HW 1 **Due October 11 by 16:40**

Please make sure your solutions are complete and nicely explained. Simply writing down a formula will receive **ZERO POINTS** even if the answer is correct!

Problem 1: Each of ten people simultaneously requests a distinct movie from a new service that broadcasts 3D movies. These 3D movies are broadcast as a left and a right stream, which are distinct. (So 10 distinct movies, 20 distinct streams in total.) But the service is not very reliable, and the 3D effect only works if you receive a matching left and right stream. Assume that each person receives exactly two of the streams, and each stream goes to exactly one person.

(a) How many ways are there for the streams to be broadcast if the service provider makes no guarantees about which streams each person receives?

(b) How many ways if each person receives a matching left and right stream? (Not necessarily for the movie they ordered.) Assume that movie players are smart enough to sort out which stream is left and which is right, so any matching pair will play properly.

(c) How many ways if each person is guaranteed one left stream and one right stream? (Not necessarily for the same movie.)

Problem 2: If three distinct dice are rolled, what is the probability that the largest rolled value is exactly twice the smallest? (Assume the dice are unbiased, so each value is equally likely.)

Problem 3: Give an exact expression for the probability that two (or more) people in a group of 25 have the same birthday? What does the expression evaluate to? (That is, what is the probability?) Assume that each person in the room has a randomly selected birthday.

Problem 4: A coin is flipped 18 times. The result has the following pattern: one or more heads, one or more tails, one or more heads, one or more tails. Exactly one of the runs of heads has length at least eight. How many ways can this happen?

Problem 5: Consider the following pair of equations: $x_1 + x_2 + \cdots + x_6 = 20$ and $x_1 + x_2 + x_3 = 7$. These equations must be simultaneously satisfied. How many non-negative, integer solutions are there?

Problem 6: Give a combinatorial argument for the following identity:

$$\sum_{k=0}^{n} \binom{n}{k}^2 = \binom{2n}{n}$$