COMP0124 Multi-Agent Artificial Intelligence

Group Report

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1. INTRODUCTION

[1 2] Advertising delivers marketing messages aimed to promote products or services to potential customers and online advertising is a method of marketing conducted on the internet. Online advertising develops at a high speed from sponsored search and contextual advertising to ad exchange and real-time bidding since more business transactions are operating online and it is commercially viable to transform more web users as potential customers. A popular technical in recent years called Real-time bidding (RTB), a real-time online second-price auction, is operated by every impression, whose ecosystem mainly has four participants: supply side platforms (SSP), Ad exchanges(ADX), demand side platforms(DSP) and data exchanges(DX) [6].

This paper tries to figure out an optimal bidding strategy based on user response prediction which refers to users' interests on the specific advertisements. A good bidding strategy is the process of determining a bidding price which results in accurate advertising delivery when there is an opportunity of displaying advertisements [6]. Treating bidding strategy as a function, various factors such as key performance indicator (KPI), bid landscape forecasting (volume) and budget are considered to involve. Additionally, working out precise prediction of user response including click-through rate (CTR) helps to find the bidding price during an online auction [8] and to make decisions on bidding strategies [1].

To be more specific, several bidding strategies for real-time bidding from constant bidding strategy to non-linear bidding strategy will be implemented and evaluated based on five metrics, especially the number of clicks will be the most considerable one. In the initial stage, the raw data needs to be dealed with, since it is unbalanced between the class with clicks and the class without clicks. In addition, for predicting a more precise pCTR, feature preparation becomes more important. Thus, filtering out features, merging all possible features and converting to one-hot vectors are implemented. The most used method during the whole project is grid search which helps to investigate the optimization of parameters in models.

After experiments, in general, XGBoost is the most fitting classification model for predicting pCTR and Quartic bidding model is the best one winning most clicks based on the evaluation results in test set. The best bidding strategy (XGBoost+Quartic) finally wins 172 clicks in validation set when CTR is 0.1816% and the comparison among all bidding strategies is stated in "3.5.3 Final Results".

2. RELATED WORK

Real-time bidding optimisation is a relatively emerging topic in research area varying from microscopic bidding analysis to macroscopic market analysis [11]. Under the complete understanding of real-time bidding and the problems met during the whole process, all the tasks can mainly be divided into two aspects, CTR estimation and bidding strategy. For CTR estimation, quantity of models or algorithms are implemented in researches and Wang et.al (2017) [6] separate them into: linear model such as logistical regression and Bayesian probit regression; and non-linear model for example deep learning model, random forest and gradient tree models.

Logistical Regression, the commonly applied model, is simple and efficient for researchers to do initial study of CTR predicting [2]. However, linear model may not perform well during fitting data from real life due to its simplicity, so some academic turn to non-linear model. Boosting models focus on integrating weak predictors to construct a strong predictor [4]. Boosted tree model is able to investigate the importance of own features and provide a more clear explanation than logistical regression [9]. Additionally, some complex models also gradually become popular in research papers. User lookalike modelling is a new idea because of the characteristics of real-time bidding compared to sponsored search ad [6].

Another point is the bidding strategy. Zhang et.al (2015) [13] put forward a bidding function called ORTB which can be implemented in realistic data. They investigate that optimal bid follows a non-linear correlation to some evaluation method for impressions such as click through rate.

3. APPROACH AND RESULT

3.1 Evaluation Metrics

Performances of all the real-time bidding strategies will be evaluated by several metrics under the limitation of 6,250 CNY fen budget. These are popular key performance indicators (KPIs) to show whether the bidding strategy is able to work well as what the agent expected in online auctions. This paper believes that evaluation should be implemented under fully understanding of all the metrics. Therefore, concise introduction of and empirical business effects of these metrics will be discussed as following:

[1]The three authors are sorted by the first letter of the first name

[2]The codes for this project is in https://github.com/milestoneg/COMP0124-Multi-Agent-Artificial-Intelligence

- **Click**: Maximum the clicks as the goal of bidding strategy is the simplest approach. In this project, the rank of clicks is treated as the most important metric, thus the final aim of this paper it to raise up the number of click based on the best bidding strategy on test set and win the bidding competition, although other metrics are still considered in the same time.

- **Spend**: Since this project is given a budget as 6,250 CNY fen, all bidding strategies should not spend over the budget.

- **CTR**: Click-through-rate

$$CTR = \frac{Number\ of\ clicks}{Total\ number\ of\ impression} \tag{1}$$

- **average CPM**: CPM is the Cost-per-Mille which is the cost that has to be paid by agent for serving 1000 impressions. It is apparent that focusing on a high CPM will be best-suited to a agent who would like to improve its brand awareness in a short time, since a high CPM means advertisements from this agent will be seen by its potential customers as often as possible. However, the high cost needs to be pay attention if only considering to raise CPM, because a bidding strategy mainly focusing on CPM will motivate the network to show the advertisement more frequently.

$$AverageCPM = \frac{Cost}{Total\ number\ of\ impression} \tag{2}$$

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- **average CPC**: CPC is the Cost-per-Click which is the effective cost of potential customers clicking on the advertisements. This is the key consideration of those agents who desire to generate traffic to their website, since they are mainly care about the viewers with enough interested to click advertisements. The concern of only focusing on CPC is that agents need to make sure that their advertisements is enough attractive to motivate viewers to click on ads, otherwise these ads will be sidelined by the platform and stop receiving any impressions.

$$AverageCPC = \frac{Cost}{Total\ number\ of\ click} \tag{3}$$

3.2 Data Exploration

For data exploration, this paper mainly focuses on statistical analysis of training set and evaluation on validation set with the help of some metrics such as CTR, CPM and eCPC. In addition, user feedback of features varying from weekdays to slot size also is discussed concisely.

