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ISYE 6644 Project 2

Library of random variate generation routines.

I used python to create the libraries.

I started with the good random variable generator to produce 1 random variable.

The function `desert_island_generator` has 3 inputs: `distribution_size` and `seed`.

Distribution size is the number of random variables to be generated and I made sure the seed was large enough in order for the variables to be random.

It returns an array of pseudo random variables.

The function `uniform` produces an array of uniforms and I used that as my basis for obtaining the pseudo random variables of the rest of the distributions

A. Discrete Distributions

1. Geometric

Complete Source Code

```
def uniform(a = 0, b = 1, distribution_size = 1, seed = 123456789):  
    return a + (b - a) * desert_island_generator(distribution_size, seed)
```

User Guide

To generate one uniform between 0 and 1

a and b are 0 and 1 respectively since the uniforms are from zero to 1.

The seed is 123456789 but could be any number that is larger.

Add the difference of b and a to a and multiply that value by the array generated by the `desert_island_generator` function.

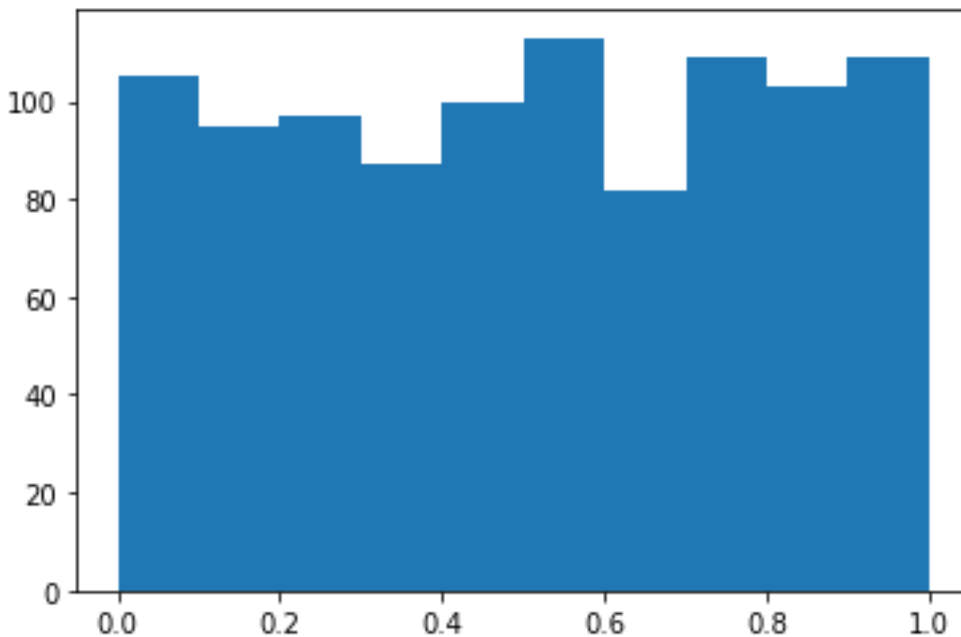
The `uniform` function returns an array

Example

```
unif = uniform(a = 0, b = 1, distribution_size = 1000, seed = 123456789)
```

The above code generates 1000 uniforms returned as an array.

The uniforms are plotted in the histogram below that shows that they are all between 0 and 1



2. Bernoulli Distribution Bern(p)

Complete Source Code

```
def bernoulli(num_coins = 1, distribution_size = 1, p = 0.5):  
    seed = 123456789  
    bern = uniform(0, 1, distribution_size, seed)  
    #check if random variable is less than or greater than probability of 0.5  
    #then convert it to integer 0 if false or 1 if true  
    bern = (bern < p).astype(int)  
    return bern
```

User Guide

The bernoulli function calls the uniform distribution function with parameters a=0, b=1, the distribution_size and the p value

The array returned has either 0 or 1 values. 0 if the value is less than p and 1 if the value is greater than p.

Example

```
bernoulli()
```

The above code calls the Bernoulli function and uses its default values to generate 1 random variable of 0 or 1.

3. Binomial Distribution

Complete Source Code

```
def binomial(num_coins = 1, distribution_size = 1, p = 0.5 ):  
    binomials = [] #list of binomial random variables  
    for _ in range(distribution_size): #run this 1 time  
        seed = random.randint(123456789, 5000000000000000000)  
        U = uniform(distribution_size = num_coins, seed = seed)  
  
        #check if each value in the array is less than/equal to p and convert to 1 or 0  
        ans = (U <= p).astype(int)  
        #Add up the values of each trial  
        binomials.append(np.sum(ans))  
    return np.array(binomials)
```

User Guide

The binomial function calls the uniform distribution function with parameters $a=0$, $b=1$, the `distribution_size` and the p value

The array returned has the sum of how many of the pseudorandom values are greater or less than p , added up.

