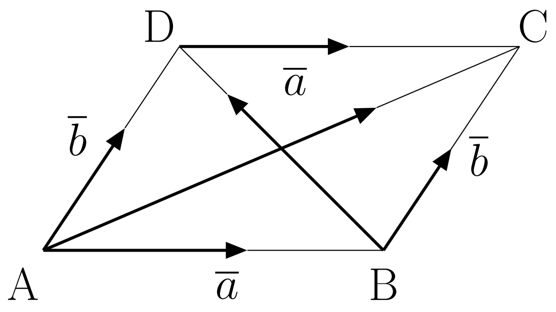
1 Question

Show by vector method that the sum of the squares of the diagonals of a parallelogram is equal to the sum of the squares of its sides.

Answer

Let ABCD be a parallelogram with
$\overline{AB} = \overline{DC} = \overline{a}$ and
$\overline{BC} = \overline{AD} = \overline{b}$



LHS

$= AC^2 + BD^2$

$= |\overline{AC}|^2 + |\overline{BD}|^2$

$= |\overline{AB} + \overline{BC}|^2 + |\overline{BC} + \overline{CD}|^2$

$= |\overline{a} + \overline{b}|^2 + |\overline{a} - \overline{b}|^2$

$= (\overline{a} + \overline{b}) \cdot (\overline{a} + \overline{b}) + (\overline{a} - \overline{b}) \cdot (\overline{a} - \overline{b})$

$= |\overline{a}|^2 + 2\overline{a} \cdot \overline{b} + |\overline{b}|^2 + |\overline{a}|^2 - 2\overline{a} \cdot \overline{b} + |\overline{b}|^2$

$= |\overline{a}|^2 + |\overline{b}|^2 + |\overline{a}|^2 + |\overline{b}|^2$

$= |\overline{AB}|^2 + |\overline{BC}|^2 + |\overline{CD}|^2 + |\overline{DA}|^2$

$= \overline{AB}^2 + \overline{BC}^2 + \overline{CD}^2 + \overline{DA}^2$

$=$ RHS

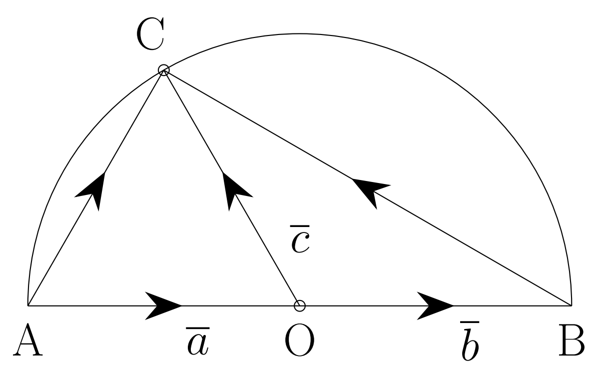
2 Question

Prove using vector method that the angle subtended on a semi-circle is a right angle.

Answer

Consider a semicircle on a diameter AB with center O and radius r. Let C be any point on the circle other than A or B.

Let $\overline{a}, \overline{b}, \overline{c}$ be the position vectors of the point A, B and C respectively.



Then

$$\begin{aligned}\overline{AC}\cdot\overline{BC} &= (\overline{c}-\overline{a})\cdot(\overline{c}-\overline{b})\\ &= (\overline{c}-\overline{a})\cdot(\overline{c}+\overline{a}) \quad \boxed{-\overline{b}=\overline{a}}\\ &= |\overline{c}|^2-|\overline{a}|^2\\ &= r^2-r^2=0\end{aligned}$$

$\therefore$ $\overline{AC}\perp\overline{BC}$

$\therefore$ Seg AB $\perp$ Seg BC

$\therefore$ Angle in Semi-circle

$\therefore$ is right angle.