3.2.1 Basic Statistics

CTR.

Learning from Table 1, most of advertisers have low CTRs which are lower than 0.1% and advertiser 2259 is the lowest one (0.032%), while advertiser 2997 shows significant difference from others with a high CTR (0.435%) much higher than 0.1%. According to Zhang et. al (2015) [13], advertisements displaying on desk-top in the real life usually have an average CTR around 0.1%. The performance of CTR of advertiser 2297 who is in the category of mobile e-commerce app install verifies that users are more likely to click the advertisement on mobile environment.

Table 1: Statistics of Training Set

Adv.	Imps	Clicks	Cost	CTR	CPM	eCPC
3427	402,806	272	30,458k	0.068%	75.62	111.98
2821	211,366	131	18,828k	0.062%	89.08	143.73
1458	492,353	385	33,969k	0.078%	68.99	88.23
2259	133,673	43	12,428k	0.032%	92.97	289.03
3386	455,041	320	34,932k	0.070%	76.77	109.16
3358	264,956	202	22,447k	0.076%	84.72	111.12
3476	310,835	187	23,919k	0.060%	76.95	127.91
2261	1,110,122	36	9,874k	0.033%	89.66	274.27
2997	49,829	217	3,129k	0.435%	62.80	14.42

CPM.

On the contrary, advertiser 2259 has the largest CPM nearly 93, while advertiser 2297 becomes the smallest one whose CPM is just 62.8, which means this advertiser pays a lower cost for one thousand impression. But in general, all of advertisers have CPMs in the interval from 60 to 100 and are similar to each other.

eCPC.

Different from CPM, eCPCs of nine advertiser are much more different varying from advertiser 2997's 14.42 to advertiser 2259's 289.03. eCPCs of two advertisers are lower than 100 and similarly other two advertisers have the eCPC more than 200. eCPC influences the rate of investment (ROI) since a lower eCPC will reduce the cost-per acquisition and all these clicks are essential for advertiser's success.

Based on the discovery mentioned before, advertiser 2259 and advertiser 2297 shows significant performances. Advertiser 2259 has a lowest CTR, largest CPM and highest eCPC while advertiser 2997 shows a highest CTR, smallest CPM and lowest eCPC.

3.2.2 User Feedback

According to Figure1 which shows several statistics of user feedback with CTR distributions of the performance of all advertisers, this paper will discuss how the CTR of all advertisers performs under different features.

Weekdays.

Advertisers gain a highest CTR (nearly 0.001%) on Tuesday which is much higher than other days, while it becomes lowest (less than 0.0006%) on Saturday. Compared to weekdays, CTR is lower on weekends.

OS and Browser.

Compared CTR on mobile and on desk-top, users with Apple devises are more likely to click the advertisements, since ios in mobile operate system and Safari in browsers show the highest CTR than others.

Region.

CTR in most of regions fluctuate around 0.001%, although CTR in region 325, 368, 394 and 395 show large changes with region 395 has the lowest CTR nearly 0.

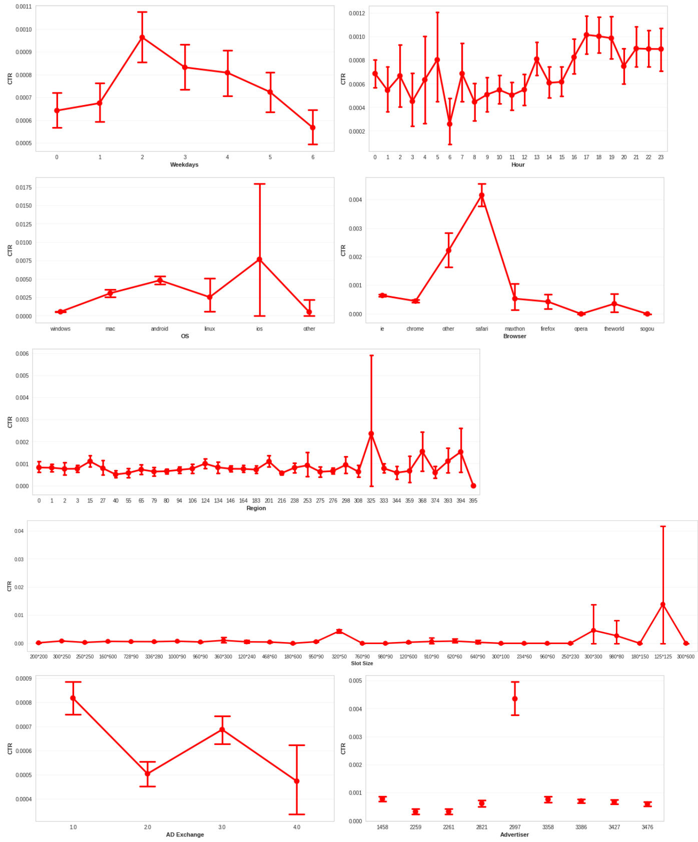


Figure 1: CTR Distribution against Different Features

Slot Size.
Zhang et.al (2015) [13] states that slot size is related to slot location in a web-page and the design. It is obvious that Slot size of 125has the highest CTR (more than 0.01%) among all slot size.

