# Homework-3

**MA-579H :** Scientific Computing          R. Alam          **Date:** August 20, 2025

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## Numerical Integration

1. True or false: Since the midpoint quadrature rule is based on interpolation by a constant, whereas the trapezoid rule is based on linear interpolation, the trapezoid rule is generally more accurate than the midpoint rule.

2. True or false: The degree of exactness of an $n$-point Newton-Cotes quadrature rule is always $n-1$.

3. Explain how the midpoint rule, which is based on interpolation by a polynomial of degree zero, can nevertheless integrate polynomials of degree one exactly.

4. Newton-Cotes and Gaussian quadrature rules are both based on polynomial interpolation.

   What specific property characterizes a Newton-Cotes quadrature rule for a given number of nodes?

   What specific property characterizes a Gaussian quadrature rule for a given number of nodes?

5. Find the number of sub-intervals $n$ necessary for the composite Simpsons Rule to approximate $\int_0^\pi \sin^2 x \, dx$ correct to six decimal places.

6. Find a quadrature rule of the form $\int_0^{2\pi} f(x)dx \approx w_0 f(0) + w_1 f(\pi)$ that is exact for any function of the form $g(x) = a + b\cos x$.

7. Find the quadrature rule of the form $\int_0^2 x f(x)dx \approx w_0 f(0) + w_1 f(1) + w_2 f(2)$ that is exact for all polynomials of degree as high as possible. What is the degree of exactness?

8. Is there a quadrature rule of the form $\int_0^1 f(x)dx \approx \alpha[f(x_0) + f(x_1)]$ that is exact for all quadratic polynomials?

9. Determine $\alpha$ for which the quadrature rule $\int_{-1}^1 f(x)dx \approx f(\alpha) + f(-\alpha)$ is exact for all quadratic polynomials.

10. Determine the quadratic Newton interpolating polynomial $p(x)$ that interpolates $f(x)$ at $x = 0$ and $x = 1$ and $f'(x)$ at $x = 0$. Based on $p(x)$ determine a quadrature rule of the form $\int_0^1 f(x)dx \approx w_0 f(0) + w_1 f(1) + w_3 f'(0)$.

11. Use the method of undetermined coefficients to construct a quadrature rule of the form $\int_0^1 f(x)dx \approx af(0) + bf(1) + cf''(d)$ having the maximum degree of exactness.

12. Use the method of undetermined coefficients to construct a quadrature rule of the form $\int_0^1 f(x)dx \approx -af'(0) + bf(1/2) + cf'(1)$ having the maximum degree of exactness.

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