

# ENERGY 295 Lab Report Equations

Jessica, Tom, Miles

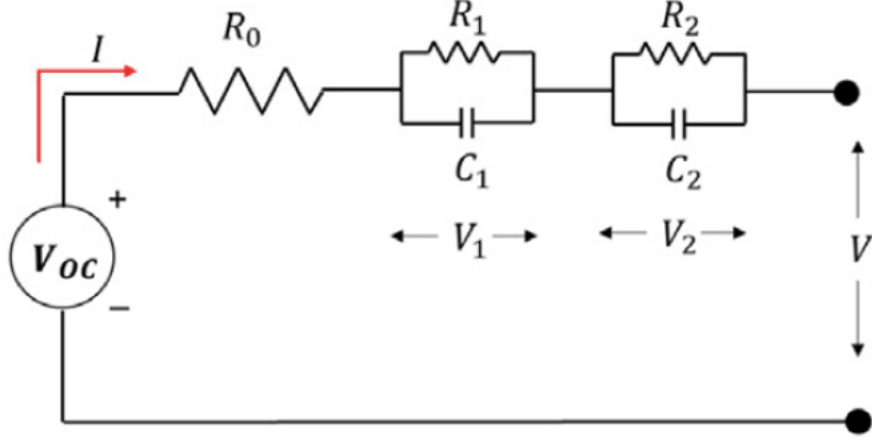
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# 1 State Equations

Beginning with a second-order equivalent circuit model for a Li-ion battery,



The cell voltage,  $V$ , is mathematically defined as,

$$V(t) = V_{OC}(SOC) - V_1(t) - V_2(t) - R_0(SOC) I(t) \quad (1)$$

and applying a general state-space representation to the system, such that

$$\dot{x}(t) = Ax(t) + Bu(t) \quad (2)$$

$$y(t) = Cx(t) + Du(t) \quad (3)$$

where

$$\dot{x}(t) = \begin{bmatrix} \dot{SOC}(t) \\ \dot{V}_1(t) \\ \dot{V}_2(t) \\ \dot{R}_0(SOC(t)) \end{bmatrix} = \begin{bmatrix} \frac{-I(t)}{Q_n} \\ \frac{-1}{R_1(SOC(t))C_1(SOC(t))}V_1(t) + \frac{1}{C_1(SOC(t))}I(t) \\ \frac{-1}{R_2(SOC(t))C_2(SOC(t))}V_2(t) + \frac{1}{C_2(SOC(t))}I(t) \\ 0 \end{bmatrix} \quad (4)$$

and our state vector,  $x(t)$  can be described as

$$x(t) = \begin{bmatrix} SOC \\ V_1 \\ V_2 \\ R_0 \end{bmatrix} \quad (5)$$

thus,

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{-1}{R_1(SOC(t))C_1(SOC(t))} & 0 & 0 \\ 0 & 0 & \frac{-1}{R_2(SOC(t))C_2(SOC(t))} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} \frac{-1}{Q_n} \\ \frac{1}{C_1} \\ \frac{1}{C_2} \\ 0 \end{bmatrix} \quad (6)$$

Assuming there is some sort of process noise in the state vector  $\dot{x}$  and observation noise at time instance  $k$  that are both Gaussian in their distribution with a mean of 0, the discrete time linear system can be described as

$$x(k) = Ax(k-1) + Bu(k-1) + w \quad y(k) = Cx(k) + \nu \quad (7)$$

The covariance of  $w$  is defined as  $Q$  and the covariance of  $\nu$  is defined as  $R$ .

By applying the Jacobian to linearize the system dynamics and measure the , the linearized matrices are developed

$$A_k = \begin{bmatrix} 0 & 0 & 0 & 0 \\ \frac{C_1 \frac{\partial R_2}{\partial SOC} + R_1 \frac{\partial C_1}{\partial SOC}}{(R_1 C_1)^2} & \frac{-1}{R_1 C_1} & 0 & 0 \\ \frac{C_2 \frac{\partial R_2}{\partial SOC} + R_2 \frac{\partial C_2}{\partial SOC}}{(R_2 C_2)^2} & 0 & \frac{-1}{R_1 C_1} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad B_k = \begin{bmatrix} \frac{-1}{Q_n} \\ \frac{1}{C_1} \\ \frac{1}{C_2} \\ 0 \end{bmatrix} \quad (8)$$

$$C_k = \begin{bmatrix} (\frac{\partial V_{OC}}{\partial SOC} - \frac{\partial R_0}{\partial SOC} I(t)) & -1 & -1 & -I(t) \end{bmatrix} \quad D_k = \begin{bmatrix} 0 \end{bmatrix}$$

## 2 Extended Kalman Filter

To estimate the state variables,  $\hat{x}_0$  and  $\hat{P}_0$  need to be initialized. These were initialized as

$$\hat{x}_0 = \begin{bmatrix} SOC_0 & 0 & 0 & 0 \\ 0 & V_{1,0} & 0 & 0 \\ 0 & 0 & V_{2,0} & 0 \\ 0 & 0 & 0 & R_{0,0} \end{bmatrix} \quad \hat{P}_0 = \begin{bmatrix} 0.1 & 0 & 0 & 0 \\ 0 & 0.001 & 0 & 0 \\ 0 & 0 & 0.001 & 0 \\ 0 & 0 & 0 & 0.001 \end{bmatrix} \quad (9)$$

The process noise,  $Q$  and its dependent measurement noise  $R$  are also initialized. The measurement noise,  $R$ , is set as

$$R = 1 \times 10^{-4} \quad (10)$$

and the process noise matrix,  $Q$ , is set as

$$Q = \begin{bmatrix} 100R & 0 & 0 & 0 \\ 0 & 0.1R & 0 & 0 \\ 0 & 0 & 0.01R & 0 \\ 0 & 0 & 0 & 0.1R \end{bmatrix} \quad (11)$$

Using the prior estimation (k-1) of the states the EKF will estimate the state variables and process noise at discrete time instance, k. The initial prediction of the states and process noise covariance at time instance k are described as,

$$\hat{x}_{k|k-1} = A_k \hat{x}_{k-1|k-1} + B_k u_{k-1} \quad (12)$$

$$P_{k|k-1} = A_k P_{k-1|k-1} A_k^T + Q$$

To correct the system difference between the prediction step and the true value, a gain (Kalman Gain,  $L$  is estimated for a time instance k, such that

$$L_k = \frac{P_{k|k-1} C_{k-1}^T}{C_k P_{k|k-1} C_k^T + R} \quad (13)$$

Thus, the corrected estimations of  $\hat{x}_k, \hat{y}_k, P_k$  are described as

$$\hat{x}_k = \hat{x}_{k|k-1} + L_k (y_k - \hat{y}_{k|k-1})$$

$$\hat{y}_k = C_k \hat{x}_k \quad (14)$$

$$P_k = (\mathbf{I} - L_k C_{k-1}) P_{k|k-1}$$

## 2.1 Adaptive Extended Kalman Filter

Modifying the system to an adaptive EKF expands upon a traditional EKF by adding an innovation covariance matrix at each time instance,  $\hat{D}_k$ , and an adaptive noise covariance matrix,  $\hat{Q}_k$ , such that,

$$\hat{D}_k = \frac{1}{N} \sum_{i=i_0}^k d_i d_i^T \quad (15)$$

$$\hat{Q}_k = L_k \hat{D}_k L_k^T$$

where,

$$d_{i,k} = y_k - \hat{y}_{k|k-1} \quad (16)$$

$$i_0 = k - N + 1 \tag{17}$$