Electrostatics

Introduction to Electric Field

- a bunch of positives (negatives) would repel away from each other.
- the opposite pieces would attract each other.
- the net result is a balance! Balance is formed by tight fine mixtures of positives and negatives.
- there is no attraction/repulsion between them

What we described is exactly electrical force. All matter is a mixture of positive protons and negative electrons in a perfect balance. What is the expression for the strength and direction of this force? Coulombs law.

$$\overrightarrow{\mathbf{F_e}} = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2}\hat{R_{12}} \tag{1}$$

In the above equation, ε_0 is electrical permittivity, q_1, q_2 electrical charge and r is the distance between charges.

How perfect is this balance?

EXAMPLE

Lets calculate the repulsive force if there was a little bit of unbalance. Say that each of these two tables had 100 extra electrons. Lets try to calculate the repulsive force.

Electromagnetic force is one of four we know today. Lets discover the other forces.

Learning outcomes: Electrostatic fields.
Author(s): Milica Markovic

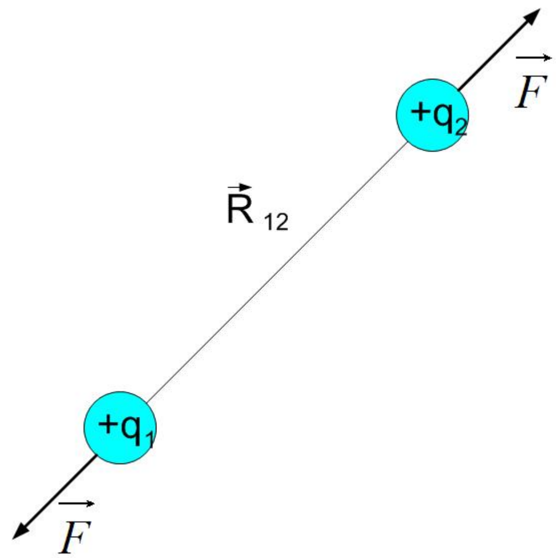

Figure 1: Vector representation of Coulomb's force between two static charges.

Why is it that the atomic nucleus stays together when it is made out of the same kind of matter?

We just elaborated that if two charges are of the same kind, the electrostatic force will push them away from each other. It seems that there needs to be another kind of force that keeps the nucleus together. This force is called the nuclear force. This is the strongest force, but acts at a short distance. For example if we have a lot of protons in the nucleus such as in radioactive elements the nucleus can split by just lightly tapping it.

The last force is a weak-interaction force that plays role in radioactive decay.

Which four forces did we talk about today?

- Nuclear Force
- Electromagnetic Force
- Weak-Interaction Force
- Gravitational Force.

Coulomb's Law

Lets review the Coulomb's Law that governs the behavior of electrostatic force.

(a) Like charges repel

(b) Opposite charges attract

(c) The force acts along the line that joins the charges

(d) The strength of the force is given by the expression .

Whats the difference between the terms force and field?

Another bunch of questions could be:

- How do we now if we are in a gravitational field.
- What is the difference between the gravitational field and gravitational force?
- More specifically what is the meaning of the term field anyway?

Lets try to answer some of these questions.

More about the gravitational force and field

How do we know that we are in the gravitational field and not in zero gravity field? No matter how hard we try to launch ourselves in the outer space by jumping, we still come back to mother earth. If we drop a pencil, where will it go? Why is that so? The gravitational force attracts the pencil (and us). Another way to say that a gravitational force exists is to say that there is the field of force acting on an object. This is our first answer to the question: What is that term **field** anyways. Let's talk more about fields.

We know that the gravitational force acts at distance. There is no giant muscle that attracts our pencil. Earth induces a gravitational field, it's influence exists at every point in space around it. This phenomenon of direct action on a distance has given rise to the concept of fields.

Lets see an example of gravitational force.

What is the source of the earth's gravitational force? Earth (of course). It would be good if we can define a quantity to show what is the strength of this force at any point in space.

