Derivatives

Implicit and Logarithmic

Milind Oka

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Question : If $x + \sqrt{xy} + y = 1$, find $\dfrac{dy}{dx}$

Answer:

Given $x + \sqrt{xy} + y = 1$

Differentiating w.r.t. x

$$1 + \frac{1}{2\sqrt{xy}}\left(x\frac{dy}{dx} + y\right) + \frac{dy}{dx} = 0$$

$$\therefore 1 + \frac{x}{2\sqrt{xy}}\frac{dy}{dx} + \frac{y}{2\sqrt{xy}} + \frac{dy}{dx} = 0$$

$$\therefore \left(\frac{x}{2\sqrt{xy}} + 1\right)\frac{dy}{dx} = -1 - \frac{y}{2\sqrt{xy}}$$

$$\therefore \left(\frac{x + 2\sqrt{xy}}{2\sqrt{xy}}\right)\frac{dy}{dx} = -\frac{2\sqrt{xy} + y}{2\sqrt{xy}}$$

$$\therefore (x + 2\sqrt{xy}\,)\frac{dy}{dx} = -2\sqrt{xy} - y$$

$$\therefore \frac{dy}{dx} = -\frac{x + 2\sqrt{xy}}{2\sqrt{xy} + y}$$

Question : If $x^p y^q = (x+y)^{p+q}$, the show that $\dfrac{dy}{dx} = \dfrac{y}{x}$

Answer:

$$\text{Given} \quad x^p y^q = (x+y)^{p+q}$$
$$\therefore \log(x^p y^q) = \log(x+y)^{p+q}$$
$$\therefore p\log x + q\log y) = (p+q)\log(x+y)$$
Differentiating w.r.t. x
$$\therefore \frac{p}{x} + \frac{q}{y}\frac{dy}{dx} = \frac{p+q}{x+y}\left(1 + \frac{dy}{dx}\right)$$
$$\therefore \frac{p}{x} + \frac{q}{y}\frac{dy}{dx} = \frac{p+q}{x+y} + \frac{p+q}{x+y}\frac{dy}{dx}$$
$$\therefore \left(\frac{q}{y} - \frac{p+q}{x+y}\right)\frac{dy}{dx} = \frac{p+q}{x+y} - \frac{p}{x}$$

Question : If $x^p y^q = (x+y)^{p+q}$, the show that $\dfrac{dy}{dx} = \dfrac{y}{x}$

Answer: Contd

$$\therefore \frac{q(x+y)-(p+q)y}{y(x+y)}\frac{dy}{dx} = \frac{(p+q)x-p(x+y)}{(x+y)x}$$

$$\therefore \frac{qx+qy-py-qy}{y}\frac{dy}{dx} = \frac{px+qx-px-py}{x}$$

$$\therefore \frac{\cancel{qx-py}}{y}\frac{dy}{dx} = \frac{\cancel{qx-py}}{x}$$

$$\therefore \frac{dy}{dx} = \frac{y}{x}$$