Forward voltage on a transmission line

Again, we will look at a transmission line circuit in Figure 1 to find the input impedance on a transmission line.

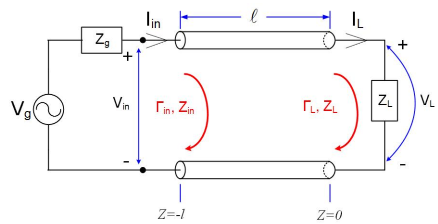

Figure 1: Transmission Line connects generator and the load.

The equations for the voltage and current anywhere (any z) on a transmission line are

$$\tilde{V}(z) = \tilde{V}_0^+(e^{-j\beta z} - \Gamma_L e^{j\beta z}) \tag{1}$$

$$I(z) = \frac{\tilde{V}_0^+}{Z_0}(e^{-j\beta z} - \Gamma_L e^{j\beta z}) \tag{2}$$

Using the equations from the previous section, we can replace the transmission line with its input impedance, Figure 2.

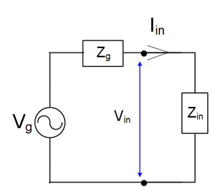

Figure 2: Transmission Line connects generator and the load.

Learning outcomes: Calculate phasors of forward going voltage at the load.
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Forward voltage phasor as a function of load impedance

From Figure 2, we can find the input voltage on a transmission line using the voltage divider.

$$\tilde{V}_{in} = \frac{Z_{in}}{Z_{in} + Z_g}\tilde{V}_g \tag{3}$$

Using Equation 2, we can also find the input voltage. The input voltage equation at the generator $z = -l$ is:

$$\tilde{V}_{in} = \tilde{V}(z = -l) = \tilde{V}_0^+(e^{j\beta l} + \Gamma_L e^{-j\beta l}) \tag{4}$$

Since these two equations represent the same input voltage we can make them equal.

$$\tilde{V}_0^+(e^{j\beta l} + \Gamma_L e^{-j\beta l}) = \frac{Z_{in}}{Z_{in} + Z_g}\tilde{V}_g \tag{5}$$

Rearranging the equation, we find $\tilde{V}_0^+$.

$$\tilde{V}_0^+ = \frac{\tilde{V}_g Z_{in}}{Z_g + Z_{in}} \frac{1}{e^{j\beta l} + \Gamma_L e^{-j\beta l}} \tag{6}$$

$$\tag{7}$$

Forward voltage phasor as a function of input reflection coefficient

There is another way to find the input impedance as a function of the input reflection coefficient.

We write KVL for the circuit in Figure 2.

$$\tilde{V}_g = Z_g \tilde{I}_{in} + \tilde{V}_{in} \tag{8}$$

Using Equations 2-2, we can also find the input voltage and current. Input voltage and current equation at the generator $z = -l$ are:

$$\tilde{V}_{in} = \tilde{V}(z = -l) = \tilde{V}_0^+(e^{j\beta l} + \Gamma_L e^{-j\beta l}) \tag{9}$$

$$\tilde{I}_{in} = I(z = -l) = \frac{\tilde{V}_0^+}{Z_0}(e^{j\beta l} - \Gamma e^{-j\beta l}) \tag{10}$$

Substituting these two equations in Equation 8 we get

$$\tilde{V}_g = Z_g \frac{\tilde{V}_0^+}{Z_0}(e^{j\beta l} - \Gamma_L e^{-j\beta l}) + \tilde{V}_0^+(e^{j\beta l} + \Gamma_L e^{-j\beta l}) \tag{11}$$

We can re-write this equation as follows.

$$\tilde{V}_g = Z_g e^{j\beta l} \frac{\tilde{V}_0^+}{Z_0}(1 - \Gamma_L e^{-2j\beta l}) + \tilde{V}_0^+ e^{j\beta l}(1 + \Gamma_L e^{-2j\beta l}) \tag{12}$$

Using that $\Gamma_{in} = \Gamma_L e^{-2j\beta l}$ is the input reflection coefficient, and multiplying through with Z_0.

$$\tilde{V}_g Z_0 = Z_g e^{j\beta l} \frac{\tilde{V}_0^+}{1}(1 - \Gamma_{in}) + \tilde{V}_0^+ Z_0 e^{j\beta l}(1 + \Gamma_{in}) \tag{13}$$

Rearranging the equation, we get $\tilde{V}_0^+$

$$\tilde{V}_0^+ = \tilde{V}_g e^{-j\beta l} \frac{Z_0}{Z_0(1 + \Gamma_{in}) + Z_g(1 - \Gamma_{in})} \tag{14}$$

$\Gamma_{in} = \Gamma_L e^{-2j\beta l}$ is the input reflection coefficient.

Special case - forward voltage when the generator and transmission-line impedance are equal

Because the generator's impedance is equal to the transmission line impedance, we will use the second equation. When $Z_g = Z_0$ we see that the denominator simplifies into $Z_0(1 + \Gamma_{in}) + Z_g(1 - \Gamma_{in}) = Z_0(1 + \Gamma_{in} + 1 - \Gamma_{in}) = 2$, and we can further simplify the fraction to get the final value of $\tilde{V}_0^+ = \frac{V_g}{2} e^{-j\beta l}$.