Recitation 10

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Q1

The following two languages is the complement of a simpler language. Construct a DFA for the simpler language, then use it to give the state diagram of a DFA for the language given. $\Sigma = \{a, b\}$.

$\{w \mid w$ is any string not in $(ab^+)^*\}$

$\{w \mid w$ is any string not in $a^* \cup b^*\}$

Q2

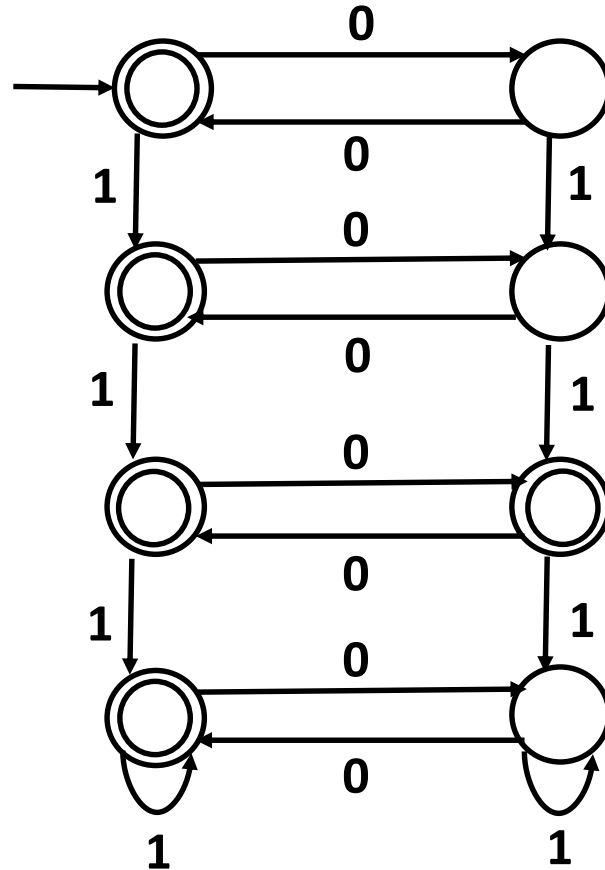- Give state diagrams of DFAs recognizing the following languages. In all parts, the alphabet is {0,1}.

{w | w starts with 0 and has odd length, or starts with 1 and has even length}

{w | w is any string except 11 and 111}

{w | w contains an even number of 0s, or contain exactly two 1s}

Q2 Answer:

{w | w contains an even number of 0s, or contain exactly two 1s}

Q3

- Give state diagram of NFAs with the specific number of states recognizing each of the following language. In all parts, the alphabet is {0, 1}.

  {w | w contains an even number of 0s, or contain exactly two 1s} with 6 states

  The language 0*1*0$^+$ with 3 states.

Q3 Answer:

{w | w contains an even number of 0s, or contain exactly two 1s} with 6 states
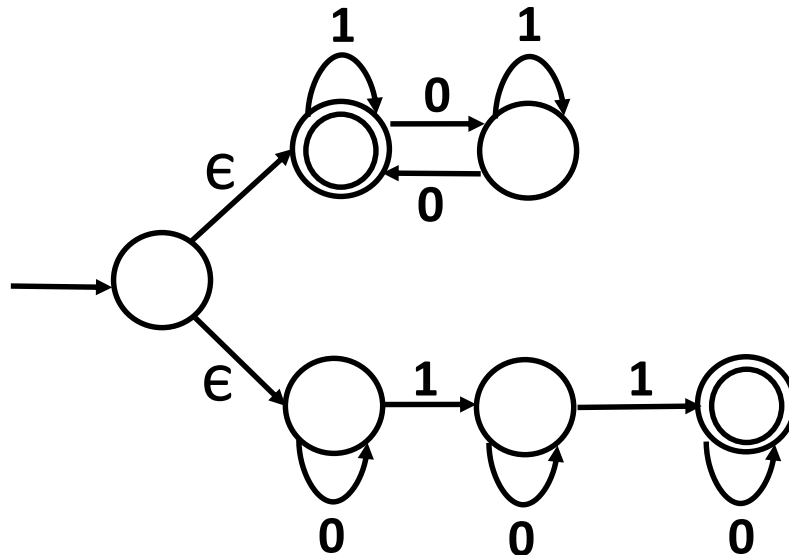
Q4

