1 Question

If
$$y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$$
, find $\frac{dy}{dx}$

Answer

$$y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) \underbrace{\text{Put } x = \tan \theta}_{} = 2\theta$$

$$= 2\tan^{-1} x$$

$$= \cos^{-1}\left(\frac{1-\tan^2 \theta}{1+\tan^2 \theta}\right)$$

$$= \cos^{-1}(\cos 2\theta)$$

$$= 2\tan^{-1} x$$

$$\therefore \frac{dy}{dx} = \frac{2}{1+x^2}$$

2 Question

If
$$y = \tan^{-1}\left(\frac{1+x\sin x}{x-\sin x}\right)$$
, find $\frac{dy}{dx}$

Answer

$$y = \tan^{-1}\left(\frac{1+x\sin x}{x-\sin x}\right) = \tan^{-1}\left(\frac{\frac{1}{x}+\sin x}{1-\frac{1}{x}\sin x}\right) = \tan^{-1}\left(\frac{\frac{1}{x}+\sin x}{1-\frac{1}{x}\sin x}\right) = \tan^{-1}\frac{1}{x}+\tan^{-1}(\sin x)$$