Example Transmission Line Problem

Example 1. *A transmitter operated at 20MHz, Vg=100V with* $Z_g = 50\Omega$ *internal impedance is connected to an antenna load through l=6.33m of the line. The line is a lossless* $Z_0 = 50\Omega$, $\beta = 0.595 rad/m$. *The antenna impedance at 20MHz measures* $Z_L = 36 + j20\Omega$. *Set the beginning of the z-axis at the load, as shown in Figure 1.*

(a) *What is the electrical length of the line?*

(b) *What is the input impedance of the line* Z_{in}*?*

(c) *What is the forward going voltage at the load* $\tilde{V}_0^+$*?*

(d) *Find the expression for forward voltage anywhere on the line.*

(e) *Find the expression for reflected voltage anywhere on the line.*

(f) *Find the total voltage anywhere on the line.*

(g) *Find the expression for forward current anywhere on the line.*

(h) *Find the expression for reflected current anywhere on the line.*

(i) *Find the total current anywhere on the line.*

(j) *Instead of the antenna, a load impedance* $Z_l = 50\Omega$ *is connected to this 50 Ohm line. How will that change the equations above?*

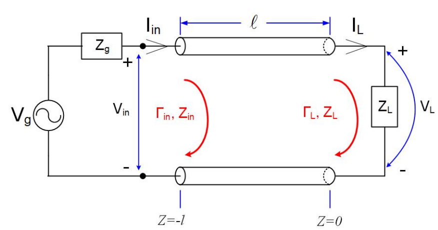

Figure 1: Transmission Line connects generator and the load.

Learning outcomes: Solve typical transmission-line problem.
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Explanation. *The equations for the voltage and current anywhere (any z) on a transmission line are*

$$\tilde{V}(z) = \tilde{V}_0^+ (e^{-j\beta z} + |\Gamma| e^{j\beta z + \Theta_r}) \tag{1}$$
$$I(z) = \frac{\tilde{V}_0^+}{Z_0} (e^{-j\beta z} - \Gamma e^{j\beta z}) \tag{2}$$

We are given phase constant β, and we have to find the other unknowns: phasor of voltage at the load $\tilde{V}_0^+$, and the reflection coefficient Γ.

Since we know the load impedance Z_L, and the transmission-line impedance Z_0, we can find the reflection coefficient Γ using Equation 3.

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = 0.279\, e^{j112^o} = 0.279\, e^{j1.95\,\mathrm{rad}} \tag{3}$$

To find the input impedance of the line, we use the equation

$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} = 70.8 + j27.1\Omega \tag{4}$$

We can use one of the following two equations to find the forward going voltage at the load:

$$\tilde{V}_0^+ = \frac{\tilde{V}_g Z_{in}}{Z_g + Z_{in}} \frac{1}{e^{j\beta l} + \Gamma e^{-j\beta l}} \tag{5}$$
$$\tilde{V}_0^+ = V_g e^{-j\beta l} \frac{Z_0}{Z_0(1 + \Gamma_{in}) + Z_g(1 - \Gamma_{in})} \tag{6}$$

Because the generator's impedance is equal to the transmission line impedance, we will use the second equation. When $Z_g = Z_0$ we see that the denominator simplifies into $Z_0(1 + \Gamma_{in}) + Z_g(1 - \Gamma_{in}) = Z_0(1 + \Gamma_{in} + 1 - \Gamma_{in})$ and we can further simplify the fraction to get the final value of $\tilde{V}_0^+ = \frac{V_g}{2} e^{-j\beta l}$. Since $\beta l = \frac{2\pi}{\lambda} 0.6\lambda$, the forward going voltage at the load is $\tilde{V}_0^+ = 50\, e^{-j1.2\pi} = 50\, e^{-j3.768} = 50\, e^{-j216^o}$.

The equations of the voltage and current anywhere on the line are therefore

$$\tilde{V}(z) = 50 e^{-j3.768} (e^{-j0.595z} + 0.279 e^{j0.595z + 1.95}) \tag{7}$$
$$I(z) = e^{-j3.768} (e^{-j0.595z} - 0.279 e^{j0.595z + 1.95}) \tag{8}$$

Suppose we replace the antenna with another load of impedance 50Ω. *In that case, the reflection coefficient from the load will be zero, and the reflected voltages will disappear, so the voltage and current will be equal to the forward-going voltage on the transmission line.*

$$\tilde{V}(z) = \tilde{V}_0^+ e^{-j\beta z} \tag{9}$$

$$I(z) = \frac{\tilde{V}_0^+}{Z_0} e^{-j\beta z} \tag{10}$$