We can define gravitational field at any point in space through the gravitational force: If an object with mass m_m existed at the point r away from earth, it

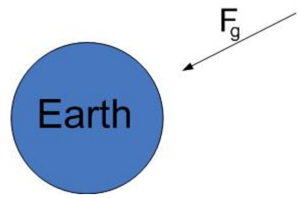

Figure 2: Gravitational force.

would experience the force F_g, we can say that the gravitational field at that point is equal to

$$\Psi = \frac{\overrightarrow{\mathbf{F_g}}}{m_m} \tag{2}$$

$$\Psi = \gamma \frac{m_e}{r^2}\hat{r} \tag{3}$$

We dont need the moon in any particular spot to know what would be the gravitational field at that particular point. Note that the field does not depend on the moons mass! It depends only on the earths mass, gravitational constant and the distance to the point we want to find the field.

Electrostatics

More about the Electrical force and field

The same situation we have with the electrical force and field. The electric field is defined as the force that a charge would experience divided by the charge.

$$\overrightarrow{\mathbf{E}} = \frac{\overrightarrow{\mathbf{F_e}}}{q_2} \tag{4}$$

$$\overrightarrow{\mathbf{E}} = \frac{q_e}{4\pi\varepsilon_0 r^2}\hat{r} \tag{5}$$

What is the source of the electrical force or field? Electric charge (of course).

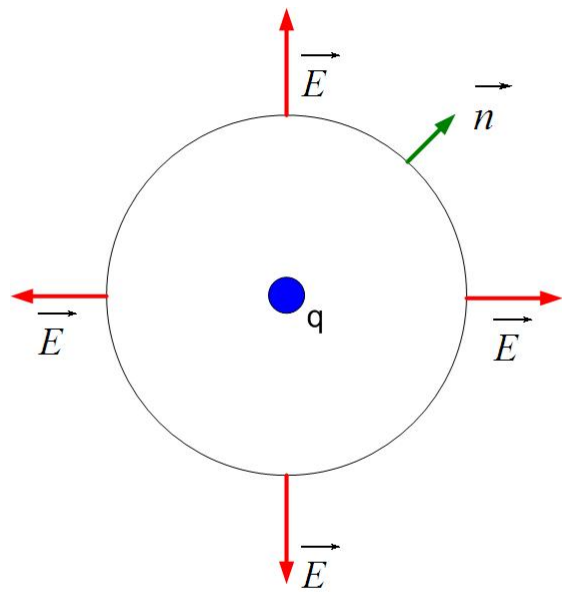

Figure 3: Electric field due to a unit charge q.

Properties of Electric Charge

Electric charge cannot be created or destroyed.

If the total net charge of an object is q, and if that object has n_e electrons and n_p protons, then the total charge is $q = n_p e - n_e e$.

What is the electric field if we have more than one charge?

The total electric field at a point in space from the two charges is equal to the sum of the electric fields from the individual charges at that point.

What if the charge is not in air?

Lets look at Coulombs law again.

$$\overrightarrow{\mathbf{F_e}} = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2}\hat{R_{12}} \tag{6}$$

Which quantity in this formula depends on the material? ε_0. If the charge is within a dielectric material, then we need to account for that by changing this

ε_0 somehow. If we place the charge inside a dielectric material what do you think will happen with the atoms in the material? The atoms will get distorted and polarized. Such a polarized atom we call an electric dipole. The distortion process is called polarization. Because the material acts in such a way, the electric field around this point charge is different than if there was no material. In any dielectric medium, the electric field is defined as

$$\overrightarrow{\mathbf{F_e}} = \frac{q_1 q_2}{4\pi\varepsilon r^2}\hat{R_{12}} \tag{7}$$

$$\varepsilon = \varepsilon_0 \varepsilon_r \tag{8}$$

We added unitless quantity ε_r, relative dielectric constant. ε_r values for different materials is shown in one of the tables in the book. Lets see its values for different materials. ε_r varies from 1 (air), to 2.2 (Teflon) to 80 (water).

