

Mathematics QB From 2019-20 onwards

1 Angles and Their Measurement

- Determine which of the following pairs of angles are co-terminal i) 210° , 150° ii) 360° , -30° iii) -180° , 540° iv) -405° , 675° v) 860° , 580° vi) 900° , -900°
- Draw the angles of the following measures and determine their quadrants. i) -140° ii) 250° iii) 420° iv) 750° v) 945° vi) 1120° vii) -80° viii) -330° ix) -500° x) -820°
- Convert the following angles in to radian. i) 85° ii) 250° iii) -132° iv) $65^\circ 30'$ v) $75^\circ 30'$ vi) $40^\circ 48'$
- Convert the following angles in degree i) $\frac{7\pi}{12}^c$ ii) $\frac{-5\pi}{3}^c$ iii) 5^c iv) $\frac{11\pi}{18}^c$ v) $(\frac{-1}{4})^c$
- The sum of two angles is $5\pi^c$ and their difference is 60° . Find their measures in degree.
- Find the length of an arc of a circle which subtends an angle of 108° at the centre, if the radius of the circle is 15 cm.
- Find the angle in degree subtended at the centre of a circle by an arc whose length is 15 cm, if the radius of the circle is 25 cm.
- The area of a circle is 25π sq.cm. Find the length of its arc subtending an angle of 144° at the centre. Also find the area of the corresponding sector.
- The perimeter of a sector of the circle of area 25π sq.cm is 20 cm. Find the area of sector.

2 Trigonometry-I

- State the signs of i) $\tan 380^\circ$ ii) $\cot 230^\circ$ iii) $\sec 468^\circ$
- Evaluate each of the following : i) $\sin 30^\circ + \cos 45^\circ + \tan 180^\circ$ ii) $\operatorname{cosec} 45^\circ + \cot 45^\circ + \tan 0^\circ$ iii) $\sin 30^\circ \times \cos 45^\circ \times \tan 360^\circ$
- If $\cos \theta = \frac{12}{13}$, $0 < \theta < \frac{\pi}{2}$, find the value of i) $\frac{\sin^2 \theta - \cos^2 \theta}{2 \sin \theta \cos \theta}$, $\frac{1}{\tan^2 \theta}$
- Using tables evaluate the following : i) $4 \cot 45^\circ - \sec^2 60^\circ + \sin 30^\circ$ ii) $\cos^2 0 + \cos^2 \frac{\pi}{6} + \cos^2 \frac{\pi}{3} + \cos^2 \frac{\pi}{2}$
- If $\frac{\sin A}{3} = \frac{\sin B}{4} = \frac{1}{5}$ and A, B are angles in the second quadrant then prove that $4 \cos A + 3 \cos B = -5$
- If $\tan \theta = \frac{1}{2}$, evaluate $\frac{2 \sin \theta + 3 \cos \theta}{4 \cos \theta + 3 \sin \theta}$
- Eliminate θ from i) $x = 3 \sec \theta$, $y = 4 \tan \theta$. ii) $x = 6 \operatorname{cosec} \theta$, $y = 8 \cot \theta$.
- Find the acute angle θ such that $5 \tan^2 \theta + 3 = 9 \sec \theta$.
- If $2 \cos^2 \theta - 11 \cos \theta + 5 = 0$ then find the possible values of $\cos \theta$

- Find the Cartesian co-ordinates of points whose polar coordinates are : i) $(3, 90^\circ)$ ii) $(1, 180^\circ)$
- Prove the following identities: i) $(\cos^A - 1)(\cot^A + 1) = -1$ ii) $\frac{\sin \theta}{1 + \cos \theta} + \frac{1 + \cos \theta}{\sin \theta} = 2 \cos \theta$ iii) $\frac{\tan \theta}{\sec \theta - 1} = \frac{\sec \theta + 1}{\tan \theta}$ iv) $(\sec A + \cos A)(\sec A - \cos A) = \tan^2 A + \sin^2 A$

3 Trigonometry-II

- Find the values of i) $\sin 15^\circ$ ii) $\cos 75^\circ$ iii) $\tan 105^\circ$ iv) $\cot 225^\circ$
- Prove the following : i) $\tan(\frac{\pi}{4} + \theta) = \frac{1 - \tan \theta}{1 + \tan \theta}$ ii) $\sqrt{2} \cos(\frac{\pi}{4} - A) = \cos A + \sin A$ iii) $\tan 50^\circ = \tan 40^\circ + 2 \tan 10^\circ$
- If $\tan A = \frac{5}{6}$, $\tan B = \frac{1}{11}$, prove that $A + B = \frac{\pi}{4}$
- Find the value of i) $\sin 690^\circ$ ii) $\cos 315^\circ$ iii) $\tan 225^\circ$
- Prove the following : i) $\frac{\cos(\pi + x) \cos(-x)}{\sin(\pi - x) \cos(\frac{\pi}{2} + x)} = \cot^2 x$ ii) $\sec 840^\circ \cdot \cot(-945^\circ) + \sin 600^\circ \tan(-690^\circ) = \frac{3}{2}$
- Prove the following : i) $\frac{1 - \cos 2\theta}{1 + \cos 2\theta} = \tan^2 \theta$ ii) $\tan x + \cot x = 2 \operatorname{cosec} 2x$

