\left(\frac{5}{z}\right) - \frac{w_0^2}{\epsilon_0 c^2} P_n e^{i\Delta k z}$$

☆Identity☆

$$2ik_n \frac{\partial A_n}{\partial z} + \nabla_T^2 A_n = -\frac{\omega_n^2}{\epsilon_0 c^2} P_n e^{i\Delta k z} \;;\; A(r,z) = \frac{A}{1+i\zeta} e^{-r^2/w_0^2(1+i\zeta)}$$

$$\frac{\partial A_n}{\partial z} = \left(\frac{A}{1+i\zeta}\right)' e^{-r^2/w_0^2(1+i\zeta)} + \left(\frac{A}{1+i\zeta}\right)\left[e^{-r^2/w_0^2(1+i\zeta)}\right]'$$

$$= A\left[\left(\frac{w_0}{w(z)}\right)'\left(1-i\frac{2z}{b}\right) + \left(\frac{w_0}{w(z)}\right)\left(1-i\frac{2z}{b}\right)'\right] e^{-r^2/w_0^2(1+i\zeta)} + \left(\frac{A}{1+i\zeta}\right)\left(\frac{-r^2}{w_0^2}\right)\left[\left(\frac{w_0}{w(z)}\right)'\left(1-i\frac{2z}{b}\right) + \left(\frac{w_0}{w(z)}\right)\left(1-i\frac{2z}{b}\right)'\right] \times e^{-r^2/w_0^2(1+i\zeta)}$$

$$= A\left[\left(\frac{w_0}{w(z)}\right)^3 \frac{\zeta^2}{z}(1-i\zeta) + \frac{2i}{b}\left(\frac{w_0}{w(z)}\right)\right] e^{-r^2/w_0^2(1+i\zeta)} + \left(\frac{A}{1+i\zeta}\right)\left(\frac{-r^2}{w_0^2}\right)\left[\left(\frac{w_0}{w(z)}\right)^3 \frac{\zeta^2}{z}(1-i\zeta) - \frac{2i}{b}\left(\frac{w_0}{w(z)}\right)\right] e^{-r^2/w_0^2(1+i\zeta)}$$

$$\nabla_T^2 A_n = A\left[\frac{-3}{z}\left(\frac{w_0}{w(z)}\right)^3\frac{\zeta^2}{z}(1-i\zeta) + \left(\frac{w_0}{w(z)}\right)^3\left[\frac{4}{b^2}(1-i\zeta) + \frac{\zeta}{z}\left(1-\frac{2i}{b}\right)\right] - \frac{2i}{b}\frac{\zeta^2}{z}\left(\frac{w_0}{w(z)}\right)^3\right] e^{-r^2/w_0^2(1+i\zeta)}$$

$$+ A\left[\left(\frac{w_0}{w(z)}\right)^3\frac{\zeta^2}{z}(1-i\zeta) \cancel{- \frac{2i}{b}\left(\frac{w_0}{w(z)}\right)}\right]\left(\frac{-r^2}{w_0^2}\right)\left[\left(\frac{w_0}{w(z)}\right)^3\frac{\zeta^2}{z}(1-i\zeta) - \frac{2i}{b}\left(\frac{w_0}{w(z)}\right)\right] e^{-r^2/w_0^2(1+i\zeta)}$$

$$\cancel{A\left[\left(\frac{w_0}{w(z)}\right)^3 \frac{\zeta^2}{z}(1-i\zeta) - \frac{2i}{b}\left(\frac{w_0}{w(z)}\right)\right]\left(\frac{-r^2}{w_0^2}\right)\left[\left(\frac{w_0}{w(z)}\right)^3\frac{\zeta^2}{z}(1-i\zeta) - \frac{2i}{b}\left(\frac{w_0}{w(z)}\right)\right] e^{-r^2/w_0^2(1+i\zeta)}}$$

$$+ \frac{A}{(1+i\zeta)}\left(\frac{-r^2}{w_0^2}\right)\left[\frac{-3}{z}\left(\frac{w_0}{w(z)}\right)^3\frac{\zeta^2}{z}(1-i\zeta) + \left(\frac{w_0}{w(z)}\right)^3\left[\frac{4}{b^2}(1-i\zeta) + \frac{\zeta}{z}\left(1-\frac{2i}{b}\right)\right] - \frac{2i}{b}\frac{\zeta^2}{z}\left(\frac{w_0}{w(z)}\right)^3\right] e^{-r^2/w_0^2(1+i\zeta)}$$

$$+ A\left[\left(\frac{w_0}{w(z)}\right)^3\frac{\zeta^2}{z}(1-i\zeta) - \frac{2i}{b}\left(\frac{w_0}{w(z)}\right)\right]\left(\frac{-r^2}{w_0^2}\right)\left[\left(\frac{w_0}{w(z)}\right)^3\frac{\zeta^2}{z}(1-i\zeta) - \frac{2i}{b}\left(\frac{w_0}{w(z)}\right)\right] e^{-r^2/w_0^2(1+i\zeta)}$$

$$2ik_n \frac{\partial A_n}{\partial z} + \nabla_T^2 A_n = \frac{-\omega_n^2}{\epsilon_0 c^2} P_n e^{i\Delta k z}$$

12. Evaluate $J_q(\Delta k, z_0, z) = \int_{z_0}^{z} e^{i\Delta k z'} dz' = \frac{e^{i\Delta k z} - e^{i\Delta k z_0}}{i\cdot\Delta k} = \frac{\cos(\Delta k z) + i\sin(\Delta k z) - \cos(\Delta k z_0) - i\sin(\Delta k z_0)}{i\cdot\Delta k}$

$$= \frac{-2\sin\left(\frac{\Delta k(z+z_0)}{2}\right)\sin\left(\frac{\Delta k(z-z_0)}{2}\right) + i\cdot 2\cos\left(\frac{\Delta k(z+z_0)}{2}\right)\cos\left(\frac{\Delta k(z-z_0)}{2}\right)}{i\Delta k}$$

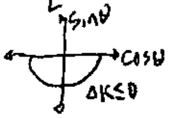
$$= \frac{+2\sin^2\left(\frac{\Delta k L}{2}\right) + 2i\cos^2\left(\frac{\Delta k L}{2}\right)}{i\Delta k} = \frac{\sin^2\left(\frac{\Delta k L}{2}\right)}{(i\Delta k/2)}$$

Evaluate

$$J_q(\Delta k, z_0, z) = \int_{-\infty}^{\infty} \frac{e^{i\Delta k z'} dz'}{(1+2iz'/b)^{q-1}} = \begin{cases} 0 & \Delta k \le 0 \\ \frac{b}{2}\frac{2\pi}{(q-2)!}\left(\frac{b\Delta k}{2}\right)^{q-2} e^{-\Delta k b/2} & \Delta k > 0 \end{cases}$$

$\Delta k \le 0$: $\Delta k = -a^2$

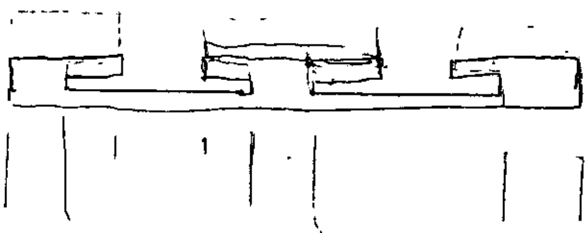
$$J_q(\Delta k, z_0, z) = \int_{-\infty}^{\infty} \frac{e^{ia^2 z} dz}{(1+2iz/b)^{q-1}} = \int_{-\infty}^{\infty}\int_0^{\pi} \frac{e^{-ia^2 Re^{i\theta}}\, d\theta\, dR}{(1+2iRe^{i\theta}/b)^{q-1}} = \iint_0^{\pi} \frac{e^{-ia^2 R\cos\theta}\, e^{a^2 R\sin\theta}\, d\theta\, dR}{(1+2iRe^{i\theta}/b)^{q-1}} \;;\; \lim_{R\to\infty} |F(R,\theta)| = \lim_{R\to\infty} \frac{e^{-a^2 R|\sin\theta|}}{(1+2iRe^{i\theta}/b)^{q-1}} = 0$$



Therefore, $\int_{-\infty}^{\infty} F(z)dz = \oint_{C_a} F(z)dz$; $z = \frac{-b}{2i} = \frac{ib}{2}$ = "upper half of plane" $= 0$;

$\Delta k > 0$: $\Delta k = a^2$; $J_q(\Delta k, z_0, z) = \int_{-\infty}^{\infty} \frac{e^{ia^2 z}}{(1+2iz/b)^{q-1}} = \int_0^R\int_0^{2\pi} \frac{e^{ia^2 Re^{i\theta}}\, dR\, d\theta}{(1+2iRe^{i\theta}/b)^{q-1}}$

$$= \int_0^R\int_0^{2\pi} \frac{e^{ia^2 R\cos\theta}\, e^{-a^2 R\sin\theta}}{(1+2iRe^{i\theta}/b)^{q-1}}$$

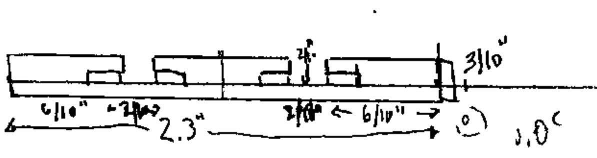


$\lim_{R\to\infty} |F(R,\theta)| = 0$; $\int_{-\infty}^{\infty} F(z)dz = \oint F(z)dz$

Applying Residue Theorem: $\oint_C F(z)dz = 2\pi i\cdot \text{Residue}\left[F(z), z = z_0, \text{order} = p-1\right]$

$$= 2\pi i \cdot \frac{1}{(p-2)!}\lim_{z\to z_0} \frac{d^{p-2}}{dz^{p-2}}\left\{F(z)\cdot(z-z_0)^{p-1}\right\} = \frac{2\pi i}{(p-2)!}\lim_{z\to z_0}\frac{d^{p-2}}{dz^{p-2}}\left\{\frac{z_0^{q-1}\, e^{ia^2 z}}{(-1)^{q-1}(z-z_0)^{q-1}}(z-z_0)^{q-1}\right\}$$

$$= \frac{2\pi i}{(p-2)!}\cdot\frac{z_0^{q-1}}{(-1)^{q-1}}\lim_{z\to z_0}\frac{d^{p-2}}{dz^{p-2}}\left\{e^{ia^2 z}\right\} = \frac{2\pi i}{(p-2)!}\frac{z_0^{q-1}}{(-1)^{q-1}}\lim_{z\to z_0}\left\{e^{i\Delta k z}\right\} = \frac{2\pi i}{(p-2)!}\frac{z_0^{q-1}}{(-1)^{q-1}}\lim_{z\to z_0}(i\Delta k)^{q-2} e^{i\Delta k z}$$



$$= \frac{2\pi i}{(p-2)!}\left(\frac{ib}{2}\right)^{q-1}(-1)^{q-1}(i\Delta k)^{q-2} e^{i\Delta k ib/2} = \frac{b}{2}\frac{2\pi}{(p-2)!}\left(\frac{b\Delta k}{2}\right)^{q-2}(-1)^{2q-1}(-1)^{q-1} e^{-b\Delta k/2}$$

= ans.

Paraxial Wave Equation: $2ik_n \frac{\partial A_n}{\partial z} + \nabla_T^2 A_n = -\frac{\omega_n^2}{\epsilon_0 c^2} p_n e^{i\Delta k z}$

$\boxed{\text{First Equation}}$ $A(r,z) = A\frac{w_0}{w(z)} e^{-r^2/w(z)^2} e^{ikr^2/2R(z)} e^{i\phi(z)}$

$$2ik_n \frac{\partial}{\partial z} \frac{Aw_0}{w(z)} e^{-r^2/w(z)^2} \cdot e^{ikr^2/2R(z)} \cdot e^{i\phi(z)}$$

$$+ \frac{\partial^2}{\partial^2 z} \frac{Aw_0}{w(z)} e^{-r^2/w(z)^2} e^{ikr^2/2R(z)} e^{i\phi(z)} = \frac{-\omega_n^2}{\epsilon_0 c^2} p_n e^{i\Delta k z}$$

$(ABCD)' = A'BCD + AB$

$ABCD = A'BCD + A(BCD)'$

$= A'BCD + A(B'CD + B(CD)')$

$= A'BCD + A(B'CD + B(C'D + CD'))$

$\boxed{\frac{d}{dz}}$ $-\frac{1}{2}\left[2(\lambda z/\pi w_0^2)\left(\frac{\lambda}{\pi w_0^2}\right)\right]^{-3/2} e^{-r^2/w(z)^2} \cdot e^{ikr^2/2R(z)} \cdot e^{i\phi(z)}$

$+ \frac{w_0}{w(z)}\left[\frac{2r^2}{w(z)^3} \cdot \frac{w_0}{2}\left[1+(\lambda z/\pi w_0^2)^2\right]^{-1/2}\left[2(\lambda z/\pi w_0^2)\left(\frac{\lambda}{\pi w_0^2}\right)\right]\right] e^{-r^2/w(z)^2} \cdot e^{ikr^2/2R(z)} \cdot e^{i\phi(z)}$

$+ e^{-r^2/w(z)^2}\left[\frac{-ikr^2}{2R(z)^2} \cdot \left[(1+(\pi w_0^2/\lambda z)^2) + z\left[2(\pi w_0^2/\lambda z)(\pi w_0^2/\lambda z^2)\right]\right]\right] e^{ikr^2/2R(z)} \cdot e^{i\phi(z)}$

$A[B \cdot + e^{ikr^2/2R(z)} \cdot i\left(\frac{1}{1+\left(\frac{\lambda z}{\pi w_0^2}\right)^2}\right)\left(\frac{\lambda}{\pi w_0^2}\right) e^{i\phi(z)}]$ 5

$\left(\frac{\lambda}{\pi w_0^2}\right)^2 \cdot z\left(\frac{w_0}{w(z)}\right)^3 + \left(\frac{w_0^3}{w(z)^5}\right)\left(2\left(\frac{\lambda}{\pi w_0^2}\right)^2 z\right) + \frac{w_0}{w(z)}\left[\frac{-ikr^2}{2R(z)^2}\left(\frac{w_0}{w(z)}\right)^4 \frac{1}{z}\left[\frac{2}{z^3}\left(\frac{\pi w_0^2}{\lambda}\right)^2\right]\right.$

$\left. - i\left(\frac{w_0}{w(z)}\right)^2\left(\frac{\lambda}{\pi w_0^2}\right)\right] e^{-r^2/w(z)^2} \cdot e^{ikr^2/2R(z)} \cdot e^{i\phi(z)}$

$$\left[\underbrace{-\frac{2}{z^2}\left(\frac{\pi w_0^2}{\lambda}\right)^2\left(\frac{w_0}{w(z)}\right)}_{1^{st}} + \underbrace{\left[\left(\frac{\lambda}{\pi w_0^2}\right)^2 \cdot z - i\left(\frac{\lambda}{\pi w_0^2}\right)\right]\left(\frac{w_0}{w(z)}\right)^3}_{3^{rd}} - \underbrace{\left[2\left(\frac{\lambda}{\pi w_0^2}\right)^2 z + \frac{ikr^2}{2R(z)^2} w_0\right]\left(\frac{w_0^3}{w(z)^5}\right)}_{5^{th}}\right] e^{-r^2/w(z)^2} e^{ikr^2/2R(z)} e^{i\phi(z)} = A$$

$\boxed{\frac{\partial^2}{\partial z^2}}$

1^{st} $\frac{6}{z^3}\left(\frac{\pi w_0^2}{\lambda}\right)^2\left(\frac{w_0}{w(z)}\right) + \frac{2}{z^2}\left(\frac{\pi w_0^2}{\lambda}\right)^2\left(\frac{w_0}{w(z)^2}\right)\frac{w_0}{2}\left(\frac{w_0}{w(z)}\right) \cdot 2\left(\frac{\lambda z}{\pi w_0^2}\right)\left(\frac{\lambda}{\pi w_0^2}\right)$

2^{nd} $= \frac{3}{4}\left(\frac{\pi w_0^2}{\lambda}\right)\left(\frac{w_0}{w(z)}\right) + \frac{2}{z}\left(\frac{w_0}{w(z)}\right)^3$

3^{rd} $\left(\frac{\lambda}{\pi w_0^2}\right)^2\left(\frac{w_0}{w(z)}\right)^3 + \left[\left(\frac{\lambda}{\pi w_0^2}\right)^2 z - i\left(\frac{\lambda}{\pi w_0^2}\right)\right] \cdot 3\left(\frac{w_0}{w(z)}\right)^2\left(\frac{w_0(-1)}{w(z)^2}\right) \cdot \frac{w_0}{2}\left(\frac{w_0}{w(z)}\right) \cdot 2\left(\frac{\lambda z}{\pi w_0^2}\right)\left(\frac{\lambda}{\pi w_0^2}\right)$

$= \left(\frac{\lambda}{\pi w_0^2}\right)^2\left(\frac{w_0}{w(z)}\right)^3 - \left[\left(\frac{\lambda}{\pi w_0^2}\right)^2 z - i\left(\frac{\lambda}{\pi w_0^2}\right)\right] 3\left(\frac{\lambda}{\pi w_0^2}\right)^2 \cdot z\left(\frac{w_0}{w(z)}\right)^5$

5^{th} $-\left[2\left(\frac{\lambda}{\pi w_0^2}\right) - \frac{2ikr^2}{2R(z)^3} w_0\left[\left[1+(\pi w_0^2/\lambda z)^2\right] + z \cdot 2\left(\frac{\pi w_0^2}{\lambda z}\right)\left(\frac{\pi w_0^2}{\lambda z^2}\right)\right]\right]\left(\frac{w_0^2}{w(z)^5}\right)$

$+ \left[2\left(\frac{\lambda}{\pi w_0^2}\right)^2 z + \frac{ikr^2}{2R(z)^2} w_0\right] 5\left(\frac{w_0^3}{w(z)^6}\right)\frac{w_0}{2}\left(\frac{w_0}{w(z)}\right) 2\left(\frac{\lambda z}{\pi w_0^2}\right)\left(\frac{\lambda}{\pi w_0^2}\right)$

$= \left[2\left(\frac{\pi w_0^2}{\lambda z}\right)^2 - \frac{ikr^2}{R(z)^3}\frac{w(z)^2}{w_0} - 2\left(\frac{\lambda}{\pi w_0^2}\right)\right]\left(\frac{w_0^2}{w(z)^5}\right)$

$+ \left[2 \cdot \left(\frac{\lambda}{\pi w_0^2}\right)^2 z + \frac{ikr^2}{2R(z)^2} w_0\right] \cdot 5\left(\frac{w_0^5}{w(z)^7}\right)\left(\frac{\lambda}{\pi w_0^2}\right)^2 \cdot z$

$[1^{st} + 3^{rd} + 5^{th}]' eee + [1^{st} + 3^{rd} + 5^{th}] e' +$

Rewritten as: $\frac{d\rho_1}{dz} = -(2w^2K/K_1\cos^2\alpha_1)\rho_1\rho_2\sin\theta$; $\frac{d\rho_2}{dz} = (4w^2K/K_2\cos^2\alpha_2)\rho_1^2\sin\theta$

where $\theta = 2\phi_1(z) - \phi_2(z) + \Delta K z$

$\Delta K = 2K_1 - K_2$

$\frac{d\theta}{dz} = \Delta K - 4w^2K\left[\rho_2/K_1\cos^2\alpha_1 - \rho_1^2/\rho_2K_2\cos^2\alpha_2\right]\cos\theta$

~~Manley-Rowe Relations~~ ~~Power of a lossless dielectric~~: $W = (c^2/8\pi w)\left[K\rho_1^2\cos^2\alpha_1 + \frac{1}{2}K_2\rho_2^2\cos^2\alpha_2\right]$

Q. Cavity length of L
Length of medium L_c
$[L_c < L]$

$\Delta\nu_c = \frac{1}{n^{(g)}}\frac{c}{2L_c}$; where $n^{(g)} = n + \nu\frac{dn}{d\nu}$

$\Delta\nu_L = \frac{c}{2L(n+\nu\frac{dn}{d\nu})} = \frac{v_g}{2L_c} = \frac{c}{2n_gL_c}$

$\Delta\nu_{L_c} = \frac{c}{2L_c(n+\nu\frac{dn}{d\nu})} = \frac{v_g}{2L_c} = \frac{c}{2n_gL_c}$

$\Delta\nu_L < \Delta\nu_{L_c}$

A generalization of that category would only compress the optical parametric oscillations by a factor of L/L_c.

~~Solving the coupled differential equations.~~

$\frac{d\theta}{dz} = \Delta K + \cos\theta/\sin\theta\,(d/dz)\ln(\rho_1^2\rho_2)$

With substitutions, $u = [c^2K_1\cos^2\alpha_1/8\pi wW]^{1/2}\rho_1$

$v = [c^2K_2\cos^2\alpha_2/16\pi wW]^{1/2}\rho_2$

$\zeta = (2w^2K/K_1\cos^2\alpha_1)(16\pi wW/c^2K_2\cos^2\alpha_2)^{1/2}z$

$\frac{du}{d\zeta} = -uv\sin\theta$; $\frac{dv}{d\zeta} = u^2\sin\theta$; $\frac{d\theta}{d\zeta} = \Delta s + (\cos\theta/\sin\theta)(d/d\zeta)\ln(u^2v)$

$\Delta s = \frac{\Delta K}{(2w^2K/K_1\cos^2\alpha_1)(16\pi wW/c^2K_2\cos^2\alpha_2)^{1/2}}$

where $\frac{d\theta}{d\zeta}$ can be rewritten: $\frac{d}{d\zeta}\ln(\underbrace{u_1^2u_2\cos\theta}_{\Gamma}) = 0$

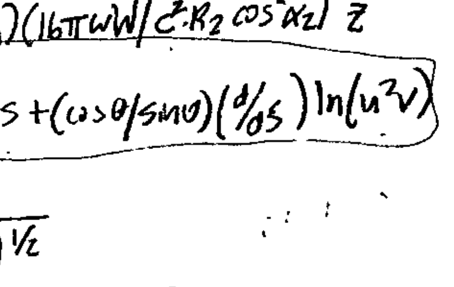
where $\frac{du_2}{d\zeta} = (1-u_2^2)(1-\cos^2\theta)^{1/2} = \pm(1-u_2^2)\left(1-\frac{\Gamma^2}{u_1^4u_2^2}\right)^{1/2} = \pm(1-u_2^2)\left(1-\frac{\Gamma^2}{(1-u_2^2)^2u_2^2}\right)^{1/2}$

$u_2\frac{du_2}{d\zeta} = \pm[(1-u_2^2)^2u_2^2 - \Gamma^2]^{1/2}$; $\frac{du_2^2}{d\zeta} = \pm2[(1-u_2^2)^2u_2^2 - \Gamma^2]^{1/2}$

Solved with Jacobi elliptic functions

$u = F(\phi, K) = \int_0^\phi \frac{dt}{\sqrt{1-K^2\sin^2 t}}$

When $\Gamma = 0$; $\cos\theta = 0$;



[Armstrong et. al 1910]

Q. Eqn 2.10.4a

$A(r,z) = A\frac{w_0}{w(z)}e^{-r^2/w(z)^2}e^{ikr^2/2R(z)}e^{i\phi(z)}$

Eqn 2.10.5a

$A(r,z) = \frac{A}{1+i\zeta}e^{-r^2/w_0^2(1+i\zeta)}$

where $w(z) = w_0[1+(\lambda z/\pi w_0^2)^2]^{1/2}$ [Beam Waist Radius]

$R(z) = z[1+(\pi w_0^2/\lambda z)^2]$ [Wavefront Radius]

$\Phi(z) = -\arctan(\lambda z/\pi w_0^2)$ [Spatial variation of phase]

Prove equivalent to Gaussian laser beam and Eq 2.10.3

$\zeta = 2z/b$ $\boxed{A(r,z) = A\frac{w_0}{w(z)}e^{-r^2/w^2(z)}e^{ikr^2/2R(z)}e^{i\phi(z)}}$

$b = 2\pi w_0^2/\lambda = kw_0^2$

$\frac{A}{1+i\zeta} = \frac{A}{1+i\frac{2z}{b}} = \frac{A}{1+\frac{4z^2}{b^2}}\left(1-i\frac{2z}{b}\right) = A\left(\frac{w_0}{w(z)}\right)\left(1-i\frac{2z}{b}\right) = A\left(\frac{w_0}{w(z)}\right)\sqrt{1+\frac{4z^2}{b^2}}\left(\frac{1}{\sqrt{1+\frac{4z^2}{b^2}}} - i\frac{1}{\sqrt{1+\frac{4z^2}{b^2}}}\frac{2z}{b}\right)$

$\left(\frac{w(z)}{w_0}\right)^2 = 1+\left(\frac{\lambda z}{\pi w_0^2}\right)^2 = 1+\frac{4z^2}{b^2}$;

$e^{-r^2/w_0^2(1+i\zeta)} = \exp\left(-\frac{r^2}{w_0^2(1+\frac{4z^2}{b^2})}\left(1-i\frac{2z}{b}\right)\right) = \exp\left(\frac{-r^2}{w^2(z)}\right)\exp\left(\frac{ir^2 2z}{bw(z)^2}\right)$

$\exp\left(\frac{ir^2 2z}{bw(z)^2}\right) = \exp(i\pi r^2/\lambda R(z)) = \exp(ikr^2/2R(z))$

$= A\left(\frac{w_0}{w(z)}\right)[\cos\phi(z) + i\sin\phi(z)] = A\left(\frac{w_0}{w(z)}\right)e^{i\phi(z)}$

[14] $\underbrace{\tilde{D}=\epsilon_0\tilde{E}+\tilde{P}}_{\text{"Electric field with polarization"}}$ with $\underbrace{\tilde{P}=\epsilon_0[\chi^{(1)}\tilde{E}+\chi^{(2)}E^2+\chi^{(3)}E^3]}_{\text{"Polarization"}}$; $\underbrace{\tilde{B}=\mu_0\tilde{H}}_{\text{"Magnetic Field"}}$

Poynting's Theorem

$$P=(E\times H)=\frac{1}{\mu}(E\times B)$$

Maxwell's Equations: $\vec\nabla\times\vec H=\vec J+\frac{\partial \vec D}{\partial t}$; $E\cdot(\vec\nabla\times\vec H)=EJ+E\frac{dD}{dt}$; Assuming value at zero:

$\vec\nabla\times\vec E=-\frac{dB}{dt}$

$$\nabla\cdot(E\times H)=H\cdot(\nabla\cdot E)-E(\nabla\times H)$$

$$\vec E\cdot(\nabla\times H)=H\underline{(\nabla\times E)}-\nabla\underline{(E\times H)}$$

$$H\cdot(\nabla\times E)-\nabla\cdot(E\times H)=E\cdot J+E\frac{\partial D}{dt} ;\ \nabla\cdot(E\times H)=H(\nabla\times E)-E\cdot J-E\frac{\partial B}{dt}$$

Taking the volume Integral:

$$\int_V \nabla(E\times H)dV=-\frac{\mu}{2}\int\frac{dH^2}{dt}dV-\int(E\cdot J)dV-\frac{\epsilon}{2}\int\frac{dE^2}{dt}dV$$

$$H\cdot(\nabla\times E)=-\left(\vec H\cdot\frac{\partial H}{\partial t}\right)\mu.$$

$$\nabla\cdot(E\times H)=-\left(H\cdot\frac{\partial H}{\partial t}\right)\mu-E\cdot J-\left(E\cdot\frac{\partial E}{\partial t}\right)\epsilon$$

$$=-\frac{\mu}{2}\frac{dH}{dt}-E\cdot J-\frac{\epsilon}{2}\frac{dE}{dt}$$

Gausss Divergence Theorem:

$$\underbrace{\oint(E\times H)\cdot dS}_{\text{Pointing}}=\int_V(E\cdot J)dV-\frac{d}{dt}\int_V\left(\frac{\mu H^2}{2}+\frac{\epsilon E^2}{2}\right)dV$$

$$\boxed{\int_V\left(E\frac{D-P}{\epsilon_0}\cdot\left(\frac{\partial D}{\partial t}\right)\right)dV-\frac{d}{dt}\int_V\left(\frac{\mu B^2}{2\mu_0^2}+\epsilon\left(\frac{D-P}{\epsilon_0}\right)^2\right)dV}$$

16. Second Harmonic Generation: $\delta K=0$ and $\Delta K\neq 0$

i) The fundamental and second-harmonic fields periodically exchange energy when they are in-phase and non-zero, for $\Delta K=0$ & $\Delta K\neq 0$.

ii) When second-harmonic field acquires all of the energy when the length (or ζ) approaches infinity in the case of $\Delta K=0$. Albeit for arbitrary ΔK the second-harmonic field acquires the energy when $\Delta S=0$, the normalized phase mismatch parameter is null.

iii) The fundamental field asymptotically acquires all of the energy for the case ($\Delta K=0$, $\Delta S=0$, and $\zeta=0$), but for wavevector mismatch ($\delta K>0$, $\Delta S\gg 0$, $\tilde{\zeta}<\zeta<\infty$).

iv) Energy resides in each component of second-harmonic generation when ($\Delta K=0$, $\Delta S=0$, $\zeta\in\{0,\infty\}$) and ($\Delta K>0$, $\Delta S\in\{0,\infty\}$, $\Delta\zeta\in\{0,\infty\}$).