Example

```
num_coins = 50
```

```
distribution_size = 100
```

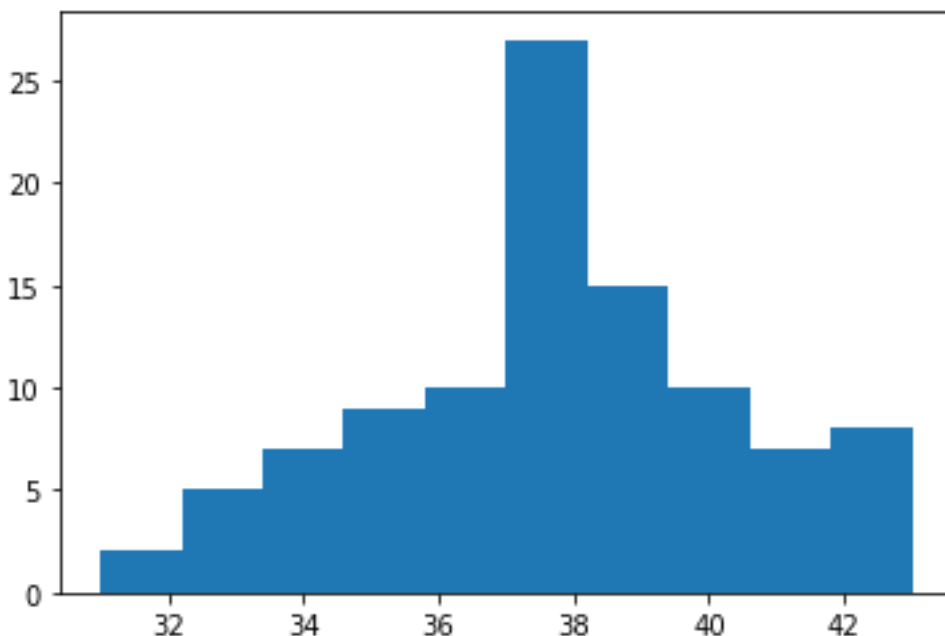
```
p = 0.75
```

```
binom = binomial(3,20,0.55)
```

```
binom
```

The example code shows 50 weighted coins flipped 100 times with probability of 0.75 of getting heads

Following is the resulting histogram



4. Geometric Distribution

Complete Source Code

```
def geometric(distribution_size = 1, p = 0.3):  
    seed = random.randint(123456789, 5000000000000000000)  
    U = uniform(0, 1, distribution_size = distribution_size, seed = seed)  
    geometric = np.log(1 - U) / np.log(1 - p)  
    return geometric
```

User Guide

The geometric function calls the uniform distribution function with parameters $a=0$, $b=1$, the `distribution_size` and the seed.

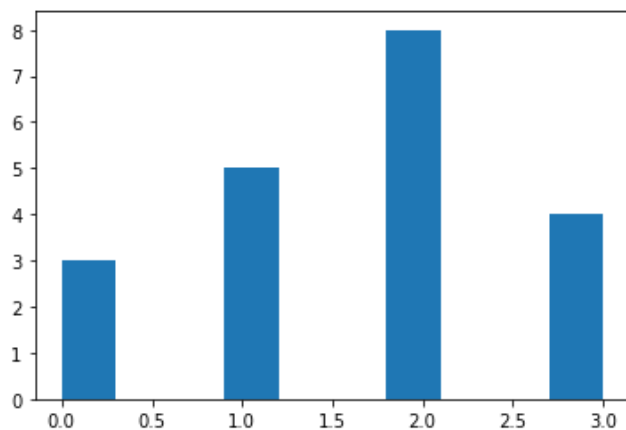
The seed is a large number generated using `random.randint` and is between the 2 large numbers shown.

Each pseudo random number in the array returned by the uniform function is run through the formula above to generate a geometric distribution

Example

```
#Histogram of geometric distribution of size 10 with  $p = 0.3$   
geometric(10, 0.3)
```

Following is the histogram for the geometric distribution



B. Continuous Distributions

1. Exponential Distribution

Complete Source Code

```
def exponential(lambd = 5,distribution_size = 1):  
    seed = random.randint(123456789,5000000000000000)  
    U = uniform(0,1,distribution_size = distribution_size,seed = seed)  
    U = U.tolist()  
    exponential = []  
    for u in U: #loop through list  
        exp = -(1/lambd) * np.log(1 - u)  
        exponential.append(exp) #append values in list  
    return exponential
```

User Guide

The parameters of the exponential function are lambda and distribution_size.

