Function's

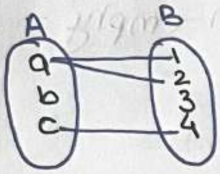
(mapping / Transformation)

Special type of relation

i.e

Relation from set A to set B

$R: A \to B$



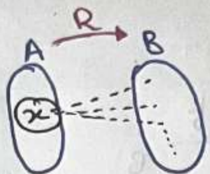
$R \subseteq A \times B$

$a \longrightarrow 1, 2$
$b \not\longrightarrow$
$c \longrightarrow 4$

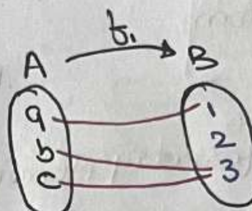
ex: $|A| = 5$, $|B| = 10$. Relation $R: 5 \to B$

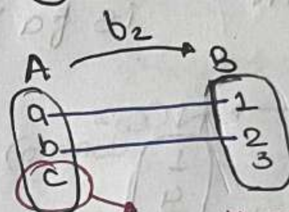
$x \in A$ then $|\{y \mid xRy,\ y \in B\}| = ?$ 0 to 10

Set of those element of B to which x is Related.



- Function from A to B is a Relation in which "Every" element of A (individually) Related to exactly one element of B.



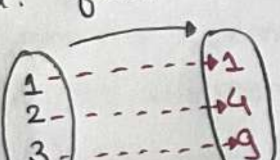


b/c of these it's not functn but Relation.

$f: A \longrightarrow B$

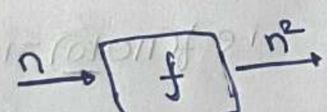
Domain   Codomain

ex: $f(n) = n^2$



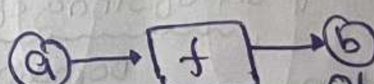
$f(n) = n^2$

Transformation



$f(1) = 1^2$; $f(2) = 2^2$; $f(3) = 3^2$

functn f is like a Transformation



I/P ∈ Domain   O/P ∈ Co-Domain

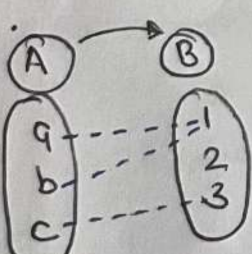
$f(a) = b$

Image of "a" (under f) is "b"

Pre-image of b = a

- pre-image : Pre-image of x (under f) $x \in$ Co-domain.

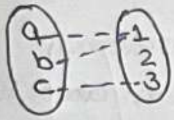
$f: A \longrightarrow B$



- Image of a = 1
- Preimage of 1 = {a, b}
- Image of 3 ✗ nonsense.
- preimage of 2 = ϕ (none)

Image of a: {1} × b/c image will always be unique

Image of a: 1 ✓

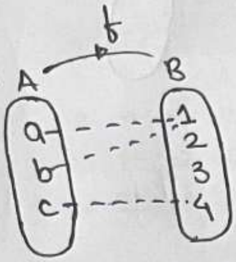


$\boxed{\text{Range of a funct}^n}$

Range ≡ Reach

Which element of Codomain are Reachable from domain.

ex:



Range(f) = {1, 4}

Range = {y | y ∈ co-domain; preimage(y) is non-empty}

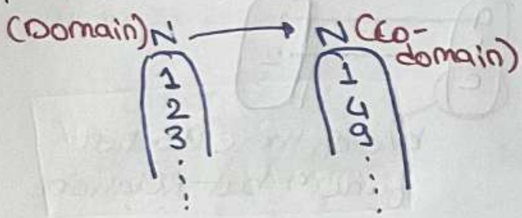
Or

Range(f): {y | y ∈ Co-domain; ∃x ∈ Domain, f(x) = y}

• for every element of A there exist exactly one y.

ex: $f: N \longrightarrow N$ is f a function?

$f(x) = x^2$ yes.



Range(f) = $\{1, 2^2, 3^2, 4^2, 5^2, 6^2, \dots\}$

7 ∉ Range(f)

8 ∉ Range(f)

9 ∈ Range.

ex: f: $\underbrace{\text{set of string of length} \geq 2}_{\text{Domain}}$ i.e f(110<u>10</u>) = 10

then

Range: {00, 01, 10, 11}

ex: $f(x) = x^2$ given, N = {1, 2, 3, ...}.

$f: \underbrace{Z \longrightarrow N}$ ×

not a funct^n

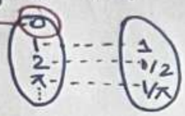
"0" has no image

① why is f not a funct^n from R to R if $\infty \notin R$

ⓐ $f(x) = \frac{1}{x}$; b/c no Image of "0"

$f: R \longrightarrow R$

no Image



ⓑ $f(x) = \sqrt{x}$; b/c no image for (-ve) no.

image of $-1 = \sqrt{-1} \notin R$

$\boxed{\sqrt{-1} = i}$

ⓒ $f(x) = \pm\sqrt{(x^2-1)}$?

b/c of these every element has two Image.

note

- $\sqrt{25} = 5$
- $x^2 = 25$ then $x = \pm 5$
- $x^2 = a^2$ then $x = \pm a$

$x^2 = a$ then $x = \pm\sqrt{a}$

② which one is funct^n from Z to R.

