

Section 3.3.4 - Linear model with interaction

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03/12/2023

Computing the analytical allocations

```
# Model: G(X) = X1 + alpha*X2 + (1-a)X1X2
#####
# Function to compute the analytical Shapley and PME Values

computeAllocs<-function(alpha, rho){

  #Output variance
  vy=2+(1-alpha)^2+2*(1-alpha)*rho + rho^2

  #(Normalized) Shapley effects
  sh1=(3+rho^2*(1-alpha)^2+2*rho*(1-alpha))/(2*vy)
  sh2=(1+2*rho^2+(2-rho^2)*(1-alpha)^2 +2*rho*(1-alpha))/(2*vy)

  #(Normalized) PME
  denom=3+(1-alpha)^2
  pme1=2/denom
  pme2=((1-alpha)^2 +1)/denom

  res=cbind(sh1, sh2, pme1, pme2)
  return(res)
}

#####
# Sequence of correlation values
rhos<-seq(-0.999,0.999,0.001)

#####
# Sequence of beta values
alphas<-seq(0,1,0.001)

#####
# Grid of crossed rho/beta values to compute
grid.alpharho<-expand.grid(alphas, rhos)

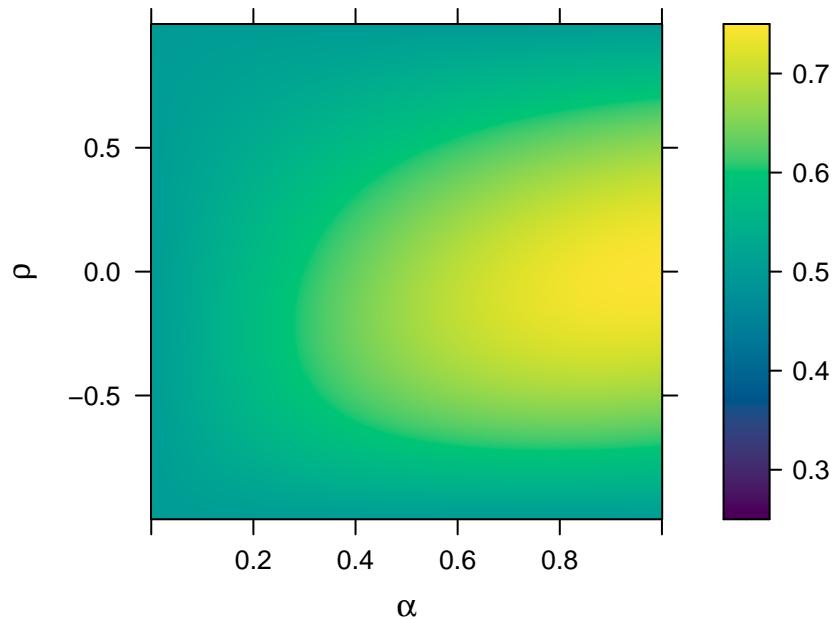
resAllocs=cbind(grid.alpharho, computeAllocs(grid.alpharho[,1], grid.alpharho[,2]))
colnames(resAllocs)[c(1,2)]=c("alpha", "rho")
```

```

par(mfrow=c(2, 1),
    cex=2)
levelplot(sh1~alpha*rho,
          data=resAllocs,
          col.regions=hcl.colors(10000),
          cuts=10000,
          main=expression(Sh[1]),
          ylab=expression(rho),
          xlab=expression(alpha),
          at=seq(0.25,.75,0.0001),
          cex=2)

