$Ax = b$ is consistent

① row pivots : all (of A) ⇒ soln ✓
② col pivots : all but last ⇒ soln unique
③ " : all but last + another ⇒ ∞ soln
④ " : last is pivot ⇒ no soln

$[A \; b]$ aug

· free variable : column doesn't have a pivot

Rank A = Rank $[A|b]$

$Ax = 0$
only trivial : all pivots are non-zero for every variable
trivial + ∞ : number of non-zero pivot < no of variables ⇒ free variables exist

area of $T(S)$ = abs($|A|$) x area of S

$A_{m \times n}$
· A has pivot in every row
· A has at least 1 soln $\forall b \in R^m$
· A's cols span R^m

th. 1·2

columns of A → basis for col space of A (↳ range)
· for $A \xrightarrow{ERO} B$, $\lvert A\rvert = \lvert B\rvert$, ERO : $R_i = R_i + nR_j$

· $Ax = 0$ has $x = 0$ ⇒ A is inv only if it is square

- A & B are inv ⇔ AB inv

. num. of non-zero pivots = $\dim C(A)$ (col) = $\dim C(A^T)$ (row) = rank(A) = rank(A^T)
· $\dim(N(A)) = n - \dim C(A)$
· rank : no of independent col = no of pivot cols
- dimension : no of free variables

rank dim

R^m
$m \times n$
$C(A)$ $N(A)$

LU transformation:
$A \xrightarrow{e_1} \xrightarrow{e_2} \cdots \xrightarrow{e_k} U$ (upper Δ)
$L = e_1^{-1} e_2^{-1} \cdots e_k^{-1}$ (lower Δ)
$Ax = b$; $LUx = b$; $Ux = y$;
$Ly = b$;

· rows ⊥ → null space ($N(A)$)
· cols ⊥ → nullspace (A^T)
· orthogonal matrix :
□ , orthogonal cols, $\|col\| = 1$

cols of A span R^n
$AD = I$
$Ax = b$ has at least 1 soln $\forall b \in R$
A is inv
A^T is inv
A is row eq to I
$CA = I$
A has n pivot pos
$Ax = 0$ has only the trivial soln
cols of A are linearly dep

$Ax = b$ is inconsistent $\rightarrow$ Rank A < Rank(A|b)

- A ← orthogonal cols : projection : $\hat{b} = \sum_{i=0}^{n} \frac{a_i \cdot b}{a_i a_i} a_i$

↑ if not

- $A = QR$ → best method : $Q_{m \times n}$, $R_{n \times n}$ $QRx = b$, $Rx = Q^T b$

$Q_{m \times n}$: using gram-schmidt
orthogonalise + normalise
$col(Q) = col(A)$
$Q^T Q = I$, QQ^T = Projection.
$\hat{b} = QQ^T b$
orthonormal basis for col(A)

o/c its inv

$R_{n \times n}$: upper Δ matrix

$$\begin{bmatrix} q_1 a_1 & q_1 a_2 & \cdots & \\ 0 & q_2 a_2 & q_2 a_3 & \cdots \\ 0 & 0 & \ddots & \\ \vdots & \vdots & & \ddots \end{bmatrix}$$

R is inv if A is inv

- QQ^T : for Q m×n
  - rank = n
  - orthonormal
  - dim(col) = n
  - dimensions : m×m

- an orthonormal basis U → $UU^T y$ give proj y onto space spanned by U

- normal equation : $A^T A x = A^T b$ (use when A is full rank)
  A m×n
  - $Ax = b$ has a unique least square soln for each b in R^m
  - cols of A are independent
  - $A^T A$ is inv
  - $\hat{x} = (A^T A)^{-1} A^T b$

- defective matrix : has an eigen value w/ alg mul > geo mul
  ↓
  not diagnonalizable

- P has cols of eigenvectors, D has eigenvalues
  $A = PDP^{-1}$
- if $\lambda = 0$, A is not inv
- inv and diagnalisability aren't related
- if n independent eigenvectors ⇒ diagnalise