

Derivatives

Implicit and Logarithmic

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Question : If $x + \sqrt{xy} + y = 1$, find $\frac{dy}{dx}$

Answer:

Given $x + \sqrt{xy} + y = 1$

Differentiating w.r.t. x

$$1 + \frac{1}{2\sqrt{xy}} \left(x \frac{dy}{dx} + y \right) + \frac{dy}{dx} = 0$$

$$\therefore 1 + \frac{x}{2\sqrt{xy}} \frac{dy}{dx} + \frac{y}{2\sqrt{xy}} + \frac{dy}{dx} = 0$$

$$\therefore \left(\frac{x}{2\sqrt{xy}} + 1 \right) \frac{dy}{dx} = -1 - \frac{y}{2\sqrt{xy}}$$

$$\therefore \left(\frac{x + 2\sqrt{xy}}{2\sqrt{xy}} \right) \frac{dy}{dx} = -\frac{2\sqrt{xy} + y}{2\sqrt{xy}}$$

$$\therefore (x + 2\sqrt{xy}) \frac{dy}{dx} = -2\sqrt{xy} - y$$

$$\therefore \frac{dy}{dx} = -\frac{x + 2\sqrt{xy}}{2\sqrt{xy} + y}$$

Question : If $x^p y^q = (x + y)^{p+q}$, then show that $\frac{dy}{dx} = \frac{y}{x}$

Answer:

$$\text{Given } x^p y^q = (x + y)^{p+q}$$

$$\therefore \log(x^p y^q) = \log(x + y)^{p+q}$$

$$\therefore p \log x + q \log y = (p + q) \log(x + y)$$

Differentiating w.r.t. x

$$\therefore \frac{p}{x} + \frac{q}{y} \frac{dy}{dx} = \frac{p + q}{x + y} \left(1 + \frac{dy}{dx} \right)$$

$$\therefore \frac{p}{x} + \frac{q}{y} \frac{dy}{dx} = \frac{p + q}{x + y} + \frac{p + q}{x + y} \frac{dy}{dx}$$

$$\therefore \left(\frac{q}{y} - \frac{p + q}{x + y} \right) \frac{dy}{dx} = \frac{p + q}{x + y} - \frac{p}{x}$$

Question : If $x^p y^q = (x + y)^{p+q}$, then show that $\frac{dy}{dx} = \frac{y}{x}$

Answer: Contd

$$\therefore \frac{q(x+y) - (p+q)y}{y(x+y)} \frac{dy}{dx} = \frac{(p+q)x - p(x+y)}{(x+y)x}$$

$$\therefore \frac{qx + qy - py - qy}{y} \frac{dy}{dx} = \frac{px + qx - px - py}{x}$$

$$\therefore \frac{\cancel{qx} - \cancel{py}}{y} \frac{dy}{dx} = \frac{\cancel{qx} - \cancel{py}}{x}$$

$$\therefore \frac{dy}{dx} = \frac{y}{x}$$