

Part 3 : Gradient Descent Manual Calculations

X	Y	$m_i = -1$	$\hat{y}_i = mx + b$
1	3	$b_i = 1$	$MSE = \frac{1}{n} \sum (y - \hat{y})^2$
3	6	$d = 0.1$	$m_{\text{new}} = m_{\text{old}} - d \frac{\partial MSE}{\partial m}$ $b_{\text{new}} = b_{\text{old}} - d \frac{\partial MSE}{\partial b}$

Step 1 : derive MSE with respect to m and b

* with m

$$MSE = \frac{1}{n} \sum (y - \hat{y})^2$$

$$= \frac{1}{n} \sum (y - (mx + b))^2$$

$$\text{Let } u = y - (mx + b)$$

$$\frac{\partial u}{\partial m} = -x$$

$$\text{So } u^2 = \frac{\partial u}{\partial m}$$

$$\text{hence } \frac{\partial MSE}{\partial m} = \frac{1}{n} \sum 2u \cdot (-x)$$

$$= \frac{1}{n} \sum 2(y - (mx + b)) \cdot (-x)$$

$$= \frac{2}{n} \sum ((mx + b) - y_i)(x)$$

with b

$$MSE = \frac{1}{n} \sum (y - \hat{y})^2$$

$$= \frac{1}{n} \sum (y - (mx + b))^2$$

$$\text{Let } u = y - (mx + b)$$

$$\frac{\partial u}{\partial b} = -1$$

$$\text{So } u^2 = \frac{\partial u}{\partial b}$$

$$\frac{\partial MSE}{\partial b} = \frac{1}{n} \sum 2u \cdot (-1)$$

$$= \frac{1}{n} \sum 2(y - (mx + b)) \cdot (-1)$$

$$= \frac{1}{n} \sum 2((mx + b) - y_i)$$

$$\boxed{\frac{\partial MSE}{\partial m} = \frac{2}{n} \sum ((mx + b) - y_i)(x)}$$

$$\boxed{\frac{\partial MSE}{\partial b} = \frac{2}{n} \sum ((mx + b) - y_i)}$$

$$m_{\text{new}} = m_{\text{old}} - d \left(\frac{2}{n} \sum ((mx + b) - y_i)(x) \right)$$

$$b_{\text{new}} = b_{\text{old}} - d \left(\frac{2}{n} \sum ((mx + b) - y_i) \right)$$

Step 2 : Iterations

Iteration 1

$$\hat{y}_1 = -1(1) + 1 = 0$$

$$\hat{y}_2 = -1(3) + 1 = -2$$

$$MSE = \frac{((3-0)^2 + (6-(-2))^2)}{2} = \boxed{\frac{20}{2}}$$



$$= \frac{9+64}{2} = \frac{73}{2}$$

$$= 36.5$$

Iteration 2

$$\hat{y}_1 = 1.7(1) + 2.1 = 3.8$$

$$\hat{y}_2 = 1.7(3) + 2.1 = 7.2$$

$$MSE = \frac{((3-3.8)^2 + (6-7.2)^2)}{2}$$

$$= \frac{(-0.8)^2 + (-1.2)^2}{2} = 1$$

$$= \frac{0.64 + 1.44}{2} = 1.04$$

Iteration 3

$$\hat{y}_1 = 1.26(1) + 1.8 = 3.16$$

$$\hat{y}_2 = 1.26(3) + 1.8 = 5.68$$

$$MSE = \frac{((3-3.16)^2 + (6-5.68)^2)}{2}$$

$$= \frac{(-0.16)^2 + (0.32)^2}{2} = 0.0956 + 0.1024$$

$$= \frac{0.0956 + 0.1024}{2} = 0.094$$

update values of m and b

$$m \rightarrow -1 - (0.1)(-2)$$

$$\frac{\partial MSE}{\partial m} = \frac{2}{2} ((0-3)(1) + (-2-6)(3))$$

$$= (-3 + (-24)) = -27$$

$$\frac{\partial MSE}{\partial b} = \frac{2}{2} ((0-3) + (-2-6))$$

$$= -3 + (-8) = -11$$

$$m_{new} = -1 - (0.1)(-27) = 1.7$$

$$b_{new} = 1 - (0.1)(-11) = 2.1$$

update values m and b

$$\frac{\partial MSE}{\partial m} = \frac{2}{2} ((3.8-3)(1) + (7.2-6)(3))$$

$$= (0.8 \times 1) + (1.2 \times 3) = 4.4$$

$$\frac{\partial MSE}{\partial b} = \frac{2}{2} ((3.8-3) + (7.2-6))$$

$$= 0.8 + 1.2 = 2$$

$$m_{new} = 1.7 - (0.1)(4.4) = 1.26$$

$$b_{new} = 2.1 - (0.1)(2) = 1.9$$

update values m and b

$$\frac{\partial MSE}{\partial m} = \frac{2}{2} ((3.16-3)(1) + (5.68-6)(3))$$

$$= 0.16 + (-0.86) = -0.8$$

$$\frac{\partial MSE}{\partial b} = \frac{2}{2} ((3.16-3) + (5.68-6))$$

$$= 0.16 + (-0.32) = -0.16$$

$$m_{new} = 1.26 - (0.1)(-0.8)$$

$$= 1.26 + 0.08 = 1.34$$

$$b_{new} = 1.8 - (0.1)(-0.16)$$

$$= 1.8 + 0.016 = 1.816$$

Iteration 4

$$\hat{y}_1 = 1.34(1) + 1.816 = 3.256$$

$$\hat{y}_2 = 1.34(3) + 1.816 = 5.836$$

$$MSE = \frac{(3 - 3.256)^2 + (6 - 5.836)^2}{2}$$

$$= \frac{0.065536}{2} + 0.004096$$

$$\Rightarrow = 0.034816$$

Iteration 5

$$\hat{y}_1 = 1.3336(1) + 1.8868 = 3.2304$$

$$\hat{y}_2 = 1.3336(3) + 1.8868 = 5.8876$$

$$MSE = \frac{(3 - 3.2304)^2 + (6 - 5.8876)^2}{2}$$

$$= \frac{0.053084}{2} + 0.010448$$

$$= 0.031785$$

Update Values m and b

$$\frac{\partial MSE}{\partial m} = \frac{1}{2} ((3.256 - 3)(1) + (5.836 - 6)) \\ = 0.256 + (-0.182) = 0.064$$

$$\frac{\partial MSE}{\partial b} = \frac{1}{2} (3.256 - 3) + (5.836 - 6) \\ = 0.256 - 0.064 \\ = 0.192$$

$$m_{new} = 1.34 - (0.1)(0.064)$$

$$= \underline{1.3336}$$

$$b_{new} = 1.816 - (0.1)(0.192) \\ = \underline{1.8868}$$

Conclusion :

- MSE Trend : After each iteration, the values of MSE decreases which indicates that our gradient descent is updating m and b well.

- m and b : both m and b are reducing the MSE after each iteration when we update them.