~~ⓐ~~ $f(n) = \pm n$

b/c of these it's not

~~ⓑ~~ $f(n) = \frac{1}{(n^2-4)}$

b/c no image of 2

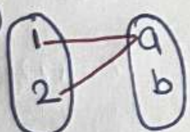
ⓒ $f(n) = \sqrt{n^2+1}$

is a funct^n

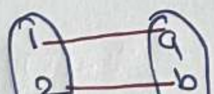
ex: write down all the funct^n from the 2 element set $\{1,2\}$ to the 2 element set $\{a,b\}$

$f: \{1,2\} \longrightarrow \{a,b\}$

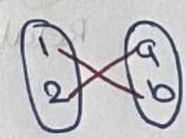
①



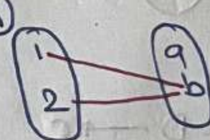
②



③



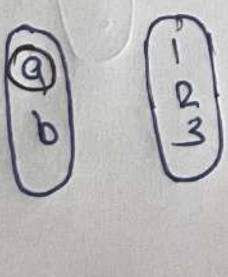
④



ex: no. of funct^n from two element set to 3 element set?

A: $\{a,b\}$ to $B = \{1,2,3\}$

$f: A \longrightarrow B$



funct^n possible.

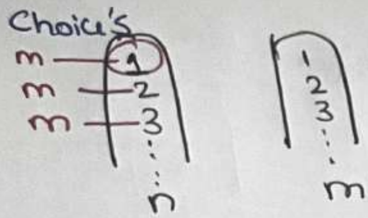
ⓐ × ⓑ

3 Choices 3 choices.

$\Rightarrow 9.$

Generalize

$|Domain| = n$; $|co\text{-}Domain| = m$



#funct^n:

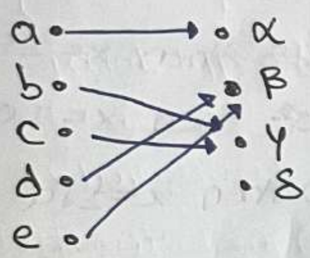
$\underbrace{m \times m \times m \cdots \times m}_{n \text{ times}}$

$\Rightarrow m^n$

$\boxed{|CoDomain|^{|Domain|}}$

Representation of funct^n

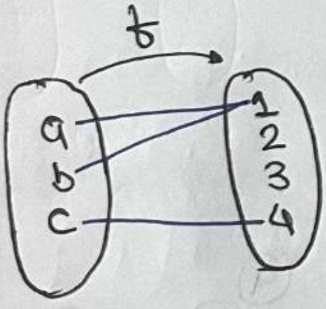
every funct^n is Relation ; but some relation are not funct^n



$\Rightarrow$ set Representation

$\{(a,\alpha), (b,\beta), (c,\gamma), (d,\beta), (e,\beta)\}$

Image for a Subset of Domain



$f(a) = 1$

$f(\{a,b\}) = \{1\}$

$f(\{a,c\}) = \{1,4\}$

a (or) c
Related

$f(Domain) = Range$
$\{1,4\}$

$f: D \longrightarrow C$

Let: $S \subseteq D$

$f(S) = \{y \ / \ y \in C ; \exists x \in S \ f(x) = y\}$

ex:

$f(\{a,b,c\}) = \{y \ / \ f(a) = y \text{ OR } f(b) = y \text{ OR } f(c) = y\}$

ex: $f: Z \to Z$; $f(x) = x^2+2$

$S \subseteq Z$, $S = \{-10, 0, 5\}$

$f(S) = \{102, 2, 27\}$
image of S.

$\boxed{\text{Real valued funct}^n}$

Co-Domain: R

- Integer-valued : co-domain : Z
- natural-valued : co-domain : N
- Boolean-valued : co-domain : {0,1}

$f: \{1,2,3\} \to R$ $f(x) = x^2$

$g: \{1,2,3\} \to R$ $g(x) = x+1$

• for these there domain must be same & they must be Real value (or) Integer value funct^n

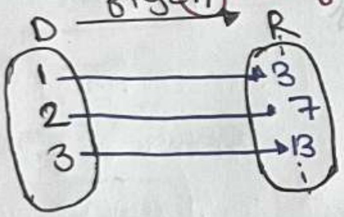
$f+g : \{1,2,3\} \to R$

$(f+g)_x = f(x) + g(x)$

$f \cdot g : \{1,2,3\} \to R$

$(f \cdot g)(x) = f(x) * g(x)$

i.e. $f+g = h$ new funct^n



$h(1) = f(1) + g(1)$
$= 1+2 = 3$

$h(2) = f(2) + g(2)$
$= 4+3 = 7.$

Range $(f+g) = \{3, 7, 13\}$

Range $(f) = \{1, 4, 9\}$

Range $(g) = \{2, 3, 4\}$

① f_1 & f_2 be funct^n from R to R.

$f_1(x) = x^2$ & $f_2(x) = x - x^2$

$f_1 + f_2$ & $f_1 \times f_2$

$h_1 : (f_1+f_2)_x = f_1(x) + f_2(x) = x^2 + x - x^2$
$= x.$

$h_1 = f_1 + f_2$; $h_1(x) = x.$

$h_2 : f_1 \times f_2$

$(f_1 \times f_2)_x : f_1(x) \times f_2(x)$

$\Rightarrow (x^2)(x - x^2)$

$h_2(x) \Rightarrow x^3 - x^4$

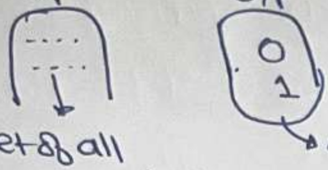
In Digital logic

Boolean Expression $\equiv$ Boolean functⁿ

Boolean functⁿ

is a funct. whose

(domain) I/P (codomain) O/P



Set of all ordered 'n' tuple of '0''s & 1's.

Set {0, 1}

Type of functⁿ

Special type of functⁿ

① one-one functⁿ (injection)

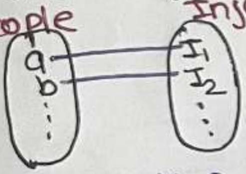
(Different element should have diff' image's)

② onto function (surjective)

③ bijection functⁿ

ex: one-one functⁿ

people Injection/Syringe.



