Sample Solution

Fall 2008, Test 2, Models of Computation

Name:
Section:
Email id:

3rd November, 2008

Answer all Seven questions. You have 100 minutes to complete the exam.

1. **Context Free Grammar**

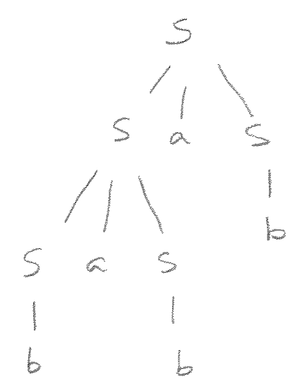
(a) What language is generated by the following context grammar with the indicated productions. $S \;\rightarrow\; a\,S\,a \;\mid\; b\,S\,b \;\mid\; a \;\mid\; b$

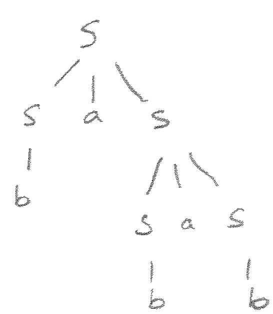
$\{ w \mid w = w^R,\ w \in \{a, b\}^* \ \ |w| \text{ is odd} \}$

(b) What language is generated by the followinbg grammar with the indicated productions? Draw a parse tree for *babab*

$S \;\rightarrow\; SaS \;\mid\; b$

$\{(ba)^* b\}$





2. **Construction of Context Free Grammar**

(a) Construct a context-free grammar for the following language: $\{w \in \{a, b\}^* :$ every prefix of w has at least as many a's as b's$\}$ [**5 points**]

$S \to aS \mid S_1$

$S_1 \to aS_1bS_1 \mid \lambda$

(a is like '('

and b is like ')')

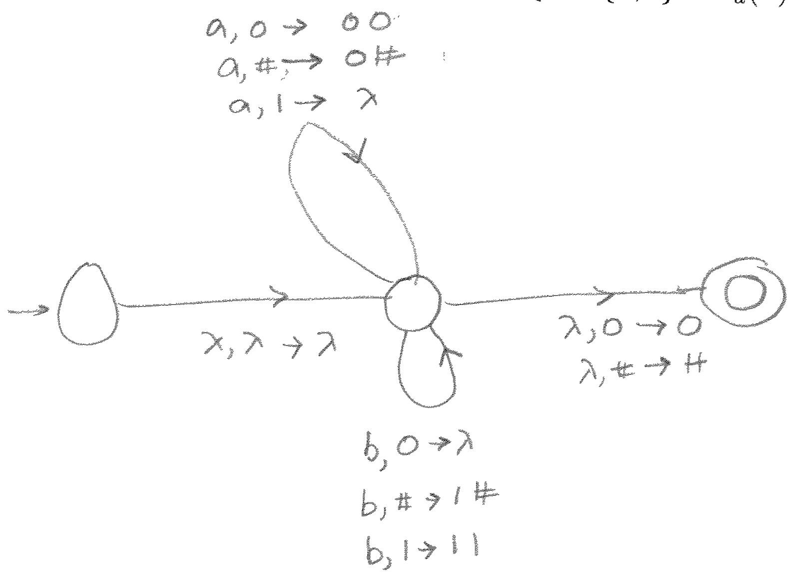
(b) Construct a context-free grammar for the following language: $\{a^n b^m :$ $n \geq m$ and $n - m$is even$\}$

$S \to aaS \mid S_1$

$S_1 \to aS_1b$

3. **PDA**

Construct a PDA that recognizes strings over $\{a, b\}$ such that the number of a's is more than the number of b's. $L = \{x \in \{a, b\}^* : n_a(x) \geq n_b(x)\}$ **[10 points]**



4. **Context Free Grammar for Boolean Expression**

Let $L = \{w \in \{[A-Z], \neg, \vee, \wedge, \rightarrow, (,), \}^* \mid \text{w is a syntactically legal Boolean Expression}\}$.

(a) Write an unambiguous context free grammar that generates L that [**6 points**]

- Associates left given operators of equal precedence, and
- Corrspond to assigning the following precedence levels to the operators (from highest to lowest): $\neg$, $\wedge$, $\vee$, $\rightarrow$.

(b) Show the parse tree for the following string in L

$$\neg P \vee R \rightarrow Q \rightarrow S$$

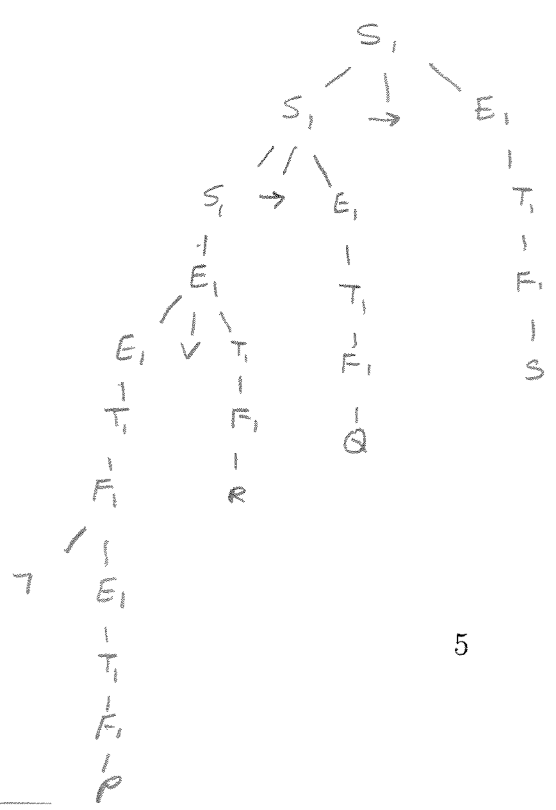
[**4 points**]