18. Second-harmonic generation can be efficient only when the phase-matching relation (ΔK) is zero. In medium, the fundamental and second-harmonic fields interchange energy periodically, and causes amplitude changes for the case $\Delta K!=0$. While two photon absorption does not periodically exchange energy, so $\Delta K=0$ or $\Delta K!=0$.

20. Cascaded optical nonlinearities means propagation through a medium where second-order nonlinearity can mimic third-order nonlinearity.

- Calculate phase shift of optical wave propagating through a second-order nonlinear optical material for nearly phase-matched second harmonic generation.
- Calculate conditions which the phase shifts acquired by the fundamental wave are proportional to path length and intensity.

Eqn (2.7.10) $\frac{dA_1}{dz} = \frac{2 i w_1^2 d_{eff}}{K_1 c^2} A_2 A_1^* e^{-i\Delta K z}$

$\frac{d^2A_1}{d\cdot} = \frac{2 i w_1^2 d_{eff}}{K_1 c^2} A_2 A_1^* (-i\Delta K) e^{-i\Delta K z}$

Eqn (2.7.11) $\frac{dA_2}{dz} = \frac{i w_2^2 d_{eff}}{K_2 c^2} A_1^2 e^{i\Delta K z}$

$A_2 = \frac{i w_2^2 d_{eff}}{K_2 c^2} A_1^2 \cdot \frac{e^{i\Delta K z}}{-i\Delta K}$

$A_2 = -\frac{w_2^2 d_{eff}}{K_2 \cdot \Delta K c^2} A_1^2 e^{i\Delta K z}$

$\frac{d^2 A_1}{d^2 z} = \left[\frac{\cancel{2 i w_1^2 d_{eff}}}{i K_1 c^2}\right] \left[\ldots e^{-i\Delta K z} + A_2^* (-i\Delta K) e^{-i\Delta K z}\right]$

$= \left[-\frac{dA_1}{i\Delta K}\right]\left[\frac{dA_1}{dz} \ldots \frac{2 i w_1^2 d_{eff}}{K_1 c^2} A_1\right]\left[\frac{i w_2^2 d_{eff}}{K_2 c^2} A_1^2 e^{i\Delta K z}\right] e^{-i\Delta K z}$

$\frac{d^2 A_1}{d^2 z} + i\Delta K \frac{dA_1}{dz} = \left[\frac{i w_2^2 d_{eff}}{K_2 c^2} A_1^2 e^{i\Delta K z}\right] A_1$

$$\boxed{\frac{d^2 A_1}{d^2 z} + i\Delta K \frac{dA_1}{dz} - \left[\frac{i w_2^2 d_{eff}}{K_2 c^2} A_1^2 e^{i\Delta K z}\right] A_1 = 0};\quad \Gamma^2(1-2|A_1/A_0|^2) = \frac{i w_2^2 d_{eff}}{K_2 c^2} A_1 e^{i\Delta K z}$$

$\Gamma = \sqrt{\frac{i w_2^2 d_{eff}}{K_2 c^2}} A_1 e^{i\Delta K z/2} \; / \; (1-2|A_1|A_0|)$

$A_0^2 A_2^2$

$$\boxed{\frac{d^2 A_1}{d^2 z} + i\Delta K \frac{dA_1}{dz} - \Gamma^2 (1-2|A_1|A_0|^2) A_1 = 0}$$

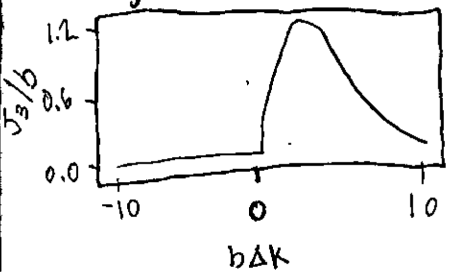
Trial solution: $A_1 = \left(\frac{I}{2n_1 \epsilon_0 c}\right)^{1/2} \operatorname{sech}\left(\frac{z}{l}\right) e^{i\phi_1}$

$i[\theta + \phi_2 - \Delta K z] \quad \theta = 0, \phi_2 = 0$

$= \left(\frac{I_1 + I_2}{2n_1 \epsilon_0 c}\right)^{1/2} \operatorname{sech}\left(\frac{z}{l}\right) e$

$$\boxed{= \left(\frac{I_1 + I_2}{2n_1 \epsilon_0 c}\right)^{1/2} \operatorname{sech}\left(\frac{z}{l}\right) e^{-i\Delta K z}}$$

22. $\boxed{\text{Fig 2.10.3}}$



"Dependence of phase-matching factor J_3 for third-harmonic generation confocal parameter $b\Delta K$"

When evaluating $J_3(\Delta K, z_0, z) = \int_{-\infty}^{\infty} \frac{e^{i\Delta K z'} \cdot dz'}{(1+2iz'/b)^{2}}$

$= \int_{-\infty}^{\infty} \frac{\cos(\Delta K z') + i\sin(\Delta K z')\, dz'}{(1+2iz'/b)^2}$

if $\Delta K z'$ is zero, then the $\int_0 \cos(\Delta K z')$ is ... zero, ...

24.
Eqn (2.10.14)

$P_{2w} = K\left[\frac{128\pi^3 w_1^3 d_{eff}^2 L}{c^4 n_1 n_2}\right] P_w^2$

would the $\cos_{\Delta K z'}$ becomes imaginary for $i\sin(\Delta K z')$

The final representation of $|J_3(\Delta K, z_0 z)|^2 = L^2 \operatorname{sinc}^2\left(\frac{\Delta K L}{2}\right)$ is real.

Boyd and Kleinman (1968); $\Delta K = 2K_1 - K_2 = (2w_1/c)(n_1 - n_2)$

$P = d \cdot E \cdot E$ ← Amplitude of the Electric Field

Amplitude of Polarization

Material lacking inversion symmetry "coefficient"

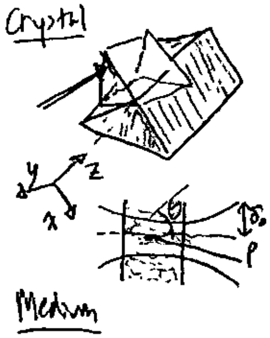
$\Delta K = 2K_1 - K_2 = (2W_1/c)(n_1 - n_2)$

Maximum from adjusting crystal, changing temperature, and applying electric field.

Focusing Parameter: $\xi = l/b$

Double Refraction Theory: $B = \rho(lk_1)^{1/2}/2$

Characteristic length: $l_p = (cn/2\pi w_3) d^{-1} E_3^{-1}$



Optimization of SHG Power: $w_0^2 K_1 = b$; $\delta_0 = 2w_0/b = 2/w_0 K_1 = 2/(bK_1)^{1/2}$

Gaussian Parameters: Focal Parameter $= b/n_1$.

Diffraction Angle $= n_1\delta_0$

Propagation Constant $= K_1/n_1$

Beam Radius $= W_n$

Focal Position $= f/n_1$

Fundamental Electric Field:

$$E_1(x', y', z') = E_0\,[1/(1+i\tau')]\exp(iK_1 z')$$
$$\times\exp[-(x'^2+y'^2)/W_0^2(1+i\tau')]\exp(-\tfrac{1}{2}\alpha_1 z')$$
$$\tau' = 2(z'-f)/b$$

Harmonic Polarization : $P(x', y', z') = P_0\,[1/(1+i\tau')^2]\exp(2iK_1 z' - \alpha_1 z')$

$$\times\exp[-2(x'^2+y'^2)/W_0^2(1+i\tau')]B(z') \;;\; B(z') = 1 \quad 0<z'<l$$
$$= 0 \quad z'<0,\ z'>l$$

Harmonic Field: $E_2(x, y, z) = A_2(x, y, z)\exp(ik_2 z)$; $x' = x-\rho(z-z')$ $(z<l)$

Harmonic Amplitude

$x' = x-\rho(l-z')$ $(z>l)$

$y' = y$ $\quad 0\le z'\le l$

$$dA_2(x', y', z') = (2\pi i w_2/cn_2)P_x(x', y', z')\exp(-ik_2 z')\,dz' = (2\pi i w_2/cn_2)[P_{0x}/(1+i\tau')]$$
$$\times\exp(i\Delta K z' - \alpha_1 z')\cdot\{[1/(1+i\tau')]$$
$$\times\exp[-2(x'^2+y'^2)/W_0^2(1+i\tau')]\,dz'$$

Note: $[1/(1+i\tau')]$ is the solution to the wave equation.

$\tau = 2(z-f)/b$ with subs above

$$dA_2(x, y, z) = (2\pi i w_2/cn_2)[P_{0x}/(1+i\tau')]\times\exp[i\Delta K z' - \alpha_1 z' - \tfrac{1}{2}\alpha_2(l-z')]\cdot[1/(1+i\tau)]$$
$$\times\exp(-2\{[x-\rho(l-z')]^2+y^2\}/W_0^2(1+i\tau))\,dz'$$

Total Harmonic Field:

$$E_2(x, y, z) = \frac{2\pi i w_2 P_{0x}}{cn_2(1+i\tau)}\exp(-\tfrac{1}{2}\alpha_2 l + 2ik_1 z)\cdot\int_0^l dz'\,\frac{\exp(-\alpha z' + i\Delta K z')}{1+i\tau'}$$

In the limit $\tau\to\infty$

$$\frac{1}{W_0^2(1+i\tau)} = (1-i\tau)/W_0^2(1+\tau^2) = [(1-i\tau)/W_0^2\tau^2]\times(1-\tau^{-2}+\tau^{-4}+\dots)\ \times\exp\left(\frac{-2\{[x-\rho(l-z')]^2+y^2\}}{W_0^2(1+i\tau)}\right)$$
$$\longrightarrow (1-i\tau)/W_0^2\tau^2$$

where $\alpha = \alpha_1 - \tfrac{1}{2}\alpha_2$

$s = [x-\rho(l-f)]/W_0\tau$, $s' = y/W_0\tau$, $\beta = \rho/\delta_0$, so that,

$$[x-\rho(l-z')]^2/W_0^2(1+i\tau) = (1-i\tau)[s+\beta(\tau'/\tau)]^2\times(1-\tau^{-2}+\dots)\to s^2(1-i\tau)-2i\beta s\tau'$$

$$E_2(x, y, z)\to(2\pi w_2 P_{0x}/cn_2\tau)\exp(-\tfrac{1}{2}\alpha_2 l+2ik_1 z)\cdot\exp[-2(1-i\tau)(s^2+s'^2)]\cdot\int_0^l dz'\,\frac{\exp(-\alpha z'+i\Delta K z')}{1+i\tau'}$$
$$\times\exp(4i\beta s\tau')$$

$$H(\sigma', \kappa, \xi, \mu) = (2\pi)^{-1}\int_{-\xi(1-\mu)}^{\xi(1+\mu)}\frac{d\tau'}{1+i\tau'}e^{-\kappa\tau'}e^{i\sigma'\tau'}$$

where $\sigma = \tfrac{1}{2}b\Delta K$

$\sigma' = \sigma + 4\beta s$

$\xi = l/b$

$\mu = (l-2f)/l$

$\kappa = \tfrac{1}{2}\alpha b$

In terms of laser power, $P_{0x} = dE_0^2 = (16P_1/n_1 c w_0^2)d$

Second Harmonic Intensity can be written: $S(\varsigma, \varsigma') = 4\pi K (P_1^2 K_1^2/\tau^2) \exp[-\alpha' l + \mu \alpha (l - 4(\varsigma' b))] \cdot |H(\sigma', K, \xi, \mu)|^2$

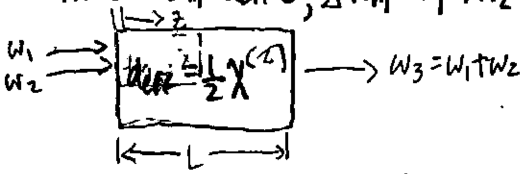
with $\boxed{K = (128\pi^2 \omega_1^2 / c^3 n_1^2 n_2) d^2}$

and $\alpha' = \alpha + \frac{1}{2}\alpha_2$.

26. Derive $\frac{dA_1}{dz} = \frac{2i\omega_1 d_m}{n_1 c} A_3 A_2^* e^{-i(\Delta K_m - 2K_m)z}$; $\frac{dA_2}{dz} = \frac{2i\omega_2 d_m}{n_2 c} A_3 A_1^* e^{-i(\Delta K_m - 2K_m)z}$

where $d_m = d_{eff} G$, $\Delta K_m = K_1 + K_2 - K_3 + K_m$

$\frac{dA_3}{dz} = \frac{2i\omega_3 d_m}{n_3 c} A_1 A_2 e^{i\Delta K_m z}$



$\tilde{E}_3(z,t) = A_3 e^{i(k_3 z - \omega_3 t)} + c.c$ [output]

where $k_3 = \frac{n_3 \omega_3}{c}$, $n_3^2 = \epsilon^{(1)}(\omega_3)$

$\tilde{P}_3(z,t) = P_3 e^{-i\omega_3 t} + c.c.$ [output polarization amplitude]

where $P_3 = 4\epsilon_0 d_{eff} E_1 E_2$

$\tilde{E}_i(z,t) = E_i e^{-i\omega_i t} + c.c.$ [Input]

where $E_i = A_i e^{ik_i z}$

$P_3 = 4\epsilon_0 d_{eff} A_1 A_2 e^{i(k_1+k_2)z} \equiv P_3 e^{i(k_1+k_2)z}$

Wave Equation

$$\nabla^2 \tilde{E}_n - \frac{\epsilon^{(1)}(\omega_n)}{c^2} \cdot \frac{\partial^2 \tilde{E}_n}{\partial t^2} = \frac{1}{\epsilon_0 c^2} \cdot \frac{\partial^2 P_n^{NL}}{\partial t^2}$$

$$\left[\frac{d^2}{dz^2} A_3 + 2ik_3 \frac{dA_3}{dz} - k_3^2 A_3 + \frac{\epsilon^{(1)}(\omega_3)\omega_3^2 A_3}{c^2}\right] e^{i(k_3 z - \omega_3 t)} + c.c$$

$$= \frac{-4 d_{eff} \omega_3^2}{c^2} A_1 A_2 e^{i[(k_1+k_2)z - \omega_3 t]} + c.c.$$

Where $k_3^2 = \epsilon^{(1)}(\omega_3)\omega_3^2/c^2$

$$\frac{d^2 A_3}{dz^2} + 2ik_3 \frac{dA_3}{dz} = \frac{-4 d_{eff} \omega_3^2}{c^2} A_1 A_2 e^{i(k_1+k_2-k_3)z}$$

small

$\frac{dA_3}{dz} = \frac{2i d_{eff} \omega_3}{n_3 c} A_1 A_2 e^{i\Delta k z}$; $\Delta K = K_1 + K_2 - K_3$

[Output]

Input Waves $\frac{dA_1}{dz} = \frac{2i d_{eff} \omega_1}{n_1 c} A_3 A_2^* e^{-i\Delta k z}$; $\frac{dA_2}{dz} = \frac{2i d_{eff} \omega_2}{n_2 c} A_3 A_1^* e^{-i\Delta k z}$

2, 3, 4, 7, 6

2. $\chi_{ij}^{(1)}(\omega_p) = \frac{N}{\epsilon_0 \hbar} \sum_n \left[\underbrace{\frac{\mu_{an}^i \mu_{na}^j}{(\omega_{na} - \omega_p) - i\gamma_{na}}}_{\text{"resonant"}} + \underbrace{\frac{\mu_{na}^i \mu_{an}^j}{(\omega_{na} + \omega_p) + i\gamma_{na}}}_{\text{"antiresonant"}} \right]$

Dropping antiresonant term of susceptibility

$$\chi_{ij}^{(1)}(\omega_p) = \frac{N}{\epsilon_0 \hbar} \frac{\mu_{an}^i \mu_{na}^j}{(\omega_{na} - \omega_p) - i\gamma_{na}} = \frac{N}{\epsilon_0 \hbar} \mu_{an}^i \mu_{na}^j \frac{(\omega_{na} - \omega_p) + i\gamma_{na}}{(\omega_{na} - \omega_p)^2 + \gamma_{na}^2}$$

Index of Refraction is described as:

$$n = \sqrt{\epsilon_0^{(1)}(\omega)} = \sqrt{1 + \chi^{(1)}(\omega)} = \sqrt{1 + \frac{N}{\epsilon_0 \hbar} \mu_{an}^i \mu_{na}^j \frac{(\omega_{na} - \omega_p) + i\gamma_{na}}{(\omega_{na} - \omega_p)^2 + \gamma_{na}^2}}$$

Assuming atomic number density $N \ll 1$

dipole transition moment μ_{an} of NaCl - 9.0070

Detuning from resonance ($\omega_{na} - \omega_p$) = 1KHz

Dampening Rate [γ_{na}] = 1×10^{-9}/s

Transition Frequency [ω_{na}] = 101KHz

Resonant Frequency [ω_p] = 100KHz

$$n = \sqrt{1 + \frac{10 \times 10^{-6}}{8.85 \times 10^{-12} \frac{F}{m} \cdot \frac{6.626 \times 10^{-34} Js}{2\pi}} (9.0070 D)^2 \frac{1KHz}{1KHz^2 + 1 \times 10^{-9} /s^2}}$$

$D = 3.336 \times 10^{-30} \frac{C}{m}$ $F = 96485$ C/mol

$$n = \sqrt{1 + 1.33 \times 10^{33} \frac{m}{F} \cdot \frac{F}{Kg m^2/s} \ldots (3.336 \times 10^{-3} \frac{C}{m})^2 \cdot \frac{1}{s}}$$

$\boxed{n = 162}$

31. Verify (Eqn 3.5.42): $\sigma_{max} = \frac{g_b}{g_a}\frac{\lambda^2}{2\pi}$; where $g_b = 2J_b + 1$ and $g_a = 2J_a + 1$

Siegman (1968)

$$2\alpha_m(\omega) = \Delta N_{12}\sigma_{21}(\omega);\ \alpha_m = \alpha_m(\omega) = (\beta/2)\chi''(\omega)$$ "Atomic Gain or Loss Coefficient"

$$\alpha_0 = \frac{\beta}{2}\frac{\sigma}{\omega\epsilon} = \frac{\sigma}{2\pi c}$$ "Atomic Background loss coefficient"

$$2\alpha_m(\omega) = \Delta N\sigma(\omega) = \frac{2\pi}{\lambda}\chi''(\omega) = \frac{3}{2\pi\lambda}\frac{\Delta N\lambda^3\gamma_{rad}}{\Delta\omega_a}\times\frac{1}{1+[2(\omega-\omega_a)/\Delta\omega_a]^2}$$

4. Show Eqn 3.6.24

$$\sigma(\omega) = \frac{3}{2}\frac{\gamma_{rad}}{\Delta\omega_a}\lambda^2 = \frac{g_0}{g_j}\sigma_{ij}(\omega_a) = \frac{3}{2\pi}\frac{\gamma_{rad,j0}}{\Delta\omega_a}\lambda_{ij}^2$$

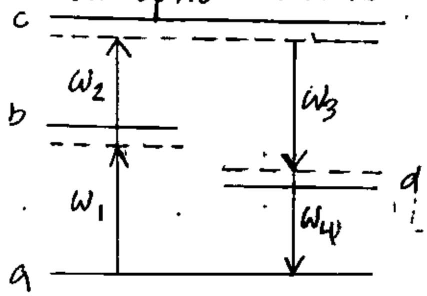
possesses full permutation symmetry:

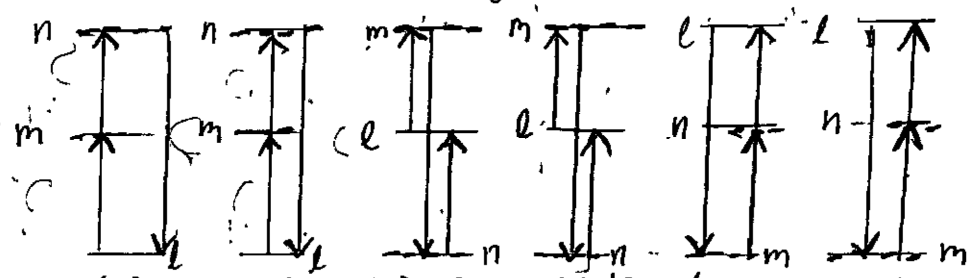
$$\chi^{(2)}_{ijk}(\omega_p+\omega_q,\omega_q,\omega_p) = \frac{N}{2\epsilon_0\hbar^2}\sum_{lmn}\rho_{ll}^{(0)}\left\{\frac{\mu^i_{ln}\mu^j_{nm}\mu^k_{ml}}{(\omega_{nl}-\omega_p-\omega_q)(\omega_{ml}-\omega_p)} + \frac{\mu^i_{ln}\mu^k_{nm}\mu^j_{ml}}{(\omega_{nl}-\omega_p-\omega_q)(\omega_{ml}-\omega_q)} + \frac{\mu^j_{ln}\mu^i_{nm}\mu^k_{ml}}{(\omega_{nl}+\omega_q)(\omega_{ml}-\omega_p)} + \frac{\mu^k_{ln}\mu^i_{nm}\mu^j_{ml}}{(\omega_{ml}+\omega_p)(\omega_{nl}-\omega_q)} + \frac{\mu^k_{ln}\mu^j_{nm}\mu^i_{ml}}{(\omega_{ml}+\omega_p+\omega_q)(\omega_{nl}+\omega_p)} + \frac{\mu^j_{ln}\mu^k_{nm}\mu^i_{ml}}{(\omega_{ml}+\omega_p+\omega_q)(\omega_{nl}+\omega_q)}\right\}$$

Full permutation symmetry:
- all frequency arguments are freely interchanged, as long as the corresponding Cartesian indices are interchanged simultaneously.

Equation
$$\chi^{(2)}_{ijk}(\omega_p+\omega_q,\omega_q,\omega_p),\ \chi^{(2)}_{jik}(\cancel{\omega_p+\omega_q},\omega_q),\ \omega_p)$$
$$\chi^{(2)}_{ijk}(\omega_q,\omega_q,-(\omega_p+\omega_q)),\ \chi^{(2)}_{ijk}(-\omega_q,-(\omega_p+\omega_q),\omega_p)$$
$$\chi^{(2)}_{ijk}(\omega_p,(\omega_p+\omega_q),\omega_q),\ \chi^{(2)}_{ijk}(\omega_p,\omega_q,(\omega_p+\omega_q))$$

6. Mutual Interaction of Four Optical Fields





(a$_1$) (a$_2$) (b$_1$)(a$_2'$) (b$_2$)(a$_1'$) (b$_1'$) (b$_2'$)

Note: Same polarization, and thermal equilibrium.

Eqn 3.7.11

$$\chi^{(3)}_{kjih}(\omega_p+\omega_q+\omega_r,\omega_r,\omega_q,\omega_p) = \frac{N}{\epsilon_0\hbar^3}P_e\sum_{vnml}\rho_{ll}^{(0)}\times\left\{\frac{\mu^k_{lv}\mu^j_{vn}\mu^i_{nm}\mu^h_{ml}}{[(\omega_{vl}-\omega_p-\omega_q-\omega_r)-i\gamma_{vl}][(\omega_{nl}-\omega_p-\omega_q)-i\gamma_{nl}][(\omega_{ml}-\omega_p)-i\gamma_{ml}]}\right.$$

$$+\frac{\mu^h_{lv}\mu^k_{vm}\mu^j_{mn}\mu^i_{nl}}{[(\omega_{nv}-\omega_p-\omega_q-\omega_r)-i\gamma_{nv}][(\omega_{mv}-\omega_p-\omega_q)-i\gamma_{mv}][(\omega_{vl}+\omega_p)-i\gamma_{vl}]} + \frac{\mu^i_{lv}\mu^k_{vm}\mu^j_{mn}\mu^h_{nl}}{[(\omega_{nv}-\omega_p-\omega_q-\omega_r)-i\gamma_{nv}][(\omega_{ml}+\omega_p+\omega_q)-i\gamma_{ml}][(\omega_{vl}-\omega_p)-i\gamma_{vl}]}$$

$$+\frac{\mu^h_{lv}\mu^i_{vm}\mu^k_{mn}\mu^j_{nl}}{[(\omega_{nm}-\omega_p-\omega_q-\omega_r)-i\gamma_{nm}][(\omega_{ml}+\omega_p+\omega_q)-i\gamma_{ml}][(\omega_{vl}+\omega_p)-i\gamma_{vl}]} + \frac{\mu^j_{lv}\mu^i_{vm}\mu^k_{mn}\mu^h_{nl}}{[(\omega_{mn}+\omega_p+\omega_q+\omega_r)-i\gamma_{mn}][(\omega_{nl}-\omega_p-\omega_q)-i\gamma_{nl}][(\omega_{ml}-\omega_p)-i\gamma_{ml}]}$$

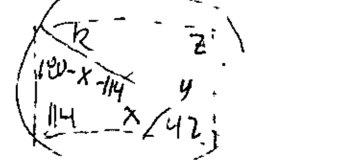
$$+\frac{\mu^h_{lv}\mu^j_{vm}\mu^k_{mn}\mu^i_{nl}}{[(\omega_{nm}+\omega_p+\omega_q+\omega_r)-i\gamma_{nm}][(\omega_{vl}+\omega_p+\omega_q)-i\gamma_{vl}][(\omega_{nl}+\omega_p)-i\gamma_{nl}]} + \frac{\mu^i_{lv}\mu^j_{vm}\mu^k_{mn}\mu^h_{nl}}{[(\omega_{nm}+\omega_p+\omega_q+\omega_r)-i\gamma_{nm}][(\omega_{mn}-\omega_p-\omega_q)-i\gamma_{mn}][(\omega_{vl}-\omega_p)-i\gamma_{vl}]}$$

$$\left.+\frac{\mu^h_{lv}\mu^i_{vn}\mu^j_{nm}\mu^k_{ml}}{[(\omega_{ml}+\omega_p+\omega_q+\omega_r)-i\gamma_{ml}][(\omega_{nl}+\omega_p+\omega_q)-i\gamma_{nl}][(\omega_{vl}+\omega_p)-i\gamma_{vl}]}\right\}$$

a) Calculate the four nonlinear susceptibilities.

$$\chi^{(3)}(\omega_4=\omega_1+\omega_2-\omega_3) \qquad \chi^{(3)}(\omega_3=\omega_1+\omega_2-\omega_4)$$

$$\chi^{(3)}(\omega_1=\omega_3+\omega_4-\omega_2) \qquad \chi^{(3)}(\omega_2=\omega_3+\omega_4-\omega_1)$$



Assuming $\omega_p=\omega_1=1KHz$, $\omega_q=\omega_2=1.1KHz$, $\omega_r=\omega_3=0.9KHz$;

$N=1\times10^{-7}$; $\epsilon_0=8.85\times10^{-12}\frac{F}{m}$; $F=96485\ C/mol$; $\mu=(9.0070\ Debye)$

$$\chi^{(3)}(\omega_4=\omega_1+\omega_2-\omega_3)=\frac{N}{\epsilon_0\hbar^3}P_I\sum_{vnml}\rho_{ll}^{(0)}\times\Big\{\frac{\mu^4}{(-2\omega_p-i\gamma)(\omega_p-i\gamma)(\omega_q-\omega_r-i\gamma)}$$

$$+\frac{\mu^4}{(-2\omega_p-i\gamma)(-\omega_p-i\gamma)(2\omega_p+\omega_q-\omega_r+i\gamma)}$$

$$+\frac{\mu^4}{(-2\omega_p-i\gamma)(2\omega_p+2\omega_q-\omega_r+i\gamma)(\omega_q-\omega_r-i\gamma)}$$

$$+\frac{\mu^4}{(-2\omega_p-i\gamma)(2\omega_p+2\omega_q-\omega_r+i\gamma)(2\omega_p+\omega_q-\omega_r+i\gamma)}$$

$$+\frac{\mu^4}{(2\omega_p+\omega_q-i\gamma)(-\omega_r-i\gamma)(+\omega_q-\omega_r-i\gamma)}$$

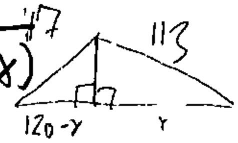
$$+\frac{\mu^4}{(2\omega_p+2\omega_q+i\gamma)(+\omega_r-i\gamma)(2\omega_p+\omega_q-\omega_r-i\gamma)}$$

$$+\frac{\mu^4}{(2\omega_p+2\omega_q+i\gamma)(2\omega_p+2\omega_q-\omega_r+i\gamma)(\omega_q-\omega_r-i\gamma)}$$

$$+\frac{\mu^4}{[(2\omega_p+\omega_q)+i\gamma][2\omega_p+2\omega_q-\omega_r+i\gamma][2\omega_p+\omega_q-\omega_r+i\gamma]}\Big]$$

15

18



$17^2=(120-x)^2+y^2$

$113^2=x^2+y^2$

Example of the first symmetry real on non-re-17 terms

$$\mu^4\left[\frac{-2\omega_p^2(\omega_q-\omega_r)+\omega_p\gamma^2+\gamma^2(\omega_r-\omega_q)-i\cdot\gamma\cdot(2\omega_p^2+\omega_p(\omega_q-\omega_r)+\gamma^2)}{(\omega_p^2+\gamma^2)(4\omega_p^2+\gamma^2)(\omega_q^2-2\omega_q\omega_r+\omega_r^2+\gamma^2)}\right] \quad 18$$

... unresolved because of deep computation per term.