3.3 Basic Bidding Strategy

This paper evaluate three bidding strategies with no predictions under the constrain of budget 6250 CNY fen and these basic strategies can be treated as benchmark for further bidding strategies based on predictions. The random bidding helps to understand the statistics of auctions under similar budget constraint. Not only the single agent situation is considered, a basic thought of multi-agent biding strategy is implemented the third sub-section. The type of auction used in the set is second-price auction which means the final winner of real-time bidding can pay the offer at a price that is less than the original one they give. For constant bidding strategy and random bidding strategy, the winning criterion is that if the bid price is larger than pay price, then this agent win the bid (Winning criterion #1). For homogeneous random bidding agents, the agent will win the bid when its bid price is not only larger than pay price but also higher than other agents' bid prices (Winning criterion #2).

3.3.1 Constant Bidding Strategy

Initial Idea.
The initial idea it to capture the optimal constant bidding price by directly finding the price corresponding to the highest number of click in training set. The clicks of all the impressions with the same pay price are summed up and the list of total number of clicks for each unique pay price is sorted in order to find the highest number of click. Accordingly, the pay price which is also the bid price of the agent is 70 CNY fen. However, this simple idea works terribly in validation set when evaluating the bid price of highest number of clicks. Based on the experiments tried before, this paper decides to apply grid search to find the optimal constant value.

Improving.
The purpose of finding optimal bid price is to win more effective impressions such as having high CTR under the budget. According to observation, the maxima and minima of pay prices in training set fall in a interval of [1, 300]. Therefore, for each bid price picked from 1 to 300, grid search is used to check each impression and do further calculations of total cost, total impressions, CTR, CPC, CPM on those whose pay price is lower than the given bid price. Firstly, after sorting all the results by click as priority and CTR as secondary level in descending order, the maximum number of click is 134 and the corresponding bid price is 24 fen and 25 fen. Considering CTR at the same time, bidding price at 25 CNY fen shows better performance. (Table2)

Table 2: Comparison on Statistics of First Two of Constant Bidding Price in Decending Order

Bid	Clicks	Cost	CTR	CPM	CPC
25	134	6,249.98	0.0331%	15.45	46.64
24	134	6,249.99	0.0325%	15.17	46.64

Additionally, this bidding strategy with 25 fen bidding price outperforms on total cost, CPC and CPM (Table below). Figure 2 reveals distributions of five metrics when bidding price varying from 1 to 300, which helps to strengthen the credibility that optimal constant bidding price is 25 fen. The number of click continuous increasing until the bid price reach 25 fen and begins to decrease beyond this optimal bid price. Impression has a similar trend as number of click which shows a highest turning point when the price is 25 fen. As can be seen, CTR overall reveals a upper trend but with large fluctuation, when the constant bidding strategy is carried out. Thus, a tentative idea is put forward that significant improvements on the bidding strategy model can be figured out in order to maximize the CTR in real-time bidding.

Evaluating.
This paper takes two steps in evaluating constant bidding strategy on validation set. Firstly, 25 CNY fen is assumed as the same optimal bid price and tested with all impressions in validation set. The corresponding total number of click is gained as 16 which seems far from what has obtained in training set 134 (Table 3), but needs to be further verified since 16 clicks may be the highest due to different sets.

Thus this paper tries to test the optimal bidding price gained from training set through grid search. Similarly, five metrics are calculated when bid price changing from 1 to 300 and the bid price of 79 CNY fen which has the highest

Table 3: Comparison on Statistics of Bidding Price at 25 CNY fen in Two sets

Data	Clicks	Cost	CTR	CPM	CPC
Tra	134	6,249.98	0.0331%	15.25	46.64
Val	16	934.66	0.0265%	15.46	58.42

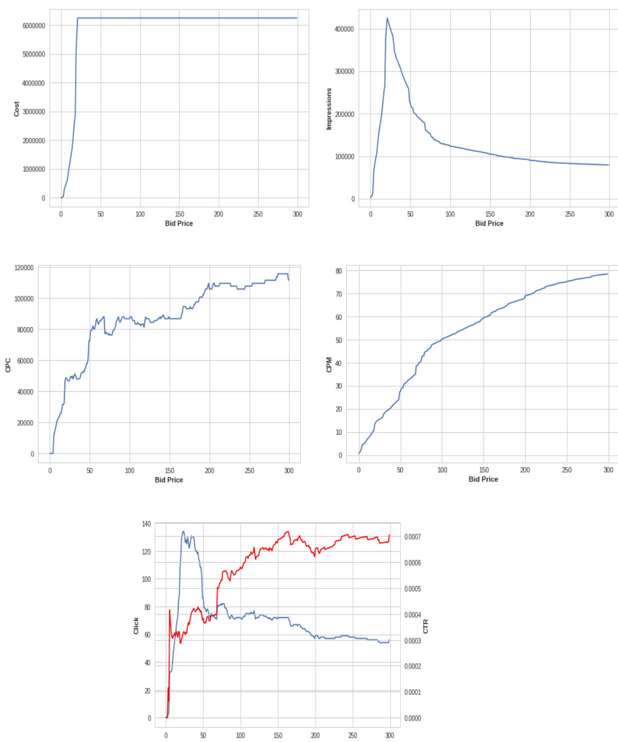


Figure 2: Evaluation of Constant Bidding Strategy with Training Set

number of click is found out. To conclude constant bidding strategy works terribly and has inconsistency in different data sets. Figure3 display distributions of number of click and CTR with bid price from 1 to 300. It proves that the number of click shows the maxima when bid price is 79 CNY fen.

3.3.2 *Random Bidding Strategy*

Finding Optimization.

