1 Question

If $y = \cot^{-1}\left(\dfrac{3+4\tan x}{4-3\tan x}\right)$, find $\dfrac{dy}{dx}$

Answer

$$
\begin{aligned}
y &= \cot^{-1}\left(\frac{3+4\tan x}{4-3\tan x}\right) \\
&= \tan^{-1}\left(\frac{4-3\tan x}{3+4\tan x}\right) \\
&= \tan^{-1}\left(\frac{\frac{4}{3}-\tan x}{1+\frac{4}{3}\tan x}\right)
\end{aligned}
\quad\Bigg|\quad
\begin{aligned}
&= \tan^{-1}\tfrac{4}{3} + \tan^{-1}(\tan x) \\
&= \frac{4}{3} + x \\
\therefore\ \frac{dy}{dx} &= 1
\end{aligned}
$$

2 Question

If $y = \tan^{-1}\left(\dfrac{1+x\sin x}{x-\sin x}\right)$, find $\dfrac{dy}{dx}$

Answer

$$
\begin{aligned}
y &= \tan^{-1}\left(\frac{1+x\sin x}{x-\sin x}\right) \\
&= \tan^{-1}\left(\frac{\frac{1}{x}+\sin x}{1-\frac{1}{x}\sin x}\right) \\
&= \tan^{-1}\tfrac{1}{x} + \tan^{-1}(\sin x)
\end{aligned}
\quad\Bigg|\quad
\begin{aligned}
&= \cot^{-1} x + \tan^{-1}(\sin x) \\
\therefore\ \frac{dy}{dx} &= \frac{-1}{1+x^2} + \frac{1}{1+\sin^2 x}\ \frac{d}{dx}(\sin x) \\
&= \frac{-1}{1+x^2} + \frac{\cos x}{1+\sin^2 x}
\end{aligned}
$$