CS410 HW4

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2.14 The geometric distrubution arises as the distribution of the number of times we flip a coin until it comes up heads. Consider now the distribution of the number of flips X until the kth head appears, where each coin flip comes up heads independently with probability p. Prove that this distribution is given by

$$\Pr(X = n) = \binom{n-1}{k-1} p^k (1-p)^{n-k}$$

$$\text{for } n \geq k.$$

We would like to know when the kth head is flipped. Let X_1 be a geometric random variable that is the distribution of the number of times we flip the coin until the first heads, and so on for all values of $i = \{1, 2, ..., k\}$.

$$X = \sum_{i=1}^{k} X_i$$

For each of these geometric random variables X_i, the distribution for the jth head is

$$\Pr(X_i = j) = (1-p)^{j-1} p$$

So the distribution of X would be

$$\Pr(X = n) = \sum_{k=1}^{n} (1-p)^{k-1} p$$

SO LOST