STEP 1

Team Member A: Stochastic Volatility Modeler

Q5. Price ATM European call and put with corr = -0.30, using Heston Model & Monte-Carlo simulation

From the question, we found out the parameters that we will use to price the european call and put are as follows:

- $\theta_v = 0.045$
- $\kappa_v = 1.85$
- $v_0 = 0.032$
- $S_0 = 80$
- $K = 80\ (we\ want\ to\ price\ ATM\ Option)$
- $r = 0.055$
- $\sigma = 0.35$
- $T = 3/12 = 0.25$
- $Correlation\ Value = \rho = -0.30$

The Heston Model is an options pricing model that utilizes stochastic volatility. It assumes that volatility is arbitrary, in contrast to Black-Scholes model that holds volatility constant (Ganti). Mathematically, Heston Model is a closed-form solution for pricing options different from Black-Scholes option pricing model (Ganti). The equation of Heston model is as follows (Ganti):

$$dS_t = rS_t dt + \sqrt{V_t} S_t dW_{1t}$$

$$dV_t = k(\theta - V_t)dt + \sigma\sqrt{V_t} dW_{2t}$$

The Heston model is already powerful tool to price options, to enhance it, many practitioners combine Heston model with Monte Carlo because it helps the model to accommodate more flexible payoff structure or constraints and with monte carlo we can utilize the law of big numbers to increase the accuracy due to numerical stability in long run. To price the option in question with Monte Carlo Model and Heston Model, the main steps we will do are (Anderson):

A. Generate Price Paths: like other monte carlo option pricing method, the first part is to generate price paths of the underlying asset that we want to do option pricing. With monte carlo we can generate numerous simulated paths for the underlying asset price S_t and the volatility V_t using the Heston equation that we have defined above. Because the Heston model is working in continuous space, then to use Heston SDE we need to discretize the equations and simulate step by step over time.

B. Valuing options: After we have generated the S_t and the V_t we can evaluate the payoff of the options, we will evaluate the payoff for each path and discounted to the present value to determine the option price.

The correlation value that is defined in the question plays a pivotal role to accurately model the interaction between the asset price S_t and the volatility V_t when pricing the options. The Heston model assumes that the S_t and V_t are driven by two stochastic process W_{1t} and W_{2t}. Those two process is correlated by ρ. Mathematically, we can write down that (Heston):

$Corr(dW_{1t}, dW_{2t}) = \rho$

Using python we price the european call and put option at the money (ATM) with correlation values = -0.3 and the result is as follows:

	European Call Option	**European Put Option**
Price	$3.86	$2.72

Q6. Price ATM European call and put with corr = -0.70, using Heston Model & Monte-Carlo simulation

With the same theory from number 5, we will calculate the price of european call and put option using python. We priced the european call and put option at the money (ATM) with correlation values = -0.7 and the result is as follows:

	European Call Option	**European Put Option**
Price	$3.86	$2.72

We found out that with the change of correlation values from -0.30 to -0.70 there's no significant change in european call option and european put option price.

Q7. Delta and gamma for each of the options in Questions 5 and 6.

Delta is a measure of the option price change with respect to underlying asset price change. We approximate the delta with a forward finite difference method. This is done by calculating the option price for current stock price and for a slightly higher stock price and then finding differences in option prices.

$$\delta = \frac{Option\ Price\ when\ the\ stock\ price\ slightly\ higher\ - option\ price\ in\ current\ stock\ price}{Change\ in\ Price}$$

Gamma is a measure of how much delta changes with respect to underlying asset price change. We use second-order central finite difference method to approximate Gamma value. This is done by calculating the option prices at three points: the current stock price, a slightly higher stock price, and a slightly lower stock price. Then, we use the following equation to calculate the Gamma:

$$\gamma = \frac{Option\ Price\ when\ the\ stock\ price\ slightly\ higher\ - 2\ x\ option\ price\ in\ current\ stock\ price\ + option\ price\ when\ price\ lower}{Change\ in\ Price^{2}}$$

Using Python we calculate the delta and gamma of number 5 and 6. The results are as follows:

	Correlation Values = -0.3	Correlation Values = -0.7
Delta for European Call option	**0.574**	**0.575**
Delta for European Put option	**-0.426**	**-0.425**
Gamma for European options	**0.047**	**0.051**

Team Member B: Jump Modeler

From the lesson 7 from WorldQuant, we learnt that the Stochastic Differential Equation for the stock price St in Merton's jump-diffusion model is given by (WorldQuant):

$dS\_t \; = \; (r \; - \; r\_j)\, S\_t\, dt \; + \; \sigma\, S\_t\, dZ\_t \; + \; J\_t\, S\_t\, dN\_t\, dSt$

where:

r**:** The risk-free interest rate.

r_j**:** The adjustment to the drift term to ensure the risk-neutral measure, defined as:
$rj \; = \; \lambda(e\mu j \; + \; \delta 22 \; - \; 1). rj \; = \; \lambda(e\mu j \; + \; 2\delta 2 \; - \; 1)$
This accounts for the expected effect of jumps on the price.

