MIMO Detection: A report of ELE-851 Project

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***Abstract*—In the past few decades, as one of the key technologies to improve spectrum efficiency, multiple input multiple output (MIMO) systems have been extensively studied. Compared with traditional signal detection using maximum likelihood detector, linear detectors such as MMSE detector and ZF detector are most commonly used for MIMO detection due to the relatively lower complexity and near-optimal performances. However, with the increasing number of antennas in massive MIMO systems, the computational complexity of traditional linear detectors increases rapidly. Therefore, different approximate methods are proposed in the literature. In this project, we first studied MMSE and ZF detectors in a 2×2 small-scale MIMO system. Then we apply the MMSE detector to massive MIMO systems and use several different approximation methods to reduce the computational complexity.**

Keywords—MIMO, signal detection, minimum mean square error(MMSE), zero-forcing(ZF), matrix approximation

I. Introduction

Multi-Input-Multi-Output (MIMO) technology, which can significantly improve the capacity and reliability of wireless systems, has been widely studied and applied to many wireless standards. As shown in Fig.1, the idea of MIMO is to use multiple antennas in the transmitter and the receiver to increase the spectral efficiency, range, and reliability[5]. However, since multiple interference messages are sent from different antennas, the MIMO receiver is expected to use a detection mechanism to separate the symbols which are corrupted by interference and noise. Comparing with the traditional Single-Input-Single-Output (SISO) systems, MIMO systems are supposed to detect multiple signals jointly.

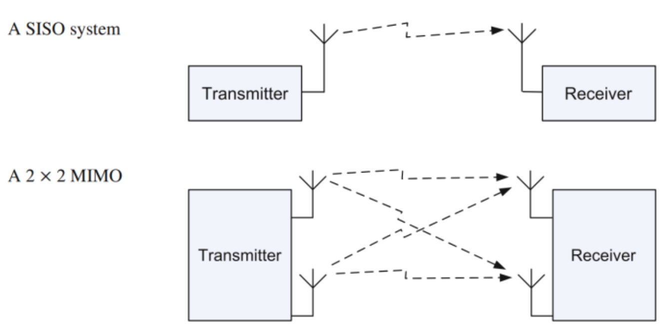

Fig. 1 SISO system and MMO system

Due to the curse of dimension, the traditional Maximum Likelihood (ML) criterion or Maximum A posteriori Probability (MAP) criterion suffers a higher computational complexity when the number of antennas increases. In order to solve the problem, many different detection methods are proposed, which can be classified into different families: Linear Detectors, Tree Search Based Detectors, Lattice Reduction Aided Detectors, etc. [2]. The most conventional and commonly used detectors are the linear detectors: Zero-Forcing (ZF) detector and Minimum Mean Square Error (MMSE) detector.

With the development of 5G communication technology, massive MIMO or large-scale antenna systems have been proposed. Compared with small-scale MIMO, the number of large-scale MIMO antennas is several orders of magnitude larger. As the number of antennas in the MIMO system grows to infinity, the effects of irrelevant noise and small-scale fading are eliminated, and the required transmission energy per bit also disappears[1].

On the other hand, with the increase in the number of antennas, the computational complexity is also increased rapidly since the conventional linear detector requires matrix inversion. Hence, some matrix approximation methods are used in the detection of massive MIMO.

In this report, we studied linear detection from small-scale MIMO to large-scale MIMO. In section II, ZF and MMSE detection models are provided. Section III illustrates the approximate techniques for massive MIMO. Simulation results are given in section IV. Finally, section VII concludes the report.

