UNIVERSITY OF WEST LONDON

Mathematics of Signal Processing

(Master in Audio Digital Engineering Assessment 1)
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ASSESSMENT ONE

Contents

ABSTRACT

The report is a technical assessment in MSc audio digital signal processing. This report discusses topics like harmonics of related members of complex exponential fundamental functions, unit step functions, zero crossings, and the graphical and tabular representation of signals.

OVERVIEW

This report is the first assessment for the master's degree of mathematics of audio digital signal processing at West London University, St. Mary's Road, London. The writer assumed that readers whose intention is to understand this paper must have some basic knowledge in linear algebra, calculus and complex analysis. The writer shall discuss topics in complex signal analysis, derivations of members of **signal distortions** (harmonics) of a basic periodic function. The writer shall, furthermore, discuss topics on **zero crossing** and its algorithm. The primary questions from this assessment will serve as a road map for the writer to further explore and discuss some of the basic's concepts behind each question and the mathematic descriptions and notations.

According to Albert Einstein, *'If you can't explain it simply you don't understand it well enough'*. Therefore, the aim of this report is to show an understanding of the posted questions of this report/assessment and furthermore, how to mathematically or algorithmically resolve them.

QUESTION 1

Determine the members of the family of harmonically related complex exponential signals in discrete time that share a common period of N = 7.

The concept behind the family members of harmonic related complex exponential functions has be proofed, that if a signal with a fundamental frequency $\boldsymbol{\omega}$ is scaled with a constant value **k**, the resultant frequency $\boldsymbol{R_\omega} = \boldsymbol{\omega} * \boldsymbol{k}$ is a member of the fundamental signal frequency with the same samples per period. Therefore, if $\boldsymbol{k} = 1$ the resultant frequency $\boldsymbol{R_\omega}$ is the same as the fundamental frequency. The fundamental frequency can also be referred to as the 1st harmonic members of the fundamental basic signal wave. The study of signal harmonics is referred to as harmonic analysis; *a branch of mathematics that is conventionally base on the Fourier transform series, which is a way of expressing a signal as a weight sum of sine and cosine waves*. Folorunso Oladipo and Ojo Adebayo O. in 2015, defined harmonic as *"distortion of the electrical signal waveform as a result of non-linear load"* which can be mathematically describe as shown below in its complex form.

$$\mathrm{X}(\omega) = \frac{1}{\mathrm{k}}\sum_{\mathrm{k}=1}^{\infty} [\cos(k\omega) + jsin(k\omega)] \quad 1.1$$

Fourier transforms of harmonic series

∴ ∞ is the infinite number of the members of related harmonic complex exponential signals (distortions) within the fundamental frequency ω . A wave frequency can be defined as the number of cycles of oscillations that has occurred per seconds. A signal period is said to be fundamental if and only if it produce 1 cycle per its period length.

Since signal are mathematically described as a wave, therefore, a signal **period** is the time required for a signal to complete a cycle. In a periodic signal, the fundamental period T is said to be repeated as the signal propagates with time. In a nutshell, *a* periodic signal can be said to have periodic behaviour if the below mathematical description is satisfied.

Periodic Signal Function

$$\mathrm{X}(\mathrm{n} + \mathbf{N}) = \mathrm{X}(\mathrm{n}) \quad 1.2$$

where **N** is the fundamental period of a discrete time signal, while **n** is the linear time interval of which every sample of the signal is sampled.

From equation 1.2 the formular can be further re-express as $X(n + \boldsymbol{N}) = X(n) * X(N)$ if N is the fundamental period, therefore $N = k\frac{2\pi}{\omega}$ where k is the number of cycles that has be oscillated.

In a periodic signal function, the number of cycles to be oscillated must be a positive integer value, therefore a complete cycle can be mathematic said to be $2\pi \cong e^{j2\pi} \cong = 1$. the fundamental period can be denoted as

$$N = k\frac{1}{\omega} \cong k\frac{2\pi}{\omega} \qquad 1.3$$

In complex trigonometric analysis, a periodic function can be described in its complex form as shown below, following Pythagoras theory of right-angle triangle, the imaginary part is $\mathrm{I} = \mathrm{jsin}(\omega \mathrm{n})$ and the real part $\mathbb{R} = \cos(\omega \mathrm{n})$

$$\mathrm{X}(\mathrm{n}) = \cos(\omega \mathrm{n}) + \mathrm{jsin}(\omega \mathrm{n}) \qquad 1.4$$

The relationship between the fundamental period, fundamental frequency and the number of cycles in a periodic signal is describe in equation 1.3. The complex exponential using Euler Formula can also be used to describe a complex signal as

$$X(n) = e^{j\omega n} \qquad 1.5$$

Where $e\ is\ the\ expression(x) = 1 + x + \frac{1}{x^{2}!} + \frac{1}{x^{3}!} + \frac{1}{x^{4}!} \ldots + \frac{1}{x^{n}!}$ which is sometimes referred to as the Euler number = **2.71828** or the base of a natural logarithm. Furthermore, equation 1.5 is also equivalent to $\cos(\omega n) + jsin(\omega n)$ *in its complex form.*

Fundamental Frequency of a Signal Function

The fundamental frequency of periodic signal function can be derived from equation **1.3** by making ω the subject formula

$$\omega = k\frac{2\pi}{N} \qquad 1.6$$

Harmonically Relationship of Complex Exponential Signals Solution

Since the writer have already used the letter **k** for the number of cycles. Let **M** be the index of the harmonic member of the fundamental periodic signal and **A** the scale value of the signal amplitude. Therefore, to find the member of the related harmonic complex exponential signal **M.** The constant M will be introduced into the equation **1.5** as:

$$X(n) = Ae^{jM\omega n} \qquad 1.7$$

Note: for a any periodic signal X(n), **M** must be a positive integer value representing the harmonic member of the fundamental signal. where $\mathrm{M} \neq 0$, to the write understanding **M** can also be referred to frequency scaling of the original fundamental signal.

