Mathematics QB From 2019-20 onwards

1 Angles and Their Measurement

1. Determine which of the following pairs of angles are co-terminal i) 210° , 150° ii) 360° , -30° iii) -180° , 540° iv) -405° , 675° v) 860° , 580° vi) 900° , -900°
2. Draw the angles of the following measures and determine their quadrants. i) –140° ii) 250° iii) 420° iv) 750° v) 945° vi) 1120° vii) –80° viii) –330° ix) –500° x) –820°
3. Convert the following angles in to radian. i) 85° ii) 250° iii) -132° iv) 65° 30' v) 75° 30' vi) 40° 48'
4. Convert the following angles in degree i) $\frac{7\pi}{12}^c$ ii) $\frac{-5\pi}{3}^c$ iii) 5^c iv) $\frac{11\pi}{18}^c$ v) $\left(\frac{-1}{4}\right)^c$
5. The sum of two angles is $5\pi^c$ and their difference is 60° . Find their measures in degree.
6. Find the length of an arc of a circle which subtends an angle of 108° at the centre, if the radius of the circle is 15 cm.
7. Find the angle in degree subtended at the centre of a circle by an arc whose length is 15 cm, if the radius of the circle is 25 cm.
8. The area of a circle is 25π sq.cm. Find the length of its arc subtending an angle of 144° at the centre. Also find the area of the corresponding sector.
9. The perimeter of a sector of the circle of area 25π sq.cm is 20 cm. Find the area of sector.

2 Trigonometry-I

10. State the signs of i) tan 380° ii) cot 230° iii) sec 468°
11. Evaluate each of the following : i) sin30° + cos 45° + tan180° ii) cosec45° + cot 45° + tan0° iii) sin30° × cos 45° × tan360°
12. If $cos\ \theta = \frac{12}{13}$, $0 < \theta < \frac{\pi}{2}$, find the value of i) $\frac{sin^2\theta - cos^2\theta}{2sin\ \theta\ cos\theta}$, $\frac{1}{tan^2\theta}$
13. Using tables evaluate the following : i) $4cot\ 45° - sec^2 60° + sin\ 30°$ ii) $cos^2 0 + cos^2\frac{\pi}{6} + cos^2\frac{\pi}{3} + cos^2\frac{\pi}{2}$
14. If $\frac{sin\,A}{3} = \frac{sin\,B}{4} = \frac{1}{5}$ and A, B are angles in the second quadrant then prove that $4cos\,A + 3cos\,B = -5$
15. If$tan\,\theta = \frac{1}{2}$, evaluate $\frac{2sin\,\theta + 3cos\,\theta}{4cos\,\theta + 3sin\,\theta}$
16. Eliminate θ from i) $x = 3sec\,\theta,\ y = 4tan\,\theta$. ii) $x = 6cosec\,\theta,\ y = 8cot\,\theta$.
17. Find the acute angle θ such that $5tan^2\theta + 3 = 9sec\,\theta$.
18. If $2cos^2\theta - 11cos\,\theta + 5 = 0$ then find the possible values of $cos\,\theta$
19. Find the Cartesian co-ordinates of points whose polar coordinates are : i) (3,90°) ii) (1,180°)
20. Prove the following identities: i) $(cos^A - 1)(cot^A + 1) = -1$ ii) $\frac{sin\,\theta}{1+cos\,\theta} + \frac{1+cos\,\theta}{sin\,\theta} = 2cos\,\theta$ iii) $\frac{tan\,\theta}{sec\,\theta - 1} = \frac{sec\,\theta + 1}{tan\,\theta}$ iv) $(sec\,A + cos\,A)(sec\,A - cos\,A) = tan^2 A + sin^2 A$