II. Small-Scale MIMO Detection

A. System Model

Suppose a MIMO detection system has N base station antennas and K single-antenna users ($K \leq N$). Assuming frequency-flat channel, $\boldsymbol{H}$ denotes the channel coefficients between K users and N receivers. (hence $\boldsymbol{H}$ is an $N \times K$ matrix) $\boldsymbol{X}$ denotes the transmitted vector, and $\boldsymbol{Y}$ represents the received vector. The linear relationship between $\boldsymbol{Y}$ and $\boldsymbol{X}$ can be characterized as

$$\boldsymbol{Y} = \boldsymbol{H}\boldsymbol{X} + \boldsymbol{n} \tag{1}$$

where $\boldsymbol{n}$ is additive white Gaussian noise. The task of MIMO detection is to estimate the vector $\boldsymbol{X}$ relying on the received $\boldsymbol{Y}$ and the channel matrix $\boldsymbol{H}$. Although we cannot predict the noise vector $\boldsymbol{n}$, we have the knowledge of all the possible combinations of $\boldsymbol{X}$. The ML detection can be carried out by exhaustively searching for all the candidate vectors and selecting the maximum likely one with the smallest error probability[4][5].

$$\widehat{\boldsymbol{X}}_{ML} = \underset{\boldsymbol{X} \epsilon O^K}{\arg\min} \|\boldsymbol{Y} - \boldsymbol{H}\boldsymbol{X}\|_2^2 \tag{2}$$

The ML criterion is related to optimal performance. However, for K transmitting antennas, the complexity of ML is exponential in the number of decision variables O^K. Therefore, even for a small-scale MIMO system, the complexity of the ML detector can be prohibitive.

A conventional method that can reduce the complexity while achieving near-optimal performance is using linear detectors.

B. ZF Detector

$$\boldsymbol{W}_{ZF} = (\boldsymbol{H}^H\boldsymbol{H})^{-1}\boldsymbol{H}^H \tag{3}$$

$$\widehat{\boldsymbol{X}}_{ZF} = \boldsymbol{W}_{ZF}\boldsymbol{Y} \tag{4}$$

$(\cdot)^H$ denotes the Hermitian transpose. $\widehat{\boldsymbol{X}}_{ZF}$ denotes the estimation of $\boldsymbol{X}$. $\boldsymbol{W}_{ZF}$ denotes weight matrix. The ZF detector aims to maximize the signal-to-interference ratio (SINR). $\boldsymbol{W}_{ZF}$ is actually the pseudo-inverse of $\boldsymbol{H}$. Thus, the effect of the channel interference can be removed by multiply the inverse matrix. However, the detector ignores the effect of noise, and it may produce a noise enhancement in the case of a small-valued coefficient channel.

C. MMSE Detector

$$W_{MMSE} = (\boldsymbol{H}^H\boldsymbol{H}+\sigma_N^2\boldsymbol{I})^{-1}\boldsymbol{H}^H \tag{5}$$

$$\widehat{\boldsymbol{X}}_{MMSE} = \boldsymbol{W}_{MMSE}\boldsymbol{Y} \tag{6}$$

σ_N^2 denotes the noise variance. $\boldsymbol{I}$ denotes the identity matrix. The main idea of the MMSE detector is to minimize the mean-square error (MSE) between the transmitted $\boldsymbol{X}$ and matched filtering signal $\boldsymbol{H}^{\mathrm{H}}\boldsymbol{Y}$. Unlike the ZF detector, MMSE takes the noise into consideration. It performs better than ZF when the noise power is large.

III. MASSIVE MIMO DETECTION

In 2013, a massive MIMO detector based on the approximate inversion method was proposed in [6]. After that, detectors based on approximate inversion became the most popular category of detectors. This type of massive MIMO detector is designed for a specific massive MIMO configuration, that is, when the number of antennas is greater than the number of users. For such a massive MIMO configuration, the effect of channel hardening is higher. Therefore, a low-complexity detector can achieve high performance.

