MA 589 | Quiz I
Statistical Foundation for Data Science | 12:10 – 12:50 IST
IIT GUWAHATI | August 11, 2025

Model Answers of Quiz I (Total points:13)

1. (2 points) Let E and F be two events with $0 < P(E) < 1$, $0 < P(F) < 1$, and $P(E|F) > P(E)$. Then, which of the following statements is/are TRUE?

   1. $P(F|E) > P(F)$
   2. $P(F|E^c) < P(F)$
   3. $P(E|F^c) > P(E)$
   4. E and F are independent

   Solution: Option 1 is true as

   $$P(E|F) > P(E) \implies P(E \cap F) > P(E)P(F) \implies P(F|E) > P(F).$$

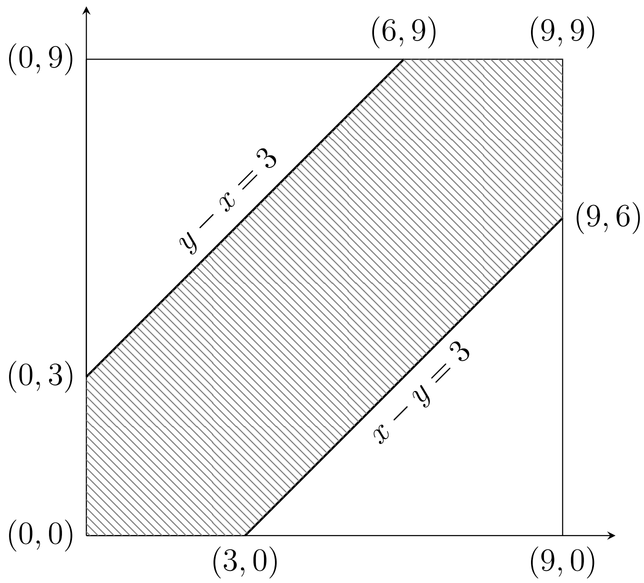
   Option 2 is also true as

   $$P(F|E^c) = \frac{P(E^c \cap F)}{P(E^c)} = \frac{P(F) - P(E \cap F)}{P(E^c)} < \frac{P(F) - P(E)P(F)}{P(E^c)} = P(F).$$

   Using Option 2, Option 3 is false. Option 4 is also false.

2. (2 points) Two points are chosen at random on a line segment of length 9 cm. Then find the probability that the distance between these two points is less than 3 cm.

   Solution: Let the randomly selected points are X and Y. Then we are interested in the probability of the even $|X - Y| < 3$.



   The required probability is

   $$\frac{\text{Area of shaded region}}{\text{Area of the square with side 9 cm}} = \frac{81-36}{81} = \frac{5}{9}.$$

3. (2 points) Consider a sample space $\mathcal{S} = \{1, 2, 3, 4\}$ and a σ-algebra

$$\mathcal{G} = \{\emptyset, \mathcal{S}, \{1\}, \{4\}, \{2, 3\}, \{1, 4\}, \{1, 2, 3\}, \{2, 3, 4\}\}$$

on $\mathcal{S}$. Let P be a probability defined on the σ-algebra $\mathcal{G}$ such that $P(\{1\}) = \frac{1}{4}$. Let $X : \mathcal{S} \to \mathbb{R}$ be the random variable defined as $X(1) = 1$, $X(2) = X(3) = 2$, $X(4) = 3$. If the value of the cumulative distribution function of X at 2 is $\frac{3}{4}$, then find $P(\{1, 4\})$.

Solution: Note that $P(\{4\}) = P(X = 3) = P(X > 2) = 1 - P(X \leq 2) = 1 - F_X(2) = \frac{1}{4}$. As $\{1,\, 4\} = \{1\} \cup \{4\}$ with $\{1\}$ and $\{4\}$ are disjoint,

$$P\left(\{1,\, 4\}\right) = P\left(\{1\}\right) + P\left(\{4\}\right) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}.$$

4. (2 points) Let X be a random variable with CDF

$$F_X(x) = \begin{cases} 0 & \text{if } x < 0 \\ 1 - e^{-x} & \text{if } 0 \leq x < \ln 2 \\ \frac{7}{4} - 2e^{-x} & \text{if } \ln 2 \leq x < \ln\left(\frac{8}{3}\right) \\ 1 & \text{if } x \geq \ln\left(\frac{8}{3}\right). \end{cases}$$

Then which of the following statements is/are TRUE?