3 Trigonometry-II

21. Find the values of i) sin 15° ii) cos 75° iii) tan 105° iv) cot 225°
22. Prove the following : i) $tan\left(\frac{\pi}{4} + \theta\right) = \frac{1 - tan\,\theta}{1 + tan\,\theta}$ ii) $\sqrt{2}cos\left(\frac{\pi}{4} - A\right) = cosA + sinA$ iii) tan 50° = tan 40° +2tan 10°
23. If $tan\,A = \frac{5}{6}$, $tan\,B = \frac{1}{11}$, prove that $A + B = \frac{\pi}{4}$
24. Find the value of i) sin 690° ii) cos 315° iii) tan 225°
25. Prove the following : i) $\frac{cos\,(\pi + x)cos(-x)}{sin(\pi - x)cos\left(\frac{\pi}{2} + x\right)} = cot^2 x$ ii) sec 840° .cot (- 945°) + sin 600° tan (-690°)= $\frac{3}{2}$
26. Prove the following : i) $\frac{1 - cos\,2\theta}{1 + cos\,2\theta} = tan^2\theta$ ii) $tan\,x + cot\,x = 2\,cosec\,2x$

4 Determinants and Matrices

27. Find the value of i) $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$ ii) $\begin{vmatrix} 3 & -4 & 5 \\ 1 & 1 & -2 \\ 2 & 3 & 1 \end{vmatrix}$
28. Find x. i) $\begin{vmatrix} x^2 - x + 1 & x + 1 \\ x + 1 & x + 1 \end{vmatrix} = 0$ ii) $\begin{vmatrix} x & -1 & 2 \\ 2x & 1 & 3 \\ 3 & -4 & 5 \end{vmatrix} = 29$
29. Find minor and cofactors of $\begin{vmatrix} 2 & -1 & 3 \\ 1 & 2 & -1 \\ 5 & 7 & 2 \end{vmatrix}$
30. Using properties show that $\begin{vmatrix} a+b & a & b \\ a & a+c & c \\ b & c & b+c \end{vmatrix} = 4abc$
31. Solve : $\begin{vmatrix} x+2 & x+6 & x-1 \\ x+6 & x-1 & x+2 \\ x-1 & x+2 & x+6 \end{vmatrix} = 0$
32. Solve by Cramer's Rule : $x + y + z = 6, x – y + z = 2, x + 2y – z = 2$
33. Find k if the following matrices are singular. i) $\begin{bmatrix} 7 & 3 \\ -2 & k \end{bmatrix}$ ii) $\begin{bmatrix} x+2 & x+6 & x-1 \\ x+6 & x-1 & x+2 \\ x-1 & x+2 & x+6 \end{bmatrix}$
34. Find a, b, c if the matrix $\begin{bmatrix} 1 & \frac{3}{5} & a \\ b & -5 & 7 \\ -4 & c & 0 \end{bmatrix}$ is symmetric.

35. Solve for X and Y : $3X - Y = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ and $X - 3Y = \begin{bmatrix} 0 & -1 \\ 0 & -1 \end{bmatrix}$

36. If $A = \begin{bmatrix} 4 & 8 \\ -2 & -4 \end{bmatrix}$, prove that $A^2 = 0$.

37. If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, show that $A^2 - 4A$ is a scalar matrix.

38. If $A = \begin{bmatrix} 3 & 4 \\ -4 & 3 \end{bmatrix}$, prove that $A^2 = 0$. and $B = \begin{bmatrix} 2 & 1 \\ -1 & 2 \end{bmatrix}$, show that $(A + B)(A - B) = A^2 - B^2$.

5 Straight Line

39. If A(2, 0) and B(0, 3) are two points, find the equation of the locus of point P such that AP = 2BP.

40. If A(4, 1) and B(5, 4), find the equation of the locus of point P if $PA^2 = 3PB^2$.

41. Obtain the new equations of the following loci if the origin is shifted to the point O'(2, 2), the direction of axes remaining the same : (a) $3x - y + 2 = 0$ (b) $x^2 + y^2 - 3x = 7$

42. A line makes intercepts 3 and 3 on the co-ordiante axes. Find the inclination of the line.

43. Without using Pythagoras theorem show that points A(4,4), B(3, 5) and C(1, 1) are the vertices of a right angled triangle.

