

# BEYOND SHAPLEY VALUES

## COOPERATIVE GAMES FOR THE INTERPRETATION OF MACHINE LEARNING MODELS

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# Introduction

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In this presentation:

**How can we leverage the theory of cooperative games for the interpretation of black-box machine learning models?**

And (better) understand **what the Shapley values are**

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Two ingredients:

- $D = \{1, \dots, d\}$ , a **set of players**, and  $\mathcal{P}_D$  the **set of coalitions of players**
- $v : \mathcal{P}_D \rightarrow \mathbb{R}$ , a **value function**, that **assigns a value to each coalition**

☞  $(D, v)$  formally defines a **cooperative game**

☞  $v(D)$  is the value of the “grand coalition” (the cake)

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Main question:

How can we redistribute  $v(D)$  to each of the  $d$  individual players?



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**Cooperative game:**

Features indices  $D$  and value function

$$v(A) = R_Y^2(X_A)$$

i.e., the  $R^2$  coefficient of the linear regression of  $X_A$  on  $Y$

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**How can we aggregate the information of all these  $R^2$ s into something more manageable?**

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$$\textbf{Efficiency: } \sum_{i \in D} \phi(i) = v(D)$$

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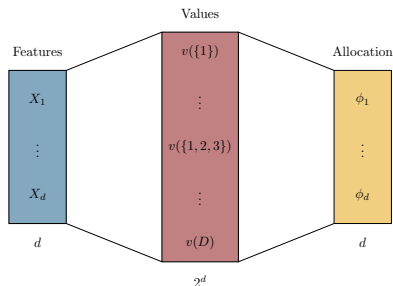
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In a nutshell:

- Start with a learned model with  $d$  input features
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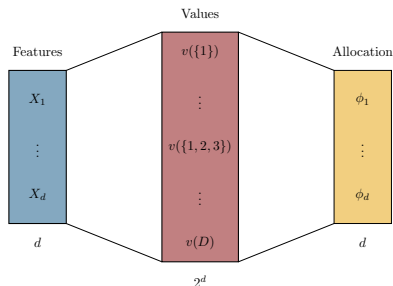
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**How can we define efficient allocations?**

Value function evaluation

Value function aggregation

# Allocations as a dividend sharing mechanism

The **Harsanyi (1963) dividends** of a cooperative game  $(D, v)$  are defined as:

$$\mathcal{D}_v(A) = \sum_{B \in \mathcal{P}_A} (-1)^{|A|-|B|} v(B), \quad \text{or equivalently,} \quad \mathcal{D}_v(A) = v(A) - \sum_{B \in \mathcal{P}_A} \mathcal{D}_v(B)$$

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The **Harsanyi set** is a family of efficient allocations that **aggregate of the Harsanyi dividends**:

$$\phi(i) = \sum_{A \in \mathcal{P}_D : i \in A} \lambda_i(A) \mathcal{D}_v(A), \quad \text{where} \quad \begin{cases} \forall i \in D, \forall A \in \mathcal{P}_D, \lambda_i(A) \geq 0, \\ \forall A \in \mathcal{P}_D, \sum_{i \in A} \lambda_i(A) = 1 \end{cases}$$

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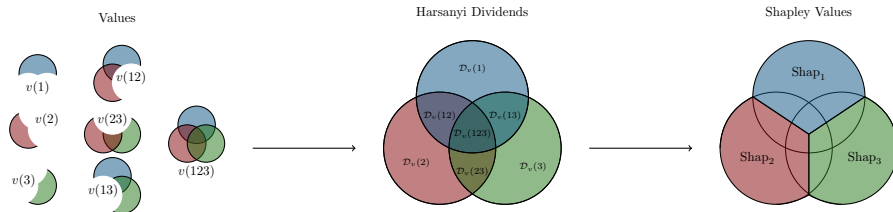
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In this setting, the **Shapley values are the egalitarian redistribution**, i.e.,  $\lambda_i(A) = 1/|A|$



