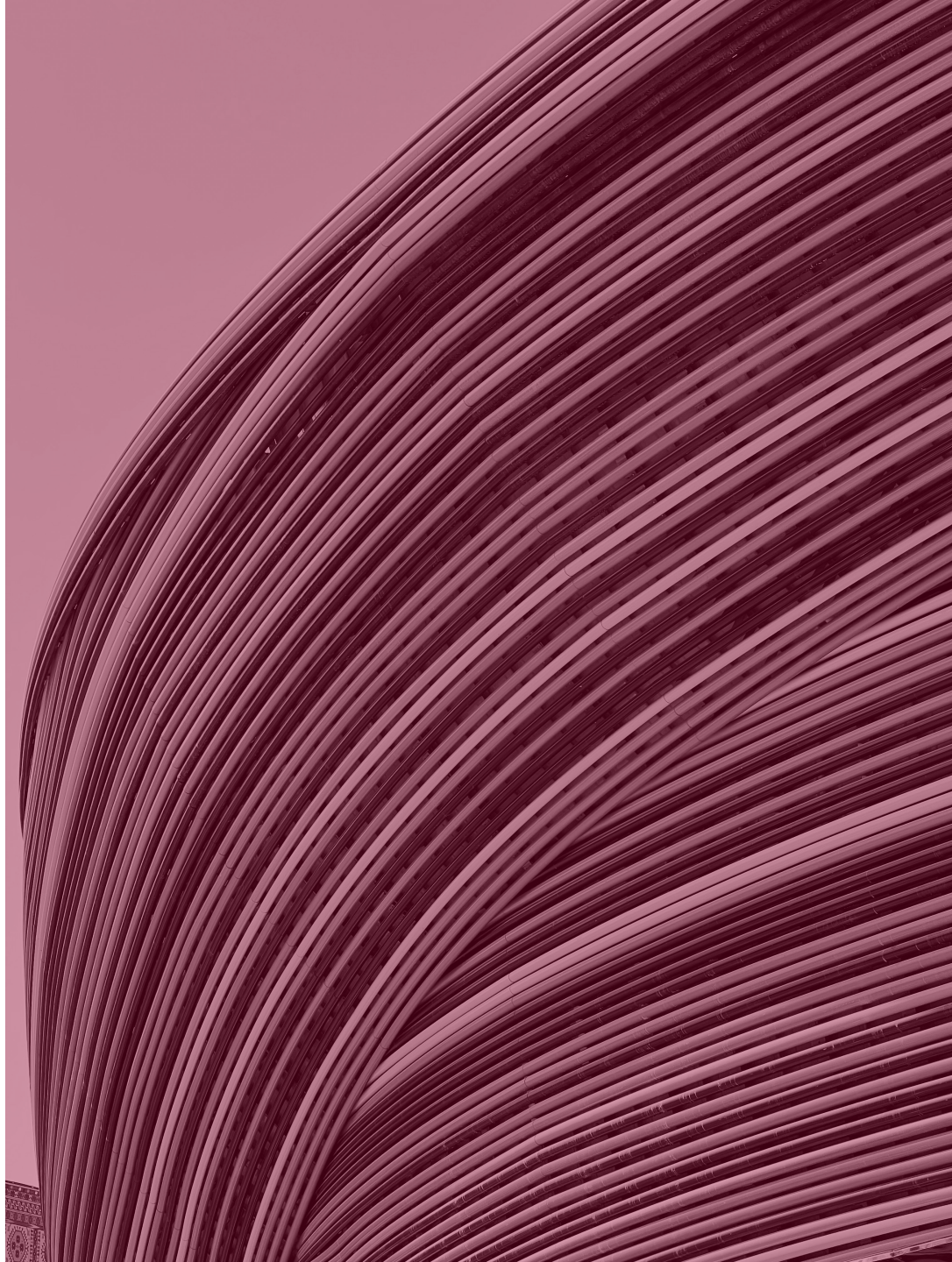


Econometrics I

Workshop III

Feb 21, 2023

Multicollinearity

BLUE

Best

Linear

Unbiased

Estimator

Given assumptions of linear regression model, estimation of Least Squares own optimal properties which are referred to Gauss-Markov Theorem