σ**:** The volatility of the stock's returns (diffusive component).

dZt**:** The standard Brownian motion component, modeling continuous price changes.

dNt**:** The Poisson jump process, modeling the occurrence of jumps, with:

- λ**:** The intensity (rate) of the Poisson process, representing the average number of jumps per unit time.

Jt**:** The relative jump size, distributed as:
$log(1 \; + \; Jt) \sim N(log(1 \; + \; \mu j) \; - \; \delta 22, \delta 2), log(1 \; + \; Jt) \sim N(log(1 \; + \; \mu j) \; - \; 2\delta 2, \delta 2),$
where:

- μj: The average jump size.
- δ: The standard deviation of the jump size.

From the above mathematical function, the python code is written to check the Merton Model performance. The next sections shall be pricing an European call and put with defined parameters. For the Merton model, we use the following parameters:

- μ =− 0. 5

- δ = 0. 22

Other parameters are as the above questions.

Notice that, the question 11 - parity check is performed together with Question 8 and 9, so it is easier to track the results.

Q8. Using the Merton Jump Model and Monte-Carlo simulation, price an ATM European call and put, using a correlation value of -0.30.

The output from python code is as below:

Jump intensity (lambda): 0.75

	European Call Option	**European Put Option**
Price	$8.31	$7.22

The Call - Put parity is checked and it shows that the parity is held with very small difference (-0.00152982).

Q9. Using the Merton Jump Model, price an ATM European call and put, using a correlation value of -0.70.

The output from python code is as below:

Jump intensity (lambda): 0.25

	European Call Option	**European Put Option**
Price	$6.82	$5.73

The Call - Put parity is checked and it shows that the parity is held with very small difference (-0.00079382).

Q10. Calculate delta and gamma for each of the options in Questions 8 and 9

Delta is a measure of the option price change with respect to underlying asset price change. We approximate the delta with a forward finite difference method. This is done by calculating the option price for current stock price and for a slightly higher stock price and then finding differences in option prices.

$$\delta = \frac{Option\ Price\ when\ the\ stock\ price\ slightly\ higher\ - option\ price\ in\ current\ stock\ price}{Change\ in\ Price}$$

Gamma is a measure of how much delta changes with respect to underlying asset price change. We use second-order central finite difference method to approximate Gamma value. This is done by calculating the option prices at three points: the current stock price, a slightly higher stock price, and a slightly lower stock price. Then, we use the following equation to calculate the Gamma:

$$\gamma = \frac{Option\ Price\ when\ the\ stock\ price\ slightly\ higher\ - 2\ x\ option\ price\ in\ current\ stock\ price\ + option\ price\ when\ price\ lower}{Change\ in\ Price^{\ 2}}$$

Using Python we calculate the delta and gamma of number 8 and 9. The results are as follows:

	Jump intensity (lambda): 0.75	Jump intensity (lambda): 0.25
Delta for European Call option	0.66	0.61
Delta for European Put option	0.36	-0.41
Gamma for European options	0.02	0.03

Team Member C: Model Validator

Q11. For Questions 5, 6, 8, and 9, use put-call Parity to determine if the prices of the put and call from the Heston Model and Merton Model satisfy put-call parity.

The put-call parity of options prices are defined by the following equation:

$$Call\ Price\ -\ Put\ Price\ =\ Initial\ Stock\ Price\ -\ Strike\ Price\ x\ e^{(-risk\ free\ rate\ x\ time\ to\ mature)}$$

Usually the difference between the right side and the left side is not = 0 but approximately 0, this is achieved by using tolerance values. In checking the result of number 5, 6, 8, and 9 we are using tolerance levels of 5 x 10^-2. We consider that already near 0.

Q11.1 Put-call parity of Heston Model

The results of put-call parity of number 5 and 6 are as follows:

Put-Call Parity Check for Correlation -0.30:

LHS (C - P): 1.14

*RHS (S0 - K * exp(-r * T)): 1.09*

Discrepancy: ***4.75e-02***

Put-Call Parity Check for Correlation -0.70:

LHS (C - P): 1.14

*RHS (S0 - K * exp(-r * T)): 1.09*

Discrepancy: ***0.05***

Based on the result, the put-call parity is held in number 5 and 6.

Q11.2 Put-call parity of Merton Model

The results of put-call parity of number 8 and 9 are as follows:

Jump intensity (lambda): 0.75

Put-Call Parity Check ***Passed*** *(Difference: -0.00152982)*

Jump intensity (lambda): 0.25

*Put-Call Parity Check **Passed** (Difference: -0.00079382)*

Based on the result, the put-call parity is held in number 8 and 9.

