Nicholas Miller

MIS HW4

#8.3 $f(x) = Ax$ where

$$A = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix}$$

#8.4 (a)

$$x = \begin{bmatrix} 1 \\ 2 \\ \vdots \\ 9 \end{bmatrix} = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix} \Leftrightarrow \begin{bmatrix} 3 & 6 & 9 \\ 2 & 5 & 8 \\ 1 & 4 & 7 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 1 \\ 6 \\ 5 \\ 4 \\ 9 \\ 8 \\ 7 \end{bmatrix} = y$$

$y = Ax$ where

column unit vectors

$$A = \begin{bmatrix} \begin{smallmatrix} 0&0&1 \\ 0&1&0 \\ 1&0&0 \end{smallmatrix} & 0 & 0 \\ 0 & \begin{smallmatrix} 0&0&1 \\ 0&1&0 \\ 1&0&0 \end{smallmatrix} & 0 \\ 0 & 0 & \begin{smallmatrix} 0&0&1 \\ 0&1&0 \\ 1&0&0 \end{smallmatrix} \end{bmatrix} = \begin{bmatrix} e_3 & e_2 & e_1 & e_6 & e_5 & e_4 & e_9 & e_8 & e_7 \end{bmatrix}$$

(b)

$$x = \begin{bmatrix} 1 \\ 2 \\ \vdots \\ 9 \end{bmatrix} = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix} \Leftrightarrow \begin{bmatrix} 3 & 2 & 1 \\ 6 & 5 & 4 \\ 9 & 8 & 7 \end{bmatrix} = \begin{bmatrix} 3 \\ 6 \\ 9 \\ 2 \\ 5 \\ 8 \\ 1 \\ 4 \\ 7 \end{bmatrix} = y$$

$y = Ax$ where

$$A = \begin{bmatrix} 0&0&1&0&0&0&0&0&0 \\ 0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&0&0&1 \\ 0&1&0&0&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0 \\ 0&0&0&0&0&0&0&1&0 \\ 1&0&0&0&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&0&0&1&0&0 \end{bmatrix} = \begin{bmatrix} e_7 & e_4 & e_1 & e_8 & e_5 & e_2 & e_9 & e_6 & e_3 \end{bmatrix}$$

(c)

$$x = \begin{bmatrix} 1 \\ 2 \\ \vdots \\ 9 \end{bmatrix} = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix} \Longleftrightarrow \begin{bmatrix} 0 & 2 & 5 \\ 0 & 3 & 6 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 2 \\ 3 \\ 0 \\ 5 \\ 6 \\ 0 \end{bmatrix} = y$$

$y = Ax$ where

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0_9 & e_4 & e_5 & 0_9 & e_7 & e_8 & 0_9 & 0_9 & 0_9 \end{bmatrix}$$

(d)

$$X = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix} \Longleftrightarrow \begin{bmatrix} \frac{2+4}{2} & \frac{1+7+5}{3} & \frac{4+8}{2} \\ \frac{1+5+3}{3} & \frac{2+4+8+6}{4} & \frac{7+5+9}{3} \\ \frac{2+6}{2} & \frac{3+5+9}{3} & \frac{6+8}{2} \end{bmatrix} = Y$$

$y = Ax$ where

$$A = \begin{bmatrix} 0 & 1/2 & 0 & 1/2 & 0 & 0 & 0 & 0 & 0 \\ 1/3 & 0 & 1/3 & 0 & 1/3 & 0 & 0 & 0 & 0 \\ 0 & 1/2 & 0 & 0 & 0 & 1/2 & 0 & 0 & 0 \\ 1/3 & 0 & 0 & 0 & 1/3 & 0 & 1/3 & 0 & 0 \\ 0 & 1/4 & 0 & 1/4 & 0 & 1/4 & 0 & 1/4 & 0 \\ 0 & 0 & 1/3 & 0 & 1/3 & 0 & 0 & 0 & 1/3 \\ 0 & 0 & 0 & 1/2 & 0 & 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 0 & 1/3 & 0 & 1/3 & 0 & 1/3 \\ 0 & 0 & 0 & 0 & 0 & 1/2 & 0 & 1/2 & 0 \end{bmatrix}$$

#8.12 (a) Given that $x = \int_{-1}^{1} f(x)\,dx$ & $\hat{x} = \sum_{i=1}^{n} w_i f(t_i)$

let f be a polynomial of degree d

then

$$f = \sum_{j=0}^{d} a_j x^j$$

Now
$$\int_{-1}^{1} f(x)\,dx = \int_{-1}^{1} (a_0 + a_1 x + \cdots + a_d x^d)\,dx$$

$$= \int_{-1}^{1} a_0\,dx + \int_{-1}^{1} a_1 x\,dx + \cdots + \int_{-1}^{1} a_d x^d\,dx$$

$$= a_0 \int_{-1}^{1} dx + a_1 \int_{-1}^{1} x\,dx + \cdots + a_d \int_{-1}^{1} x^d\,dx$$

$$= a_0 \sum_{i=1}^{w} w_i + a_1 \sum_{i=1}^{w} w_i t_i + \cdots + a_d \sum_{i=1}^{w} w_i t_i^d$$

For this to satisfy for $(d+1)$ arbitrary coefficients a_i, we must have $(d+1)$

$$\sum_{i=1}^{w} w_i t_i^j = \int_{-1}^{1} x^j\,dx \quad \text{for } j = 0, 1, 2, \ldots, d$$