Best

Linear

Unbiased

Estimator

Linear function of a random variable

Best

Linear

Unbiased

Estimator

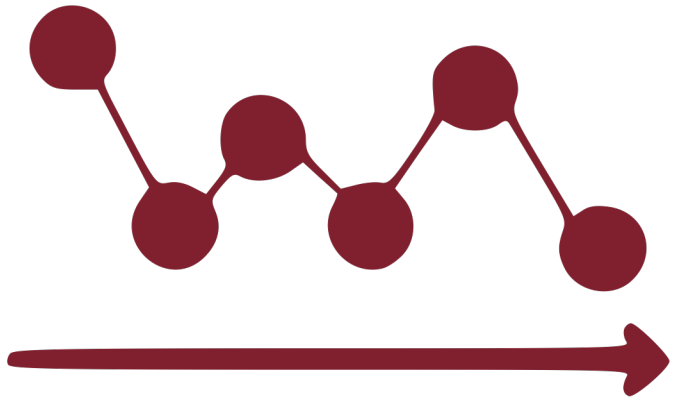
Expected value $E(\beta^2)$
equal to true value β

Best

Linear

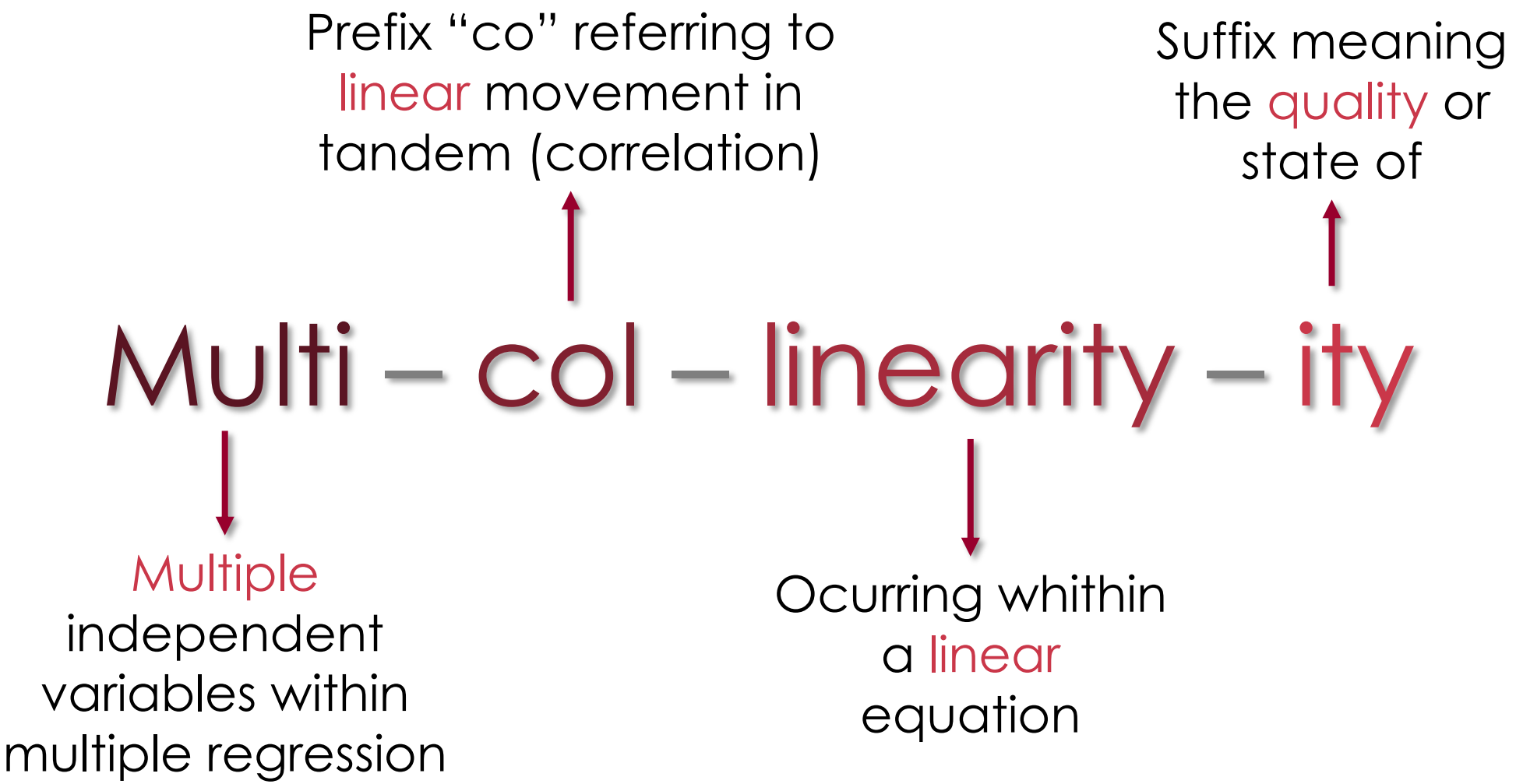
Unbiased

Estimator



Minimum variance

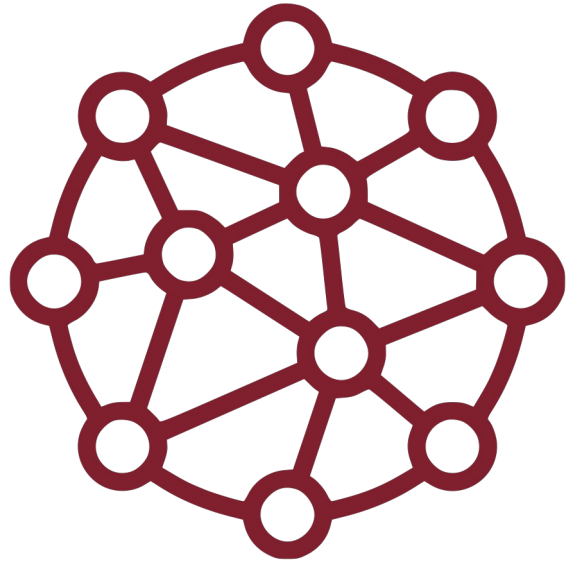
To **remember**



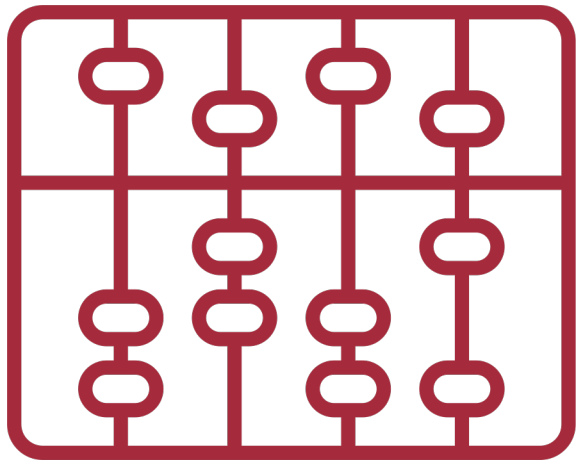
Taken from Andrew Ozbun (2021)

Empirical studies have shown that finding correlation levels between independent variables is quite usual.

"Linear 'perfect' relationship among some or all independent variables from a regression model."



The existence of any level of association among them has effects when estimating parameters and their variances.



For multicollinearity to exist, it is not feasible to separate into neatly, the effects on the dependent variable of each of the explanatory variables

One of the basic assumptions of the general linear model states that the explanatory variables are linearly independent

When independent variables are correlated in such a way that any of the columns of the matrix explanatory variables can be written as a linear combination of the others it is not possible to get the matrix inverse of $(X'X)1$

There is perfect multicollinearity when columns from matrix X are linearly dependent.

$$X = \begin{pmatrix} 1 & 3 & 7 \\ 1 & 2 & 5 \\ 1 & 4 & 9 \end{pmatrix}$$

Do you notice something?

