Impedance and admittance circles on the Smith Chart

Reflection Coefficient and Impedance

Reflection coefficient and impedance are related through Equation 1. We can find an impedance that corresponds to the reflection coefficient, Equation 2. Every point on the Smith Chart represents one reflection coefficient Γ and one impedance Z_L.

$$\Gamma = \frac{Z_L - Z_\circ}{Z_L + Z_\circ} \tag{1}$$

$$Z_L = Z_0 \frac{1+\Gamma}{1-\Gamma} \tag{2}$$

All impedances on the Smith Chart are normalized to the transmission line impedance Z_0. The normalized impedance is denoted in Equation 3 with lowercase z_L.

$$z_L = \frac{Z_L}{Z_0} = \frac{1+\Gamma}{1-\Gamma} \tag{3}$$

Derivation of Impedance and Admittance Circles on the Smith Chart

Impedance and reflection coefficient are complex numbers. The normalized impedance has a real and imaginary part $z_L = r_L + jx_L$, and the reflection coefficient can also be shown in Cartesian coordinates as $\Gamma = \Gamma_r + j\Gamma_i$. We can now substitute these equations into Equation 4.

$$r_L + jx_L = \frac{1 + \Gamma_r + j\Gamma_i}{1 - (\Gamma_r + j\Gamma_i)} \tag{4}$$

Learning outcomes: Explain how was Smith Chart developed.
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We can equate the real and imaginary parts on the left and right side of Equation 4 to get the equations of constant r_L and x_L.

$$\left(\Gamma_r - \frac{r_L}{1+r_L}\right)^2 + \Gamma_i^2 = \left(\frac{1}{1+r_L}\right)^2 \tag{5}$$

$$(\Gamma_r - 1)^2 + \left(\Gamma_i - \frac{1}{x_L}\right)^2 = \left(\frac{1}{x_L}\right)^2 \tag{6}$$

These are equations of a circle, with the constant resistance circle's center at $(\frac{r_L}{1+r_L}, 0)$ and radius $\frac{1}{1+r_L}$; and the constant reactance imaginary circle center at $(1, \frac{1}{x_L})$ and radius of $\frac{1}{x_L}$.

Figures 1- 2 show circles on the Smith Chart that represent constant (normalized) reactances, and resistances.

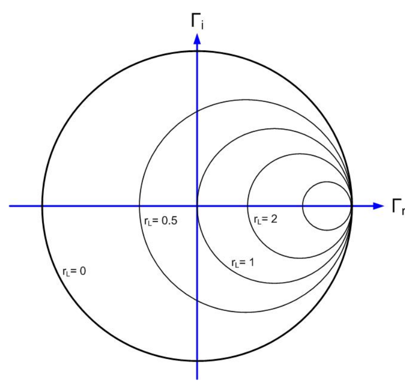

Figure 1: All points on the circle have the constant real part of the impedance (resistance). Normalized resistance circles.

Admittance circles can be similarly derived using the fact that $Y_L = \frac{1}{Z_L}$ and the Equation 3

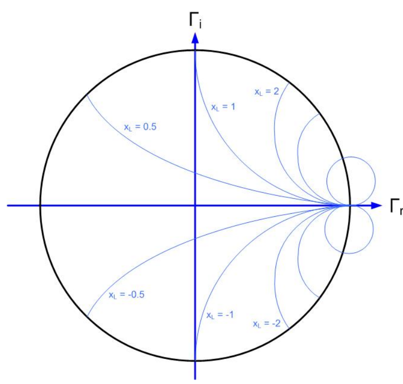

Figure 2: All points on the circle have the constant imaginary part of the impedance (reactance). Normalized reactance circles.