18.905: Problem Set IV

Due November 2, 2016, in class.

Homework is an important part of this class. I hope you gain from the struggle. Collaboration can be effective, but be sure that you grapple with each problem on your own as well. If you do work with others, you must indicate with whom on your solution sheet. Scores will be posted on the Stellar website.

15. **(a)** Provide the Euclidean space $\mathbb{R}^n$ with the structure of a CW complex.

(b) Provide each compact surface with the structure of a CW complex with just a single 2-cell.

16. Let p and q be relatively prime positive integers. Define a space $L(p,q)$ as the quotient of S^3, the unit sphere in $\mathbb{C}^2$, by the action of the group of p<u>th</u> roots of unity given by

$$\zeta \cdot (z_1, z_2) = (\zeta z_1, \zeta^q z_2)\,.$$

Impose on $L(p,q)$ the structure of a finite cell complex with one cell in each dimension between 0 and 3. The cell complex structure is just the filtration, but you should specify the characteristic maps as well. Then compute the homology of $L(p,q)$.

17. Prove that the collapse map induces an isomorphism

$$H_*\left(\coprod_\alpha D^n_\alpha, \coprod_\alpha S^{n-1}_\alpha\right) \to H_*\left(\bigvee_\alpha (D^n/S^{n-1}), *\right).$$

18. **(a)** Let m, n be positive integers and consider the cyclic groups $\mathbb{Z}/m$ and $\mathbb{Z}/n$. Compute the tensor product $\mathbb{Z}/m \otimes \mathbb{Z}/n$. (Hint: Problem **7 (b)**.)

(b) Compute $H_*(\mathbb{R}P^n; M)$ where $M = Z/p$ (p a prime number) and where $M = \mathbb{Q}$.

(c) Let $0 \to L \to M \to N \to 0$ be a short exact sequence. Construct a natural transformation $\partial : H_n(X; N) \to H_{n-1}(X; L)$ that fits into a ("coefficient") long exact sequence

$$\cdots \to H_{n+1}(X; N) \to H_n(X; L) \to H_n(X; M) \to H_n(X; N) \to H_{n-1}(X; L) \to \cdots\,.$$

(d) Describe this long exact sequence for $X = \mathbb{R}P^n$ and the long exact sequence $0 \to \mathbb{Z} \to \mathbb{Z} \to \mathbb{Z}/2 \to 0$.