Introduction of Transformations

- A Cartographer can change the size of charts and topographical maps. So if graphics images are coded as numbers, the numbers can be stored in memory. These numbers are modified by mathematical operations called as **Transformation.**
- The purpose of using computers for drawing is to provide facility to user to view the object from different angles, enlarging or reducing the scale or shape of object called as Transformation.
- **Two essential aspects of transformation are given below:**
    - Each transformation is a single entity. It can be denoted by a unique name or symbol.
    - It is possible to combine two transformations, after connecting a single transformation is obtained, e.g., A is a transformation for translation. The B transformation performs scaling. The combination of two is C=AB. So C is obtained by concatenation property.
- There are two complementary points of view for describing object transformation.
    - **Geometric Transformation:** The object itself is transformed relative to the coordinate system or background. The mathematical statement of this viewpoint is defined by geometric transformations applied to each point of the object.
    - **Coordinate Transformation:** The object is held stationary while the coordinate system is transformed relative to the object. This effect is attained through the application of coordinate transformations.
- An example that helps to distinguish these two viewpoints:
    - The movement of an automobile against a scenic background we can simulate this by
    - Moving the automobile while keeping the background fixed-(Geometric Transformation)
    - We can keep the car fixed while moving the background scenery-(Coordinate Transformation)
- **Example of Computer Graphics Packages:**

- LOGO
- COREL DRAW
- AUTO CAD
- 3D STUDIO
- CORE
- GKS (Graphics Kernel System)
- PHIGS
- CAM (Computer Graphics Metafile)
- CGI (Computer Graphics Interface)

TRANSFORMATIONS

It is defined as changing shape, size, position of an object on display.
Changes in orientations, size, and shape are accomplished with geometric transformations that alter the coordinate descriptions of objects.

Basic geometric transformations are:

- Translation
- Rotation
- Scaling

By using combination of above three transformations we can obtain any transformation , hence they are called as Basic Transformations

Other transformations:

- Reflection
- Shear

TRANSLATION

• It is a transformation that used to reposition the object along the straight line path from one coordinate location to another
We translate 2 D point by adding translation distance tx and ty to the original coordinate (x, y) to move at new position (x′ , y′) as :

$$x' = x + tx$$

$$y' = y + ty$$

• Translation distance pair (tx, ty) is called as a Translation vector or Shift vector
• We can represent it into single matrix equation in column vector as:

$$P' = P + T$$

ROTATION

• It is a transformation that is used to reposition the object along the circular path
• To perform rotation we specify a rotation angle ⍰ and the position of rotation point (pivot point)
(xr,yr) about which the rotation is to be performed
• Positive value of rotation angle - Counter clockwise rotation
• Negative value of rotation angle - Clockwise rotation
• From figure we can write.

$$x = r \cos \varnothing$$
$$y = r \sin \varnothing$$
$$x' = r\cos(\theta + \varnothing) = r\cos\varnothing\cos\theta - r\sin\varnothing\sin\theta$$
$$y' = r\sin(\varnothing + \theta) = r\cos\varnothing\sin\theta + r\sin\varnothing\cos\theta$$

Now replace rcos∅ with x and rsin∅ with y in above equation.

$$x' = x\cos\theta - y\sin\theta$$
$$y' = x\sin\theta + y\cos\theta$$

• We can write it in the form of column vector matrix equation as;

$$P' = R \cdot P$$

SCALING

• This transformation is used to alter the size of an object
• This operation is carried out by multiplying coordinate value (x,y) with scaling factor (sx, sy) respectively
• Scaling equations are written as:

$$x'=x+sx$$
$$y'=y+sy$$

• These equations can be written in column matrix as:

$$P'=S*P$$

Matrix Representation

• Sequence of geometric transformations are involved in many computer graphics applications.

• We have matrix representation of basic transformation and we can express it in the general matrix
form as:

$$P' = M1 \cdot P + M2$$

Where,
P=initial position
P′ =final point position
M1= rotation and scaling (Multiplicative)terms
M2= translational terms associated with pivot point, fixed point and reposition.

• So according to the general matrix form following are matrix expression

- P’=P+T
- P’=R*P
- P’=S*P

1. Scaling $\begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix}$

2. Rotation (clockwise) $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

3. Rotation (anti-clock) $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

4. Translation $\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ t_x & t_y \end{bmatrix}$

Homogeneous Coordinate System

• The rotation of a point, straight line or an entire image on the screen, about a point other than origin, is achieved by first moving the image until the point of rotation occupies the origin, then performing rotation, then finally moving the image to its original position.

