Exercise 1 Suppose that $\overrightarrow{\mathbf{u}} = \langle 0, -1, 3 \rangle$ and $\overrightarrow{\mathbf{v}} = \langle -1, 2, 0 \rangle$. Find a vector $\overrightarrow{\mathbf{w}}$ with a positive y-component of magnitude 7 that is parallel to $\overrightarrow{\mathbf{u}} + 2\overrightarrow{\mathbf{v}}$.

$$\overrightarrow{\mathbf{w}} = \left\langle \left\lceil \frac{-14}{\sqrt{22}} \right\rceil, \left\lceil \frac{21}{\sqrt{22}} \right\rceil, \left\lceil \frac{21}{\sqrt{22}} \right\rceil \right\rangle$$

Hint: To begin, let's find a vector in the same direction as $\overrightarrow{\mathbf{u}} + 2\overrightarrow{\mathbf{v}}$. Using the rules of addition and scalar multiplication, we find:

$$\overrightarrow{\mathbf{u}} + 2\overrightarrow{\mathbf{v}} = \langle \boxed{-2}, \boxed{3}, \boxed{3} \rangle$$

How should we proceed?

Multiple Choice:

- (a) Multiply this result by 7; that is, $\overrightarrow{\mathbf{w}} = \langle -14, 21, 21 \rangle$.
- (b) Find the magnitude of $\langle -2, 3, 3 \rangle$ and scale it appropriately if necessary. \checkmark

We compute:

$$\left|\overrightarrow{\mathbf{u}} + 2\overrightarrow{\mathbf{v}}\right| = \sqrt{\left(-2\right)^2 + \left(3\right)^2 + \left(3\right)^2} = \sqrt{22}$$

(type the components in the order of $\overrightarrow{\mathbf{u}} + 2\overrightarrow{\mathbf{v}}$)

A unit vector in the direction of $\overrightarrow{\mathbf{w}}$ is thus $\frac{\overrightarrow{\mathbf{u}} + 2\overrightarrow{\mathbf{v}}}{\left| \overrightarrow{\mathbf{u}} + 2\overrightarrow{\mathbf{v}} \right|}$, so:

$$\hat{\mathbf{w}} = \left\langle \left\lceil \frac{-2}{\sqrt{22}} \right\rceil, \left\lceil \frac{3}{\sqrt{22}} \right\rceil, \left\lceil \frac{3}{\sqrt{22}} \right\rceil \right\rangle$$

and $\overrightarrow{\mathbf{w}} = \boxed{7} \hat{\mathbf{w}}$.