7. Again, unresolved because of deep computation per term. i.e Rationalizing complex polynomial
And weirdly frequency dependent

8. $$\lim_{\gamma\to0}\chi^{(3)}_{kjih}(\omega_p+\omega_q+\omega_r,\omega_r,\omega_q,\omega_p)=\frac{N}{\epsilon_0\hbar^3}P_I\sum_{vnml}\rho_{ll}^{(0)}\times\Big\{\frac{\mu^4}{(+2\omega_p\cdot\omega_p(\omega_q-\omega_r)}+\frac{\mu^4}{(+2\omega_p\omega_p(2\omega_p+\omega_q-\omega_r))}$$

$$+\frac{\mu^4}{(-2\omega_p(2\omega_p+2\omega_q-\omega_r)(\omega_q-\omega_r)}+\frac{\mu^4}{(-2\omega_p(\cancel{2\omega_p+2\omega_q-\omega_r})(2\omega_p+\omega_q-\omega_r)}+\frac{\mu^4}{(2\omega_p+\omega_q)(-\omega_r)(\omega_q-\omega_r)}$$

$$+\frac{\mu^4}{(2\omega_p+2\omega_q)(-\omega_r)(2\omega_p+\omega_q-\omega_r)}+\frac{\mu^4}{(2\omega_p+2\omega_q)(2\omega_p+2\omega_q-\omega_r)(\omega_q-\omega_r)}+\frac{\mu^4}{(2\omega_p+\omega_q)(2\omega_p+2\omega_q-\omega_r)(2\omega_p+\omega_q-\omega_r)}$$

$$\boxed{\lim_{\gamma\to 0} \chi^{(3)}_{R j i h}(\omega_p+\omega_q+\omega_r, \omega_r, \omega_q, \omega_p) = \frac{N}{\epsilon_0 \hbar^3} P_I \sum_{\nu n m \ell}' \rho^{(0)}_{\ell\ell} \times \left\{ \frac{\mu^4}{(\omega_q-\omega_r)} + \frac{\mu^4}{(\omega_q+\omega_q)} \right\}}$$ 1.15

When dampening is not negligable, Permutation wave-mixing represents half of the tensors, each with 8 cartesian components and $(R!\cdot 8^3)$ interactions. Thus, $\chi^{(3)}_{ijkl}$ would have $(4!)\cdot 2 = 48$ tensors, with 64 cartesian components and $(R!\cdot 8^4) = 1944$ light-matter interactions.

Chapter 4: 2. Derive Eqn 4.2.2 : $\chi_{1111} = \chi_{1122} + \chi_{1212} + \chi_{1221}$

Dial Transformations: $1\to 1; 2\to -2; 3\to -3$

Reflections: $1\to \pm 1; 2 \to \mp 2; 3 \to \mp 3$

$90°$ Rotation: $1\to 2; 2\to -1; 3\to 3$

$$R = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}; \mu=\alpha=\beta=\gamma=1; n=3$$

$$\chi^{(3)}_{1111} = R_{11}\cdot R_{22}\cdot R_{33}\cdot \chi^{(3)}_{111} + 2...$$

$$= (\cos^4\theta + \sin^4\theta)\chi^{(3)}_{1111} + 2\cos^2\theta\sin^2\theta\left(\chi^{(3)}_{1122} + \chi_{xyxy} + \chi_{xyyx}\right)$$

$$\boxed{\chi^{(3)}_{1111} = \chi^{(3)}_{1222} + \chi^{(3)}_{1212} + \chi^{(3)}_{1221}}$$

4. $$\chi_{1111} = \frac{2N}{3\epsilon_0\hbar^3}\sum_{\ell m n}' \frac{\mu^x_{gn}\cdot\mu^x_{nm}\cdot\mu^x_{m\ell}\cdot\mu^x_{\ell g}}{(\omega_{ng}-\omega)(\omega_{mg}-2\omega)(\omega_{\ell g}-\omega)} - \cancel{\frac{2N}{3\epsilon_0\hbar^3}\sum_{\ell n}' \frac{\mu^x_{gn}\mu^x_{ng}\mu^x_{g\ell}\mu^x_{\ell g}}{(\omega_{ng}-\omega)(\omega_{\ell g}-\omega)(\omega_{\ell g}-\omega)}}$$ ← two-photon contribution

$N>0; \epsilon_0>0; \hbar^3>0; \mu^x_{gn}=\mu^x_{nm}=\mu^x_{m\ell}=\mu^x_{\ell g}>0$

if $\omega_{ng}, \omega_{mg}, \omega_{\ell g} > \omega$; then $(\omega_{ng}-\omega)\wedge(\omega_{mg}-2\omega)\wedge(\omega_{\ell g}-\omega) > 0$

6. Diameter $[D_1]$ Laser ; Si Rod Diameter $[D_2]$ Transverse Intensity
Power $[P] = \pi R^2 I_0$; External Temp $[T_0]$

Heat-transport equation: $\underbrace{(\rho_0 C)\frac{\partial \tilde{T}_1}{\partial t}}_{} - K\nabla^2\tilde{T}_1 = \alpha\cdot\tilde{I}(r)$ Steady-state Conditions

Silica $[\rho_0 C](J/cm^3) = 1.67$

Silica $[K](W/m\cdot K) = 1.4$; Silica $[dn/dT](K^{-1}) = 1.2\times10^{-5}$

$T_1^{(max)} = \frac{\alpha I^{(max)} R^2}{K}$; $\Delta n = \left(\frac{dn}{dT}\right)\frac{\alpha I^{(max)}\cdot R^2}{K}$;

$I = 2n_0'\epsilon_0 c|E(\omega)|^2$; $E(\omega) = \frac{A}{1+i\zeta} e^{-r^2/w_0^2(1+i\zeta)}$; n_0 = silica refractive index $= 1.47$

α = Mean Polarizability $= 5.38\times10^{-24}\,cm^3$; $\zeta = 2z/b$; $b = 2\pi N_0^2/\lambda$; $\epsilon_0 = 8.854\times10^{-12}\,F/m$

K = Thermal Conductivity $= 1.4\,W/m\cdot K$

Assuming Steady-state conditions:

$-K\nabla^2\tilde{T}_1 = \alpha\cdot\tilde{I}(r)$

$+K\tilde{T}_1/R^2 = \alpha\cdot\tilde{I}(r)$

$$T_1^{(max)} = \frac{\alpha\,\tilde{I}(r)\,R^2}{K}$$

$$= \frac{\alpha\cdot 2n_0\epsilon_0 c|E(\omega)|^2\cdot R^2}{K}$$

$$T^{max} = 5.38\times10^{-24}\,cm^3 \cdot 2\cdot 1.47\cdot 8.854\times10^{-12}\,F/m \cdot 2.998\times10^{8}\frac{m}{s}\ \frac{1}{1+i\cdot\frac{2z\cdot\lambda}{2\pi w_0^2}}\ e^{-(D_1/2)^2/w_0^2(1+i\frac{2z\cdot\lambda}{2\pi w_0^2})}\cdot(D_2/2)^2$$

$1.4\ W/m\cdot K$

$$= 2.997\times10^{-26}\,cm^3\left(\frac{1m}{100cm}\right)^3\cdot F\left(\frac{96500\,C}{1K}\right)\cdot\frac{m/s}{1+i\left(\frac{z\cdot\lambda}{\pi\cdot w_0^2}\right)}\ e^{-(D_1/2)^2/w_0^2(1+i(\frac{z\cdot\lambda}{\pi w_0^2}))}\cdot(D_2/2)^2\cdot\frac{K\cdot s}{J}$$

$$\cancel{= 2.81\times10^{-27}\cdot m^4\frac{C\cdot K}{J}\ e^{-(D_1/2)^2/w_0^2(1+i(z\cdot\lambda/\pi w_0^2))}\ (D_2/2)^2\times\frac{1J}{1.602\times10^{-19}C}\cdot\frac{1}{1+i\left(\frac{z\lambda}{\pi w_0^2}\right)}}$$

$$\boxed{= \frac{4.509\times10^{-9}\cdot D_2^2}{1+i\left(\frac{z\cdot\lambda}{\pi w_0^2}\right)}\cdot e^{-(D_1/2)^2/w_0^2(1+i(\frac{z\lambda}{\pi w_0^2}))}\ (m^2\cdot K)}$$

$-b = 2 = 3$ $\quad a-2=2$

4

$2b = 3$

$-(D_1/2)^2/w_0^2(1+i(\frac{z\lambda}{\pi w_0^2}))$

$$\Delta n = \left(\frac{dn}{dT}\right)T_1 = \left(\frac{dn}{dT}\right)\frac{\alpha I^{(max)}\cdot R^2}{K} = n_2^{(th)} I_1^{(max)} = \frac{3.67\times10^{-20}\cdot 4.509\times10^{-9}\cdot D_2^2\cdot e\ \ m^2\cdot K}{1+i\left(\frac{z\lambda}{\pi w_0^2}\right)}$$

$$\left| = \frac{1.65\times10^{-28}\cdot D_2^2\cdot e^{-(D_1/2)^2/w_0^2(1+i(\frac{z\lambda}{\pi w_0^2}))}\cdot m^2K}{1+i\left(\frac{z\lambda}{\pi w_0^2}\right)}\right.$$

8. Nonlinear Phase Shift

$I = 2n_0'\cdot\epsilon_0\cdot c\,|E(w)|^2$

$E(w) = \frac{A}{1+i\zeta}e^{-r^2/w_0^2(1+i\zeta)}$; $\zeta = 2z/b$

$b = 2\pi w_0^2/\lambda$

$$\Phi_{NL} = \int_{-\infty}^{\infty} ... \,dz = 2n_2\left(\frac{w}{c}\right)\int_0^{\infty} I(z)\,dz = 2n_2\left(\frac{w}{c}\right)\int_0^{\infty}2 n_0'\epsilon_0 c\,|E(w)|^2 dz$$

$$= 2n_2\left(\frac{w}{c}\right)\int_0^{\infty}2n_0'\cdot\epsilon_0 c\left|\frac{A}{1+i\zeta}e^{-r^2/w_0^2(1+i\zeta)}\right|^2 dz = 4n_2\left(\frac{w}{c}\right)n_0\epsilon_0\int\frac{A^2}{(1+i\zeta)^2}e^{-2r^2/w_0^2(1+i\zeta)}dz$$

$$= 4n_0'\cdot n_2\cdot\epsilon_0\cdot A^2\int_0^{\infty}\frac{e^{-2r^2/w_0^2(1+i\zeta)}}{(1+i\zeta)^2}dz = 4n_0'\cdot n_2\cdot\epsilon_0\cdot A^2\int_0^{\infty}\frac{e^{-2r^2/w_0^2(1+2z/b)}}{(1+i\frac{2z}{b})^2}dz$$

$$= 4n_0'\cdot n_2\cdot\epsilon_0\cdot A^2\left(\int_0^{\infty}e^{-2r^2/w_0^2(1+2z/b)}\left[-\frac{4z^2}{b^2}\right]dz + \int_0^{\infty}e^{-2r^2/w_0^2(1+2z/b)}\left[\frac{2iz^2}{b}\right] + \int_0^{\infty}e^{-2r^2/w_0^2(1+2z/b)}dz\right)$$

≡ Does not converge. The accuracy of this method would change from ...

Another representation using Laguerre's polynomial... for convergence

$$9.\quad \cancel{\Phi_{NL} = n_2(w/c)\left[\int_0^{1cm}e^{-2r^2/w_0^2(1+2z/b)}\left[\frac{4z^2}{b^2}\right]dz + \int_0^{1cm}e^{-2r^2/w_0^2(1+2z/b)}\left[\frac{2iz^2}{b}\right] + \int_0^{1cm}e^{-2r^2/w_0^2(1+2z/b)}dz\right]\cdot 4n_0'\epsilon_0 A^2}$$

Intensity with absorption coefficient

$$\Phi_{NL} = n_2(w/c)\int_0^{1cm}I_0 e^{-\kappa z} = 3.67\times10^{-20}\frac{m^2}{W}\left(\frac{w}{c}\right)10\,GW/cm^2\cdot\left[-\kappa\right]\left[e^{-\kappa}-e^{0}\right] = \boxed{\frac{1.22\times10^{-15}\,F\,w\cdot(1-e^{-\kappa})}{\kappa}}$$

$$= \frac{6.34\times10^{-26}\cdot w}{\alpha}(1-e^{-\kappa});\ \frac{6.34\times10^{-26}\cdot w\ F/m}{5.38\times10^{-24}\,cm^3\cdot C}\left(1-e^{-5.38\times10^{-24}cm^3}\right) = \boxed{1.1\times10^{-4}\cdot w\ F/cm^2}$$

Small phase shift 10^{-4} order

11. Derive (4.6.5) $n_2 = -\frac{e^2 \alpha \tau_R}{2\epsilon_b \cdot n_o m \hbar \omega^3}$ <u>Semiconductor Nonlinearities</u>

$\hbar\omega > E_g$: Nonlinear response from transfer of electrons

$\hbar\omega < E_g$: Nonlinear response from parametric processes.

$$\frac{dN_c}{dt} = \frac{\alpha I}{\hbar\omega} - \frac{(N_c - N_c^{(0)})}{\tau_R} \leftarrow \text{Population per electron-hole recombination time} \quad ; N_c = N_c^{(0)} + \frac{\alpha I \tau_R}{\hbar\omega}$$

change of band population — Normal energy transfer

Reasons: 1) Free-Electron Response

$$\epsilon(\omega) : \epsilon_b - \frac{\omega_p^2}{\omega(\omega + i\tau)} \; ; \omega_p^2 = N_c e^2/\epsilon_o m$$

$$n_o^2 = \epsilon_b - \frac{N_c(0) e^2}{\epsilon_o m \omega(\omega + i\tau)} \; ; \; n_c = \sqrt{\epsilon_o(\omega)}$$

$$; \; n_o + n_2 I = \sqrt{\epsilon(\omega)}$$

$$n_2 = -\frac{e^2 \kappa \tau_R}{2\epsilon_b n_o m \hbar\omega^3} \; ; \; n_2 = \frac{\sqrt{\epsilon(\omega)} - n_o}{I}$$

$$= \frac{\sqrt{\epsilon_b - \frac{\omega_p^2}{\omega(\omega + i/\tau)}} - \sqrt{\epsilon_b - \frac{N_c(0)e^2}{\epsilon_o m \omega(\omega + i/\tau)}}}{I}$$

$$= \frac{\sqrt{\epsilon_b - \frac{N_c e^2}{\epsilon_o m \omega(\omega + i/\tau)}} - \sqrt{\epsilon_b - \frac{N_c(0) e^2}{\epsilon_o m \omega(\omega + i/\tau)}}}{2 n_o \underline{\tau c} |\hbar\omega|^2}$$

$$= \frac{\sqrt{\epsilon_b \epsilon_o m \omega(\omega + i/\tau) - N_c e^2} - \sqrt{\epsilon_b \epsilon_o m \omega(\omega + i/\tau) - N_c(0) e^2}}{\sqrt{\epsilon_o m \omega(\omega + i/\tau)} \cdot 2 n_o c |\hbar\omega|^2}$$

$$\frac{1}{\omega + i\tau} \quad \frac{(\omega - i\tau)}{\omega^2 + \tau^2}$$

$$= \frac{\sqrt{\epsilon_b \epsilon_o m \omega(\omega^2 + 1/\tau^2) - N_c e^2 (\omega - i/\tau)} - \sqrt{\epsilon_b \epsilon_o m \omega(\omega^2 + 1/\tau^2) - N_c(0) e^2 (\omega - i/\tau)}}{\sqrt{\epsilon_o m \omega^3(\omega^2 + 1/\tau^2)} \cdot 2 n_o c |\hbar\omega|^2}$$

$$= \frac{\sqrt{\epsilon_b \epsilon_o m \omega(\omega^2 + 1/\tau^2) - N_c e^2 \omega + i N_c e^2 \tau} - \sqrt{\epsilon_b \epsilon_o m \omega(\omega^2 + 1/\tau^2) - N_c(0) e^2 \omega + i N_c(0) e^2 \tau}}{\sqrt{\epsilon_o m \omega(\omega^2 + \tau^2)} \cdot 2 n_o c |\hbar\omega|^2}$$

$$= \frac{\sqrt{\epsilon_b \epsilon_o m \omega^3 + \epsilon_b \epsilon_o m \omega/\tau^2 - N_c e^2 \omega + i N_c e^2/\tau} - \sqrt{\epsilon_b \epsilon_o m \omega^3 + \epsilon_b \epsilon_o m/\tau^2 - N_c(0) e^2 \omega + i N_c(0) e^2/\tau}}{\sqrt{\epsilon_o m \omega(\omega^2 + \tau^2)} \cdot 2 n_o c |\hbar\omega|^2}$$

$$\epsilon(\omega) = \epsilon_b - \frac{\omega_p^2}{\omega(\omega + i/\tau)} = \epsilon_b - \frac{\omega_p^2(\omega - i/\tau)}{\omega^3 - \omega/\tau^2} = \underbrace{\epsilon_b - \frac{\omega_p^2}{\omega^3 - \omega/\tau^2}}_{\text{Real}} + \underbrace{\frac{\omega_p^2 \cdot i/\tau}{\omega^3 - \omega/\tau^2}}_{\text{Imaginary}}$$

$$n_b^2 = n_o^2 + 2 n_o n_2 I + n_2^2 I^2 = \epsilon_b - \frac{\omega_p^2}{\omega(\omega + i/\tau)} = \epsilon_b - \frac{N_c e^2/\epsilon_o m}{\omega(\omega + i/\tau)} = \epsilon_b - \frac{(N_c^{0} \hbar\omega + \alpha I \tau_R/\hbar\omega) e^2/\epsilon_o m}{\omega(\omega + i/\tau)}$$

$$= \epsilon_b - \frac{N_c^{(1)}\hbar\omega e^2 + \kappa I \tau_R e^2/\hbar\omega}{\omega(\omega + i/\tau)\epsilon_0 m} = \underbrace{\epsilon_b - \frac{N_c^{(0)}\hbar\omega e^2}{\epsilon_0 m\hbar\omega(\omega+i/\tau)}} + \frac{\kappa I \tau_R e^2}{\omega^2(\omega+i/\tau)\epsilon_0 m\,\hbar\omega} = n_0^2 + \frac{e^2\kappa\tau_R I}{\hbar\omega^2\epsilon_0 m(\omega+i/\tau)}$$

$$= n_0^2 + \frac{e^2\kappa\tau_R(\omega+i/\tau) I}{\hbar\omega^2\epsilon_0 m(\omega^2+1/\tau^2)} = n_0^2 + \frac{e^2\kappa\tau_R\,\omega\, I}{\hbar\omega^3\epsilon_0 m} + \frac{i e^2\kappa\tau_R/\tau\, I}{\hbar\omega^2\epsilon_0 m/\tau^2} = n_0^2 + \boxed{\frac{e^2\kappa\tau_R}{\hbar\omega^3\epsilon_0 m}} I$$

n_2

Chapter 5:

2. Nonlinear Response of the Square-well potential:

Polarization $\tilde{P} = \epsilon_0\chi^{(1)}\tilde{E} + \epsilon_0\chi^{(2)}\tilde{E}^2 + \epsilon_0\chi^{(3)}\tilde{E}^3 + \dots$

Energy $W = -\int_0^E \tilde{P}(\tilde{E}')dE' = -\frac{1}{2}\chi^{(1)}E^2 - \frac{1}{3}\chi^{(2)}E^3 - \frac{1}{4}\chi^{(3)}E^4 \dots$

$= W^{(2)} + W^{(3)} + W^{(4)} + \dots$

Susceptibility $\chi^{(n-1)} = -\frac{nW^{(n)}}{\epsilon_0\tilde{E}^n} = \frac{-1}{\epsilon_0(n-1)!}\frac{\partial^n W}{\partial\tilde{E}^n}\Big|_{E=0}$

Hydrogen Atom

$$\frac{W}{2R} = -\frac{1}{2} - \frac{9}{4}\left(\frac{E}{E_{at}}\right)^2 - \frac{3555}{64}\left(\frac{E}{E_{at}}\right)^4 + \dots$$

where $R = me^4/32\pi^2\epsilon_0^2\hbar^2 = 13.6\,eV$

$\chi^{(1)} = N\alpha$; where $\alpha = \frac{9}{2}a_0^3$

$\chi^{(3)} = N\gamma$; where $\gamma = \frac{3555}{16}\frac{a_0^7}{e^6}$

where $a_0 = 4\pi\epsilon_0\hbar^2/me^2$

α = atomic volume V

$\gamma \propto V^{7/3}$

General Expression for Nonlinear Susceptibility

$\hat{H} = \hat{H}_0 + \hat{V}$; $\hat{V} = -\mu\tilde{E}$; $\hat{H}|\psi_n\rangle = W_n|\psi_n\rangle$; where $W_n = W_n^{(0)} + W_n^{(1)} + W_n^{(2)} + \dots$

$|\psi_n\rangle = |\psi_n^{(0)}\rangle + |\psi_n^{(1)}\rangle + |\psi_n^{(2)}\rangle + \dots$

$= (W_n^{(0)} + W_n^{(1)} + W_n^{(2)} + \dots)(|\psi_n^{(0)}\rangle + |\psi_n^{(1)}\rangle + |\psi_n^{(2)}\rangle + \dots)$

$W_n^{(1)} = e\hat{E}\langle n|x|n\rangle$

$W_n^{(2)} = e^2\hat{E}^2\sum_s \frac{\langle n|x|s\rangle\langle s|x|n\rangle}{W_s^{(0)} - W_n^{(0)}}$

$W_n^{(3)} = e^3\hat{E}^3\sum_{st}\frac{\langle n|x|s\rangle\langle s|x|t\rangle\langle t|x|n\rangle}{(W_s^{(0)} - W_n^{(0)})(W_t^{(0)} - W_n^{(0)})}$

$W_n^{(4)} = e^4E^4\sum_{stu}\frac{\langle n|x|s\rangle\langle s|x|t\rangle\langle t|x|u\rangle\langle u|x|n\rangle}{(W_s^{(0)} - W_n^{(0)})(W_t^{(0)} - W_n^{(0)})(W_u^{(0)} - W_n^{(0)})} - e^2E^2W_n^{(2)}\sum\frac{\langle n|x|u\rangle\langle u|x|n\rangle}{(W_u^{(0)} - W_n^{(0)})^2}$

$\chi^{(1)} = N\alpha$; $\alpha = \alpha_{xx} = \frac{2e^2}{\hbar}\sum_{s\neq g}\frac{x_{gs}\cdot x_{sg}}{\omega_{sg}}$

$\chi^{(2)} = N\beta$; $\beta = \beta_{xxx} = \frac{3e^3}{\hbar^2}\sum\frac{x_{gt}, x_{ts}, x_{sg}}{\omega_{tg}\cdot\omega_{sg}}$

$\chi^{(3)} = N\gamma$; $\gamma = \gamma_{xxxx} = \frac{4e^4}{\hbar^3}\left(\sum\frac{x_{gu}x_{ut}x_{ts}\cdot x_{sg}}{\omega_{ug}\cdot\omega_{tg}\cdot\omega_{sg}} - \sum\frac{x_{gt}\cdot x_{tg}\cdot x_{gs}x_{sg}}{\omega_{tg}\cdot\omega_{sg}^2}\right)$

$\alpha = \frac{2e^2}{\hbar\omega_0}\sum_s'\langle g|x|s\rangle\langle s|x|g\rangle$; $\alpha = \frac{2e^2}{\hbar\omega_0}\langle g|x\hat{O}x|g\rangle$; where $\sum'|s\rangle\langle s|$

$= \frac{2e^2}{\hbar\omega_0}\langle x^2\rangle$; $\beta = -\frac{3e^3}{\hbar^2\omega_0^2}\langle x^3\rangle$; $\gamma = \frac{4e^4}{\hbar^3\omega_0^3}[\langle x^4\rangle - 2\langle x^2\rangle^2]$

These hyperpolarizabilities represent various higher-order measures of the electron-cloud. (optional/nonlinear)

$\gamma = \alpha^2 \frac{g}{\hbar\omega_0}$; $g = \left[\frac{\langle x^4\rangle}{\langle x^2\rangle^2} - 2\right]$; Thomas-Reiche sum Rule $= \frac{2m}{\hbar}\sum \omega_{kg}|x_{kg}|^2 = Z$ "optically active electrons"

$\omega_0 = \frac{Z\hbar}{2m\langle x^2\rangle}$; $\alpha = \frac{4e^2 m}{Z\hbar^2}\langle x^2\rangle^2$; $\beta = -\frac{12 e^3 m^2}{Z^2\hbar^4}\langle x^2\rangle^2\langle x^3\rangle$; $\gamma = \frac{32 e^4 m^3}{Z^3\hbar^6}\langle x^2\rangle^3\left(\langle x^4\rangle - 2\langle x^2\rangle^2\right)$

(Rustagi & Ducuing 1974): Third order polarizability of conjugated organic molecules

$$\left(H^{(0)} - E^{(0)}\right)\psi^{(n)} = E^{(n)}\psi^{(0)} + E^{(n-1)}\psi^{(1)} + \cdots \left(E^{(1)} - H^{(1)}\right)\psi^{(n-1)}$$

$$\langle\psi^{(s)}|H^{(1)}|\psi^{(r)}\rangle = \sum_{q=0}^{s}\sum_{p=0}^{r} E^{(p+q+1)}\langle\psi^{(s-q)}|\psi^{(r-p)}\rangle \;;\; \alpha = -2\sum_{n=1}^{N}\frac{\partial^2 E_n^{(2)}}{\partial E^2} = \frac{4L^4}{a_0}\sum_{n=1}^{N}\left(\frac{-2}{3\pi^2 n^2} + \frac{10}{\pi^4 n^4}\right)$$

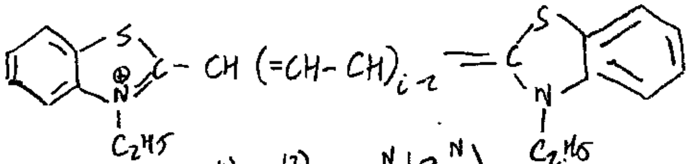
Two Limits: A) $N \to \infty$

B) $n \gg 1$

$$\gamma = -2\sum_{n=1}^{N}\frac{1}{6}\frac{\partial^4 E_n^{(4)}}{\partial E^4} = \frac{128 L^{10}}{a_0^3 e^2}\sum_{n=1}^{N}\left(\frac{-2}{9\pi^6 n^6} + \frac{140}{3\pi^8 n^8} - \frac{440}{\pi^{10} n^{10}}\right)$$

$(CH_3)_2 - \overset{\oplus}{N} = CH - (CH = CH)_{j-1} - N - (CH_3)_2$

Dipole Moment: $\mu/E = \alpha + \gamma E^2$



$$\hat{H}|\psi_n\rangle = \sum^{N} \omega_n^{(N)} \Big| \sum^{N} \psi_n \rangle$$

$$= \left(\omega_n^{(0)} + \omega_n^{(1)} + \omega_n^{(2)} + \cdots\right)\left(|\psi_n^{(0)}\rangle + |\psi_n^{(1)}\rangle + \cdots |\psi_n^{(N)}\rangle\right) = E_n^{(0)} + E_n^{(1)} + E_n^{(2)} + \cdots E_n^{N}|\psi_n^{N}\rangle$$

$$= E_n^{(0)}\psi_n^{(0)} + E^{(1)}\psi_n^{(1)} + E^{(2)}\psi_n^{(2)} + \cdots$$

$\omega_n^{(1)} = eE\langle n|x|n\rangle$

$$= E_n^{(0)}\psi_n^{(0)} + \cancel{eE\langle n|x|n\rangle} + e\tilde{E}^2\frac{\langle n|x|s\rangle\langle t|x|n\rangle}{(\omega_s^{(0)} - \omega_n^{(0)})(\omega_t^{(1)} - \omega_n^{(1)})}$$

$\omega_n^{(2)} = e^2E^2\sum \frac{\langle n|x|s\rangle\langle s|x|n\rangle}{\omega_s^{(0)} - \omega_n^{(0)}}$

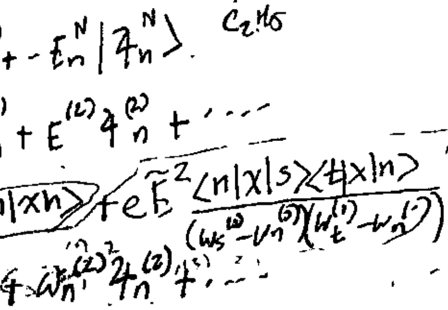
$$= \omega_n^{(0)}\psi_n^{(0)} + \omega_n^{(1)}\psi_n^{(1)} + \omega_n^{(2)}\psi_n^{(2)} + \cdots$$

$\omega_n^{(3)} = eE^3\sum \frac{\langle n|x|s\rangle\langle s|x|t\rangle\langle t|x|n\rangle}{(\omega_s^{(0)} - \omega_n^{(0)})(\omega_t^{(1)} - \omega_n^{(1)})}$

$$\omega_n^{(4)} = eE^4\sum\frac{\langle n|x|s\rangle\langle s|x|t\rangle\langle t|x|u\rangle\langle u|x|u\rangle}{(\omega_s^{(0)} - \omega_n^{(0)})(\omega_t^{(1)} - \omega_n^{(1)})(\omega_n^{(1)} - \omega_n^{(1)})} - e^2E^2\omega_n^{(2)}\sum\frac{\langle n|x|u\rangle\langle u|x|n\rangle}{(\omega_n^{(1)} - \omega_n^{(1)})^2}$$

$$\boxed{\chi^{(1)} = \frac{-n\, W^{(n)}}{\epsilon_0 \tilde{E}^n} = \frac{+Ze^2}{\hbar}\sum\frac{x_{gs}x_{sg}}{\omega_{sg}}}$$

$$\boxed{\chi^{(3)} = \frac{4e^4}{\hbar^3}\left(\sum\frac{x_{gu}x_{ut}x_{ts}x_{sg}}{\omega_{ug}\,\omega_{tg}\,\omega_{sg}} - \sum\frac{x_{gt}x_{tg}x_{gs}x_{sg}}{\omega_{tg}\,\omega_{sg}}\right)}$$



Chapter 6: 1. Open two-level Atom: $\dot{\rho}_{ba} = -\left(i\omega_{ba} + \frac{1}{T_2}\right)\rho_{ba} + \frac{i}{\hbar}V_{ba}(\rho_{bb} - \rho_{aa})$

$\dot{\rho}_{bb} = \lambda_b - T_b\left(\rho_{bb} - \rho_{bb}^{(eq)}\right) - \frac{i}{\hbar}\left(V_{ba}\rho_{ab} - \rho_{ba}V_{ab}\right)$

$\dot{\rho}_{aa} = \lambda_a - T_a\left(\rho_{aa} - \rho_{aa}^{(eq)}\right) + \frac{i}{\hbar}\left(V_{ba}\cdot\rho_{ab} - \rho_{ba}\cdot V_{ab}\right)$

Attempt!

