Estimating camera inactivity periods from capture histories

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Abstract

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1 Data generating process and Models

1.1 Data Generating Process

1.1.1 Latent Camera Process

Let n cameras be placed within the study area, and let E_i denote the total length of time camera $i = 1, \cdots, n$ is in place. By convention we will let time zero denote the time at which the cameras are first operational. Cameras can be operating in one of three states; normal, broken, or continuously mis-firing, which we call states 1, 2, and 3 respectively. The camera can transition between the three states. We apply the following restrictions on the latent process:

- Once a camera is continuously mis-firing, it must then transition to broken,
- Once a camera is broken, if it is fixed, it must transition to functioning normally,
- The camera always begins by operating normally.

1.1.2 Detection Process

Each image taken by camera i has a timestamp, so our detection histories are $\mathbf{x_i} = (x_{i,1}, \cdots, x_{i,k_i})$, where $x_{i,k} \in (0, E_i)$ is the length of time between set up of camera i and image k. By K_i we denote the total number of images taken by camera i. If $K_i = 0$ then we let $\mathbf{x_i} = \emptyset$.

The camera takes images at different rates depending on the state that it is in. Whilst the camera is in state s, the time between consecutive detections is exponentially distributed with rate λ_s. In state 2, the camera is broken so we take $\lambda_2 = 0$. In state 3, the camera is continuously mis-firing, and would therefore expect $\lambda_3 >> \lambda_1$.

1.1.3 Marks

The detection process may be marked, wherein each detection is tagged as a 'true positive' (containing an image of an individual) or a 'false positive' (no individual can be seen). Let $\gamma_{i,k} \in (0, 1)$ denote whether detection k by camera i is a true positive (1) or a false positive (0). If we have this information, we can impose the following structure on λ_1 and λ_3 given the underlying process behind the mark generation.

Let λ_+ denote the rate at which the camera makes true positive detections. Let λ_-^1 and λ_-^3 denote the rates at which the camera makes false negative detections in states 1 and 3 respectively. The detection rates in each state can then be expressed as $\lambda_1 = \lambda_+ + \lambda_-^1$ and $\lambda_3 = \lambda_+ + \lambda_-^3 s$. The probability that an image is a true positive is then λ_+/λ_1 for state 1 (and λ_+/λ_3 for state 3). Since we expect that $\lambda_-^3 >> \lambda_-^1$, the distribution of marks is indicative of the state the camera is in, as we would expect far more false positives in state 3.

1.2 Models

We can model this process discretely using a Hidden Markov Model (HMM) or continuously using a Markov Modulated Marked Poisson Process (MMMPP), which we describe in Sections 1.2.1 and 1.2.2 respectively.

1.2.1 Hidden Markov Model

We can model the process discretely using a Hidden Markov Model (HMM). Across the study define a 'unit of time' (for example an hour or a day), so that the period $(0, E_i)$ is divided into mutually exclusive time periods. Let there be T_i periods for camera i, and we denote by $s_{i,t}^D, t = 1, \cdots, T_i$, the state of camera i during period t.

The state transitions can be described by the following transition matrix:

$$\Theta = \begin{pmatrix} \theta_{11} & \theta_{12} & \theta_{13} \\ \theta_{21} & \theta_{22} & 0 \\ 0 & \theta_{32} & \theta_{33}, \end{pmatrix} \tag{1}$$

where $\theta_{xy} = \mathbb{P}(s^D_{i,t+1} = y | s^D_{i,t} = x)$ for camera i and time points t. Note that $\theta_{23}, \theta_{31} = 0$ by the assumptions in Section 1.1.1.

Let $C^1_{i,t}$ and $C^0_{i,t}$ respectively denote the counts of true positive and false positive images taken by camera i during time period t. Given the detection rates λ_+, λ^1_-, and λ^3_-, the counts $C^1_{i,t}$ and $C^0_{i,t}$ are Poisson distributed and for $s^D_{i,t} = s \in (1,3)$ we have the following:

$$\mathbb{P}(C^0_{i,t}, C^1_{i,t} | s^D_{i,t} = s) = \mathbb{P}(C^0_{i,t} | s^D_{i,t} = s)\mathbb{P}(C^1_{i,t} | s^D_{i,t} = s) \tag{2}$$

$$= \frac{(\lambda^s_-)^{C^0_{i,t}} \exp(\lambda^s_-)}{C^0_{i,t}!} \frac{(\lambda_+)^{C^1_{i,t}} \exp(\lambda_+)}{C^1_{i,t}!}, \tag{3}$$

and for $s^D_{i,t} = 2$ we have:

$$\mathbb{P}(C^0_{i,t}, C^1_{i,t} | s^D_{i,t} = 2) = \begin{cases} 1 & \text{if } C^0_{i,t}, C^1_{i,t} = 0, \\ 0 & \text{otherwise.} \end{cases} \tag{4}$$

The likelihood function for camera i can be expressed as:

$$\mathcal{L}_i = [1,0,0] \left(\prod_{t=1}^{T_i - 1} \operatorname{diag}(P_{i,t}) \Theta \right) P_{i,T_i}, \tag{5}$$

where entry s of $P_{i,t}$ denotes $\mathbb{P}(C^0_{i,t}, C^1_{i,t} | s^D_{i,t} = s)$ for $s = 1, 2, 3$.

1.2.2 Markov Modulated Mark Poisson Process

We can model the process continuously using a Markov Modulated Mark Poisson Process. In this case, the camera can move between the three states in continuous time, according to a continuous time Markov Process, where the time spent in each state is exponentially distributed. This has the following generator/transition matrix Q:

$$Q = \begin{pmatrix} -\mu_{12} - \mu_{13} & \mu_{12} & \mu_{13} \\ \mu_{21} & -\mu_{21} & 0 \\ 0 & \mu_{32} & -\mu_{32} \end{pmatrix} \tag{6}$$

where μ_{ij} is the rate at which the camera transitions to state j given that it is in state i. Analogously to the HMM, $\mu_{23}, \mu_{31} = 0$ due to the assumptions in Section 1.1.1.

The likelihood function for camera i is then expressed as:

$$\mathcal{L}_i = [1,0,0] \left\{ \prod_{k=1}^{K_i} \exp(C[x_{i,k} - x_{i,k-1}]) \Lambda \operatorname{diag}(P_{i,k}) \right\} \exp(C[E_i - x_{i,K_i}]) \mathbf{e},$$

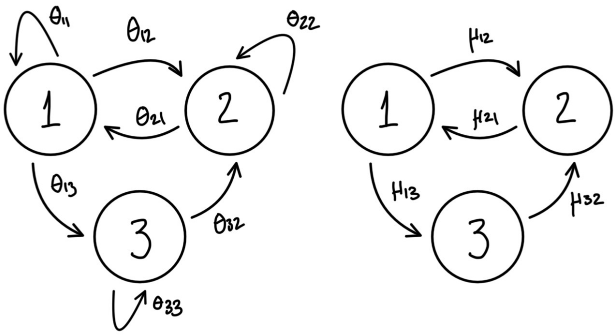
where $x_{i,0} = 0$, $\mathbf{e}$ is a vector of ones, $C = Q - \Lambda$, and $\Lambda = \operatorname{diag}(\lambda_1, \lambda_2, \lambda_3)$. If $s_{i,k}$ denotes the state of camera i at detection k, then entry s of $P_{i,k}$ is $\mathbb{P}(\gamma_{i,k} | s_{i,k} = s)$, where:

$$P(\gamma_{i,k}|s_{i,k}=s) = \begin{cases} \lambda_+/\lambda_s & \text{if } \gamma_{i,k}=1, \\ \lambda_-^s/\lambda_s & \text{otherwise.} \end{cases} \tag{7}$$

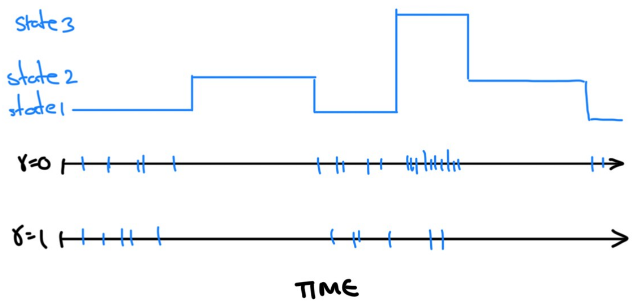
1.2.3 Heterogeneous Detection Rates

Camera detection rate heterogeneity and how it can be added. Addition of covariates.

Figures 1 and 2 demonstrate the underlying processes for both the MMMPP and HMM models. Note that in the HMM, transitions can only occur between distinct time periods t, whereas transitions may occur at any time in the MMMPP modelling framework. In the MMMPP, the model data are the distances between consecutive detections, whereas in the HMM, model data are counts of detections per time period t.