1. $P\left(X = \ln 2\right) = \frac{1}{2}$
2. $P\left(0.5 \leq X \leq \ln 2\right) = e^{-0.5} - 0.25$
3. $P\left(0.6 \leq X \leq 1\right) = e^{-0.6}$
4. $P\left(\ln 2 \leq X \leq 0.8\right) = 1.25 - 2e^{-0.8}$

Solution: Option 1 is incorrect as

$$P\left(X = \ln 2\right) = F_X(\ln 2) - F_X(\ln 2-) = \frac{3}{4} - \frac{1}{2} = \frac{1}{4}.$$

Option 2 is correct as

$$P\left(0.5 \leq X \leq \ln 2\right) = F_X(\ln 2) - F_X(0.5-) = e^{-0.5} - 0.25.$$

Option 3 is correct as

$$P\left(0.6 \leq X \leq 1\right) = F_X(1) - F_X(0.6-) = e^{-0.6}.$$

Option 4 is correct as

$$P\left(\ln 2 \leq X \leq 0.8\right) = F_X(0.8) - F_X(\ln 2-) = 1.25 - 2e^{-0.8}.$$

5. (2 points) A box contains a certain number of balls out of which 80% are white, 15% are blue and 5% are red. All the balls of the same color are indistinguishable. Among all the white balls, α% are marked defective, among all the blue balls, 6% are marked defective and among all the red balls, 9% are marked defective. A ball is chosen at random from the box. If the conditional probability that the chosen ball is white, given that it is defective, is 0.4, find the value of α.

Solution: Let us define

$$\begin{aligned} W &= \text{ The ball chosen is white,} \\ B &= \text{ The ball chosen is blue,} \\ R &= \text{ The ball chosen is red,} \end{aligned}$$

and

$$D = \text{ The chosen ball is defective.}$$

It is given that $P\left(W|D\right) = 0.4$. Thus, using Bayes theorem, we have

$$\begin{aligned} & \frac{P(D|W)P(W)}{P(D|W)P(W) + P(D|B)P(B) + P(D|R)P(R)} = 0.4 \\ \implies & \frac{0.008\alpha}{0.008\alpha + 0.0090 + 0.0045} = 0.4 \\ \implies & \alpha = 1.125. \end{aligned}$$

6. (1 point) An experiment yields 3 mutually exclusive and exhaustive events A, B and C. If $P(A) = 2P(B) = 3P(C)$, then find $P(A)$.

Solution: As the events are exhaustive, $P(A \cup B \cup C) = 1$. As the events are mutually exclusive, $P\left(A \cup B \cup C\right) = P\left(A\right) + P(B) + P(C)$. As $P(A) = 2P(B) = 3P(C)$,

$$P\left(A\right) + \frac{1}{2}P\left(A\right) + \frac{1}{3}P\left(A\right) = 1 \implies P(A) = \frac{6}{11}.$$

7. (2 points) If two fair die are tossed independently, then find the probability that the sum of the numbers on the upper face of the dice is strictly greater than 7.

Solution:

Sum	Combination of upper faces	Count
8	$(2,6),\ (3,5),\ (4,4),\ (5,3),\ (6,2)$	5
9	$(3,6),\ (4,5),\ (5,4),\ (6,3)$	4
10	$(4,6),\ (5,5),\ (6,4)$	3
11	$(5,6),\ (6,5)$	2
12	$(6,6)$	1
	Total	15

Hence, the required probability is $\dfrac{15}{36} = \dfrac{5}{12}$.