Electric field density is the quantity that we introduce here:

$$\overrightarrow{\mathbf{D}} = \varepsilon \overrightarrow{\mathbf{E}} \tag{9}$$

Principle of Superposition

If we have two charges, the total field due to both charges is equal to the vector sum of the fields due to individual charges, see Figure **??**. The field at

The fields or charges q_1 and q_2 are:

$$\overrightarrow{\mathbf{E_1}} = \frac{q_1}{4\pi\varepsilon_0 {r_a}^2}\hat{r_a} \tag{10}$$

$$\overrightarrow{\mathbf{E_2}} = \frac{q_1}{4\pi\varepsilon_0 {r_b}^2}\hat{r_b} \tag{11}$$

Where $\hat{r_a}$ and $\hat{r_b}$ are unit vectors in the direction of r_a and r_b. The total field due to both charges is

$$\overrightarrow{\mathbf{E}} = \overrightarrow{\mathbf{E_1}} + \overrightarrow{\mathbf{E_2}} \tag{12}$$

Electric Field in Rectangular Coordinates

In general equation for the electric field is given as

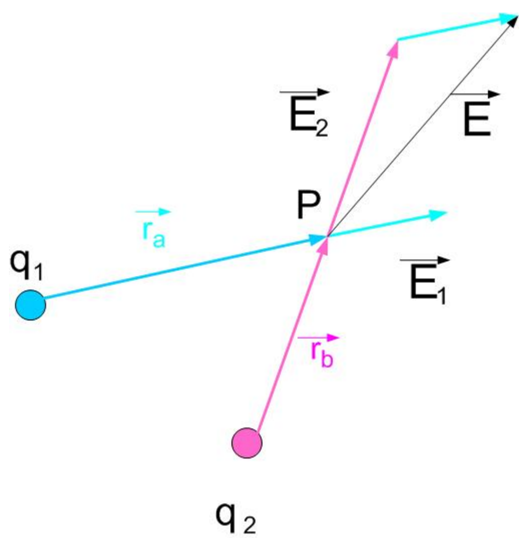

Figure 4: Electric Field due to two charges.

$$\overrightarrow{\mathbf{E}} = \frac{q_1}{4\pi\varepsilon_0 {r_a}^2}\hat{r_a} \tag{13}$$

The electric field at a point $P(x, y, z)$ due to a charge q_1 positioned at a point $P_{q_1}(x_1, y_1, z_1)$ in the rectangular coordinate system is shown in Figure 6. The position vector of the point P_{q_1} is

$$\overrightarrow{\mathbf{r_1}} = x_1\overrightarrow{\mathbf{x}} + y_1\overrightarrow{\mathbf{y}} + z_1\overrightarrow{\mathbf{z}} \tag{14}$$

The position vector of point P is equal to

$$\overrightarrow{\mathbf{r_p}} = x\overrightarrow{\mathbf{x}} + y\overrightarrow{\mathbf{y}} + z\overrightarrow{\mathbf{z}} \tag{15}$$

The two vectors mark the beginning and the end of the distance vector $\overrightarrow{\mathbf{r_a}}$ between points P_{q_1} and P. The vector $\overrightarrow{\mathbf{r_a}}$ is the sum of vectors $-\overrightarrow{\mathbf{r_p}}$ and $\overrightarrow{\mathbf{r_1}}$

$$\overrightarrow{\mathbf{r_a}} = \overrightarrow{\mathbf{r_p}} + (-\overrightarrow{\mathbf{r_1}}) \tag{16}$$

When we substitute position vectors r_1 and r_p:

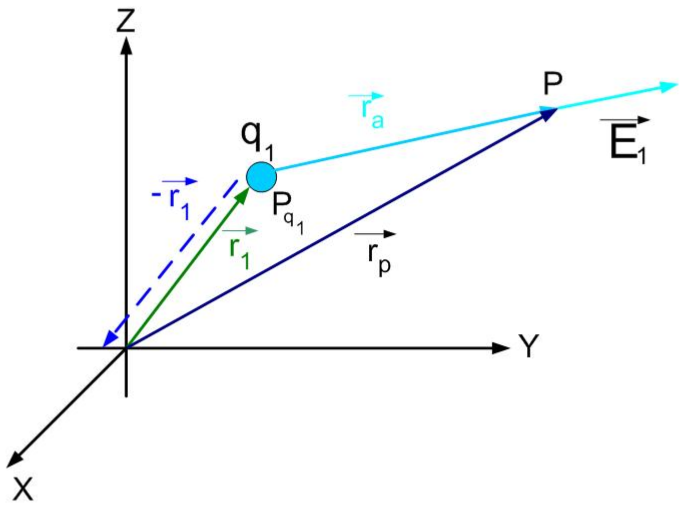

Figure 5: Electric Field due to a unit charge in Rectangular coordinate system.

$$\overrightarrow{\mathbf{r_a}} = (x - x_1)\overrightarrow{\mathbf{x}} + (y - y_1)\overrightarrow{\mathbf{y}} + (z - z_1)\overrightarrow{\mathbf{z}} \tag{17}$$

Vector $\overrightarrow{\mathbf{r_a}}$ has the magnitude of:

$$|\overrightarrow{\mathbf{r_a}}| = \sqrt{(x - x_1)^2 + (y - y_1)^2 + (z - z_1)^2} \tag{18}$$

Unit vector in the direction of vector $\overrightarrow{\mathbf{r_a}}$ is:

$$\hat{r_a} = \frac{\overrightarrow{\mathbf{r_a}}}{|\overrightarrow{\mathbf{r_a}}|} \tag{19}$$

$$\hat{r_a} = \frac{\overrightarrow{\mathbf{r_a}}}{\sqrt{(x - x_1)^2 + (y - y_1)^2 + (z - z_1)^2}} \tag{20}$$

$$\overrightarrow{\mathbf{E_1}} = \frac{q_1}{4\pi\varepsilon_0 {r_a}^2}\hat{r_a} \tag{21}$$

Where r_a is the distance between the charge q_1 and the point P. Substituting expressions for $\hat{r_a}$, and $|\overrightarrow{\mathbf{r_a}}|$ in equation 13 we get

$$\overrightarrow{\mathbf{E_1}} = \frac{q_1}{4\pi\varepsilon_0\sqrt{(x-x_1)^2+(y-y_1)^2+(z-z_1)^2}^3}\overrightarrow{\mathbf{r_a}} \tag{22}$$

Substituting

For two charges, as shown in Figure **??** equation 22 becomes

$$\overrightarrow{\mathbf{E}} = \frac{q_1}{4\pi\varepsilon_0\sqrt{(x-x_1)^2+(y-y_1)^2+(z-z_1)^2}^3}\overrightarrow{\mathbf{r_a}} + \frac{q_2}{4\pi\varepsilon_0\sqrt{(x-x_2)^2+(y-y_2)^2+(z-z_2)^2}^3}\overrightarrow{\mathbf{r_b}} \tag{23}$$