44. Find the value of k for which points P(k,1),Q(2,1) and R(4,5) are collinear.

45. Find the equation of the line a) passing through the points A (2 , 0) and B(3,4). b) passing through the points P(2,1) and Q(2,-1)

46. Find the equation of the line a) containing the origin and having inclination 60° . b) passing through the origin and parallel to AB, where A is (2,4) and B is (1,7). c) having slope $\frac{1}{2}$ and containing the point (3,2).

47. The vertices of a triangle are A(3,4), B(2,0) and C(1,6). Find the equations of the lines containing (a) side BC (b) the median AD (c) the mid points of sides AB and BC.

48. Show that lines $x - 2y - 7 = 0$ and $2x - 4y + 15 = 0$ are parallel to each other.

49. Show that lines $x - 2y - 7 = 0$ and $2x + y + 1 = 0$ are perpendicular to each other. Find their point of intersection.

50. If the line $3x+4y = p$ makes a triangle of area 24 square unit with the co-ordinate axes then find the value of p.

51. Find the distance of the point A (-2,3) from the line $12x - 5y - 13 = 0$

52. Find the distance between parallel lines $4x-3y+5 = 0$ and $4x - 3y + 7 = 0$,

6 Complex Numbers

53. Find a and b if i) $a+2b+2ai = 4+6i$ ii) $(a-b)+(a+b)i = a+5i$ iii) $(a+b)(2+i) = b+1+(10+2a)i$ iv) $\frac{1}{a+ib} = 3-2i$

54. Express the following in the form of a+ib i) $(1+2i)(-2+i)$ ii) $\frac{2+i}{(3-i)(1+2i)}$ iii) $\left(\frac{1+i}{1-i}\right)^2$ vii) $(1+i)^{-3}$

55. Show that $(-1+3i)^3$ is a real number.

56. Evaluate : i) i^{35} ii) i^{-888} iii) i^{93} iv) i^{116}

57. Find the value of x and y if $(x+2y)+(2x-3y)i+4i = 5$

58. Find the square root of the following complex numbers. i) $-8-6i$ ii) $7+24i$ iii) $1+4\sqrt{3}i$

59. Find the value of i) $x^3 - x^2 + x + 46$, if $x = 2 + 3i$ ii) $x^3 + x^2 - x + 22$, if $x = \frac{5}{1-2i}$ iii) $x^3 - x^2 + 2x + 10$ if $1 + \sqrt{3}i$ (Ans , ,6)

7 Sequences and Series

60. Check whether the following sequences are G.P. If so, write t_n : i) 2,6,18,54,... ii) 1,-5,25, -125 ...

61. For a GP i) If $r = \frac{1}{3}, a = 9$, find t_7 ii) If $a = \frac{7}{243}, r = 3$ find t_6 iii) If $r = -3$ and $t_6 = 1701$, find a.

62. Find three numbers in G.P. such that their sum is 21 and sum of their squares is 189.

63. For a GP i) If $a = 2, r = -\frac{2}{3}$, find S_6 ii) If $S_5 = 1023, r = 4$ find a iii) If $r = -3$ and $t_6 = 1701$, find a.

64. For a G.P. sum of first 3 terms is 125 and sum of next 3 terms is 27, find the value of r .

65. Find the sum to n terms i) 3 + 33 + 333 + 3333 + ... ii) 8 +88 + 888 + 8888 + ... iii) 0.4 + 0.44 + 0.444 + ... iv) 0.7 + 0.77+ 0.777 + ...