The writer's understanding of question **1** above, is to find the first harmonic related members of the fundamental signal whose fundamental period $\mathbf{N_M} = \mathbf{N}$ This simply means that when a harmonic signal whose fundamental period $\mathbf{N_M} \neq \mathbf{N}$, we shall automatically stop the

iteration and list all the first members and its fundamental frequency $\boldsymbol{\omega}_{\mathrm{M}}$ and period $\mathbf{N}_{\mathrm{M}}$ in its complex exponential form.
Therefore, knowing that **N** = 7, substitute **N** into equation 1.6 to represent the fundamental frequency of the basic parent signal for the assessment.

$$\omega = k\frac{2\pi}{7} \tag{1.7}$$

Note: *A signal is said to be a fundamental periodic signal if the number of cycle k = 1.*

Recall equation 1.6 and substitute equation 1.7 replacing ω in equation 1.6 the resultant formula will be the basic signal function where $k = 1$, N = 7.

$$X(n) = e^{jM\left(\frac{2\pi}{7}\right)n} \tag{1.8}$$

With equation **1.8** we can now derive the harmonic related member signal function that have same $\mathbf{N}_{\mathrm{M}} = \mathbf{N}$, in the range $-\infty < \mathrm{M} < \infty, where\ M \neq 0$ substitute the equation **1.9** into equation 1.3

$$\omega = \frac{2\pi}{7} \tag{1.9}$$

Therefore, from equation 1.8 we can now derive the harmonic members of the periodic final whose fundamental period is $\mathbf{N}_{\mathrm{M}} = 7$.

Calculating the Harmonic Related Complex Exponential Signal

If $-\infty < \mathrm{M} < \infty, where\ M \neq 0$ we can assume that M starts from 1, therefore:

1st Harmonic Related Frequency where M= 1

In equation 1.8 replace M with one so that the equation will result to

$$X(n) = e^{j\left(\frac{2\pi}{7}\right)n} = \cos\left(\left(\frac{2\pi}{7}\right)n\right) + j\sin\left(\left(\frac{2\pi}{7}\right)n\right) \tag{2.0}$$

1st Member of the Harmonic Signal Function.

Step 1

Find the fundamental period of the first harmonic signal $\mathbf{N}_{\mathrm{M=1}} = k\frac{2\pi}{\omega_M}$ substitute equation **1.9** to replace ω_M.

$$\mathbf{N}_{\mathrm{M=1}} = k\frac{2\pi}{\frac{2\pi}{7}}$$

Step 2

Simply the equation above

$\mathbf{N}_{\mathrm{M}=1} = k\frac{14\pi}{2\pi} = 7k$ if k = 1, then, $\mathbf{N}_{\mathrm{M}=1} = 7$ which have the same fundamental frequency as the parent fundamental period **N**, however since this signal also have k = 1, means it is the fundament basic signal itself.

$$\omega_{M=1} = \frac{2\pi}{7} \quad , \qquad \mathbf{N}_{\mathrm{M}=1} = 7, k = 1$$

2nd Harmonic Related Frequency if M= 2

If **M = 2**, substitute M into equation 1.8 as shown below

$$X(n, M) = e^{j\left(\frac{4\pi}{7}\right)n} = \cos\left(\left(\frac{4\pi}{7}\right)n\right) + jsin\left(\left(\frac{4\pi}{7}\right)n\right) \qquad 2.1$$

2nd Periodic Member of the Harmonic Signal

Step 1

Find the fundamental period of the first harmonic signal $\mathbf{N}_{\mathrm{M}=2} = k\frac{2\pi}{\omega_M}$ substitute equation **1.9** to replace ω_M.

$$\mathbf{N}_{\mathrm{M}=2} = k\frac{2\pi}{\frac{4\pi}{7}}$$

Step 2

Simply the equation above

$\mathbf{N}_{\mathrm{M}=2} = k\frac{14\pi}{4\pi} = k\frac{7}{2}$ if k = 2, then, $\mathbf{N}_{\mathrm{M}=2} = 7$ which have the same fundamental frequency as the parent fundamental period **N** which make it a member of the parent signal that meets the criteria of the question.

$$\omega_{M=2} = \frac{4\pi}{7} \quad , \qquad \mathbf{N}_{\mathrm{M}=2} = 7, k = 2$$

Repeat the sample steps for $2 < M < 7$

3rd Harmonic Related Frequency if M= 3

If M = 3, substitute M into equation 1.8 will result to equation

$$X(n,M) = e^{j\left(\frac{6\pi}{7}\right)n} = \cos\left(\left(\frac{6\pi}{7}\right)n\right) + jsin\left(\left(\frac{6\pi}{7}\right)n\right) \quad 2.3$$

3rd Periodic Member of the Harmonic Signal

Finding the number of cycles in the 3rd harmonic function:

$$\omega_{M=2} = \frac{6\pi}{7} \quad , \mathbf{N}_{\mathrm{M=3}} = 7, k = 3$$

4th Harmonic Related Frequency if M= 4

If M = 4, substitute M into equation 1.8 will result to below equation:

$$X(n.M) = e^{j\left(\frac{8\pi}{7}\right)n} = \cos\left(\left(\frac{8\pi}{7}\right)n\right) + jsin\left(\left(\frac{8\pi}{7}\right)n\right) \quad 2.4$$

4th Periodic Member of the Harmonic Signal

$$\omega_{M=4} = \frac{8\pi}{7} \quad , \mathbf{N}_{\mathrm{M=4}} = 7, k = 4$$

5th Harmonic Related Frequency if M= 5

If M = 5, substitute M into equation *1.8* is will result to below equation:

$$X(n) = e^{j\left(\frac{10\pi}{7}\right)n} = \cos\left(\left(\frac{10\pi}{7}\right)n\right) + jsin\left(\left(\frac{10\pi}{7}\right)n\right) \quad 2.5$$

5th Periodic Member of the Harmonic Signal

$$\omega_{M=5} = \frac{10\pi}{7} \quad , \qquad \mathbf{N}_{\mathrm{M=5}} = 7, k = 5$$