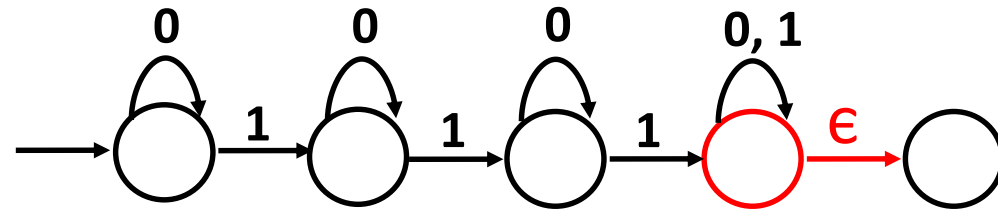- Give the state diagrams of NFAs recognizing the concatenation of the two languages:

{w | w contains at least three 1s}
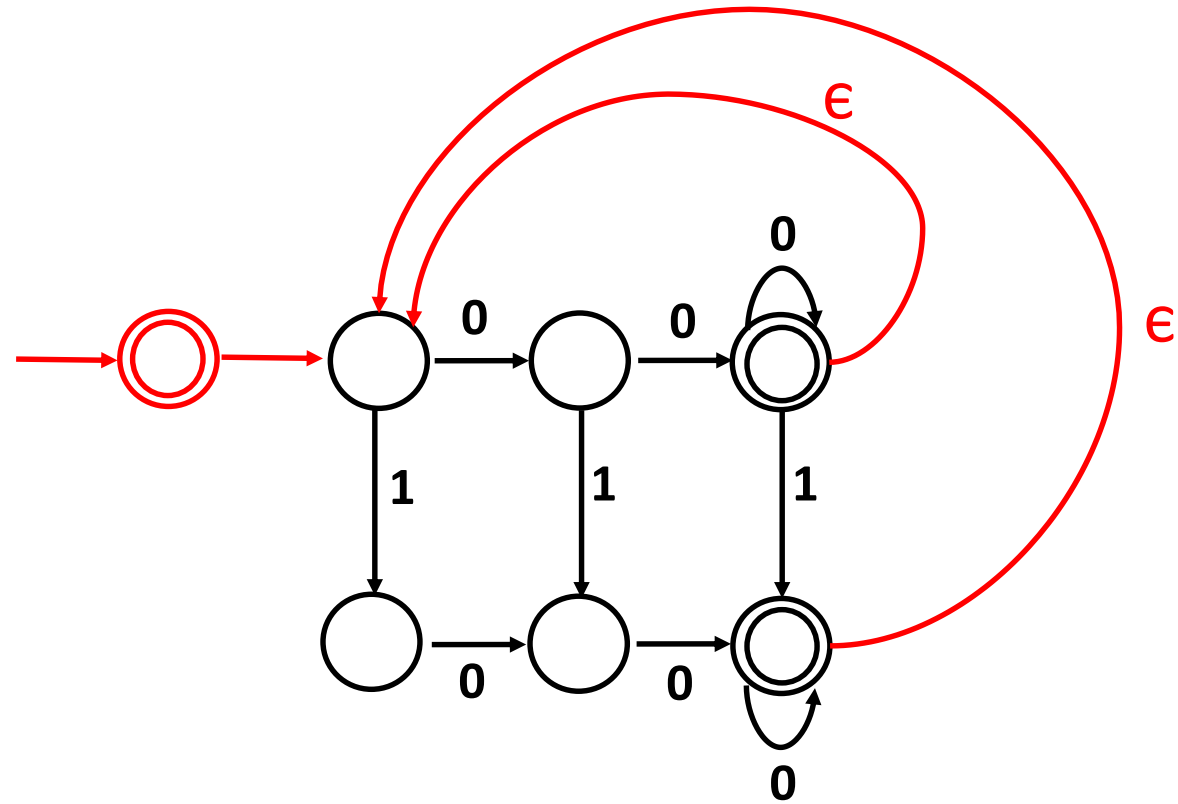
The empty set

