ENERGY 295 Lab Report Equations

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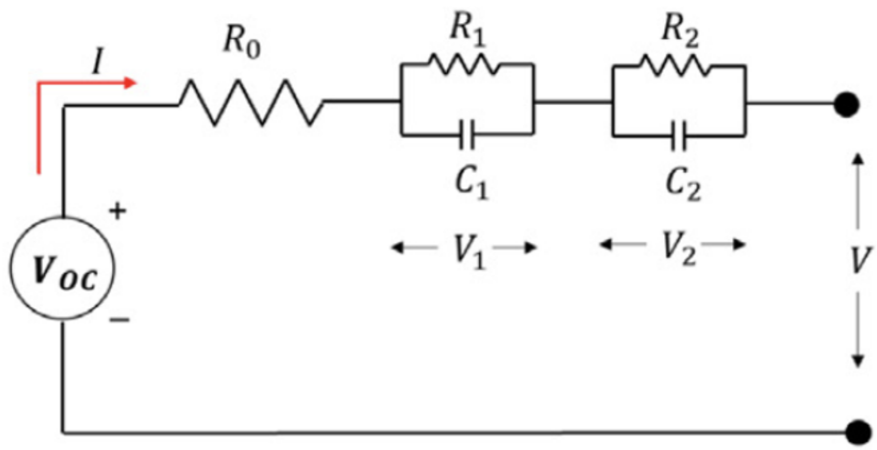
1 State Equations

Beginning with a second-order equivalent circuit model for a Li-ion battery,



The cell voltage, V, is mathematically defined as,

$$V(t) = V_{OC}(SOC) - V_1(t) - V_2(t) - R_0(SOC)\,I(t) \tag{1}$$

and applying a general state-space representation to the system, such that

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{2}$$

$$y(t) = Cx(t) + Du(t) \tag{3}$$

In this model, we initially do not consider R_0 as a state variable that responds to system noise. Rather it is considered to be a pure function of SOC, so our state vectors are considered to be

$$x(t) = \begin{bmatrix} SOC \\ V_1 \\ V_2 \end{bmatrix} \tag{4}$$

thus, our equations of state are

$$\dot{x}(t) = \begin{bmatrix} S\dot{O}C(t) \\ \dot{V}_1(t) \\ \dot{V}_2(t) \end{bmatrix} = \begin{bmatrix} \frac{-I(t)}{Q_n} \\ \frac{-1}{R_1(SOC(t))C_1(SOC(t))}V_1(t) + \frac{1}{C_1(SOC(t))}I(t) \\ \frac{-1}{R_2(SOC(t))C_2(SOC(t))}V_2(t) + \frac{1}{C_2(SOC(t))}I(t) \end{bmatrix} \tag{5}$$

Thus our coefficient matrices, A, B, C, and D, can be defined as

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{-1}{R_1(SOC(t))C_1(SOC(t))} & 0 \\ 0 & 0 & \frac{-1}{R_2(SOC(t))C_2(SOC(t))} \\ 0 & 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} \frac{-1}{Q_n} \\ \frac{1}{C_1} \\ \frac{1}{C_2} \end{bmatrix}$$
$$C = \begin{bmatrix} (\frac{\partial V_O C}{\partial SOC} - \frac{\partial R_0}{\partial SOC} I(t)) & -1 & -1 \end{bmatrix}^* \qquad D = \begin{bmatrix} \mathbf{0} \end{bmatrix} \tag{6}$$

Assuming there is some sort of process noise in the state vector $\dot{x}$ and observation noise at time instance k that are both Gaussian in their distribution with a mean of 0, the discrete time linear system can be described as

$$x(k) = Ax(k-1) + Bu(k-1) + w \qquad y(k) = Cx(k) + \nu \tag{7}$$

The covariance of w is defined as Q and the covariance of ν is defined as R. By applying the Jacobian to linearize the coefficient matrices, we get the matrices in their discrete form, such that

$$A_k = \begin{bmatrix} 0 & 0 & 0 \\ \frac{C_1 \frac{\partial R_2}{\partial SOC} + R_1 \frac{\partial C_1}{\partial SOC}}{(R_1C_1)^2} & \frac{-1}{R_1C_1} & 0 \\ \frac{C_2 \frac{\partial R_2}{\partial SOC} + R_2 \frac{\partial C_2}{\partial SOC}}{(R_2C_2)^2} & 0 & \frac{-1}{R_1C_1} \end{bmatrix} \quad B_k = \begin{bmatrix} \frac{-1}{Q_n} \\ \frac{1}{C_1} \\ \frac{1}{C_2} \end{bmatrix} \tag{8}$$

