Proof. Prove

$$\frac{1}{1} + \frac{1}{3} + \frac{1}{6} + \frac{1}{10} + \ldots + \frac{1}{t_n} < 2. \tag{1}$$

Triangular numbers are the sum of series' of integers from 1 to k such that

$$\begin{aligned} t_k &= 1 + 2 + 3 + \ldots + k \\ &= \frac{1}{2}k(k+1). \end{aligned}$$

With some algebra, we can see that

$$\begin{aligned} \frac{1}{t_k} &= \frac{2}{k(k+1)} \\ &= 2\frac{k+1-k}{k(k+1)} \\ &= 2(\frac{k+1}{k(k+1)} - \frac{k}{k(k+1)}) \\ &= 2(\frac{1}{k} - \frac{1}{k+1}). \end{aligned}$$

Now we can see (1) is equivalent to

$$2(\frac{1}{1} - \frac{1}{2}) + 2(\frac{1}{2} - \frac{1}{3}) + 2(\frac{1}{3} - \frac{1}{4}) + 2(\frac{1}{4} - \frac{1}{5}) + \ldots + 2(\frac{1}{n} - \frac{1}{n+1}).$$

We can see that the second part of each term cancels the first of the next, leaving us only with the first part of the first term and the second of the last, or

$$\frac{1}{1} + \frac{1}{3} + \frac{1}{6} + \frac{1}{10} + \ldots + \frac{1}{t_n} = 2(1 - \frac{1}{n+1}). \tag{2}$$

We know $n \geq 1$, so

$$\begin{aligned} \frac{1}{n+1} &> 0 \\ -\frac{1}{n+1} &< 0 \\ 1 - \frac{1}{n+1} &< 1 \\ 2(1 - \frac{1}{n+1}) &< 2. \end{aligned}$$

This combined with (2) proves (1). □