Homework Set 1

Miles Van de Wetering Cierra Shawe Charles Hill

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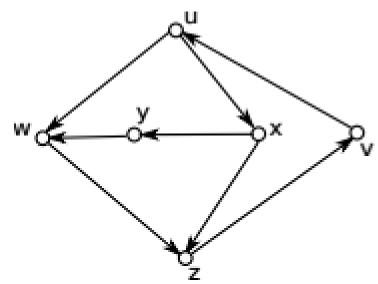
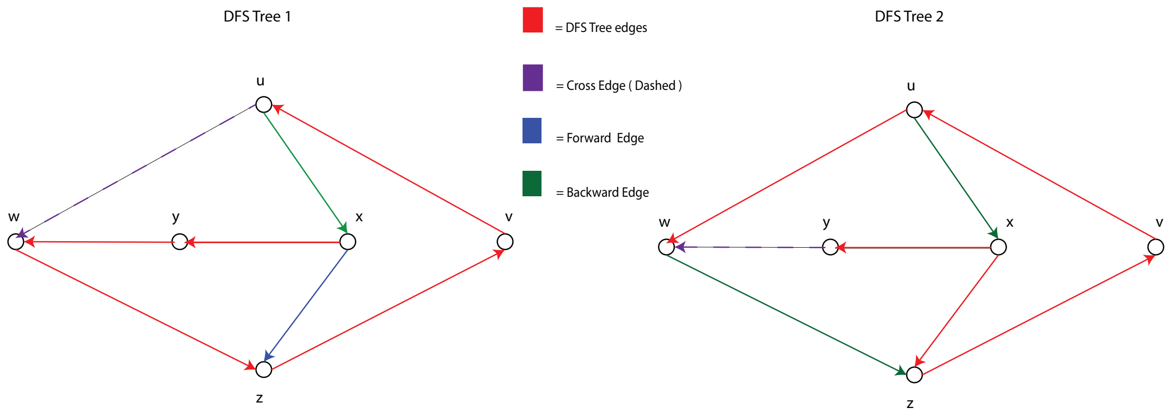
Problem 1

Draw all possible DFS trees rooted at x for the following graph. Mark, tree, forward, backward, and cross edges





Problem 3

Let G = (V, E) be an undirected unweighted graph, and let s, t ∈ V. Show that at least one of the following conditions hold:

1. The distance between s and t is at most $\sqrt{V}+1$.
2. There is a subset S ⊂ V of cardinality at most $\sqrt{V}$ whose removal disconnects s and t.

In order to show this we will build an argument based on three truths:

1. Any single path, in any graph, may be destroyed by removing a single vertex along that path.
2. If condition (1) is false, then every path between s and t must pass through at least $\sqrt{V}+3$ vertices.
3. The most vertices that we may need to remove in order to disconnect every path from s to t is the number of paths between s and t, assuming that these paths share no vertices except s and t. If they share more vertices, then we can remove fewer nodes and still succeed. (Menger Theorem)

From these three premises, we may conclude that the maximum number of unique paths that can exist between s and t is less than or equal to the number of non start-end nodes available in the graph, divided by the number of non start-end nodes required to construct a unique path:

$$\frac{V-2}{\sqrt{V}+1} \leq \frac{V}{\sqrt{V}+1} \leq \frac{V}{\sqrt{V}} = \sqrt{V}$$

Therefore, the number of unique paths that may exist is less than or equal to $\sqrt{V}$, and therefore the maximum number of nodes that we must remove in order to disconnect s and t, given (1), is strictly bounded by $\sqrt{V}$, thus (2) holds.

Since ¬(1) implies (2), ¬(2) must imply (1).