Kendall Tau Distance

In this Jupyter notebook, we will calculate the Kendall τ distance for a slightly complex scenario described below.

Scenario

Suppose there are two individuals, A and B. Each person needs to rank their top 5 favorite fruits from a list of 11 fruits. How can we calculate the Kendall τ distance in this case?

Procedure: To calculate Kendall's Tau for two persons, A and B, who have ranked their 5 most favorite fruits out of a list of 11 fruits, follow these steps:

1. **Construct the rankings:** Create two rankings for the 5 fruits each person has ranked. Since not all 11 fruits are ranked by each person, you will have partial rankings. You need to handle the unranked fruits by assigning them a lower rank or ignoring them in the comparison.
2. **Extend the rankings:** For simplicity, extend the rankings to include all 11 fruits. Assign a rank of 6 (or any consistent low rank) to the unranked fruits.
3. **Create pairs:** Form pairs of the fruits for comparison. Since there are 11 fruits, you'll form $\binom{11}{2} = 55$ pairs.
4. **Count concordant and discordant pairs:**
   - A pair (i, j) is concordant if the ranking order of the fruits is the same for both persons. That is, if fruit i is ranked higher than fruit j by both A and B or lower by both A and B.
   - A pair (i, j) is discordant if the ranking order of the fruits is different for the two persons. That is, if fruit i is ranked higher than fruit j by one person but lower by the other.
5. **Calculate Kendall's Tau:**

$$\tau = \frac{C - D}{\binom{11}{2}}$$

   where C is the number of concordant pairs, D is the number of discordant pairs, and n is the number of pairs (which is 55 in this case).

Example

Suppose the list of 11 fruits is: [Apple, Banana, Cherry, Date, Elderberry, Fig, Grape, Honeydew, Kiwi, Lemon, Mango].

Person A ranks: Apple (1), Banana (2), Cherry (3), Date (4), Elderberry (5)
Person B ranks: Apple (1), Banana (2), Grape (3), Kiwi (4), Lemon (5)

Step 1: Construct the rankings:

Fruit	A	B
Apple	1	1
Banana	2	2
Cherry	3	6
Date	4	6
Elderberry	5	6
Fig	6	6
Grape	6	3
Honeydew	6	6
Kiwi	6	4
Lemon	6	5
Mango	6	6

Step 2: Form pairs and count concordant and discordant pairs:

Pair	A's Rank (i, j)	B's Rank (i, j)	Concordant/Discordant
(Apple, Banana)	(1, 2)	(1, 2)	Concordant
(Apple, Cherry)	(1, 3)	(1, 6)	Concordant
(Apple, Date)	(1, 4)	(1, 6)	Concordant
(Apple, Elderberry)	(1, 5)	(1, 6)	Concordant
(Apple, Fig)	(1, 6)	(1, 6)	Concordant
(Apple, Grape)	(1, 6)	(1, 3)	Concordant
(Apple, Honeydew)	(1, 6)	(1, 6)	Concordant
(Apple, Kiwi)	(1, 6)	(1, 4)	Concordant
(Apple, Lemon)	(1, 6)	(1, 5)	Concordant
(Apple, Mango)	(1, 6)	(1, 6)	Concordant
(Banana, Cherry)	(2, 3)	(2, 6)	Concordant
(Banana, Date)	(2, 4)	(2, 6)	Concordant
(Banana, Elderberry)	(2, 5)	(2, 6)	Concordant
(Banana, Fig)	(2, 6)	(2, 6)	Concordant
(Banana, Grape)	(2, 6)	(2, 3)	Concordant
(Banana, Honeydew)	(2, 6)	(2, 6)	Concordant
(Banana, Kiwi)	(2, 6)	(2, 4)	Concordant
(Banana, Lemon)	(2, 6)	(2, 5)	Concordant
(Banana, Mango)	(2, 6)	(2, 6)	Concordant
(Cherry, Date)	(3, 4)	(6, 6)	Concordant
(Cherry, Elderberry)	(3, 5)	(6, 6)	Concordant
(Cherry, Fig)	(3, 6)	(6, 6)	Concordant

Pair	A's Rank (i, j)	B's Rank (i, j)	Concordant/Discordant
(Cherry, Grape)	(3, 6)	(6, 3)	Discordant
(Cherry, Honeydew)	(3, 6)	(6, 6)	Concordant
(Cherry, Kiwi)	(3, 6)	(6, 4)	Discordant
(Cherry, Lemon)	(3, 6)	(6, 5)	Discordant
(Cherry, Mango)	(3, 6)	(6, 6)	Concordant
(Date, Elderberry)	(4, 5)	(6, 6)	Concordant
(Date, Fig)	(4, 6)	(6, 6)	Concordant
(Date, Grape)	(4, 6)	(6, 3)	Discordant
(Date, Honeydew)	(4, 6)	(6, 6)	Concordant
(Date, Kiwi)	(4, 6)	(6, 4)	Discordant
(Date, Lemon)	(4, 6)	(6, 5)	Discordant
(Date, Mango)	(4, 6)	(6, 6)	Concordant
(Elderberry, Fig)	(5, 6)	(6, 6)	Concordant
(Elderberry, Grape)	(5, 6)	(6, 3)	Discordant
(Elderberry, Honeydew)	(5, 6)	(6, 6)	Concordant
(Elderberry, Kiwi)	(5, 6)	(6, 4)	Discordant
(Elderberry, Lemon)	(5, 6)	(6, 5)	Discordant
(Elderberry, Mango)	(5, 6)	(6, 6)	Concordant
(Fig, Grape)	(6, 6)	(6, 3)	Concordant
(Fig, Honeydew)	(6, 6)	(6, 6)	Concordant
(Fig, Kiwi)	(6, 6)	(6, 4)	Concordant
(Fig, Lemon)	(6, 6)	(6, 5)	Concordant
(Fig, Mango)	(6, 6)	(6, 6)	Concordant
(Grape, Honeydew)	(6, 6)	(3, 6)	Discordant
(Grape, Kiwi)	(6, 6)	(3, 4)	Discordant
(Grape, Lemon)	(6, 6)	(3, 5)	Discordant
(Grape, Mango)	(6, 6)	(3, 6)	Discordant
(Honeydew, Kiwi)	(6, 6)	(6, 4)	Concordant
(Honeydew, Lemon)	(6, 6)	(6, 5)	Concordant
(Honeydew, Mango)	(6, 6)	(6, 6)	Concordant
(Kiwi, Lemon)	(6, 6)	(4, 5)	Concordant
(Kiwi, Mango)	(6, 6)	(4, 6)	Concordant
(Lemon, Mango)	(6, 6)	(5, 6)	Concordant

Step 3: Calculate Kendall's Tau:
Count of concordant pairs $C = 43$
Count of discordant pairs $D = 12$

Total pairs $n = 55$

$$\tau = \frac{C - D}{\binom{11}{2}} = \frac{42 - 13}{55} \approx 0.53$$

So, Kendall's Tau for the two persons' rankings is approximately 0.53.