$$C_k = \begin{bmatrix} (\frac{\partial V_O C}{\partial SOC} - \frac{\partial R_0}{\partial SOC} I(t)) & -1 & -1 \end{bmatrix} \qquad D_k = \begin{bmatrix} \mathbf{0} \end{bmatrix}$$

the system can be now be considered using linear modeling techniques.

1.1 Considering R_0 as a state variable

Expanding on the previous system, R_0 can be considered as a state variable that adapts based on the observation and process noise of the system. The new state vector of the system now is considered to be

$$x(t) = \begin{bmatrix} SOC \\ V_1 \\ V_2 \\ R_0 \end{bmatrix} \tag{9}$$

with state variables that are defined as

$$\dot{x}(t) = \begin{bmatrix} S\dot{O}C(t) \\ \dot{V}_1(t) \\ \dot{V}_2(t) \\ \dot{R}_0(SOC(t)) \end{bmatrix} = \begin{bmatrix} \frac{-I(t)}{Q_n} \\ \frac{-1}{R_1(SOC(t))C_1(SOC(t))}V_1(t) + \frac{1}{C_1(SOC(t))}I(t) \\ \frac{-1}{R_2(SOC(t))C_2(SOC(t))}V_2(t) + \frac{1}{C_2(SOC(t))}I(t) \\ 0 \end{bmatrix} \tag{10}$$

The non-linear coefficients matrices are defined as,

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{-1}{R_1(SOC(t))C_1(SOC(t))} & 0 & 0 \\ 0 & 0 & \frac{-1}{R_2(SOC(t))C_2(SOC(t))} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} \frac{-1}{Q_n} \\ \frac{1}{C_1} \\ \frac{1}{C_2} \\ 0 \end{bmatrix} \tag{11}$$

$$C = \begin{bmatrix} (\frac{\partial V_O C}{\partial SOC} - \frac{\partial R_0}{\partial SOC}I(t)) & -1 & -1 & -I(t) \end{bmatrix}^* \qquad D = \begin{bmatrix} \mathbf{0} \end{bmatrix}$$

By applying the Jacobian to linearize the system dynamics, the discretized coefficient matrices are developed for each time-instance k

$$A_k = \begin{bmatrix} 0 & 0 & 0 & 0 \\ \frac{C_1 \frac{\partial R_2}{\partial SOC} + R_1 \frac{\partial C_1}{\partial SOC}}{(R_1 C_1)^2} & \frac{-1}{R_1 C_1} & 0 & 0 \\ \frac{C_2 \frac{\partial R_2}{\partial SOC} + R_2 \frac{\partial C_2}{\partial SOC}}{(R_2 C_2)^2} & 0 & \frac{-1}{R_1 C_1} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad B_k = \begin{bmatrix} \frac{-1}{Q_n} \\ \frac{1}{C_1} \\ \frac{1}{C_2} \\ 0 \end{bmatrix} \tag{12}$$

$$C_k = \begin{bmatrix} (\frac{\partial V_O C}{\partial SOC} - \frac{\partial R_0}{\partial SOC}I(t)) & -1 & -1 & -I(t) \end{bmatrix} \quad D_k = \begin{bmatrix} \mathbf{0} \end{bmatrix}$$

2 Extended Kalman Filter

To estimate the state variables, $\hat{x}_0$ and $\hat{P}_0$ need to be initialized. These were initialized as

$$\hat{x}_0 = \begin{bmatrix} SOC_0 & 0 & 0 & 0 \\ 0 & V_{1,0} & 0 & 0 \\ 0 & 0 & V_{2,0} & 0 \\ 0 & 0 & 0 & R_{0,0} \end{bmatrix} \quad \hat{P}_0 = \begin{bmatrix} 0.1 & 0 & 0 & 0 \\ 0 & 0.001 & 0 & 0 \\ 0 & 0 & 0.001 & 0 \\ 0 & 0 & 0 & 0.001 \end{bmatrix} \tag{13}$$