4 Determinants and Matrices

- Find the value of i) $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$ ii) $\begin{vmatrix} 3 & -4 & 5 \\ 1 & 1 & -2 \\ 2 & 3 & 1 \end{vmatrix}$
- Find x. i) $\begin{vmatrix} x^2 - x + 1 & x + 1 \\ x + 1 & x + 1 \end{vmatrix} = 0$ ii) $\begin{vmatrix} x & -1 & 2 \\ 2x & 1 & 3 \\ 3 & -4 & 5 \end{vmatrix} = 29$
- Find minor and cofactors of $\begin{vmatrix} 2 & -1 & 3 \\ 1 & 2 & -1 \\ 5 & 7 & 2 \end{vmatrix}$
- Using properties show that $\begin{vmatrix} a+b & a & b \\ a & a+c & c \\ b & c & b+c \end{vmatrix} = 4abc$
- Solve : $\begin{vmatrix} x+2 & x+6 & x-1 \\ x+6 & x-1 & x+2 \\ x-1 & x+2 & x+6 \end{vmatrix} = 0$
- Solve by Cramer's Rule : $x + y + z = 6$, $x - y + z = 2$, $x + 2y - z = 2$
- Find k if the following matrices are singular. i) $\begin{bmatrix} 7 & 3 \\ -2 & k \end{bmatrix}$ ii) $\begin{bmatrix} x+2 & x+6 & x-1 \\ x+6 & x-1 & x+2 \\ x-1 & x+2 & x+6 \end{bmatrix}$
- Find a, b, c if the matrix $\begin{bmatrix} 1 & \frac{3}{5} & a \\ b & -5 & 7 \\ -4 & c & 0 \end{bmatrix}$ is symmetric.

35. Solve for X and Y : $3X - Y = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ and $X - 3Y = \begin{bmatrix} 0 & -1 \\ 0 & -1 \end{bmatrix}$
36. If $A = \begin{bmatrix} 4 & 8 \\ -2 & -4 \end{bmatrix}$, prove that $A^2 = 0$.
37. If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, show that $A^2 - 4A$ is a scalar matrix.
38. If $A = \begin{bmatrix} 3 & 4 \\ -4 & 3 \end{bmatrix}$, prove that $A^2 = 0$. and $B = \begin{bmatrix} 2 & 1 \\ -1 & 2 \end{bmatrix}$, show that $(A + B)(A - B) = A^2 - B^2$.

5 Straight Line

39. If A(2, 0) and B(0, 3) are two points, find the equation of the locus of point P such that $AP = 2BP$.
40. If A(4, 1) and B(5, 4), find the equation of the locus of point P if $PA^2 = 3PB^2$.
41. Obtain the new equations of the following loci if the origin is shifted to the point O'(2, 2), the direction of axes remaining the same : (a) $3x - y + 2 = 0$ (b) $x^2 + y^2 - 3x = 7$
42. A line makes intercepts 3 and 3 on the co-ordinate axes. Find the inclination of the line.
43. Without using Pythagoras theorem show that points A(4,4), B(3, 5) and C(1, 1) are the vertices of a right angled triangle.
44. Find the value of k for which points P(k,1), Q(2,1) and R(4,5) are collinear.
45. Find the equation of the line a) passing through the points A (2, 0) and B(3,4). b) passing through the points P(2,1) and Q(2,-1)
46. Find the equation of the line a) containing the origin and having inclination 60° . b) passing through the origin and parallel to AB, where A is (2,4) and B is (1,7). c) having slope $\frac{1}{2}$ and containing the point (3,2).
47. The vertices of a triangle are A(3,4), B(2,0) and C(1,6). Find the equations of the lines containing (a) side BC (b) the median AD (c) the mid points of sides AB and BC.
48. Show that lines $x - 2y - 7 = 0$ and $2x - 4y + 15 = 0$ are parallel to each other.
49. Show that lines $x - 2y - 7 = 0$ and $2x + y + 1 = 0$ are perpendicular to each other. Find their point of intersection.
50. If the line $3x + 4y = p$ makes a triangle of area 24 square unit with the co-ordinate axes then find the value of p .
51. Find the distance of the point A (-2,3) from the line $12x - 5y - 13 = 0$
52. Find the distance between parallel lines $4x - 3y + 5 = 0$ and $4x - 3y + 7 = 0$,