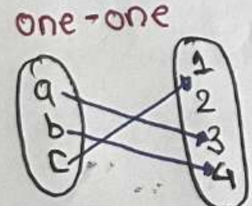
one injection should go to atmost one person only.

ex onto functⁿ

CoDomain = Range

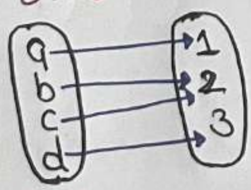
(Covering the complete co-domain.)

ex: one-one



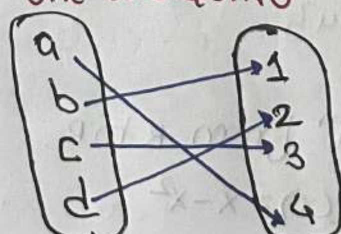
but not onto

ex. onto

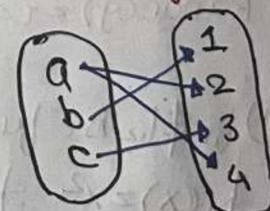


but not one-one

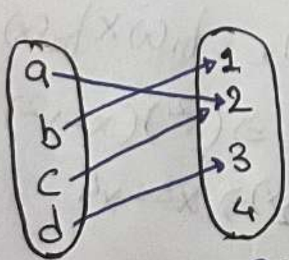
ex: one-one & onto



ex not a funct.



ex



neither one-one, nor onto.

① one-one function

$f: D \xrightarrow{1\text{-}1} C$

↳ if $a \neq b \in D$ then $f(a) \neq f(b)$

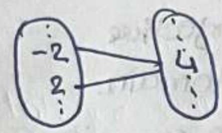
* ↳ if $f(a) = f(b)$ then $\boxed{a=b}$

$\boxed{|D| \leq |C|}$

ex: $f: Z \to Z$; $f(x) = x^2$

not one-one

$f(-2) = f(2) = 4$



ex: $f(x) = x^2$

① $f: N \to N$; one-one

② $f: N \to Z$; one-one.

ex: $f: R \to R$; $f(x) = 3x+7$

is f one-one?

assume $f(a) = f(b)$

$\Rightarrow 3a+7 = 3b+7$

$\boxed{a=b}$

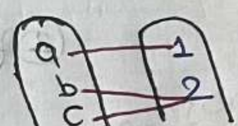
■ From codomain point of view.

① Injective:

Def. Every element of codomain has atmost one pre image

② Surjective:

Def. Every element of codomain has atleast one preImage



onto

Pre-Image(1) = {a}

Pre-Image(2) = {b, c}

② Surjective function $\boxed{|D| \geq |C|}$

$f: D \to C$

$\forall_{y \in C} \to \exists_{x \in D}$

s.t $\boxed{f(x) = y}$

ex: $f: N \to N$, $f(x) = x^2$ } not onto (no pre image of 3).

③ Surjective & Injective.

Def. Every element of "co-domain" has exactly one preimage

③ Bijections functⁿ $\boxed{|D| == |C|}$

means

Surjective + Injective

(one-one Correspondance).

$\boxed{x \in A ; \ |\{y \mid f(x) = y\}| = 1}$

Practice Question

① $f(n) = 2$ for $n \leq 2$

$f(n) = f(n-1) + f(n-2) + 1$

$f_1(2) = 2$ & $f_2(2) = 2$

$f_3 = f_2 + f_1 + 1 \Rightarrow 2+2+1$
$= 5$

$f_4 = 5 + 2 + 1 = 8$

$f_5 = 8 + 5 + 1 = 14$

$f_6 = 14 + 8 + 1$
$= 23$

$f : A \longrightarrow B$

① To prove f is Injective

$f(a) = f(b) \longrightarrow \boxed{a = b}$

② To prove f is not Injective

Some a, b; $a \neq b$ but $f(a) \neq f(b)$

③ To prove f is surjective

arbitrary $y \in$ codomain

then find $x \in$ Domain s.t

$f(x) = y$.

④ To prove f is not onto:

find some $y \in$ co-domain;

y doesn't have preimage.

ex: $f : Z \longrightarrow Z$

① $f(x) : x^2 - 1$

$f(-1) = f(1) = 0$ × not one-one

There is no $x \in Z$

s.t $f(x) = 5$ × not onto

Preimage of $8 = \{-3, 3\}$

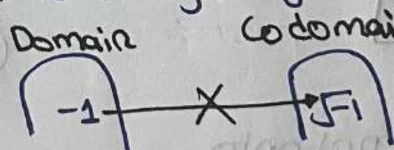
So, it's not bijection.

② $f(x) = x^2 + 1$

not bijection

③ $f(x) = \sqrt{x}$ not even a functⁿ

↳ no image of -1.



$f(-1) = \sqrt{-1} \notin$ codomain

④ $f : Z \longrightarrow Z$

① $f(x) : 5$ — function.

$f(0) = 5$; $f(-2) = 5$; $f(10) = 5$ . . . .

one-one ×

onto ×

↳ will be onto if

codomain is $\{5\}$

② $f: \mathbb{Z} \to \mathbb{Z}$

$f(x) = 2^x$ ... function. ✗

$f(-1) = 2^{-1} = \frac{1}{2} = 0.5 \notin$ co-domain

③ $f: \mathbb{R} \to \mathbb{R}$

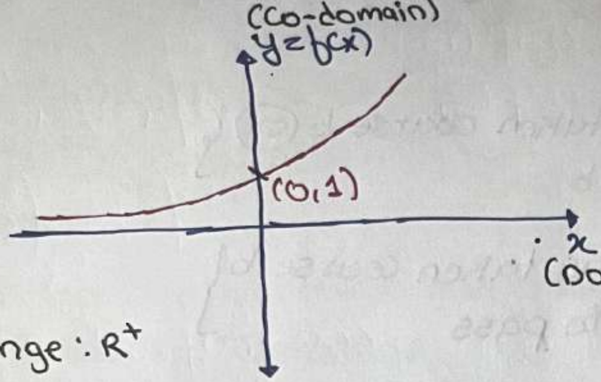
$f(x) = e^x$ .... function ✓

one-one ✓

onto ✗

no preimage of 0, (-ve) no.

a^x:
$(a > 1)$



Range: $\mathbb{R}^+$

$f: \mathbb{R} \to \mathbb{R}^+$

$f(x) = 2^x, e^x, 5^x$ .....