Q4 Answer

Q5

- Give the state diagrams of NFAs recognizing the star of the language describe in

$$\{w \mid w \text{ contains at least two 0s and at most one 1}\}$$
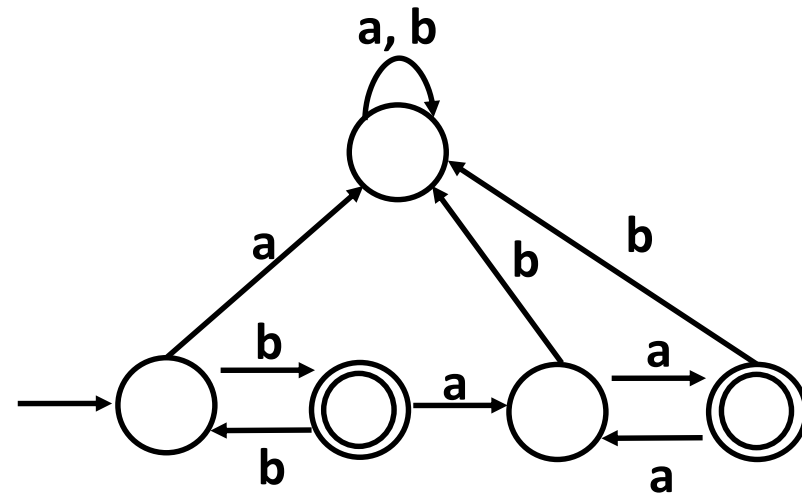
Q5 Answer:

Q6

Let D = {w | w contains an even number of a's and an odd number of b's and does not contain the substring ab}. Give a DFA that recognizes D and a regular expression that generate D.

Regular expression: b(bb)*(aa)*

Q7 – 1.14

a. Show that if M is a DFA that recognizes language B, swapping the accept and non-accept states in M yields a new DFA recognizing the complement of B. Conclude that the class of regular languages is closed under complement.

Answer: let *M'* be the **DFA** *M* with the accept and non-accept states swapped. We show that *M'* recognizes the complement of *B*, where *B* is the language recognized by *M*. Suppose *M'* accepts *x*. If we run *M'* on *x* we end in an accept state of *M'*. Because *M* and *M'* have swapped accept/non-accept states, if we run *M* on *x*, we would end in a non-accept state. Therefore, $x \notin B$. Similarly, if *x* is not accepted by *M'*, then it would be accepted by *M*. So *M'* accept exactly those strings not accepted by *M*. Therefore, *M'* recognizes the complement of *B*.

Since *B* could be any arbitrary regular language and our construction shows how to build an automation to recognize its complement, it follows that the complement of any regular language is also regular. Therefore, the class of regular languages is closed under complement.

Q7 – 1.14

b. Show by giving an example that if M is an NFA that recognizes language C, swapping the accept and non-accept states in M doesn't necessarily yield a new NFA that recognizes the complement of C. Is the class of languages recognized by NFAs closed under complement? Explain your answer.

Answer: Consider the **NFA** in Exercise 1.16(a). The string a is accepted by this automation. If we swap the accept and reject states, the string a is still accepted. This shows that swapping the accept and non-accept states of an **NFA** doesn't necessarily yield a new **NFA** recognizing the complementary language. The class of languages recognized by **NFA** is, however, closed under complement. This follows from the fact that the class of languages recognized by **NFA**s is precisely the class of languages recognized by **DFA**s which we know is closed under complement from part (a).