3.3.2a) Prove that $1 + p_1 p_2$ is always a perfect square given that p_1, p_2 are twin primes.

Proof. We have $p_1 + 2 = p_2$.
This means we can show p_1 as $n - 1$ and p_2 as $n + 1$.
We have

$$\begin{aligned}
&p_1 p_2 + 1 \\
=&(n-1)(n+1) + 1 \\
=&n^2 + 1 - 1 \\
=&n^2.
\end{aligned}$$

Therefore, the square of any integer is one more than the product of the numbers on either side of it, implying $1 + p_1 p_2 = n^2$ for all twin primes p_1, p_2. $\square$

3.3.2b) Prove that the sum of twin primes p_1, p_2 is divisible by 12 given $p_1, p_2 > 3$.

Proof. We have $p_1 + 2 = p_2$.
We know by the division algorithm all primes $p > 3$ can be shown as $p = 6k + 1$ or $p = 6k + 5$ for some $k \in \mathbb{Z}$.
We know $p_1 + 2 = p_2$ so $p_1 \neq 6k + 1$ because $6k + 1 + 2 = 6k + 3 \neq 6s + 1$ or $6s + 5 \ \forall \ s \in \mathbb{Z}$.
Therefore we know $p_1 = 6k + 5$.
This implies

$$p_2 = p_1 + 2 = 6k + 5 + 2 = 6k + 7 = 6(k + 1) + 1$$

which can represent a prime number.
We have

$$\begin{aligned}
&p_1 + p_2 \\
=&p_1 + p_1 + 2 \\
=&2(6k + 5) + 2 \\
=&12k + 10 + 2 \\
=&12k + 12 \\
=&12(k + 1).
\end{aligned}$$

It is clear that $12 \mid 12(k + 1) \implies 12 \mid p_1 + p_2$. $\square$