Nonlinear Dynamics in the Periodically Forced Bouncing Car: An Experimental and Theoretical Study.

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ABSTRACT

We consider, experimentally and theoretically, a mechanical system consisting of a sliding car on an inclined plane that bounces on an oscillating piston. The main aim of our study is to reproduce the different types of orbits displayed by nonlinear dynamical systems. In particular, we are looking to identify the parameters and initial conditions for which periodic and chaotic (irregular) behavior are exhibited. The data in our experimental model is collected by using infrared lights that are monitored by a Nintendo Wii remote which is linked to a laptop computer via Bluetooth. By varying the piston's frequency and amplitude, it is possible to produce, for relatively small amplitudes, periodic orbits. As the amplitude of the piston is increased (for a fixed piston frequency) we observe bifurcations where the original, stable, periodic orbit is destabilized and replaced by a higher order periodic orbit. For larger amplitudes, periodic orbits are destabilized and replaced by chaotic trajectories. After carefully measuring all experimental parameters, we are able to successfully produce periodic orbits (up to period 3), sticking solutions (the car does not bounce, but gets stuck to the piston), and seemingly chaotic (irregular/ unpredictable) behavior that are in very good agreement with the model for the same parameter values.

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INTRODUCTION

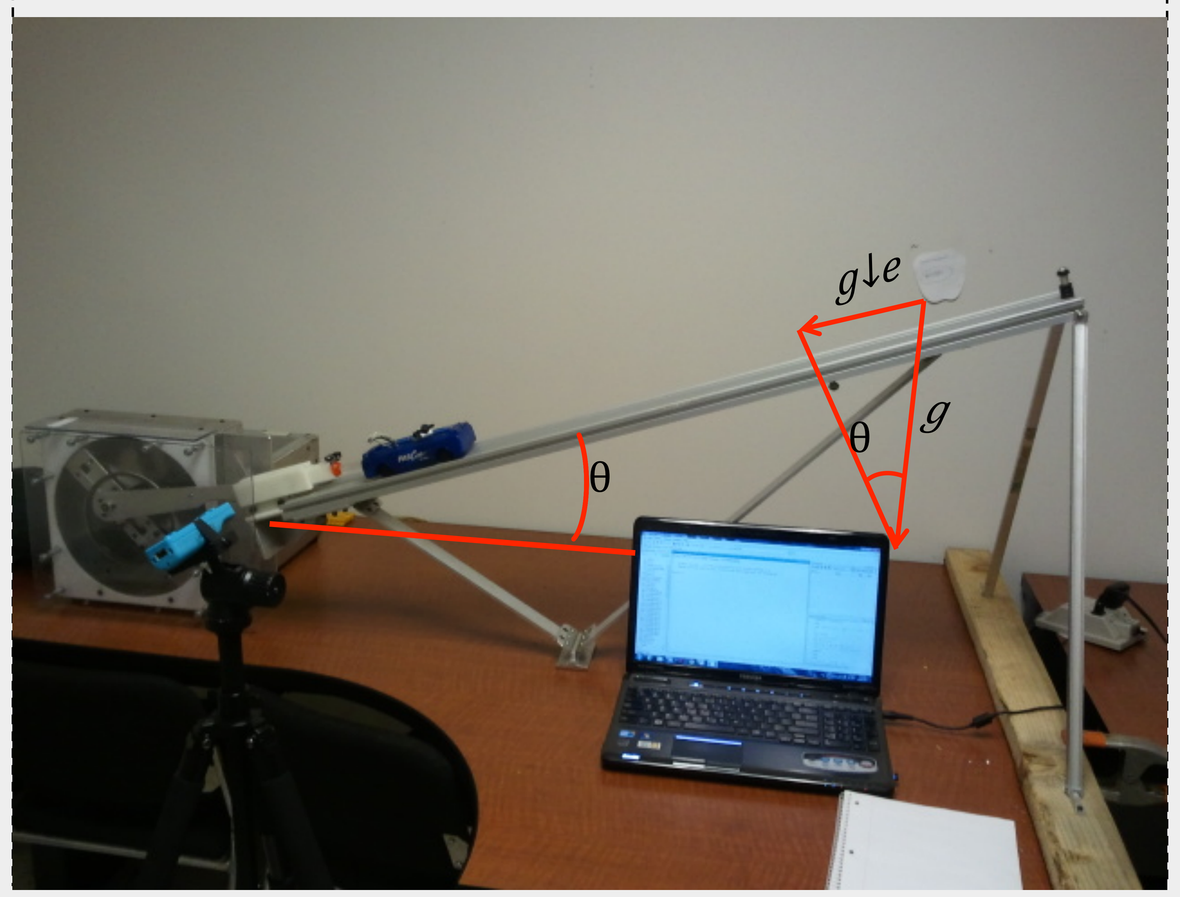
Chaos: In mathematics, chaos is defined as seemingly stochastic behavior occurring in deterministic systems. This seemingly stochastic behavior is found to be caused by sensitive dependence to initial conditions; namely, small (infinitesimal) errors in the prescription of the initial conditions are amplified exponentially. This sensitive dependence to initial conditions is the major obstacle when dealing with chaotic systems since it precludes our ability to obtain long-term predictions. Chaos theory, in general, tries to explain naturally occurring phenomena such as population growth, which displays seemingly random behavior although its dynamics are inherently deterministic.