6 Complex Numbers

53. Find a and b if i) $a + 2b + 2ai = 4 + 6i$ ii) $(a - b) + (a + b)i = a + 5i$ iii) $(a + b)(2 + i) = b + 1 + (10 + 2a)i$ iv) $\frac{1}{a + ib} = 3 - 2i$
54. Express the following in the form of $a + ib$ i) $(1 + 2i)(-2 + i)$ ii) $\frac{2 + i}{(3 - i)(1 + 2i)}$ iii) $\left(\frac{1 + i}{1 - i}\right)^2$ vii) $(1 + i)^{-3}$
55. Show that $(-1 + 3i)^3$ is a real number.
56. Evaluate : i) i^{35} ii) i^{-888} iii) i^{93} iv) i^{116}
57. Find the value of x and y if $(x + 2y) + (2x - 3y)i + 4i = 5$
58. Find the square root of the following complex numbers. i) $-8 - 6i$ ii) $7 + 24i$ iii) $1 + 4\sqrt{3}i$
59. Find the value of i) $x^3 - x^2 + x + 46$, if $x = 2 + 3i$ ii) $x^3 + x^2 - x + 22$, if $x = \frac{5}{1 - 2i}$ iii) $x^3 - x^2 + 2x + 10$ if $1 + \sqrt{3}i$ (Ans , 6)

7 Sequences and Series

60. Check whether the following sequences are G.P. If so, write t_n : i) 2, 6, 18, 54, ... ii) 1, -5, 25, -125 ...
61. For a GP i) If $r = \frac{1}{3}$, $a = 9$, find t_7 ii) If $a = \frac{7}{243}$, $r = 3$ find t_6 iii) If $r = -3$ and $t_6 = 1701$, find a .
62. Find three numbers in G.P. such that their sum is 21 and sum of their squares is 189.
63. For a GP i) If $a = 2$, $r = -\frac{2}{3}$, find S_6 ii) If $S_5 = 1023$, $r = 4$ find a iii) If $r = -3$ and $t_6 = 1701$, find a .
64. For a G.P. sum of first 3 terms is 125 and sum of next 3 terms is 27, find the value of r .
65. Find the sum to n terms i) $3 + 33 + 333 + 3333 + \dots$ ii) $8 + 88 + 888 + 8888 + \dots$ iii) $0.4 + 0.44 + 0.444 + \dots$ iv) $0.7 + 0.77 + 0.777 + \dots$
66. Find the n^{th} term and hence find the 8^{th} term of the following HPs i) $\frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \frac{1}{9}$, ii) $\frac{1}{3}, \frac{1}{6}, \frac{1}{12}, \frac{1}{24}$,
67. Find the sum i) $\sum_{r=1}^n (r + 1)(2r - 1)$ $\sum_{r=1}^n (3r^2 - 2r + 1)$ iii) $\sum_{r=1}^n \frac{1 + 2 + 3 + \dots + r}{r}$ iv) $\sum_{r=1}^n \frac{1^3 + 2^3 + 3^3 + \dots + r^3}{r(r + 1)}$

8 Permutations and Combinations

68. How many two letter words can be formed using letters from the word SPACE, when repetition of letters (i) is allowed, (ii) is not allowed?
69. How many three-digit numbers can be formed from the digits 0, 1, 3, 5, 6 if repetitions of digits (i) are allowed, (ii) are not allowed?
70. How many numbers between 100 and 1000 have 4 in the units place?
71. Write in terms of factorials (i) $5 \times 6 \times 7 \times 8 \times 9 \times 10$ (ii) $3 \times 6 \times 9 \times 12 \times 15$ (iii) $6 \times 7 \times 8 \times 9$ (iv) $5 \times 10 \times 15 \times 20$
72. Evaluate : $\frac{n!}{r!(n-r)!}$ for (i) $n = 8, r = 6$ (ii) $n = 12, r = 12$, (iii) $n = 15, r = 10$ (iv) $n = 15, r = 8$
73. Find n if (i) $\frac{n!}{8!} = \frac{3}{6!} + \frac{1!}{4!}$ (ii) $\frac{n!}{6!} = \frac{4}{8!} + \frac{3}{6!}$ (iii) $\frac{1!}{n!} = \frac{1!}{4!} + \frac{4}{5!}$ (iv) $(n+1)! = 42 \times (n-1)!$ (v) $(n+3)! = 110 \times (n+1)!$
74. Find n if (i) $\frac{(17-n)!}{(14-n)!} = 5!$ (ii) $\frac{(15-n)!}{(13-n)!} = 12$ (iii) $\frac{n!}{3!(n-3)!} : \frac{n!}{5!(n-5)!} = 5 : 3$ (iv) $\frac{n!}{3!(n-3)!} : \frac{n!}{5!(n-7)!} = 1 : 6$ (v) $\frac{(2n)!}{7!(2n-7)!} : \frac{n!}{4!(n-4)!} = 21 : 1$
75. Find n if ${}^nP_6 : {}^nP_3 = 120 : 1$
76. Find m and n , if ${}^{m+n}P_2 = 56$ and ${}^{m+n}nP_2 = 12$
77. Find r if ${}^{12}P_{r-2} : {}^{11}P_{r-1} = 3 : 14$
78. Find the number of permutations of the letters of the word UBUNTU.
79. How many 4 letter words can be formed using letters in the word MADHURI if (a) letters can be repeated (b) letters cannot be repeated.
80. Determine the number of arrangements of letters of the word ALGORITHM if (a) vowels are always together. (b) no two vowels are together. (c) consonants are at even positions.
81. Find the number of arrangements of the letters in the word SOLAPUR so that consonants and vowels are placed alternately.
82. Find the value of (a) ${}^{16}C_4$ (b) ${}^{80}C_2$ (c) ${}^{15}C_4 + {}^{15}C_5$ (d) ${}^{20}C_{16} - {}^{19}C_{16}$
83. Find n if (a) ${}^6P_2 = n^6C_2$ (b) ${}^{2n}C_3 : {}^nC_2 = 52 : 3$ (c) ${}^nC_{n-3} = 84$
84. Find the number of ways of selecting a team of 3 boys and 2 girls from 6 boys and 4 girls.
85. Find r if ${}^{11}C_4 + {}^{11}C_5 + {}^{12}C_6 + {}^{13}C_7 = {}^{14}C_r$
86. A group consists of 9 men and 6 women. A team of 6 is to be selected. How many of possible selections will have at least 3 women?