As shown in (5) and (6), the estimated signal $\widehat{\boldsymbol{X}}$ can be written as the inner product of an inverse matrix $\boldsymbol{A}^{-1}$ and a vector $\boldsymbol{b}$, where $\boldsymbol{A}$ is the Gram matrix $\boldsymbol{H}^H\boldsymbol{H}$ plus a noise factor, and b is the matched filtering signal $\boldsymbol{H}^{\mathrm{H}}\boldsymbol{Y}$.

$$\widehat{\boldsymbol{X}}_{MMSE} = \boldsymbol{A}^{-1}\boldsymbol{b} \tag{7}$$

$$\boldsymbol{A} = \boldsymbol{H}^H\boldsymbol{H} + \sigma_N^2\boldsymbol{I} \tag{8}$$

$$\boldsymbol{b} = \boldsymbol{H}^H\boldsymbol{Y} \tag{9}$$

However, in massive MIMO systems, directly inverting matrix $\boldsymbol{A}$ exhibits a high computational complexity since the size of the matrix is related to the number of antennas. Thus, several methods have been proposed to reduce complexity by approximating the inverse of a matrix rather than computing it[3].

A. Newton Iteration (NI)

Newton Iteration is an iterative method for finding the approximation of the inverse matrix. For a matrix $\boldsymbol{A}$, estimation of the inverse at n^{th} iteration is given by

$$\boldsymbol{X}_n^{-1} = \boldsymbol{X}_{n-1}^{-1}(2\boldsymbol{I} - \boldsymbol{A}\boldsymbol{X}_{n-1}^{-1}), \tag{10}$$

which converges quadratically if

$$\|\boldsymbol{I} - \boldsymbol{A}\boldsymbol{X}_0^{-1}\| < 1. \tag{11}$$

Usually due to channel hardening phenomenon, the matrix **A** is a diagonal dominant matrix. We can choose the diagonal element matrix **D** of **A** as the initial estimate $\mathbf{X_0}$.

B. Neumann Series (NS)

The Neumann Series is a popular method for approximating the matrix inversion, which subsequently reduces the complexity of the linear detector. $\boldsymbol{A}$ can be decomposed into a diagonal matrix $\boldsymbol{D}$ and a non-diagonal matrix $\boldsymbol{E}$. The NS expansion of $\boldsymbol{A}$ is given by

$$\boldsymbol{A}^{-1} = \textstyle\sum_{n=0}^{\infty}(-\boldsymbol{D}^{-1}\boldsymbol{E})^n\boldsymbol{D}^{-1}, \tag{12}$$

the expansion in (12) converges if

$$\lim_{n\to\infty}(-\boldsymbol{D}^{-1}\boldsymbol{E})^n = 0. \tag{13}$$

As the iteration increases, high precision of the matrix inverse will be achieved. However, the complexity can be reduced only when n is small. In practice, only a finite number of terms is used.

C. Gauss-Seidel (GS)

The GS method is used to solve the linear function $\boldsymbol{A}\boldsymbol{X} = \boldsymbol{b}$. The matrix $\boldsymbol{A}$ can be decomposed into a diagonal matrix $\boldsymbol{D}$, an upper triangle matrix $\boldsymbol{U}$ and a lower triangle matrix $\boldsymbol{L}$. estimate of $\boldsymbol{X}$ at the n^{th} iteration is characterized by

$$\widehat{\boldsymbol{X}}^{(n)} = (\boldsymbol{D} + \boldsymbol{L})^{-1}(\boldsymbol{b} - \boldsymbol{U}\widehat{\boldsymbol{X}}^{(n-1)}). \tag{14}$$

If there is no prior information about the initial solution $\widehat{\boldsymbol{X}}^{(0)}$, it can be considered as zero.

D. Jacobi Method

Similar to the GS method, the Jacobi method is also an approximation method for solving linear equations by iteration.

$$\widehat{\boldsymbol{X}}^{(n)} = \boldsymbol{D}^{-1}[\boldsymbol{b} + (\boldsymbol{D} - \boldsymbol{A})\widehat{\boldsymbol{X}}^{(n-1)}] \tag{15}$$

The initial estimate can be identified as

$$\widehat{\boldsymbol{X}}^{(0)} = \boldsymbol{D}^{-1}\boldsymbol{b} \tag{16}$$

E. Conjugate Gradient (CG)

The conjugate gradients method is another effective method to solve the linear equation. The estimated signal can be obtained using

$$\widehat{\boldsymbol{X}}^{(n+1)} = \widehat{\boldsymbol{X}}^{(n)} + \alpha^{(n)}\boldsymbol{P}^{(n)} \tag{17}$$

where $\boldsymbol{P}^{(\mathrm{n})}$ is the conjugate direction with respect to $\boldsymbol{A}$,

$$(\boldsymbol{P}^{(n)})^{H}\boldsymbol{A}\boldsymbol{P}^{(j)} = 0,\ for\ n \neq j, \tag{18}$$

where $\alpha^{(n)}$ is a scalar parameter.