Our Goal: We hope to identify the parameters for which the different types of orbits (periodic, chaotic, etc.) in nonlinear dynamical systems are exhibited. Furthermore, we are also working on finding, experimentally, orbits that co-exist for the same parameter values by conducting frequency sweeps of our system. Understanding how and when chaos emerges in simple systems will be very useful to understanding other, more complex, phenomena.

MATERIALS

Our experimental system consists of:

- A Nintendo Wii remote
- Bluetooth reciever
- Infrared lights
- A computer
- A sliding car on an inclined plane
- An oscillating piston.



Equations

The following equations were used in our theoretical model:

- The trajectory of the car, due to gravity, is given by: $x(t)=-g_e(t^2/2)+v_0t+x_0$. (1)
- The piston follows the harmonic oscillation: $s(t)=A[\sin(\omega t+\theta_0)+1]$. (2)
- The distance between the car and piston is: $d(t)=x(t)-s(t)$. (3)
- Substituting $x(t)$ and $s(t)$ for an impact time ($d(t_k)=0$) yields:
- $0=A[\sin(\theta_k)+1]+v_k[1/\omega(\theta_{k+1}-\theta_k)]-1/2\,g_e[1/\omega(\theta_{k+1}-\theta_k)]^2-A[\sin(\theta_{k+1})+1]$. (4)
- This equation implicitly gives θ_{k+1} .
- After each collision, the velocity has to be adjusted (due to dissipation during impact) by using:
- $v_k=(1+\alpha)u_k-\alpha v'_k$ (where u_k is the velocity of the piston). (5)
- Taking the derivative of the piston's position and the car's position, and substituting their values into equation (5): $v_{k+1}=(1+\alpha)A\omega\cos(\theta_{k+1})-\alpha\{v_k-g_e[1/\omega(\theta_{k+1}-\theta_k)]\}$ (6)
- This equation gives the next impact velocity
- From equations (4) and (6) we get (θ_{k+1} , v_{k+1}), the next impact between the car and piston. These impacts are plotted on an impact map. A Period-n orbit will have n points on its impact map.

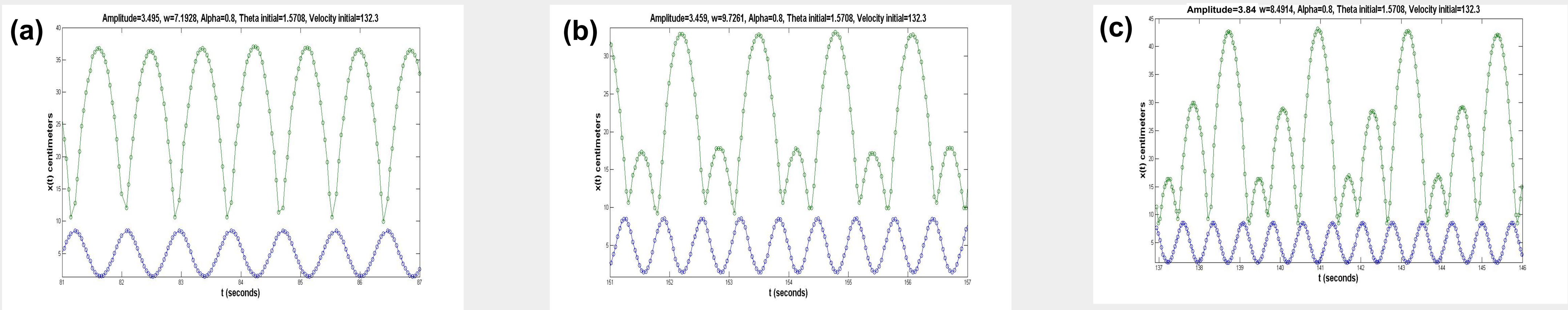


Figure 1. Experimental results. (a) Period-1 orbit. (b) Period-2 orbit and (c) Period-3 orbit