Sometimes columns from matrix X are almost linearly dependent

$$\lambda_1 X_1 + \lambda_1 X_1 + \cdots + \lambda_k X_k \approx 0$$

- Matrix X has a rank equal to k
- Matrix $X'X$ is not singular
- OLS can be calculated

There is an approximated multicollinearity when columns from matrix X are almost linearly dependent

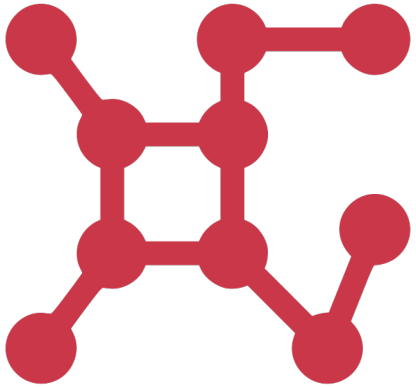
For example, columns from matrix X are almost linearly dependent.

$$X = \begin{pmatrix} 1 & 3 & 7.01 \\ 1 & 2 & 5 \\ 1 & 4 & 9 \end{pmatrix}$$

$$|X'X| = 0.02$$

Presence of approximated multicollinearity allows a better coefficient estimate but variance will be higher

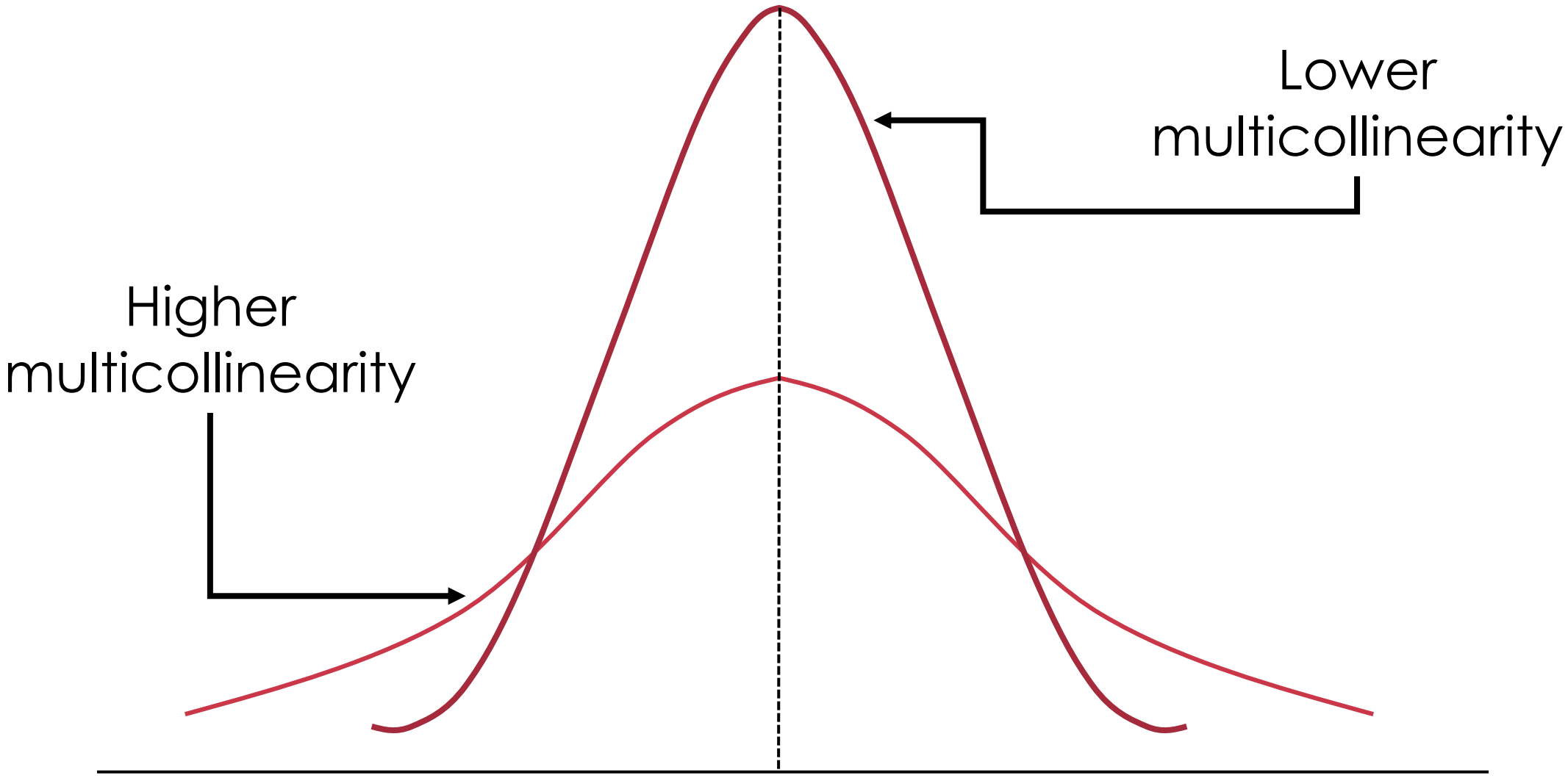
Structural multicollinearity



Mathematical artifact caused by creating new predictors from other predictors

Data-Based Multicollinearity

Results from a poorly designed experiment, reliance on purely observational data, or the inability to manipulate the system on which the data are collected.


Lower
multicollinearity
Higher
multicollinearity

How to detect multicollinearity?