IV. SIMULATION AND RESULT

A. Small-scale MIMO detection

We implement the ZF detector and MMSE detector in a 2×2 MIMO, BPSK modulation, and frequency-flat channel model. Since the ML detector corresponds to optimal performance, it is also implemented as a baseline.

The simulation result is shown in Fig.2. According to the result, MMSE performs better than ZF, which is consistent with the theoretical analysis.

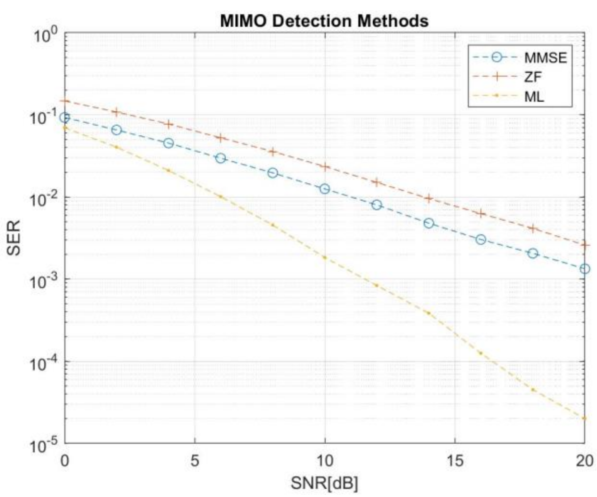

Fig. 2 small-scale MIMO detection

B. Large-scale MIMO detection

Fig.3(a)-(d) shows the simulation results of large-scale MIMO detection. We Implement MMSE detectors in 64×8, 64×16, 128×8, and 128×16 MIMO systems, using 64-QAM modulation and frequency-flat channel model. Different approximate methods including (a).Neumann Series (iteration number n=2), (b.)Newton Iteration (n=3), (c).Gauss-Seidel (n=3), (d).Jacobi Method (n=4), (e).Conjugate Gradient (n=3) are also implemented to compare the performance.

