

Example Transmission Line Problem

Example 1. A transmitter operated at 20MHz, $V_g=100V$ with $Z_g = 50\Omega$ internal impedance is connected to an antenna load through $l=6.33m$ of the line. The line is a lossless $Z_0 = 50\Omega$, $\beta = 0.595\text{rad/m}$. The antenna impedance at 20MHz measures $Z_L = 36 + j20\Omega$. Set the beginning of the z -axis at the load, as shown in Figure 1.

- What is the electrical length of the line?
- What is the input impedance of the line Z_{in} ?
- What is the forward going voltage at the load \tilde{V}_0^+ ?
- Find the expression for forward voltage anywhere on the line.
- Find the expression for reflected voltage anywhere on the line.
- Find the total voltage anywhere on the line.
- Find the expression for forward current anywhere on the line.
- Find the expression for reflected current anywhere on the line.
- Find the total current anywhere on the line.
- We now remove the antenna and connect load impedance $Z_L = 50\Omega$ to this 50 Ohm line, how will that change the equations above?

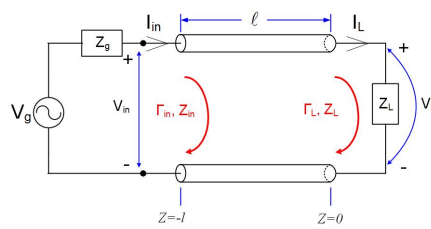


Figure 1: Transmission Line connects generator and the load.

Learning outcomes: Solve a typical transmission-line problem.
 Author(s): Milica Markovic

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Explanation. The equations for the voltage and current anywhere (any z) on a transmission line are

$$\tilde{V}(z) = \tilde{V}_0^+ (e^{-j\beta z} - |\Gamma_L| e^{j\beta z + \Theta_r}) \quad (1)$$

$$I(z) = \frac{\tilde{V}_0^+}{Z_0} (e^{-j\beta z} - \Gamma_L e^{j\beta z}) \quad (2)$$

We are given phase constant β , and we have to find the other unknowns: phasor of voltage at the load \tilde{V}_0^+ , and the reflection coefficient Γ_L .

Since we know the load impedance Z_L , and the transmission-line impedance Z_0 , we can find the reflection coefficient Γ_L using Equation 3.

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = 0.279 e^{j112^\circ} = 0.279 e^{j1.95 \text{ rad}} \quad (3)$$

To find the input impedance of the line, we use the equation.

$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} = 70.8 + j27.1 \Omega \quad (4)$$

We can use one of the following two equations to find the forward going voltage at the load:

$$\tilde{V}_0^+ = \frac{\tilde{V}_g Z_{in}}{Z_g + Z_{in}} \frac{1}{e^{j\beta l} + \Gamma_L e^{-j\beta l}} \quad (5)$$

$$\tilde{V}_0^+ = V_g e^{-j\beta l} \frac{Z_0}{Z_0(1 + \Gamma_{in}) + Z_g(1 - \Gamma_{in})} \quad (6)$$

Because the generator's impedance is equal to the transmission line impedance, we will use the second equation. When $Z_g = Z_0$ we see that the denominator simplifies into $Z_0(1 + \Gamma_{in}) + Z_g(1 - \Gamma_{in}) = Z_0(1 + \Gamma_{in} + 1\Gamma_{in})$ and we can further simplify the fraction to get the final value of $\tilde{V}_0^+ = \frac{V_g}{2} e^{-j\beta l}$. Since $\beta l = \frac{2\pi}{\lambda} 0.6\lambda$, the forward going voltage at the load is $\tilde{V}_0^+ = 50 e^{-j1.2\pi} = 50 e^{-j3.768} = 50 e^{-j216^\circ}$.

The equations of the voltage and current anywhere on the line are therefore

$$\tilde{V}(z) = 50 e^{-j3.768} (e^{-j0.595z} - 0.279 e^{j0.595z + 1.95}) \quad (7)$$

$$I(z) = e^{-j3.768} (e^{-j0.595z} - 0.279 e^{j0.595z + 1.95}) \quad (8)$$

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If we replace the antenna with another load of impedance 50Ω , the reflection coefficient from the load will now be zero, and the reflected voltages will disappear, so the voltage and current will be

$$\tilde{V}(z) = \tilde{V}_0^+ e^{-j\beta z} \quad (9)$$

$$I(z) = \frac{\tilde{V}_0^+}{Z_0} e^{-j\beta z} \quad (10)$$