$\frac{1}{T_2} = \frac{1}{2}\left(T_b + T_a\right) + \gamma_c$

$$\left(\dot{\rho}_{bb} - \dot{\rho}_{aa}\right) = (\lambda_b - \lambda_a) + T_a\left(\rho_{aa} - \rho_{aa}^{(eq)}\right) - T_b\left(\rho_{bb} - \rho_{bb}^{(eq)}\right) + \frac{2i}{\hbar}\left(V_{ba}\,\rho_{ab} - \rho_{ba}\,V_{ab}\right)$$

$= (\lambda_a \; \lambda_b)\, T$

$V_{ba} = -\mu_{ba}\left(Ee^{-i\omega t} + E^{*}e^{i\omega t}\right)$

$$\dot\rho_{ba} = -\left(i\omega_{ba} + \frac{1}{T_2}\right)\rho_{ba} - \frac{i}{\hbar}\mu_{ba} E e^{-i\omega t}(\rho_{bb}-\rho_{aa})$$

$G = H - T\Delta S$

$\dot\mu = \bar\mu\,\Delta 3 = \frac{PV\Delta S}{nR}$

$$(\dot\rho_{bb}-\dot\rho_{aa}) = (\lambda_b-\lambda_a) + T_a(\rho_{aa}-\rho_{aa}^{(eq)}) - T_b(\rho_{bb}-\rho_{bb}^{(eq)}) + \frac{2i}{\hbar}\left(\mu_{ba} E e^{-i\omega t}\rho_{ba} - \mu_{ab} E^* e^{i\omega t}\rho_{ab}\right)$$

Goal: Steady State Solution with $\rho_{ba}(t) = \sigma_{ba} e^{-i\omega t}$

$$P = \left(\frac{nR}{V}\right)\frac{\Delta H}{\Delta S}$$

$$\dot\sigma_{ba} = \left[i(\omega-\omega_{ba}) - \frac{1}{T_2}\right]\sigma_{ba} - \frac{i}{\hbar}\mu_{ba}\cdot E(\rho_{bb}-\rho_{aa})$$

$$0 = \left[i(\omega-\omega_{ba}) - \frac{1}{T_2}\right]\sigma_{ba} - \frac{i}{\hbar}\mu_{ba}\cdot E(\rho_{bb}-\rho_{aa}) \;\; \cancel{\rho_{bb}-\rho_{aa} = \frac{\left(i(\omega-\omega_{ba}) - \frac{1}{T_2}\right)\hbar\,\sigma_{ba}}{\mu_{ba}\cdot E\cdot i}}$$

ρ_{ba}:
$$\boxed{\rho_{ba}} = \sigma_{ba}e^{-i\omega t} = \frac{i(\rho_{bb}-\rho_{aa})\mu_{ba}E e^{-i\omega t}}{\left(i(\omega-\omega_{ba}) - \frac{1}{T_2}\right)\hbar} = \frac{(\rho_{bb}-\rho_{aa})\mu_{ba}E\, e^{-i\omega t}}{\hbar\left(\omega-\omega_{ba}+\frac{i}{T_2}\right)}$$

$$\frac{dP}{dT} = \frac{\Delta H}{T\Delta V}$$

$\rho_{bb}-\rho_{aa}$:
$$0 = -\frac{(\rho_{bb}-\rho_{aa}) - (\rho_{bb}-\rho_{aa})^{(eq)}}{T_1} + \frac{2i}{\hbar}\left(\mu_{ba}E\sigma_{ab} - \mu_{ba}\cdot E^*\sigma_{ba}\right)$$

$$dP = \frac{\Delta H}{T\Delta V}dT$$

$$T_{bb}\rho_{bb} - T_{aa}\rho_{aa} = (\lambda_b-\lambda_a) + T_{bb}\rho_{bb}^{(eq)} - T_{aa}\rho_{aa}^{(eq)} + \frac{2i\cdot T_1}{\hbar}\left(\mu_{ba}\cdot E\cdot\sigma_{ab} - \mu_{ba}\cdot E^*\sigma_{ba}\right)$$
$$= (\lambda_b-\lambda_a) + T_{bb}^{(eq)}\rho_{bb} - T_{aa}^{(eq)}\rho_{aa} + \frac{4iT_1|\mu_{ba}|^2|E|^2(\rho_{bb}-\rho_{aa})}{\hbar^2\left(i(\omega-\omega_{ba})+\frac{1}{T_2}\right)}$$

$PV = nRT$

Assuming $T_{bb} = T_{aa}$

$ab - cd = R(b+d)$

$$(\rho_{bb}-\rho_{aa}) = \frac{(\lambda_b-\lambda_a)}{T_a} + \frac{T_a}{T_a}(\rho_{bb}-\rho_{aa})^{(eq)} + \frac{4i\cdot T_1|\mu_{ba}|^2|E|^2(\rho_{bb}-\rho_{aa})}{\hbar^2 T_a\left(i(\omega-\omega_{ba})+\frac{i}{T_2}\right)}$$

$$\frac{1}{r}\left(\frac{dP}{dT}\right) = \frac{\Delta H}{RT}$$

$$(\rho_{bb}-\rho_{aa})\left[1 - \frac{4\cdot T_1|\mu_{ba}|^2|E|^2(\omega-\omega_{ba}-i/T_2)}{\hbar^2T_a\left((\omega-\omega_{ba})^2+1/T_2^2\right)}\right] = \frac{(\lambda_b-\lambda_a)}{T_a}$$

Note: The value ...

$$(\rho_{bb}-\rho_{aa}) = \frac{(\lambda_b-\lambda_a)\hbar^2T_a[(\omega-\omega_{ba})^2T_2^2+1]/T_2^2}{\hbar^2T_a[(\omega-\omega_{ba})^2T_2^2+1] - 4|\mu_{ba}|^2|E|^2T_1T_2(\omega-\omega_{ba}-i/T_2)} = \frac{\left[(\lambda_b-\lambda_a)\frac{1}{T_a}(\rho_{bb}-\rho_{aa})^{(eq)}\right][(\omega-\omega_b)^2T_2^2+1]/T_2^2}{(\omega-\omega_b)^2T_2^2+1 - (4/\hbar^2)|\mu_{ba}|^2|E|^2T_1T_2/T_a \times(\omega-\omega_b-i/T_2)}$$

Polarization: $P(t) = N\langle\mu\rangle = N\,\mathrm{Tr}(\hat\rho\cdot\hat\mu) = N(\mu_{ab}\rho_{ba}+\mu_{ba}\rho_{ab}) = Pe^{-i\omega t} + c.c. = \epsilon_0\chi E$

Susceptibility:
$$\chi = \frac{(\rho_{bb}-\rho_{aa})|\mu_{ba}|^2 N}{\epsilon_0\hbar[(\omega-\omega_{ba})+i/T_2]} = \frac{[\lambda_b-\lambda_a+\rho_{bb}^{(eq)}-\rho_{aa}^{(eq)}][(\omega-\omega_b)^2T_2^2+1]|\mu_{ba}|^2 N/T_2^2}{\epsilon_0\hbar[(\omega-\omega_{ba})+i/T_2]\left[(\omega-\omega_b)^2T_2^2+1-(4/\hbar^2)|\mu_{ba}|^2|E|^2T_1T_2/T_a\times(\omega-\omega_b-i/T_2)\right]}$$

Rabi Frequency: $\Omega = 2|\mu_{ba}||E|/\hbar$; $\Delta = \omega - \omega_{ba}$

$$= \frac{[N(\lambda_b-\lambda_a+\rho_{bb}^{(eq)}-\rho_{aa}^{(eq)})]|\mu_{ba}|^2}{\epsilon_0\hbar}\cdot\frac{(\Delta^2T_2+1)T_2}{[\Delta T_2+i]\left[\Delta^2T_2^3+\frac{1}{T_2}-\Omega^2T_1T_2^2/T_a(\Delta T_2-i)\right]}$$

Note factor issue [complex]

$$= \left[\frac{N(\lambda_b-\lambda_a+\rho_{bb}^{(eq)}-\rho_{aa}^{(eq)})|\mu_{ba}|^2}{\epsilon_0\hbar}\right]\frac{\Delta^2T_2+1}{1+\Delta^2T_2^2+\Omega^2T_1T_2/T_a}$$

Absorption coefficient: $\alpha = \frac{2\omega}{c} Im(n) = \frac{2\omega}{c} Im[(1+\chi)^{1/2}]$: Assumption $|\chi| \ll 1$; Binomial $(a+b)^{1/2}$

$$\cong \frac{\omega}{c} Im\,\chi$$

$$\alpha(\Delta) = \frac{\alpha_0(0)}{1+\Delta^2 T_2^2} \; ; \; \alpha(0) = \frac{-\omega_{ba}}{c}\left[N(\lambda_b - \lambda_a + \rho_{bb}^{(eq)} - \rho_{aa}^{(eq)})|\mu_{ba}|^2 \frac{T_2}{\epsilon_0 \hbar}\right]$$

The susceptibility becomes: $\chi = \frac{\alpha_0(0)}{\omega_{ba}/c} \frac{\Delta T_2 - i}{1+\Delta^2 T_2^2 + \Omega^2 T_1 T_2}$ = Real + Imaginary $= \chi' + i\chi''$

$$\chi' = -\frac{\alpha_0(0)}{\omega_{ba}/c} \frac{1}{\sqrt{1+\Omega^2 T_1 T_2/T_a}} \frac{\Delta T_2/\sqrt{1+\Omega^2 T_1 T_2/T_a}}{1+\Delta^2 T_2^2/(1+\Omega^2 T_1 T_2/T_a)}$$

$$\chi'' = \frac{\alpha_0(0)}{\omega_{ba}/c}\left(\frac{1}{1+\Omega^2 T_1 T_2/T_a}\right) \frac{1}{1+\Delta^2 T_2^2/(1+\Omega^2 T_1 T_2/T_a)}$$

Line-center Saturation field strength: $|E_s| \ldots$; $\Omega^2 T_1 T_2/T_a = \frac{|E|^2}{|E_s^0|^2}$

Saturation Intensity: $I_s = 2\epsilon_0 c \ldots |\mu_{ba}|^2 |E|^2/\hbar^2 \ldots T_1 T_2/T_a = \frac{|E|^2}{|E_s^0|^2}$; $|E_s|^2 = \frac{\hbar^2}{4|\mu_{ba}|^2 T_1 T_2/T_a}$

Susceptibility of a saturated field: $\chi = \frac{-\alpha_0(0)}{\omega_{ba}/c} \frac{\Delta T_2 - i}{1+\Delta^2 T_2^2 + |E|^2/|E_s^0|^2 \cdot T_a}$

Saturation field strength for arbitrary detuning:

Saturation Intensity: $I_s^0 = 2\epsilon_0 c |E_s^0|^2$ $\quad |E_s^\Delta| = |E_s^0|^2 (1+\Delta^2 T_2)^2 \cdot T_a$

Saturation Intensity for Arbitrary detuning: $\boxed{I_s^\Delta = 2\epsilon_0 c |E_s^\Delta|^2 = I_s^0(1+\Delta^2 T_2^2)\cdot T_a}$ $= I_s(1+\Delta^2 T_2^2) T_a$

Two-Level Atom with non-radiative coupled intermediate levels:

Relaxation Processes:
$$\dot{\rho}_{ba} = -(i\omega_{ba} + \gamma_{ba})\rho_{ba} + \frac{i}{\hbar} V_{ba}(\rho_{bb} - \rho_{aa})$$
$$\dot{\rho}_{bb} = -(T_{ba} + T_{bc})\rho_{bb} - \frac{i}{\hbar}(V_{ba}\rho_{ab} - \rho_{ba}V_{ab})$$
$$\dot{\rho}_{cc} = T_{bc}\rho_{bb} - T_{ca}\cdot\rho_{cc}$$
$$\dot{\rho}_{aa} = T_{ba}\cdot\rho_{bb} + T_{ca}\cdot\rho_{cc} + \frac{i}{\hbar}(V_{ba}\rho_{ab} - \rho_{ba}V_{ab})$$
$$\dot{\rho}_{aa} + \dot{\rho}_{bb} + \dot{\rho}_{cc} = 0$$

Substituting Dipole Moment:
$$\hat{V} = -\mu \tilde{E}(t) = -\mu(Ee^{-i\omega t} + E^* e^{-i\omega t})$$
$$\dot{\rho}_{ba} = -(i\omega_{ba} + \gamma_{ba})\rho_{ba} - \frac{i}{\hbar}\mu_{ba} E e^{-i\omega t}(\rho_{bb} - \rho_{aa})$$

bb − aa − cc

$$\dot{\rho}_{bb} = -(T_{ba} + T_{bc})\rho_{bb} + \frac{i}{\hbar}(\mu_{ba} E e^{-i\omega t}\rho_{ab} - \mu_{ba} E^* e^{i\omega t}\rho_{ba})$$
$$\dot{\rho}_{cc} = T_{bc}\rho_{bb} - T_{ca}\cdot\rho_{cc}$$
$$\dot{\rho}_{aa} = T_{ba}\rho_{bb} + T_{ca}\rho_{cc} + \frac{-i}{\hbar}(\mu_{ba} E e^{-i\omega t}\rho_{ab} + \mu_{ba} E^* e^{i\omega t}\rho_{bc})$$
$$(\dot{\rho}_{bb} - \dot{\rho}_{aa} - \dot{\rho}_{cc}) = -2(T_{ba} + T_{bc})\rho_{bb} + \frac{2i}{\hbar}(\mu_{ba} E e^{-i\omega t}\rho_{ab} - \mu_{ba} E^* e^{-i\omega t}\rho_{ba})$$

Substituting a varying coefficient: $\rho_{ba}(t) = \sigma_{ba}(t)e^{-i\omega t}$

$$\dot{\sigma}_{ba} = [i(\omega-\omega_{ba}) - \gamma_{ba}]\sigma_{ba} - \frac{i}{\hbar}\mu_{ba}E(\rho_{bb}-\rho_{aa})$$

$$(\dot{\rho}_{bb} - \dot{\rho}_{aa} \cdot \dot{\rho}_{cc}) = -2(T_{ba}+T_{bc})\rho_{bb} + \frac{2i}{\hbar}(\mu_{ba}E\sigma_{ab} - \mu_{ab}E^{*}\sigma_{ba})$$

Setting the equations equal to zero:

$$0 = [i(\omega-\omega_{ba}) - \gamma_{ba}]\sigma_{ba} - \frac{i}{\hbar}\mu_{ba}E(\rho_{bb}-\rho_{aa})$$

$$(\rho_{bb}-\rho_{aa}) = \frac{[(\omega-\omega_{ba})-\gamma_{ba}]\cdot\hbar\cdot\sigma_{ba}}{\mu_{ba}\cdot E}$$

$i(T_{ba} + T_{bc})\rho_{bb} \;\; \cdots$

$$\sigma_{ba} = \frac{(\rho_{bb}-\rho_{aa})\mu_{ba}\cdot E}{[(\omega-\omega_{ba})-\gamma_{ba}]\hbar}$$

$$(T_{ba}+T_{bc})\;\; \boxed{\rho_{ba} = \frac{i}{\hbar}\frac{(\rho_{bb}-\rho_{aa})\mu_{ba}Ee^{-i\omega t}}{[(\omega-\omega_{ba})-\gamma_{ba}]\hbar}}$$

$$(T_{ba}+T_{bc})\rho_{bb} = \frac{i}{\hbar}\left[\frac{2\cdot|\mu_{ba}|^2|E|^2(\rho_{bb}-\rho_{aa})}{[(\omega-\omega_{ba})-\gamma_{ba}]\hbar}\right]$$

$$\boxed{(i\rho_{bb}-\rho_{aa}) = \frac{-\hbar^2[(\omega-\omega_{ba})-\gamma_{ba}](T_{ba}+T_{bc})\rho_{bb}}{2|\mu_{ba}|^2|E|^2(\;\;)}}$$

Polarization Equation: $\tilde{P}(t) = N\langle\mu\rangle = N\,\mathrm{Tr}(\hat{\rho}\hat{\mu}) = N(\mu_{ab}\rho_{ba} + \mu_{ba}\rho_{ab}) = Pe^{-i\omega t} + c.c.$
$= \epsilon_0 \chi E$

Susceptibility: $\chi = \frac{N(\mu_{ab}\rho_{ba}+\mu_{ba}\rho_{ab})}{\epsilon_0 E e^{-i\omega t}} = \frac{N(\rho_{bb}-\rho_{aa})|\mu_{ba}|^2}{\epsilon_0[(\omega-\omega_{ba})-\gamma_{ba}]\hbar}$

$$= \frac{-N\hbar^{\cancel{2}}\cancel{[(\omega-\omega_{ba})-\gamma_{ba}]}(T_{ba}+T_{bc})\rho_{cc}\cancel{|\mu_{ba}|^2}}{\epsilon_0\cancel{[(\omega-\omega_{ba})-\gamma_{ba}]}\hbar\cdot 2\cancel{|\mu_{ba}|^2}|E|^2(} = \frac{-N\hbar\,(T_{ba}+T_{bc})\rho_{cc}}{\epsilon_0\cdot 2\cdot|E|^2}$$

Rabi Frequency: $\Omega = 2|\mu_{ba}||E|/\hbar$; $\Delta = \omega - \omega_{ba}$

$$\chi = \frac{-N\cancel{\hbar}(T_{ba}+T_{bc})\rho_{cc}}{2\epsilon_0\hbar^2\Omega^2/4|\mu_{ba}|^2} = \frac{-2N(T_{ba}+T_{bc})\rho_{cc}|\mu_{ba}|^2}{\epsilon_0\hbar\,\Omega^2}$$

Absorption Coefficient: $\alpha = \frac{2\omega}{c} - \frac{2\omega}{c}\mathrm{Im}\left[(1+\chi)^{1/2}\right] = \frac{\omega}{c}\mathrm{Im}\,\chi$

$$\alpha_0(\Delta) = +\frac{\omega_{ba}}{c}\left[\frac{2N(T_{ba}+T_{bc})\rho_{cc}|\mu_{ba}|^2}{\epsilon_0\hbar}\right]$$

$$P(\omega) = \epsilon_0 \chi^{1}_{eff} E_1$$

$$\boxed{\begin{aligned} P(\omega+\delta) &= \epsilon_0 \chi^{(1)}_{eff}(\omega+\delta) E_1 + 3\epsilon_0 \chi^{(3)}_{eff} E_0^2 E_{+1}^{*} \\ P(\omega-\delta) &= \epsilon_0 \chi^{(1)}_{eff}(\omega-\delta) E_{-1} + 3\epsilon_0 \chi^{(3)}_{eff} E_0^2 E_1^{*} \end{aligned}}$$

Chap 7 1, 2, 4, 5, 6, 7, 8 = 30

Chapter 7: 1. Verify 7.1.19 through 7.1.21 do satisfy 7.1.18

Eqn 7.1.16: $2ik_0 \frac{\partial A}{\partial z} + \nabla_T^2 A = -\frac{\omega^2}{\epsilon_0 c^2} p_{NL}$ "Paraxial Equation"

Eqn 7.1.17: $p_{NL} = 3\epsilon_0 \chi^{(3)} |A|^2 A$ "Third Order Nonlinear Polarization"

Eqn 7.1.18: $2ik_0 \frac{\partial A}{\partial z} + \frac{\partial^2 A}{\partial x^2} = -3\chi^{(3)} \frac{\omega^2}{c^2} |A|^2 A$ "Paraxial with transverse variance" 3.30

Eqn 7.1.19: $A(x,z) = A_0 \operatorname{sech}(x/x_0) e^{i\gamma z}$ "solution to Paraxial with transverse variance"

Eqn 7.1.20: $x_0 = \frac{1}{k_0}\sqrt{\frac{n_0}{2\bar{n}_2 |A_0|^2}} = \frac{1}{k_0}\sqrt{\frac{n_0}{n_2 I}}$ "width of field distribution"

Eqn 7.1.21 $\gamma = k_0 \bar{n}_2 |A_0|^2 / n_0 = k_0 n_2 I / (2n_0)$ "Rate of nonlinear phase acquisition"

$$\bar{n}_2 = 3\chi^{(3)}/4n_0 \,;\ \bar{n}_2 I = 2\bar{n}_2 |A_0|^2$$

Verifying solution: $2ik_0 \frac{\partial A}{\partial z} + \frac{\partial^2 A}{\partial x^2} = -3\chi^{(3)} \frac{\omega^2}{c^2} |A|^2 A \,;\ 2ik_0 \frac{\partial}{\partial z}\left[A_0 \operatorname{sech}(x/x_0) e^{i\gamma z}\right] + \frac{\partial^2}{\partial x^2}\left[A_0 \operatorname{sech}(x/x_0) e^{i\gamma z}\right]$

$$\boxed{\begin{aligned} \tfrac{d}{dx}\operatorname{sech} x &= -\tanh x \operatorname{sech} x \\ \tfrac{d}{dx}\tanh x &= \operatorname{sech}^2 x \end{aligned}}$$

$$= 2ik_0\left[i\gamma \cdot A_0 \operatorname{sech}(x/x_0) e^{i\gamma z}\right] + \frac{\partial}{\partial x}\left[A_0 \frac{\partial}{\partial x}\left(\frac{1}{\cosh(\,)}\right) e^{i\gamma z}\right]$$

$$= -2k_0 A_0 \cdot \gamma \operatorname{sech}(x/x_0) e^{i\gamma z} + \frac{\partial}{\partial x} A_0 \cdot \frac{\partial}{\partial x}\left[\frac{2}{e^{x/x_0} + e^{-x/x_0}}\right] e^{i\gamma z}$$

$$= -2k_0 A_0 \cdot \gamma \operatorname{sech}(x/x_0) e^{i\gamma z} + 2A_0 \frac{\partial}{\partial x}\left[-\tanh\left(\tfrac{x}{x_0}\right)\operatorname{sech}\left(\tfrac{x}{x_0}\right)\right]\left[\tfrac{1}{x_0}\right] e^{i\gamma z}$$

$$= -2k_0 A_0 \cdot \gamma \operatorname{sech}(x/x_0) e^{i\gamma z} + 2A_0\left[\operatorname{sech}^{3}(x/x_0) - \tanh^2(x/x_0)\operatorname{sech}(x/x_0)\right]\left[\tfrac{1}{x_0}\right]^2 e^{i\gamma z}$$

$$= -2\cancel{A_0}\cancel{\operatorname{sech}(x/x_0)}\left[\gamma k_0 + \left(\operatorname{sech}^2(x/x_0) - \tanh^2(x/x_0)\operatorname{sech}(x/x_0)\right)\left[\tfrac{1}{x_0}\right]^2\right]\cancel{e^{i\gamma z}}$$

$$= -3\chi^{(3)} \frac{\omega^2}{c^2}\left[A_0 \operatorname{sech}(x/x_0) e^{i\gamma z}\right]^2 \cancel{A_0}\cancel{\operatorname{sech}}(x/x_0)\cancel{e^{i\gamma z}}$$

$$= \gamma k_0 + \left(\operatorname{sech}^2(x/x_0) - \tanh^2(x/x_0)\operatorname{sech}(x/x_0)\right)\left[\tfrac{1}{x_0}\right]^2 = -3\chi^{(3)}\frac{\omega^2}{c^2} A_0^2 \operatorname{sech}^2(x/x_0) e^{2i\gamma z}$$

$$\cancel{\gamma} k_0 = \tanh^2(x/x_0)\operatorname{sech}(x/x_0) - \operatorname{sech}^2(x/x_0) - 3\chi^{(3)}\frac{\omega^2}{c^2} A_0^2 \operatorname{sech}^2(x/x_0) e^{2i\gamma z}$$

$$= \operatorname{sech}^2(x/x_0)\left[\tanh(x/x_0)\sinh(x/x_0) - \left(1 + 3\chi^{(3)}\frac{\omega^2}{c^2} A_0^2 e^{2i\gamma z}\right)\right]$$

$$\boxed{k = \frac{2\pi}{\lambda} = \frac{2\pi f}{v} = \frac{\omega}{v}}$$

$\frac{n_2 I}{2\bar{n}_2} = |A_0|^2$

$\frac{n_2 I}{2 \cdot 3\chi^{(3)}/4n_0} = |A_0|^2$

$$k_0 \bar{n}_2 |A_0|^2 / n_0 \cdot k_0 = \frac{k_0^2 \cdot 3 \cdot \chi^{(3)} |A_0|^2}{4 n_0\, n_0} = \frac{3 k_0^2 \chi^{(3)} |A_0|^2}{4 \cdot n_0^2} = \operatorname{sech}^2\left(\tfrac{x}{x_0}\right)\left[\tanh\left(\tfrac{x}{x_0}\right)\sinh\left(\tfrac{x}{x_0}\right) - \left(1 + 3\chi^{(3)} k_0^2 A_0^2 e^{2i\gamma z}\right)\right]$$

$$\frac{k_0^2 n_2 I}{2 n_0} = \operatorname{sech}^2\left(\tfrac{x}{x_0}\right)\left[\tanh\left(\tfrac{x}{x_0}\right)\sinh\left(\tfrac{x}{x_0}\right) - \left(1 + 3\chi^{(3)} k_0^2 \frac{n_2 I \cdot n_0}{\chi^{(3)}}\left(\tfrac{2}{3}\right) e^{2i\gamma z}\right)\right]$$

$$\frac{k_0^2 n_2 I}{2 n_0} = \operatorname{sech}^2\left(\tfrac{x}{x_0}\right)\left[\tanh\left(\tfrac{x}{x_0}\right)\sinh\left(\tfrac{x}{x_0}\right) - \left(1 + k_0^2 n_2 I n_0 2 e^{2i\gamma z}\right)\right]$$

$$\frac{k_0^2 n_2 I}{2 n_0} = \tanh^2(x/x_0)\operatorname{sech}(x/x_0) - \operatorname{sech}^2(x/x_0) - \operatorname{sech}^2(x/x_0) k_0^2 n_2 I\, 2 n_0 e^{2i\gamma z}$$

$$\frac{k_0^2 n_2 I}{2 n_0}\left(1 + 4 n_0^2 \operatorname{sech}^2(x/x_0) e^{2i\gamma z}\right) = \tanh^2\left(\tfrac{x}{x_0}\right)\operatorname{sech}\left(\tfrac{x}{x_0}\right) - \operatorname{sech}^2\left(\tfrac{x}{x_0}\right) \quad \boxed{\text{Try again}}$$

G 10^{-3}

3.3×10^{-3} cm/W

$\chi = \chi' + i\chi''$; $\chi' = -\frac{\alpha_0(0)}{\omega_{ba}/c}\left(\frac{1}{\Omega^2}\right)$; $\chi'' = 0$; "Only real values"

$\Omega^2 = |E|^2/|E_s^0|^2$; $|E_s^0|^2 = \frac{\hbar^2\Omega^2}{4|\mu_{ba}|^2}$; $\chi = \frac{-\alpha_0(0)}{\omega_{ba}/c}\frac{|E_s^0|^2}{|E|^2}$

$$\boxed{I_s^0 = 2\epsilon_0 c|E_s^0|^2 = \frac{2\epsilon_0 c\,\hbar^2\Omega^2}{4|\mu_{ba}|^2} = \frac{\epsilon_0\hbar^2 c\,\Omega^2}{2|\mu_{ba}|^2}}$$

$$\boxed{I_s^\Delta = I_s^0}$$

3. verify (Eq 6.5.43) $\langle \Psi_+|\Psi_-\rangle = 0$; $\Psi_\pm = N_\pm\{u_a(r)\exp[-i(\omega_a - \frac{1}{2}\Delta \pm \frac{1}{2}\Omega')t] + \frac{\Delta \mp \Omega'}{\Omega^*}u_b(r)\exp[-i(\omega_b + \frac{1}{2}\Delta \pm \frac{1}{2}\Omega')t]\}$

$$\langle\Psi_+|\Psi_-\rangle = \int \Psi_+\Psi_- d\tau = \int N_+\{u_a(r)e^{-i(\omega_a - \frac{1}{2}\Delta + \frac{1}{2}\Omega')t} + \frac{\Delta - \Omega'}{\Omega^*}u_b(r)e^{-i(\omega_b + \frac{1}{2}\Delta + \frac{1}{2}\Omega')t}\} \times N_-\{u_a(r)e^{-i(\omega_a - \frac{1}{2}\Delta - \frac{1}{2}\Omega')t} + \frac{\Delta + \Omega'}{\Omega^*}u_b(r)\exp^{-i(\omega_b + \frac{1}{2}\Delta - \frac{1}{2}\Omega')t}\}d\tau$$

$$= N_+ N_-\left[\underbrace{\int u_a^2(r)\cdots d\tau}_{\text{Even} = 0} + \underbrace{2\int u_a(r)\cdot u_b(r)\cdots d\tau}_{=0} + \underbrace{\int u_b^2(r)\cdots d\tau}_{\text{Even} = 0}\right]$$

5. Estimating the Response Time of NonResonant Electronic Nonlinearities

Section 4.3: Nonresonant Electronic Nonlinearities: $\tau = 2\pi a_0/v \approx 100$ attosec.

