a11 Part 1

by ____________________

Reading Questions

1. According to Pierce, the first type systems in computer science were introduced to improve?

   (a) Improve efficiency
   (b) Dectect errors
   (c) Enforce abstraction
   (d) Provide documentation

2. According to Pierce, static typing is usually insufficient to ensure run-time safety. He offers array-bounds checking as an example of something that safe languages considered statically typed languages still check dynamically. He suggests though, that in principle array bounds could be enforced through the type system using mechanisms based on what?

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3. Which of the following does Pierce not list as a statically-checked programming language?

   (a) ML
   (b) Haskell
   (c) Scheme
   (d) Java

Typing Inference

Using the rules found on page 3, on the next page, build the complete trees that allow you to infer the types of the following expressions. You should produce both the completed tree, and identify each rule as you use it. We provide an example below. Note that the rules on the next page may be slightly different from those used in class.

$$
\begin{prooftree}
\AxiomC{$\emptyset,\ y : Nat,\ x : Nat(y) = Nat$}
\RightLabel{Var}
\UnaryInfC{$\emptyset,\ y : Nat,\ x : Nat \vdash y : Nat$}
\RightLabel{$sub1$}
\UnaryInfC{$\emptyset,\ y : Nat,\ x : Nat \vdash (sub1\ y) : Nat$}
\RightLabel{$Abstr$}
\UnaryInfC{$\emptyset,\ y : Nat \vdash (\lambda\ (x)\ (sub1\ y)) : Nat \to Nat$}
\AxiomC{}
\RightLabel{$Num-Axiom$}
\UnaryInfC{$\emptyset,\ y : Nat \vdash 4 : Nat$}
\RightLabel{App}
\BinaryInfC{$\emptyset,\ y : Nat \vdash ((\lambda\ (x)\ (sub1\ y))\ 4) : Nat$}
\RightLabel{$zero?$}
\UnaryInfC{$\emptyset,\ y : Nat \vdash (zero?\ ((\lambda\ (x)\ (sub1\ y))\ 4)) : Bool$}
\RightLabel{$Abstr$}
\UnaryInfC{$\emptyset \vdash (\lambda\ (y)\ (zero?\ ((\lambda\ (x)\ (sub1\ y))\ 4))) : Nat \to Bool$}
\end{prooftree}
$$

4. $(((\lambda\ (x)\ (\lambda\ (y)\ (sub1\ y)))\ 5)\ 6)$

5. $(\lambda\ (!)\ (\lambda\ (n)\ (if\ (zero?\ n)\ 1\ (*\ n\ (!\ (sub1\ n))))))$

6. $(fix\ (\lambda\ (k1)\ (\lambda\ (n)\ (if\ (zero?\ n)\ 1\ (k1\ (sub1\ n))))))$

Typing Rules

The following are the typing rules for the λ-calculus and some additional forms. The variables we use in our expressions do have meanings. Where e, and subscripted versions thereof, represent arbitrary terms of the language, b represents only booleans, n represents only natural numbers, and x represents only variables. These induce restrictions on the application of these rules.

$$\frac{\Gamma(x) = \tau}{\Gamma \vdash x : \tau}\, Var$$

$$\frac{}{\Gamma \vdash n : Nat}\, Nat - Axiom$$

$$\frac{}{\Gamma \vdash b : Bool}\, Bool - Axiom$$

$$\frac{\Gamma,\ x\ : \tau_1 \vdash e : \tau}{\Gamma \vdash (\lambda\ (x)\ \ e) : \tau_1 \rightarrow\ \tau}\, Abstr$$

$$\frac{\Gamma \vdash e_1 : \tau_1 \rightarrow\ \tau \qquad \Gamma \vdash e_2 : \tau_1}{\Gamma \vdash (e_1\ e_2) : \tau}\, App$$

$$\frac{\Gamma \vdash e_1 : Nat \qquad \Gamma \vdash e_2 : Nat}{\Gamma \vdash (+\ e_1\ e_2) : Nat}\, +$$

$$\frac{\Gamma \vdash e_1 : Nat \qquad \Gamma \vdash e_2 : Nat}{\Gamma \vdash (*\ e_1\ e_2) : Nat}\, *$$

$$\frac{\Gamma \vdash e : Nat}{\Gamma \vdash (sub_1\ e) : Nat}\, sub1$$

$$\frac{\Gamma \vdash e : Nat}{\Gamma \vdash (zero?\ e) : Bool}\, zero?$$

$$\frac{\Gamma \vdash e_1 : Bool \qquad \Gamma \vdash e_2 : \tau \qquad \Gamma \vdash e_3 : \tau}{\Gamma \vdash (if\ e_1\ e_2\ e_3) : \tau}\, if$$

$$\frac{\Gamma,\ x\ :\ \tau \vdash e : \tau}{\Gamma \vdash (fix\ (\lambda\ (x)\ e)) : \tau}\, fix$$