

Proof. 2.3.4b

Prove $15|2^{4n} - 1$.

Assume $n = 0$.

We have $2^0 - 1 = 0$.

By Theorem 2.2a we know $15|0$.

Assume $n = k$ holds. This means

$$15|2^{4k} - 1. \quad (1)$$

Assume $n = k + 1$.

We have

$$\begin{aligned} & 2^4(k+1) - 1 \\ &= 2^{4k+4} - 1 \\ &= 16(2^{4k}) - 1 \\ &= 16(2^{4k} - 1) - 1 + 16 \\ &= 16(2^{4k} - 1) + 15. \end{aligned}$$

We know $15|16(2^{4k} + 1)$ by (1) and $15|15$ by Theorem 2.2a.

By Theorem 2.2g, this implies $15|2^{4(k+1)} - 1$.

By FPMI we can imply $15|2^{4n} - 1 \forall n \geq 0$. □