Kirchoff's Laws

Phasors are essential tool in circuit analysis.

In this section, we apply the phasor transformation to an RC circuit shown in Figure 21. To solve this circuit in the time domain, we apply Kirchoff's voltage law, as shown in Equation 1 -2.

The circuit in Figure 1 is a simple RC circuit. Equation 1 shows the KVL the time domain.

$$v_s(t) = v_R(t) + v_C(t) \tag{1}$$

$$v_s(t) = Ri + \frac{1}{C}\int i(t)dt \tag{2}$$

As we discussed in the previous section, we will be using the principle of superposition, and add another generator to the circuit, as shown in Figure 1.

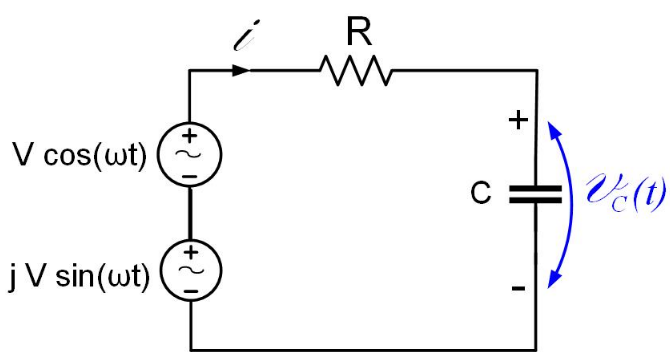

Figure 1: Using superposition to find phasors of voltages and currents in an RC circuit.

The generator we originally had in the circuit is now just the real part of the phasor expression shown in Equation 3.

$$\begin{aligned} v_s(t) = V\cos(\omega t + \Theta_V) = \Re\{V cos(\omega t + \Theta) + jV sin(\omega t + \Theta)\} = \\ = \Re\{Ve^{j(\omega t + \Theta)}\} = \Re\{Ve^{j\Theta}e^{j\omega t}\} \end{aligned} \tag{3}$$

Learning outcomes: Apply phasor transformation to Kirchoff's Laws.
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We can now use the analysis from the previous section to replace the time-domain quantities in equation 2 with these newly developed expressions.

$$v_s(t) = v_R(t) + v_C(t) \tag{4}$$

$$Re\{V_s e^{j\Theta_V} e^{j\omega t}\} = \Re\{R\, I e^{j\Theta_I} e^{j\omega t}\} + \Re\{\frac{1}{j\omega C} I e^{j\Theta_I} e^{j\omega t}\} \tag{5}$$

A common term in the previous equation is $e^{j\omega t}$, and we can now drop Re, as long as we later remember to take only the real part of the expression for the voltage and current phasors to get the time domain expression. We can now write the equation as

$$V_s e^{j\Theta_V} = R\, I e^{j\Theta_I} + \frac{1}{j\omega C} I e^{j\Theta_I} \tag{6}$$

$$\tilde{V}_s = R\tilde{I} + \frac{\tilde{I}}{j\omega C} \tag{7}$$

Since this is a linear equation, we can easily solve it:

$$\tilde{I} = \frac{\tilde{V}_s}{R + \frac{1}{j\omega C}} \tag{8}$$

Converting the phasor back to the time domain

In general, if the phasor is $\tilde{V} = |V|e^{j\Theta_V}$, to find the time-domain signal, we first multiply the phasor with $e^{j\omega t}$ term, and then take the real part of it.

$$v(t) = \Re\{\tilde{V} e^{j\omega t}\} \tag{9}$$

$$v(t) = \Re\{|\tilde{V}| e^{j(\omega t + \Theta_V)}\} \tag{10}$$

$$v(t) = \Re\{|\tilde{V}| cos(\omega t + \Theta_V) + j|\tilde{V}| sin(\omega t + \Theta_V)\} \tag{11}$$

$$v(t) = V cos(\omega t + \Theta_V) \tag{12}$$

Example 1. *The phasor of current is $I = 3e^{j45^o}$. Obtain the signal in the time domain.*

Explanation. *To obtain the time-domain signal from the phasor:*

(a) *multiply the phasor $\tilde{I}$ with the $e^{j\omega t}$ term,*

(b) *use Euler's formula*

(c) *take the real part of the expression.*

$$
\begin{aligned}
i(t) = \Re\{3e^{j45^o} e^{j\omega t}\} = \Re\{3e^{j\omega t + 45^o}\} = \\
= \Re\{3\cos(\omega t + 45^o) + j3\sin(\omega t + 45^o\} = 3\cos(\omega t + 45^o)
\end{aligned}
\tag{13}
$$

Question 1 *The phasor for the voltageis given as* $\tilde{V} = 5e^{j\beta}$. *Find the expression for the phasor in the time domain.*

Multiple Choice:

(a) $v(t) = 5\cos\beta t$

(b) $v(t) = 5\cos(\omega t + \beta)$ ✓

(c) $v(t) = 5\sin\beta t$