CS 581 Assignment 2

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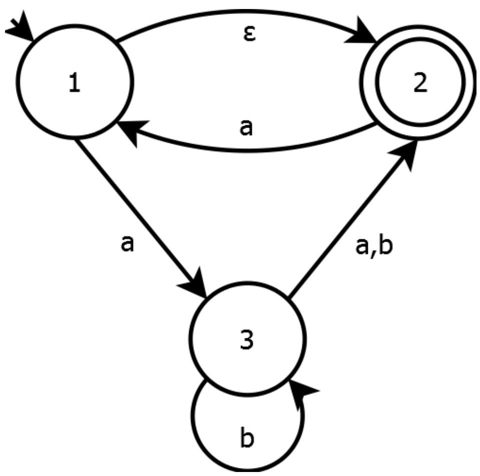
1.16(b) Convert the following NFA to its equivalent DFA.

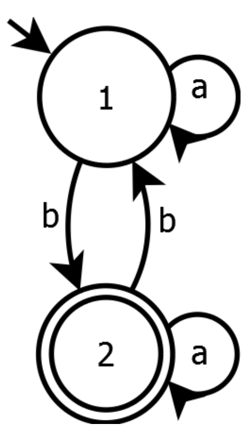


For this NFA $Q = 1, 2, 3$. The states of the equivalent DFA are the members of the powerset of Q:
$P(Q) = Q' = \{\phi, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$.
The transition table for the DFA is thus:

δ'	a	b
ϕ	ϕ	ϕ
$\{1\}$	$\{3\}$	ϕ
$\{2\}$	$\{1,2\}$	ϕ
$\{3\}$	$\{2\}$	$\{2,3\}$
$\{1,2\}$	$\{1,2,3\}$	ϕ
$\{1,3\}$	$\{2,3\}$	$\{2,3\}$
$\{2,3\}$	$\{1,2\}$	$\{2,3\}$
$\{1,2,3\}$	$\{1,2,3\}$	$\{2,3\}$

The DFA,M, equivalent to this NFA is given by:
$M = (Q', \Sigma, \delta', \{1,2\}, \{\{2\}, \{1,2\}, \{2,3\}, \{1,2,3\}\})$

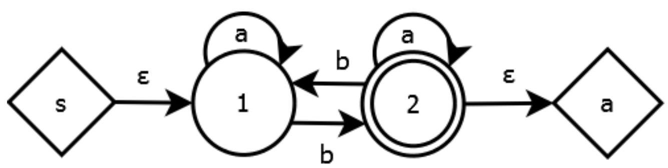
1.21(a) Convert this finite automaton to a regular expression.



The transition table of the DFA is given by δ:

δ	a	b
1	1	2
2	2	1

Let us include start state s and accept state a in the following manner:



δ_0	1	2	a
s	ϵ	ϕ	ϕ
1	a	b	ϕ
2	b	a	ϵ

This finite automaton is assumed to be fully connected; any transitions that were not previously possible are now said to be possible only on input of ϕ. The table at left describes its transitions..

δ_0	2	a
s	a^*b	ϕ
2	$(a \cup ba^*b)^*$	ϵ

The removal of state 1 from this system requires that transitions become independent of state 1, and system solutions do not change. These new transitions are shown in the table at left.

The removal of state 2 has the same requirements as the removal of state 1. The new transition is shown in the table to the right.

δ_0	a
s	$a^*b(a \cup ba^*b)^*$

$a^*b(a \cup ba^*b)^*$ is the equivalent regular expression for this finite automaton.

1.46(a) Prove that language $L = \{0^n1^m0^n | m, n \geq 0\}$ is not regular.

Proof by Contradiction: If L is a regular language then pumping length p exists such that:

$$\forall s, |s| \geq p, s\epsilon L \quad \text{and} \quad \exists x, y, z| \quad 0.s = xyz \quad 1.\forall i xy^i z\epsilon L \quad 2.|xy| \leq p \quad 3.|y| > 0$$

Consider $s = 0^p1^q0^p$ where $|s| \geq p, s\epsilon L$. Because L is regular, the characteristics above must hold, thus $\exists x, y, z$ such that they do. By (2.) it follows that $xy\exists 0^*$. By (3.) it follows that $y = 0^j$ for some $j > 0$. By (1.) it follows that $xy^0z\epsilon L$. These definitions imply that $z\epsilon 1^*0^*$.

Contradiction: In order to belong to this language, the number of preceeding 0s must match the number of ending 0s. However, the conditions of the Pumping Lemma allow for the possibility of xy^0z in the language L if it is regular. Because y must consist of at least one 0 in this case to be valid, and because this Pumping Lemma property allows for y to be omitted (in the case of xy^0z), the number of 0s in the beginning of the string can be at least one less than the number of 0s at the end of the string.

1.51 Show that $\equiv_L$ is an equivalence relation.

Let x and y be strings, and L be any language. If $\forall z|yz\epsilon L \rightarrow xz\epsilon L$ then $x \equiv_L y$. In order to establish that this relation is an equivalence relation, we must establish that it is reflexive, symmetric, and transitive:

Reflexive: For any language the string $xz = xz$, thus $xz\epsilon L \rightarrow xz\epsilon L$ is trivial and true. Because despite the value of z, xz is equivalent to and therefore implicative of itself, this relation is reflexive.

Symmetric: By definition, x and y are distinguishable only if $\exists z|xz \bigoplus yz\epsilon L$. Thus x and y are not distinguishable in two cases: (1) when some z causes both xz and yz to belong to L, and (2) when some z causes neither xz nor yz to belong to L. This classification is disjoint from $\equiv_L$, so wherever x and y are not distinguishable, they are indistinguishable. Because in order to be indistinguishable, x and y are either both present in L or both absent from L, their presence is mutually dependent and thus $x \equiv_L y \leftrightarrow y \equiv_L x$. This indicates that this relation is symmetric.

Transitive: Consider $w \equiv_L x$ and $x \equiv_L y$; by definition,

$$\text{if } \forall z|xz\epsilon L \rightarrow wz\epsilon L \text{ then } w \equiv_L x\text{, and if } \forall z|yz\epsilon L \rightarrow xz\epsilon L \text{ then } x \equiv_L y.$$

By the transitivity of entailment/implication it holds that $yz\epsilon L \rightarrow xz\epsilon L \rightarrow wz\epsilon L = yz\epsilon L \rightarrow wz\epsilon L$. Thus, if $w \equiv_L x$ and $x \equiv_L y$, then $\forall z|yz\epsilon L \rightarrow wz\epsilon L$, which indicates that $w \equiv_L y$ as well. This relation is thus also transitive.