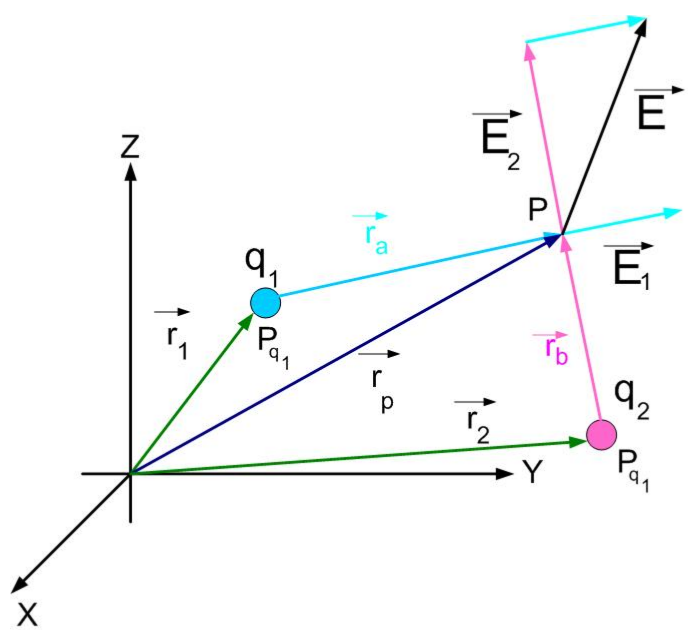

Figure 6: Electric field due to two charges in Rectangular coordinate system.

Electric Field Distributions

Example of Line Charge Distribution

Find the field at the z-axis of a loop of charge. Charge is uniformly distributed along the loop with line charge density of ρ_l.

Line charge distribution

Surface charge distribution

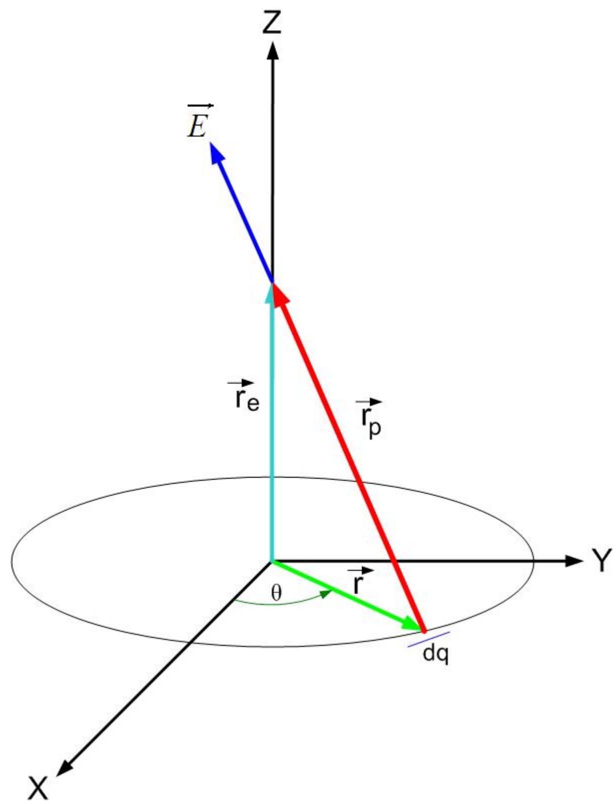

Figure 7: Loop of wire uniformly charged with line charge density ρ_l. Electric field is shown due to a very small section of the loop.

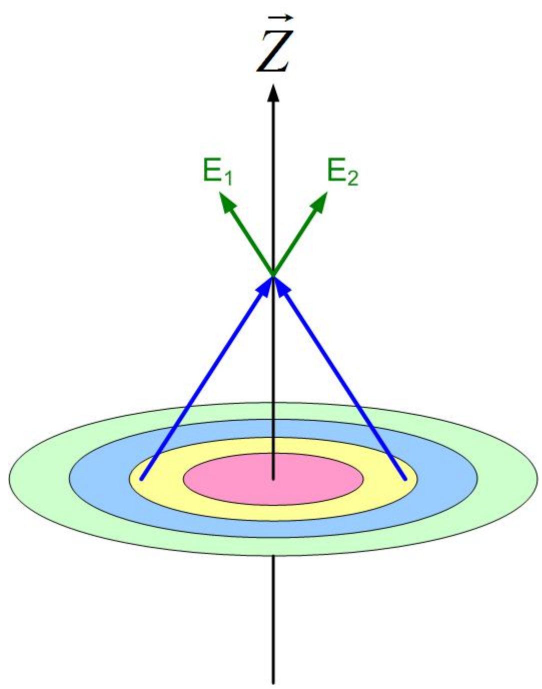

Figure 8: Disk of charge can be regarded as an infinite number of concentric rings of charge.