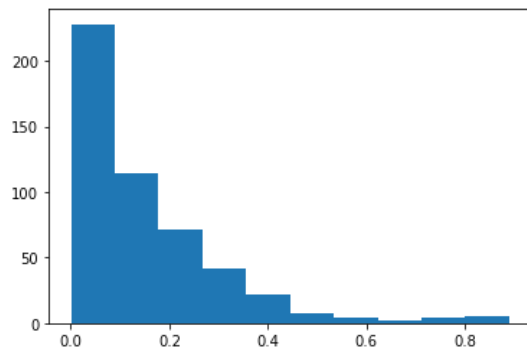
The exponential distribution calls the uniform distribution .

When the array of uniforms is returned by the uniform function they are run through the formula above to generate an exponential distribution

Example

```
exp = exponential(7,500)  
exp  
plt.hist(exp)  
plt.show()
```

Following is the histogram for the exponential distribution



2. Normal Distribution $\text{Normal}(\mu, \sigma^2)$

Complete Source Code

```
def normal(mu = 0.0, stdev = 1.0, distribution_size = 1):  
    seed1 = random.randint(123456789, 5000000000000000000)  
    seed2 = random.randint(123456789, 5000000000000000000)  
    U1 = uniform(0, 1, distribution_size = distribution_size, seed = seed1)  
    U2 = uniform(0, 1, distribution_size = distribution_size, seed = seed2)  
  
    #print(U1)  
    z0 = (np.sqrt(-2 * np.log(U1))) * (np.cos(2 * np.pi * U2))  
    z1 = np.sqrt(-2 * np.log(U1)) * np.sin(2 * np.pi * U2)  
    #z0 with a mean of 0 and standard deviation of 1  
    z0 = z0 * stdev + mu  
    z1 = z1 * stdev + mu  
  
    return z0
```

User Guide

The parameters of the normal function are mean, standard deviation and distribution_size.

The normal distribution function calls the uniform distribution

The array of uniforms is ran through the Box-Muller transform and values with a normal distribution are returned

Example

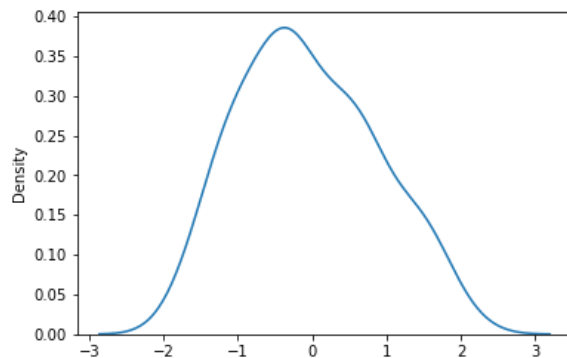
```
z0 = normal(0,1,100)
```

100 random variables with mean zero and standard deviation of 1

```
#Display distribution
```

```
sns.kdeplot(z0)
```

```
plt.show()
```



3. Uniform Distribution

Complete Source Code

```
def uniform(a = 0, b = 1, distribution_size = 1, seed = 123456789):  
    return a + (b - a) * desert_island_generator(distribution_size, seed)
```


User Guide

The uniform function calls the `desert_island_generator` function which produced pseudo random numbers. The returned array is then manipulated using the formula $a + (b - a)$

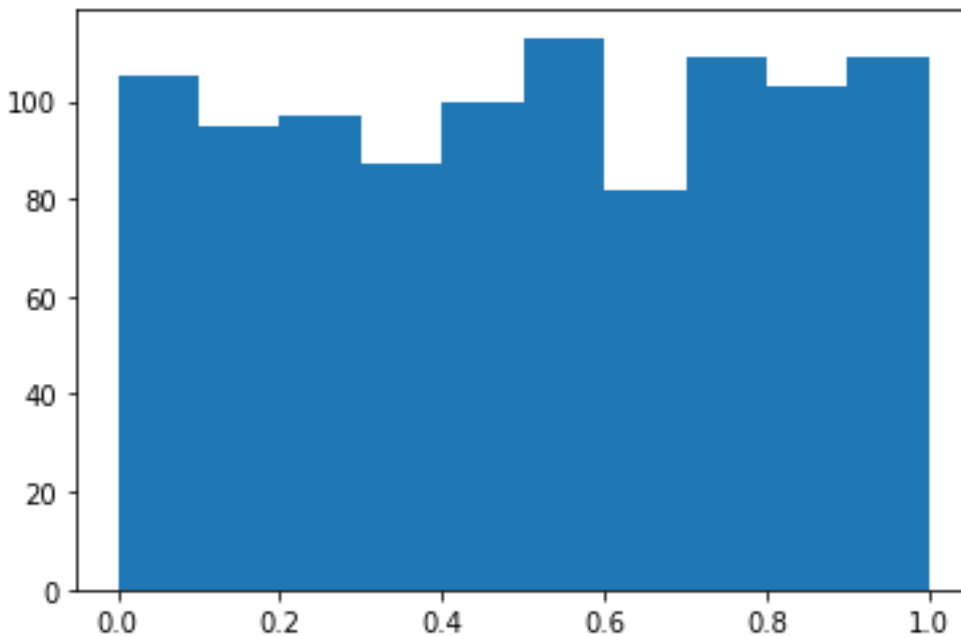
To ensure all the numbers in the array are between 0 and 1

