


Time complexity of Matrix + Array

$T(\text{accessing element in Matrix}) \rightarrow O(1)$

$T(\text{accessing element in array}) \rightarrow O(1)$

Time complexity of the methods

① $T(\text{create PQ}) = O(V) \quad [\text{best, worst}] \quad T(V)$

$V \times O(1) \rightarrow \text{insert each element}$

② $T(\text{insert}) = T(\text{fix})$

③ $T(\text{fix}) = O(\text{index})^{\text{PQ size}} [\text{worst}], O(1) [\text{best}]$

\therefore there is only 1 element out of place (at index) when this method is called.

and it always has to be moved up (towards 0) so a max of index - 1

④ $T(\text{delete}) = O(\text{PQ size}) + T(\text{fix}) = \max(O(n), O(h)) = O(n)$

finding the index of the vertex to delete is $O(\text{PQ size} [\text{worst}])$ due to the array of indexes used. ($O(1) \rightarrow \text{best}^2$)
index + $T(\text{index})$

⑤ $T(\text{empty PQ}) = O(1)$

⑥ $T(\text{initialise Algo}) = O(4V) \rightarrow O(V)$

⑦ $T(\text{fill graph}) = O(V^2)$

⑦ Dijkstra $\rightarrow O(V^2) + O(E \cdot V)$
 in worst case: $E = V(V-1)$
 $\therefore O(V^3)$

① initialise Algo

② create PQ $\rightarrow O(V)$

③ fix $\rightarrow O(V)$

$O(V^2)$
 $+$
 $O(EV)$

$\left\{ \begin{array}{l} \text{while (!emptyPQ)} \rightarrow \text{runs } V \text{ times } (\because \text{PQ has } V \text{ elements} \\ \text{1 is removed per iteration}) \\ \quad \text{① delete} \rightarrow O(V) \\ \quad \text{for (every neighbour not in } S) \{ \\ \quad \quad \text{③ comp} \leftarrow \text{constant} \\ \quad \quad \text{① delete} \\ \quad \quad \text{② insert} \\ \quad \quad \} \\ \quad \} \end{array} \right\} \left\{ \begin{array}{l} O(V) \\ O(\frac{E}{V} \cdot V) \\ = O(E) \end{array} \right\}$

worst case :

$$g \cdot E = (g \cdot V)(g \cdot V - 1) \text{ (connected graph)}$$

⑦ Dijkstra $\rightarrow O(V^2) + O(EV)$ best case: $E = V - 1$
 $\therefore O(V^2) + O(V \cdot V - 1)$
 $= O(V^2)$

① initialise Algo

② create PQ $\rightarrow O(V)$

③ fix $\rightarrow O(V)$

while (!empty PQ) \rightarrow runs V times (\because PQ has V elements
 1 is removed per iteration)

① delete $\rightarrow O(V)$

for (every neighbour not in S) {

3 comp \leftarrow constant

① delete

② insert

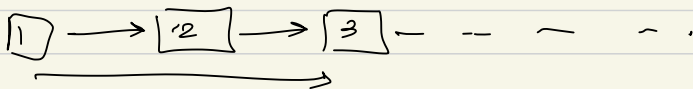
} } $O(V)$ $\left\{ \begin{array}{l} O\left(\frac{E}{V} \cdot V\right) \\ = O(E) \end{array} \right.$

$O(V^2)$
 $+$
 $O(E \cdot V)$

best case:

✓ $g: E = g: V - 1$ (a list)

\rightarrow only one path \rightarrow by default, shortest path
 each vertex has 1 adjacent vertex



Time complexity of List + Heap

$T(\text{accessing element in list}) \rightarrow O(1)$

$T(\text{accessing element in heap}) \rightarrow O(1)$

we are only accessing one next's due to indexes

Time complexity of the methods

① $T(\text{create PQ}) = O(V)$ [best, worst] $T(V)$

$V \times O(1) \rightarrow$ insert each element

③ $T(\text{rearrange}) = O(\log_2 \text{index})$ [worst], $O(1)$ [best]

$\rightarrow \text{PQ size} \rightarrow V$

only one element in the heap is out of place. It has to be moved up. Max \rightarrow its depth $\rightarrow (\log_2 \text{index})$
min $\rightarrow 1$

worst case index

= PQ size

worst case PQ size : V

④ $T(\text{delete Min}) = T(\text{fix-Heap})$

$T(\text{fix-Heap})$

$\rightarrow O(2 \log_2 V) = O(\log_2 V)$

⑤ $T(\text{empty PQ}) = O(1)$

⑥ $T(\text{initialise Algo}) = O(4V) \rightarrow O(V)$

⑦ $T(\text{fill Graph}) = O(V + E)$

⑦ Dijkstra $\rightarrow O(V \log V) + O(E \log V)$

① initialise Algo $\rightarrow O(V)$

② create PQ $\rightarrow O(V)$

③ rearrange $\rightarrow O(\log_2 V)$

$O(V \log V) + O(E \log V)$

$\left\{ \begin{array}{l} \text{while (!empty PQ)} \rightarrow \text{runs } V \text{ times } (\because \text{PQ has } V \text{ elements} \\ \text{1 is removed per iteration}) \\ \quad \text{① deleteMin} \rightarrow O(\log_2 V) \\ \quad \text{for (every neighbour not in } S) \{ \\ \quad \quad \begin{array}{l} \text{3 comp} \leftarrow \text{constant} \\ \text{① rearrange} \end{array} \} O(\log_2 V) \\ \quad \} \end{array} \right\} O\left(\frac{E}{V} \log_2 V\right)$

worst case : $E = V \cdot (V-1)$

$O(V^2 \log V)$

best case : $E = V-1$
 $O(V \log V)$