$\boxed{\text{Student A}}$ "results from: bound electrons in an applied optical field"

$p = \frac{h}{mv}$

$\boxed{\text{Student B}}$ Relationship of T_1 and T_2 ; $T_1 = \frac{1}{T_a + T_b}$; $T_2 = \frac{2}{T_a + T_b + 2\gamma}$ "Dephasing rate"

Time-Derivative of Momentum: $\dot{p} = (i\Delta - \frac{1}{T_2})p - \frac{i}{\hbar}|\mu|^2 E W$

$\downarrow$ "Bernoulli"

$$p(t) = \left(1 - e^{-(i(\omega+\Delta) - \frac{1}{T_2})t}\right)\frac{(i\Delta - \frac{1}{T_2})}{\hbar}|\mu|^2 E W$$

$-\frac{1}{2}\omega_a\Delta + \frac{1}{4}\Delta^2 - \frac{1}{4}\Omega^2$

where $W(t) = W_0 - (1 + W_0)e^{-t/T_2}\left[\cos\Omega' t + \frac{1}{\Omega' T_2}\sin\Omega' t\right]$

$$W_0 = \frac{-(1 + \Delta^2 T_2^2)}{1 + \Delta^2 T_2^2 + \Omega^2 T_1 T_2}$$

When $\Delta \gg \Omega$ and $\Delta T_2 \gg 1$;

$W_0 \approx -1$; $W(t) = -1 - (1-1)\cdots = -1$

$$p(t) = \left(1 - e^{-(i(\Delta-1) - \frac{1}{T_2})t}\right)\cdot\frac{i\Delta}{\hbar}|\mu|^2 E$$

$$= \left(1 + \left[\frac{\Delta}{\hbar}|\mu|^2 E\sin([\Delta-1]t) + \frac{i\Delta}{\hbar}|\mu|^2 E\cos([\Delta-1]t)\right]e^{\frac{t}{T_2}}\right)$$

Student A's argument to response being "the order of the reciprocal of the detuning of the laser field from the nearest atomic resonance" relates to $|\omega-\omega_{ba}|=\Delta$, and within adiabatic limits $|\omega-\omega_{ba}| = \Delta \gg \frac{1}{T_2}$, also is associated with Student B's outcomes of T_1 and T_2: When the pulse length (τ_p) is much less than T_1 or T_2 relaxation. In addition, assume the laser is detuned sufficiently far from resonance $|\omega-\omega_{ba}| \gg T_2^{-1}, \tau_p^{-1}, \mu_{ba}E/\hbar$. Both answers are correct when considering transition linewidth.

7. Verify 6.6.37a) $P(\omega+\delta) = \epsilon_0 \chi^{(1)}_{eff}(\omega+\delta)E_1 + 3\epsilon_0 \chi^{(3)}_{eff}[\omega+\delta=\omega+\omega-(\omega-\delta)]E_0^2 E_1^*$

6.6.38 b) $P(\omega-\delta) = \epsilon_0 \chi^{(1)}_{eff}(\omega-\delta)E_{-1} + 3\epsilon_0 \chi^{(3)}_{eff}[\omega-\delta=\omega+\omega-(\omega+\delta)]E_0^2 E_1^*$

6.6.37a) Polarization / Power Series: $p = \frac{-\Delta\Omega}{\Delta^2}\frac{-\frac{1}{2}N\mu_{AB}}{(1+|\Omega|^2/\Delta^2)^{1/2}}$

$\chi^{(1)}_{eff}(\omega+\delta) = Np_1/\epsilon_0 E_1$

Derivation of Pump and Probe Fields:

$\tilde{E}(t) = Ee^{-i\omega t}+c.c.$; $\tilde{p}(t) = pe^{-i\omega t}+c.c.$

$\frac{dp}{dt} = (i\Delta - \frac{1}{T_2})p - \frac{i}{\hbar}|\mu_{ba}|^2 E w$

$\frac{dw}{dt} = -\frac{w-w^{(eq)}}{T_1} - \frac{4}{\hbar}\,\mathrm{Im}(Ep^*)$

Assuming the electric field is:

$\tilde{E}(t) = E_0 e^{-i\omega t} + E_1 e^{-i(\omega+\delta)t} + c.c.$

$p = p_0 + p_1 e^{-i\delta t} + p_{-1}e^{i\delta t}$

$w = w_0 + w_1 e^{-i\delta t} + w_{-1}e^{i\delta t}$

Binomial Expansion: $|\Delta| \gg |\Omega|$

$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \dots$

$p = -\frac{\Delta\Omega}{\Delta^2}\left(\frac{1}{2}N\mu_{AB}\right)\left[1 - \frac{1}{2}\frac{|\Omega|^2}{\Delta^2} + \frac{3}{8}\left(\frac{|\Omega|^2}{\Delta^2}\right)^2 - \dots\right]$

$p^{(1)} = \frac{-\Delta\Omega}{\Delta^2}\left(\frac{1}{2}N\mu_{AB}\right)$

$p^{(3)} = \frac{-\Delta\Omega}{\Delta^2}\left(\frac{1}{2}N\mu_{AB}\right)\left(-\frac{1}{2}\frac{|\Omega|^2}{\Delta^2}\right)$; $\Omega(t) = 2\mu_{ba}E(t)/\hbar$

$= \frac{\Delta\Omega|\Omega|^2 N\mu_{AB}}{4\Delta^4} = \frac{2N|\mu_{ab}|^4}{\hbar^3\Delta^3}|E|^2E$

Coefficient for $P^{(3)}$ is $3\epsilon_0\chi^{(3)}$:

$\chi^{(3)} = \frac{2N|\mu_{ba}|^4}{3\epsilon_0\hbar^3\Delta^3}$

Equation of Motion:

$0 = (i\Delta - \frac{1}{T_2})p_0 - \frac{i}{\hbar}|\mu_{ba}|^2 E_0 w_0$ (#1)

$p_0 = \frac{\hbar^{-1}|\mu_{ba}|^2 E_0 w_0}{\Delta + i/T_2}$

$-i\delta p_1 = (i\Delta - \frac{1}{T_2})p_1 - \frac{i}{\hbar}|\mu_{ba}|^2(E_0 w_1 + E w_0)$ (#2)

$p_1 = \frac{\hbar^{-1}|\mu_{ba}|^2(E_0 w_1 + E_1 w_0)}{(\Delta-\delta)+i/T_2}$

$-i\delta p_{-1} = (i\Delta - \frac{1}{T_2})p_{-1} - \frac{i}{\hbar}|\mu_{ba}|^2(E_0 w_{-1})$ (#3)

$p_{-1} = \frac{\hbar^{-1}|\mu_{ba}|^2 E_0 w_{-1}}{(\Delta-\delta)+i/T_2}$

$p_1 = \frac{\hbar^{-1}|\mu_{ba}|^2 w_0 E_1}{D(\delta)}\left[(\delta + \frac{i}{T_1})(\delta - \Delta + \frac{i}{T_2}) - \frac{1}{2}\Omega^2\frac{\delta}{\Delta - i/T_2}\right]$

$\chi^{(1)}_{eff}(\omega+\delta) = \frac{N|\mu_{ba}|^2 w_0}{\epsilon_0\hbar D(\delta)}\left[(\delta+\frac{i}{T_1})(\delta-\Delta+\frac{i}{T_2}) - \frac{1}{2}\Omega^2\frac{\delta}{\Delta-i/T_2}\right]$; $\chi^{(3)}_{eff} = \frac{2Nw_0|\mu_{ba}|^4(\delta-\Delta-i/T_2)(\delta+2i/T_2)(\Delta+i/T_2)^{-1}}{3\epsilon_0\hbar^3(\Delta-\delta+i/T_2)D^*(\delta)}$

$$2ik_0\frac{\partial A}{\partial z}+\frac{\partial^2 A}{\partial x^2}=-3\chi^{(3)}\frac{\omega^2}{c^2}|A|^2A = 2ik_0\left[i\gamma A_0\,\mathrm{sech}(x/x_0)e^{i\gamma z}\right]+\left[\frac{\tanh^2(\frac{x}{x_0})\mathrm{sech}(\frac{x}{x_0})-\mathrm{sech}^3(\frac{x}{x_0})}{x_0^2}\right]A_0e^{i\gamma z}$$

$$-2k_0\gamma A_0\,\mathrm{sech}\left(\frac{x}{x_0}\right)e^{i\gamma z}+\left[\frac{\tanh^2(\frac{x}{x_0})\mathrm{sech}(\frac{x}{x_0})-\mathrm{sech}^3(\frac{x}{x_0})}{x_0^2}\right]A_0e^{i\gamma z}=-3\chi^{(3)}k_0^2A_0^3\,\mathrm{sech}^3\left(\frac{x}{x_0}\right)e^{3i\gamma z}$$

$$-2k_0\gamma+\left[\frac{\tanh^2(\frac{x}{x_0})-\mathrm{sech}^2(\frac{x}{x_0})}{x_0^2}\right]=-3\chi^{(3)}k_0^2\cdot A_0^2\,\mathrm{sech}^2\left(\frac{x}{x_0}\right)e^{2i\gamma z}$$

$$-2k_0\gamma+\frac{\tanh^2(\frac{x}{x_0})}{x_0^2}=\left[-3\chi^{(3)}k_0^2A_0^2e^{2i\gamma z}+\frac{1}{x_0^2}\right]\mathrm{sech}^2\left(\frac{x}{x_0}\right)=\left[-3\chi^{(3)}k_0^2A_0^2e^{2i\gamma z}+\frac{k_0^2(2\bar{n}_2|A_0|^2)}{n_0}\right]\mathrm{sech}^2\left(\frac{x}{x_0}\right)$$

$$-2k_0k_0\bar{n}_2|A_0|^2/n_0+\frac{k_0^2(2\bar{n}_2|A_0|^2)}{n_0}\tanh^2\left(\frac{x}{x_0}\right)=k_0^2|A_0|^2\left[-3\chi^{(3)}e^{2i\gamma z}+2\bar{n}_2/n_0\right]\mathrm{sech}^2\left(\frac{x}{x_0}\right)$$

$$-2\left(\frac{\bar{n}_2}{n_0}\right)+2\left(\frac{\bar{n}_2}{n_0}\right)\tanh^2\left(\frac{x}{x_0}\right)=\left[-3\boxed{\left(\tfrac{4}{3}\bar{n}_2 n_0\right)}e^{2i\gamma z}+2\left(\frac{\bar{n}_2}{n_0}\right)\right]\mathrm{sech}^2\left(\frac{x}{x_0}\right)$$

unusual part

$$\left(\frac{1}{n_0}\right)\left[\tanh^2\left(\frac{x}{x_0}\right)-1\right]=\left[2n_0e^{2i\gamma z}+2\left(\frac{1}{n_0}\right)\right]\mathrm{sech}^2\left(\frac{x}{x_0}\right)$$

Note(?) $\rightarrow\left(+\frac{1}{n_0}\right)=\left[2n_0e^{2i\gamma z}+2\left(\frac{1}{n_0}\right)\right];\ +2n_0e^{2i\gamma z}=\frac{1}{n_0};\ +2n_0^2e^{2i\gamma z}=1$

$$+2n_0^2\left[\cos^2\gamma z+\sin^2\gamma z\right]\boxed{\neq 1}$$

2. Eqn 7.1.42 and 7.1.44

Eqn 7.1.42 ; $Z_{br}=\dfrac{G}{n_2 k_{vac} I}$ ‖ Eqn. 7.1.43: $Z_{sp}=\dfrac{2n_0w_0^3}{\lambda_0}\dfrac{1}{\sqrt{P/P_{cr}-1}}$ ‖ Eqn 7.1.43 $P/P_{cr}=4G^2$

Write a paragraph about beam breakup in producing a 10nsec pulse, and utilizing the nonlinear response of carbon disulfide [CS_2]. Issues (in)of pulse energy required, length of interaction, and focusing characteristics.

Paragraph I: Optical Beam Breakup

Topic sentence: Optical beam breakup is a process induced by the wave-front interaction.

pg 208 Concrete Detail: The spatial sidemodes [E_1 & E_{-1}] generate intensity patterns.

Supporting: Both random and regular patterns have been measured [Bennick 2002]

Concrete Detail: By solving the spatial light evolution, real and imaginary components show attenuation.

Supporting: The paraxial solution is described as a nonvanishing condition.

Concrete Detail: Orders of 10^{-9}s pulses inside Carbon disulfide would require large powers.

Supporting: When a supply of 27KW is achieved @ 1μm, then 10ns pulses are achieved.

Thesis: Optical beam breakup provides avenues of literature research.

Topic sentence: Research for self-focusing is developing.

Concrete Detail: As earlier mentioned, CS_2 provides focusing characteristics.

Supporting: Self-focusing models predict a 1cm distance to focus.

Concrete Detail: Nd:Yag laser operating at 1.06μm produce correct responses.

Supporting: Q-switching is required for self-focusing models.

Conclusion: Optical filaments are produced in the presence of self-focusing

4. Boundary Conditions: $A_3(0)$ and $A_4(L)$ $\boxed{\text{Amplitudes only}}$

$$\frac{dA_3}{dz} = -\kappa_3 A_3 - i k_4^* A_4^* \;;\; \frac{dA_4}{dz} = \kappa_4 A_4 + i k_3^* A_3^*$$

"Bernoulli Equation": $y' + p(x)y = Q(x)$; $I = e^{\int p(x)dz}$; $y = \frac{1}{I(x)}\left[\int I(x)Q(x)dx + c\right]$

Assuming a constant A_4^* ; $\frac{dA_3}{dz} + \kappa_3 A_3 = -i k_4^* A_4^*$; $p(x) = \kappa_3$; $Q(x) = -i k_4^* A_4^*$

$$A_4 = \frac{\text{Numerator}}{\text{Denominator}} = \frac{\int I(x)Q(x)dx + C}{I(x)}$$

Numerator: $\int e^{-\kappa_4 z + k_3^* k_4\left[\frac{ze^{-\kappa_3 z}}{\kappa_3} - \frac{(1-e^{-\kappa_3 z})}{\kappa_3^2}\right]e^{\kappa_3}} \cdot i k_3^* A_{3,c}\, dz$

$\log A_4 = \log \int I(x)Q(x)dx + c$

$$\int -\kappa_4 z + k_3^* k_4\left[\frac{ze^{-\kappa z}}{\kappa_3} - \frac{(1-e^{-\kappa_3 z})}{\kappa_3^2}\right]e^{\kappa_3} + \log[ik_3^* A_{3,c}]z$$

$$= -\frac{\kappa_4 z^2}{2} + k_3^* k_4\left[-\frac{ze^{-\kappa_3 z}}{\kappa_3^2} - \frac{(1-e^{-\kappa_3 z})}{\kappa_3^3} + \frac{z}{\kappa_3^2} + \frac{e^{-\kappa_3 z}}{\kappa_3^3}\right]e^{\kappa_3}$$

$$= -\frac{\kappa_4 z^2}{2} + k_3^* k_4\left[\frac{ze^{-\kappa_3 z}}{\kappa_3^2} + \frac{1}{\kappa_3^3} + \frac{e^{-\kappa_3 z}}{\kappa_3^3} + \frac{z}{\kappa_3^2} + \frac{e^{-\kappa_3 z}}{\kappa_3^2}\right]e^{\kappa_3}$$

$$= -\frac{\kappa_4 z^2}{2} - k_3^* k_4\left[\frac{1}{\kappa_3} + z(1+e^{-\kappa_3 z})\right]\frac{e^{\kappa_3}}{\kappa_3^2}$$

$$\boxed{A_4 = \frac{\exp\left(-\frac{\kappa_4 z^2}{2} - k_3^* k_4\left[\frac{1}{\kappa_3} + z(1+e^{-\kappa_3 z})\right]\frac{e^{\kappa_3}}{\kappa_3^2}\right)\left[ik_3^* A_{3,c}\right]z}{\exp\left(-\kappa_4 z + k_3^* k_4\left[\frac{ze^{-\kappa_3 z}}{\kappa_3} - \frac{(1-e^{-\kappa_3 z})}{\kappa_3^2}\right]e^{\kappa_3}\right)} + A_{4,c}}$$

5. $\frac{dA_3}{dz} = -\kappa_3 A_3 - i k_4^* A_4^* e^{i\Delta k z}$

$$\frac{dA_4}{dz} = \kappa_4 A_4 + i k_3^* A_3^* e^{-i\Delta k z}$$

Where $\Delta k = (k_1 + k_2 - k_3 - k_4)\cdot\hat{z}$

$\boxed{\text{Amplitudes and phase}}$

Assuming A_3 is constant

$\frac{dA_4}{dz} - \kappa_4 A_4 = i k_3 A_3^* e^{-i\Delta k z}$

Bernoulli Differential: $p(x) = -\kappa_4$; $Q(x) = i k_3 A_3^* e^{-i\Delta k z}$

$I(x) = e^{-\kappa\int dz} = e^{-\kappa_4 z}$;

$$A_4 = \frac{1}{I(x)}\left[\int I(x)Q(x)dx\right]$$

$I = e^{\int \kappa_3 dz} = e^{\kappa_3 \cdot z}$

$$A_3 = \frac{1}{e^{\kappa_3 z}}\left[\int e^{\kappa_3}\cdot(-ik_4^* A_4^*)dz\right]$$

$$= (-ik_4^* A_4^*)e^{\kappa_3(1-z)}\cdot z + C$$

$A_{3,c} = C$

$$\boxed{A_3 = (-ik_4^* A_4^*)e^{\kappa_3(1-z)}\cdot z + A_{3,c}}$$

$$\frac{dA_4}{dz} - \kappa_4 A_4 = i k_3^* A_3^* = i k_3^*\left[(+i k_4 A_4)e^{\kappa_3(1-z)}\cdot z + A_{3,c}\right]$$

$$\frac{dA_4}{dz} - \kappa_4 A_4 = k_3^* k_4 A_4 e^{\kappa_3(1-z)}\cdot z + i k_3^* A_{3,c}$$

$$\frac{dA_4}{dz} - \left[\kappa_4 + k_3^* k_4 e^{\kappa_3(1-z)}\cdot z\right]A_4 = i k_3^* A_{3,c}$$

$p(x) = -\left[\kappa_4 + k_3^* k_4 e^{\kappa_3(1-z)}\cdot z\right]$; $Q(x) = i k_3^* A_{3,c}$

$$I = e^{-\int\left[\kappa_4 + k_3^* k_4 e^{\kappa_3(1-z)}\cdot z\right]dz}$$

$$= e^{-\kappa_4\cdot z - k_3^* k_4 e^{\kappa_3}\underbrace{\int e^{-\kappa_3 z}\cdot z\,dz}_{a}}$$

Integration by Parts: $\int u\,dv = uv| - \int v\,du$

$u = z$; $du = dz$

$dv = e^{-\kappa_3 z}$; $v = -\frac{e^{-\kappa_3 z}}{\kappa_3}$

$$a = \int u\,dv = -\frac{z\cdot e^{-\kappa_3 z}}{\kappa_3}\Big|_0^z + \int_0^z \frac{e^{-\kappa_3 z}}{\kappa_3}\cdot dz$$

$$= -\frac{ze^{-\kappa_3 z}}{\kappa_3} - \left[\frac{e^{-\kappa_3 z}}{\kappa_3^2} - \frac{1}{\kappa_3^2}\right]$$

$$= -\frac{ze^{-\kappa_3 z}}{\kappa_3} - \frac{(1-e^{-\kappa_3 z})}{\kappa_3^2}$$

$$I = e^{-\kappa_4\cdot z - k_3^* k_4\left(-\frac{ze^{-\kappa_3 z}}{\kappa_3} - \frac{(1-e^{-\kappa_3 z})}{\kappa_3^2}\right)e^{\kappa_3}}$$

$$= e^{-\kappa_4 z + k_3^* k_4\left[\frac{ze^{-\kappa_3 z}}{\kappa_3} - \frac{(1-e^{-\kappa_3 z})}{\kappa_3^2}\right]e^{\kappa_3}}$$

$$A_4 = \frac{1}{e^{-\kappa_4 z}}\left[\int e^{-\kappa_4 z}\cdot(ik_3^* A_3^* e^{-i\Delta k z})\,dz\right] = \frac{1}{e^{-\kappa_4 z}}\left[\int e^{-(\kappa_4 + i\Delta k)z}(ik_3^* A_3^*)\,dz\right] = \frac{ik_3^* A_3^*}{e^{-\kappa_4 z}}\,\frac{e^{-(\kappa_4+i\Delta k)z}}{-(\kappa_4 + i\Delta k)} + c$$

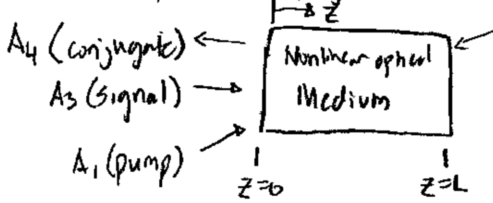
$$\frac{dA_3}{dz} + \alpha_3 A_3 = -ik_4^* A_4^* e^{i\Delta k z} = -ik_4^*\left[\frac{-k_3^* A_3^*(\Delta k + i\kappa_4)}{(\kappa_4^2 + \Delta k^2)} e^{-i\Delta k z}\right] e^{i\Delta k z} \qquad -\frac{ik_3^* A_3^* \cdot e^{-i\Delta k z}}{(\kappa_4 + i\Delta k)}\,\frac{(\kappa_4 - i\Delta k)}{(\kappa_4 - i\Delta k)} + c$$

$$\frac{dA_3}{dz} + \kappa_3 A_3 = \left[\frac{-ik_4^* k_3 A_3 (i\kappa_4 + i\Delta k)}{(\alpha_4^2 + \Delta k^2)}\right] e \qquad \boxed{A_4 = \frac{-k_3^* A_3^*(\Delta k + i\kappa_4)}{(\alpha_4^2 + \Delta k^2)}\, e^{-i\Delta k z} + \cancel{c}}$$

$$\frac{dA_3}{dz} + \left[\frac{\kappa_3 + k_4^* k_3(\kappa_4 + i\Delta k)}{(\kappa_4^2 + \Delta k^2)}\right] A_3 = 0 \;;\; A_3 = \frac{1}{I(x)}\int I(x)Q(x)\,dx = \boxed{0}$$

"Phase demonstrated amplitudes approach zero when coupled".

6. Derive a phase-conjugate reflexivity for four-wave mixing of "two-level" atom.



Approximations: 1) Rotating wave: Neglecting terms of amplitude because of model considerations [e.g. One photon absorption, SE]
2) Slow-varying Amplitude: the model for coupled equations considers a constant value of mathematical description.
3) Neglect Pump waves: considers pump does not modify the nonlinear interaction.

$$\frac{dA_3}{dz} = \frac{3i\omega}{nc}\chi^{(3)}\left[(|A_1|^2 + |A_2|^2)A_3 + A_1 A_2 A_4^*\right];\; \frac{dA_4}{dz} = \frac{-3i\omega}{nc}\chi^{(3)}\left[(|A_1|^2 + |A_2|^2)A_4 + A_1 A_2 A_3^*\right]$$

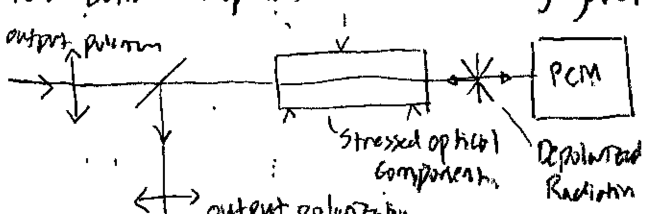
$$\boxed{\frac{dA_3}{dz} = \frac{3i\omega}{nc}\chi^{(3)}\left[2A_3 + A_4^*\right];\; \frac{dA_4}{dz} = \frac{-3i\omega}{nc}\chi^{(3)}\left[2A_4 + A_3^*\right]}$$

Even hill 15

7. Verify (7.2.41)

$$\begin{bmatrix}P_x\\P_y\end{bmatrix} = 6\epsilon_0\begin{bmatrix}\chi_{1111}B_xF_x + \chi_{1221}B_yF_y & \chi_{1122}(B_xF_y + B_yF_x)\\ \chi_{1122}(B_yF_x + B_xF_y) & \chi_{1111}B_yF_y + \chi_{1221}B_xF_x\end{bmatrix}\begin{bmatrix}S_x^*\\S_y^*\end{bmatrix}$$

Polarization Properties of Phase-conjugation.



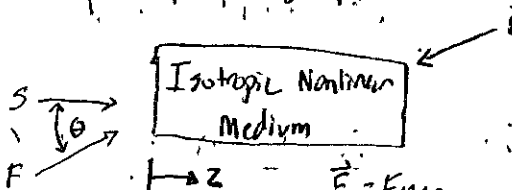
$$\chi^{(3)}_{ijkl} = \chi_{1122}(\delta_{ij}\delta_{kl} + \delta_{ik}\delta_{jl}) + \chi_{1221}\delta_{il}\delta_{jk}$$

$$P = 6\epsilon_0\chi_{1122}(E\cdot E^*)E + 3\epsilon_0\chi_{1221}(E\cdot E)E^*$$

$$= \epsilon_0 A(E\cdot E^*)E + \tfrac{1}{2}\epsilon_0 B(E\cdot E)E^*$$

$$= 6\epsilon_0\chi_{1122}(F+B+S)(F+B+S)^*(F+B+S) + 3\epsilon_0\chi_{1221}(F+B+S)(F+B+S)^*(F+B+S)^*$$

$$= 6\epsilon_0\left[\chi_{1122}(F+B+S)^2(F+B+S)^*(F+B+S) + \tfrac{1}{2}\chi_{1221}(F+B+S)^2(F+B+S)^*\right]$$



$\vec{E} = \vec{F} + \vec{B} + \vec{S}$: $\vec{E}$ = Energy, $\vec{F}$ = Forward Pump, $\vec{B}$ = Backward Pump, $\vec{S}$ = Signal.

Assumptions: ∘ θ is small; x-only incident Amplitudes.

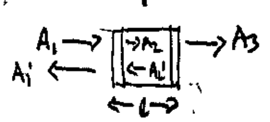
∘ Isotropic nonlinear Material.

$$\chi^{(3)}_{ijkl} = \chi^{(3)}_{ijkl}(\omega = \omega + \omega - \omega)$$

8. Optical Bistability: two output intensities are possible for a given input intensity. (1969)

$R = |\rho|^2$; $T = |\tau|^2$; $R + T = 1$

$A_2' = \rho A_2 e^{2ikl - \kappa l}$; $A_2 = \tau A_1 + \rho A_2'$; $k = n\omega/c$



$$A_2 = \frac{\tau A_1}{1 - \rho^2 e^{2ikl - \kappa l}} = \frac{\tau A_1}{1 - Re^{i\delta_0}}$$

Absorptive Bistability: case where α depends on nonlinear behavior...

