Electrical Length

Electrical Length of the line in meters

We can express the physical length of the line in meters. However, in high-frequency electronics (microwave engineering), we usually convert this length to the fraction of a wavelength of a signal that is traveling on the line.

$$l = N\lambda \tag{1}$$

Typically, but not always, N is a fraction, for example, $N = \frac{1}{2} = 0.5$, $N = \frac{1}{4} = 0.25$ or $N = \frac{1}{8} = 0.125$; although it can be any The lenght of the line is then written as

$$l = \frac{\lambda}{2} \tag{2}$$

$$l\frac{\lambda}{4} \tag{3}$$

$$l = \frac{\lambda}{8} \tag{4}$$

If the physical length of the line is $l = \frac{\lambda}{4}$, we say: This line is quarter-wavelength long at 1GHz, meaning one-quarter of the wavelength fits on the line. We could also say that the line is 7.5cm long, as wavelength is $\lambda = 30\,\text{cm}$ at 1GHz.

When we say quarter-wavelength long, we refer to the lines physical length at a specific frequency.

Electrical length of the line in degrees

The phase shift between input and output signal on a transmission line is $\Theta = \beta * l$. *beta* is called the phase constant. It represents the spatial frequency of the

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signal. $\Theta = \beta * L$ is the phase in degrees or radians (related is a time delay in seconds). Θ can be, for example $\Theta = 45^0$, $\Theta = 90^0$, $\Theta = 180^0$. Θ is a function of frequency, because β is a function of frequency. If $\Theta = 90^0$, we say: The line is 90 degrees long at 1GHz, meaning the output signal at 1GHz will be shifted for 90^0 with respect to the input signal. When we say 90^0, we refer to the lines electrical length, representing the number of degrees that the line introduces between the input and the output signal.

Example 1. *What is the electrical length of a 30 cm line in terms of the fraction of wavelength at 1 GHz? What is the electrical length of the line at 1GHz?*

Explanation. *Wavelength at 1 GHz, assuming the wave is propagating in air is* $\lambda = \frac{c}{f} = 30$ *cm. Since the line is also l=30 cm long, the length of the line in terms of wavelength is* $\frac{l}{\lambda} = 1$*, or* $l = \lambda$.

The electrical length of the line is $\theta = \beta l = \frac{2\pi}{\lambda}\lambda = 2\pi = 360^0$.

Example 2. *What is the electrical length of a 15 cm line in terms of the fraction of wavelength at 1 GHz? What is the electrical length of the line at 1GHz?*

Explanation. *Wavelenght at 1 GHz, assuming the wave is propagating in air is* $\lambda = \frac{c}{f} = 30$ *cm. Since the line is 15 cm long, the length of the line in terms of wavelenth is* $\frac{l}{\lambda} = \frac{1}{2}$*, or* $l = \frac{\lambda}{2}$.

The electrical length of the line is $\theta = \beta l = \frac{2\pi}{\lambda}\frac{\lambda}{2} = \pi = 180^0$.

Example 3. *What is the electrical length of a 7.5 cm line in terms of the fraction of wavelength at 1 GHz? What is the electrical length of the line at 1GHz?*

Explanation. *Wavelength at 1 GHz, assuming the wave is propagating in air is* $\lambda = \frac{c}{f} = 30$ *cm. Since the line is 7.5 cm long, the line's length in terms of wavelenth is* $l = \frac{\lambda}{4}$.

The electrical length of the line is $\theta = \beta l = \frac{2\pi}{\lambda}\frac{\lambda}{4} = \pi/2 = 90^0$.