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Harmonic Analysis and its Mathematical Philosophy

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Abstract- This paper looks into the mathematics of harmonics. It is of no doubt that engineering without mathematics is a like well without water. Knowing the mathematics behind the study of harmonics could pave way to knowing its effects in power network. Consequently, this paper work derived the mathematics involving harmonics from the study of previous work done, and teaching. It concludes by calculating the Total Harmonic Distortion on electrical devices.

Keywords- *Harmonic, mathematics, electrical power system*

I. INTRODUCTION

Harmonic is as old as electrical power system itself. Since the establishment of AC generator, harmonics has been a threat to electrical power system at level in different degrees. It is of no doubt that this occurrence contributes negatively to the economy advantage of power network. Therefore, its study cannot be over emphasized.

Power system quality is a daily questionable occurrence in a country like Nigeria. The problem of insufficient power generation poses challenges on its own. But beside these are numerous factors contributing to the imbalance of sinusoidal waveform of the total power generation. Harmonics distortion, flickers, voltage imbalances, transient interruptions, voltage swells, etc., are various issues that determines the power system quality of any region (Farooq et al., 2012). Among these entire aforementioned unavoidable occurrence in power system, harmonics distortion seems to be the most experienced due to technological advancement in electronics equipment. The non-linear loads and sophisticated flexible AC transmission system (FACTs) devices do contribute immensely to power quality problem. These modern days equipment introduces non-sinusoidal currents into the entire system. Electrical loads which often introduce harmonics includes motor drives rectifying circuits, energy saving lamps, personal computers, and many other devices which can be found in any home today.

Harmonic simply mean distortion of the electrical signal waveform as a result of non-linear load. The harmonic currents generated by non-linear electronic loads, increase power system losses due to heat, and invariably causes increase in power bills of end-users. These harmonic related losses reduce system efficiency, causes apparatus/equipment overheating. (Lalotra, 2013)

Harmonics in power systems mean the existence of signals, superimposed on the fundamental signal, whose frequencies are integer numbers of the fundamental frequency. The presence of harmonics in the voltage or current waveform leads to a distorted signal for voltage or current, and the signal becomes non-sinusoidal signal which it should not be (Soliman, and Ahmad, 2011).

II. THE HARMONIC ANALYSIS

The process of determining the magnitude, order, and phase of several harmonic present in a complex periodic function is called harmonic analysis. From electrical power generation to distribution, the various source of harmonics include the following; flux distortion in the synchronous machine from sudden load changes, tooth ripple or ripples in the voltage waveform of rotating machines, variations in air-gap reluctance over synchronous machine pole-pitch, non-sinusoidal of the flux in the gap of synchronous machines, transformer magnetizing currents, network non-linearity from loads such as rectifiers, inverters, welder, arc furnaces, voltage compensators, frequency converters, etc. (Soliman, and Ahmad, 2011).

Estimation of harmonic in static cases assumed that

1. Waveform consists of a fundamental frequency and harmonic components with order of integral multiples of the fundamental frequency.
2. The frequency is known and constant during the estimation period

Considering a non-sinusoidal voltage given by a Fourier type equation;

$$x(t) = \sum_{k=0}^{N} x_k sin({}_k w_0\, t + \emptyset_k) \quad (1)$$

Expanding equation 1 gives:

$$x(t) = x_k \sum_{k=0}^{N} \sin({}_k w_o t)\, cos\emptyset_k + cos\,({}_k w_0 t) sin\emptyset_k \quad (2)$$

where N = total number of harmonics

k = order of harmonic

w_0 = the fundamental frequency

$\emptyset_k$ = the phase angle of harmonic k

$x(t)$ = electrical signal

x_k = electrical amplitude of harmonic k.

The constants of equation 2 can be represented by some letters so as to reduce the equation to desired form. Let $x_k sin\emptyset_k\ be\ A$ and $x_k\ cos\emptyset_k be\ B$. Substituting A and B into equation 2, gives:

$$x(t) = \sum_{k=0}^{N} A\sin({}_{k}w_o t) + \ Bcos\ ({}_{k}w_0 t) \quad (3)$$

At time t = 0, equation 3 reduces further to

$$x(0) = A(0) \quad (4)$$

Consequently, the desired equation required to analyze the harmonic is given as

$$x(t) = A(0) + \sum_{k=1}^{N} A\sin({}_{k}w_o t) + \ Bcos\ ({}_{k}w_0 t) \quad (5)$$

Equation 5 is a trigonometric form of a finite series.