• The moving of an image from one place to another in a straight line is called a translation. A translation may be done by adding or subtracting to each point, the amount, by which picture is required to be shifted.
• Translation of point by the change of coordinate cannot be combined with other transformation by using simple matrix application. Such a combination is essential if we wish to rotate an image about a point other than origin by translation, rotation again translation.
• To combine these three transformations into a single transformation, homogeneous coordinates are used. In homogeneous coordinate system, two-dimensional coordinate positions (x, y) are represented by triple-coordinates.
• Homogeneous coordinates are generally used in design and construction applications. Here we perform translations, rotations, scaling to fit the picture into proper position.
• Expressing positions in homogeneous coordinates allow us to represent all geometric transformation equations as a matrix multiplications.

Translation In Homogeneous coordinate System

$$P' = T_{(t_x,t_y)} \cdot P$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Scaling In Homogeneous coordinate System

$$P' = S_{(s_x,s_y)} \cdot P$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Rotation In Homogeneous coordinate System

1. Clockwise

$$R = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

2. Anticlockwise

$$R = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Composite Transformation Introduction:

• We can set up a matrix for any sequence of transformations as a composite transformation matrix by calculating the matrix product of individual transformation.

• For column matrix representation of coordinate positions, we form composite transformations by multiplying matrices in order from right to left.
• A number of transformations or sequence of transformations can be combined into single one called as composition. The resulting matrix is called as composite matrix.

Successive Translations

• If two successive translation vectors (tx1, ty1) and (tx2, ty2)are applied to a coordinate position P, the final transformed location P' is calculated as

P′ = T(tx2, ty2) · {T(tx1, ty1) · P}

P′ = {T(tx2, ty2) · T(tx1, ty1)} · P

1 0 tx2		1 0 tx1		1 0 tx1 + tx2
0 1 ty2	.	0 1 ty1	=	0 1 ty1 + ty2
0 0 1		0 0 1		0 0 1

T(tx2, ty2) · T(tx1, ty1) = T(tx2 + tx1, ty2 + ty1)

Successive Rotations

• Two successive rotations applied to point P produce the transformed position.

P′ = R(θ 2)· {R(θ 1)· P}

P′ = {R(θ 2) · R(θ 1)}· P

P′ = R(θ 1+ θ 2) ·P

Successive Scaling

• Two successive scaling are performed as:

P′ = S(sx2, sy2) · {S(sx1, sy1) · P}

P′ = {S(sx2, sy2) · S(sx1, sy1)} · P

sx2 0 0 sx1 0 0 sx1 sx2 0 0

0	sy2	0	.	0	sy1	0	=	0	sy1 sy2	0
0	0	1		0	0	1		0	0	1

• By multiplying two Scaling matrices we can verify that two successive scaling transformations are multiplicative

$$P' = S(sx1 \cdot sx2, sy1 \cdot sy2) \cdot P$$

General Pivot point Rotation

Steps:

• Translate the object so that the pivot-point coincides with the coordinate origin.

• Rotate the object about the coordinate origin with specified angle.

• Translate the object so that the pivot-point is returned to its original position (i.e. Inverse of step - 1).

General Fixed point Scaling

Steps:

• Translate the object so that the fixed-point coincides with the coordinate origin.

• Scale the object with respect to the coordinate origin with specified scale factors.

• Translate the object so that the fixed-point is returned to its original position (i.e. Inverse ofstep-1).

Other Transformations

Some additional transformations that are useful in certain applications are Reflection and Shear.

Reflection : Gives Mirror image

Shear: slants the shape of an object

REFLECTION

A reflection is a transformation that produces a mirror image of an object.

The mirror image can be either about x-axis or y-axis.
Reflection gives image based on position of axis of reflection.

Types of Reflection

Reflection about the x-axis -

In this transformation value of x will remain same whereas the value of y will become negative.

The object will lie another side of the x-axis.

The object can be reflected about x-axis with the help of the following matrix

Reflection about the y-axis -

In this transformation value of y will remain same whereas the value of x will become negative.

The object will lie another side of the y-axis.

The object can be reflected about y-axis with the help of the following matrix

Reflection about an axis perpendicular to xy plane and passing through the origin -

In this value of x and y both will be reversed. This is also called as half revolution about the origin.

The reflected object can be represented with the help of the following matrix

Reflection about line y = x -

The object may be reflected about line y = x with the help of following transformation matrix

SHEAR

It is transformation which changes the shape of object.
The sliding of layers of object occur.
The shear can be in one direction or in two directions.
Types of Shear

X - Shear -

(Shearing in the X-direction)

In this horizontal shearing sliding of layers occur.

The homogeneous matrix for shearing in the x-direction is shown below:

Y - Shear -

(Shearing in the Y direction)

Here shearing is done by sliding along vertical or y-axis.

The homogeneous matrix for shearing in the y-direction is shown below:

XY - Shear -

(Shearing in both the directions)

Here layers will be slide in both x as well as y direction.

The sliding will be in horizontal as well as vertical direction.

The shape of the object will be distorted.

The matrix of shear in both directions is given by: