Homework 9, Your Name.

Problem #1 (Central Limit Theorem)

In similar fashion to what I did for sample proportion in the lecture code, proceed to use R and generate the following values:

$$X_1, X_2, ..., X_{10.000} \sim Uniform(-2, 2)$$

subsequently demonstrating (once again, using R) the aspects of sampling distribution for \bar{X} , such as its

- shape (is it bell-shaped, or not quite?)
- mean (is it unbiased?)
- standard deviation (does it correspond to theoretical st. dev.?)

For more details on Uniform distribution (e.g. its mean, variance, etc) check https://en.wikipedia.org/wiki/Uniform_distribution_(continuous).

Do it for:

a. n = 2;

b. n = 50.

Problem #2 (Coverage of Confidence Interval)

- 1. Write your own function that takes as arguments:
- total # of observations,
- number of successes
- confidence level (default at 0.95)

and calculates the confidence interval for population proportion, outputting a 2-element vector (left and right end of the interval, respectively).

2. Proceed to generate 10,000 values from Bin(n = 1000, p = 0.6) distribution, and use your function from part 1 in order to calculate & record the 95% and 90% intervals for each of those 10,000 values. Calculate the % of times your confidence intervals actually contain the true population proportion p = 0.6. Is it what you expect for 95% and 90% intervals, accordingly?

Some code to start you off:

```
set.seed(1)
n.sim <- 10000
prob <- 0.6
size <- 100
# Placeholders for left (first column) and right (second column) ends
# of our GIs.
ci.95 <- matrix(0, nrow=n.sim, ncol=2)
ci.90 <- matrix(0, nrow=n.sim, ncol=2)

# Here you will need to
# 1) generate the 10,000 values from Bin(1000,0.6),
# 2) loop through those and feed them as input to your confidence level function from part 1
# (for cases of 95% and 90%)
#...
# That's an example of how you calculate the % of times your confidence interval
# contains the true parameter
mean(ci.95[,1] < prob & ci.95[,2] > prob)
#...
```

Problem #3

We will now consider the Boston housing data set, from the MASS library.

- (a) Based on this data set, provide an estimate for the population mean of medv. Call this estimate $\hat{\mu}$.
- (b) Provide an estimate of the standard error for $\hat{\mu}$ via
 - central limit theorem (use theoretical formula);
 - bootstrap (over 10,000 replicates).

How do these compare to each other?

- (c) Based on this data set, provide an estimate, $\hat{\mu}_{med}$, for the median value of medv in the population.
- (d) We now would like to estimate the standard error of $\hat{\mu}_{med}$. Unfortunately, there is no simple formula for computing the standard error of the median. Instead, estimate the standard error of the median using the bootstrap.

Problem #4 (Make sure to use R when appropriate for carrying out the calculations, showing your work)

```
7.7
7.14
7.15 (not necessary to provide the graph, but make sure to play with the app at https://istats.shinyapps.io/sampdist_cont/)
7.20
8.6
```

8.13

8.16

8.25 - 8.26

8.29-8.30

8.37