$S_1 \rightarrow S_1 \text{ '}\!\rightarrow\!\text{' } E_1 \mid E_1$

$E_1 \rightarrow E_1 \vee T_1 \mid T_1$

$T_1 \rightarrow T_1 \wedge F_1 \mid F_1$

$F_1 \rightarrow \neg E_1 \mid (E_1) \mid [A-Z]$



5. **Closure Properties**

(a) Prove that Context-free Languages are closed under union and concatenation [**5 points**]

Let G_1 be the Grammar for L_1
(N_1, T_1, S_1, P_1)

Let G_2 (N_2, T_2, S_2, P_2) be the grammar for L_2

For Union: $S \rightarrow S_1 \mid S_2$

For Concatenation $S \rightarrow S_1 S_2$

(b) Give a counter example to show Context-free Languages are **Not** closed under intersection. [**5 points**]

$L_1 = \{a^n b^n c^m \mid n, m \geq 0\}$

$L_2 = \{a^m b^n c^n \mid n, m \geq 0\}$

$L_1 \cap L_2 = \{a^n b^n c^n \mid n \geq 0\}$

$L_1 \cap L_2$ is not Context-Free

6. **Pumping Lemma**

Show that the language a^{n^2}, is not context-free. **[10 points]**

Choose ~~$~~string with which to find a counterexample $w = a^\wedge(p^\wedge 2)$, where p is the pumping length (magic number).

Must show that for any partition $uvxyz$, duplicating v and y by constant i-many times will produce a string not the language.

For any partition, $|vy| \geq 1$

$|vxy| \leq p$

for $i=2$, we will add atleast 1 symbol and at most p symbols.

$a^\wedge(p^\wedge 2 + k)$ cannot be equal to any $a^\wedge(n^\wedge 2)$ because of the constrainst on k.

$$|a^\wedge(p^\wedge 2)| < |a^\wedge(p^\wedge 2+k)| < |a^\wedge(p^\wedge 2+2p+1)| < |a^\wedge((p+1)^\wedge 2)|$$

arriving at a contradiction

7. **Normal Forms and Simplied Grammars**

(a) Convert the following grammar to Chomsky Normal Form.[**5 points**]

$S \to aACa$
$A \to B \mid a$
$B \to C \mid c$
$C \to cC \mid \epsilon$

1. removing null production $C \to \epsilon$

$S \to aACa \mid aAa$
$A \to B \mid a$
$B \to C \mid c \mid \epsilon$
$C \to cC \mid c$

Same as Q 5a in sample Exam 2

2. Removing null production $B \to \epsilon$

$S \to aACa \mid aAa$
$A \to B \mid a \mid \epsilon$
$B \to C \mid c$
$C \to cC \mid c$

Removing $A \to \epsilon$

$S \to aACa \mid aAa \mid aCa \mid aa$
$A \to B \mid a$
$B \to C \mid c$
$C \to cC \mid c$

Removing unit production $B \to C$ & $A \to B$

$S \to aACa \mid aAa \mid aCa \mid aa$
$A \to a \mid cC \mid c$
$B \to cC \mid c$
$C \to cC \mid c$

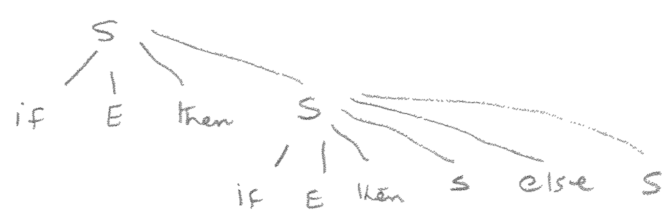
CNF

$S \to T_1V_1 \mid T_1V_2 \mid T_1V_3 \mid T_1T_1$
$V_1 \to AV_4$
$V_2 \to AT_1$
$V_3 \to CT_1$
$V_4 - CT_1$
$A \to T_1 \mid T_2C \mid c$
$B \to T_2C \mid c$
$C \to T_2C \mid c$
$T_1 \to a$
$T_2 \to c$

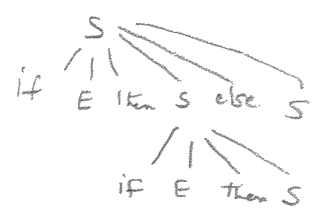
(b) Explain with an example of two different (possible) parse trees in a nested if statements. How does compiler avoid this ambiguity or what is the assumption in parsing if statements? [**5 points**]

if E then if E then S else S

Parse trees



Tree 1



Tree 2

Compiler always associates the else clause with the closest if. Hence parse Tree 1 is used instead of Tree 2.