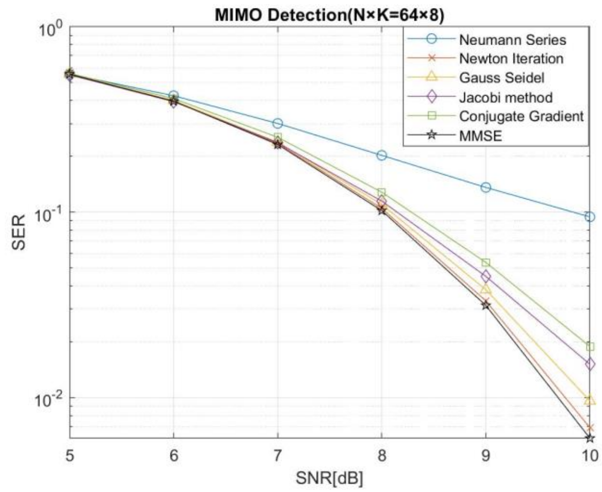

Fig. 3(a) 64x8 MIMO detection

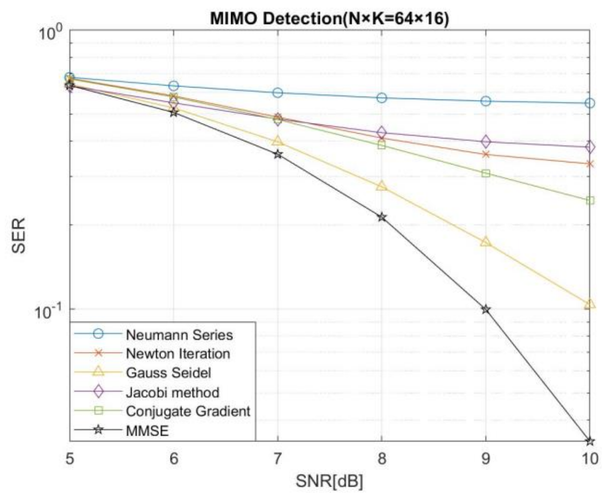

Fig. 4(b) 64x16 MIMO detection

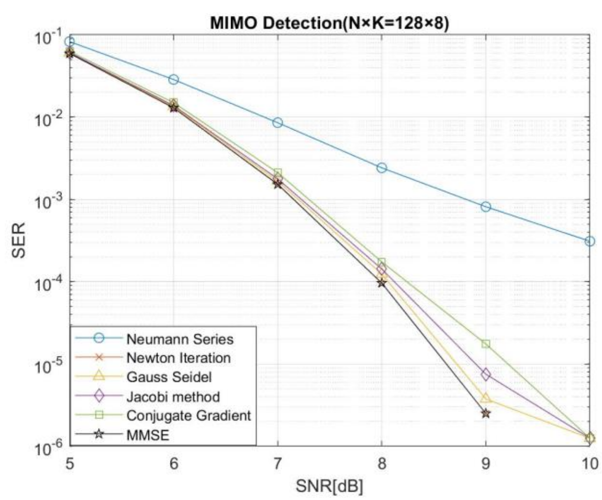

Fig. 5(c) 128x8 MIMO detection

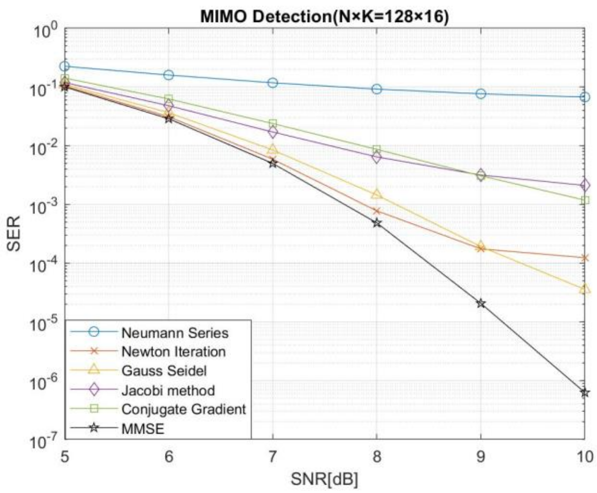

Fig. 6(d) 128x16 MIMO detection

According to the result:

- When the ratio of N (the number of receive antennas) and K (the number of user equipment) increases, all approximations perform better.
- GS method is most precise in In the 64 × 16 system (N/K = 4). As the ratio of N and K increases, NI performs slightly better.
- The reason for the rapid improvement in NI performance is that as the antenna ratio increases, the interference between channels will decrease. The diagonal elements of matrix A dominate. The error of the initial estimate $||\boldsymbol{I} - \boldsymbol{A}\boldsymbol{X}_0^{-1}||$ is reduced.
- NS method performs worst in all scenarios. One possible reason may be that we only use the first three terms (n=2) in the simulation, and the result does not converge. However, notice that as the iteration number n increases, the computational complexity will increase too.
- Although Jacobi Method requires more iterations (n=4) to achieve a precise result, due to the low complexity of the algorithm, it can still reduce the computation.

V. CONCLUSIONS

As a crucial part of the MIMO system, MIMO detection techniques have received extensive attention in the past few decades. In this report, we first give a formal discussion of the MIMO system. Then, we have studied linear detectors (ZF detector and MMSE detector) in small MIMO systems. After that, several matrix inverse approximation methods in massive MIMO detection are discussed by regarding the linear detection process as solving linear equation problem. Finally, the simulation results are given and analyzed.

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