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Math for Intelligent Systems
HW 3

MIS HW #3

#5.2 No, the supervisor is wrong. By independence-dimension theorm since we are comparing 400 stocks over a 250 day period since $m > n$ the set of stock vectors must be linearly dependent on each other.

#5.8 Based on the early termination of Gram-schmidt algorithm vectors $a_1, \ldots, a_4$ must be non-zero linearly independent that form a linear combination for a_5. Thus, choices a, c, d are true.

#5.9 Gram-schmidt takes $\approx 2nk^2$ flops
So, $2nk^2 = 2 \times 10000 \times (1000)^2 = 2 \times 10^{10}$ exec

2×10^{10} exec$/2$sec $= 10^{10}$ exe/sec

Then, $2nk^2 = 2 \times 1000 \times 500^2 = 5 \times 10^8$ exec

Thus, $5 \times 10^8 \text{ exec} \left| \frac{\text{sec}}{10^{10} \text{ exec}} \right. = 5 \times 10^{-2} \text{ sec} = 0.05 \text{ sec}$

#6.8 $b_1 = c_1$
$b_2 = (1+r)c_1 + c_2$
$b_3 = (1+r)^2 c_1 + (1+r)c_2 + c_3$

Thus, A must be a lower triangular matrix with $A_{ij} = 0$ for $i < j$ & $A_{ij} = (1+r)^{i-j}$ for $i \geq j$.

#6.13

$$D = \begin{bmatrix} 0 & 1 & 0 & \cdots & & 0 \\ 0 & 0 & 2 & \cdots & & 0 \\ \vdots & \vdots & \vdots & \ddots & & \vdots \\ 0 & 0 & 0 & \cdots & 0 & j-1 \end{bmatrix}$$

D is a $(n-1)\times m$ matrix with $D_{ij} = (j-1)$ for $j-i=1$, & $A_{ij}=0$ for $j-i \neq 1$

#6.18 Consider $c_0v_0 + c_1v_1 + \cdots + c_{n-1}v_{n-1} = 0$, where v_i is the ith column of the vandermonde matrix with $c_0, c_1, \ldots, c_n \in \mathbb{R}$

We look at the jth row of the vandermonde matrix namely

$$c_0 + c_1x_j + c_2x_j^2 + \cdots + c_{n-1}x_j^{n-1} = 0$$

This means x_j is a root of the polynomial $p(x) = c_0 + c_1x + c_2x^2 + \cdots + c_{n-1}x^{n-1}$

Now if $p(x)$ is of degree $n-1$ has m roots where $m > n$ namely $t_1, t_2, \ldots, t_m$ it must mean all coefficents are zero.

Therefore, $c_0 = c_1 = \cdots = c_{n-1} = 0$ & thus the columns of the vandermonde matrix are linearly independent.

#6.22 (a) $\underset{\text{M-M add}}{mn} + \underset{\text{V-V add}}{n} + \underset{\text{M-V mult}}{2mn} = 3mn + n$

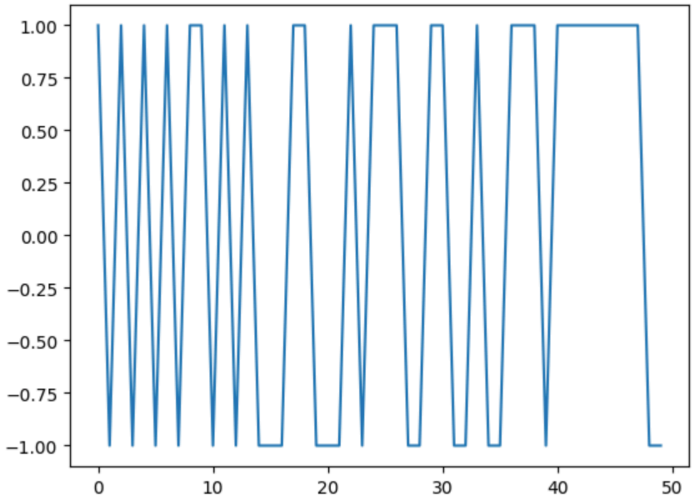
(b) $\underset{\text{4 M-V mult}}{4(2mn)} + \underset{\text{3 MM add}}{3mn} = 11mn$

(c) Since we are considering coefficients to be significant, (b) will require on average about 8 times more flops. So, part (a) will require fewer flops.

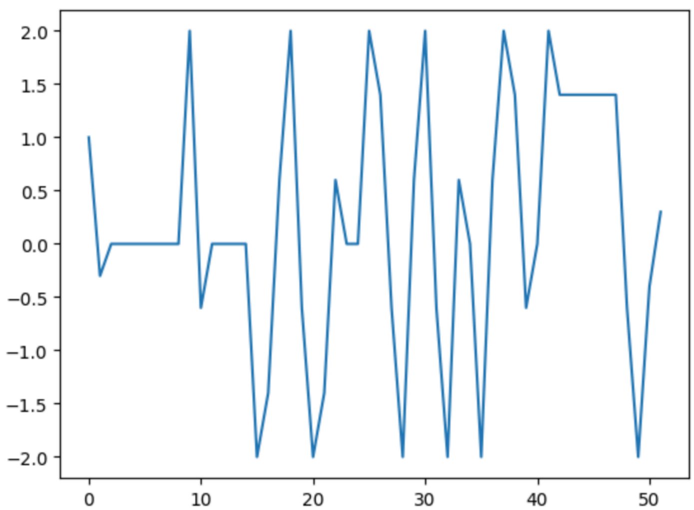
#7.15 $z = h*y = h*(c*u) = (h*c)*u \approx e_1 * u$

So, z is approximately equal to the convolution of u with the c_1 vector of size $n+k-1$.

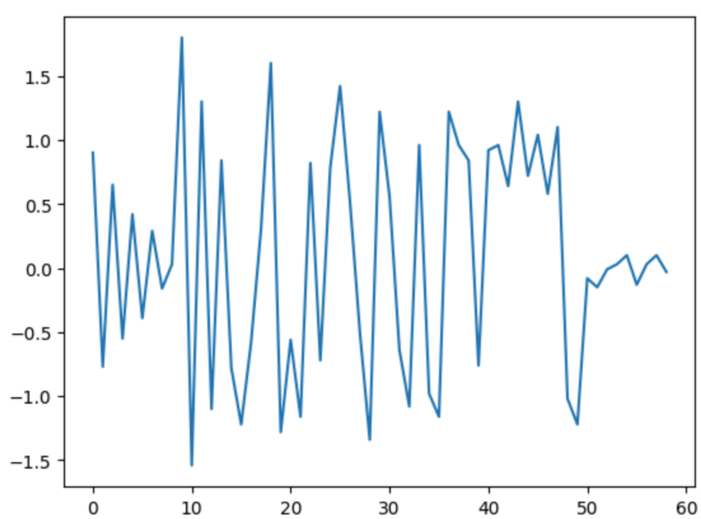
Plot of u



Plot of y = c*u



Plot of z = h*y



Julia Code:

```
# creating arrays
u = rand([-1, 1], 50);
c = [1.0, 0.7, -0.3];
h = [0.9, -0.5, 0.5, -0.4, 0.3, -0.3, 0.2, -0.1];
reshape(u, (1, 50));
reshape(c, (1, 3));
reshape(h, (1, 8));

# creating convolutions
using DSP
y_conv = conv(c, u);
z_conv = conv(h, y_conv);

# ploting u and convolutions
using PyPlot;
plot(u)
plot(y_conv)
plot(z_conv)
```