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For any  $i \in D$ , denote  $\pi(i)$  the **position of player  $i$  in the permutation  $\pi$**  (i.e.,  $\pi_{\pi(i)} = i$ )

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$$\begin{aligned}\phi(i) &= \mathbb{E}_{\pi \sim p} [v(\{\pi_1, \dots, \pi_{\pi(i)}\}) - v(\{\pi_1, \dots, \pi_{\pi(i)-1}\})] \\ &= \sum_{\pi \in \mathcal{S}_D} p(\pi) [v(\{\pi_1, \dots, \pi_{\pi(i)}\}) - v(\{\pi_1, \dots, \pi_{\pi(i)-1}\})]\end{aligned}$$

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In this setting, the **Shapley values are the uniform distribution over the permutations**, i.e.,  $p(\pi) = 1/d!$

$$\text{Shap}(i) = \frac{1}{d!} \sum_{\pi \in \mathcal{S}_D} [v(\{\pi_1, \dots, \pi_{\pi(i)}\}) - v(\{\pi_1, \dots, \pi_{\pi(i)-1}\})]$$



# The recipe

Overall blueprint for using cooperative games for XAI:

1. **Step 1: Identify a quantity of interest**

Choose **a cake worth cutting**, e.g., point predictions  $f(x)$ , model variance  $\mathbb{V}(f(X))\dots$

🔍 Guides the **interpretation of the extracted insights**

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## 2. Step 2: Pick a **value function** $v$

And make sure that  **$v(D)$  is equal to the quantity of interest**, e.g.,

$\mathbb{E}[f(X) \mid X_A = x_A]$  for  $f(x)$ ,  $\mathbb{V}(\mathbb{E}[f(X) \mid X_A])$  for  $\mathbb{V}(f(X))$ ...

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## 3. Step 3: Pick an efficient allocation

In order to summarize the information of the  $2^d$  evaluations of  $v$

☞ Less crucial and can **highlight some model behavior**

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A **bad choice** can lead to **misleading insights**

e.g., **correlation/concurvity identifiability issues** (Zhang, Martinelli, and John 2024), **lack of purity** Köhler, Rügamer, and Schmid (2024)...

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## Conditional expectation

$$v(A) = \mathbb{E}[f(X) \mid X_A = x_A]$$

$$\mathcal{D}_v(1) = x_1 + \rho(x_1 + x_1^2 - 1) \quad \mathcal{D}_v(2) = x_2 + \rho(x_2 + x_2^2 - 1)$$

$$\mathcal{D}_v(12) = x_1x_2 - \rho(x_1 + x_1^2 + x_2 + x_2^2 - 1)$$

$$\text{Shap}_v(\{1\}) = x_1 + \frac{\rho}{2}(x_1 + x_1^2 - x_2 - x_2^2 - 1) + \frac{x_1x_2}{2}$$

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## Oblique projections

Il Idrissi et al. (2025)

$$\mathcal{D}_v(1) = x_1 \quad \mathcal{D}_v(2) = x_2$$

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**Example:** Proportional Marginal Effects (Herin et al. [2024](#))

- **Quantity of interest:**  $\mathbb{V}(f(X))$
- **Value function:**  $v(A) = \mathbb{E}[\mathbb{V}(f(X) \mid X_{D \setminus A})]$
- **Allocation:** *Proportional values*

$$p(\pi) = \frac{L(\pi)}{\sum_{\sigma \in \mathcal{S}_D} L(\sigma)}, \quad L(\pi) = \exp \left( - \sum_{j \in D} \log (v(\{\pi_1, \dots, \pi_{\pi(j)}\})) \right)$$

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**Proposition** (*Exogeneity detection*).

$$PME_i = 0 \iff X_i \text{ is not in the model.}$$

Key take-aways:

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- The **choice of value function is crucial**
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- The **choice of allocation is secondary**
  - ☞ It amounts to choosing an aggregation flavor

# References i

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**THANK YOU FOR YOUR ATTENTION!**

**ANY QUESTIONS?**

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