Math 489

Midterm Reveiw

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Conditioning

Probability Convolution
Let $Z = X+Y$ with respective densities f_ϕ then:

$$f_Z(z) = \int_{-\infty}^{\infty} f_X(z-y)f_Y(Y)\ dY$$

$$f_Z(z) = \int_{-\infty}^{\infty} f_Y(z-x)f_X(X)\ dX$$

Geometric Dist
For $k \in \{1,\ 2,\ \ldots,\ \mathbb{I}\}$ we have:

$$\text{PMF}:\ (1-p)^{k-1} \cdot p$$

$$\text{CDF}:\ 1-(1-p)^k$$

$$\mu = \frac{1}{p}$$

$$\sigma = \frac{1-p}{p^2}$$

Law of Total Probability

$$P(X=x) = \sum_{y=0}^{\infty} p_{XY}(x\ |y)p_Y(y)$$

$$p_{XY}(x \mid y) = \frac{P(X=x \wedge Y=y)}{P(Y=y)}$$

$$p_Y(y) = \sum_X p_{XY}(x,\ y)$$

$$\mathbb{E}(g(X) \mid Y=y) = \sum_X g(x) \cdot p_{XY}(x \mid y)$$

$$\mathbb{E}(g(X)) = \sum_Y \mathbb{E}(g(X) \mid Y=y)p_Y(y)$$

Exponential Distribution
For $x \in [0,\ \infty)$ and $\mu = \frac{1}{\lambda}$

$$\text{PMF}:\ P(X=x) = \lambda e^{-\lambda x}$$

$$\text{CDF}:\ P(X=x) = 1-e^{-\lambda x}$$

Markov Chains

Basic Rules
Suppose you want to find $P(X_1 = k \mid X_\circ = \ell)$ for $\mathbb{P}$. We must find:

$$\mathbb{P}_{\ell k}$$

Suppose the difference between steps is $i = j - m$, it would follow that:

$$P(X_j = k \mid X_m = \ell) = \mathbb{P}^i_{\ell k}$$

Stationary Distribution
For

$$\mathbb{P} = \begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array} \left\| \begin{array}{ccc} 1 & 2 & 3 \\ a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{array} \right\|$$

$$\pi_1 = a_{11}\pi_1 + a_{21}\pi_2 + a_{31}\pi_3$$

$\ldots$

Long term frequency of transition from state i to j is $\pi_i \cdot P_{ij}$ (mnmonic: $\pi's \equiv$ cols, $v's \equiv$ cols and $+1$).

Ergotic
Reccurent (can return to state with non-zero prob). Transcient (returns with 0 probability).