Based on the information gained from processing constant bidding strategy, this paper tries to find an optimal interval in which the number of total click is highest. Lower bound and upper bound of bid price are set under the limitation of [1, 300] and step size of 10 to gradually assess each possible interval in the training set. For each interval of bid price, this paper randomly picks a bid price out for every impression and filters out those impressions whose pay price is lower than bid price. With limited total budget, total number of click are counted and sorted according to number of click and CTR. Finally the best interval [11, 81] of bid price is discovered with the highest total number of clicks 46 (Table 4) higher than 44 hold by [11, 101]. From Figure4, the heatmap shows the quantity of clicks in each interval with different color standing for different number of click. As

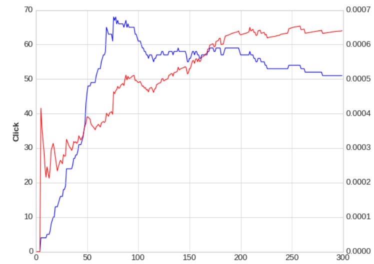


Figure 3: Evaluation of Constant Bidding Strategy with Validation Set

the legend shown , lightener the color block is , higher the number of click is, so the brightest color block is [11, 81] which is the optimal interval for random bidding strategy.

Table 4: Comparison on Metrics of First Two Intervals of Random Strategy in Descending Order

Interval	Clicks	Cost	CTR	CPM	CPC
[11,81]	46	5,958.37	0.0439%	56.84	129.53
[11,101]	44	6,250.01	0.0492%	69.83	142.05

Evaluating.

The performance of the optimal interval gained from training set is evaluated in validation set in order to check whether it also has the highest total number of click. Given lower bound of 11 CNY fen and upper bound of 81 CNY fen, bid price is randomly chose from the interval for each impressions and similar following steps as what has done in training set are processed. There are totally 36 clicks when bid price is from 11 CNY fen and 81 CNY fen in validation set.

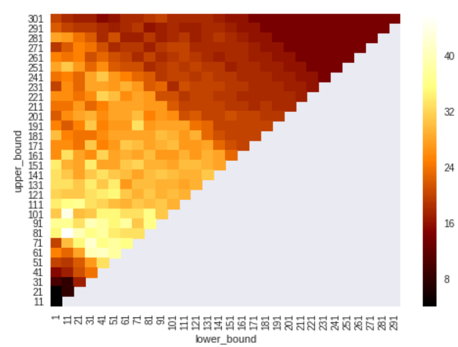


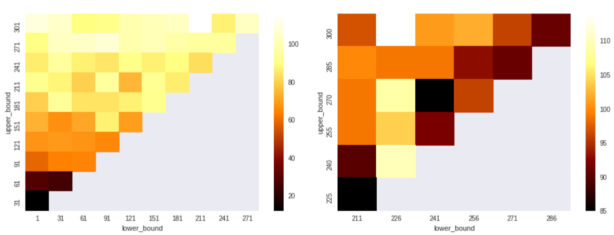
Figure 4: Random Strategy with Training Set

3.3.3 *Homogeneous Random Bidding Agents*

A real-time bidding server is constructed for multi-agents to campaign based on random bidding strategy. Totally 75 agents appear in the auction because this paper is concerned about the impact of special value and use the intermediate value of [50, 100] as the number of agents. Due to the limited computational expense and ability, 50% of the training set is randomly selected for each auction.

Initial Stage.

Firstly, with the help of grid search, all possible intervals are constructed with the limitation of falling in [1, 300] and taking 30 as step size, so a rough optimal interval of bid price can be picked out. Initially taking this larger step size is helpful to reduce the computational expense and save time. When a real-time bidding is processing for one impression, every agent randomly pick a bid price from the given interval of bid price and all these bid prices will be compared with each other to discover the highest one. If this highest bid price is larger than the pay price set for this impression, the owner of this bid price wins the auction. Since each of 75 agents is limited by given budget 6,250 CNY fen, the total number of click obtained by each agent is counted when they buy impressions as many as they can. Result of all number of click when the interval of bid price changes is plotted in a heatmap (The left one in Figure5) which shows the interval of [211, 300] has the highest number of click (116).



(a) Step Size 30 (b) Step Size 15

Figure 5: Multi-Agent Random Bidding Strategy

Narrow Down the Interval.
After gaining the rough interval [211, 300], this paper considers to narrow down it to a more precision one, so step size of 15 is applied to work out the optimal interval for random bid price. On the basic of similar processing, it is found that the interval of bid prices with the highest number of click is from 226 to 300 which is a bit more accurate than the previous one. Figure 5 (The right one) reveals that comparing the number of click of all different intervals, the interval of bid price at [226, 300] is the highest with the most lighten color.

Comparing to Single Agent.
In contrast to the case of single agent, interval of bid price with multi-agent is much higher, even the lower bound of multi-agent is much higher than the upper bound of single agent (Table 5). For the whole interval [1, 300], the optimal interval for single agent mainly falls in the first half of [1, 300] and the optimal interval for multi-agent falls in the second half. This paper supposes that different winning criteria may be one of important reasons, since the bid price given by a agent not only should be higher than the pay price of the impression but also needs to be higher than all the other bid price in the case of multi-agent. Additionally, it is much easier to raise the bid price to a high level than the case of single agent, because there is competitions among all agents.

Impact of N.
From 50 to 100 of agents, this paper uses step size of 2 to check all situations of different number of agents attending

Table 5: Comparison on Statistics of Intervals in Case of Single Agent and Multi-agent(!!!)

Interval	Clicks	Cost	CTR	CPM	CPC
[11,81]	46	5,958.37	0.0439%	56.84	129.5k
[226,300]	116	4,5607	0.492%	69.83	142

the action in the same time. All the other processes are not changed except that the interval of bid price is set as the optimal interval gained before [226, 300] and are the same as what has done when the number of agents is 75. When the number of agents changing, the number of clicks fluctuates dramatically. The distribution of number of click and CTR are shown in Figure6 from which the highest click and CTR fall in the number of agents varying from 80 to 90.