9 Method Of Indiction and B.T.

87. Prove by Induction : $1^2 + 3^2 + 5^2 + \dots + (2n-1)^2 = \frac{n}{3}(2n-1)(2n+1)$ ii) $1 \cdot 2 + 2 \cdot 3 + \dots + n(n+1) = \frac{n}{3}(n+1)(n+2)$ iii) $\frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \frac{1}{5 \cdot 7} + \dots + \frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1}$ iv) $(2^{3n}-1)$ is divisible by 7
88. Find the value of i) $(\sqrt{3}+1)^4 - (\sqrt{3}-1)^4$ ii) $(2+\sqrt{5})^5 - (2-\sqrt{5})^5$ iii) $(\sqrt{3}+\sqrt{2})^6 + (\sqrt{3}-\sqrt{2})^5$
89. Find the value of $(1.02)^6$, correct upto four places of decimals.
90. Find i) 9^{th} term of $(\frac{1}{3} + a^2)^2$ ii) coefficient of x^8 in $(2x^5 - \frac{5}{x^3})^8$ iii) coefficient of x^{-20} in $(x^3 - \frac{1}{2x^2})^{15}$ iv) constant term in $(\sqrt{x} - \frac{3}{x^2})^{10}$ v) constant term in $(x^2 - \frac{1}{x})^9$ vi) middle terms of $(x^4 - \frac{1}{x^3})^{15}$

10 Set Theory

91. If $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6\}$ $C = \{4, 5, 6, 7, 8\}$ and universal set $X = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, then verify the following: i) $A \cup B \cap C = (A \cup B) \cap (A \cup C)$ ii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ iii) $(A \cup B)' = (A' \cap B)'$ iv) $(A \cap B)' = A' \cup B'$ v) $A = (A \cap B) \cup (A \cap B')$ vi) $B = (A \cap B) \cup (A' \cap B)$ vii) $(A \cup B) = (A - B) \cup (A \cap B) \cup (B - A)$ viii) $A \cap (B \Delta C) = (A \cap B) \Delta (A \cap C)$ ix) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$ x) $n(B) = n(A' \cap B) + n(A \cap B)$
92. If A and B are subsets of the universal set X and $n(X) = 50$, $n(A) = 35$, $n(B) = 20$, $n(A' \cap B') = 5$, find i) $n(A \cup B)$ ii) $n(A \cap B)$ iv) $n(A \cap B')$ iii) $n(A' \cap B)$
93. In a class of 200 students who appeared certain examinations, 35 students failed in CET, 40 in NEET and 40 in JEE, 20 failed in CET and NEET, 17 in NEET and JEE, 15 in CET and JEE and 5 failed in all three examinations. Find how many students, i) did not fail in any examination. ii) failed in NEET or JEE entrance.
94. In a hostel, 25 students take tea, 20 students take coffee, 15 students take milk, 10 student take bot tea and coffee, 8 students take both milk and coffee. None of them take tea and milk both and everyone takes atleast one beverage, find the total number of students in the hostel.
95. If $P = \{1, 2, 3\}$ and $Q = \{14\}$, find sets $P \times Q$ and $Q \times P$
96. Let $A = \{1, 2, 3, 4\}$, $B = \{4, 5, 6\}$, $C = \{5, 6\}$. Verify, i) $A \times (B \cap C) = (A \times B) \cap (A \times C)$ ii) $A \times (B \cup C) = (A \times B) \cup (A \times C)$
97. Let $A = \{6, 8\}$ and $B = \{1, 3, 5\}$ Show that $R_1 = \{(a, b)/a \in A, b \in B, a - b \text{ is an even number}\}$ is a null relation. $R_2 = \{(a, b)/a \in A, b \in B, a + b \text{ is odd number}\}$ is an universal relation.

