Equilibrium solver for a dipole equilibrium:

$\vec{B}_{toroidal} = 0$, so Grad-Shafranov (GS) reduces to:

$\Delta^* \psi = -\mu_0 R^2 \frac{dP}{d\psi}$, R is the cylindrical radial coordinate.

I plan on solving this in cylindrical coordinates, then calculating the flux coordinates from the solution.

$$\Rightarrow R \frac{\partial}{\partial R}\left(\frac{1}{R}\frac{\partial \psi}{\partial R}\right) + \frac{\partial^2 \psi}{\partial Z^2} = -\mu_0 R^2 \frac{dP}{d\psi}$$

→ a form for this will be guessed

$\psi_{i,j}^{n}$ ~ iteration level.

↳ $R_i = i \times \delta R$, $Z_i = j \times \delta Z$

$$, \quad \frac{\partial \psi_{ij}^n}{\partial R} = \frac{\psi_{i+1,j}^n - \psi_{i-1,j}^n}{2\delta R}, \quad \frac{\partial^2 \psi_{ij}^n}{\partial R^2} = \frac{\psi_{i+1,j}^n - 2\psi_{ij}^n + \psi_{i-1,j}^n}{\delta R^2}$$

$$\Rightarrow \Delta^* \psi_{ij}^n = \frac{\psi_{i+1,j}^n - 2\psi_{ij}^n + \psi_{i-1,j}^n}{\delta R^2} + \frac{\psi_{i,j+1}^n - 2\psi_{ij}^n + \psi_{i,j-1}^n}{\delta Z^2} - \frac{1}{R_i}\frac{\psi_{i+1,j}^n - \psi_{i-1,j}^n}{2\delta R} = \Gamma_{ij}$$

for $\Gamma_{ij} = \mu_0 R_i^2 \left.\frac{dP}{d\psi}\right|_{ij}$

$$\Rightarrow -\psi_{ij}^n\left(\frac{2}{\delta R^2} + \frac{2}{\delta Z^2}\right) = \Gamma_{ij} - \frac{\psi_{i+1,j}^n}{\delta R^2} - \frac{\psi_{i-1,j}^n}{\delta R^2} - \frac{\psi_{i,j+1}^n}{\delta Z^2} - \frac{\psi_{i,j-1}^n}{\delta Z^2} + \frac{1}{R_i}\frac{\psi_{i+1,j}^n - \psi_{i-1,j}^n}{2\delta R}$$

$$\boxed{\Rightarrow \tilde{\psi}_{i,j}^n = \frac{1}{\frac{2}{\delta R^2} + \frac{2}{\delta Z^2}}\left\{\frac{\psi_{i+1,j}^n}{\delta R^2} + \frac{\psi_{i-1,j}^n}{\delta R^2} + \frac{\psi_{i,j+1}^n}{\delta Z^2} + \frac{\psi_{i,j-1}^n}{\delta Z^2} - \frac{1}{R_i}\frac{\psi_{i+1,j}^n - \psi_{i-1,j}^n}{2\delta R} - \Gamma_{ij}^n\right\}}$$

re-calculated using previous ψ_{ij}

So now I can apply successive over-relaxation:

γ = blending parameter:

$$\psi_{i,j}^{n+1} = \gamma\, \psi_{i,j}^n + (1-\gamma)\,\tilde{\psi}_{i,j}^n$$

Guess $\Psi_{i,j}^{0}$, calculate Γ_{ij}^{0} from $\Psi_{i,j}^{0}$

$\downarrow$

Calculate $\tilde{\Psi}_{i,j}^{n+1}$ from Ψ_{ij}^{n}, Γ_{ij}^{n}

$\downarrow$

$$\Psi_{i,j}^{n+1} = \gamma\, \Psi_{i,j}^{n} + (1-\gamma)\, \tilde{\Psi}_{i,j}^{n}$$

$\downarrow$

Calculate Γ_{ij}^{n+1} from $\Psi_{i,j}^{n+1}$

$\downarrow$

Check if $|\Psi_{i,j}^{n+1} - \Psi_{i,j}^{n}| = \delta\Psi < \epsilon$

$\delta\Psi > \epsilon$

$\delta\Psi \leq \epsilon$

$\downarrow$

Output Ψ_{ij}, Γ_{ij}