Exactness can be satisfied by

$$\int_{-1}^{1} dx = \sum_{i=1}^{w} w_i \cdot 1$$

$$\int_{-1}^{1} x^d\,dx = \sum_{i=1}^{w} w_i t_i^d$$

$$\Rightarrow \begin{bmatrix} 1 & 1 & \cdots & 1 \\ t_1 & t_2 & \cdots & t_w \\ \vdots & & \ddots & \vdots \\ t_1^d & t_2^d & \cdots & t_w^d \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix} = \begin{bmatrix} \int_{-1}^{1} dx \\ \vdots \\ \int_{-1}^{1} x^d\,dx \end{bmatrix}$$

$\Rightarrow Ax = b$

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(b) Trap $f(x) = ax + b$

$$\alpha = \int_{-1}^{1} ax + b\,dx = \frac{ax^3}{2} + bx\Big|_{-1}^{1} = \left(\frac{a}{2} + b\right) - \left(\frac{a}{2} - b\right) = 2b$$

$$\hat{\alpha} = (1)(-a+b) + (1)(a+b) = 2b$$

order 1, since $\alpha = \hat{\alpha}$

Simp $f(x) = ax^2 + bx + c$

$$\alpha = \int_{-1}^{1} ax^2 + bx + c\,dx = \frac{ax^3}{3} + \frac{bx^2}{2} + cx\Big|_{-1}^{1} = \frac{2}{3}a + 2c$$

$$\hat{\alpha} = \tfrac{1}{3}(a - b + c) + \tfrac{4}{3}(c) + \tfrac{1}{3}(a + b + c)$$
$$= \tfrac{2}{3}a + 2c$$

order 2, since $\alpha = \hat{\alpha}$

Simp 3/8 $f(x) = ax^3 + bx^2 + cx + d$

$$\int_{-1}^{1} f(x)\,dx = \tfrac{2}{3}b + 2d$$

$$\hat{\alpha} = \tfrac{1}{4}(-a+b-c+d) + \tfrac{3}{4}\left(-\frac{a}{3^3} + \frac{b}{3^2} - \frac{c}{3} + d\right)$$
$$+ \tfrac{3}{4}\left(\frac{a}{3^3} + \frac{b}{3^2} + \frac{c}{3} + d\right) + \tfrac{1}{4}(a + b + c + d)$$
$$= \tfrac{4}{6}b + \tfrac{4}{2}d = \tfrac{2}{3}b + 2d$$

order 3, since $\alpha = \hat{\alpha}$

$\#8.16$ $A\Theta = b$ where

$$A = \begin{bmatrix} 1 & x_1 & y_1 & x_1y_1 \\ 1 & x_1 & y_2 & x_1y_2 \\ 1 & x_2 & y_1 & x_2y_1 \\ 1 & x_2 & y_2 & x_2y_2 \end{bmatrix} \text{ \& } b = \begin{bmatrix} F_{11} \\ F_{12} \\ F_{21} \\ F_{22} \end{bmatrix}$$

$\#9.1$ $x_{t+1} = Ax_t$ where

$$A = \begin{bmatrix} 0.90 & 0.00 & 0.05 \\ 0.10 & 0.95 & 0.00 \\ 0.00 & 0.05 & 0.90 \end{bmatrix}$$

$\#9.3$ $x_{t+1} = Ax_t + c$

Assume z is a equilibrium point then

$z = Az + c$ multiply z by I_n

$I_n z = Az + c$ subtract Az from both sides

$I_n z - Az = c$ factor out z

$(I_n - A)z = c$

Since z is an equilibrium point $(I_n - A)z = c$ is in the same form as $Fz = g$

Thus $F = (I_n - A)$ \& $g = c$

#10.11 (a)

$$tr(A^TB) = \sum_{i=1}^{m}(A^TB)_{ii} = \sum_{i=1}^{m}\sum_{j=1}^{n} A_{ij}^T B_{ji} = \sum_{i=1}^{m}\sum_{j=1}^{n} A_{ji} B_{ji}$$

complexity $O(mn)$

(b)

$$tr(A^TB) = \sum_{i=1}^{m}\sum_{j=1}^{n} A_{ij}B_{ij} = \sum_{i=1}^{m}\sum_{j=1}^{n} A_{ij} B_{ji}^T = \sum_{i=1}^{m} (AB^T)_{ii} = tr(AB^T)$$

(c) $$tr(A^TA) = \sum_{i=1}^{m}\sum_{j=1}^{n} A_{ij}A_{ij} = \sum_{i=1}^{m}\sum_{j=1}^{n} A_{ij}^2 = \|A\|^2$$

(d) $$tr(A^TB) = \sum_{i=1}^{m}\sum_{j=1}^{n} A_{ij}B_{ij} = \sum_{i=1}^{m}\sum_{j=1}^{n} B_{ij}A_{ij} = \sum_{i=1}^{m}\sum_{j=1}^{n} B_{ij}A_{ji}^T = \sum_{i=1}^{m}(BA^T)_{ii} = tr(BA^T)$$

#10.13 (a) $$D(v) = \|A^Tv\|^2 = (A^Tv)^T(A^Tv) = (v^TA)(A^Tv)$$
$$= v^T(AA^T)v = v^TLv$$

(b) $$L_{i,j} = \begin{cases} deg(v_i) & \text{if } i=j \\ -1 & \text{if } i \neq j \text{ \& } v_i \text{ is adjacent to } v_j \\ 0 & \text{otherwise} \end{cases}$$