The process noise, Q and its dependent measurement noise R are also initialized. The measurement noise, R, is set as

$$R = 1 \times 10^{-4} \tag{14}$$

and the process noise matrix, Q, is set as

$$Q = \begin{bmatrix} 100R & 0 & 0 & 0 \\ 0 & 0.1R & 0 & 0 \\ 0 & 0 & 0.01R & 0 \\ 0 & 0 & 0 & 0.1R \end{bmatrix} \tag{15}$$

Using the prior estimation (k-1) of the states the EKF will estimate the state variables and process noise at discrete time instance, k. The initial prediction of the states and process noise covariance at time instance k are described as,

$$\begin{gathered} \hat{x}_{k|k-1} = A_k \hat{x}_{k-1|k-1} + B_k u_{k-1} \\ P_{k|k-1} = A_k P_{k-1|k-1} A_k^T + Q \end{gathered} \tag{16}$$

To correct the system difference between the prediction step and the true value, a gain (Kalman Gain, L is estimated for a time instance k, such that

$$L_k = \frac{P_{k|k-1} C_{k-1}}{C_k P_{k|k-1} C_{k-1}^T + R} \tag{17}$$

Thus, the corrected estimations of $\hat{x}_k$, $\hat{y}_k$, P_k are described as

$$\begin{gathered} \hat{x}_k = \hat{x}_{k|k-1} + L_k (y_k - \hat{y}_{k|k-1}) \\ \hat{y}_k = C_k \hat{x}_k \\ P_k = (\mathbf{I} - L_k C_{k-1}) P_{k|k-1} \end{gathered} \tag{18}$$

2.1 Adaptive Extended Kalman Filter

Modifying the system to an adaptive EKF expands upon a traditional EKF by adding an innovation covariance matrix at each time instance, $\hat{D}_k$, and an adaptive noise covariance matrix, $\hat{Q}_k$, such that,

$$\begin{gathered} \hat{D}_k = \tfrac{1}{N} \sum_{i=i_0}^{k} d_i d_i^T \\ \hat{Q}_k = L_k \hat{D}_k L_k^T \end{gathered} \tag{19}$$