Figure 6: Distribution of Clicks and CTR with Different Number of Agents

3.4 Linear Bidding Strategy

Experimenting on those former bidding strategies, historic data of bidding and use feedback is helpful to predict and optimize the KPI such as number of clicks and CTR for every advertisement impression in auction [12]. The defined linear bidding strategy is based on base_bid adjusted by the ratio of pCTR (need to be predict) and avgCTR. The formula is showing below:

$$Bid\ price = Base\_bid \times \frac{pCTR}{avgCTR} \quad (4)$$

To obtain the optimal base_bid and more accurate pCTR, each step of implementation is firstly explained by concise process to show the whole structure and details of four significant steps is stated including Data Balancing, Feature Engineering, pCTR Optimisation and Base_bid Optimisation. Evaluation of linear strategy is also claimed at the end of this section.

3.4.1 Concise Process

1. Balance data (under-sampling and up-sampling) in training set to the situation that the number of impressions with click to the number of impressions with no-click is 1 to 10.

2. Select proper features and construct some combined features to improve future prediction of user response (pCTR).

3. Pick out three predicting models (logistical regression, XGboost, and random forest) and train these mod-

els on training set to predict optimal user response (pCTR) to online advertising

4. Find out the most precise model with the help of Area Under the Curve (AUC) and tune hyper parameters of models to raise pCTR as high as possible

5. Check the accuracy of predicting CTR in validation set .

6. Use each base_bid in a given interval gained from previous observation and calculate bid price for each impression.

7. Compare the calculated bid price and the pay price of each impression and accumulate all number of clicks when the situation is winning the bidding (bid price is larger than pay price) and is under the constraint of 6,250 CNY fen budget.

8. Figure out the optimal base_bid corresponding to the one with highest number of click.

9. Evaluate whether the linear bidding strategy works well through calculating the number of clicks when winning impressions with budget, CTR, total amount of money paid, average CPM and average CPC based on the optimal base_bid and pCTR.

3.4.2 Data Balancing

This paper finds a significant imbalance between the class of clicks and the class of none click, which may lead a problem that the majority class may have much more impact on further analysis than the minority class, even the minority class will be ignore by learning algorithms [3]. To solve it, under-sampling is implemented to reduce quantity of impressions with no click until the ratio of majority and minority reaches 1 to 10, since under-sampling is efficient when dealing with class imbalance learning[7]. In this state, although two classes of data-set are not perfectly balanced, the data-set becomes more balanced and can be processed much faster.

3.4.3 Feature Preparation

According to the observation of feature data after exploring data and considering the computational expense, this paper filters out some of features which provide unhelpful information when classifying, such as bid_id or user_id. Additionally, features like IP in which almost all values are unique are eliminated so the feature space can be reduced and models are less likely to over-fitting. This paper takes the feature ”user tag” out firstly both in training set and validation set, and processes each tag as a new feature added into feature space, owing to too many types of user tag. Moreover, in order to gain a more comprehensive feature space suitable for universal use, this paper merges all remaining features on both training set and validation set, except for repeated features. Having observed the large quantity of data, data splitting is processed in slot price data depending on several intervals [0, 20], [21, 70], [71, 200] and [201, 5000]. Afterward, one hot encoding is applied to convert all feature variables to a form which is more suitable for classification algorithms to work with.

3.4.4 pCTR Optimisation

Commonly used models containing Logistic Regression, Random Forest and XGBoost are considered in predicting CTR. All models will learn from training set and parameters of each model will be searched. AUC will be applied to measure the performance of fitting, because AUC performs better as a measurement than accuracy empirically and formally when evaluating learning algorithms of classifying [5]. Thus, to obtain the most accurate prediction of pCTR, this paper struggles to improve AUC through utilizing validation set and grid search to help tune hyper parameters gained from training set. Results are displayed in Figure7 in which AUC of XGBoost is the highest among three models.

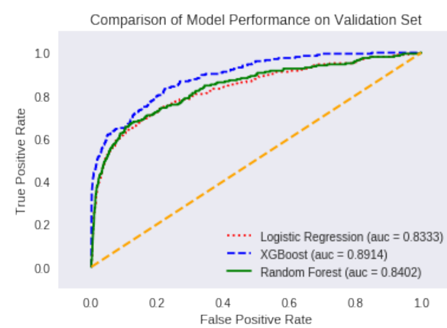


Figure 7: Comparison on Models of Predicting CTR

3.4.5 Base Bid Optimisation

As mentioned in concise process, with the help of optimal pCTR (xxx) and observed value interval of base_bid [1,600], this paper compares the bid price of each impression which is obtained from one of base_bid from [1, 600] and pCTR of each impression to the pay price of each impression. All the winning prices are filtered out and the total number of clicks is summed up. After working out every total number of click corresponding to each base_bid, Figure8 is gained and it is easy to figure out that the optimal base_bid 18 CNY fen is the one who has the largest number of click 165.

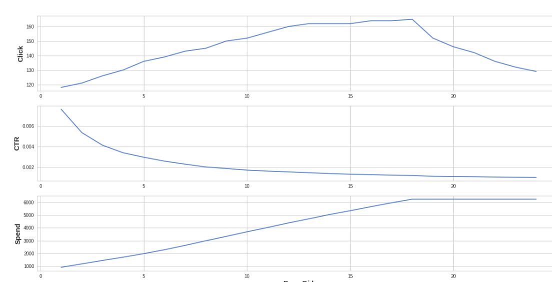


Figure 8: Results of Linear Strategy with Different Value of Base_bid on Validation set

3.4.6 Strategy Evaluating

Table 6 shows that linear bidding strategy (165) wins a much higher number of clicks than random bidding strategy (46) and lightly higher than constant bidding strategy (134). Compared the cost of each strategy, constant strategy and linear strategy both spend as much as they can under the budget, while random strategy leaves nearly 300 CNY fen. A significant difference is that linear bidding strategy (0.12%) shows a much higher CTR than other two strategies (0.031% and 0.044%).

