

# Leading and Lagging Signals

## Review of Sinusoidal Signals

**Definition 1.** How do we recognize lagging and leading on a graph?

In Figure 1 we observe two step functions,  $V(t)$  and  $V(t-T)$ . Function  $V(t)$  step occurs at  $t=0$ , and  $V(t-T)$  step occurs at  $t=T$ . The function  $V(t-T)$  is shifted to the right, the step occurs later, at  $t=T$ , and is, therefore, lagging function  $V(t)$ .

Similarly, if the step function is  $V(t+T)$ , the function  $v(t)$  is shifted to the left. The step occurs earlier at  $t=-T$ , and therefore  $V(t+T)$  is leading  $V(t)$ .

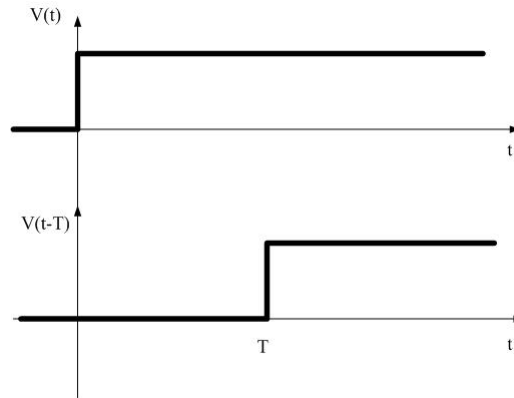


Figure 1: Voltage as a function of time at the generator side (top) and the load side (bottom) of a transmission line, if the switch closes at  $t=0$  the voltage arrives at  $t=l/c=T$  at the load. These graphs can be obtained by observing the voltage on an oscilloscope at the load and at the generator side.

**Example 1.** What if we have a sinusoidal signal? We will observe a specific point on the signal, such as the maximum value, and determine if it shifted left or right on the graph.

When the phase of a signal is positive as in Figure 2  $\sin(\omega t + 45^\circ)$ , we say that the signal is leading with respect to the signal  $v(t) = \sin(\omega t)$ , because it is shifted to the left for  $45^\circ$  ( $\pi/4$ ). The maximum of the function now occurs at  $t=-T$ , or  $\omega t = -45^\circ$ , and we can write the new function as the original sinusoidal function  $V(t)$  shifted left for a time  $T$ ,  $V(t+T)$ . The phase of the signal is  $45^\circ$ , and the time-delay is  $T$ .

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Learning outcomes: Recognize leading and lagging signals. Explain why is a signal leading or lagging.

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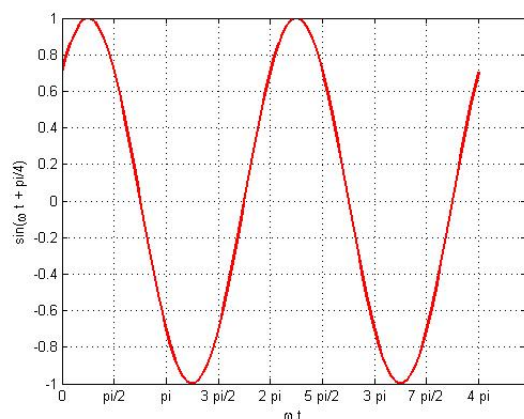


Figure 2: Sinusoidal signal as a function of angle  $\omega t$  with a phase shift of  $+\pi/4$

### Example 2.

When the phase of a signal is negative as in Figure 4, 3,  $\sin(\omega t - 45^\circ)$ , we say that the signal is lagging with respect to the signal  $\sin(\omega t)$ , because it is shifted to the right for  $45^\circ$  ( $\pi/4$ ), or  $\tau = -\frac{\pi/4}{\omega}$ . The lagging function's peak occurs later in time, and therefore it is lagging. The phase of the signal is  $-45^\circ$ .

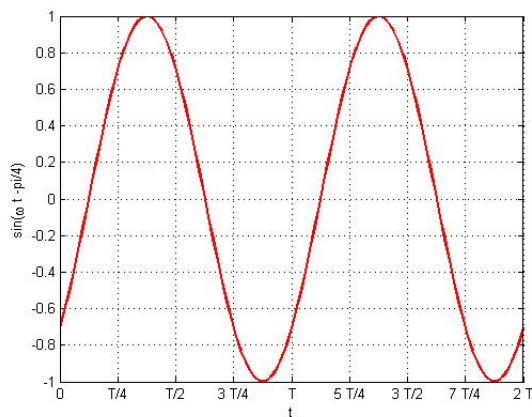


Figure 3: Sinusoidal signal shifted for time delay  $-\frac{\pi/4}{\omega}$

**Question 1** Sinusoidal signal  $v_1 = \cos(\omega t - 25^\circ)$  is given. Compared to  $v = \cos(\omega t)$ , signal  $v_1$

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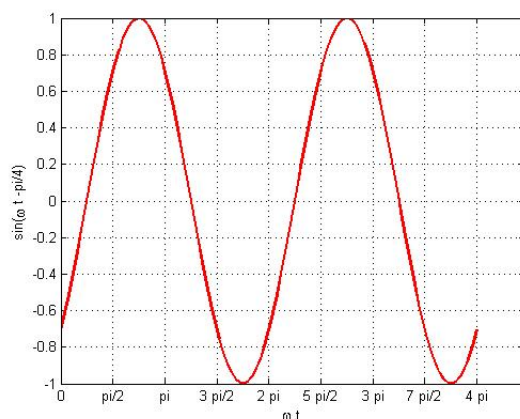
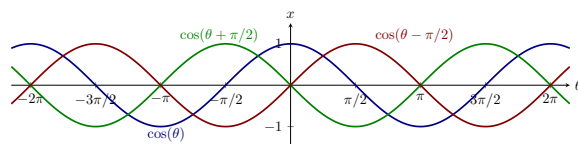


Figure 4: Sinusoidal signal with phase shift  $-\pi/4$

**Multiple Choice:**

- (a) Leads signal  $v$
- (b) Lags signal  $v$  ✓

**Question 2** Observe three signals in Figure below



Which of the following functions leads  $\cos(\omega t)$ ?

**Multiple Choice:**

- (a) The green signal. ✓
- (b) The red signal.
- (c) The blue signal.