Mid Semester Examination

MA-581 : Numerical Computing Lab **Time : 3 hours** **40 marks**

October 14, 2025

Answer ALL question.

1. The MATLAB function below computes Lagrange interpolant.

```
function v = polyinterp(x,y,u)

%   v = polyinterp(x,y,u) computes v(j) = P(u(j)) where P is the
%   polynomial of degree d = length(x)-1 with P(x(i)) = y(i).

n = length(x);
v = zeros(size(u));
for k = 1:n
w = ones(size(u));
for j = [1:k-1 k+1:n]
w = (u-x(j))./(x(k)-x(j)).*w;
end
v = v + w*y(k);
end
```

Consider the Runge function $g(x) := 1/(1+25x^2)$ for $x \in [-1,1]$. Interpolate g at n equally spaced points in the interval $x \in [-1,1]$ using `polyinterp` function. Plot $\|g - p_n\|_\infty$ vs. n on "`semilogy`" axes for $n = 10, 15$. You should estimate $\|g - p_n\|_\infty$ by taking maximum of $|g(x) - p(x)|$ at 1000 equispaced points in $[-1,1]$. Separately plot (in single plot) $f(x), p_{10}(x)$ and $p_{15}(x)$.

Repeat the experiment for the Chebyshev nodes $x_j := \cos\left(\frac{(2j-1)\pi}{2n}\right)$ for $j = 1 : n$. What is your observation?

Finally, consider the function $h(x) := e^{\cos(6x)}$ for $x \in [0, 2\pi]$ and repeat the experiment above by considering equispaced nodes and Chebyshev nodes in $[0, 2\pi]$. The Chebyshev nodes in $[0, 2\pi]$ are given by $x_j := \pi/2\left(1 + \cos\left(\frac{(2j-1)\pi}{2n}\right)\right) = \pi\cos^2\left(\frac{(2j-1)\pi}{4n}\right)$ for $j = 1 : n$. What is your observation? **11 marks**

2. Write a MATLAB function `y = trapezoid(f,a,b,n)` to evaluate $\int_a^b f(x)dx$ by composite trapezoid rule with n equi-spaced nodes on $[a, b]$.

Since $\displaystyle\int_0^1 \frac{4}{1+x^2}dx = \pi$, one can compute an approximate value of π using numerical integration of the given function. Use composite trapezoid quadrature rule to compute approximate values of π for various step sizes h. Display the accuracy of the rule (based on the known value of π) by plotting the errors for $n = 10, \ldots, 200$ in semilogy scale. Is there any point beyond which decreasing h yields no further improvement? **5 marks**

3. Consider the problem of finding the smallest positive solution of the nonlinear equation $\cos(x)+1/(1+e^{-2x}) = 0$. Investigate numerically the following iterative schemes for solving this problem using the starting point $x_0 := 3$.

   (a) $x_{k+1} = \arccos(-1/(1+e^{-2x_k}))$.

   (b) Newton's method.

   For each scheme, determine whether it is locally convergent. Determine the expected convergence rate and order of convergence based on your results.

   7 marks

4. The function $\int_0^x \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt$ is related to the distribution function of the standard normal random variable - a very important distribution in probability and statistics. For a fixed $\alpha \in [0, 1]$, many applications require solution of the equation

$$\int_0^x \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt = \alpha.$$

   Newton's method can be used to solve the equation $f(x) := \int_0^x \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt - \alpha = 0$. For each iteration, Newton's method will require evaluation of the integral

$$\phi(x_j) := \int_0^{x_j} \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt, \tag{1}$$

   where x_j is the j-th iterate. Write a MATLAB program implementing Newton's method for solving $f(x) = 0$ and use the MATLAB command `quad` for approximating the integral (1). For Newton's method set tolerance `tol` $= 10^{-6}$ and $x_0 = 0.5$. Run your code for $\alpha = 0.4$ and $\alpha = 0.1$, and report your output.

   6 marks

5. The intensity of diffracted light near a straight edge is determined by the values of the Fresnel integrals

$$C(x) := \int_0^x \cos\left(\frac{\pi t^2}{2}\right) dt \quad \text{and} \quad S(x) := \int_0^x \sin\left(\frac{\pi t^2}{2}\right) dt.$$

   Use MATLAB function **`integral`** to evaluate these integrals for enough values of x to draw a smooth plot of $C(x)$ and $S(x)$ over the range $0 \le x \le 5$. **4 marks**

6. The error function $\text{erf}(x)$ is defined $\text{erf}(x) := \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt$ but it can also be defined as the solution of the initial value problem (IVP)

$$y' = \frac{2}{\sqrt{\pi}}e^{-x^2}, \quad y(0) = 0.$$

The MATLAB command **`erf(x)`** generates $\text{erf}(x)$.

Write a MATLAB function program `[x,y] = Eulerforward(f,[0,4],0,m)` to solve the IVP by forward Euler method on the interval `[0,4]`, where `y(0) = 0` and `m` is the number of steps. Further, `x` is a vector with m components $x_1, \dots, x_m$ such that $x_1 = 0$ and $x_m = 4$. The vector `y` is such that $y(x_j) \approx y_j$ for $j = 1 : m$. Solve the IVP for `m = 65` and `m=100`. Plot the function $\text{erf}(x)$ on the interval $[0, 4]$ and the points (x_j, y_j) returned by **`Eulerforward`** for `m=65` and `m=100` in a single plot and comment on the results. **7 marks**

*************End************

Instruction: Prepare a livescript program that contains all comments, answers and codes necessary to produce the required output in it. Also the answers should be correctly numbered. The filename of the program should be **LABrollnumberMA581.mlx.**