Table 6: Comparison on Basic Bidding Strategies

Strategy	Bid	Clicks	Cost	CTR	CPM	CPC
Constant	25	134	6,249.98	0.031%	15.45	46.64
Random	[11,81]	46	5,958.37	0.0439%	56.84	129.53
Linear	18	165	6,249.97	0.1204%	45.62	37.88

3.5 None Linear Bidding Strategy

Learning from the linear bidding strategy, the result is not good as what has expected. This paper starts to think about how will non-linear bidding strategy perform on real-time bidding. This section will introduce mainly four types of non-linear bidding strategy and try to find out the most fitting one from which agent is benefit in ad bidding. Methods used for data balancing, feature selecting and optimal pCTR discovering are the same as what have done for linear bidding strategy, so this paper mainly discuss about how to do with none linear bidding function.

3.5.1 Strategy: Optimal Real-Time Bidding

This strategy is proposed in a paper called "optimal real-time bidding for display advertising" [12] which is considered to outperform on bidding. Based on advertiser's target rules, the statistics of impressions, feature vectors, KPI for impressions and budget, Zhang et.al (2014) put forward a formula of generating optimal bid:

$$b()_{ORTB} = \underset{b()}{\arg\max}\, N_T \int_x \theta(\mathbf{x}) w(b(\theta(\mathbf{x}), \mathbf{x}), \mathbf{x}) p_x(\mathbf{x}) d\mathbf{x} \quad (5)$$

where $\mathbf{x}$ is feature vector, $b(\theta(\mathbf{x}), \mathbf{x})$ is bidding function, $\theta(\mathbf{x})$ is predicted CTR and $w(b, \mathbf{x})$ is the estimated winning rate when bid price is b with feature vector $\mathbf{x}$. With two assumptions: $b(\theta(\mathbf{x}), \mathbf{x}) \equiv b(\theta(\mathbf{x}))$ and $w(b, \mathbf{x}) \equiv w(b)$ and information that $\mathbf{x}$ and $\theta(\mathbf{x})$ have a deterministic relationship defined by $p_\theta(\theta(\mathbf{x})) = \frac{p_x(\mathbf{x})}{\|\triangledown\theta(\mathbf{x})\|}$, the integration referring to θ is rewritten as:

$$b()_{ORTB} = \underset{b()}{\arg\max}\, N_T \int_\theta \theta w(b(\theta)) p_\theta(\theta) d\theta \quad (6)$$

With the help of Lagrangian of equation (5) (λ denotes Lagrangian multiplier) and calculus of variations, the Euler-Lagrange condition of $b(\theta)$ is:

$$\lambda w(b(\theta)) = [\theta - \lambda b(\theta)] \frac{\partial w(b(\theta))}{\partial b(\theta)} \quad (7)$$

According to equation (7), it is apparent that the bidding function mainly relies on winning function. Thus, taking different winning function leads to different bidding function.

Winning Function 1.

Due to the concave shape of winning rate $w(b)$, a simple wining function formula is $w(b(\theta)) = \frac{b(\theta)}{c+b(\theta)}$ with a constant c and the derivative formula of bid is $\frac{\partial w(b(\theta))}{\partial b(\theta)} = \frac{c}{(c+b(\theta))^2}$. Implementing these two formulas into equation (7) and processing some calculation, the final bidding strategy is:

$$b_{ORTB1}(\theta) = \sqrt{\frac{c}{\lambda}\theta + c^2} - c \quad (8)$$

where θ stands for the pCTR and other two parameters (c and λ) needs to search the optimal ones. The aim is to search for the optimal pair value of c and λ which can help the ORTB1 strategy (equation (8)) achieve the highest number of clicks. λ is initially set value from a list of number [e^{-7}, e^{-6}, e^{-5}, e^{-4}, e^{-3}] and c is randomly selected with step size of 0.0001. Thus, on the basic of each pair value of c and λ, grid search is applied to try every possible pair value and all gained results are compared depending on the number of clicks. After efforts, it is found that highest number of clicks can obtained through ORTB1 strategy is 149 when c is 0.0672 and 0.0671 with λ of e^{-6}, stating in Table7. By comparison of metrics, $c = 0.0672$ is a bit better than $c = 0.0671$, although these two are very similar to each other.

Table 7: Comparison on Statistics of First Two of ORTB1 Strategy in Descending Order

c	Click	Cost	CTR	CPM	CPC
0.0672	149	6249.99	0.0952%	39947.54	41.95
0.0671	149	6249.97	0.0952%	39935.43	41.95

Winning Function 2.

For the purpose of containing the feature that the probability of winning the campaigns will not raise up in at a high speed if the bid price is about 0, while when the bid price is larger than non-zero values, the probability of winning will increase rapidly [14], the winning function is modified as $w(b(\theta)) = \frac{b^2(\theta)}{c^2+b^2(\theta)}$ with curve's increasing point is controlled by c. Therefore, on the basic of new winning function, the new final bidding strategy is:

$$b_{ORTB2}(\theta) = c \cdot \left[\left(\frac{\theta + \sqrt{c^2\lambda^2 + \theta^2}}{c\lambda} \right)^{\frac{1}{3}} - \left(\frac{c\lambda}{\theta + \sqrt{c^2\lambda^2 + \theta^2}} \right)^{\frac{1}{3}} \right] \quad (9)$$

λ is still set value from [e^{-7}, e^{-6}, e^{-5}, e^{-4}, e^{-3}] and similar operations as what have done in ORTB1 strategy are implemented, except that the step size for finding c changes to 1 and the bidding function changes to ORTB2 as equation (9). With the help of grid search, the highest number of clicks based on ORTB2 strategy is 134 when c is 18 under λ is e^{-4}. Table8 reveals the metrics of ORTB2 strategy and the number of click is higher when $c = 18$ than when $c = 19$, although it does not spend the full budget, which means some improvements may be made.