DISK of charge

Infinite plane

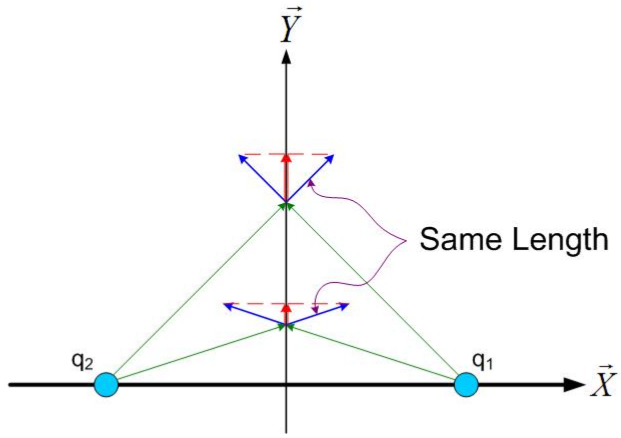

Figure 9: Electric field from two rings located on the infinite plane.

Volume charge distribution

EXAMPLE Line charge distribution Loop of wire

EXAMPLE Surface charge distribution Disk

EXAMPLE Volume charge distribution diode

Gauss Law

EXAMPLE Wire

Figure 10: Application of Gauss's Law to find Electric Field of wire.

EXAMPLE Infinite Plane

EXAMPLE Two Infinite Planes

Definition of Potential and Voltage

EXAMPLE Potential due to unit charge

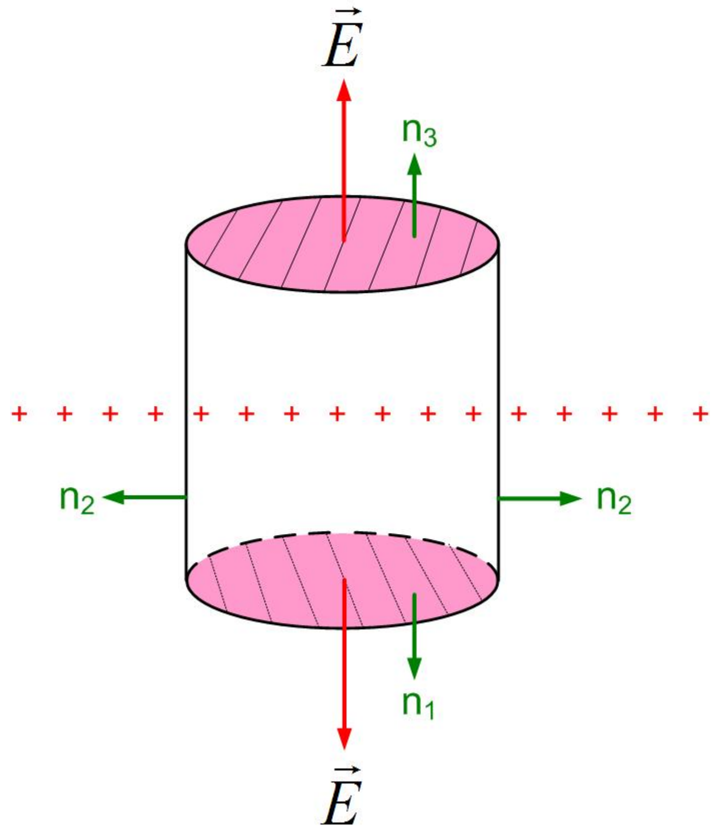

Figure 11: Infinite plane charged with positive surface charge density ρ_S.