Traditional

$$A_2 = \frac{\tau A_1}{1-R(1-\kappa l)}$$

$$I_i = 2n\epsilon_0 |A_i|^2$$

$$I_2 = \frac{T_1 I_1}{[1-R(1-\kappa l)]^2}$$

$$C = \frac{R\kappa l}{1-R}$$

$$I_2 = \frac{1}{T}\frac{I_1}{(1+C)^2}$$

Absorption coefficient

$$\kappa = \frac{\alpha_0}{1+I/I_S}$$

$$C = \frac{C_0}{1+2I_2/I_S}$$

$$I_1 = T I_2\left(1+\frac{C_0}{1+2I_2/I_S}\right)^2$$

Adjusted:

$$A_2 = \frac{\tau A_1}{1-Re^{i\delta_0}(1-\kappa l)}$$

$$I_2 = \frac{T I_i}{[1-Re^{i\delta_0}(1-\kappa l)]^2}$$

$$C = Re^{i\delta_0} \Big/ (1-Re^{i\delta_0}\kappa l)$$

$$I_2 = \frac{1}{T}\frac{I_1}{(1+C)^2}$$

$$\kappa = \frac{\kappa_0}{1+I/I_S}$$

$$C = \frac{C_0 T/R}{1+2I_2/I_S}$$

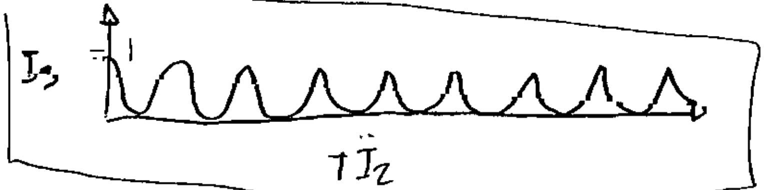
$$T I_1 = I_2\left[1-Re^{i\delta_0}(1-\kappa l)\right]^2$$

$$= I_2\left[1-2Re^{i\delta_0}(1-\kappa l)+R^2e^{2i\delta_0}(1-\kappa l)^2\right]$$

$$\boxed{= I_2\left[1-2Re^{i\delta_0}\left(1-\frac{C_0T/R}{1+2I_2/I_S}\right)+R^2e^{2i\delta_0}\left(1-\frac{C_0T/R}{1+2I_2/I_S}\right)^2\right]}$$

10. $$I_3 = TI_2 = T\frac{T^2 I_1}{(1-Re^{i\delta})(1-Re^{-i\delta})} = \frac{T^2 I_1}{1+R^2-2R\cos\delta}$$

$$= \frac{T I_1}{1+(4R/T^2)\sin^2\frac{1}{2}\delta} \; ; \; \delta = \delta_0 + (4n_2\omega l/c) I_2$$



12. Reflectance Intensity: $R = |\rho|^2$; $T = |\tau|^2$: Transmittance Intensity

$$R + T = 1$$

Boundary conditions: $A_2' = \rho A_2 e^{2\kappa l - \kappa l}$

$$A_2 = \tau A_1 + \rho A_2'$$

$\boxed{\text{Optical Bistability}}$

$\boxed{\text{optical switching}}$

reflection coefficient : $r = i\sqrt{R}$

transmission coefficient : $t = \sqrt{T}$

$$R + T = 1.$$

$R = |\rho|^2$; $\sqrt{R} = |\rho|$

$\boxed{i\sqrt{R} = \rho = r}$

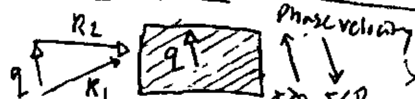
$T = |\tau|^2$; $\sqrt{T} = |\tau|$

$\sqrt{T} = \tau$

Real Imaginary $= 1$

$\sqrt{T} + R = 1$

#1. Interference Pattern in medium



"Interacting waves"

$\vec{q} = \vec{k}_1 - \vec{k}_2$; $\tau\frac{dn_{NL}}{dt} + n_{NL} = n_2 I$ $\boxed{\text{Debye Relaxation}}$

response time

$$n_{NL} = \frac{n_2}{\tau}\int_{-\infty}^{t} I(t')e^{(t'-t)/\tau}\,dt''$$

spatial and temporal conditions

$$= \frac{n_2}{\tau}\left[\frac{e^{-i\delta t}}{-i\delta+1/\tau}\right]$$

$$\frac{d^2A_2}{dz^2} + 2ik_2\frac{dA_2}{dz} - k_2^2A_2 + \frac{n_0^2\omega_2^2}{c^2}A_2$$

$$= \underbrace{-\frac{4n_0^2n_2\omega_2^2\epsilon_0}{c}(|A_1|^2+|A_2|^2)A_2}_{\text{Stationary refractive index component.}} - \underbrace{\frac{4n_0^2n_2\omega_2^2}{c}\frac{|A_1|^2A_2}{1+i\delta\tau}}_{\text{time varying component}}$$

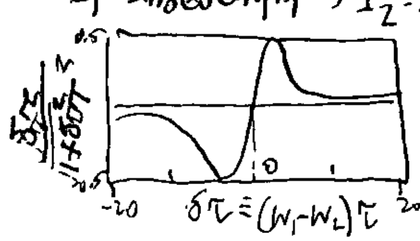
$$= 2n_0n_2\epsilon_0 c\left[(A_1A_1^* + A_2A_2^*) + \frac{A_1A_2^* e^{i(qr-\delta t)}}{1-i\delta\tau} + \frac{A_1^*A_2 e^{-i(qr-\delta t)}}{1+i\delta\tau}\right]$$

$$\nabla^2 E - \frac{n^2}{c^2}\frac{\partial^2 E}{\partial t^2} = 0 \; ; \; n = n_0 + n_{NL}$$

$$n^2 = n_0^2 + 2n_0 n_{NL}$$

Slow-vary amplitude Approximation: $\frac{dA_2}{dz} = 2in_0n_2(\omega/c)\left[(|A_1|^2+|A_2|^2)A_2 + \frac{|A_1|^2A_2}{1+i\delta\tau}\right]$

$I_1 = 2n_0\epsilon_0 c A_1A_1^*$; $I_2 = 2n_0\epsilon_0 c A_2A_2^*$; $\frac{dI_2}{dz} = 2n_0\epsilon_0 c\left(A_2^*\frac{dA_2}{dz} + A_2\frac{dA_2^*}{dz}\right)$; $\frac{dI_2}{dz} = \frac{2n_2\omega}{c}\frac{\delta\tau\, I_1 I_2}{1+\delta^2\tau^2}$



Pulse Propagation and Temporal Solutions

7.5.1: Self-phase Modulation: $E(z,t) = A(z,t)e^{i(k_0 z - \omega_0 t)} + c.c.$; $n(t) = n_0 + n_2 I(t)$; $I(t) = 2n_0\epsilon_0 c|A(z,t)|^2$

$\phi_{NL}(t) = -n_2 I(t)\omega_0 L/c$ $\boxed{\text{"changes broadness of pulse"}}$

Spectrum of Self-phase Modulation: $S(\omega) = \left|\int_{-\infty}^{\infty} A(t)e^{-i\omega_0 t - i\phi_{NL}(t)} e^{i\omega t}\,dt\right|^2$

Instantaneous Frequency: $\omega(t) = \omega_0 + \delta\omega(t)$ where $\delta\omega(t) = \frac{d}{dt}\phi_{NL}(t)$

Intensity of waveform: $I(t) = I_0 \operatorname{sech}^2(t/\tau_0)$

Nonlinear phase shift: $\phi_{NL}(t) = -n_2 \frac{\omega_0}{c} L I_0 \operatorname{sech}^2(t/\tau_0)$

Instantaneous Frequency: $\delta\omega(t) = 2n_2 \frac{\omega_0}{c\tau_0} L I_0 \operatorname{sech}^2(t/\tau_0) \tanh(t/\tau_0)$

The spectrum of two beam coupling is described by $\frac{dI_2}{dz} = \frac{2n_2\omega}{c}\frac{\delta\tau}{1+\delta^2\tau^2} I_1 I_2$. If the spectral pulse duration increased to a much longer value, then the intensity of these secondary beam may never change with propagation. For self-phase modulation, the spectral broadening may change with $|\omega_0 t - \phi_{NL}(t)| < 1$ or $|\omega_0 - \omega| \ll 1$ or $|\phi_{NL} - \omega| \ll 1$ or larger τ_0.

$\boxed{|k_0 z - \omega_0 t| \ll 1}$

$\boxed{\Delta\omega = \phi_{NL}^{max}/T}$

Sawtooth Function is similar to $\delta(\omega - \omega_0)$.

15. Pulse Propagation: $E(z,t) = A(z,t)e^{i(k_0 z - \omega_0 t)} + c.c.$

Pulse envelope: $\frac{\partial^2 E}{\partial z^2} - \frac{1}{\epsilon_0 c^2}\frac{\partial^2 D}{\partial t^2} = 0$

Wave equation: $\frac{\partial^2 E(z,\omega)}{dz^2} + \epsilon(\omega)\frac{\omega^2}{c^2}E(z,\omega) = 0$ $\boxed{\text{Helmholtz Equation}}$

$E(z,\omega) \cong A(z,\omega-\omega_0)e^{ik_0 z}$; $2ik_0\frac{\partial A(z,\omega-\omega_0)}{\partial z} + \underbrace{(k^2 - k_0^2)}_{k(\omega)=\sqrt{\epsilon(\omega)}\,\omega/c}A(z,\omega-\omega_0) = 0$ $= (k^2 - k_0^2)$

$\frac{\partial A(z,\omega-\omega_0)}{\partial z} + i(k - k_0)A(z,\omega-\omega_0) = 0$

$2k_0(k - k_0) = (k^2 - k_0^2)$ "Not this constant" $-2s+1$ $\quad 2(2s)+1$ $\sum m_L$ $s+L$

$2ik_0\frac{\partial A(z,\omega-\omega_0)}{\partial z} + (k^2 - k_0^2)A(z,\omega-\omega_0) = \left(2ik_0\frac{\partial}{\partial z} + \Big[\underbrace{\big(k_0 + \Delta k_{NL} + k_1(\omega-\omega_0) + \tfrac{1}{2}(\omega-\omega_0)^2\big)^2}_{} - k_0^2\Big]\right)A(z,\omega-\omega_0) = 0$

$k_1 = \left(\frac{\partial k}{\partial \omega}\right) = \frac{\partial}{\partial\omega}\left[\frac{n_{lin}(\omega)\omega}{c}\right] = \frac{1}{c}\left[n_{lin}(\omega) + \omega\frac{dn_{lin}(\omega)}{d\omega}\right] \equiv \underbrace{\frac{n_g}{c}}_{\text{"Power Series"}} \equiv \frac{1}{V_g(\omega_0)}$; $k_2 = \left(\frac{\partial^2 k}{\partial\omega^2}\right) = \frac{d}{d\omega}\left[\frac{1}{v_g(\omega)}\right] = \frac{-1}{v_g^2}\left(\frac{dv}{d\omega}\right)$

$\Big(2ik_0\frac{\partial}{\partial z} + \cancel{k_0^2} + k_0\Delta k_{NL} + k_0 k_1(\omega-\omega_0) + \frac{k_0 k_2(\omega-\omega_0)^2}{2} + \Delta k_{NL}^2 + \Delta k_{NL}k_1(\omega-\omega_0) + \frac{\Delta k_{NL}k_2(\omega-\omega_0)^2}{2} + k_1^2(\omega-\omega_0)^2 + k_1 k_2(\omega-\omega_0)^2/2$ $+\Delta k_{NL}k_0$ $+k_1(\omega-\omega_0)k_0$ $+k_1(\omega-\omega_0)\Delta k_{NL}$

$+ \frac{k_2^2(\omega-\omega_0)^4}{4} + \frac{k_0 k_2(\omega-\omega_0)^2}{2} + \frac{\Delta k_{NL}k_2(\omega-\omega_0)^2}{2} + \frac{k_1 k_2(\omega-\omega_0)^2}{2} - \cancel{k_0^2}\Big)A(z,\omega-\omega_0) = 0$

$\Big(2ik_0\frac{\partial}{\partial z} + 2k_0\Delta k_{NL} + 2k_0 k_1(\omega-\omega_0) + \Delta k_{NL}^2 + 2\Delta k_{NL}k_1(\omega-\omega_0) + \Delta k_{NL}k_2(\omega-\omega_0)^2 + k_1^2(\omega-\omega_0)^2 + k_1 k_2(\omega-\omega_0)^2$

$+ \frac{k_2^2(\omega-\omega_0)^2}{4}\Big)A(z,\omega-\omega_0) = 0$ $\quad 2ab + 2ac + b^2 + 2bc +$

Fourier Transform:

$\int_{-\infty}^{\infty} A(z,\omega-\omega_0)e^{-i(\omega-\omega_0)t}\frac{d(\omega-\omega_0)}{2\pi} = A(z,t)$;

Derivative of Fourier Transform

$\int_{-\infty}^{\infty}(\omega-\omega_0)A(z,\omega-\omega_0)e^{-i(\omega-\omega_0)t}\frac{d(\omega-\omega_0)}{2\pi} = \frac{1}{i}\frac{d}{dt}A(z,t)$

$$\int_{-\infty}^{\infty}(w-w_0)^2 A(z,w-w_0)\, e^{-i(w-w_0)t}\, \frac{d(w-w_0)}{2\pi} = -\frac{\partial^2}{\partial t^2}\tilde{A}(z,t)$$

$$\left(2ik_0\frac{\partial A}{\partial z} + 2k_0k_1\cdot i\frac{dA(z,t)}{dt} + 2\Delta k_{nl}k_1\cdot i\frac{dA(z,t)}{dt} + \Delta k_{nl}\cdot k_2\frac{\partial^2}{\partial t^2}A(z,t) - k_1^2\frac{\partial^2 A(z,t)}{\partial t^2} - k_1k_2\frac{\partial^2 A(z,t)}{\partial t^2} + k_2^2\frac{\partial A(z,t)}{\partial t^2}\right) + 2k_0k_{NL}\tilde{A} + \Delta k_{nl}^2\tilde{A} = 0$$

$$\left(2ik_0\frac{\partial \tilde{A}}{\partial z} + (2k_0k_1 i + 2\Delta k_{NL}k_1)\frac{dA(z,t)}{dt} - (\Delta k_{NL}\cdot k_2 + k_1^2 + k_1k_2^2 + k_2^2)\frac{\partial^2 \tilde{A}(z,t)}{dt^2}\right) + (2k_0k_{NL} + k_{NL}^2)A = 0$$

$\boxed{\text{Coordinate Transformation}}$ $\tau = t - \frac{z}{v_g} = t - k_1 z$; $A_s(z,\tau) = \tilde{A}(z,t)$

$$\frac{dA}{dz} = \frac{dA_s}{dz} + \frac{dA}{d\tau}\frac{d\tau}{dz} = \frac{\partial A_s}{\partial z} - k_1\frac{\partial A_s}{\partial \tau} \ ; \ \frac{dA}{dt} = \frac{dA_s}{d\tau}$$

$$\left[2ik_0\left(\frac{\partial A_s}{\partial z} - k_1\frac{\partial A_s}{\partial \tau}\right) + \cancel{(2ik_0k_1 + 2\Delta k_{NL}k_1)\frac{\partial A}{\partial \tau} - (\Delta k_{NL}k_2 + k_1^2 + k_1k_2^2 + k_2^2)\left[\frac{\partial A_s}{\partial z} - k_1\frac{\partial A_s}{\partial \tau}\right]} + (2k_0k_{NL} + k_{NL}^2)A = 0\right.$$

$$\cancel{\left[2ik_0\frac{\partial A}{\partial z} - 2ik_0k_1\frac{\partial A_s}{\partial \tau} + 2ik_0k_1\frac{\partial A}{\partial \tau} + 2i\Delta k_{NL}k_1\frac{\partial A}{\partial \tau} - \Delta k_{NL}k_2\frac{\partial A}{\partial z} - k_1^2\frac{\partial A_s}{\partial z} - k_1k_2^2\frac{\partial A_s}{\partial z} - k_2\frac{\partial A_s}{\partial z} + \Delta k_{NL}k_1k_2\frac{\partial A_s}{d\tau}\right.}$$

$$\cancel{+ k_1^3\frac{\partial A_s}{\partial \tau} + k_1^2k_2^2\frac{\partial A_s}{\partial \tau} + k_2^2k_1\frac{\partial A_s}{\partial \tau}\Big] + (2k_0\Delta k_{NL} + \Delta k_{NL}^2)A = 0}$$

$$\left[2ik_0\frac{\partial A}{\partial z} - k_1^2\frac{\partial A_s}{\partial z} - k_2\frac{\partial A_s}{\partial z} + k_1^3\frac{\partial A_s}{\partial \tau} + k_1^2k_2^2\frac{\partial A_s}{\partial \tau}\right] + \cancel{\left[2ik_1\frac{\partial A}{\partial \tau} - k_2\frac{\partial A}{\partial z} + k_1k_2\frac{\partial A_s}{\partial \tau}\right]\Delta k_{NL} + (2k_0\Delta k_{NL} + \Delta k_{NL}^2)A = 0}$$

$$\left[2ik_0\left(\frac{\partial A_s}{\partial z} - k_1\cancel{\frac{\partial A_s}{\partial \tau}}\right) + (2k_0k_1 i + 2\Delta k_{NL}\cdot k_1)\frac{\partial A(z,t)}{\partial \tau} - (\Delta k_{NL}\cdot k_2 + k_1^2 + k_1k_2^2 + k_2^2)\frac{\partial^2 A(z,t)}{\partial \tau^2} + (2k_0k_{NL} + k_{NL}^2)A = 0\right.$$

$$2ik_0\left[\frac{\partial A_s}{\partial z}\right] + 2\Delta k_{NL}\cdot k_1\left[\frac{\partial A(z,t)}{\partial \tau}\right] - (\Delta k_{NL}\cdot k_2 + k_1^2 + k_1k_2^2 + k_2^2)\left[\frac{\partial^2 A(z,t)}{\partial \tau^2}\right] = -(2k_0\Delta k_{NL} + \Delta k_{NL}^2)A$$

$$2ik_0\left[\frac{\partial A_s}{\partial z}\right] + 2\gamma|A_s|^2k_1\left[\frac{\partial A(z,\tau)}{\partial \tau}\right] - (k_1^2 + k_1k_2^2 + k_2^2)\left[\frac{\partial^2 A(z,t)}{\partial \tau^2}\right] = +\gamma|A_s|^2\left[\frac{\partial^2 A(z,\tau)}{\partial \tau^2}\right] - (2k_0\Delta k_{NL} + \Delta k_{NL}^2)A$$

$$\boxed{2ik_0\left[\frac{\partial A_s}{\partial z}\right] \underbrace{- (k_1^2 + k_1k_2^2 + k_2^2)\left[\frac{\partial^2 A(z,t)}{\partial \tau^2}\right]}_{\text{"Broadening in time"}} = \underbrace{\left[\left[\frac{\partial^2 A(z,\tau)}{\partial \tau^2}\right] - 2k_1\left[\frac{\partial A(z,\tau)}{\partial \tau^2}\right] - (2k_0 + \gamma|A|^2)A\right]\gamma|A|^2}_{\text{"How pulse broadens with frequency with self phase modulation"}}}$$

18. Verify 7.5.33 satisfies 7.5.32. (7.5.33) $A_s(z,\tau) = A_s^0\,\mathrm{sech}(\tau/\tau_0)e^{ikz}$

(7.5.32) (7.5.33) $|A_s^0|^2 = \frac{-k_2}{\gamma\tau_0^2} = \frac{-k_2}{2n_0\epsilon_0 n_2\omega_0\tau_0^2}$

$\frac{\partial A_s(z,\tau)}{\partial z} + \frac{1}{2}ik_2\frac{\partial^2 A(z,\tau)}{\partial \tau^2} = i\gamma|A_s(z,\tau)|^2 A_s(z,\tau)$ (7.5.33) $k = -k_2/2\tau_0^2 = \frac{1}{2}\gamma|A_s^0|^2$

$$\frac{\partial A_s(z,\tau)}{\partial z} = iK A_s^0 \operatorname{sech}(\tau/\tau_0) e^{ikz}; \quad \frac{\partial A_s(z,\tau)}{\partial \tau} = \frac{A_s^0}{\tau_0} \tanh(\tau/\tau_0)\operatorname{sech}(\tau/\tau_0) e^{ikz}$$

check

$$\frac{\partial^2 A_s(z,\tau)}{\partial \tau} = \frac{A_s^0}{\tau_0^2}\left[\operatorname{sech}^3(\tau/\tau_0) - \tanh^2(\tau/\tau_0)\operatorname{sech}^2(\tau/\tau_0)\right] e^{ikz}$$

$$iK A_s^0 \operatorname{sech}(\tau/\tau_0) e^{ikz} + \frac{1}{2} i k_2 \frac{A_s^0}{\tau_0^2}\left(\operatorname{sech}^{2}(\tau/\tau_0) - \tanh^2(\tau/\tau_0)\operatorname{sech}(\tau/\tau_0)\right)e^{ikz} = i\gamma |A_s^0|^2 A_s^0 \operatorname{sech}^3(\tau/\tau_0) e^{3ikz}$$

$$ik + \frac{1}{2} i k_2 \frac{1}{\tau_0^2}\left(\operatorname{sech}^2(\tau/\tau_0) - \tanh^2(\tau/\tau_0)\cancel{\operatorname{sech}(\tau/\tau_0)}\right) = i\gamma |A_s^0|^2 \operatorname{sech}^2(\tau/\tau_0) e^{2ikz}$$

$$\frac{-k_2}{2\tau_0^2} + \frac{k_2}{2\tau_0^2}\left(\operatorname{sech}^2(\tau/\tau_0) - \tanh^2(\tau/\tau_0)\cancel{\operatorname{sech}(\tau/\tau_0)}\right) = \gamma \frac{k_2}{2\tau_0^2} k_2 \operatorname{sech}^2(\tau/\tau_0) e^{2ikz}$$

$$-\frac{1}{2} + \frac{1}{2}\left(\operatorname{sech}^2(\tau/\tau_0) - \tanh^2(\tau/\tau_0)\cancel{\operatorname{sech}(\tau/\tau_0)}\right) = \gamma \frac{k_2}{2\tau_0^2}\operatorname{sech}^2(\tau/\tau_0) e^{2ikz}$$

$$-2 = \frac{\gamma k_2}{2\tau_0^2}\operatorname{sech}^2(\tau/\tau_0) e^{2ikz}; \quad \frac{-4\tau_0^2}{\gamma k_2} = \frac{2}{k\gamma} = \operatorname{sech}^2(\tau/\tau_0) e^{-2ikz} = \frac{2 e^{2ikz}}{(e^x + e^{-x})^2}; \quad \frac{1}{k\gamma} = \frac{e^{2ikz}}{e^{\tau/\tau_0} + e^{-\tau/\tau_0}}$$

$$\boxed{\frac{e^{\tau/\tau_0} + e^{-\tau/\tau_0}}{k\gamma} = e^{2ikz}}$$

$$\boxed{e^{(t-k_1 z)/\tau_0} + e^{-(t-k_1 z)/\tau_0} = k\gamma e^{2ikz}}$$

(B) $\tau_0 = 10\,psec$; $D = 8\,\mu m$; $\lambda = 1.55\,\mu m$; $|A_s^0|^2 = \frac{-2\cdot(k_2)k_2}{2 n_0 \epsilon_0 n_2 w_0 \tau_0^2}$

Assuming $k_2 = -20\,ps^2/km$ $\quad w = \frac{2\pi c}{\lambda} = 2\pi\nu$

$n_2 = 3\times10^{-6}\,\frac{cm^2}{W}$ $\quad = 1.2\times10^{15}\,\frac{1}{s}$

$$|A_s^0| = \sqrt{\frac{-20\,ps^2/km \cdot (10\,ps)^2}{2\cdot 1.5\cdot 8.85\times10^{-12}(3\times10^{-6}\,cm^2/W)(10\,ps)^2}}$$

$$= \sqrt{-2.51\times10^{15}\frac{ps^2/km}{\left(\frac{F}{m}\right)\left(\frac{cm^2}{W}\right)ps^2}}$$

$$= \sqrt{-2.51\times10^{16}\,W/m\ldots}$$

$$\frac{t - k_1 z}{\tau_0} = 2ikz + \log k\gamma$$

$$\frac{1}{\tau_0} = \ldots$$

$$\frac{1}{\tau_0} = -\frac{k_2 z i}{\tau_0} + \log k\gamma$$

$$\boxed{1 = e^{2ikz}(k\gamma)^{\tau_0}}$$

$$\boxed{1 = k\gamma^{\tau_0}} \text{ close}$$

$$\boxed{1 = k\gamma = 1}$$

$\tilde{E}(z,t) = \tilde{A}(z,t) e^{i(k_0 z - wt)}$ $\quad n = n_2$

$\tilde{A}_s(z,\tau) = A_s^0 \operatorname{sech}(\tau/\tau_0) e^{ikz}$

$P(t) = P_p \operatorname{sech}^2(\tau/\tau_0)$

$$\tilde{A}_s^2(z,\tau) = |A_s^0|^2 \operatorname{sech}^2(\tau/\tau_0) e^{2ikz} = \frac{-k_2}{2 n_0 \epsilon_0 n_2 w_0 \tau_0^2}\operatorname{sech}^2(\tau/\tau_0) e^{2i\left(\frac{-k_2}{2\tau_0^2}\right)\cdot z}$$

$$= \frac{+20\,ps^2/km \cdot \operatorname{sech}^2(\tau/\tau_0)\exp\left[2i\left(\frac{-20\,ps^2/km}{2(10\,ps)^2}\right)\cdot z\right]}{2\cdot1\cdot8.85\times10^{-12}\frac{F}{m}\left(3\times10^{-6}\frac{cm^2}{W}\right)\cdot1.2\times10^{15}\frac{1}{s}(10\,ps)^2}$$

$$= \frac{+20\,\frac{1}{km}\cdot\ldots\left(\frac{1\,km}{1000\,m}\right)}{2\cdot1\cdot8.85\times10^{-12}\frac{F}{m}\left(3\times10^{-6}\frac{cm^2}{W}\times\left(\frac{1\,m}{100\,cm}\right)^2\right)\left(1.2\times10^{15}\frac{1}{s}\right)(100)} = +31.39\,\frac{1}{\frac{F\cdot m^2}{W}\cdot\frac{1}{s}}$$

$$\boxed{= +31.39\,\frac{W\cdot s}{F\cdot m^2}}$$

Power = Intensity x Area.

$$P(t) = I = \frac{1}{2}\epsilon_0 c \cdot 31.39\,\frac{W\cdot s}{F\cdot m^2}\times\pi(4\,\mu m)^2 = \frac{1}{2}\,8.85\times10^{-12}\frac{F}{m}\cdot 2.998\times10^{8}\frac{m}{s}\cdot 31.39\,\frac{W\cdot s}{F\cdot m^2}\,\pi\left(4\,\mu m\times\frac{1\,m}{10^6\,\mu m}\right)^2$$

$$\frac{F}{m}\cdot\frac{K}{s}\cdot J^2$$

$$= 2.09\times10^{-12}\,W = \boxed{2.09\,pW}$$

$$\boxed{P = 0.083\,W = 83\,mW}$$

$$\boxed{E = 1.05\,pJ}$$

10. Optical Block Equations: $\frac{dp}{dt} = (i\Delta - \frac{1}{T_2})p - \frac{\hbar}{4} i |K|^2 E w$; $\frac{dw}{dt} = -\frac{w - w^{eq}}{T_1} - \frac{4}{\hbar} \mathrm{Im}(E p^*)$

$\frac{I\!P}{\hbar} = \hat{A}$, and $|K|^2 = |\mu_{AB}|^2$

$$\boxed{\frac{dp}{dt} = -\frac{\hbar}{4} i |\mu_{AB}|^2 A w}$$

$$\boxed{\frac{dw}{dt} = \frac{-4i}{\hbar} \tilde{A} p}$$

Fourier Transform Derivation of the Paraxial Equation: $\frac{\partial \tilde{A}}{\partial z} + k_1 \frac{\partial A}{\partial t} + \frac{1}{2} i k_2 \frac{\partial^2 A}{\partial t^2} - i \Delta k_{NL} \tilde{A} = 0$

$$\boxed{\frac{\partial \tilde{A}}{\partial z} + k_1 \frac{\partial \tilde{A}}{\partial t} = i \Delta k_{NL} \tilde{A} = \frac{i \Delta n w_0}{c} \cdot \tilde{A}}$$

Amplitude: $\tilde{A}\left(\frac{\partial A}{\partial z}\right) + \frac{\mu A}{\hbar c_s} \frac{\partial \tilde{A}}{\partial t} = \frac{2\pi i \omega N}{c} p$ "First order linear partial Differential Equation"

Solution: $A(z,t) = \dfrac{bc\left(\frac{bt - az}{b}\right) + cz}{b}$

$= \dfrac{\frac{1}{c}\left[\frac{2\pi i \omega N}{c} p\right]\left(\frac{t/c - z}{1/c}\right) + \left[\frac{2\pi i \omega N}{c} p\right] z}{1/c}$

Momentum: $\frac{dp}{dt} = -\frac{i}{\hbar} |\mu|^2 \tilde{A} w$; $p = \frac{-i}{\hbar} |\mu|^2 A \int A w dt$

Energy: $\frac{dw}{dt} = \frac{-4i}{\hbar} \tilde{A} p$; $w = -\frac{4i}{\hbar} \int A p \, dt$

Assume a general solution for the "First order linear partial Differential Equation"

$A(z,t) = a \cdot \mathrm{sech}\left(\frac{t - zv}{\tau_0}\right)$

$\frac{\partial A}{\partial z} = a\left(\frac{-v}{\tau_0}\right) \mathrm{sech}\left(\frac{t - zv}{\tau_0}\right) \tanh\left(\frac{t - zv}{\tau_0}\right)$

$\frac{\partial A}{\partial t} = a\left(\frac{1}{\tau_0}\right) \mathrm{sech}\left(\frac{t - zv}{\tau_0}\right) \tanh\left(\frac{t - zv}{\tau_0}\right)$

$a \,\mathrm{sech}\left(\frac{t - zv}{\tau_0}\right) \tanh\left(\frac{t - zv}{\tau_0}\right)\left[\left(\frac{-v}{\tau_0}\right)\left(\frac{1}{\tau_0}\right)\right] = \frac{2\pi i \omega N}{c} p$

$p = \frac{c/v}{m \cdot v}$; $\lambda = \frac{c}{\nu}$; $p = \frac{2\pi N \mu^2 w \tau_0^2}{\hbar} \Big/ m \cdot v$

Leslie, Eberly (1975)

Optical Resonance and two-level atoms

$\dot{p} = \kappa \mathcal{E} w$; $\dot{w} = -\kappa \mathcal{E} p$

$p(t, z; 0) = -\sin\theta(t, z)$; $w(t, z; 0) = \cos\theta(t, z)$

$\theta(t, z) = \kappa \int_{-\infty}^{t} \mathcal{E}(t', z) dt'$

$p(t, z; \Delta) = p(t, z; 0) F(\Delta)$; $\ddot{\theta} = \frac{\Delta^2 F(\Delta)}{\sin F(\Delta)} \sin\theta$; $\ddot{\theta} - \frac{1}{\tau^2} \sin\theta = 0$

