Leading and Lagging Signals

Review of Sinusoidal Signals

Definition 1. *How do we recognize lagging and leading on a graph?*

In Figure 1 we observe two step functions, V(t) and V(t-T). Function V(t) step occurs at t=0, and V(t-T) step occurs at t=T. The function V(t-T) is shifted to the right, the step occurs later, at t=T, and is, therefore, lagging function V(t).

Similarly, if the step function is V(t+T), the function v(t) is shifted to the left. The step occurs earlier at t=-T, and therefore V(t+T) is leading V(t).

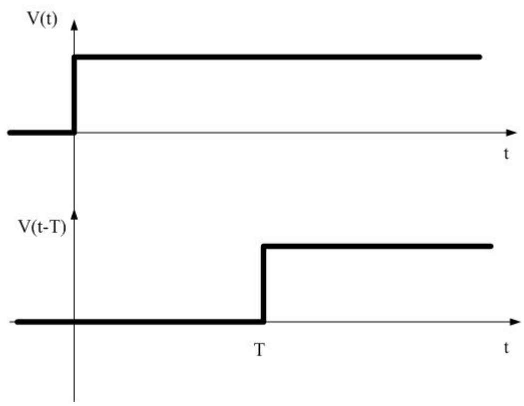


Figure 1: Voltage as a function of time at the generator side (top) and the load side (bottom) of a transmission line, if the switch closes at t=0 the voltage arrives at t=l/c=T at the load. These graphs can be obtained by observing the voltage on an oscilloscope at the load and at the generator side.

Example 1. *What if we have a sinusoidal signal? We will observe a specific point on the signal, such as the maximum value, and determine if it shifted left or right on the graph.*

When the phase of a signal is positive as in Figure 2 $\sin(\omega t + 45^o)$*, we say that the signal is leading with respect to the signal* $v(t) = \sin(\omega t)$*, because it is shifted to the left for* 45^o *(pi/4). The maximum of the function now occurs at t=-T, or* $\omega t = -45^o$*, and we can write the new function as the original sinusoidal function V(t) shifted left for a time T, V(t+T). The phase of the signal is* 45^o*, and the time-delay is T.*

Learning outcomes: Recognize leading and lagging signals. Explain why is a signal leading or lagging.

Author(s): Milica Markovic

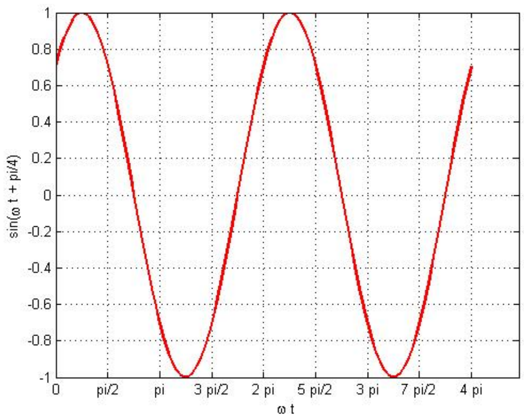


Figure 2: Sinusoidal signal as a function of angle ωt with a phase shift of $+\pi/4$

Example 2.

When the phase of a signal is negative as in Figure 4, 3, $\sin(\omega t - 45^o)$*, we say that the signal is lagging with respect to the signal* $\sin(\omega t)$*, because it is shifted to the right for* 45^o *(pi/4), or* $\tau = -\dfrac{pi/4}{\omega}$*. The lagging function's peak occurs later in time, and therefore it is lagging. The phase of the signal is* -45^o*.*

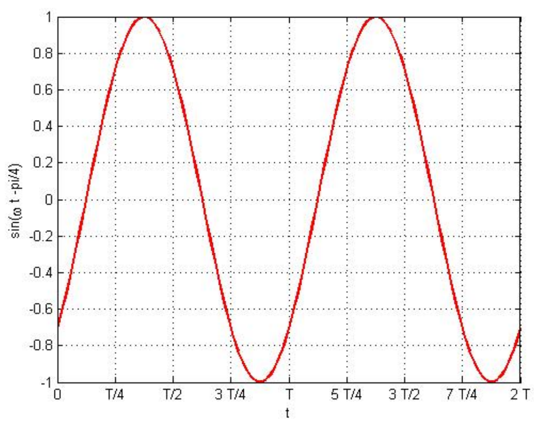


Figure 3: Sinusoidal signal shifted for time delay $-\dfrac{\pi/4}{\omega}$

Question 1 *Sinusoidal signal* $v_1 = \cos(\omega t - 25^o)$ *is given. Compared to* $v = \cos(\omega t)$*, signal* v_1

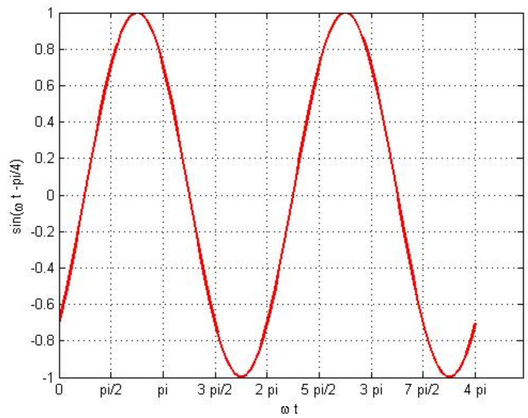


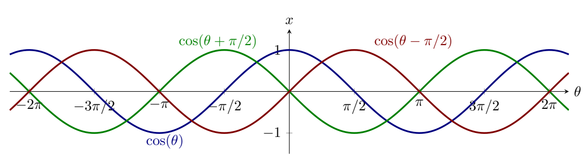
Figure 4: Sinusoidal signal with phase shift $-\pi/4$

Multiple Choice:

(a) *Leads signal v*

(b) *Lags signal v* ✓

Question 2 *Observe three signals in Figure below*



Which of the following functions leads $\cos(\omega t)$*?*

Multiple Choice:

(a) *The green signal.* ✓

(b) *The red signal.*

(c) *The blue signal.*