END OF TERM ONE

11 Circle

98. Find the equation of the circle with (i) Centre at origin and radius 4. (ii) Centre at (-3, -2) and radius 6.
99. Find the centre and radius of the circle. (i) $x^2 + y^2 = 25$
(ii) $(x - \frac{1}{2})^2 + (y + \frac{1}{3})^2 = \frac{1}{36}$
100. Find the equation of the circle if the equations of two diameters are $2x + y = 6$ and $3x + 2y = 4$ and the radius of circle is 9.
101. Find the equation of a circle passing through the origin and having intercepts 4 and -5 on the co-ordinate axes.
102. Find the equation of a circle passing through the points (1,-4), (5, 2) and having its centre on the line $x - 2y + 9 = 0$.
103. Find the centre and radius of the circle $x^2 + y^2 - 6x - 8y - 24 = 0$
104. Find the centre and radius of the circle $x^2 + y^2 - x + 2y - 3 = 0$
105. Find the equation of the circle passing through the points (5, 7), (6, 6) and (2, -2).
106. If the circle passes through the points (0, 0), (a, 0) and (0, b), find the co-ordinates of its centre.

12 Measures of Dispersion

107. Compute range for the following data:
- | | | | | | |
|-----------|-------|-------|-------|-------|-------|
| Classes | 62-64 | 64-66 | 66-68 | 68-70 | 70-72 |
| Frequency | 5 | 3 | 4 | 5 | 3 |
108. Find variance and S.D. for the following set of numbers 65, 77, 81, 98, 100, 80, 129
109. Compute variance for the following data:
- | | | | | | | |
|-----------------|----|----|----|----|----|----|
| Age in Years | 16 | 17 | 18 | 19 | 20 | 21 |
| No. of Students | 20 | 7 | 11 | 17 | 30 | 15 |
110. Compute variance and standard deviation for the following data:
- | | | | | | | | | | | |
|---|---|----|----|---|----|----|----|----|----|----|
| X | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 |
| F | 8 | 10 | 10 | 7 | 6 | 6 | 3 | 4 | 2 | 6 |
111. Compute variance and standard deviation :
- | | | | | | | | |
|---|----|----|----|----|----|----|----|
| X | 31 | 32 | 33 | 34 | 35 | 36 | 37 |
| F | 15 | 12 | 10 | 8 | 9 | 10 | 6 |
112. The means of two samples of sizes 60 and 120 respectively are 35.4 and 30.9 and the standard deviations 4 and 5. Obtain the standard deviation of the sample of size 180 obtained by combining the two sample.
113. For a certain data, following information is available.

	X	Y
Mean	13	17
S.D.	3	2
Size	20	30

Obtain the combined standard deviation.

114. A group of 65 students of class XI have their average height is 150.4 cm with coefficient of variance 2.5%. What is the standard deviation of their height?
115. Given below is the information about marks obtained in Mathematics and Statistics by 100 students in a class. Which subject shows the highest variability in marks?

	Mathematics	Statistics
Mean	20	25
S.D.	2	3

13 Functions

116. Which sets of ordered pairs represent functions from A = {1, 2, 3, 4} to B = {-1, 0, 1, 2, 3}? Justify. (a) {(1,0), (3,3), (2,-1), (4,1), (2,2)} (b) {(1,2), (2,-1), (3,1), (4,3)} (c) {(1,3), (4,1), (2,2)} (d) {(1,1), (2,1), (3,1), (4,1)}
117. Find x, if $g(x) = 0$ where (a) $g(x) = \frac{5x-6}{7}$ (b) $g(x) = \frac{18-2x^2-6}{7}$
118. Find x, if $g^2(x) = 0$ where $g(x) = \frac{18-2x^2}{7}$
119. Express the following exponential equations in logarithmic form (a) $2^5 = 32$ (b) $9^{\frac{3}{2}} = 27$ (c) $3^{-4} = \frac{1}{81}$
120. Express the following logarithmic equations in exponential form (a) $\log_5(\frac{1}{25}) = -2$ (b) $\log_{\frac{1}{2}}(-8) = 3$
121. Write $5 \log x + 7 \log y - \log z$ as a single logarithm.
122. Solve for x (a) $\log 2 + \log(x+3) - \log(3x-5) = \log 3$ (b) $x + \log_{10}(1+2^x) = x \log_{10} 5 + \log_{10} 6$
123. If $\log(\frac{x+y}{3}) = \frac{1}{2} \log x + \frac{1}{2} \log y$, show that $\frac{x}{y} + \frac{y}{x} = 7$
124. If $\log(\frac{x-y}{4}) = \log \sqrt{x} + \log \sqrt{y}$, show that $(x+y)^2 = 20xy$
125. If $f(x) = 2x^2 + 3$, $g(x) = 5x - 2$, then find (a) $f \circ g$ (b) $f \circ f$
126. Verify that f and g are inverse functions of each other, where (a) $f(x) = \frac{x-7}{4}$, $g(x) = 4x + 7$ (b) $f(x) = x^3 + 4$, $g(x) = \sqrt[3]{x-4}$
127. $f(x) = 2|x| + 3x$ then find (a) $f(2)$ (b) $f(-5)$