$w(t, z; \Delta) = F(\Delta) \cos\theta + F(\Delta) - 1$; $\theta(t, z) = 4 \tan^{-1}\left[\exp\left(\frac{t - t_0}{\tau}\right)\right]$; $\mathcal{E}(t, z) = \left(\frac{2}{\kappa \tau}\right) \mathrm{sech}\left(\frac{t - t_0}{\tau}\right)$

"Hyperbolic Secant pulse"

$F(\Delta) = \frac{1}{1 + (\Delta\tau)^2}$; $A = \frac{2\Delta\tau}{1 + (\Delta\tau)^2} \mathrm{sech}\left(\frac{t - t_0}{\tau}\right)$

$p = \frac{2}{1 + (\Delta\tau)^2} \mathrm{sech}\left(\frac{t - t_0}{\tau}\right) \tanh\left(\frac{t - t_0}{\tau}\right)$; $W = -1 + \frac{2}{1 + (\Delta\tau)^2} \mathrm{sech}^2\left(\frac{t - t_0}{\tau}\right)$

$\int_{-\infty}^{\infty} \frac{2\mu}{\hbar} A(z, t) dt$ = Power of the wave

11. Nonlinear Schrödinger Equation: $\frac{\partial A_s(z, \tau)}{\partial z} + \frac{1}{2} i k_2 \frac{\partial^2 A_s(z, \tau)}{\partial \tau^2} = i \gamma |\tilde{A}_s(z, \tau)|^2 A_s(z, \tau)$

Total Field: $A(z, \tau) = A_0(z) + A_1(z) e^{-i\delta t} + A_2(z) e^{-i\delta\tau}$

$$\frac{\partial A(z,\tau)}{dz} = \frac{\partial A_0(z)}{dz} + \left[\frac{\partial A_1(z)}{\partial z} + \frac{\partial A_2(z)}{\partial z}\right] e^{-i\delta t} - i\delta k_1 \left[A_1(z) + A_2(z)\right] e^{-i\delta t} A_2(z) \ldots$$

$$\frac{\partial A_S(z,\tau)}{d\tau} = \left[A_1(z) + A_2(z)\right] e^{-i\delta(\tau + k_1 z)} (i\delta) \; ; \; \frac{\partial^2 A(z,\tau)}{\partial^2 \tau} = -\delta^2 \left[A_1(z) + A_2(z)\right] e^{-i\delta(\tau + k_1 z)}$$

$$\frac{\partial A_0(z)}{dz} + \left[\frac{\partial A_1(z)}{\partial z} + \frac{\partial A_2(z)}{\partial z}\right] e^{-i\delta t} - i\delta k_1 \left[A_1(z) + A_2(z)\right] e^{-i\delta t} - \frac{1}{2} i k_2 \left[A_1(z) + A_2(z)\right] e^{-i\delta(\tau + k_1 z)} = i\gamma |A_S(z,\tau)|^2 A_S$$

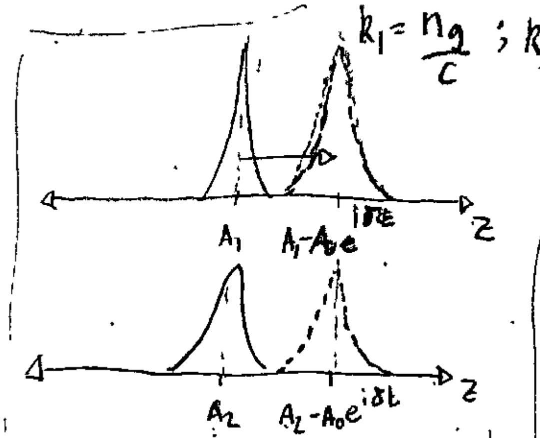
$$\frac{\partial A_1(z)}{dz} = e^{i\delta t}\left[-\frac{\partial A_0(z)}{\partial z} - \frac{\partial A_2(z)}{\partial z} e^{-i\delta t} + i\delta k_1 \left[A_1(z) + A_2(z)\right] e^{-i\delta t} + \underline{\frac{1}{2} k_2 \left[A_1 + A_2\right] e^{-i\delta(\tau + k_1 z)}} + i\gamma |A|^2 A_j\right]$$

$$= \frac{1}{2} k_2 \left[A_1 + A_2\right] - \frac{\partial A_2}{\partial z} + i\delta k_1 \underline{\left[A_1 + A_2\right]} - \frac{\partial A_0}{\partial z} e^{i\delta t} + i\gamma |A|^2 A \cdot e^{i\delta t}$$

$$\boxed{\frac{\partial A_1(z)}{dz} = \left(\frac{1}{2} k_2 + i\delta k_1\right)\left[A_1 + A_2\right] - \frac{\partial}{\partial z}\left[A_2 - A_0 e^{i\delta t}\right] + i\gamma |A|^2 A e^{i\delta t}}$$

$$\boxed{\frac{\partial A_2(z)}{\partial z} = \left(\frac{1}{2} k_2 + i\delta k_1\right)\left[A_1 + A_2\right] - \frac{\partial}{\partial z}\left[A_1 - A_0 e^{i\delta t}\right] + i\gamma |A|^2 A e^{i\delta t}}$$

$k_1 = \frac{n_g}{c}$; $k_2 = \frac{-1}{v_g^2}\left(\frac{dv_g}{dw}\right)$; The values of n_2 and k_2 require opposite signs.



- Exponential growth of amplitude occurs when the δt approaches $n\pi$, where $n = 0, 1, 2, 3 \ldots$
- The modulation of the nonlinear Schrödinger equation is oscillatory as frequency changes.

Group Velocity: $\frac{d}{d\lambda}\left(\frac{1}{v_g}\right) = k_1$; Phase Velocity : $\boxed{v_p = \frac{\lambda}{T} = \frac{w}{k}}$

$\boxed{v_g = \frac{\partial w}{\partial k}}$

Group velocity is a portion of the equation because of the change of the frequencies of the monochromatic lights.

Chapter 8: Spontaneous Light Scattering and Acoustooptics:

1. Lorentz model of the Atom:

$$\alpha(w) = \frac{e^2/m\epsilon_0}{w_0^2 - w^2 - 2iw\gamma}$$

Estimate σ_{N_2} for visible light of 500 nm, $\epsilon_{N_2} = 1.0005480$, $\epsilon_{Air} = 1.00021$

$$\alpha = 4\pi(1.0005430)\frac{1.00021 - 1.0005430}{1.00021 + 2(1.0005430)} \; 1.097469\,\text{Å}$$

$$= -1.87 \times 10^{-3}\,\text{Å}^3$$

$$\sigma = \frac{8\pi}{3}\frac{\left(\frac{2\pi c}{\lambda}\right)^4}{c^4} a^6 \epsilon^2 \left(\frac{\epsilon_1-\epsilon}{\epsilon_1+2\epsilon}\right)^2 = 4.64\times10^{-23}\,Å^2 = \boxed{0.46\,m^2}$$ $\boxed{\text{Handbook of Chemistry and Physics}}$

Otherwise, $R = \frac{N}{16\pi^2}\left(\frac{\omega^4}{c^4}\right)|\alpha(\omega)|^2 \sin^2\phi \Rightarrow |\alpha(\omega)|^2 = \frac{16\pi^2}{N}\left(\frac{c^4}{\omega^4}\right)\frac{1}{\sin^2\phi} = \cdots$

$\nu = \frac{c}{\lambda}$

$\therefore$ Attenuating Distance: "Penetration Depth" • Beer-Lambert's: $I(z) = I_0 e^{-\alpha z}$

• Penetration Depth: $\boxed{\delta_R = \frac{1}{\alpha} = 33\,Km}$

2. $I_s = I_0 \frac{\omega\, \omega^4 V}{16\pi^2 L^2 c^4}\gamma_e^2 C_T kT\sin^2\phi$

Where $R = \frac{\omega^4}{16\pi^2 c^4}\gamma_e^2 C_T kT\sin^2\phi$; $I_s = \frac{I_0 RV}{L^2}$; H_2O Values: γ_e Symbol Quantity

Electrostrictive constant: $\gamma_e = \rho\left(\frac{\partial\epsilon}{\partial\rho}\right)$

Assuming book values, $C_T = 4.5\times10^{-11}\,cm^2/dyne$ and

$$\gamma_e = \rho\frac{\partial\epsilon}{\partial\rho} \cong \frac{\epsilon_1-\epsilon}{\epsilon_1+2\epsilon} = \frac{n^2-1}{n^2+2\cdot1}\left(\frac{1}{3}\right)$$

Isothermal Compressibility $C_T = 50\times10^{-6}\,atm^{-1} = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)$

ACS Handbook of Chemistry and Physics.

$T = 300K$, $n = 1.33$.

$$R = \frac{\left(\frac{2\pi c}{\lambda}\right)^4}{16\pi^2 c^4}\left(\frac{1}{3}\right)\left[\frac{n^2-1}{n^2+2}\right]^2 C_T\, k\, T\cdot\sin^2 90^\circ$$

$$= \frac{\pi^2}{\lambda^4}\left(\frac{1}{3}\right)\left[\frac{(1.33)^2-1}{(1.33)^2+2}\right]^2 4.5\times10^{-11}\,cm^2/dyne\,(1.38\times10^{-23}\,J/K)\,300K\cdot1^2$$

$$= 4.08\times10^{-7}\,\frac{1}{m^4}\cdot\frac{cm^2}{dyne}\cdot J = 4.08\times10^{-11}\,\frac{J}{m^2\,dyne}\times\frac{dyne}{0.00002\,J/m} = 2.04\times10^{-7}\,\frac{1}{cm}$$

$R \rightarrow R_{90}^u$; $2\pi\nu = \omega = 2\pi c$; $R_{90}^u = 2.04\times10^{-7}\sin^2\phi\ 1/cm = 2.04\times10^{-7}\left(\frac{1}{2}\right)1/cm = \boxed{1.02\times10^{-7}\,cm^{-1}}$

$$\alpha = \left(\frac{16\pi}{3}\right)R_{90}^u = \frac{16\pi}{3}(1.02\times10^{-7}\,cm^{-1}) = 1.71\times10^{-6}\,cm^{-1} = \boxed{(585\,m)^{-1}}$$

3. Verify Eqn 8.2.12: $\alpha = 4\pi\epsilon\frac{\epsilon_1-\epsilon}{\epsilon_1+2\epsilon}a^3$

$\boxed{\text{Stratton 1941 pg 206 "Electromagnetic Theory"}}$

Jackson 1962 pg 158 "Classical Electrodynamics"

Sphere in a Parallel Field: $\phi_0 = -E_0 z = -E_0 r\cos\theta = -E_0 r P_1(\cos\theta)$; $\boxed{\text{Primary Potential}}$

$\phi_1^+ = \sum_{n=0}^{\infty} b_n \frac{P_n\cos\theta}{r^{n+1}}$ $\boxed{\text{Potential outside sphere}}$

Dipole Moment $p = 4\pi\epsilon_2 E_0 r_1^3$

$a_0 = b_0 = 0$; $a_1 = \frac{-3\epsilon_2}{\epsilon_1+2\epsilon_2}E_0$, $b_1 = \frac{\epsilon_1-\epsilon_2}{\epsilon_1+2\epsilon_2}r_1^3 E_0$

$a_n = b_n = 0$; $n > 1$

$\phi_2 = -E_0 r_1 P_1(\cos\theta) + \sum_{n=0}^{\infty} b_n\frac{P_n(\cos\theta)}{r_1^{n+1}}$

Whence $b_0 = r_1\phi_2$; $b_1 = r_1^3 E_0$; $b_n = 0$

Resultant $\phi^+ = -E_0 r\cos\theta + \frac{\epsilon_1-\epsilon_2}{\epsilon_1+2\epsilon_2}r_1^3 E_0\frac{\cos\theta}{r^2}$

$\phi^+ = -E_0 r\cos\theta + E_0 r_1^3\frac{\cos\theta}{r^2} + \phi_2\frac{r_1}{r}$

$$\phi^- = -\frac{3\epsilon_2}{\epsilon_1+2\epsilon_2}E_0 r\cos\theta$$

Charge density: $\omega = 3\epsilon_2 E_0\cos\theta + \frac{\epsilon_2\phi_2}{r_1}$; $q_1 = 4\pi r_1\epsilon_2\phi_2$

Sphere is parallel and uniform: $E = -\frac{\partial\phi}{\partial z} = \frac{3\epsilon_2}{\epsilon_1+2\epsilon_2}E_0$

$E^{\wedge} = -\frac{\partial d}{\partial z} = \frac{3\epsilon_2}{\epsilon_1 + 2\epsilon_2} E_0 > E_0$; Dipole oriented along z-axis $\boxed{p = 4\pi\epsilon_2 \frac{\epsilon_1 - \epsilon_2}{\epsilon_1 + 2\epsilon_2} r^3 E_0}$

4. Acoustic Absorption coefficient [α_s] in H_2O @ $\nu = 10^3, 10^6, 10^9$ Hz

$= \frac{q^2 \Gamma'}{\nu} = \frac{\Gamma}{\nu}$; Γ = phonon decay rate ; Γ' = Dampening Parameter ; ν = velocity ; q = Propagation coefficient

$\Gamma' = \frac{1}{\rho}\left[\frac{4}{3}\eta_s + \eta_b + \frac{K}{C_p}(\gamma - 1)\right]$

"Density" "Shear Viscosity Coefficient" "Bulk Viscosity Coefficient" "Thermal Conductivity" "Adiabatic Index"

Sound velocity [H_2O] = 1.50×10^3 m/s

Shear Viscosity Coefficient [η_s] = 0.01 dyne sec/cm^2

Stokes Relation [η_b] = $-\left(\frac{2}{3}\right)\eta_s$

$\Gamma' = \frac{1}{1\,kg/m^3}\left[\frac{4}{3}(0.01\,dynes\,sec/cm^2) + \left[\frac{2}{3}(0.01\,dyne\,sec/cm^2) + \left(\frac{-2}{3}\right)(0.01\,dyne\,sec/cm^2)\right]\right] + \frac{0.6062\frac{W}{mK}}{4.18\,J/K\cdot m}(0.00654)$

$= 1.33 \times 10^{-5}\,\frac{dynes\,sec\cdot m}{kg} \times \frac{0.0002\,J/m}{1\,dyne} + 9.56 \times 10^{-4}\,\frac{m^3}{kg}\,s\cdot m = 2.66 \times 10^{-2}\,\frac{m^2}{s} + 9.56 \times 10^{-4}\,\frac{m^2}{s}$

$= 2.756 \times 10^{-2}\,m^2/s$

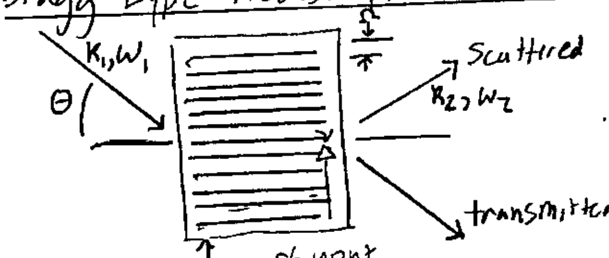
$q = Real(q) = \frac{\Omega}{\nu} = \frac{\Omega}{1.50 \times 10^3\,m/s}$

$\boxed{10^3 Hz: \quad \alpha_s = \frac{q^2\Gamma'}{\nu} = \frac{10^3 Hz^2 \cdot 2.756 \times 10^{-2}\,m^2/s}{(1.50 \times 10^3\,m/s)^3} = 8.16 \times 10^{-3}\,\frac{1}{m}}$; 10^6: $\alpha_s = 8.16 \times 10^{20}\,\frac{1}{m}$; 10^9: $\alpha_s = 8.16 \times 10^{60}\,\frac{1}{m}$

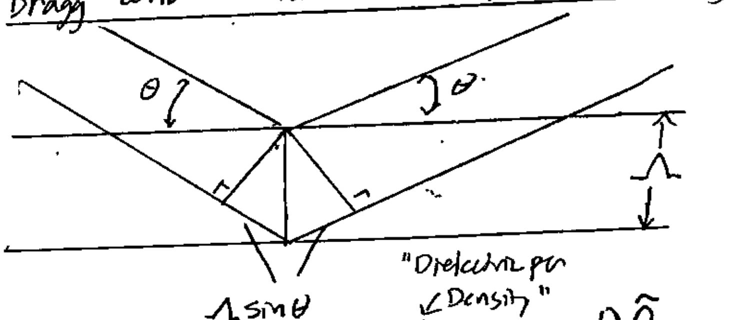
5. Verify 8.4.8: Bragg Scattering by Sound Waves:

$\lambda = 2\Lambda\sin\theta$; $\Lambda = 2\pi\nu/\Omega$; ν = velocity of sound ; Ω = Acoustic Wave Frequency

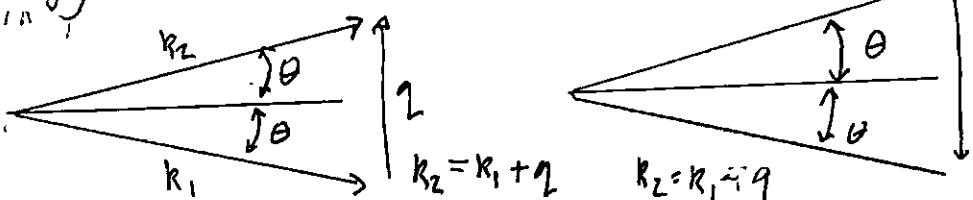
Bragg-type Acoustooptic modulator



Bragg condition for acoustooptic scattering



Bragg condition as a phase-matching relation



$k_2 = k_1 + q$ $\quad$ $k_2 = k_1 - q$

$w_2 = w_1 + \Omega$ $\quad$ $w_2 = w_1 - \Omega$

$\Delta\tilde{\epsilon} = \frac{\partial\epsilon}{\partial\rho}\Delta\tilde{\rho} = \gamma_e \cdot \frac{\Delta\tilde{\rho}}{\rho_0}$

"Dielectric per Density" ; Dielectric constant ; "Electrostriction constant"

$[\Delta(\epsilon^{-1})]_{ij} = \sum p_{ijkl}\, S_{kl}$

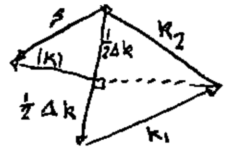
"Strainoptic tensor" "Strain tensor"

$S_{kl} = \frac{1}{2}\left[\frac{\partial d_k}{\partial x_l} + \frac{\partial d_l}{\partial x_k}\right]$

$(\Delta\epsilon)_{il} = -\sum_{jk} p_{ij} \sum p_{jkil}\, \frac{1}{2}\left[\frac{\partial d_i}{\partial x_l} + \frac{\partial d_l}{\partial x_i}\right] \sum p_{klij}$

$\boxed{= +\sum p_{ilkj}\,\frac{1}{2}\left[\frac{\partial d_i}{\partial x_l} + \frac{\partial d_l}{\partial x_i}\right]}$

6. Verify Eqns (8.4.31a) to (9.4.35b)



$\frac{\partial A_1}{\partial x} = i K A_2 e^{-i\Delta k x}$; $\frac{\partial A_2}{\partial x} = i K^* A_1 e^{i\Delta k x}$; Assuming A_1 is a constant

$K = \frac{\omega^2 \Delta \epsilon^*}{2 k_x c^2}$

$A_1 = \int i K A_2 [\cos \Delta k x + i \sin \Delta k x] dx$

$= i K A_2 \frac{\sin \Delta k x + i \cos \Delta k x}{\Delta k} = \frac{-K A_2 [\cos \Delta k x + i \sin \Delta k x]}{\Delta k}$

$= \frac{-K A_2 e^{i\Delta k x}}{\Delta k} = \frac{-K}{\Delta k} A_2 e^{i\frac{1}{2}\Delta k x - i\frac{1}{2}\Delta k x} \cdot e$

$= \frac{-K}{\Delta k} A_2 e^{-\frac{1}{2}\Delta k x} \left[\cos\left(\frac{1}{2}\Delta k x\right) + i \sin\left(\frac{1}{2}\Delta k x\right)\right]$

$$\eta(\Delta k) = \frac{|A_2(L)|^2}{|A_1(0)|^2} = \frac{|K|^2}{|K|^2 + (\frac{1}{2}\Delta k)^2} \sin^2\left\{\left[|K|^2 + (\tfrac{1}{2}\Delta k)^2\right]^{1/2} L\right\}$$

$$= \eta(0) + \Delta k \left.\frac{d\eta}{d(\Delta k)}\right|_{\Delta k = 0} + \frac{1}{2}(\Delta k)^2 \left.\frac{d^2\eta}{d(\Delta k)^2}\right|_{k=0} + \cdots$$

$$= \eta(0) \left[1 - \frac{(\Delta k)^2}{4|K|^2}\left(1 - \frac{|K| K \cos(|K|L)}{\sin(|K|L)}\right)\right]$$

17. θ_1 remains fixed while acoustic frequency Ω is varied by θ_2.

Derive maximum deflection angle, to 50% efficiency:

where $|K|L = \frac{\pi}{2}$, $L = 1.1\,cm$, $\Lambda = 30\,\mu m$

$$|K| = \frac{\omega \gamma_e}{2nc\cos\theta}\left(\frac{I}{2Kv}\right)^{1/2} = \frac{\omega \gamma_e}{2nc\cos\theta}\left(\frac{2Kv|\Delta\epsilon|^2/\gamma_e^2}{2Kv}\right)^{1/2} = \frac{\omega\sqrt{\gamma_e}\,|\Delta\epsilon|}{2nc\cos\theta}$$

$\frac{S^2}{\frac{1}{2}\Delta k^2} = \Delta k = S$

$I = K v \frac{\langle \Delta\tilde{\rho}^2\rangle}{\rho_0^2} = 2Kv\left|\frac{\Delta\rho}{\rho_0}\right|^2$; where $K = 1/C$ is the bulk modulus.

v: sound velocity; $\frac{\Delta\rho}{\rho_0}$: Density Disturbance.

$\frac{2S^2}{\Delta k} = \frac{2|K|^2}{\Delta k} + 1$

$\frac{2S}{\Delta k} = \frac{2K}{\Delta k}$

$$\Delta K = \frac{1}{2}\Delta\theta q \;;\quad 1 = \frac{(\Delta k)^2}{4|K|^2}\left(1 - \frac{|K|K\cos(|K|L)}{\sin(|K|L)}\right) = \frac{1}{4}\frac{\Delta\theta^2 q^2}{4|K|^2}\left(1 - \frac{|K|K\cos(|K|L)}{\sin(|K|L)}\right)$$

$$\boxed{\frac{2\sqrt{2}|K|}{q}\left(1 - \frac{|K|K\cos(|K|L)}{\sin(|K|L)}\right)^{-1/2} = \Delta\theta}$$

$1 = \frac{2|K|\Delta k}{S\,\Delta k} + \frac{\Delta K}{2S}$

$$\text{For } |K|L = \pi/2 \;;\; \frac{2\sqrt{2}|K|}{q}\left(1 - \frac{|K|K\cos(\pi/2)}{\sin(\pi/L)}\right)^{-1/2} = \frac{2\sqrt{2}\cdot\frac{\pi}{2L}}{q} = \boxed{\frac{\sqrt{2}\pi}{q \cdot L}}$$

Chapter 9: Stimulated Brillouin and Stimulated Rayleigh Scattering:

2. Generalize Section 9.3: A) Brillouin Scattering is described by a Brillouin Frequency $[\Omega_B = |q_B| v]$ which encompasses

B) The traveling acoustic wave power: amplitude, lifetime, and linewidth, with the SBS gain factor [g], with a line-center gain.

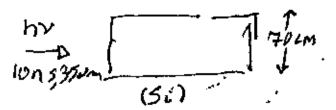
C) Intensity of the Stokes wave grows exponentially, so the pump becomes depleted.

D) SBS generators are initiated by Stokes photons to induce Brillouin scattering.

E) With lifetime of phonons, SBS generators have been produced in liquids and relationship to threshold values.

$\Omega_B = |q|v = q \sin\theta \cdot v$; $T_p = T_B^{-1} = (q^2 T')^{-1} = (q \sin\theta\, T')^{-1}$; $g_0 = \dfrac{\gamma_e^2 \omega^2}{n v c^3 \rho_0 q \sin\theta\, T'}$

3.



Substance	$\Omega_B/2\pi$ (MHz)	$T_B/2\pi$ (MHz)	g_0 (m/GW)	$g_B^a(\max)/\alpha$ (cm²/MW)
SiO_2	25,800	78	0.045	

"Full width @ - half max"

$$g = g_0 \frac{(T_B/2)^2}{(\Omega_B - \Omega)^2 + (T_B/2)^2} \; ; \; I_2(z) = I_2(0)\, e^{g I_2 z}$$

$\dfrac{hc/\lambda}{\pi (335\,\text{nm} \frac{1\text{m}}{10^9\text{nm}})^2}$ = $\boxed{1.476 \times 10^{-18} \frac{J}{m^2}}$

$-2.3\times10^{-6} \frac{m}{GW} \cdot 1.47\times10^{-18} \frac{J}{m^2} \times \frac{1\,GW}{10^9 W} \times \frac{W \cdot s}{1 J} \cdot 1\,m$

$$= 0.045 \frac{m}{GW} \cdot \frac{(78\,\text{MHz}/2)^2}{\left(25{,}800\,\text{MHz} - \frac{2\pi \cdot 3.0\times10^8 \frac{m}{s}}{335\,\text{nm} \cdot \frac{1\text{m}}{10^9\text{nm}}} \times \frac{1\,\text{MHz}}{10^6\,\text{Hz}}\right)^2 + (78\,\text{MHz}/2)^2} = 2.3\times10^{-6} \frac{m}{GW}$$

4. $I_{\text{threshold}} = 3\,GW/cm^2 \xrightarrow{50\%} 1.5\,GW/cm^2$ What is the minimum length of Silica that can be used to excite SBS?

