Lab Session 3

MA581: Numerical Computations Lab   R. Alam   August 19, 2025

Note: To be consistent with MATLAB, consider polynomial interpolation in which indexing of data (nodes and values) starts with 1 rather than 0. Thus we denote the data points as $(x_1, f_1), \ldots, (x_n, f_n)$. The interpolating polynomial $p_n(x)$ will be of degree $n-1$.

MATLAB has multiple functions that can be used for interpolation.

- `interp1` - implements 1-dimensional interpolation. Type `doc interp1` for more information.
- `polyinterp` - implements Lagrange interpolation. Type `doc polyinterp` for more information.
- `piecelin` - piecewise linear interpolant. Type `doc piecelin` for more information.
- `spline` - cubic spline interpolation. Type `doc spline` for more information.
- `pchip` - piecewise cubic Hermite interpolant. Type `doc pchip` for more information.

1. Consider the data points `x = 0:3` and y = [-5, -6, -1, 16]. To illustrate `polyinterp`, consider a vector of densely spaced evaluation point `u = -.25:.01:3.25`. Then
   ```
   v = polyinterp(x, y, u)
   plot(x, y, 'o', u, v, '-')
   ```
   creates the plot of the interpolant. You can display the polynomial in symbolic form with the following commands:
   ```
   symx = sym(x)
   P = polyinterp(x,y,symx)
   pretty(P)
   ```
   To simplify the expression, type `simplify(P)`.

   Here is another data set.
   ```
   x = 1:6;
   y = [16 18 21 17 15 12];
   disp([x; y])
   u = .75:.05:6.25;
   v = polyinterp(x,y,u);
   plot(x,y,'o',u,v,'-')
   ```
   Now replace `polyinterp` with `spline` and `pchip` and repeat the experiment.

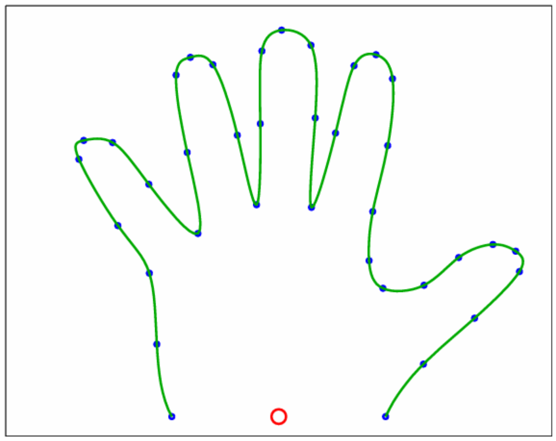
2. Interpolate the following data using `piecelin, polyinterp, spline` and `pchip` and plot the results for $-1 \le x \le 1$.
   `x = [-1.00, -.96, -0.65, .10, .40, 1.00]` and
   `y = [-1.00, -0.1512, 0.3860, 0.4802, 0.8838,1.00]`

   What are the values of each of the four interpolants at `x = 3`? Which of these values do you prefer? Why? The data was actually generated from a low-degree polynomial with integer coefficients. What is that polynomial?

3. Make a plot of your hand. Start with

```
figure( position ,get(0,screensize))
axes(position ,[0 0 1 1])
[x,y] = ginput;
```

Place your hand on the computer screen. Use the mouse to select a few dozen points outlining your hand. Terminate the `ginput` with a carriage return.



Now think of x and y as two functions of an independent variable that goes from one to the number of points you collected. You can interpolate both functions on a finer grid and plot the result with

```
n = length(x);
s = (1:n) ;
t = (1:.05:n);
u = spline(s,x,t);
v = spline(s,y,t);
clf reset
plot(x,y, '.', u,v, '-' )
```

Do the same thing with `pchip`. Which do you prefer?

4. Consider the Runge function $g(x) := 1/(1+25x^2)$ for $x \in [-1, 1]$. Interpolate g at n equally spaced points in the interval $x \in [-1, 1]$ using `polyinterp` function. Plot $\|g - p_n\|_\infty$ vs. n on "`semilogy`" axes for $n = 5, 10, 15, 20$. You should estimate $\|g - p_n\|_\infty$ by taking maximum of $|g(x) - p(x)|$ at 1000 equispaced points in $[-1, 1]$. Separately plot (in single plot) $f(x), p_{15}(x)$ and $p_{20}(x)$. Also plot the error $E_n(x) := |f(x) - p_n(x)|$ as a function of x for $n = 10, 15, 20$.

   Now repeat the experiment by replacing `polyinterp` with `spline`.

   Next, consider the Chebyshev nodes $x_j := \cos\left(\frac{(2j-1)\pi}{2n}\right)$ for $j = 1 : n$. Repeat the experiment above for the Chebyshev nodes. What is your observation?

   Finally, consider the function $h(x) := e^{\cos(6x)}$ for $x \in [0, 2\pi]$ and repeat the experiment above by considering equispaced nodes and Chebyshev nodes in $[0, 2\pi]$. The Chebyshev nodes in $[0, 2\pi]$ are given by $x_j := \pi/2\big(1 + \cos\left(\frac{(2j-1)\pi}{2n}\right)\big) = \pi \cos^2\left(\frac{(2j-1)\pi}{4n}\right)$ for $j = 1 : n$.

5. **Assignment:** Compile a list of 101 consecutive daily close prices of an exchange-traded stock from a financial data website.

(a) Plot the interpolating polynomial through every fifth point. That is, let $x = 0 : 5 : 100$ and y denote the stock prices on days $0, 5, 10, ..., 100$. Plot the degree 20 interpolating polynomial at points $x = 0 : 1 : 100$ and compare with the daily price data. What is the maximum interpolation error? Is the Runge phenomenon evident in your plot?

(b) Plot the natural cubic spline (use MATLAB command `spline`) with interpolating nodes $0 : 5 : 100$ instead of the interpolating polynomial, along with the daily data. Answer the same two questions. Compare the two approaches of representing the data. **10 marks**

Submission: Submit a livescript program that contains all comments, answers and codes necessary to produce the required output in it. Also the answers should be correctly numbered. The filename of the program should be **XrollnumberMA581.mlx,** where X is the first letter of your name. Submission through **MS Teams.**

Submission date: August 24, 2025 Time: 11:59 PM

You must not share/copy answers and MATLAB programs with others. Anyone found to have shared/copied answers and MATLAB programs will be awarded zero mark.

*** End ***