6th Harmonic Related Frequency if M= 6

If M = 6, substitute M into equation 1.8 will result to below equation:

$$X(n) = e^{j\left(\frac{12\pi}{7}\right)n} = \cos\left(\left(\frac{12\pi}{7}\right)n\right) + jsin\left(\left(\frac{12\pi}{7}\right)n\right) \quad 2.6$$

6th Periodic Member of the Harmonic Signal

$$\omega_{M=6} = \frac{12\pi}{7} \quad , \qquad \mathbf{N}_{\mathrm{M=6}} = 7, k = 6$$

7th Harmonic Related Frequency if M= 7

If M = 7, substitute M into equation *1.8* will result to below equation

$$X(n) = e^{j(2\pi)n} = \cos\big((2\pi)n\big) + jsin\big((2\pi)n\big) \tag{2.7}$$

7th Periodic Member of the Harmonic Signal

$$\omega_{M=7} = 2\pi \quad , \qquad \mathbf{N}_{\mathrm{M=7}} = 1, k = 1$$

The 7th harmonic member does not have same fundamental period **N;** therefore, we shall end the iteration here. However, the 7th harmonic possesses the characters of a parent child where k=1, $\mathbf{N}_{\mathrm{M=7}} = 1$

Moreover, the 7th harmonic member is not the 1st member, they both possess similar characteristics with different frequency and period. Moreover , both in their fundament period **N** produces a cycle.

8th Harmonic Related Frequency if M= 8

If we pretend to continue looking for harmonic members, you will notice that the 8th element satisfies the criteria above. But because we are looking for the first harmonic related members, we are going to accept $0 < M < 7$ as the first harmonic member of the fundamental signal.

If **M = 8**, substitute M into equation *1.8* is will result to below equation

$$X(n) = e^{j\left(\frac{16\pi}{7}\right)n} = \cos\left(\left(\frac{16\pi}{7}\right)n\right) + jsin\left(\left(\frac{16\pi}{7}\right)n\right) \tag{2.8}$$

$$\omega_{M=8} = \frac{16\pi}{7} \quad , \qquad \mathbf{N}_{\mathrm{M=8}} = 7, k = 8$$

QUESTION 2

Use MATLAB (or similar technical software to plot each member of Question 1.

The writer has decided to use **Python 3** and **matplotlib.pyplot** library to plot the harmonics member of the fundamental basic signal functions derived in **Question 1** from a generated time slices and complex array values.

Matplotlib.pyplot library in python have similar inbuild functions like MATLAB functions and allow developer to plot signals for x-values and y-values.

Where the X-values are the time slices, and the y-values are the real and imagine part of the complex number. In this instance, my graph shall contain the **real** and the **imagine** part of the signal distinguished with different colours.
Furthermore, waves have relationship between period and its frequencies and ability to calculate the period per cycle is given by the below formular

$$P = 1/f \qquad 2.9$$

Since the fundamental signal given is in time domain, we are going to generate the time intervals of the signal using the below algorithm.

Time Domain Interval Generator Algorithm

1) Get the basic signal frequency as $\boldsymbol{W}$ and the harmonic index **k** integral value**.**
2) Calculate the harmonic frequency $f = k * W$, which is the scaling of $\boldsymbol{W}$
3) Calculate the harmonic period per a cycle as $PeriodPerCycle \ = \ 1/f$
4) Calculate the time required per a sample $\boldsymbol{S} = \frac{PeriodPerCycle}{\boldsymbol{N}}$.
   So that time slice step = $\boldsymbol{S}$.
5) Now, generate from time slices from $-\frac{N}{2}\ to + \frac{N}{2}\ with\ step\ time\ of\ \boldsymbol{S}$
6) Collect and store the time slice values into a list called **X-Values**
7) For **t** in **x-Values** compute complex number **C** = $\boldsymbol{cos(f * t) + jsin(f * t)}$ store C into a list **Y-Values**
8) End t loop.
9) Plot **x-Values** against **Y-Values** [Real] and **x-Values** against **Y-Values** [Imaginary]
10) Terminate function.

The above pseudocode is coded using the python programming. The x-values and imaginary and real number are plot into axes to show their relationship.

The graphical representation is shown below:

1st Harmonic Graphical Representation

1st Harmonic Related Complex Signal

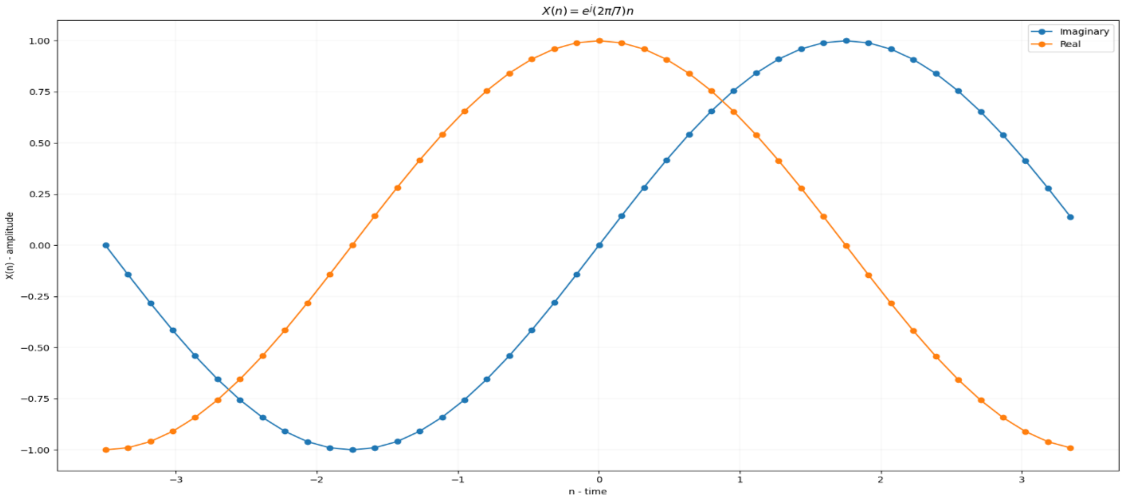

Fig 1

2nd Harmonic Graphical Representation

2nd Harmonic Related Complex Signal

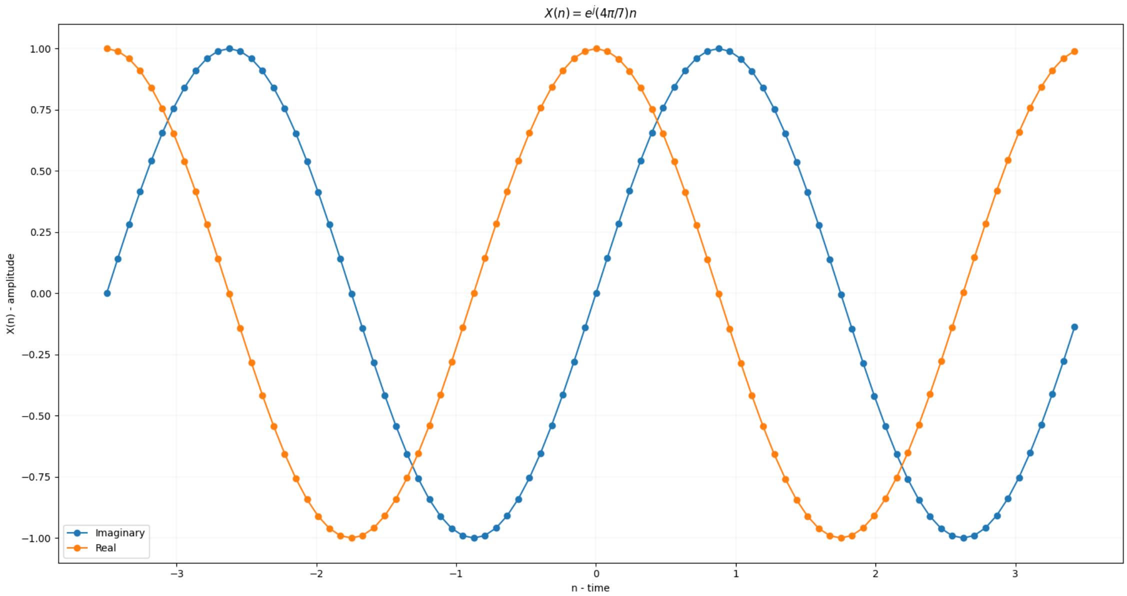

Fig 2

3rd Harmonic Graphical Representation

3rd Harmonic Related Complex Signal

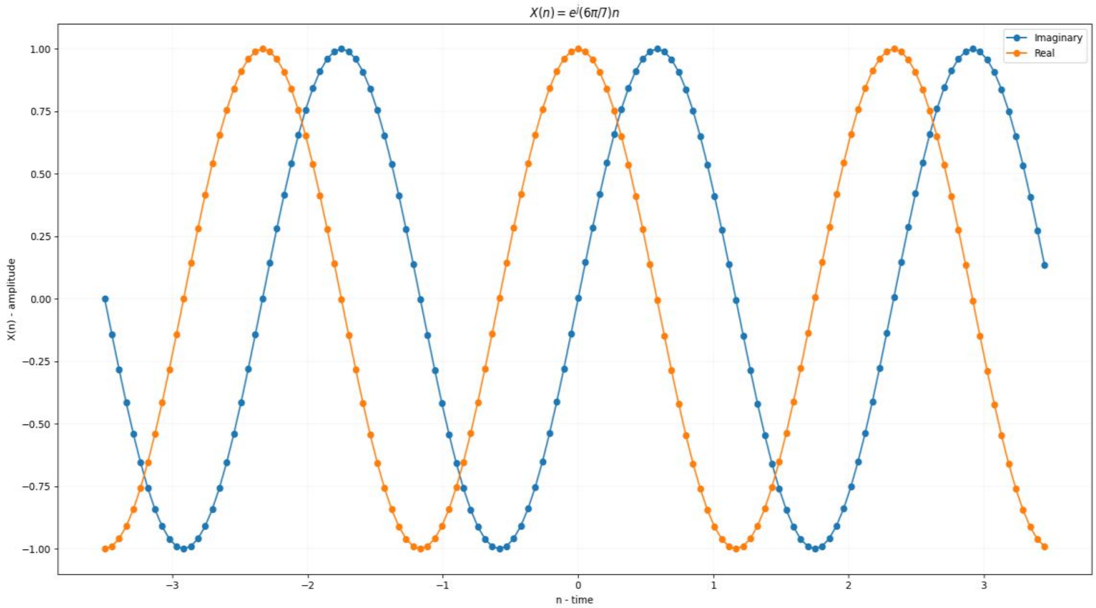

Fig 3

4th Harmonic Graphical Representation

4th Harmonic Related Complex Signal

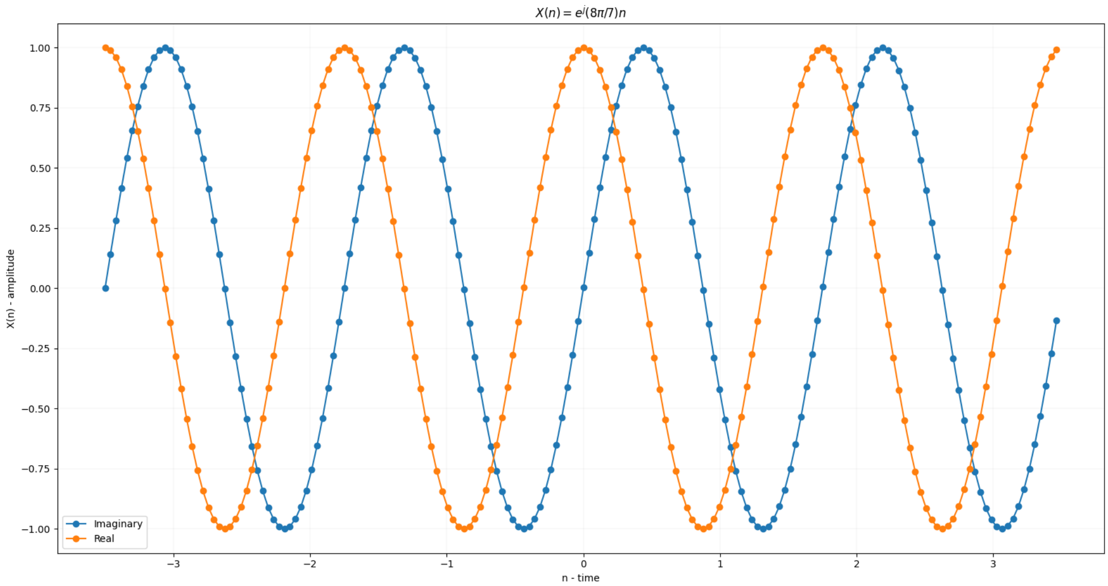

Fig 5

5th Harmonic Graphical Representation

5th Harmonic Related Complex Signal

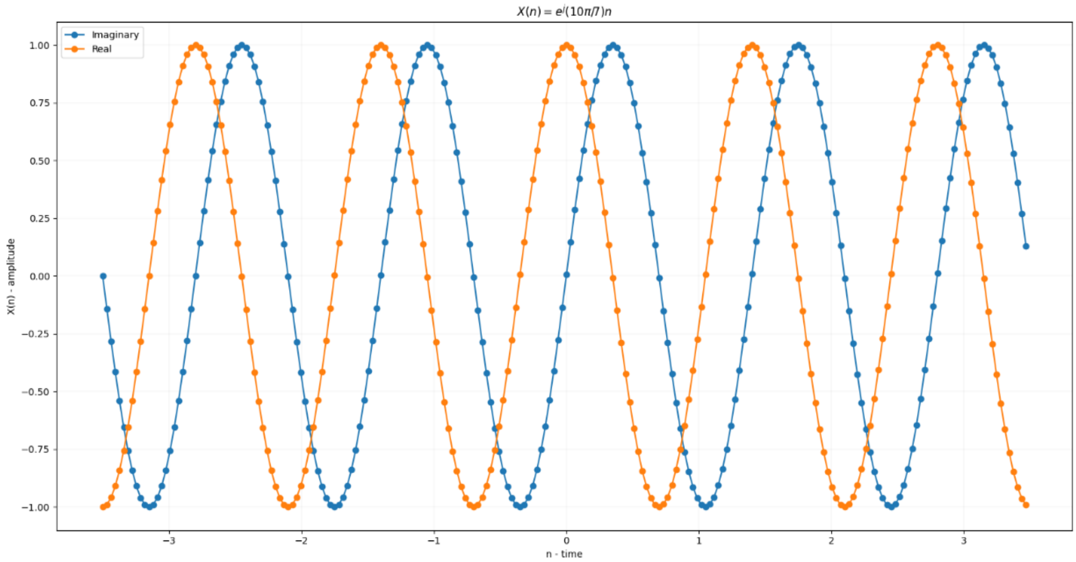

Fig 5

6th Harmonic Graphical Representation

6th Harmonic Related Complex Signal

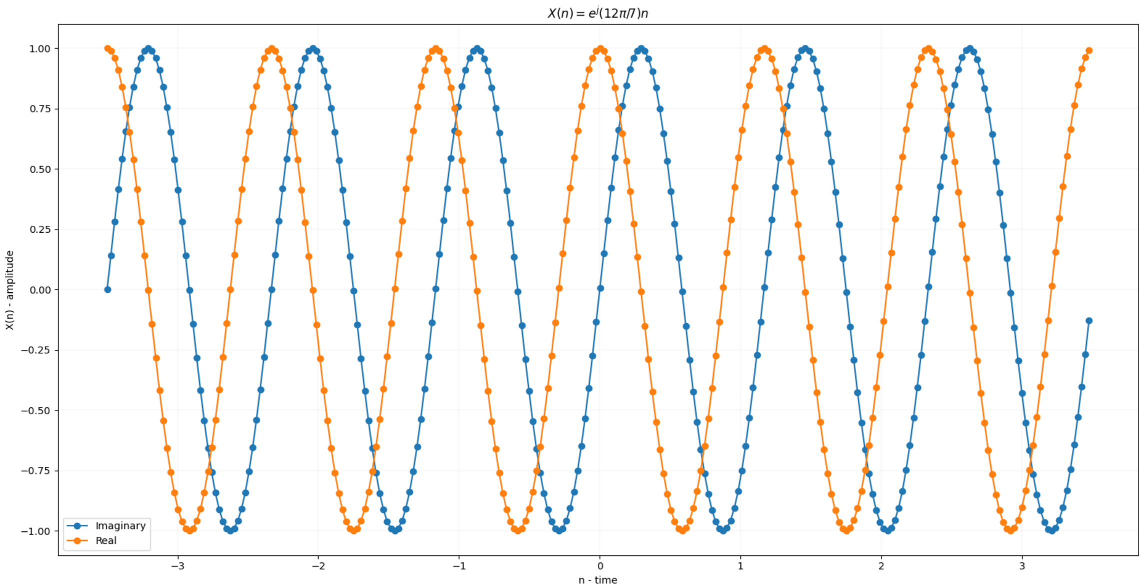

Fig 6

7th Harmonic Graphical Representation

7th Harmonic Related Complex Signal

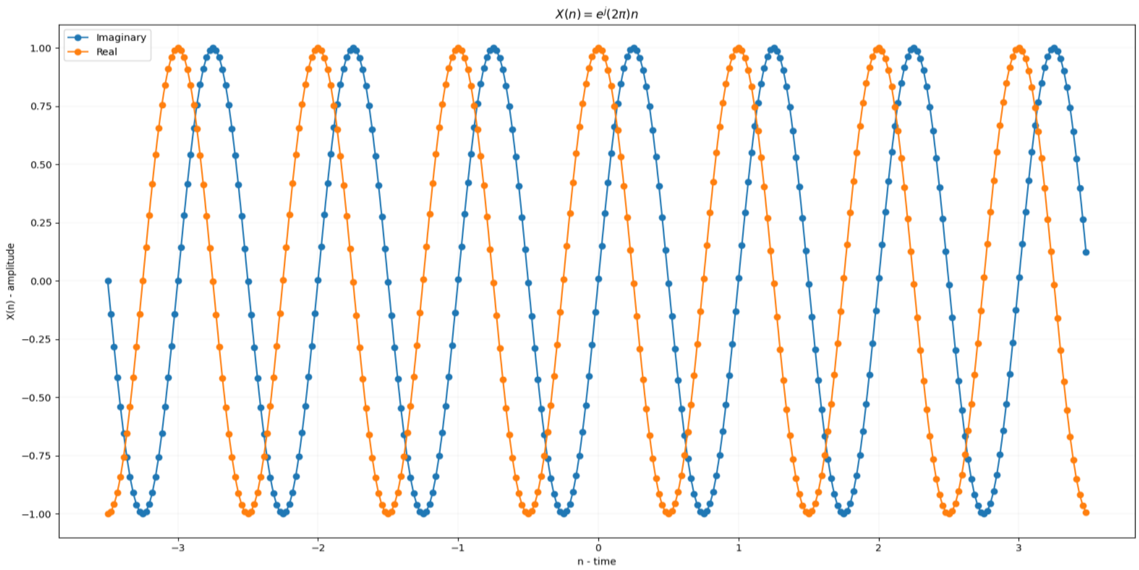

Fig 7

8th Harmonic Graphical Representation

8th Harmonic Related Complex

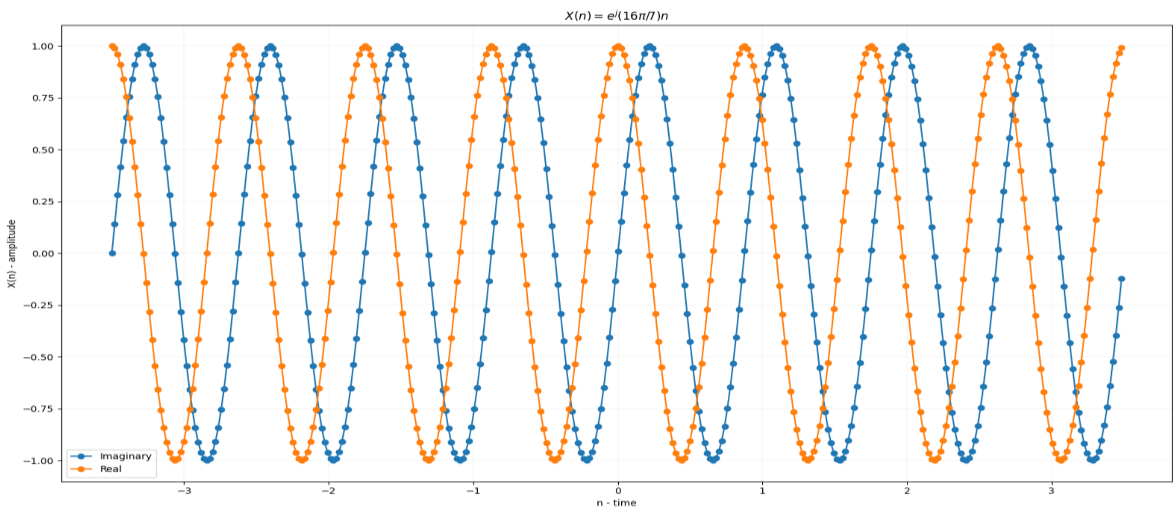

Fig 8

QUESTION 3

Compare the second (m = 2) and fifth (m = 5) members of the family, and judge which one oscillates faster. Justify your answer. Explain how this can be seen from the graphs of the 2 signals.

The is question is about the comparison between 2nd and 5th harmonic member related complex exponential function.
Even though, this member has same period per their frequency, they actual have different cycles and since the frequency is scaled the frequency are also different. From the resulting graph in question 2. We can visually and mathematical compare them as:

1) The 2nd harmonic member oscillated slower compare to the 5th harmonic member.
2) The 2nd harmonic member has longer period to complete a cycle while the 5th harmonic member has shorter period to complete a cycle.
3) 2nd harmonic is of a lower frequency compare to the 5th harmonic member.

QUESTION 4

Compare the fourth (m = 4) and fifth (m = 5) members of the family, and judge which one oscillates faster. Justify your answer. Explain how this can be seen from the graphs of the 2 signals.

Comparison between 4^{th} and 5^{th} harmonic member related complex exponential function are mostly visual and mathematic and will be listed below.

1) The 4^{th} harmonic member oscillated slower compare to the 5^{th} harmonic member by $1/5^{th}$.
2) The 4^{th} harmonic member has longer period to complete a cycle while the 5^{th} harmonic member has shorter period to complete a cycle.
3) 4^{th} harmonic member has lower frequency compare to the 5^{th} harmonic member.

From fig 4 and fig 5, you can see that more cycles where generated in fig 5 compare fig 4 and the time period to complete a cycle also diff.

QUESTION 5

Use MATLAB (or similar technical software) to write your own function to count the zero crossings of the above discrete-time harmonics. The function should be saved as a separate file (.m file in MATLAB), which you can call in order to compute the zero crossings. Present the results in a Table. Explain how these results agree or not with your conclusions in questions 3 and 4.

Solution

In signal processing and mathematical, the term “Zero crossing is used to describe the point at which a function changes sign from positive to negative or negative to positive.
Zero crossing have many advantages in audio engineering and signal processing for finding the phase angle of a signals. Detecting them can be very useful for DC offset correction.

In this question I have designed an algorithm base on my understanding of zero crossing to count the number of crossing from positive to negative using mathematical functions.

This pseudocode as listed can be implemented in any language, but I have decided to implement it in python and attached along size this assessment.

Zero Crossing Algorithm Pseudocode

1) Let **A** the amplitude signal of an input to a function COUNT(**A**).
2) Get the amplitude sign list **S** of the signals **A** and return it.
   If A_k = 0 return 0, A_k < 0 return -1, A_k > 0 return 1, where k is the position of the signal in A.
3) Differential the signs list **S** and return the list of the differentials **D**
4) From the list of **D,** count the number of values that are not $\boldsymbol{D}_k \neq 0$ called **N**
5) Return **N** as the number of zero crossing of the signal **A.**
6) Terminate the function

Testing the Zero Crossing Algorithm with the mathematical function in Question 1.