$a^x ;\ a > 1$

✓ one-one
✓ onto } bijection

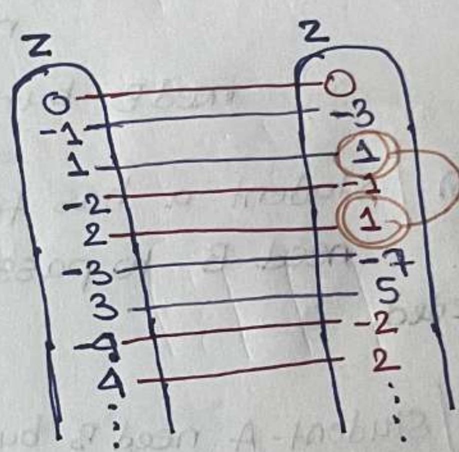
④ $f: \mathbb{Z} \to \mathbb{Z}$

$f(x) = \begin{cases} x/2 & ; x = \text{even} \\ 2x-1 & ; x = \text{odd} \end{cases}$

is onto

for $\forall y \in$ co-domain

atleast one preimage is $2y$.



$\boxed{\text{Combining Relation}}$

$R: A \to B$

$R \subseteq A \times B$

is a set
(so we can apply set operation).

∘ operation on set.

on Relation, set operation can be applied

$\{ \cup, ④, \times$
$\cap, -$

① $A = \{1,2,3\}$ & $B = \{1,2,3,4\}$

$R_1: \{(1,1),(2,2),(3,3)\}$

$R_2: \{(1,1)(1,2)(1,3),(1,4)\}$

$R_1 \cup R_1 = \{(1,1)(1,2)(1,3)(1,4)(2,2),(3,3)\}$

$R_1 \cap R_2 = \{(1,1)\}$

$R_1 - R_2 = \{(2,2)(3,3)\}$

$R_2 - R_1 = \{(1,2)(1,3)(1,4)\}$

(2) A: set of students   B = set of courses

$R_1 = \{(a,b) \mid$ student a has taken course b$\}$

$R_1 : A \longrightarrow B$

$R_2 = \{(x,y) \mid$ student x needs course y to pass$\}$

$R_2 : A \longrightarrow B$

$R_1 \cup R_2 : \{(a,b) \mid$ Student A has taken course b or need b$\}$

$R_1 \cap R_2 : \{(a,b) \mid$ Student A has taken course b and need b to pass$\}$

$R_1 \oplus R_2 : \{(a,b) \mid$ Student A has taken course B but not need B OR need B but not taken$\}$

$R_1 - R_2 : \{(a,b) \mid$ student a has taken course B But not need B to pass.$\}$

but not needed.

$R_2 - R_1 : \{(a,b) \mid$ Student A need B but not taken B$\}$

$R_1 \times R_2 : \{(\underbrace{(a,b),(x,y)}_{\text{Order pair.}}) \mid$ Stud. A takes B & Stud x need y$\}$

(3) $R_1 : \{(x,y) / x<y\}$ & $R_2 : \{(x,y) / x>y\}$

$R_1 \cup R_2 : \{(x,y) \mid \underbrace{x \neq y}_{x<y \text{ or } x>y}\}$.

$R_1 - R_2 = R_1$, $R_2 - R_1 = R_2$ B/c they are disjoint

$R_1 \cap R_2 = \{\}$

$R_1 \oplus R_2 = R_1 \cup R_2 = \{(x,y) / x \neq y\}$

$R_1 \times R_2 = \{(a,b),(x,y) \mid a<b, x>y\}$

$\boxed{\text{Composition of Relation}}$

Relation $\xrightarrow{\quad}$ Set
Relation $\not\leftarrow$ Set

Special type of set. (so we have special operations only for Relation → { Composition, Inverse

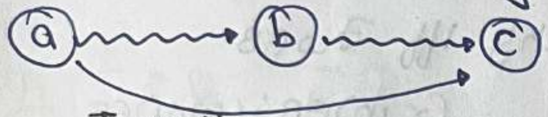
Composition $\xrightarrow{\text{also}}$ Relation operation
Create new Relation

i.e $R_1 : \{(1,1),(1,2)\}$ $R_2 : \{(2,3)\}$

$R_1 \cup R_2 : S : \{(1,1),(1,2),(2,3)\}$ new Relation

Symbol

- Composition : An operation that create's new Relation Transitively.



