Homework 8

Problem #1

6.28 (use R for calculations)

6.42

6.43

6.51

Problem #2

1. a. **(+1 bonus pt)** For an arbitrary **continuous** random variable: derive the form of $E[aX + b]$ as a function of a, b and $E[X]$.

   b. Using the results for $E[aX + b]$ and $Var[aX + b]$, proceed to show that for $Z = \frac{X-\mu}{\sigma}$, $X \sim N(\mu, \sigma^2)$, we have

   - $E[Z] = 0$,
   - $Var[Z] = 1$.

   Hint: Keeping in mind that μ, σ are **constants**, and X - random variable, represent Z as $aX + b$, where a, b **only contain constants**.

2. For Bernoulli random variable $X \sim Bernoulli(p)$, show that

   a. $E[X] = p$.
   b. $V[X] = p(1 - p)$.

Problem #3

The most recent Major League Baseball scandal involved Houston Astros being caught cheating in the 2017 World Series (it was discovered restrospectively, and revealed no earlier than 2020), where they stole opponents' pitching signs by illegally using technology during their home games. Below is a video of some former baseball player mentioning how in Game 5 of that 2017 World Series - which was played in Houston - their **opposing pitcher Clayton Kershaw** threw 51 **pitches that were either sliders or change-ups, with Astros' hitters not biting on any of them (**0/51 **of swings & misses on these pitches)**, which is a **suspiciously amazing performance for the Astros hitters**:

https://www.youtube.com/watch?v=elpGtkt64Ps

Moreover, in that video he talks about the typical % of swings and misses that Kershaw got on change-ups and sliders that 2017 season. Aggregating them (change-ups with sliders), let's say that throughout the

season leading up to the World Series, Kershaw had opponents swing-and-miss ("success" for Kershaw) at 27% of his change-up & slider pitches.

Let's use some basic probability and statistics here to test out the following hypotheses:

H_0 : "Astros did NOT cheat in Game 5 of 2017 World Series", vs H_a : "Astros cheated in Game 5 od 2017 World Series"

1. ~~Explain how, **if the H_0 hypothesis were true**, Kershaw's 51 pitches in the World Series Game 5 may constitute a binomial setting. What's a success/failure here? What's the probability of a success? How many trials? What other assumption(s) should we think about, and is it (are they) likely to be satisfied?~~

2. Presuming that H_0 hypothesis were actually true, and Kershaw's 51 pitches were indeed binomially distributed, calculate the

    a. ~~Expected value for the # of successful pitches for Kershaw out of 51 pitches thrown.~~
    b. ~~Probability of Astros having 0 swings-and-misses on his change-ups & slider pitches **just by chance**. Would it seem likely? Hence, would you reject H_0 or not?~~