QUIZ

MA-580H : Matrix computations **Time : 50 minutes** **20 marks**

October 29, 2025

Answer ALL questions

1. Prove of disprove (with counter example, if necessary) the following statements:

   (a) Let $A \in \mathbb{C}^{m\times n}$ be such that $\text{rank}(A) = n$. Let $x \in \mathbb{C}^n$ and $b \in \mathbb{C}^m$. If $(Ax - b) \perp R(A)$ then $x = (A^*A)^{-1}A^*b$, where $R(A)$ is the range space of A.

   (b) Let $A \in \mathbb{R}^{n\times n}$. Let $PA = LU$ be an LU factorization of A, where L is unit lower triangular, U is upper triangular and P is a permutation matrix. Suppose that U has $p < n$ nonzero diagonal entries. Then $\text{rank}(A) = p$.

   4 marks

2. Compute Cholesky factorization of $A := \begin{bmatrix} 16 & -16 & 0 \\ -16 & 41 & -5 \\ 0 & -5 & 5 \end{bmatrix}$, if it exists, using outer product form of Cholesky algorithm. **6 marks**

3. Let $A \in \mathbb{R}^{n\times n}$ be nonsingular. Let $u \in \mathbb{R}^n$ and $v \in \mathbb{R}^n$ be such that $A + uv^T$ is nonsingular. Assuming that the LU factorization $A = LU$ is given, use the Sherman-Morrison formula

$$(A + uv^T)^{-1} = A^{-1} - \frac{A^{-1}uv^TA^{-1}}{1 + v^TA^{-1}u}$$

   to describe an efficient algorithm (outline only the steps) for solving $(A + uv^T)x = b$ and determine the flop count. **4 marks**

4. Let $A \in \mathbb{R}^{m\times n}$ be such that $\text{rank}(A) = n$ and $b \in \mathbb{R}^m$. Describe an algorithm (outline only the steps with justification) that uses QR factorization of A to compute a unique solution of the LSP $Ax \approx b$ and determine the flop count. **6 marks**

*** End ***