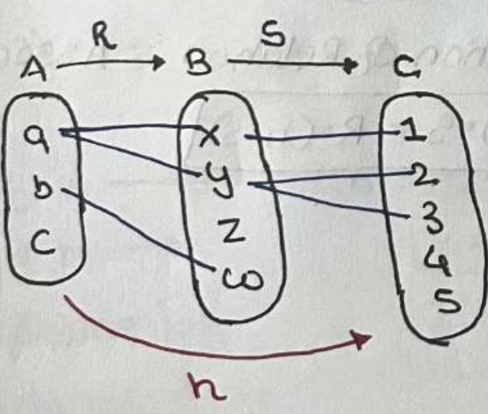
Transitive edge.

i.e $R_1 = \{(1,1),(1,2)\}$ $R_1 \circ R_2 = \{(1,4),(1,3)\ \cancel{(1,4)}$ duplicate $\}$

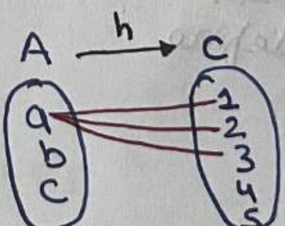
$R_2 = \{(2,3),(2,4),(1,4)\}$

1,1 − 1,4   1,2 − 2,3

ex:



$h : A \to C$
Image of $a = \{1,2,3\}$
$a \to 1,2,3$



$h = \{(a,1),(a,2),(a,3)\}$

Composition of R & S.

$h(a) = S(\underbrace{R(a)}_{\text{1st find these.}})$

$\boxed{h = S \circ R}$

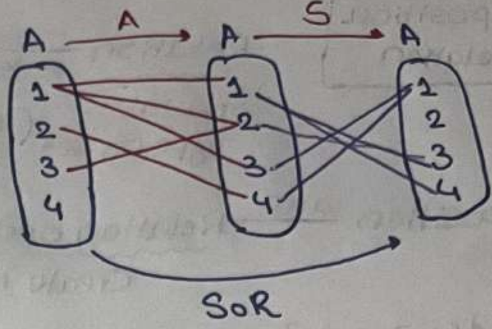
ex: $R: A \to A$; $S: A \to A$

$A = \{1,2,3,4\}$

$R = \{(1,2),(1,1),(1,3),(2,4),(3,2)\}$

$S = \{(1,4),(1,3),(2,3),(3,1),(4,1)\}$

find SoR.



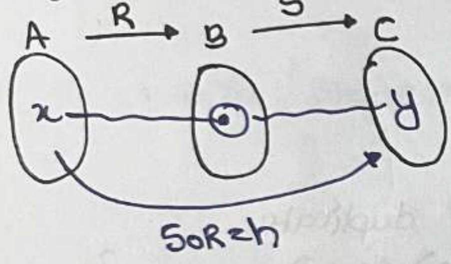
S(R)

$S \circ R: A \to A$

$\{(1,3),(1,4),(1,1),(2,1)\ (3,3)\}$

so, $|S \circ R| = 5$.

- To Define SoR

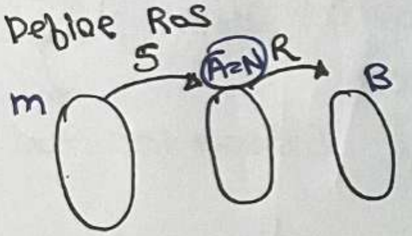


$S \circ R: x(S \circ R)y$; $x \in A$; $y \in C$

$\Rightarrow x h y$ iff $\exists\, \omega \in B$

$(x,\omega) \in R$; $(\omega,y) \in S$

- To Define RoS



$\left.\begin{array}{l} R: A \to B \\ S: M \to N \end{array}\right\}$

ex: $R_1: N \to R$

$R_2: R \to N$

then (1) $R_1 \circ R_2$ } both can be define

(2) $R_2 \circ R_1$

o- - - - - - - - note - - - - - - - - -o

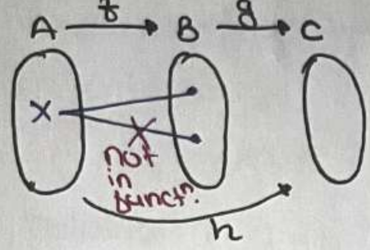
- Composition of Relation is Associative

$$\boxed{(R \circ h) \circ S = R \circ (h \circ S)}$$

Imp.

Composition of funct^n

funct^n f, g

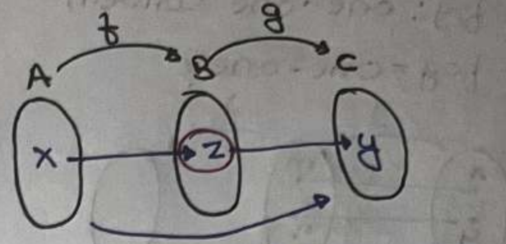


$f: A \to B$
$g: B \to C$

$h: A \to C$

$h(x) = y$ iff $\left.\begin{array}{l} f(x) = z \ \& \\ g(z) = y. \end{array}\right\}$

ex:



$f: A \to B$ & $g: B \to C$

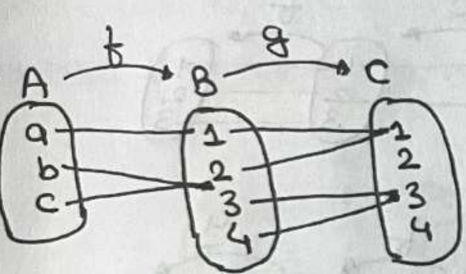
$h: A \to C$

$h(x) = g(f(x))$

$\boxed{h = g \circ f}$

$\boxed{f \circ g(x) = f(g(x))}$

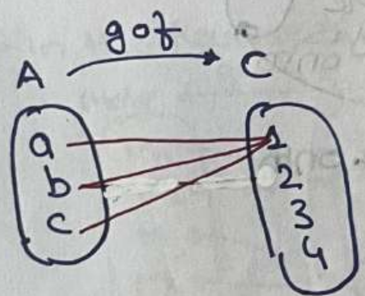
ex:



$g \circ f : A \to C$

$g \circ f(a) = g(f(a)) = g(1)$
$= 1$

$g \circ f(a) = 1$



ex: $f: A \to B$
$g: M \to N$

To define $f \circ g$

codomain of g = codomain of f.