66. Find the n^{th} term and hence find the 8^{th} term of the following HPs i) $\frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \frac{1}{9}$, ii) $\frac{1}{3}, \frac{1}{6}, \frac{1}{12}, \frac{1}{24}$,

67. Find the sum i) $\sum_{r=1}^{n}(r+1)(2r-1)$ $\sum_{r=1}^{n}(3r^2-2r+1)$ iii) $\sum_{r=1}^{n}\frac{1+2+3+..+r}{r}$ iv) $\sum_{r=1}^{n}\frac{1^3+2^3+3^3+..+r^3}{r(r+1)}$

8 Permutations and Combinations

68. How many two letter words can be formed using letters from the word SPACE, when repetition of letters (i) is allowed, (ii) is not allowed?

69. How many three-digit numbers can be formed from the digits 0, 1, 3, 5, 6 if repetitions of digits (i) are allowed, (ii) are not allowed?

70. How many numbers between 100 and 1000 have 4 in the units place?

71. Write in terms of factorials (i) $5\times6\times7\times8\times9\times10$ (ii) $3\times6\times9\times12\times15$ (iii) $6\times7\times8\times9$ (iv) $5\times10\times15\times20$

72. Evaluate : $\frac{n!}{r!(n-r)!}$ for (i) n = 8, r = 6 (ii) n = 12, r = 12, (iii) n = 15, r = 10 (iv) n = 15, r = 8

73. Find n if (i) $\frac{n}{8!}=\frac{3}{6!}+\frac{1!}{4!}$ (ii) $\frac{n}{6!}=\frac{4}{8!}+\frac{3}{6!}$ (iii) $\frac{1!}{n!}=\frac{1!}{4!}+\frac{4}{5!}$ (iv) $(n+1)!=42\times(n-1)!$ (v) $(n+3)!=110\times(n+1)!$

74. Find n if (i) $\frac{(17-n)!}{(14-n)!}=5!$ (ii) $\frac{(15-n)!}{(13-n)!}=12$ (iii) $\frac{n!}{3!(n-3)!}:\frac{n!}{5!(n-5)!}=5:3$ (iv) $\frac{n!}{3!(n-3)!}:\frac{n!}{5!(n-7)!}=1:6$ (v) $\frac{(2n)!}{7!(2n-7)!}:\frac{n!}{4!(n-4)!}=21:1$

75. Find n if ${}^nP_6 :{}^nP_3 = 120:1$

76. Find m and n, if ${}^{m+n}P_2 = 56 and {}^{m+n}nP_2 = 12$

77. Find r if ${}^12P_{r-2} :{}^1 1P_{r-1} = 3:14$

78. Find the number of permutations of the letters of the word UBUNTU.

79. How many 4 letter words can be formed using letters in the word MADHURI if (a) letters can be repeated (b) letters cannot be repeated.

80. Determine the number of arrangements of letters of the word ALGORITHM if (a) vowels are always together. (b) no two vowels are together. (c) consonants are at even positions.

81. Find the number of arrangements of the letters in the word SOLAPUR so that consonents and vowels are placed alternately.

82. Find the value of (a) ${}^{16}C_4$ (b) ${}^{80}C_2$ (c) ${}^{15}C_4+{}^{15}C_5$ (d) ${}^{20}C_{16}-{}^{19}C_{16}$

83. Find n if (a) ${}^6P_2 = n{}^6C_2$ (b) ${}^{2n}C_3 :{}^nC_2 = 52:3$ (c) ${}^nC_{n-3}=84$

84. Find the number of ways of selecting a team of 3 boys and 2 girls from 6 boys and 4 girls.

85. Find r if ${}^{11}C_4+{}^{11}C_5+{}^{12}C_6+{}^{13}C_7={}^{14}C_r$

86. A group consists of 9 men and 6 women. A team of 6 is to be selected. How many of possible selections will have at least 3 women?