Table 8: Comparison on Statistics of First Two of ORTB2 Strategy in Descending Order

c	Click	Cost	CTR	CPM	CPC
18	134	6053.93	0.0852%	38475.99	45.18
19	130	6249.99	0.0821%	39455.29	48.08

3.5.2 Strategy: Polynomial Bidding

Observing the results of ORTB1 strategy and ORTB2 strategy, this paper is unsatisfied to them and considers to improve the linear bidding strategy, since from the experience of previous machine learning related projects, it may become better fitting results if the linear model is tried to be converted to nonlinear models. Therefore, this paper tries polynomial bidding functions from quadratic function to sixth power function. The entire process of dealing with polynomial bidding strategy is completely same as what have done in linear bidding strategy, but the bidding strategy is transformed to different number of n: $\left(\frac{pCTR}{avgCTR}\right)^n$

Quadratic bidding.

This paper firstly thinks of quadratic model and implement it, but there is still a great potential for making further progress on the model after obtaining results. The power of the function constantly raises up from two to six. It is found that the performance of Quartic bidding strategy is the best when testing the result on test set, and when the power beyond four, the bidding model becomes over-fitting. Therefore, this paper decides to discuss about Quartic bidding strategy in details.

$$Bid\ price = Base\_bid \times \left(\frac{pCTR}{avgCTR}\right)^2 \tag{10}$$

Quartic bidding.

On the basic of optimal pCTR figured out by XGBoost, the highest number of clicks in Quartic bidding strategy is 172 when base_bid is from 13.1 CNY fen to 13.9 CNY fen. The results of evaluating by five metrics are displayed in Table9, which is shown in a descending order based on number of click and CTR. Although several base_bid prices meet the requirements of 172 clicks, base_bid of 13.1 has the highest CTR of 0.001816%.

$$Bid\ price = Base\_bid \times \left(\frac{pCTR}{avgCTR}\right)^4 \tag{11}$$

Table 9: Comparison on Statistics of First Two of Quartic Strategy in Descending Order

base_bid	Click	Cost	CTR	CPM	CPC
13.1	172	6,086.28	0.1816%	64273.80	35.39
13.2	172	6,104.57	0.1811%	64287.08	35.49

3.5.3 Final Results - The Best Bidding Strategy

Comparing all performances of bidding strategy models on the basic of XGBoost predicting CTR, Quartic bidding strategy is best which wins the highest number of clicks.

Table 10: Comparison of Metrics of All Bidding Strategies Combined with XGBoost in Validation set

Strategy	Click	Cost	CTR	CPM	CPC
Const	134	6,249.98	0.031%	15.45	46.64
Random	46	5,958.37	0.0439%	56.84	129.53
Linear	165	6,249.97	0.1204%	45.62	37.88
ORTB1	149	6,249.99	0.0952%	399947.54	41.95
ORTB2	134	6,053.93	0.0852%	38475.99	45.18
Quartic	172	6,086.28	0.1816%	64273.80	35.39

This paper also check the probability distribution of the click's prediction in test set, showing in Figure9. It is obvious that large proportion of prediction falls in the interval of [0.0, 0.2], which means that large quantity of predicts are in low precision.

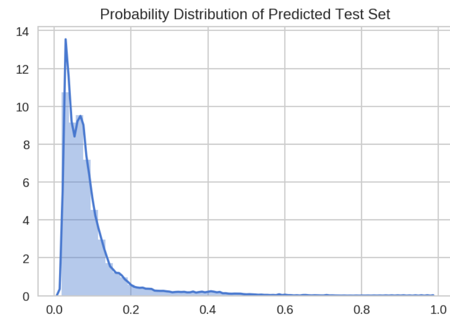


Figure 9: Probability Distribution of Predicted Test Set

3.6 Multi-agent Bidding Strategy

3.6.1 Bayesian Nash equilibrium

Game theory is a idea to optimize strategy based on the predicted actions and real actions of other players. Depending on whether there is a binding agreement between players, game theory can be considered into two types: cooperative game theory and non-cooperative game theory. In addition, based on time series of behaviour, game theory can be divided as static game theory and dynamic game theory. Moreover, game theory also can be separated as complete information game and incomplete information game on the basic of awareness of information about other participants. Learning from this project, this paper consider the real-time bidding in iPinYou dataset is a static game of incomplete information in non-cooperative game theory. Therefore, this paper raises up the idea of Bayesian Nash equilibrium.