$\boxed{A = N}$

To define $g \circ f$

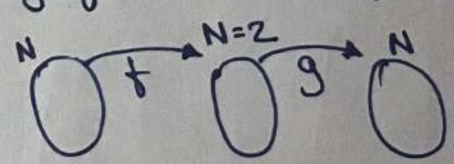
$\boxed{B = M}$

$g(f(x))$
$M \to \underline{N}(\underline{A} \to B)$

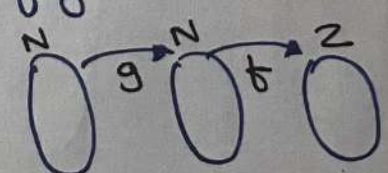
ex: $f: N \to Z$ & $g: N \to N$

$f(x) = x^2$ & $g(x) = x+1$

$g \circ f$ can't be defined.



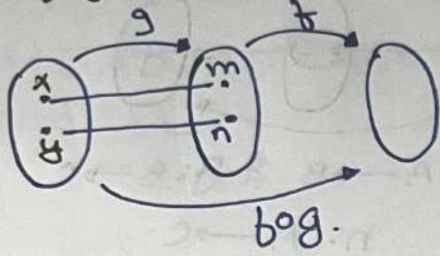
$\checkmark f \circ g$



ex: $f: A \longrightarrow A$; $g: A \longrightarrow A$

① f, g: one-one (Given)

$f \circ g$ = one-one?



$x \neq y \Rightarrow g(x) \neq g(y)$

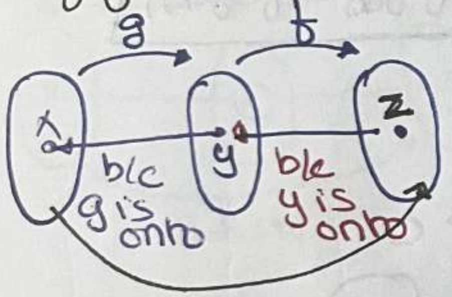
& $m \neq n$ so,

$f(m) \neq f(n)$

↳ is one-one

② f, g: onto, onto (Given)

$f \circ g$ = onto?

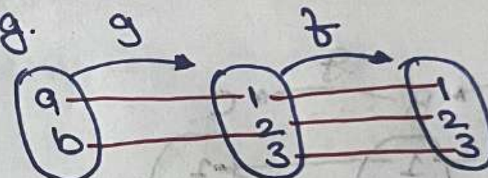


so $f \circ g$ = onto

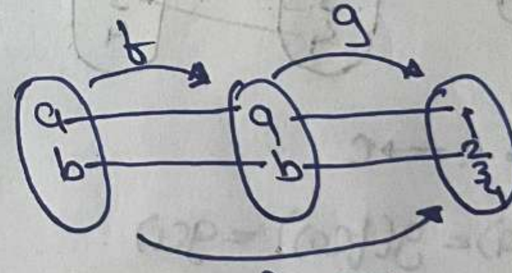
③ $f, g: A \longrightarrow A$

f: onto then $f \circ g$ = ~~onto~~ no

ex. fog.



gof



gof not onto

Inverse of a funct.

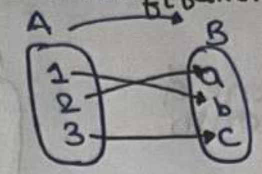
functⁿ g "Reve the mapping" given we will say;

g is Inverse of f

notation : $g = f^{-1}$

f^{-1} = Inverse of f.

Domain



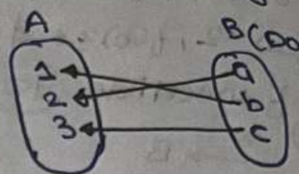
$f: A \to B$

$1 \xrightarrow{f} b$

$2 \xrightarrow{f} a$

$3 \xrightarrow{f} c$

another functⁿ g :



$g: B \to A$

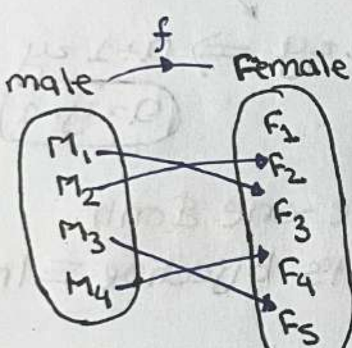
$a \xrightarrow{g} 2$

$b \xrightarrow{g} 1$

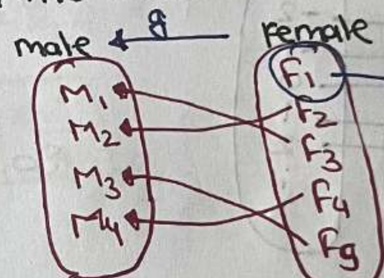
$c \xrightarrow{g} 3$.

Informal definition: functⁿ is invertible iff when we look at function f in the Reverse Direction, that also is a function.

In Reverse direction we get functⁿ iff f is bijective.



• In the Reverse direction is it a functⁿ? no



b/c from F_1 point of view F_1 has no preimage.

or b/c F_1 has no image (in g).

note

• Function f is invertible iff f is bijective.

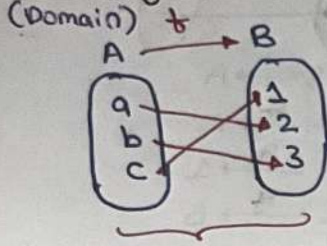
Invertible functⁿ ⇄ Bijective functⁿ (one-one & onto)

$f: A \to B$ invertible then in the Reverse direction the functⁿ we get is Inverse of f.

ex: $f: \{a,b,c\}$ to $\{1,2,3\}$

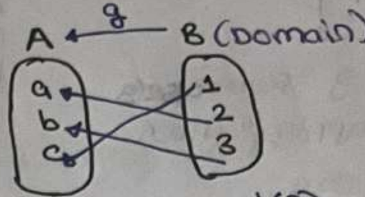
$f(a)=2,\ f(b)=3$ & $f(c)=1$

is f invertible. yes one-one & onto.



