Name _____ Student No. _____

No aids allowed. Answer all questions on test paper. Use backs of sheets for scratch work.

Total Marks: 60

[20]   1. Give a context free grammar that generates the language

$$\{a^i b^j c^k : \; i = j \text{ or } j = k \text{ where } i, j, k \geq 0 \}.$$

Solution:

$$S \longrightarrow E_{ab}C \,|\, AE_{bc}$$
$$E_{ab} \longrightarrow aE_{ab}b \,|\, \varepsilon$$
$$E_{bc} \longrightarrow bE_{bc}c \,|\, \varepsilon$$
$$C \longrightarrow Cc \,|\, \varepsilon$$
$$A \longrightarrow Aa \,|\, \varepsilon$$

[20]   2. Consider the language $L = \{ww^R : w \in \{0,1\}^*\}$, that is, L is the language of even length palindromes over $\{0,1\}$. Give a PDA for this language; explain why do we need non-determinism for this language.

Solution: The PDA should push w onto the stack, and then pop the stack while reading w^R, comparing the bits. Non-determinism is necessary in order to "guess" where is the middle of the string (on a wrong guess the computation will reject; on a right guess it will compare the two halves). The diagram is given in figure 2.19 in Sipser.

[20]   3. Define what it means for a CFG to be in Chomsky Normal Form. Show that if G is in CNF, then for any string $w \in L(G)$, where $|w| = n \geq 1$, exactly $2n - 1$ steps are required for any derivation of w.

Solution: CNF is given in definition 2.8 in Sipser. Consider a derivation of w; each application of the rule $A \longrightarrow BC$ increases the length of the sentential form by 1; so there is a need of $n - 1$ steps. Then each variable yields a single terminal, which takes n steps. Hence, $2n - 1$ steps in total.