III. THE PERIODIC FUNCTION AND THE TRIGONOMETRIC FOURIER SERIES

A function is said to be periodic if $x(t) = \ x(T+t)$ for all values of t, where T is some positive number. This T is the interval between two successive repetitions and it is called the period of $x(t)$. A sine wave having a period of $T = \frac{2\pi}{w}$ is a common example of periodic function. The function $x(t)$ represents either voltage or current waveform. The trigonometric series of $x(t)$ is given in equation 5. Putting $w = \frac{2\pi}{T}$ into equation 5, gives:

$$x(t) = A_0 + \sum_{k=1}^{N} \left[A\sin({}_{k}\frac{2\pi}{T}t) + \ Bcos\ ({}_{k}\frac{2\pi}{T}t)\ \right] \quad (6)$$

where T is the period.

The trigonometric equation 6 is equivalent to equation 7:

$$x(t) = A_0 + \sum_{k=1}^{N} A_k(\cos(k\frac{2\pi}{T}t - \ \emptyset_k) \quad (7)$$

where $A_k \ = \sqrt{A^2 + \ B^2}$ (8)

$$\emptyset_k = \ tan^{-1}\left(\frac{B}{A}\right) \quad (9)$$

IV. DEFINNING THE FOURIER COEFFICIENTS

Equation 6 can be integrated as follows:

Let $\frac{2\pi}{T}t\ be\ \theta$, for $\theta = 0, to\ \theta = 2\pi;$

$$\int_0^{2\pi} x(\theta)d\theta \ = \int_0^{2\pi} A_0\, d\theta +$$

$$\int_0^{2\pi} A_1 sin\theta d\theta + \cdots \ldots A_k \int_0^{2\pi} sin\theta d\theta \ldots + \ \int_0^{2\pi} B_1 cos\theta d\theta +$$

$$\cdots \ldots \int_0^{2\pi} B_k cos\theta d\theta$$

Consequently, $A_0 = \frac{1}{T}\int_{-T/2}^{T/2} x(\theta)d\theta$

Following the same process;

$$A_k = \frac{1}{T}\int_{-T/2}^{T/2} x(\theta)sin\theta d\theta$$

$$B_k = \frac{1}{T}\int_{-T/2}^{T/2} x(\theta)cos\theta d\theta$$

A_0 is the dc component of the signal while the $A_k and\ B_k$ are the harmonic components of the signal.

V. TOTAL HARMONIC DISTORTION

Harmonics have frequencies that are integer multiples of the waveform's fundamental frequency. For example, given a 50Hz fundamental waveform, the 2nd, 3rd, 4th and 5th harmonic components will be at 100Hz, 150Hz, 200Hz and 250Hz respectively (Bingham, 2000). The total harmonic distortion (THD) is defined as the ratio of the RMS value of the waveform excluding the fundamental components, to the RMS fundamental magnitude.

$$THD = \frac{\sum_{k=1}^{N}\sqrt{(V_n)^2}}{V_0} \quad (10)$$

where V_n is the RMS voltage of the nth harmonics, and the V_0 is the RMS value of the fundamental voltage. A typical voltage waveform doesn't exceed 5% THD (Key T and Jih-Sheng,).

If a 10V is added to the sine-wave of 230V, the THD would be equal to 4.35%.The distortion caused by the 10V is this 4.35%.

The study of Total Harmonic Distortion helps in understanding the thermal effects of harmonics on electrical devices and power systems in general.

VI. CONCLUSION

The best approach to the analysis of harmonics in electrical power system is to understand the mathematics behind it. The mathematical derivation was able to provide the information required to analyze electrical signals. The dc components and harmonic components were successfully separated and this is what is needed to calculate the total harmonic distortion on the signal.

VII. REFERENCES

[1] Bingham, R. P. (2000.). HARMONICS - Understanding the Facts.

[2] Farooq, H., Zhou, C., & Emad, M. (2012). Analyzing the Harmonic Distortion in a Distribution System Caused by the Non-Linear Residential Loads.

[3] Key, T. (n.d.). Costs And Benefits of Harmonic Current Reduction For Switch -Mode Power Supplies In A Commercial Office Building, 1–9.

[4] Lalotra, J. (2013). International Journal of Advanced Research in Investigate the Effect of Different Combination of Loads on Harmonic Distortion, *3*(8), 587–590.

[5] Soliman, S. A. (2011.). Electric Power Systems Harmonics - Identification and Measurements.