Solutions to Selected Problems
Problem Set 01

25 Let $C_i, i = 1, 2, 3$, denote the events that the car is behind door i. Let $X_i, i = 1, 2, 3$, denote the event that you choose door i in the beginning. Let $D_i, i = 1, 2, 3$, denote the event that the host opens door i. Then

$$\begin{array}{ll} P(D_3 \mid C_1, X_1) = \frac{1}{2}, & P(D_2 \mid C_1, X_1) = \frac{1}{2}, \\ P(D_3 \mid C_2, X_1) = 1, & P(D_2 \mid C_2, X_1) = 0, \\ P(D_3 \mid C_3, X_1) = 0, & P(D_3 \mid C_3, X_1) = 1, \\ P(C_1) = \frac{1}{3}, & P(D_3 \mid X_1) = \frac{1}{2}, P(D_2 \mid X_1) = \frac{1}{2}. \end{array}$$

Now

$$P(C_1 \mid D_3, X_1) = \frac{P(C_1 \cap D_3 \cap X_1)}{P(D_3 \cap X_1)} = \frac{P(D_3 \mid C_1 \cap X_1) P(C_1 \mid X_1) P(X_1)}{P(D_3 \mid X_1) P(X_1)} = \frac{\frac{1}{2} \cdot \frac{1}{3}}{\frac{1}{2}} = \frac{1}{3}.$$

$$P(C_2 \mid D_3, X_1) = \frac{P(C_2 \cap D_3 \cap X_1)}{P(D_3 \cap X_1)} = \frac{P(D_3 \mid C_2 \cap X_1) P(C_2 \mid X_1) P(X_1)}{P(D_3 \mid X_1) P(X_1)} = \frac{\frac{1}{3}}{\frac{1}{2}} = \frac{2}{3}.$$

Similarly, $P(C_1 \mid D_2 \cap X_1) = \frac{1}{3}$, $P(C_3 \mid D_2 \cap X_1) = \frac{2}{3}$. Thus winning probability increases with switching.

30 Let XY denote the event that the link between nodes X and Y is up. Then, the event that there is path between nodes A and B is given by

$$[AC \cap \{(CE \cap EB) \cup (CF \cap FB)\}] \cup [AD \cap DB].$$

Thus, the required probability is

$$\begin{aligned} & P([AC \cap \{(CE \cap EB) \cup (CF \cap FB)\}] \cup [AD \cap DB]) \\ & = P([AC \cap CE \cap EB] \cup [AC \cap CF \cap FB] \cup [AD \cap DB]) \\ & = P(AC \cap CE \cap EB) + P(AC \cap CF \cap FB) + P(AD \cap DB) \\ & \quad - P(AC \cap CE \cap EB \cap CF \cap FB) - P(AC \cap CE \cap EB \cap AD \cap DB) \\ & \quad - P(AC \cap CF \cap FB \cap AD \cap DB) + P(AC \cap CE \cap EB \cap CF \cap FB \cap AD \cap DB) \\ & = P(AC) P(CE) P(EB) + P(AC) P(CF) P(FB) + P(AD) P(DB) \\ & \quad - P(AC) P(CE) P(EB) P(CF) P(FB) - P(AC) P(CE) P(EB) P(AD) P(DB) \\ & \quad - P(AC) P(CF) P(FB) P(AD) P(DB) \\ & \quad + P(AC) P(CE) P(EB) P(CF) P(FB) P(AD) P(DB) \\ & \approx 0.957. \end{aligned}$$

The second equality is due to inclusion-exclusion principle. The third equality is due to assumption that link failures are independent.