

Proof. 2.3.4c

Prove $5|3^{3n+1} + 2^{k+1}$.

Assume $n = 0$.

We have $3^1 + 2^1 = 5$.

By Theorem 2.2a, we know $5|5$.

Assume $n = k$ holds.

We have

$$5|3^{3k+1} + 2^{k+1}. \quad (1)$$

Assume $n = k + 1$.

We have

$$\begin{aligned} & 3^{3(k+1)+1} + 2^{(k+1)+1} \\ &= 3^{3k+4} + 2^{k+2} \\ &= 3^3(3^{3k+1}) + 2(2^{k+1}) \\ &= 3^3(3^{3k+1}) + 2(2^{k+1}) + 2(3^{3k+1}) - 2(3^{3k+1}) + 3^3(2^{k+1}) - 3^3(2^{k+1}) \\ &= (3^3 + 2)(3^{3k+1} + 2^{k+1}) - 5(3^2 * 2^k + 3^{3k}) \end{aligned}$$

We know $5|(3^3 + 2)(3^{3k+1} + 2^{k+1})$ by (1) and $5|5$ by Theorem 2.2a.

By Theorem 2.2g this implies $5|3^{3(k+1)+1} + 2^{(k+1)+1}$.

By FPMI this implies $5|3^{3n+1} + 2^{k+1} \forall n \geq 0$. □