$I_2(z) = I_2(0)\, e^{-g I_2 z}$; $3\,GW/cm^2 = 1.5\,GW/cm^2 \cdot e^{g\, 1.5 GW/cm^2 \cdot z}$

$0.693 = g \cdot 1.5\,GW/cm^2 \cdot z$

$$g = 0.2 \frac{m}{GW} \times \frac{(224\,\text{MHz}/2)^2}{\left(4600\,\text{MHz} - \frac{2\pi\, 3.0\times10^8 \frac{m}{s}}{335\,\text{nm} \frac{1\text{m}}{10^9\text{nm}}} \times \frac{1\,\text{MHz}}{10^6\,\text{Hz}}\right)^2 + (224\,\text{MHz}/2)^2}$$

$z = 8.65\times10^{-17} \frac{m}{GW} \times 1.5 \frac{GW}{cm^2} \times \left(\frac{100\,cm}{1\,m}\right)^2$ $\doteq 8.65\times10^{-17} \frac{m}{GW}$

$\boxed{= 1.29\times10^{-12}\, m}$

20 nsec ; $1.5 \frac{GJ}{s\, cm^2} \times 20\,\text{nsec} \times \frac{1\,s}{10^9\,\text{nsec}} = \boxed{3.00\times10^{-8} \frac{J}{cm^2}}$

5. Stokes radiation Duration is shorter than the excitation radiation.

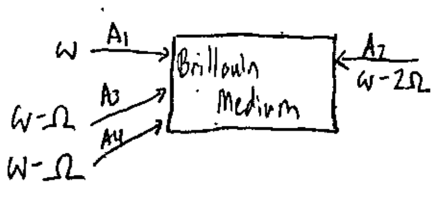
The radiation of Stokes ($\vec{k}_2$) in the backward direction, with frequency ($\omega_2 = \omega_1 - \Omega_B$), because of the Heisenberg Uncertainty Principle ($\Delta E \Delta t \leq \hbar/2$). With a theoretically larger frequency, Stokes radiation has a smaller energy than the excitation pulse, the relaxation time ($\tau = T^{-1}$) would be shorter. The physical length of the interaction region is related is related to the duration of the pulse by the decaying signal population per distance i.e. ($\tau/E = c \cdot T$). The coupled equations involve density, frequency ---

$$\frac{dA_1}{dz} = \frac{i \epsilon_0 \omega q^2 \gamma_e^2}{2 n c \rho_0} \frac{|A_2|^2 A_1}{\Omega_B^2 - \Omega^2 - i\Omega T_B}$$

$$\frac{dA_2}{dz} = \frac{-i \epsilon_0 \omega q^2 \gamma_e^2}{2 n c \rho_0} \frac{|A_1|^2 A_2}{\Omega_B^2 - \Omega^2 + i\Omega T_B}$$

The minimum value of the output pulse is determined by the frequency of input as related by Heisenberg Uncertainty

6. Brillouin-enhanced four-wave Mixing (BEFWM):



$E(z,t) = E_1(z,t) + E_2(z,t) + E_3(z,t)$

$E_1(z,t) = A_1(z,t)\, e^{i(k_1 z - \omega_1 t)} + c.c$

$E_2(z,t) = A_2(z,t)\, e^{i(-k_1 z - (\omega - 2\Omega) t)}$

$E_3(z,t) = A_3(z,t)\, e^{i(k_3 z - (\omega - \Omega) t)}$

Acoustic field in terms of Density: $\tilde{\rho}(z,t) = \rho_0 + [\rho(z,t)e^{i(qz-\Omega t)} + c.c]$ where $q = 2k$ ∴ (2)

Acoustic Wave equation: $\frac{\partial^2 \rho}{\partial t^2} - \Gamma' \nabla^2 \frac{\partial \rho}{\partial t} - v^2 \nabla^2 \rho = \nabla \cdot f$ Force per unit volume. [Divergence] ∴ (3)

Dampening Parameter. velocity of sound

Relationship to Pressure: $f = \nabla p_{st}$, $p_{st} = -\frac{1}{2}\epsilon_0 \gamma_e \langle E^2 \rangle$

$$\nabla \cdot f = \epsilon_0 \gamma_e q^2 [A_1 A_2^* A_3 e^{i(qz-\Omega t)} + c.c] \quad (4)$$

(1) & (2) into (3)

$$\frac{\partial}{\partial t}\left[\frac{\partial \rho(z,t)}{\partial t} e^{i(qz-\Omega t)} - i\Omega \rho(z,t) e^{i(qz-\Omega t)}\right] - \Gamma' \nabla\left[\frac{\partial \rho(z,t)}{\partial t} e^{i(qz-\Omega t)} - i\Omega \rho(z,t) e^{i(qz-\Omega t)}\right]$$

$$- v^2 \nabla^2 [\rho_0 + [\rho(z,t) e^{i(qz-\Omega t)}]] = \epsilon_0 \gamma_e q^2 [A_1 A_2^* A_3 e^{i(qz-\Omega t)}]$$

$$\left[\frac{\partial^2 \rho(z,t)}{\partial^2 t} e^{i(qz-\Omega t)} + i\Omega \frac{\partial \rho(z,t)}{\partial t} \rho(z,t) e^{i(qz-\Omega t)} - i\Omega\left[\frac{\partial \rho(z,t)}{\partial t} e^{i(qz-\Omega t)} - i\Omega \rho(z,t) e^{i(qz-\Omega t)}\right]\right]$$

$$- \Gamma' \nabla\left[\frac{\partial^2 \rho(z,t)}{\partial z \partial t} e^{i(qz-\Omega t)} + iq \frac{\partial \rho(z,t)}{\partial t} e^{i(qz-\Omega t)} - i\Omega\left[\frac{\partial \rho(z,t)}{\partial z} + iq \rho(z,t) e^{i(qz-\Omega t)}\right]\right]$$

$$- v^2 \nabla\left[\frac{\partial \rho(z,t)}{\partial z} e^{i(qz-\Omega t)} + iq \rho(z,t) e^{i(qz-\Omega t)}\right] = \epsilon_0 \gamma_e q^2 [A_1 A_2^* A_3 e^{i(qz-\Omega t)}]$$

$$\frac{\partial^2 \rho(z,t)}{\partial^2 t} e^{i(qz-\Omega t)} - i\Omega \frac{\partial \rho(z,t)}{\partial t} \rho(z,t) e^{i(qz-\Omega t)} - i\Omega \frac{\partial \rho(z,t)}{\partial t} e^{i(qz-\Omega t)} + i\Omega \rho(z,t) e^{i(qz-\Omega t)} - \Gamma'\left[\frac{\partial^3 \rho(z,t)}{\partial^2 z \partial t} e^{i(qz-\Omega t)}\right.$$

$$+ iq \frac{\partial^2 \rho(z,t)}{\partial z \partial t} e^{i(qz-\Omega t)} + iq \frac{\partial \rho(z,t)}{\partial t} e^{i(qz-\Omega t)} + q^2 \rho(z,t) e^{i(qz-\Omega t)} - i\Omega \frac{\partial^2 \rho(z,t)}{\partial^2 z} e^{i(qz-\Omega t)} + i\Omega q \frac{\partial \rho(z,t)}{\partial z} e^{i(qz-\Omega t)}$$

$$\left. + q \frac{\partial \rho(z,t)}{\partial z} e^{i(qz-\Omega t)} + iq \rho(z,t) e^{i(qz-\Omega t)}\right] - v^2\left[\frac{\partial^2 \rho(z,t)}{\partial^2 z} e^{i(qz-\Omega t)} + iq \frac{\partial \rho(z,t)}{\partial z} e^{i(qz-\Omega t)} + iq \frac{\partial \rho(z,t)}{\partial z} e^{i(qz-\Omega t)}\right.$$

$$\left. + iq^2 \rho(z,t) e^{i(qz-\Omega t)}\right] = \epsilon_0 \gamma_e q^2 A_1 A_2^* A_3 e^{i(qz-\Omega t)}$$

$$\frac{\partial^2 \rho(z,t)}{\partial^2 t} - i\Omega \frac{\partial \rho(z,t)}{\partial t} \rho(z,t) - i\Omega \frac{\partial \rho(z,t)}{\partial t} + i\Omega \rho(z,t) - \Gamma'\left[\frac{\partial^2 \rho(z,t)}{\partial^2 z \partial t} + iq \frac{\partial^2 \rho(z,t)}{\partial z \partial t} + iq \frac{\partial \rho(z,t)}{\partial t} - q^2 \rho(z,t)\right.$$

$$\left. - i\Omega \frac{\partial^2 \rho(z,t)}{\partial^2 z} + i\Omega q \frac{\partial \rho(z,t)}{\partial z} + q \frac{\partial \rho(z,t)}{\partial z} + iq \rho(z,t)\right] - v^2\left[\frac{\partial^2 \rho(z,t)}{\partial^2 z} + iq \frac{\partial \rho(z,t)}{\partial z} + iq \frac{\partial \rho(z,t)}{\partial z}\right.$$

$$\left. - iq^2 \rho(z,t)\right] = \epsilon_0 \gamma_e q^2 A_1 A_2^* A_3 e^{i(qz-\Omega t)}$$

$$-\frac{\partial^2 \rho(z,t)}{\partial^2 t} - (i\Omega[\rho(z,t) + 1] + iq\Gamma') \frac{\partial \rho(z,t)}{\partial t} + i(\Omega + \Gamma'(q^2 + q))\rho(z,t)$$

$$+ \Gamma'\left(\frac{\partial^2 \rho(z,t)}{\partial^2 z \partial t} + iq \frac{\partial^2 \rho(z,t)}{\partial z \partial t}\right) + \ldots - i(\Gamma' i\Omega q - v^2 q) \frac{\partial \rho(z,t)}{\partial z} - i(\Gamma' q + q) \frac{\partial \rho(z,t)}{\partial z}$$

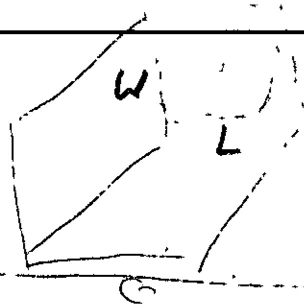
$$= \epsilon_0 \gamma_e q^2 A_1 A_2^* A_3 e^{i(qz-\Omega t)}$$

$$-i(\Omega[\rho(z,t) + 1] + q\Gamma') \frac{\partial \rho(z,t)}{\partial t} + i(\Omega + \Gamma'(q^2 + q))\rho(z,t) - i(\Gamma'(\Omega + 1)q - 2v^2) \frac{\partial \rho(z,t)}{\partial z} = \epsilon_0 \gamma_e q^2 A_1 A_2^* A_3 e^{i(qz-\Omega t)}$$

$$P = \frac{m \cdot a}{\pi r^2}\left(h^2 - \sqrt{h^2+d^2}\right) \qquad \rho = \frac{m}{V} \qquad \rho V = m$$

$$= \frac{\rho \cdot V \cdot a}{\pi r^2}\left(h^2 - \sqrt{h^2+d^2}\right)$$

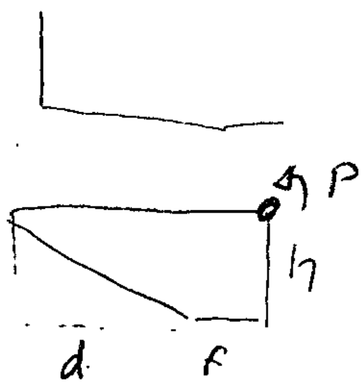
$$= \frac{\rho \cdot L \cdot w \cdot t a}{L}$$



$$= \frac{a\rho \cdot \cancel{L \cdot w}\left(h + d + \sqrt{h^2+d^2+\cancel{f}}\right)}{\cancel{L \cdot w}}\left(\sqrt{h^2}\right.$$

$$= \rho a\left(h + d + \sqrt{h^2 + d^2} + f\right) \cdot \left(h^2 - \sqrt{h^2+d^2}\right)$$

$$P = \rho\left(h + d + \sqrt{h^2+d^2} + f\right)\left(h^2 - \sqrt{h^2+d^2}\right) a$$



$@\ \rho = 1.29\,kg/m^3,\ h = 25ft,\ d = 50ft,\ f = 10ft$

$$\cancel{= 1.332 \times 10^3 \frac{kg}{m^2}} \qquad @\ V = 1m^2$$

$W = P\Delta V$

$\cancel{= 1.332 \times 10^3 kg}$

$$= 13{,}544.34 \frac{kg}{m^3} \cdot \frac{m}{s^2} \qquad @\ \Delta V = 1m^3$$

$V =$ $\quad w = \frac{V}{F}$

$$\boxed{= 13{,}544.34 \frac{kg}{m^3} \cdot \frac{1}{s} \cdot \frac{m}{s}}$$

$0.93(293 + 75)$

$W = C_v(T_2 - T_1) \qquad 13{,}544.34 \frac{kg}{m^3} = 0.93(T_2 - T_1)$ $\quad$ 15 KJ

By dropping the Spatial Derivative: $-i(\Omega[\rho(z,t)+1]+qT')\frac{\partial \rho(z,t)}{\partial t}+i(\Omega+T'(q^2+q))\rho(z,t)$

$= \epsilon_0 \gamma_e q^2 A_1 A_2^* A_3 i$

$I(t)=\exp\left[\int i(\Omega+T'(q^2+q))dt\right]$

$=\exp[i(\Omega+T'(q^2+q))t]$; $y=\frac{\int \epsilon_0\gamma_e q^2 A_1 A_2^* A_3 \exp[i(\Omega+T'(q^2+q))t]dt}{\exp[i(\Omega+T'(q^2+q))t]}$

"Bernoulli Equation" $y'+P(x)y=Q(x)y^n$; $I(x)=e^{\int[1-n]P(x)dx}$; $y^{1-n}=\frac{1}{I(x)}\int[1-n]Q(x)I(x)dx$

$$\rho(z,t)=\frac{\epsilon_0\gamma_e q^2 A_1A_2^*A_3}{i(\Omega+T'(q^2+q))}=\frac{-i\epsilon_0\gamma_e q^2 A_1A_2^*A_3}{(\Omega+T'(q^2+q))}$$

Optical Wave Equation: $\frac{\partial^2 E_i}{\partial z^2}-\frac{1}{(c/n)^2}\frac{\partial^2 E_i}{\partial t^2}=\frac{1}{\epsilon_0 c^2}\frac{\partial^2 P_i}{\partial t^2}$; $i=1,2.$

Total Nonlinear Polarization: $\tilde{P}=\epsilon_0\Delta\chi E=\epsilon_0\Delta\epsilon\tilde{E}=\epsilon_0\rho^{-1}\gamma_e\tilde{\rho}E$

To determine phase-matched sources: $\tilde{P}_1=p_1e^{i(k_1z-\omega t)}$; $\tilde{P}_2=p_2e^{i(k_2z-(\omega-2\Omega)t)}$ (or ωt)

$\tilde{P}_3=p_3e^{i(k_3z-(\omega-\Omega)t)}$

where $p_1=\epsilon_0\gamma_e\rho_0^{-1}\rho A_2A_3$; $p_2=\epsilon_0\gamma_e\rho_0^{-1}\rho^* A_1A_3$

$p_3=\epsilon_0\gamma_e\rho_0^{-1}\rho A_1A_3$

Slow varying Amplitude Approximation:

$$\frac{\partial A_1}{\partial z}+\frac{1}{c/n}\frac{\partial A_1}{\partial t}=\frac{i\omega\gamma_e}{2nc\rho_0}\rho A_2A_3$$

$$-\frac{\partial A_2}{\partial z}+\frac{1}{c/n}\frac{\partial A_2}{\partial t}=\frac{i\omega\gamma_e}{2nc\rho_0}\rho^* A_1A_3$$

$$+\frac{\partial A_3}{\partial z}+\frac{1}{c/n}\frac{\partial A_3}{\partial z}=\frac{i\omega\gamma_e}{2nc\rho_0}\rho A_3A_1$$

The coupled amplitude equations become: $\frac{\partial A_1}{\partial z}=\frac{\epsilon_0\gamma_e^2 q^2\omega A_1|A_2|A_3^2}{2nc\rho_0(\Omega+T'(q^2+q))}$

$$\frac{\partial A_2}{\partial z}=\frac{\epsilon_0\gamma_e^2 q^2\omega|A_1|^2A_2|A_3|^2}{2nc\rho_0(\Omega+T'(q^2+q))}$$

$$\frac{\partial A_3}{\partial z}=\frac{\epsilon_0\gamma_e^2 q^2\omega|A_1|^2|A_2||A_3|^2}{2nc\rho_0(\Omega+T'(q^2+q))}$$

With constant pump approximation: $\frac{\partial A_1}{\partial z}=0$; $\rightarrow$ solve $\frac{\partial A_2}{\partial z}$, $\frac{\partial A_3}{\partial z}$ ... I didn't

Skeldon, Boyd (1987) describes the function as

$\frac{\partial E_1^*}{\partial z}=-E_4^*(Q_{14}+Q_{23}e^{i\Delta kz})$ $\quad Q_{14}=g\frac{nc}{8\pi}E_1^*E_4$ $\quad Q_{23}=g\frac{nc}{8\pi}E_2E_3^*$

$\frac{\partial E_3}{\partial z}=-E_3(Q_{23}+Q_{14}e^{-i\Delta kz})$

$\frac{\partial E_3^*}{\partial z}=E_2^*(Q_{23}+Q_{14}e^{-i\Delta kz})$ $\quad \frac{\partial E_4}{\partial z}=-E_1(Q_{14}+Q_{23}e^{i\Delta kz})$

Chapter 10: 2.) Polarization Properties of Stimulated Rayleigh Wing Scattering.
Paragraph following (10.6.16) verify table (10.6.2)

Polarization Properties of Stimulated Rayleigh-Wing Scattering:

Nonlinear contribution to susceptibility $\tau \frac{d}{dt}\Delta\chi_{ik} + \Delta\chi_{ik} = C\left(\langle \tilde{E}_i \tilde{E}_k^* \rangle - \frac{1}{3}\delta_{ik}\langle \tilde{E}\cdot\tilde{E}^* \rangle\right)$

$\frac{\sqrt{2}}{\sqrt{2}} - \frac{1}{\sqrt{2}} = \frac{\sqrt{2}-1}{\sqrt{2}}$

where $C = \frac{N\epsilon_0^2(\alpha_{\parallel}-\alpha_{\perp})^2}{15kT}$

Laser Polarization: ↕ ↕ ○ ○

Stokes Polarization: ↕ ⟷ ○ ○

Gain Factor $(1-0)^2 = 1$ $\left(\frac{\sqrt{3}+\sqrt{2}}{2\sqrt{2}}\right)^2 \left(\frac{\sqrt{4}-\sqrt{1}}{\sqrt{2}}\right)^2 \left(\frac{1}{\sqrt{3}+\sqrt{2}}\right)^2$

$= \frac{3}{2}$

$\frac{\sqrt{3}+\sqrt{2}}{2\sqrt{2}}$

Description of circularly polarized light:

$\vec{E} = \underbrace{E_+\hat{\sigma}_+}_{\text{"left"}} + \underbrace{E_-\hat{\sigma}_-}_{\text{"Right"}}$; $\hat{\sigma}_\pm = \frac{\hat{x} \pm i\hat{y}}{\sqrt{2}}$

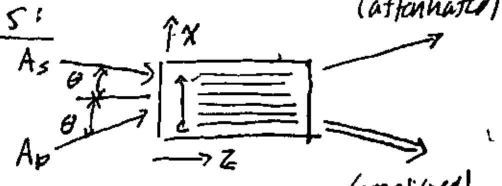
$E^* \cdot E = |E_+|^2 + |E_-|^2$; $P^{NL} = \epsilon_0 A(E_+^2 + E_-^2)\vec{E} + \epsilon_0 B(E_+^2 + E_-^2)\vec{E}^*$

$= P_+\hat{\sigma}_+ + P_-\hat{\sigma}_-$

$\nabla^2 E(z,t) = \frac{\epsilon^{(1)}}{c^2}\frac{\partial^2 E(z,t)}{\partial t^2} + \frac{1}{\epsilon_0 c^2}\frac{\partial^2}{\partial t^2}P^{NL}$

Chapter 11: Electroptic and Photorefractive Effects

2. Verify (11.6.19) Two-beam Coupling in Photorefractive Materials:



Constructive Interference: Signal (A_s) and Pump (A_p) with (A_s)
Destructive Interference: Signal (A_s) into Pump (A_p)

"Process known as attenuation"

$\tilde{E}_{opt}(r,t) = \left[A_p(z)e^{ik_p r} + A_s(z)e^{ik_s r}\right]e^{-i\omega t} + c.c.$; $I = n_0\epsilon_0 c\langle \tilde{E}_{opt}^2 \rangle$ or

$= I_0 + (I_1 e^{iqx} + c.c)$

Where $I_0 = 2n_0\epsilon_0 c(|A_p|^2 + |A_s|^2)$

$I_1 = 2n_0\epsilon_0 c(A_p A_s^*)(\hat{e}_p \cdot \hat{e}_s)$

and $\vec{q} = q\hat{x} = k_p - k_s$

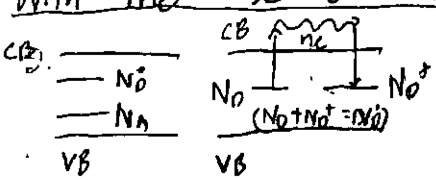
Grating wavevector. Polarization unit vectors

$I = I_0[1 + m\cos(qx+\phi)]$

where $m = 2|I_1|/I_0$ "modulation index"

$\phi = \tan^{-1}(\mathrm{Im}\, I_1 / \mathrm{Re}\, I_1)$

With the use of Kukhtarev et. al to find Static electric fields $\vec{E}$:



Number density of conduction band.

$\frac{\partial N_D^+}{\partial t} = (sI + \beta)(N_D^0 - N_D^+) - \gamma n_e N_D^+$

$\frac{\partial n_e}{\partial t} = \frac{\partial N_D^+}{\partial t} + \frac{1}{e}(\nabla \cdot \vec{j})$

s = Donor;
β = Thermal Generation Rate
γ = Recombination Coefficient

Current flow:

$j = n_e e\mu E + eD\nabla n_e + j_{ph}$

electron mobility; Diffusion constant; photovoltaic contribution

e = charge of electron.
j = current density.

Static field: $\epsilon_{dc}\nabla\cdot E = -e(n_e + N_A - N_D^+)$

crystals static Dielectric constant.

Optical-frequency of the Dielectric constant:

$\Delta\epsilon = -\epsilon^2 r_{eff}|E|$

Steady State solutions become

$E = E_0 + (E_1 e^{iqx} + c.c)$; $j = j_0 + (j_1 e^{iqx} + c.c)$

$n_e = n_{e0} + (n_{e1} e^{iqx} + c.c)$; $N_D^+ = N_{D0}^+ + (N_{D1}^+ e^{iqx} + c.c)$

Where $E = E\hat{x}$ and $j = j\hat{x}$.

$(sI_0+\beta)(N_D^0 - N_{D0}^+) = \gamma n_{e0} N_{D0}^+$; $j_0 = \text{constant}$; $j_0 = n_{e0} e \mu E_0 + j_{ph,0}$; $N_{D0}^+ = n_{e0} + N_A$

$N_{D0}^+ = N_A$; $n_{e0} = \dfrac{(sI_0+\beta)(N_D^0 - N_A)}{\gamma N_A}$; Assuming photovoltaic is negligible:

When considering the spatial dependence e^{iqx}:

$sI_1(N_D^0 - N_A) - (sI_0+\beta)N_{D1}^+ = \gamma n_{e0} N_{D1}^+ + \gamma n_{e1} N_A$; $j_1 = 0$; $-n_{e0} e E_1 = iq k_B T n_{e1}$

$iq \epsilon_0 \epsilon_{ac} E_1 = -e(n_{e1} - N_{D1}^+)$

Solving for $E_1 = \dfrac{-iqk_BTn_{e1}}{e n_{e0}} = -iE_D\left[\dfrac{n_{e1}}{n_{e0}}\right]$

$= -iE_D\left(\dfrac{sI_1(N_D^0-N_A) - (sI_0+\beta)N_{D1}^+ - \gamma\left[\frac{(sI_0+\beta)(N_D^0-N_A)}{\gamma N_A}\right]}{\gamma N_A}\right)\cdot\dfrac{\gamma N_A}{(sI_0+\beta)(N_D^0-N_A)}$

$= -iE_D\left[\dfrac{sI_1(N_D^0-N_A)}{(sI_0+\beta)(N_D^0-N_A)} - \dfrac{(sI_0+\beta)N_{D1}^+}{(sI_0+\beta)(N_D^0-N_A)} - \dfrac{1}{N_A}\right]$

$= -iE_D\left[\dfrac{sI_1}{(sI_0+\beta)} - \dfrac{N_{D1}^+}{(N_D^0-N_A)} - \dfrac{1}{N_A}\right]$

$= -iE_D\left[\dfrac{sI_1}{(sI_0+\beta)} - \dfrac{N_{D1}^+ - N_A - N_D^0 + N_A}{\cancel{N_D^0} + N_A}\dfrac{e}{\epsilon_0\epsilon_{dc}q}N_{eff}\right]$

$E_D = -iE_D\left[\dfrac{sI_1}{(sI_0+\beta)} + \dfrac{N_D^0}{(N_D^0 \ldots)}\right]\dfrac{N_A(N_D^0-N_A)}{N_D^0}$

$\dfrac{E_q}{E_q+E_D} = -iE_D\left[\dfrac{sI_1}{(sI_0+\beta)} + N_{eff}\right]$

$= -iE_D\left[\dfrac{sI_1}{(sI_0+\beta)} + N_{eff}\right]\dfrac{k_BT}{\epsilon_0\epsilon_{dc}}N_{eff}$

Assuming

$E_1 = -i\left(\dfrac{sI_1}{sI_0+\beta}\right)\left(\dfrac{E_D}{1+E_D/E_q}\right)$

where $E_D = \dfrac{qk_BT}{e}$; $\boxed{\text{Diffusion field Strength}}$

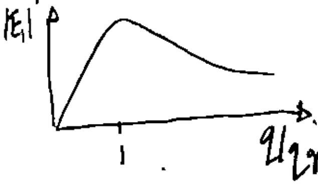
$\boxed{\text{Maximum Space charge Field}}$ $E_q = \dfrac{e}{\epsilon_0\epsilon_{dc}q}N_{eff}$

Since the diffusion field strength and space charge field depend upon the grating wavevector.

$E_1 = -i\left(\dfrac{sI_1}{sI_0+\beta}\right)E_{opt}\dfrac{2(q/q_{opt})}{1+(q/q_{opt})^2}$

Where $q_{opt} = \left(\dfrac{N_{eff}e^2}{k_BT\epsilon_0\epsilon_{dc}}\right)^{1/2}$; $E_{opt} = \left(\dfrac{N_{eff}k_BT}{4\epsilon_0\epsilon_{dc}}\right)^{1/2}$

Dependence of the modulated component:



; $q = 2n\dfrac{\omega}{c}\sin\theta$; $\boxed{\text{Controls the angle between pump and signal}}$

$E_1 = -i\dfrac{A_pA_s^*}{|A_s|^2+|A_p|^2}(\hat{e}_p\cdot\hat{e}_s)E_m$

Where $E_m = \dfrac{E_D}{1+E_D/E_q}$

The dielectric constant changes: $\Delta\epsilon = -\epsilon^2 r_{eff}E_1$; $\epsilon^2 r_{eff} = \sum r_{ijk}(\epsilon_{il}\hat{e}_l^s)(\epsilon_{jm}\hat{e}_m^p)\hat{q}_k$

Cartesian coordinates of the unit vectors

Ordinary waves: $r_{eff} = r_B\sin\left(\dfrac{\alpha_s+\alpha_p}{2}\right)$

Extraordinary waves: $r_{eff} = n^{-4}\left[n_o^4 r_{13}\cos\alpha_s\cos\alpha_p + 2n_e^2n_o^2r_{42}\cos\tfrac{1}{2}(\alpha_s+\alpha_p) + n_e^4 r_{33}\sin\alpha_s\sin\alpha_p\right]\sin\tfrac{1}{2}(\alpha_s+\alpha_p)\right]$

α_s and α_p = angles.

$P^{NL} = (\Delta\epsilon e^{iqr} + c.c)(A_se^{ik_sr} + A_pe^{ik_pr})$; $q = k_p - k_s$

$P_s^{NL} = \Delta\epsilon^* A_p e^{ik_sr} = -i\epsilon^2 r_{eff}E_m\dfrac{|A_p|^2A_s}{|A_p|^2+|A_s|^2}e^{ik_sr}$

$P_p^{NL} = \Delta\epsilon A_s e^{ik_pr} = i\epsilon^2 r_{eff}E_m\dfrac{|A_s|^2A_p}{|A_p|^2+|A_s|^2}e^{ik_pr}$

Slow varying approximation:

$2ik\dfrac{dA_s}{dz}e^{ik_sr} = -\dfrac{\omega^2}{c^2}P_s^{NL}$

$\dfrac{dA_s}{dz} = \dfrac{\omega}{2c}n^3 r_{eff}E_m\dfrac{|A_p|^2A_s}{|A_p|^2+|A_s|^2}$

With $I_s = 2n\epsilon_0 c|A_s|^2$; $\frac{dI_s}{dz} = n\epsilon_0 c(A_s^* dA_s/dz + c.c)$; $\frac{dI_s}{dz} = \Gamma \frac{I_s I_p}{I_p + I_s}$ where $\Gamma = \frac{\omega}{c} n^3 r_{eff} E_m$

$\frac{dI_p}{dz} = -\Gamma \frac{I_s I_p}{I_s + I_p}$ || $\boxed{E_1 = -i \frac{A_p A_s^*}{|A_p|^2 + |A_s|^2}(\hat{e}_p \hat{e}_s) E_m = \tau \frac{\partial E_1}{\partial t} + E_1}$

Where $\tau = \tau_p \frac{1 + E_D/E_m}{1 + E_D/E_m}$; $\tau_\lambda = \frac{\epsilon_0 \epsilon_{dc}}{e \mu n_e 0}$; $E_m = \frac{\gamma N_A}{2\mu}$.

Chapter 14:

2. If $k_r = 0$, then the frequency of the laser(ω) and plasma(ω_p) ~~are equivalent~~ and the dielectric constant $\epsilon(\omega)$ approaches zero from (Eqn 14.3.6).

$$\epsilon^{(1)}(\omega) = 1 - \frac{\omega_p^2}{\omega^2 + 2i\omega\gamma}$$

$$\frac{\partial C}{\partial t} = D\frac{\partial^2 C(x,t)}{\partial x^2}$$

$$\boxed{\langle r^2\rangle = 6Dt} \qquad \boxed{r^2 = x^2+y^2+z^2}$$

$$D = \frac{c_s RT}{3\pi\mu d_p}$$

$$\boxed{\frac{dt}{dV} = \frac{\mu\alpha C}{\Delta P\cdot A^2}\cdot V + \frac{\mu R_m}{A\cdot\Delta P}}$$

$$\frac{t}{V} = \frac{\mu\alpha C}{2\Delta P A}\cdot V + \frac{\mu R_m}{A\Delta P}$$

$$\boxed{J = -D\frac{d\phi}{dx}}$$

$$\boxed{\frac{\partial\phi}{\partial t} = D\frac{\partial^2\phi}{\partial x^2}}$$

O_2 1.78×10^{-5} m^2/s

CO_2 1.38×10^{-5} $m^2/s \cdot M$ in Air

H_2O 2.36×10^{-5} $\frac{m^2}{s}$

$$\dot{V} = D\frac{A}{t}(P_1 - P_2)$$

0.039% CO_2

78.084% N_2 : Lower

20.946% O_2 : Lower

3% H_2O : Lower

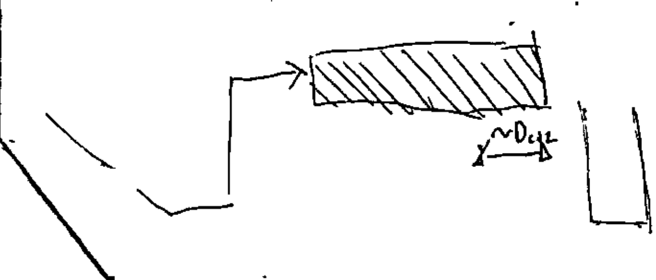
$$\frac{\partial x}{\partial t\,\partial C} = D = \frac{\Delta x}{\Delta t\cdot\Delta C}$$

$$\frac{x}{\partial C} = D\,t \qquad \frac{m^2}{Ms}$$

$$\boxed{KE = \sqrt{\tfrac{3}{2}MRT}}$$

C_{O_2} [ ]

1000ft [3.8ft] ocn



$\sim D_{O_2}$