Q12. Run the Heston Model and Merton Model for 7 different strikes (moneyness values: 0.85, 0.90, 0.95, 1, 1.05, 1.10)

Like with the previous GWP2, in this task we are going to approximate the strike price using Heston Model and Merton Model using the moneyness levels that are defined in the question. The results are:

Results for Different Strike Prices (Moneyness: 0.85 to 1.15):

A. Strike Price: 94.12 (Moneyness: 0.85)
    - Heston Call Price: 0.3055, Heston Put Price: 13.0995
    - Merton Model Lambda 0.75: Call Price: 2.8118, Put Price: 15.6342
    - Merton Model Lambda 0.25: Call Price: 1.9933, Put Price: 14.8390
B. Strike Price: 88.89 (Moneyness: 0.9)
    - Heston Call Price: 0.9009, Heston Put Price: 8.5284
    - Merton Model Lambda 0.75: Call Price: 4.3751, Put Price: 12.0239
    - Merton Model Lambda 0.25: Call Price: 3.2781, Put Price: 10.9401
C. Strike Price: 84.21 (Moneyness: 0.95)
    - Heston Call Price: 2.0491, Heston Put Price: 5.0832
    - Merton Model Lambda 0.75: Call Price: 6.2157, Put Price: 9.2778
    - Merton Model Lambda 0.25: Call Price: 4.9044, Put Price: 7.9566
D. Strike Price: 80.00 (Moneyness: 1.0)
    - Heston Call Price: 3.8567, Heston Put Price: 2.7220
    - Merton Model Lambda 0.75: Call Price: 8.3158, Put Price: 7.2260
    - Merton Model Lambda 0.25: Call Price: 6.8189, Put Price: 5.7475

E. Strike Price: 76.19 (Moneyness: 1.05)

- Heston Call Price: 6.1938, Heston Put Price: 1.3134
- Merton Model Lambda 0.75: Call Price: 10.5291, Put Price: 5.6883
- Merton Model Lambda 0.25: Call Price: 8.9531, Put Price: 4.1281

F. Strike Price: 72.73 (Moneyness: 1.1)

- Heston Call Price: 8.8738, Heston Put Price: 0.5692
- Merton Model Lambda 0.75: Call Price: 12.8468, Put Price: 4.5628
- Merton Model Lambda 0.25: Call Price: 11.2443, Put Price: 2.9714

G. Strike Price: 69.57 (Moneyness: 1.15)

- Heston Call Price: 11.6339, Heston Put Price: 0.2209
- Merton Model Lambda 0.75: Call Price: 15.1252, Put Price: 3.7174
- Merton Model Lambda 0.25: Call Price: 13.5496, Put Price: 2.1782

STEP 2

All team members:

Q13. Repeat Questions 5 and 8 for the case of an American call option. Comment on the differences you observe from original Questions 5 and 8.

Q13.5. Price ATM American call with corr = -0.30, using Heston Model & Monte-Carlo simulation

The price of the American call option at the money (ATM) using Heston Model with correlation values = -0.3 are calculated from python code as below. Besides, the European Call Option as calculated above is compared here.

	European Call Option	American Call Option
Price	$14.4	$15.12

As can be seen above, the American call price is 5% more than the European Call price. The slightly higher price of the American call option ($15.12 vs. $14.4) reflects this small early exercise premium.

Q13.8. Using the Merton Model, price an ATM American call with jump intensity parameter equal to 0.75

The price of the American call option at the money (ATM) using Merton Model with jump intensity parameter equal to 0.75 are calculated from python code as below. Besides, the European Call Option as calculated above is compared here.

	European Call Option	American Put Option
Price	$8.31	$27.27

As can be seen above, the American call price is 3.5 times the European Call price. The early exercise feature of the American put explains much of its higher price relative to the European call.

Q14. Using Heston model data from Question 6, price a European up-and-in call option (UAI) with a barrier level of $95 and a strike price of $95 as well.

The output of python code is:

```
European up-and-in-call option price under Heston model with -0.70
correlation is: $0.26
```

The price under Heston Model (Question 6) is $3.86 which is significantly higher because it does not depend on whether the stock price breaches a barrier. It can be explained that the barrier condition adds a significant restriction to the UAI option, reducing its value dramatically compared to the European call.

Q15. Using Merton model data from Question 8, price a European down-and-in put option (DAI) with a barrier level of $65 and a strike price of $65 as well.

The output of python code is:

```
European down-and-in put option price under Merton model with jump
intensity 0.75 is: $2.92
```

The European Put Option with a simple Merton Model is $7.22 which is the higher price. This reflects the absence of the barrier condition, as the option payoff is determined solely by the terminal stock price at expiration. It can be explained that the DAI option's activation condition significantly reduces its value compared to the vanilla European put option, as not all paths of the stock price result in a valid option payoff.

Reference:

[1] Ganti, A. "Heston Model: Meaning, Overview, Methodology." *Investopedia*, 18 Sept. 2022, www.investopedia.com/terms/h/heston-model.asp.

[2] Heston, "A closed-form solution for options with stochastic volatility with applications to bond and currency options", Rev. Finan. Stud. Vol. 6 (1993)

[3] Anderson, L, "Efficient Simulation of the Heston Stochastic Volatility Model", (2005).

[4] WorldQuant. *JUMP DIFFUSION MODELS*, WorldQuant, 2024,

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