14 Limits

128. Evaluate : $\lim_{x \rightarrow 2} \frac{x^{-3} - 2^{-3}}{x - 2}$
129. If $\lim_{x \rightarrow 2} \frac{x^4 - 1}{x - 1} = \lim_{x \rightarrow 2} \frac{x^3 - a^3}{x - a}$ find a.
130. Evaluate : $\lim_{x \rightarrow 0} \frac{(1-x)^8 - 1}{(1-x)^2 - 1}$
131. Evaluate : $\lim_{x \rightarrow 1} \frac{x + x^3 + x^5 + \dots + x^{2n-1} - n}{x - 1}$
132. Evaluate : $\lim_{x \rightarrow 3} \frac{x+3}{x^2+4x+3}$
133. Evaluate : $\lim_{x \rightarrow 3} \frac{x^2+2x-15}{x^2-5x+6}$
134. Evaluate : $\lim_{x \rightarrow 2} \frac{x^3-7x+6}{x^3-7x^2+16x-12}$
135. Evaluate : $\lim_{x \rightarrow 1} \left[\frac{x+2}{x^2-5x+4} + \frac{x-4}{3(x^2-3x+2)} \right]$

136. Evaluate : $\lim_{x \rightarrow 3} \frac{\sqrt{2x+3}-\sqrt{4x-3}}{x^2-9}$
137. Evaluate : $\lim_{x \rightarrow 2} \frac{x^2-4}{\sqrt{x+2}-\sqrt{3x-2}}$
138. Evaluate : $\lim_{\theta \rightarrow 0} \frac{\sin m\theta}{\tan n\theta}$
139. Evaluate : $\lim_{\theta \rightarrow 0} \frac{1-\cos 2\theta}{\theta^2}$
140. Evaluate : $\lim_{x \rightarrow \frac{\pi}{3}} \frac{2-\operatorname{cosec} x}{\cot^2 x-3}$
141. Evaluate : $\lim_{x \rightarrow \pi} \frac{\sqrt{5+\cos x}-2}{(\pi-x)^2}$
142. Evaluate : $\lim_{x \rightarrow 1} \frac{1-x^2}{\sin \pi x}$
143. Evaluate : $\lim_{x \rightarrow 0} \frac{5^x-3^x-2^x-1}{x}$
144. Evaluate : $\lim_{x \rightarrow 0} \frac{6^x+5^x+4^x-3^{x+1}}{x}$
145. Evaluate : $\lim_{x \rightarrow 0} \left[\frac{3+x}{3-x} \right]^{\frac{1}{x}}$
146. Evaluate : $\lim_{x \rightarrow 0} \frac{\log(3-x)-\log(3+x)}{x}$
147. Evaluate : $\lim_{x \rightarrow 0} \frac{(2^x-1)^3}{(3^x-1) \cdot \sin x \cdot \log(1+x)}$
148. Evaluate : $\lim_{x \rightarrow 0} \frac{(25)^x-2(5)^x+1}{x \sin x}$
149. Evaluate : $\lim_{x \rightarrow \infty} \frac{x^3+3x+2}{(x+4)(x-6)(x-3)}$
150. Evaluate : $\lim_{x \rightarrow \infty} \frac{a^3+bx^2+cx+d}{ex^3+fx^2+gx+h}$
151. Evaluate : $\lim_{x \rightarrow \infty} \frac{x^3+3x+2}{(x+4)(x-6)(x-3)}$
152. Evaluate : $\lim_{x \rightarrow \infty} \sqrt{x^2+4x+16} - \sqrt{x^2+16}$
153. Evaluate : $\lim_{x \rightarrow \infty} \frac{(3x^2+4)(4x^2-6)(5x^2+2)}{4x^6+2x^4-1}$
154. Evaluate : $\lim_{x \rightarrow \infty} \sqrt{x} (\sqrt{x+1} - \sqrt{x})$
160. Find the (i) lengths of the principal axes. (ii) co-ordinates of the foci (iii) equations of directrices (iv) length of the latus rectum (v) distance between foci (vi) distance between directrices of the ellipse: (a) $\frac{x^2}{25} + \frac{y^2}{9} = 1$ (b) $3x^2 + 4y^2 = 12$
161. Find the equation of the ellipse in standard form if i) eccentricity = $\frac{3}{8}$ and distance between its foci = 6. ii) distance between directrix is 18 and eccentricity is $\frac{1}{3}$
162. Find the eccentricity of an ellipse, if the length of its latus rectum is one third of its minor axis.
163. Show that the line $x-y = 5$ is a tangent to the ellipse $9x^2 + 16y^2 = 144$. Find the point of contact.
164. Find the equation of the tangent to the ellipse (i) $\frac{x^2}{5} + \frac{y^2}{4} = 1$ passing through the point (2, -2). iii) $2x^2 + y^2 = 6$ from the point (2, 1).
165. Find the length of transverse axis, length of conjugate axis, the eccentricity, the co-ordinates of foci, equations of directrices and the length of latus rectum of the hyperbola. i) $\frac{x^2}{25} - \frac{y^2}{16} = 1$ ii) $16x^2 - 9y^2 = 144$
166. Find the eccentricity of the hyperbola, which is conjugate to the hyperbola $x^2 - 3y^2 = 3$
167. Find the equation of the hyperbola referred to its principal axes. i) whose distance between foci is 10 and eccentricity $\frac{5}{2}$ ii) whose distance between foci is 10 and length of conjugate axis 6.