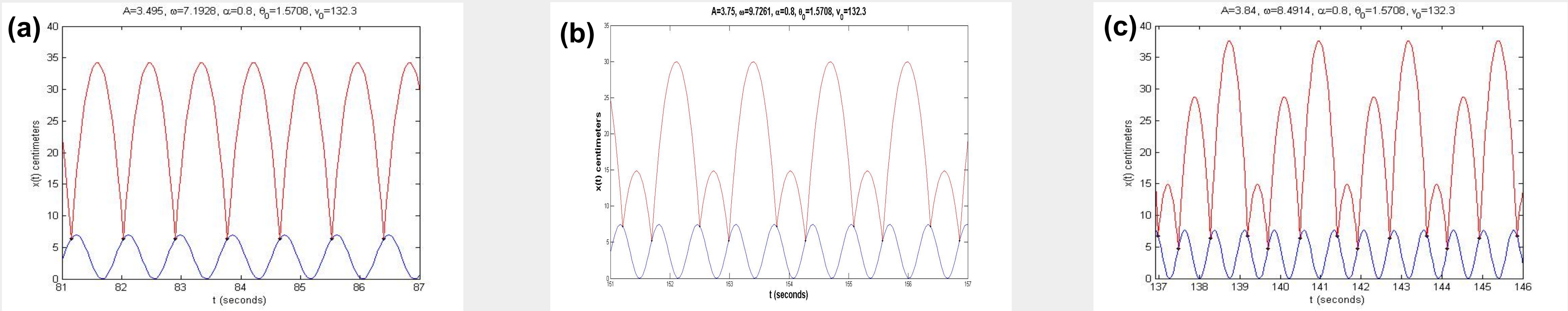


Figure 2. Same as in Figure 1 for our theoretical model.

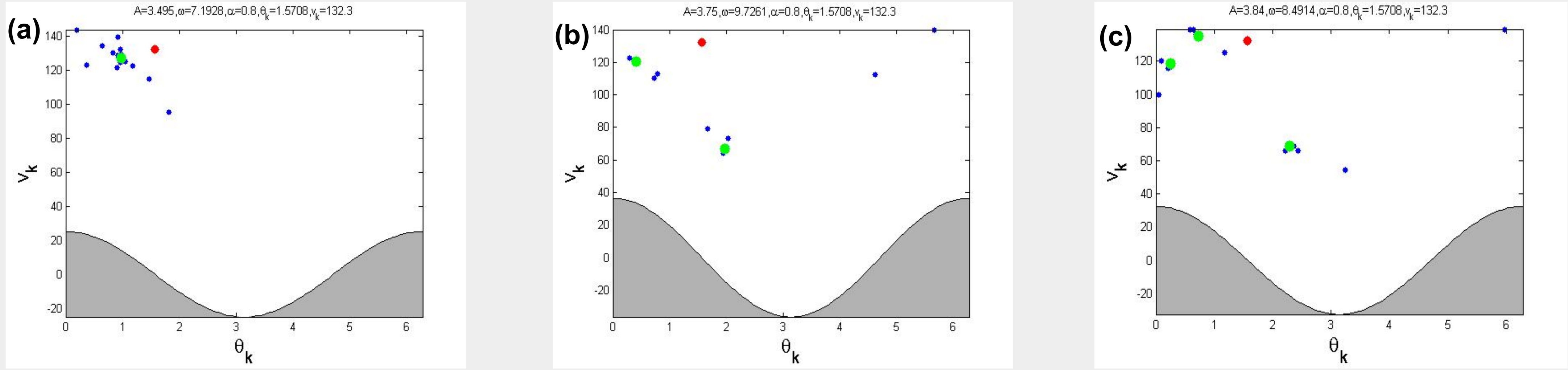


Figure 3. Impact maps corresponding to Figure 1. Red markers correspond to the initial condition, green marker/s to the final periodic orbit, and blue markers correspond to the transient orbit. n green markers correspond to Period-n orbit.

Results

- We have been able to produce experimentally period-1, period-2, period-3, sticking solutions, and seemingly chaotic behaviors.
- By fitting the experimental parameters we observe good quantitative agreement between the experimental and theoretical models as seen in Figs.1-3.
- Some of our experimental results show discrepancies in amplitude due to a spring attached to the back of the sliding car. This spring adds up to 3 centimeters of error to our experimental results as well as creates a gap between the parabolic trajectory and oscillating sine function on the graph.

Future Work

- Modeling the spring that is attached to the back of the sliding car. This will give us a better model for our system (it will also give us the experimental graphs with less errors in the amplitude).
- Identifying the parameters for which there are co-existing orbits (distinct orbits that co-exist for the same parameter values). To accomplish this, we will hold the amplitude constant while doing a frequency sweep in both directions.
- Compare the basins of attraction (These are regions of initial conditions in phase space that tend towards specific attractors) between model and experiment.

REFERENCES

Alligood, K.T., Sauer, T.D., & Yorke, J.A. (1996). Chaos: An introduction to dynamical systems. New York: Springer-Verlag.

Kaplan, D., &Glass, L. (1995). Understanding nonlinear dynamics. New York: Springer-Verlag.

Mello, T.M. & Tufillaro, N.B. (1986). Strange attractors of a bouncing ball. American Journal of Physics, 55, 316-320.

Peek, B. (2009). WiimoteLib [version 15] (software). Available from http://wiimotelib.codeplex.com/

Tufillaro, N.B., Abbot, T., & Reilly, J. (1992). An experimental approach to nonlinear dynamics and chaos. New York: Addison Wesley.

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