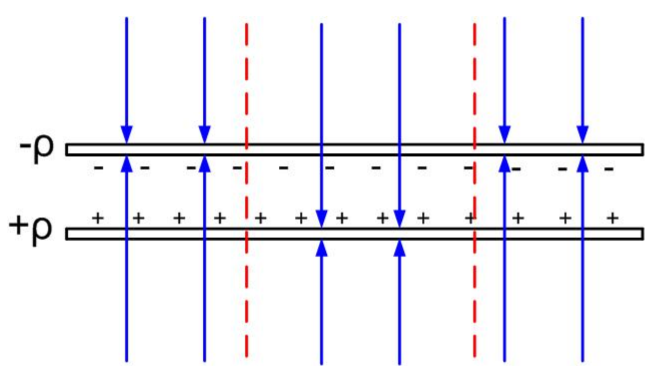

Figure 12: Two infinite planes charged with positive surface charge density ρ_S and $-\rho_S$.

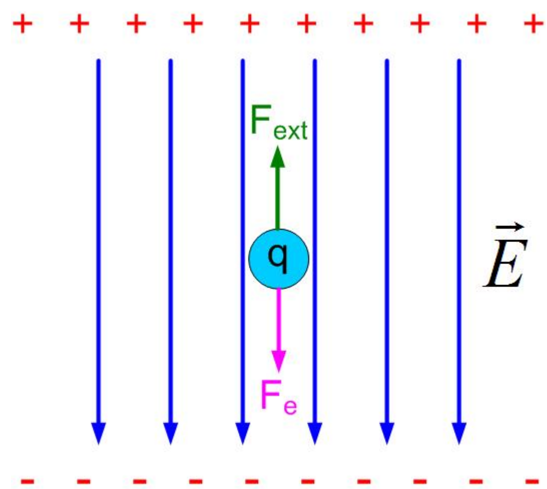

Figure 13: Potential

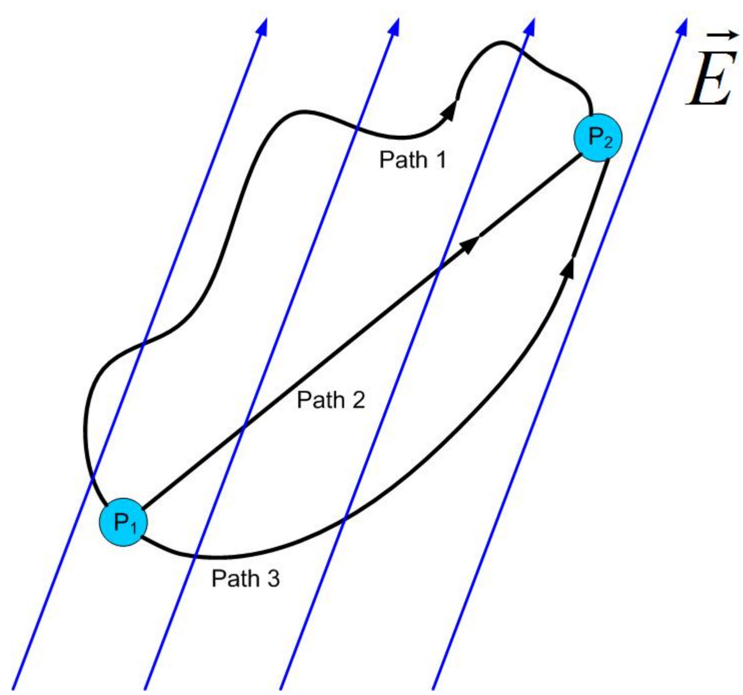

Figure 14: Potential is not dependent on the specific path.

Capacitance

What is capacitance, how does it affect circuits.

