HW #2

ECE 472: Robotics and Computer Vision
Instructor: Kristen Dana, Fall 2019
Rutgers University

Seth Karten
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Problem 3

Are the following vectors basis vectors for $\mathbb{R}^3$? Why or why not?

$$^{B}P = {}^{B}M_{A}{}^{A}P$$

We substitute,

$$^{B}M_A = \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 & 1 \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

and we append a 1 to ^{A}P to get,

$$^{A}P = \begin{bmatrix} 1 \\ 0 \\ 2 \\ 1 \end{bmatrix}.$$

By simplifying we have,

$$^{B}P = \begin{bmatrix} \frac{\sqrt{2}}{2} + 1 \\ 1 - \frac{\sqrt{2}}{2} \\ 2 \end{bmatrix}.$$

Problem 4

First we solve for $^{B}R_A$,

$$^{B}R_A = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{bmatrix}.$$

To get $^{A}R_B$, we invert $^{B}R_A$ by transposing,

$$^{A}R_B = \begin{bmatrix} 0 & 0 & -1 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}.$$

4a

The second column of ${}^{A}R_B$ gives us $\hat{y}_B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$.

4b

The second column of ${}^{B}R_A$ gives us $\hat{y}_A = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$.

4c

To find ${}^{B}P$, we must solve,

$$ {}^{B}P = {}^{B}M_A {}^{A}P = \begin{bmatrix} {}^{B}R_A & {}^{B}t_A \end{bmatrix} {}^{A}P. $$

Substituting, we have,

$$ {}^{B}P = \begin{bmatrix} 0 & -1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ -1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 10 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 11 \\ -1 \end{bmatrix}. $$

Problem 5

First we solve for ${}^{B}R_A$,

$$ {}^{B}R_A = \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ 0 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ -1 & 0 & 0 \end{bmatrix}. $$

To get ${}^{A}R_B$, we invert ${}^{B}R_A$ by transposing,

$$ {}^{A}R_B = \begin{bmatrix} 0 & 0 & -1 \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \end{bmatrix}. $$

5a

The first column of ${}^{A}R_B$ gives us $\hat{x}_B = \begin{bmatrix} 0 \\ -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \end{bmatrix}$.

5b

The first column of ${}^B R_A$ gives us $\hat{x}_B = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$.

5c

To find ${}^B P$, we must solve,

$$ {}^B P = {}^B M_A {}^A P = {}^B R_A ({}^B t_A + {}^A P). $$

Substituting, we have,

$$ {}^B P = \begin{bmatrix} 0 & -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ 0 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 10 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -\sqrt{2} \\ \sqrt{2} \\ -10 \end{bmatrix}. $$

Problem 6

6a

The rotation matrix: ${}^A R_B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

6b

$$ {}^B P = {}^B M_A {}^A P = \begin{bmatrix} {}^B R_A & {}^B t_A \end{bmatrix} {}^A P = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}. $$

Problem 7

7a

The rotation matrix: ${}^A R_B = \begin{bmatrix} -\frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \\ 0 & 1 & 0 \end{bmatrix}$.

7b

The translation vector: ${}^A t_B = \begin{bmatrix} 9 \\ 0 \\ -4 \end{bmatrix}$.