1. A first glance, we can obtain simple sampling correlation coefficients for each pair of independent variables, then we check if the degree of correlation among them is high

2. The other way around is to run a regression for each independent variable over the rest and then analyze coefficients of determination of each regression

Do not worry if you do not understand, it will be crystal clear with practicing!

Super note!

If ANY of the coefficients of determination is high, it would indicate possible presence of multicollinearity.

```
// Data: nomulti.dta
// Check the scatter matrix and correlation matrix
graph matrix y x1 x2, half
corr x1 x2

// Run regression & save them
est store yvsx1
reg y x2
est store yvsx2
reg y x1 x2
est store yvsx1x2
reg y x2 x1
est store yvsx2x1
```

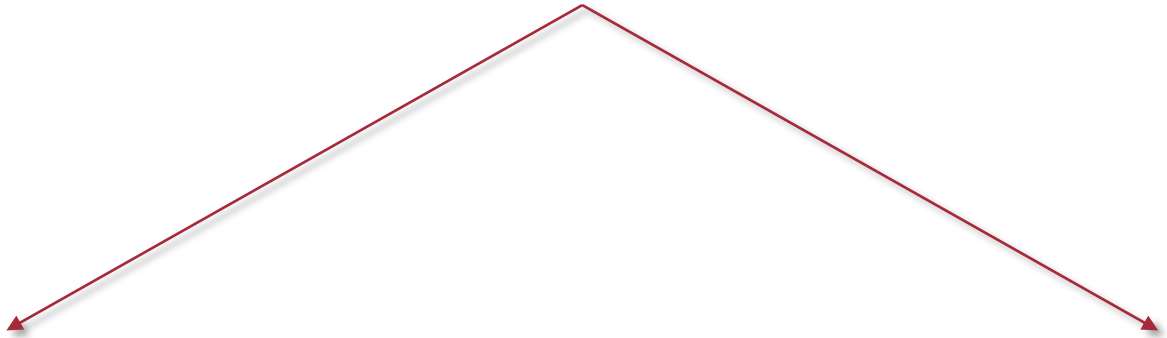
```
// Integrate them in a table
estimates table yvsx1 yvsx2 yvsx1x2 yvsx2x1, b(%9.2f) se(%9.2f)

// Obtain anova
anova y x1 x2
anova y x2 x1
```

```
// Data: presion.dta
// Calculate correlation matrix
Graph matrix bp age stress weight bsa dur pulse strees, half

// Obtain correlation
Corr bp age weight bsa dur pulse stress
```

