


Compute mean and variance from the **population** of size N :

Population Mean $\mu = E[X] = \frac{\sum X}{N}$

$$\sigma^2 = E[X^2] - \mu^2$$

Population Variance

$$\sigma^2 = E[(X - \mu)^2] = \frac{\sum (X - \mu)^2}{N} \text{ or } \frac{\sum X^2 - \frac{(\sum X)^2}{N}}{N}$$

Estimate mean and variance from a **sample** of size n :

Sample Mean $\bar{x} = \frac{\sum X}{n}$

Sample Variance $S^2 = \frac{\sum (X - \bar{x})^2}{n-1} \text{ or } \frac{\sum X^2 - \frac{(\sum X)^2}{n}}{n-1}$

Why $n-1$?

if this is n
$$s^2 = \frac{\sum (X - \bar{x})^2}{n} = \frac{1}{n} \left(\sum X^2 - \frac{(\sum X)^2}{n} \right)$$

for unbiased estimate, we expect the mean of s^2 to be equal to σ^2

$$E[S^2] = E \left[\frac{1}{n} \left(\sum X^2 - \frac{(\sum X)^2}{n} \right) \right]$$

$$= \frac{1}{n} \left(\sum E[X^2] - \frac{E[(\sum X)^2]}{n} \right)$$

if $Y = \sum X$
 $E(Y^2) = \sigma_y^2 + \mu_y^2$

$$= \frac{1}{n} \left(n\sigma^2 + n\mu^2 - \frac{E[(\sum X)^2]}{n} \right)$$

$\sigma^2 + \mu^2$
 from prev slide

$$= \frac{1}{n} \left(n\sigma^2 + n\mu^2 - \frac{\text{Var}[\sum X] + E[\sum X]^2}{n} \right)$$

$$\begin{aligned}
E[S^2] &= \frac{1}{n} \left(n\sigma^2 + n\mu^2 - \frac{\sum \text{Var}[x] + \sum E[x]^2}{n} \right) \\
&= \frac{1}{n} \left(n\sigma^2 + n\mu^2 - \frac{n\sigma^2 + (n\mu)^2}{n} \right) \\
&= \frac{1}{n} \left(n\sigma^2 + n\mu^2 - \sigma^2 - n\mu^2 \right) = \frac{1}{n} (n-1)\sigma^2
\end{aligned}$$

something about this becoming biased? idk

- linear transformation of variable

- Variance sum law I

↳ linear combo of 2 independent variables

if $t = 5x + 10y$

mean = $5(\bar{x}) + 10(\bar{y})$ ← daily income difference d

variance = $5\sigma^2$

???

Eg: Students took 2 parts of a test, each worth 50 points. Part A has a variance of 25, and Part B has a variance of 49. The correlation between the test scores is 0.6.

- (i) If the teacher adds the grades of the two parts together to form a final test grade, what would the variance of the final test grades be?
(ii) What would the variance of Part A - Part B be?

$$\begin{aligned} \text{(i) } \text{Var}(A + B) &= 25 + 49 + 2 \cdot 0.6 \cdot \sqrt{25} \cdot \sqrt{49} \\ &= 116 \end{aligned}$$

$$\begin{aligned} \text{(ii) } \text{Var}(A - B) &= 25 + 49 - 2 \cdot 0.6 \cdot \sqrt{25} \cdot \sqrt{49} \\ &= 32 \end{aligned}$$