Prove that: p prime and $p \neq 5$ and $p \neq 2 \implies 10 \mid p^2 + 1$ or $10 \mid p^2 - 1$.

Proof. 3.1.10
We know p is odd, so it must not be able to take the form $2k, k \in \mathbb{Z}$.
By the division algorithm, all $p > 5$ odd primes can be represented by $10k+r$ with $r \in [0, 9]$.
Because $2 \mid 10k$, we know by theorem 2.2 that we must have $2 \nmid r$.
Because $5 \mid 10k$, we know by theorem 2.2 that we must have $5 \nmid r$.
Therefore, $r \notin \{0, 2, 4, 5, 6, 8\} \implies r \in \{1, 3, 7, 9\}$.
We have

$$\begin{aligned}
&(10k + r)^2 \pm 1 \\
=&100k^2 + 20rk + r^2 \pm 1 \\
=&10(10k^2 + 2rk) + r^2 \pm 1.
\end{aligned}$$

We know $10 \mid 10(10k^2 + 2rk)$.
Therefore, by theorem 2.2, $10 \mid r^2 \pm 1 \implies 10 \mid p^2 \pm 1$. (1)
We have

$$\begin{aligned}
& 1^2 \pm 1 = 0 \text{ or } 2 && \text{and} && 3^2 \pm 1 = 8 \text{ or } 10 \\
\text{and} \quad & 7^2 \pm 1 = 48 \text{ or } 50 && \text{and} && 9^2 \pm 1 = 80 \text{ or } 82.
\end{aligned}$$

We have

$$\begin{aligned}
& 10 \mid 0 && \text{and} && 10 \mid 10 \\
\text{and} \quad & 10 \mid 50 && \text{and} && 10 \mid 80
\end{aligned}$$

Therefore, $10 \mid r^2 \pm 1 \; \forall \; r \in \{1, 3, 7, 9\}$.
Therefore, by (1), $10 \mid p^2 \pm 1$. $\square$