From here, we will
review two cases:



when regressors are
slightly correlated

when regressors are
highly correlated

```
// Data: presion.dta
// Calculate correlation matrix
Graph matrix bp age stress, half

// Obtain correlation
Corr bp age stress
```

```
// Run regressions and store them
reg bp stress
Est store bpstress

Reg bp age
Est store bpage

Reg bp stress age
Est store stressage

Reg bp age stress
Est store agestress
```

```
// Create a comparative table
estimates table bpstress bpage stressage agestress, b(%92.f) se(%9.2f)

// Analyze variance
anova bp stress age
anova bp age stress
```

Now, what happens when regressors are highly correlated?

When regressor is correlated, the estimated coefficient will depend on variations from another regressor on which the former maintains that relationship.

```
// Create a comparative table
corr bp age weight bsa dur pulse stress
```

```
// Again, we generate regressions
reg bp bsa
est store bsa

reg bp weight
est store weight

reg bp bsa weight
est store bsa weight

estimates table bsa weight bsa weight b(%9.2f)
```

If BSA is the only regressor, we can say that for each additional square metre in body surface (*bsa*), blood pressure increases in 34.4 mm Hg.

If we include *weight* and *bsa* in the model, it is possible to point out that for each additional square metre in body surface (bsa) keeping weight constantly, then blood pressure increases only in 5.83 mm. Hg.

We can observe that variable *BSA* is meaningful in simple regression

Weight ceases to be significant in the regression where it appears

This may be contradictory due to the conclussion that blood pressure is related with body surface

VIF quantifies how big is the variance over estimator

The closer R^2 gets to 1 or the higher the colinearity of variable X_j with the rest of variables, the greater the value of VIF and the larger the variance of estimated coefficient turns

$$VIF = \frac{1}{(1 - R_j^2)}$$

$$TOL = \frac{1}{(VIF)}$$

Multicollinearity inflate variance

If $VIF_j > 10$ then conclude that collinearity of variable X_j regarding with the rest of variables is high.

If $TOL < 0.1$ there is collinearity.

elemapi2. contains information about academic performance from elementary education

Let's prove that academic performance (*api00*) depends on the percentage of students that receive free meals (*meals*), that are learning English (*ell*), on percentage of teacher with new accreditations (*emer*), and if parents have any college degree (*some_col*)

```
// Data: elemapi2.dta
// Run regression
reg api00 meals ell emer some_col

// Obtain VIF
vif
```

Let's run a second estimation now adding the following variables:

Grad_sch: Parents' educational level. *Col_grad*: Number of parents with college degree. *Avg_ed*: Parent's educatonal level average.

```
// Run regression
reg api00 meals ell emer some_col avg_ed grad_sch col_grad

// Obtain VIF
vif
```

Drop explicative variables: it is possible there may be a problem due to specificiation error (omission of any relevant variable)

Transform data: with cross-sectional data it is adsivable to use variables quotients, such as:

$$\frac{Y_i}{X_{3i}} = \beta_1 \frac{1}{X_{3i}} + \beta_2 \frac{1}{X_{3i}} + \beta_3 \frac{1}{X_{3i}}$$

Note that with time series, using data in first differentiation is recommended

$$\Delta Y_t = \beta_2 \Delta X_{2t} + \beta_3 \Delta X_{3t} + e_t$$

```
// Drop avg_ed and run regression
reg api00 meals ell emer some_col grad_sch col_grad

// Obtain VIF
vif
```

References

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- PennState Eberly College of Science, *Reducing Structural Multicollinearity*, from https://online.stat.psu.edu/stat462/node/182/
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