Lab Session 2

MA581: Numerical Computations Lab R. Alam August 12, 2025

Note: To be consistent with MATLAB, consider polynomial interpolation in which indexing of data (nodes and values) starts with 1 rather than 0. Thus we denote the data points as $(x_1, f_1), \ldots, (x_n, f_n)$. The interpolating polynomial $p_n(x)$ will be of degree $n-1$.

1. Write a MATLAB function `v = polyinterp(x, f, u)` that constructs the Lagrange interpolant $p_n(x) = \sum_{j=1}^{n} \ell_j(x) f_j$ of the data $(x_1, f_1), \ldots, (x_n, f_n)$ and evaluates $p_n(x)$ at the points given in the vector u, that is, $v_i = p_n(u_i)$. Store x, f, u and v as column vectors. The vectors u and v are required for plotting $p_n(x)$. Also write a MATLAB function `v = barycent(x, f, u)` that generates $p_n(x)$ in barycentric form

$$p_n(x) = \frac{\sum_{j=1}^{n} \frac{w_j f_j}{x - x_j}}{\sum_{j=1}^{n} \frac{w_j}{x - x_j}}, \quad \text{where} \quad w_j = 1/\prod_{i \neq j}(x_j - x_i),$$

and computes $v_i = p_n(u_i)$. Next, write a MATLAB function `v = newton(x, f, u)` that generates Newton interpolating polynomial $p_n(x)$ and computes $v_i = p_n(u_i)$. Use divided differences for the Newton interpolating polynomial.

(a) Now consider the functions $f(x) := e^{\sin(5.5x)}$ for $x \in [-2, 2]$. Next, interpolate f at n equally spaced points in the interval $x \in [-2, 2]$ using `polyinterp` function for $n = 15, 30, 60$. Plot $f(x)$ and $p_n(x)$ for $n = 15, 30, 60$. Also plot the error $E_n(x) := f(x) - p_n(x)$ as a function of x for $n = 15, 30, 60$. The error is usually tend to be dominant towards the end nodes.

Repeat the above experiment using the functions `barycent` and `newton`. For small values of n, the interpolating polynomials returned by `polyinterp, barycent, newton` are likely to be almost the same. However, for large n, `barycent` is expected to outperform the other algorithms.

2. Consider the following population data for the United States:

Year	Population
1900	76, 212, 168
1910	92, 228, 496
1920	106, 021, 537
1930	123, 202, 624
1940	132, 164, 569
1950	151, 325, 798
1960	179, 323, 175
1970	203, 302, 031
1980	226, 542, 199

There is a unique polynomial of degree eight that interpolates these nine data points, but of course that polynomial can be represented in many different ways. Consider the following possible basis functions $\phi_j(x), j = 1:9$:

1. $\phi_j(x) := x^{j-1}$
2. $\phi_j(x) := (x - 1900)^{j-1}$

3. $\phi_j(x) := (x - 1940)^{j-1}$
4. $\phi_j(x) := ((x - 1940)/40)^{j-1}$

(a) For each of these four sets of basis functions, generate the corresponding Vandermonde matrix A and compute its condition number ($\text{cond}(A) := \|A\| \, \|A^{-1}\|$) using the MATLAB command `cond(A)`. How do the condition numbers compare?

(b) Using the best-conditioned basis (basis for which cond(A) is the smallest) found in part (a), compute the polynomial $p(x)$ that interpolates the population data. For this purpose, solve the system `Aa = f` using MATLAB "backslash" command. The vector `a` contains the coefficients of the interpolating polynomial $p(x)$. Use MATLAB function `polyval` to compute $v_i = p(u_i)$ from the coefficient vector `a`.

Plot the resulting polynomial, using MATLAB command `polyval` (type `help polyval` for help) to evaluate the polynomial at one-year intervals to obtain a smooth curve. Also plot the original data points on the same graph.

(c) Use the polynomial to extrapolate the population to 1990. How close is the computed value to the true value of 248,709,873 according to the 1990 census?

(d) Determine the Lagrange interpolating polynomial to the same nine data points and evaluate it at the same yearly intervals as in part (b). Use the polynomial to extrapolate the population to 1990. How close is the computed value to the true value of 248,709,873 according to the 1990 census? Compare the total execution time with that in (c).

(e) Determine the Newton form of the polynomial interpolating the same nine data points. Now determine the Newton polynomial of one degree higher that also interpolates the additional data point for 1990 given in part (c), without starting over from scratch (i.e., use the Newton polynomial of degree eight already computed to determine the new Newton polynomial). Plot both the resulting polynomials (of degree eight and nine) over the interval from 1900 to 1990.

(f) Round the population data for each year to the nearest million and compute the corresponding interpolating polynomial of degree eight using the same basis as in part (b). Compare the resulting coefficients with those determined in part (b). Explain your results.

3. The gamma function is defined by $\Gamma(x) := \int_0^\infty t^{x-1}e^{-x}dt$ for $x > 0$. For an integer argument n, the gamma function has the value $\Gamma(n) = (n-1)!$, so interpolating the data points

x	1	2	3	4	5
f	1	1	2	6	24

should yield an approximation to the gamma function over the given range.

Compute the polynomial of degree four that interpolates these five data points. Plot the resulting polynomial as well as the corresponding values given by the MATLAB function `gamma` over the domain $[1, \ 5]$.

*** End ***