Example

```
# Generate 1000 uniform random variables between 0 and 1
```

```
unif = uniform(a = 0, b = 1, distribution_size = 1000, seed = 123456789)
```

The plot for this uniform distribution is below.



4. Weibull

Complete Source Code

```
def weibull(lambd = 5,beta = 2,distribution_size = 1):  
    seed = random.randint(123456789,5000000000000000000)  
    U = uniform(0,1,distribution_size = distribution_size,seed = seed)  
    U = U.tolist()  
    weibull = []  
    for u in U: #loop through list  
        weib = 1/lambd * (-(np.log(1 - u)))** 1/beta  
        weibull.append(weib) #append values in list  
    return weibull
```

User Guide

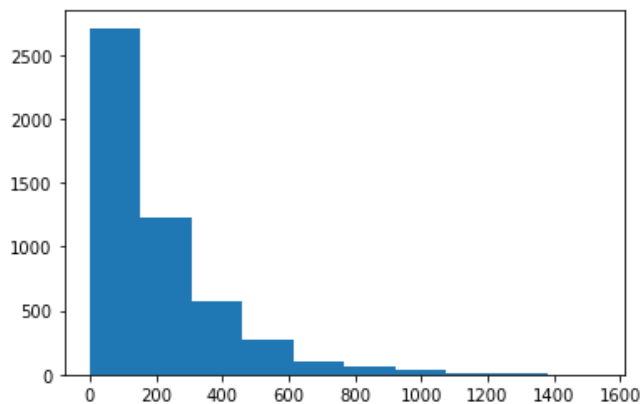
The parameters of the weibull function are lambda,beta and distribution_size.

The Weibull distribution function calls the uniform distribution

The returned array of uniforms is converted to Weibull random numbers by the formula above

Example

```
wei = weibull(lambd = 0.001,beta = 5,distribution_size = 5000)
```



5. Triangular Distribution

Complete Source Code

```
triangular(0,1,2) distribution  
  
def triangular(distribution_size = 1):  
    seed = random.randint(123456789,5000000000000000)  
    U = uniform(0,1,distribution_size = distribution_size,seed = seed)  
    triangular = []  
    for num in U:  
        if num < 0.5:  
            tr = np.sqrt(2 * num)  
        elif num > 0.5:  
            tr = 2 - np.sqrt(2-(1 - num))  
        triangular.append(tr)  
    triangular = np.asarray(triangular)  
    return triangular
```

User Guide

The parameter of the triangular function is `distribution_size`.

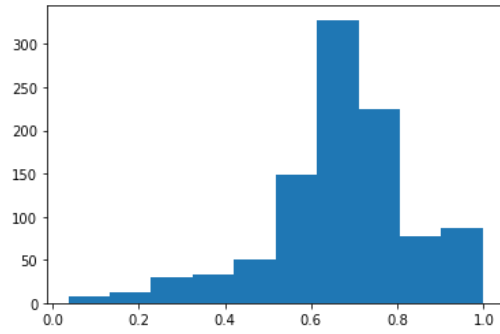
The triangular function calls the uniform function

The returned array of uniforms is converted to triangular by the if-elif statements which are obtained from calculating the cdf of a triangular(0,1,2) distribution.

Example

1000 values of a triangular(0,1,2) distribution

```
trian = triangular(1000)
```



References:

How to generate random variables from scratch (no library used)

<https://towardsdatascience.com/how-to-generate-random-variables-from-scratch-no-library-used-4b71eb3c8dc7>

Probability Distributions in Python

<https://www.datacamp.com/community/tutorials/probability-distributions-python>