Combinational logic circuits

- most common type of digital logic circuit
- combination of logic gates
- $O(t) = f[I_1(t), I_2(t), I_3(t) \dots I_n(t)]$

O/P at any time t depends on the inputs at that time

- no memory characteristics (unlike sequential circuits)
- easier to analyse and design (— " —)
- its f^n can be completely represented in a truth table
- boolean expression = boolean equation = logic f^n

forms of boolean exp.

Canonical — all variables must exist

active HIGH SOM (sum of minterms) = 1
active LOW POM (product of maxterms) = 0

Standard — all variables need not exist

SOP (sum of products)
POS (product of sums)

key.

minterms : all combinations of input variables connected by an AND operator

maxterms : all possible combinations of a given set of boolean variables formed by using the OR operator

(more on next to next page)

- simplified exp from canonical forms lead to simpler logic circuits (standard form)

↑ combinational circuits minimisation

→ no of gates

→ no of inputs per gate

ex. $f(x, y, z) = xyz' + xyz + x'y'z + xy'z$

$= xy + y'z$ → sum of products

- neither SOP or POS

$f = (xy)'z + xz'$

$f = xy(x'+z)'$

minterm because minimum number (1) of terms we need for logic-1 o/p

maxterm because max (all) number of terms needed for logic-1 o/p

	x	y	minterms (written in a way that result is 1)		maxterms (result is 0)	
dec 0	0	0	$x' \cdot y'$	$m0$	$x+y$	$M0$
dec 1	0	1	$x' \cdot y$	$m1$	$x+y'$	$M1$
dec 2	1	0	$x \cdot y'$	$m2$	$x'+y$	$M2$
dec 3	1	1	$x \cdot y$	$m3$	$x'+y'$	$M3$

→ clearly $m_i = \overline{M_i}$

→ n-inputs have 2^n min/max terms

4 : a, b, c, d

$13_{10} = 1101_2$

$M13 = a' + b' + c + d'$

$2_{10} = 0010_2$

$m2 = a'b'cd'$

→ for n-inputs for n minterms maxterms, only one minterm maxterm (for a certain set of inputs) gives one zero.
and it is order specific.

011

$xyz \longrightarrow x'yz$ $(m3)$

$zyx' \rightarrow 110$ $(m6)$

inputs			Output F	min		max	
X	Y	Z					
0	0	0	0	$x' \cdot y' \cdot z'$	m0	$X+Y+Z$	M0
0	0	1	1		m1		M1
0	1	0	1		m2		M2
0	1	1	0		m3		M3
1	0	0	1		m4		M4
1	0	1	0		m5		M5
1	1	0	0		m6		M6
1	1	1	1		m7		M7

SOm : $(x'y'z) + x'yz' + xy'z' + xyz$

$\Sigma_{xyz}(1,2,4,7)$

$F(x,y,z) = \Sigma m(1,2,4,7) = m1 + m2 + m4 + m7$

POM : $(x+y+z)(x+y'+z')(x'+y+z')(x'+y'+z)$

$\Pi_{xyz}(0,3,5,6)$

$F(x,y,z) = \Pi M(0,3,5,6) = M0 \cdot M3 \cdot M5 \cdot M6$

canonical form → std. form

→ algebraic

Karnaugh maps

→ Quine-McCluskey

Heuristic methods eg. Espresso-II

implemented in comp

$$Z = ABC + AB'(A'C')'$$
$$Z = ABC + AB'(A+C)$$
$$Z = ABC + AB' + AB'C$$
$$Z = AC + AB'$$
$$Z = A(C+B')$$

$$X = (A'+B)(A+B+D)D'$$
$$X = (A'+B)(AD'+BD'+0)$$
$$X = A'BD' + BAD' + BD'$$
$$X = BD' + BDA'$$
$$X = BD'$$

L7 practice problems

1. Construct the truth table for the following logic functions.

   a. F(X,Y,Z) = X’Y + X’Y’Z $= X'YZ + X'YZ' + X'Y'Z = \Sigma m(1,2,3)$

   b. F(A,B,C,D) = (((A+B’)’ + C)’+D)’ $= ((\overline{A'}B\cdot\overline{C})+D)' = ((A+B')C'+D)' = \overline{(A+B')C'}\cdot D'$

   $= (A'B+C)D' = A'BCD' + A'BC'D' + CD'(AB+A'B+BA'$
   $+A'B')$

2. Write the canonical sum-of-minterm expression for output F in question 1(a).

3. Write the canonical product-of-maxterm expression for output F in question 1(b).

4. The following expressions are taken from lecture slides 5.23. Use algebraic manipulations to simplify each of them and obtain the minimum cost SOP (sum-of-product) expression.

   Students are required to know and apply Boolean theorems but are not required to cite the name of the theorems used

   $X + XY = X$

   $X + X'Y = X + Y$

   $ABC + AB'(A+C)$

   $ABC + AB'C + AB'$

   $ABC + AB'$

   $A(BC + B')$

   $A(B' + C) = AC + AB'$

   a. Z = ABC + AB’(A’C’)’

   b. X = (A’ + B)(A + B + D)D’

   $(A'+B)(AD'+BD')$

   $(A'BD' + BAD')$

   BD'