9 Method Of Indiction and B.T.

87. Prove by Induction : $1^2+3^2+5^2+...+(2n-1)^2=\frac{n}{3}(2n-1)(2n+1)$ ii) $1\cdot2+2\cdot3+...+n(n+1)=\frac{n}{3}(n+1)(n+2)$ iii) $\frac{1}{1\cdot3}+\frac{1}{3\cdot5}+\frac{1}{5\cdot7}+...+\frac{1}{(2n-1)(2n+1)}=\frac{n}{2n+1}$ iv) $(2^{3n}-1)$ is divisible by 7

88. Find the value of i) $\left(\sqrt{3}+1\right)^4-\left(\sqrt{3}-1\right)^4$ ii) $\left(2+\sqrt{5}\right)^5-\left(2-\sqrt{5}\right)^5$ iii) $\left(\sqrt{3}+\sqrt{2}\right)^6+\left(\sqrt{3}-\sqrt{2}\right)^5$

89. Find the value of $(1.02)^6$, correct upto four places of decimals.

90. Find i) 9^{th} term of $\left(\frac{1}{3}+a^2\right)^2$ ii) coefficient of x^8 in $\left(2x^5-\frac{5}{x^3}\right)^8$ iii) coefficient of x^{-20} in $\left(x^3-\frac{1}{2x^2}\right)^{15}$ iv) constant term in $\left(\sqrt{x}-\frac{3}{x^2}\right)^{10}$ v) constant term in $\left(x^2-\frac{1}{x}\right)^9$ vi) middle terms of $\left(x^4-\frac{1}{x^3}\right)^{15}$

10 Set Theory

91. If A = { 1, 2, 3, 4}, B = {3, 4, 5, 6} C = {4, 5, 6, 7, 8} and universal set X = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, then verify the following: i) $A\cup B\cap C)=(A\cup B)\cap(A\cup C)$ ii) $A\cap(B\cup C)=(A\cap B)\cup(A\cup C)$ iii) $(A\cup B)'=(A'\cap B)'$ iv) $(A\cap B)'=A'\cup B'$ v) $A=(A\cap B)\cup(A\cap B')$ vi) $B=(A\cap B)\cup(A'\cap B)$ vii) $(A\cup B)=(A-B)\cup(A\cap B)\cup(B-A)$ viii) $A\cap(B\Delta C)=(A\cap B)\Delta(A\cap C)$ ix) $n(A\cup B)=n(A)+n(B)-n(A\cap B)$ x) $n(B)=n(A'\cap B)+n(A\cap B)$

92. If A and B are subsets of the universal set X and n(X) = 50, n(A) = 35, n(B) = 20,$n(A'\cap B')=5$, find i) $n(A\cup B)$ ii) $n(A\cap B)$ iv) $n(A\cap B')$ iii) $n(A'\cap B)$

93. In a class of 200 students who appeared certain examinations, 35 students failed in CET, 40 in NEET and 40 in JEE, 20 failed in CET and NEET, 17 in NEET and JEE, 15 in CET and JEE and 5 failed in all three examinations. Find how many students, i) did not fail in any examination. ii) failed in NEET or JEE entrance.

94. In a hostel, 25 students take tea, 20 students take coffee, 15 students take milk, 10 student take bot tea and coffee, 8 students take both milk and coffee. None of them take tea and milk both and everyone takes atleast one beverage, find the total number of students in the hostel.

95. If $P=\{1,2,3\}$ and $Q=\{14\}$,find sets $P\times Q$ and $Q\times P$

96. Let A = {1, 2, 3, 4}, B = {4, 5, 6}, C = { 5, 6 }. Verify, i) $A\times(B\cap C)=(A\times B)\cap(A\times C)$ ii) $A\times(B\cup C)=(A\times B)\cup(A\times C)$

97. Let A = {6, 8} and B = {1, 3, 5} Show that $R_1=\{(a,b)/a\in A,b\in B,a-b$ is an even number } is a null relation. $R_2=\{(a,b)/a\in A,b\in B$, a+b is odd number } is an universal relation.