$\underbrace{\hspace{6em}}$
f: Invertible

inverse of $f = f^{-1} = g$



$g(3) = b = f^{-1}(3)$

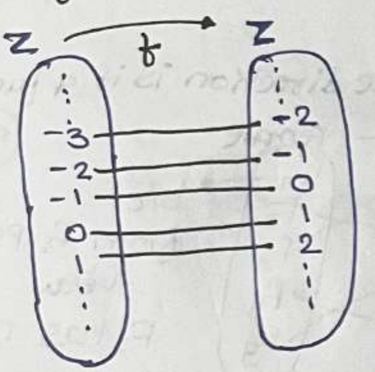
$g(1) = c = f^{-1}(1)$

$g(2) = a = f^{-1}(2)$

ex: $f: Z \to Z$

Such that $f(x) = x+1$

is f invertible.



Checking onto? yes.

Take $y \in$ Co-domain & find who map's to y Assume "a" maps to y

$f(a) = y \Rightarrow a+1 = y$

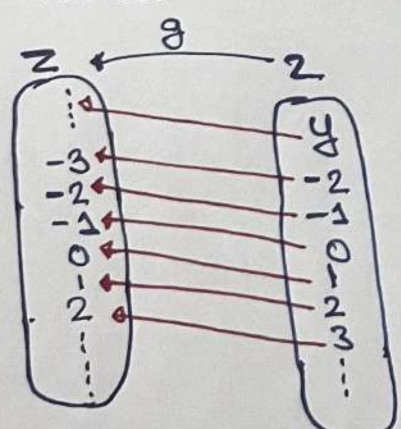
$\boxed{a = y-1}$

So, it's one-one & onto.

So, it's bijective $\equiv$ Invertible

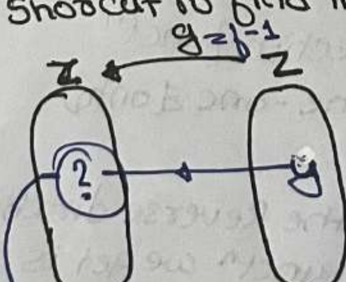
what is the Inverse?

apply the Definition of Inverse.



$\boxed{f^{-1} = g}$ → Look at f in Reverse Direction.

Shortcut to find inverse



we have to find.
assume it "a"

$g(y) = ?$

$f(a) = y$

$a+1 = y$

$\boxed{a = y-1}$

& $a = g(y)$

$g(y) = y-1$

$g(x) = x-1$

$\boxed{f^{-1}(x) = x-1}$

ex: $f: \mathbb{R} \to \mathbb{R}$ & $f(x) = x^2$ } is not invertible

is not one-one

$f(2) = f(-2) = 4.$

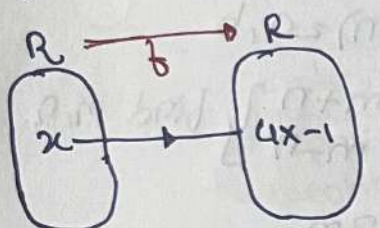
is not onto b/c -ve no. have no preimage

preimage of 3: $\sqrt{3}, -\sqrt{3}$

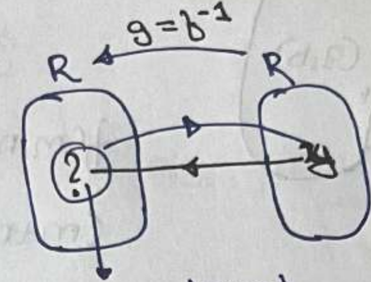
ex: $f: \mathbb{R} \to \mathbb{R}$

& $f(x) : 4x-1$

find it's inverse.



$f(x) = 4x-1$



To be found assume "b"

So, $g(y) = b$

$f(b) = y.$

$4b - 1 = y$

$\Leftarrow b = \frac{y+1}{4}$

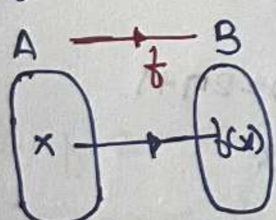
$b = g(y) = \frac{y+1}{4}$

$$\boxed{g(x) = \frac{x+1}{4}}$$

Summary:

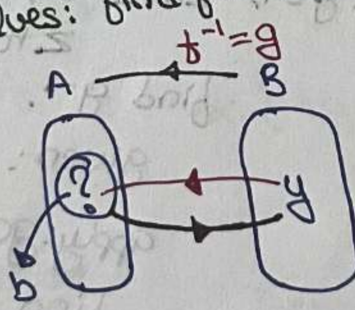
If "f" is invertible.

$f: A \to B$



$f(x)$: given.

Ques: find f^{-1}



$f(b) = y$

$g: B \to A$

To find "b"

$\boxed{b = g(y)}$

① if x, y are set & $F: x \to y$ is a one to one corresponding $F^{-1}: y \to x$ is also one to one correspondence. True.

② $(f^{-1})^{-1} = f$

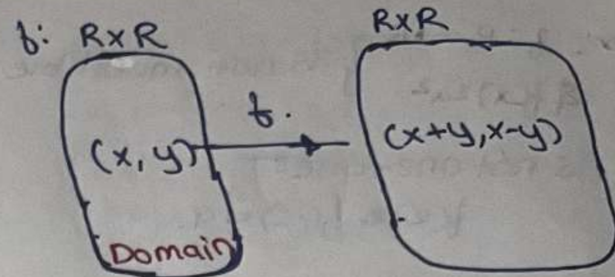
gate 1996

ex: R : Real no.