Electric Field inside Metals

Boundary Conditions

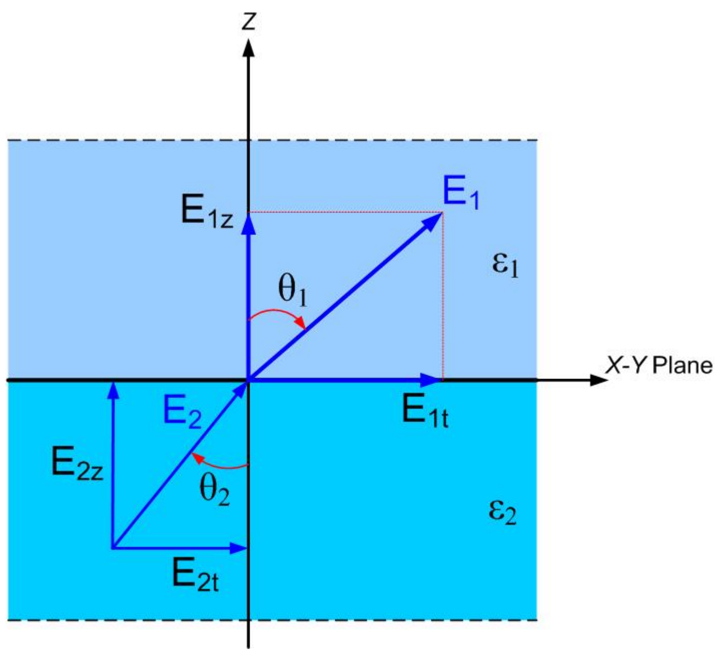

Figure 15: Boundary Conditions for Electric Field.

Image Theory

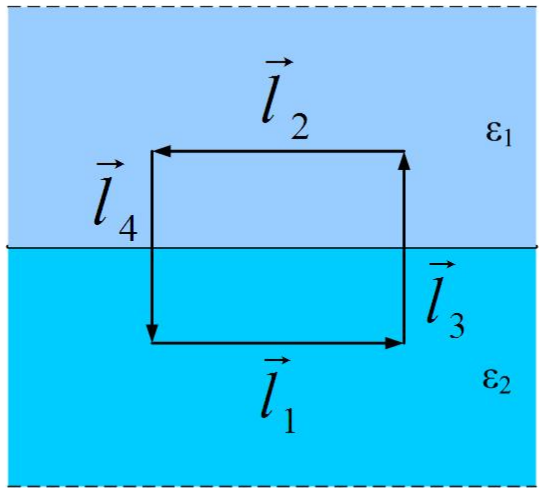

Figure 16: Integration

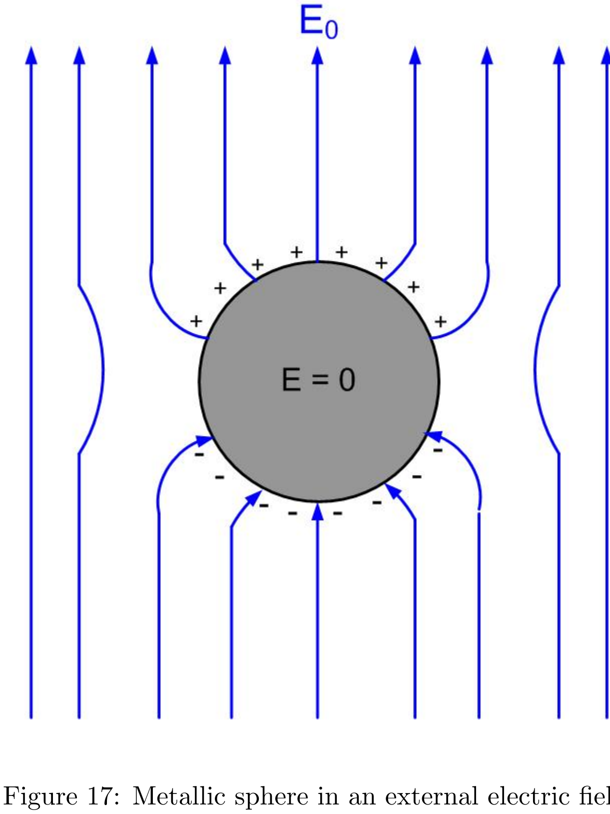

Figure 17: Metallic sphere in an external electric field.