Harmonic Related Members	Number of Zero Crossing	Number of Oscillated Cycles N=7
1st Member	2	1
2nd Member	4	2
3rd Member	6	3
4th Member	8	4
5th Member	10	5
6th Member	12	6
7th Member	14	7

Table 1

The zero-crossing shown that Question 3 & 4 are correct, the more crossing the faster in oscillation of the signal, the more cycles that are generated per the fundamental period N = 7.

QUESTION 6

Use MATLAB (or similar technical software) to plot the following two signals

1) $X(n) = u[n-4] - u[u+5]$
2) $Y(n) = u[n] * u[-u]$

Solutions

Above functions are linear combination of unit step function $u[x]$ *; a function whose amplitude is 1 for all value of* $x \geq 0$ *and 0 for all value of* $x < 0$.
This can be mathematical described as shown below.

$$u(x) = \begin{cases} 1, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

Therefore, with the understanding of unit step functions we can mathematical, and logically evaluate the two equation above.

Question 6.1 Solution

1) $X(n) = u[n-4] - u[u+5]$

X(n) is a different of two-unit function and need to be evaluated separately before we can resolve them together for understanding purpose.

$u[n-4]$ is a time shift to the right by 4 units, recall that unit function by default have amp of 1 throughout when n >= 0

Evaluation of signal $u[n-4]$

If ($-\infty < n < 0$) then *amplitude of* u = 0
If ($0 < n < (n-4)$) then *amplitude of* u = 0
If ($(n-4) < n < \infty$) then *amplitude of* u = 1

The graphically representation of the unit signal $u[n-4]$
Is shown below.

Unit Function u (n - 4)

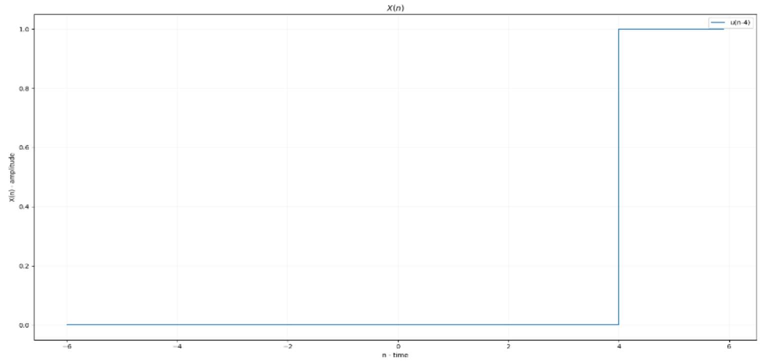

Fig 9.

Evaluation of signal $u[n+5]$, this is a left shifting of the signal u to the left by 5 unit from the origin.

If ($-\infty < n < -5$) then *amplitude* $u = 0$
If ($-5 < n < 0$)) then *amplitude* $u = 1$
If ($0 < n < \infty$) then *amplitude* $u = 1$

And can be graphically shown below

$$\boldsymbol{u[n+5]}$$

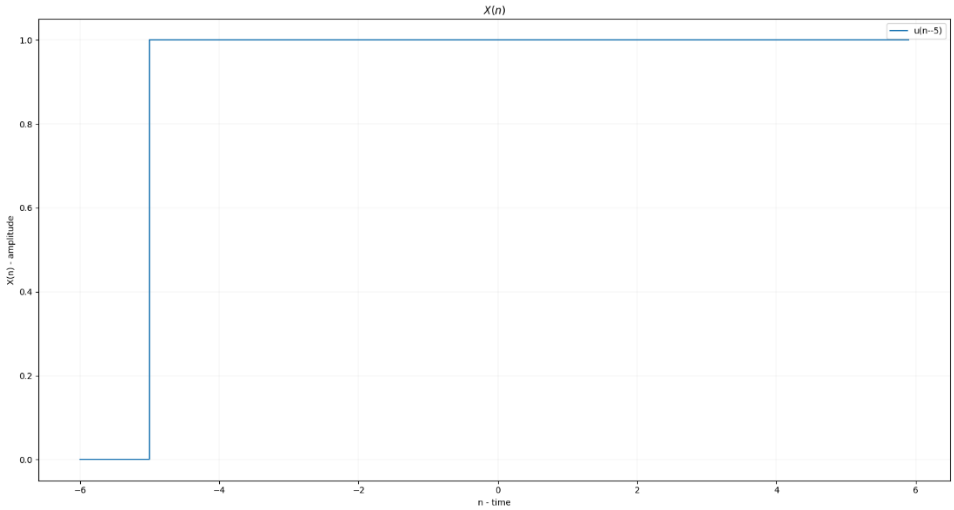

Fig 10.

Therefore $X(n) = u[n-4] - u[u+5]$ *can now be evaluated as shown below*

If ($-\infty < n < -5$) then *amplitude* $u[n-4] - u[u+5]$ = 0 + 0 = 0
If ($-5 < n < 0$)) then *amplitude* $u[n-4] - u[u+5]$ = 0 + 1 = 1
If ($0 < n < 4$)) then *amplitude* $u[n-4] - u[u+5]$ = 0 + 1 = 1
If ($4 < n < \infty$) then *amplitude* $u[n-4] - u[u+5]$ = 1 + 1 = 2

The signal can be tabularly depicted as show below:

$$\boldsymbol{X(n) = u[n-4] - u[u+5]}$$

n	-6	-5	0	4	∞
X(n)	0	1	1	2	2

Table 2

and can be graphically shown below draw using python as shown below:

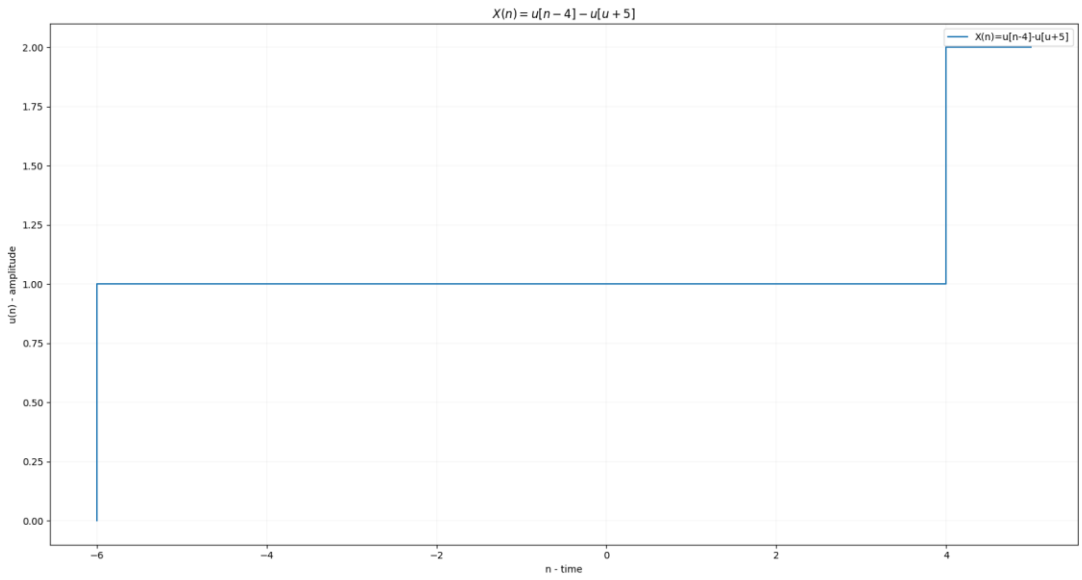

Fig 11.

Question 6.2

$Y(n) = u[n] * u[-u]$ the Y(n) signal is the linear multiplication of two-unit step function. $u[n]$ and $u[-u]$. But for we to evaluate this function, we will individually evaluate them as we did in 6.1

Equation (a) $u[n]$

a) $u[n]$; per the definition of unit step function, we can say that

If $(-\infty < n < 0)$ then *amplitude* $u[n]$ = 0
If $(0 < n < \infty))$ then *amplitude* $u[n]$ = 1

And now, let try to evaluate unit function $u[-u]$,you will agree with the writer that this is odd. But if you look at it in a different angle, you will conclude that $-u$ it's just the amplitude inverse $of\ u[n]$. Since we already knew the amplitude values of unit $u[n]$, doing the inverse is a bit simple as show below:

If ($-\infty < n < 0$) then *amplitude* $-u[n]$ = 0
If ($0 < n < \infty$)) then *amplitude* $-u[n]$ = -1

Recall that, $-u[n]$ is this $-u$ which is the input value of the function $u[-u]$ therefore we can carry the evaluation of the outermost function $\boldsymbol{u}[-u]$.

If ($-\infty < n < 0$) then *amplitude* $-u[n]$ = 0, therefore $u[(-u) = 0]$ = 1
If ($0 < n < \infty$)) then *amplitude* $-u[n]$ = -1, therefore $u[(-u) = -1]$ = 0

Now that the functions are evaluated $u[n]\ and\ u[-u]$, we can then multiply to have the function $Y(n)$ as shown below:

If ($-\infty < n < 0$) , $u[n] * u[-u]$ = 0 * 1 = 0
If ($0 < n < \infty$) , $u[n] * u[-u]$ = 1 * 0 = 0

The $Y(n) = u[n] * u[-u]$ values are all zero from $-\infty < n < \infty$ and can be depicted into a table as show below

$\boldsymbol{Y(n)}$

n	$-\infty$	n	∞
Y(n)	0	0	0

Table 3

This values in table 3 can be graphical plot using the python programming language as show below

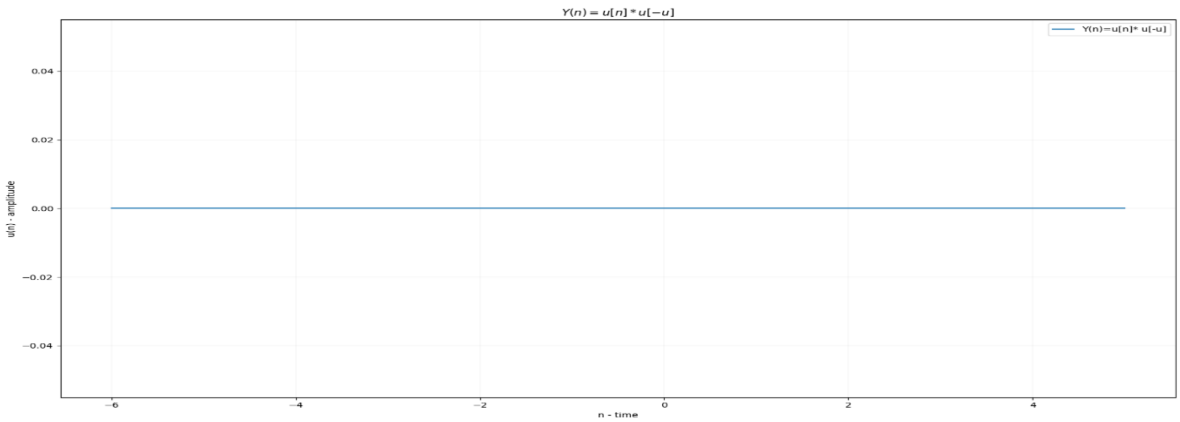

Fig 12.

Function **Y(n)** has zero pressure at all instance of n.

Conclusion

The report has demonstrate the knowledge of the writer in area of harmonic related member function of a fundament basic signal. He has concluded that harmonic related members are simply achieved by just scaling the fundamental frequency ω of the basic signal function. He has also noticed that harmonic members can either be periodic and non-periodic.
Furthermore, he has demonstrated from a practical side of view that the higher the frequencies the higher the oscillation of the signal. That, it is also true that a periodic signal T are multiply of complete cycles.

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