3.3.20) Prove p, p^2+8 are primes $\implies p^3+4$ is prime.

Proof. We know any prime $p > 3$ can take the form $p = 6k+1$ or $p = 6k+5$. Take $p = 6k+1$. We have

$$\begin{aligned} p^2+8 =& (6k+1)^2+8 \\ =& 36k^2+12k+1+8 \\ =& 36k^2+12k+9 \\ =& 6(6k^2+2k+1)+3. \end{aligned}$$

It is clear that $6(6k^2+2k+1)+3$ is not a prime.
Take $p = 6k+5$. We have

$$\begin{aligned} p^2+8 =& (6k+5)^2+8 \\ =& 36k^2+60k+25+8 \\ =& 36k^2+60k+33 \\ =& 6(6k^2+10k+5)+3. \end{aligned}$$

It is clear that $6(6k^2+10k+5)+3$ is not prime. Therefore, we can conclude $p \leq 3$. This means we have at least $p \in \{2, 3\}$.
Take $p = 2$. We have

$$\begin{aligned} p^2+8 =& 2^2+8 \\ =& 4+8 \\ =& 12. \end{aligned}$$

It is clear that 12 is not prime. Therefore $p \neq 2$.
Take $p = 3$. We have

$$\begin{aligned} p^2+8 =& 3^2+8 \\ =& 9+8 \\ =& 17. \end{aligned}$$

17 is prime, so the condition holds if and only if $p = 3$. We have $p = 3$. Take

$$\begin{aligned} p^3+4 =& 3^3+4 \\ =& 27+4 \\ =& 31. \end{aligned}$$

31 is prime. Therefore, we have that p, p^2+8 are primes $\implies p^3+4$ is prime. $\square$