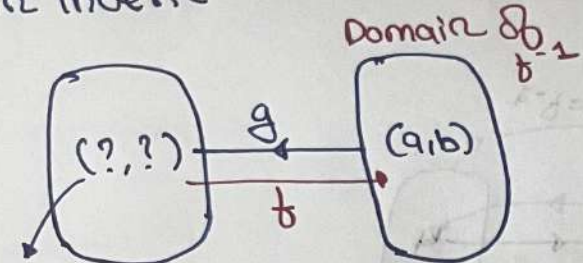
$f : R \times R \to R \times R$ be bijective
invertible

$f(x, y) = (x+y, x-y)$

then Inverse?



in inverse



To find
Assume (m, n)

note: order pair equality

$(x, y) = (P, Q)$
iff
$x = P$ & $y = Q$

find m, n?

$g(a, b) = m, n$

$f(m, n) = (m+n, m-n)$

$(m+n, m-n) = a, b$

so, $\left.\begin{array}{l} a = m+n \\ b = m-n \end{array}\right\}$ find m, n.

$a + b = 2m$

$m = \frac{a+b}{2}$ & $n = \frac{a-b}{2}$

$$\boxed{g(a, b) = \left(\frac{a+b}{2}, \frac{a-b}{2}\right)}$$

ex: $f(n) = 2n$ is a bijection from Z to the even Integers. find f^{-1}.

Algo: given: $f(x) = 2x$

apply: $f(y) = x$

$2y = x \Rightarrow y = x/2$

& $g(n) = 2n+1$ is a bijection from Z to odd Integer. find g^{-1}.

given: $g(x) = 2n+1$

apply: $g(y) = x$

find y:

$2y + 1 = x$

$$\boxed{y = \frac{x-1}{2}}$$

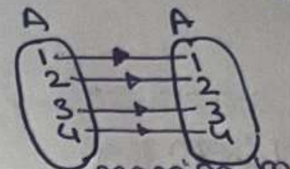
Question based on Inverse & Composition combo

• Identity function

$f : A \to A$

$f(x) = x$ Identity funct^n

$I_A : A \to A$

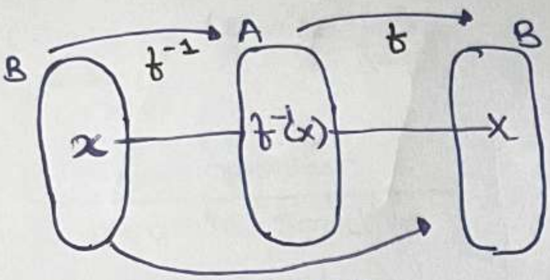


mapping to itself.

① $f : A \to B$ be an Invertible.

(i) $\underbrace{f o f^{-1}}$ & $f^{-1} o f$

$f(f^{-1}(x))$



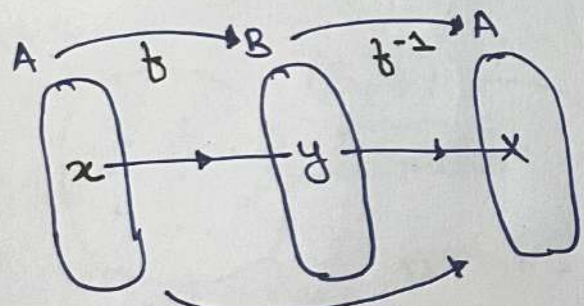
$f o f^{-1}(x)$

composition is identity funct^n from $B \to B$

So, $f o f^{-1}$ is identity funct^n $B \to B$

$$\boxed{f o f^{-1} = I_B}$$

(ii) $f^{-1} o f = f^{-1}(f(x))$



$f^{-1} o f$

So, $f^{-1} o f$ is Identity funct^n from A to A

$$\boxed{f^{-1} o f = I_A}$$

Q: $f : Z \to Z$;

$f(x) = x + 1$ is Invertible

$\underbrace{f o f^{-1}(x)} = ? = I_Z$

Identity funct^n

$$\boxed{f o f^{-1}(x) = x}$$

ex: f & g are Invertible function such that

$f : A \to B$; $g : B \to C$

is $g o f$ invertible.

Remember

if f, g both are,

① 1-1, then $g o f$ 1-1

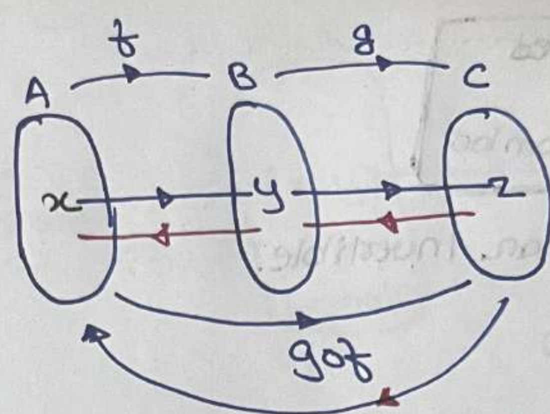
② onto, then $g o f$ onto

③ bijective, then $g o f$ bijective.

ex: f & g are invertible

$f: A \to B$; $g: B \to C$

what is $(g \circ f)^{-1}$?



$(g \circ f)^{-1} = f^{-1}(g^{-1}(x))$

$f^{-1} \circ g^{-1}$

--- note ---------

invertible funct^n f, g

$f: A \to A$; $g: A \to A$

$(f \circ g)^{-1} = g^{-1} \circ f^{-1}$

$(g \circ f)^{-1} = f^{-1} \circ g^{-1}$

$\boxed{\text{Inverse} \equiv \text{undo}}$