Referring to Wang's lecture notes, the equilibrium in second price auction can be stated in equations. Firstly, some assumptions are constructed:

- The value of bidder i is v_i which is not known by other bidders, for example v_1 is the true value of bidder 1.
- The bid price of bidder i is b_i.
- $v_1, ..., v_n$ are distributed independently and identically from F

Consider the situation of bidder 1 as the representative of all bidders, the payoff of bidder 1 is:

$$\begin{cases} v_1 - b_1,\ if\ b_1 > b_i > max[b(v_2, ..., b(v_{i-1}), b(v_{i+1}, ..., b(v_n))] \\ 0,\ if\ b_i \le max[b(v_2), ..., b(v_n)] \end{cases} \tag{12}$$

Given by v_1 and the payoff of bidder 1, the expected payoff when bidding b_1 is:

$$\begin{aligned} \pi(v_1, b_1) &= \int_0^{b_1} (v_1 - x) dF^{N-1}(x) \\ &= \int_0^{b_1} (N-1)(v_1 - x) f(x) F^{N-2}(x) dx \end{aligned} \tag{13}$$

In the case of $b_1 < v_1$, when b_1 is raised up to v_1, the expected payoff will increase by the amount as:

$$\int_{b_1}^{v_1} (N-1)(v_1 - x) f(x) F^{N-2}(x) dx \tag{14}$$

When $b_1 = v_1$, it is telling the truth and it is a Bayesian Nash equilibrium bidding strategy.

3.6.2 Observation and Analysis

Since the two leaderboards reflect the performance of the same bidding submission, we had many chances to observe the fluctuation of performance on the criterion #2 leaderboard. Because we submitted multiple bidding plans for the criterion #1 leaderboard to tune the parameters of our CTR estimation model and bidding models.

Situation 1.

Observing the criterion #2 leaderboard, two extreme cases caught our eyes. First, we noticed that some of the groups used up all the budget and achieved a fairly large yet reasonable number of impressions. Cross comparing their performance on the criterion #1 leaderboard, those with better prediction ability also earned more clicks on the criterion #2 leaderboard. Their consistency in the number of clicks, the amount of impression, and the budget usage leads us to the speculation that their bidding plans mostly rely on the accuracy of their CTR prediction model. The better their CTR prediction model, the higher price they would bid for those who have a greater chance of being clicked. But how much higher, exactly? That depends on the bidding strategy each group adopts. In our case, we used a Quartic bidding strategy developed upon the basic linear bidding function, as discussed before. However, in reality, this kind of strategies does not show optimal performance. Our CTR prediction model stayed top of the criterion #1 leaderboard, so with high confidence, we submitted our Quartic bidding plan, which showed the best performance on validation set. The result shows that we achieved a small number of clicks, far from our expectation. Based on this disappointing result, we decided to thoroughly reconsider our bidding strategy. And do some more in-depth research and speculation on other groupsâĂŹ strategies.

Situation 2.

The other case showed an opposite result. A small number of groups used only a small portion of their total budget yet achieved a disproportionate amount of impression âĂŤ even more than most of the groups who used up their budget. Furthermore, they achieved an exceptionally high number of clicks, taking the lead on the board. Based on their abnormally low CPC (about 50 times lower than others) and high CTR, we suspected that they avoided competing in the area where the impressions have the highest pCTRs and therefore higher bidding prices on them, instead, they focused on the âĂIJtailâĂİ of the pCTR distribution âĂŤ bidding the ones with the lower pCTRs. We suspected that this strategy would work because most of the groups believe in their predicting models just like we did in the first place. We will naturally bid high on those that we believe would be clicked. This result in the high competition in that area of pCTR distribution, and consequently no one could monopolise the clicks. In the opposite direction, if you bid higher on those with low pCTRs, you might be able to grab some market share. According to our research, two facts supported our speculation. Firstly, we surprisingly found that even a simple random bidding strategy could yield a fairly good result. This means our CTR prediction could matter less than our expectation. Secondly, our unmodified bidding strategies bid for all impressions, mostly proportional to the pCTRs. Therefore, if we deliberately miss the impressions whose prices were pushed up too high, and raise the bid for those less noticed, we may be able to grab more from the neglected impressions. Inspired by this idea, we worked hard to develop and tune our bidding strategies for higher place on the criterion #2 leaderboard.

We then further developed our bidding strategies to optimise our clicks and CTR. We stick to our principle that care less about the fierce-competition area and pay more attention on the less-likely overpriced section of the distribution.

3.6.3 Proposed Bidding Strategy

Concretely speaking, since bid truth is the only nash equilibra of 2nd price auction, then for this strategy we decided to use truth bidding as our basic strategy. According to analystic and observation from 3.6.2 we found that bidding truth purely is no longer the optimal strategy for this scenario. Thus, we made some modifications on the bidding price. According to the predicted probability distribution of test set, we made a linear transformation on the bidding price of probability in range 0.0 to 0.3, the function we used is y = 1/0.3*pCTR . Moreover, we did a linear transformation on the bidding price of probability in range 0.85 to 1 as well, the function we use is âĂę Beside these linear transformation, for the bidding price in probability range 0.3 to 0.85, we increase 10% on the original bidding price in order to ensure the number of impressions we get.

$$y = \frac{1}{0.3} pCTR \tag{15}$$

$$y = 0.85 pCTR + \frac{1}{0.15} \tag{16}$$

4. CONCLUSION

In this project, several bidding strategies are implemented both in the case of single agent and multi-agent. Each strategy can be separated into two main parts, CTR estimation and bidding function. With the help of several techniques such as grid search. Initially, three basic bidding strategies are evaluated and are considered as baseline for further researches on other bidding strategies. Three CTR prediction models are tested and compared with each other: Logistic Regression, Random Forest and XGBoost. In addition, four bidding functions are applied to calculate bid price for each impressions and Quartic bidding strategy outperforms on the basic of pCTR predicted by XGBoost.

Future Work.

Firstly, there is a problem in the feature preparation of this work. When slicing the slot price, the interval is chosen randomly without enough reference to support, which needs to make improvements in the future. Due to the limitation of time and computational ability, this paper suggest that multi-agent reinforcement learning can be tried in case of multi-agent and this algorithm may perform much better than the strategies used in the former sections. Additionally, this paper also considers about the method called regret value method in decision making. It will help the player to choose the one with the lowest regret value.

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