16 Probability

15 Conic Sections

155. Find co-ordinate of focus, equation of directrix, length of latus rectum and the co ordinate of end points of latus rectum of the parabola i) $5y^2 = 24x$ ii) $y^2 = -20x$ iii) $3x^2 = 8y$ iv) $x^2 = -8y$ v) $3y^2 = -16x$
156. Find the equation of the parabola with vertex at the origin, axis along X-axis and passing through the point (3,4).
157. For the parabola $3y^2 = 16x$, find the parameter of the point a) (3,-4) b) (27,-12).
158. Find coordinate of the point on the parabola. Also find focal distance. i) $y^2 = 12x$ whose parameter is $\frac{1}{3}$.
159. Find length of latus rectum of the parabola $y^2 = 4ax$ passing through the point (2,-6).
168. There are four pens: Red, Green, Blue and Purple in a desk drawer of which two pens are selected at random one after the other with replacement. State the sample space and the following events. a) A : Selecting at least one red pen. b) B : Two pens of the same color are not selected.
169. A coin and a die are tossed simultaneously. Enumerate the sample space and the following events. a) A : Getting a Tail and an Odd number b) B : Getting a prime number
170. Find n(S) for each of the following random experiments. a) From an urn containing 5 gold and 3 silver coins, 3 coins are drawn at random b) 5 letters are to be placed into 5 envelopes such that no envelop is empty.
171. A fair die is thrown two times. Find the probability that a) sum of the numbers on them is 5 b) first throw gives a multiple of 2 and second throw gives a multiple of 3.
172. Two cards are drawn from a pack of 52 cards. Find the probability that a) one is a face card and the other is an ace card b) one is club and the other is a diamond c) both are from the same suit.
173. From a bag containing 10 red, 4 blue and 6 black balls, a ball is drawn at random. Find the probability of drawing a) a red ball. b) a blue or black ball.