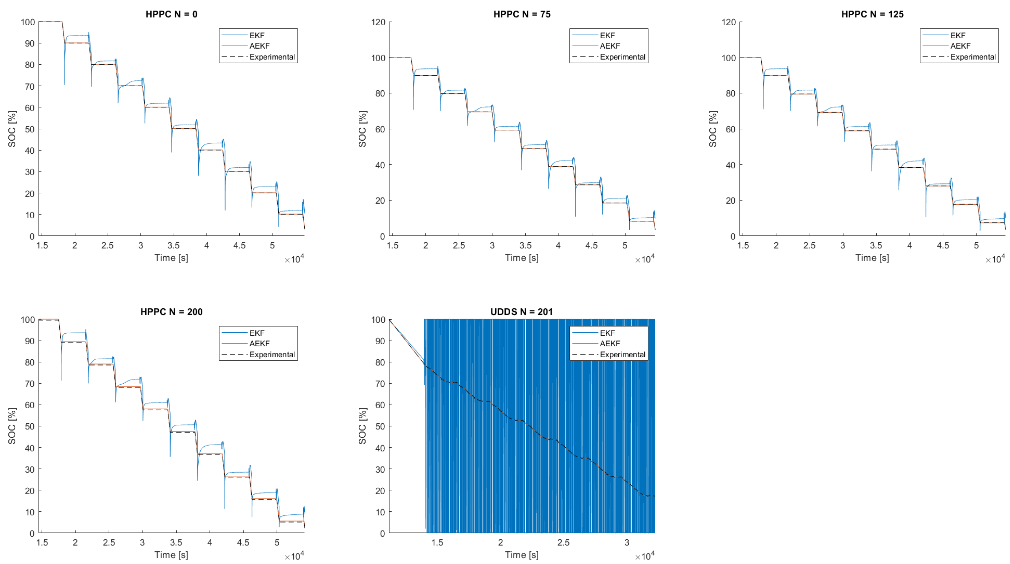
where,

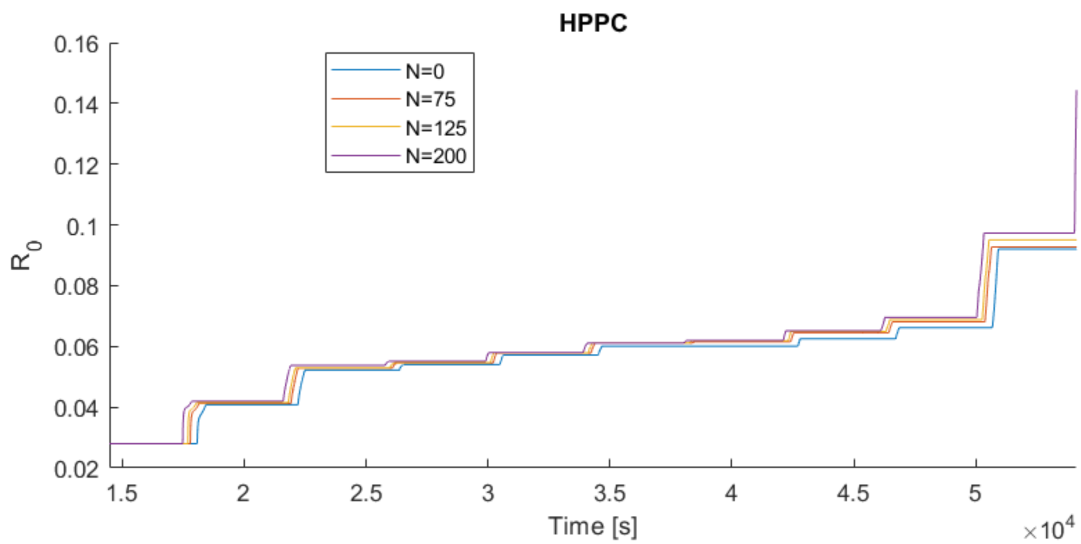
$$d_{i,k} = y_k - \hat{y}_{k|k-1} \tag{20}$$

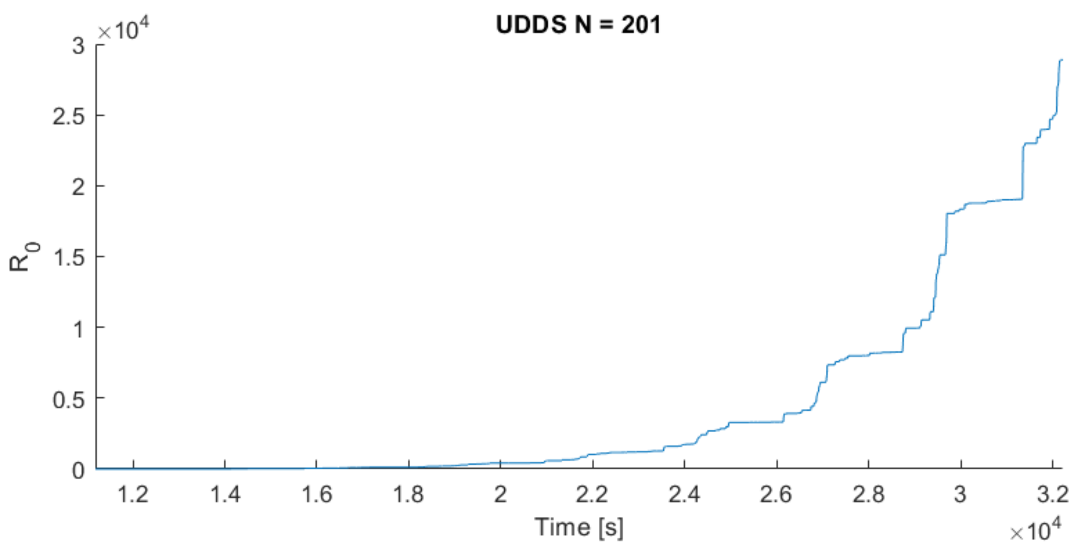
$$i_0 = k - N + 1 \tag{21}$$

which gives a noise covariance matrix, Q, that adapts for each time step based on the previous N time instances.

3 System Performance




HPPC
N=0
N=75
N=125
N=200
R_0
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
1.5
2
2.5
3
3.5
4
4.5
5
Time [s]
$\times 10^4$


UDDS N = 201
$\times 10^4$
R_0
3
2.5
2
1.5
1
0.5
0
1.2
1.4
1.6
1.8
2
2.2
2.4
2.6
2.8
3
3.2
Time [s]
$\times 10^4$