CS410 HW1

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1.8 I choose a number uniformly at random from the range [1, 1,000,000]. Using the inclusion-exclusion principle, determine the probability that the number chosen is divisible by one or more of 4, 6, and 9.

I wasn't sure exactly how to get these sets of numbers, so I wrote a script in Python that generated them. I checked each set of numbers to get intersects, and produced a size of each set. Let each variable $divX$ represent how many numbers are divisible by X, with multiple Xs being the intersects, and mil just being the number $1,000,000$. Here is the final formula I used to get this probability.

$$\begin{aligned}
P(div4 \cup div6 \cup div9) &= P(div4) + P(div6) + P(div9) - P(div4 \cap div6) - P(div4 \cap div9) - \\
&\quad P(div6 \cap div9) + P(div4 \cap div6 \cap div9) \\
&= \frac{div4}{mil} + \frac{div6}{mil} + \frac{div9}{mil} - \frac{div46}{mil} - \frac{div49}{mil} - \frac{div69}{mil} + \frac{div469}{mil} \\
&= \frac{250000}{mil} + \frac{166666}{mil} + \frac{111111}{mil} - \frac{83333}{mil} - \frac{27777}{mil} - \frac{55555}{mil} + \frac{27777}{mil} \\
&\boxed{= .3\bar{8}9}
\end{aligned}$$

See attached transcript of Pythons script.

1.10 I have a fair coin and a two-headed coin. I choose one of the two coins randomly with equal probability and flip it. Given that the flip was heads, what is the probability that I flipped the two-headed coin?

Let A be the event of choosing the two-headed coin. Let B be the event of flipping a heads. We're told that $P(A) = \frac{1}{2}$. Getting a heads while flipping the two-headed coin is 1. So $P(A \mid B) = 1$. Since there are 4 total possible outcomes (Heads, Heads, Heads, Tails), getting Heads is $\frac{3}{4}$, which is $P(B)$.

$$\begin{aligned}
P(A \mid B) &= \frac{P(A \cap B)}{P(B)} \\
&= \frac{P(B \mid A)P(A)}{P(B)} \\
&= \frac{1 \times \frac{1}{2}}{\frac{3}{4}} \\
&\boxed{= \frac{2}{3}}
\end{aligned}$$

1.15 Suppose that we roll ten standard six-sided dice. What is the probability that their sum will be divisible by 6, assuming that the rolls are independent? (*Hint:* Use the principle of deferred decisions, and consider the situation after rolling all but one of the dice.)

At the point when 9 dice have been rolled, the outcome is already determined. If the sum were 54 (all 9 rolls were 6), then the only possible roll that could result in the total sum being divisible by 6 is a 6. Similarly, any other sum that the first 9 are capable of have only one valid accompanying 10^{th} roll.

$$\boxed{\text{Thus the probability of a sum that is divisible by 6 for 10 dice rolls is at most } \frac{1}{6}.}$$