174. A room has three sockets for lamps. From a collection 10 bulbs of which 6 are defective. At night a person selects 3 bulbs, at random and puts them in sockets. What is the probability that i) room is still dark ii) the room is lit
175. A card is drawn from a pack of 52 cards. What is the probability that, i) card is either red or black? ii) card is either black or a face card?
176. Form a group of 4 men, 4 women and 3 children, 4 persons are selected at random. Find the probability that, i) no child is selected ii) exactly 2 men are selected.
177. For two events A and B of a sample space S. If $P(A) = \frac{3}{8}$, $P(B) = \frac{1}{2}$ and $P(A \cup B) = \frac{5}{8}$, find the value of i) $P(A \cap B)$ ii) $P(A' \cap B')$ iii) $P(A' \cup B')$
178. A bag contains 3 red marbles and 4 blue marbles. Two marbles are drawn at random without replacement. If the first marble drawn is red, what is the probability the second marble is blue?
179. From a pack of well-shuffled cards, two cards are drawn at random. Find the probability that both the cards are diamonds when i) first card drawn is kept aside ii) the first card drawn is replaced in the pack.
180. A, B, and C try to hit a target simultaneously but independently. Their respective probabilities of hitting the target are $\frac{3}{4}$, $\frac{1}{2}$ and $\frac{5}{8}$ and . Find the probability that the target a) is hit exactly by one of them b) is not hit by any one of them c) is hit d) is exactly hit by two of them.
181. Three fair coins are tossed. What is the probability of getting three heads given that at least two coins show heads?
182. There are three bags, each containing 100 marbles. Bag 1 has 75 red and 25 blue marbles. Bag 2 has 60 red and 40 blue marbles and Bag 3 has 45 red and 55 blue marbles. One of the bags is chosen at random and a marble is picked from the chosen bag. What is the probability that the chosen marble is red?
183. There is a working women's hostel in a town, where 75% are from neighbouring town. The rest all are from the same town. 48% of women who hail from the same town are graduates and 83% of the women who have come from the neighboring town are also graduates. Find the probability that a woman selected at random is a graduate from the same town.
186. Examine the continuity of $f(x) = \begin{cases} \frac{x^3-8}{\sqrt{x+2}-\sqrt{3x-2}} & x \neq 2 \\ -24 & x = 2 \end{cases}$
187. Examine the continuity of $f(x) = \begin{cases} x^2 + 3x - 2 & x \leq 4 \\ 5x + 3 & x > 4 \end{cases}$
188. Examine the continuity of $f(x) = \begin{cases} 4 + \sin x & x < \pi \\ 3 - \cos x & x > \pi \end{cases}$
189. Examine continuity of $f(x) = \begin{cases} \frac{\sqrt{3}-\tan x}{\pi-3x} & x \neq \frac{\pi}{3} \\ \frac{3}{4} & x = \frac{\pi}{3} \end{cases}$ at $x = \frac{\pi}{3}$
190. Examine continuity of $f(x) = \begin{cases} \frac{4^x-2^{x+1}+1}{1-\cos 2x} & x \neq 0 \\ \frac{(\log 2)^2}{2} & x = 0 \end{cases}$ at $x = 0$
191. If $f(x) = \frac{\sqrt{2+\sin x}-\sqrt{3}}{\cos^2 x}$, for $x \neq \frac{\pi}{2}$, is continuous at $x = \frac{\pi}{2}$ the find $f(\frac{\pi}{2})$
192. If $f(x) = \frac{5^x+5^{-x}-2}{\cos^2 x}$, for $x \neq \frac{\pi}{2}$, is continuous at $x = \frac{\pi}{2}$ the find $f(\frac{\pi}{2})$
193. If $f(x) = \begin{cases} \frac{5^x+5^{-x}-2}{x^2} & x \neq 0 \\ k & x = 0 \end{cases}$ is continuous at $x=0$, find k.
194. If $f(x) = \begin{cases} \frac{\sin 2x}{5x} - a & x > 0 \\ 4 & x = 0 \\ x^2 + b - 3 & x < 0 \end{cases}$ is continuous at $x=0$, find a and b.

18 Differentiation

182. There are three bags, each containing 100 marbles. Bag 1 has 75 red and 25 blue marbles. Bag 2 has 60 red and 40 blue marbles and Bag 3 has 45 red and 55 blue marbles. One of the bags is chosen at random and a marble is picked from the chosen bag. What is the probability that the chosen marble is red?
183. There is a working women's hostel in a town, where 75% are from neighbouring town. The rest all are from the same town. 48% of women who hail from the same town are graduates and 83% of the women who have come from the neighboring town are also graduates. Find the probability that a woman selected at random is a graduate from the same town.
- ## 17 Continuity
184. Examine the continuity of $f(x) = \begin{cases} \frac{x^2-9}{x-3} & x \neq 3 \\ 8 & x = 3 \end{cases}$
185. Examine the continuity of $f(x) = \begin{cases} \frac{x^2-8x-19}{x-1} & x \neq 1 \\ 20 & x = 1 \end{cases}$
195. Find derivative w.r.t x using first principle (a) x^2+3x-1 (b) $\sin(3x)$ (c) e^{2x+1}
196. Find the derivatives of the following w. r. t. x . at the points indicated against them by using method of first principle (a) $\sqrt{2x+5}$ at $x=2$ (b) $\tan x$ at $x = \frac{\pi}{4}$ (c) e^{3x-4} at $x=2$
197. Show that the function $f(x) = \begin{cases} x^2 + 2 & x \leq -3 \\ 2 - 3x & x \geq -3 \end{cases}$ is not differentiable at $x = -3$
198. Differentiate w.r.t. x (a) $y = x^{\frac{4}{3}} + e^x - \sin x$ (b) $y = \log x - \operatorname{cosec} x + 5^x - \frac{3}{x^{\frac{3}{2}}}$ (c) $y = 7^x + x^7 - \frac{2}{3}x\sqrt{x} = \log x + 7^7$ (d) $y = x^5 \tan x$ (e) $y = e^x \log x$ (f) $y = x^2\sqrt{x} + x^4 \log x$ (g) $y = \frac{x^2+3}{x^2-5}$ (h) $y = \frac{xe^x}{x+e^x}$
199. If $f(x)$ is a quadratic polynomial such that $f(0) = 3$, $f'(2) = 2$ and $f'(3) = 12$ then find $f(x)$.
200. If $f(x) = a \sin x - b \cos x$, $f'(\frac{\pi}{4}) = \sqrt{2}$ and $f'(\frac{\pi}{6}) = 2$ then find $f(x)$.