```

Sh_1

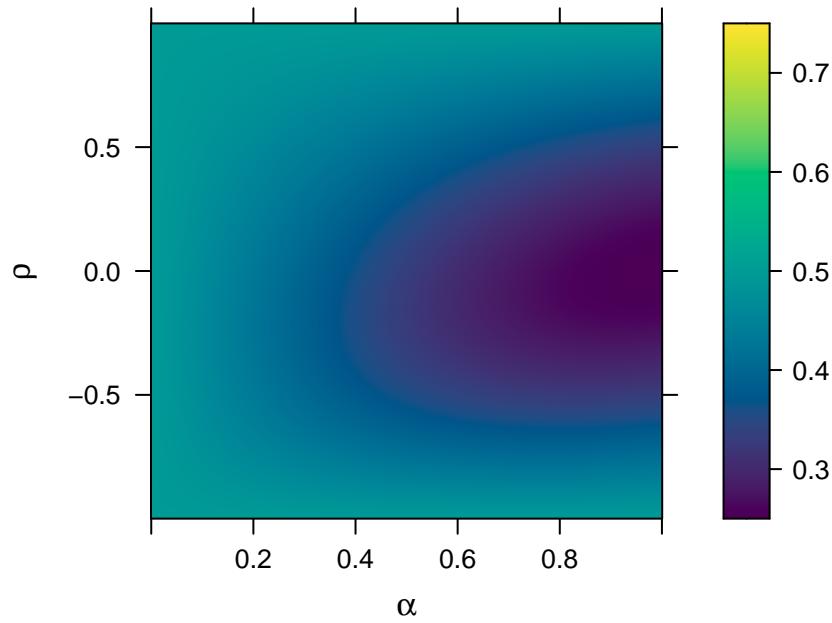


```

levelplot(sh2~alpha*rho,
          data=resAllocs,
          col.regions=hcl.colors(10000),
          cuts=10000,
          main=expression(Sh[2]),
          ylab=expression(rho),
          xlab=expression(alpha),
          at=seq(0.25,.75,0.0001),
          cex=2)

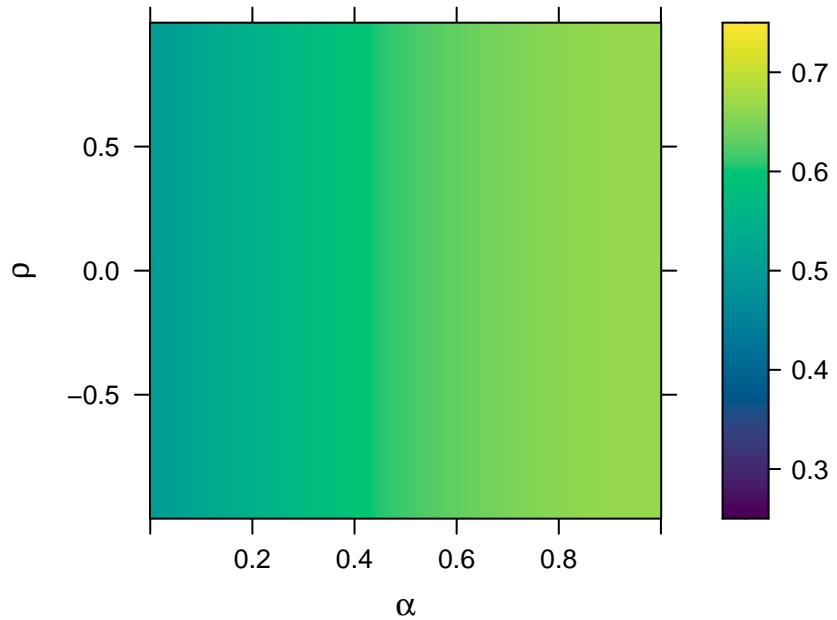
```

Sh_2



```
par(mfrow=c(2,1),
  cex=2)
levelplot(pme1~alpha*rho,
  data=resAllocs,
  col.regions=hcl.colors(10000),
  cuts=10000,
  main=expression(PME[1]),
  ylab=expression(rho),
  xlab=expression(alpha),
  at=seq(0.25,.75,0.0001),
  cex=2)
```

PME₁



```
levelplot(pme2~alpha*rho,
          data=resAllocs,
          col.regions=hcl.colors(10000),
          cuts=10000,
          main=expression(PME[2]),
          ylab=expression(rho),
          xlab=expression(alpha),
          at=seq(0.25,.75,0.0001),
          cex=2)
```

PME₂