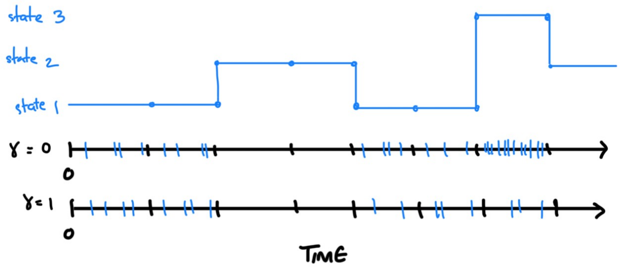


(a)

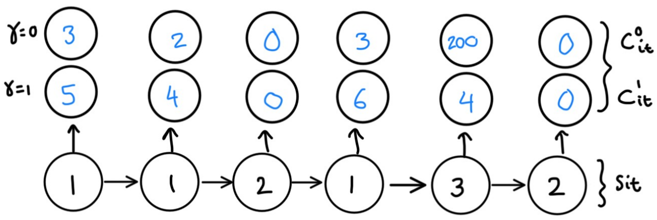


(b)

Fig. 1: (a) The transitions between the latent camera states for the HMM (left) and MMMPP (right). (b) Depiction of the MMMPP where the blue denotes the latent camera states for each time period, and the detections for true positives ($\gamma = 1$) and false positives ($\gamma = 0$) are shown below.



(a)



(b)

Fig. 2: (a) Depiction of the HMM where the blue denotes the latent camera states for each time period, and the detections for true positives ($\gamma = 1$) and false positives ($\gamma = 0$) are shown below. (b) The HMM showing transitions in the latent camera state and the observations C^0_{it}, C^1_{it}.

1.3 Estimating inactivity periods

We denote by $U_i \in (0, 1)$ the fractional effective effort of camera i over the time frame $(0, E_i)$. This is the proportion of time that camera i was in state 1 (i.e working normally) whilst the camera was up.

We can also visualise posterior estimates for the state of each camera at each time point through the study period. This enables practitioners to determine when cameras were likely not functioning normally. We go through determining these probabilities for each model separately below.

1.3.1 MMMPP

For a given set of values of transition and detection rates, the Viterbi Algorithm (ref) is used to determine the most likely sequence of states $s_{i,x_{i,k}}$ at each detection point $x_{i,k}$ for $k \in (1, K_i)$ at camera i. The following theorem can then be used to determine states in the inter-detection periods:

Theorem 1. *Let $s_{i,t}$ be the state of camera i at time-point $t \in (0, L_i)$. By $N^i_{t_1,t_2}$ we denote the number of detections between time-points t_1 and t_2 for camera i. Then for $k \in (1, K_i)$ and $t \in (x_{i,k}, x_{i,k+1})$:*

$$\mathbb{P}(s_{i,t} = s | s_{i,x_{i,k}} = j_1, s_{i,x_{i,k+1}} = j_2, N^i_{x_{i,k},x_{i,k+1}} = 0) = \tag{8}$$

$$\frac{[\exp(C(t - x_{i,k}))]_{j_1,s}[\exp(C(x_{i,k+1} - t))]_{s,j_2}}{[\exp(C(x_{i,k+1} - x_{i,k}))]_{j_1,j_2}}, \tag{9}$$

where $x_{i,0} = 0$ and $x_{i,K_i+1} = E_i$, $C = Q - \Lambda$ for $\Lambda = diag(\lambda, 0)$, and Q the generator matrix for the state switching rates.

Proof See Appendix A. □

We then determine U_i for camera i and $t \in (0, E_i)$ using:

$$U_i = \frac{1}{E_i}\sum_{k=0}^{K_i}\left(\int_{x_{i,k}}^{x_{i,k+1}} \mathbb{P}(s_{i,t} = s | s_{i,x_{i,k}} = j_1, s_{i,x_{i,k+1}} = j_2, N^i_{x_{i,k},x_{i,k+1}} = 0)dt\right) \tag{10}$$

1.3.2 HMM

We can determine posterior distributions of effective effort of each camera i by calculating:

$$U_i = \frac{1}{L_i}\sum_{t=1}^{T_i} \mathbb{1}(s^D_{i,t} = 1) \tag{11}$$

at each iteration of the MCMC.

2 Simulations

Running

3 Case Study

The data set includes $n = 121$ cameras. The cameras were set up on average for 40 days. In this study, 23 cameras were broken at time of collection. All cameras took time-lapse photos at 10am every day, and so we can get reasonable estimates for the day on which the camera broke during the study period by looking at the last day on which a time-lapse was taken.

During this study, the cameras were not repaired, and so for the MMMPP we take $\mu_{21} = 0$, and in the HMM $\theta_{21} = 0$. One camera shows evidence that it was repaired in the middle of the study, we have removed this as it violates the modelling assumptions. We allow for camera heterogeneity in the detection rates λ_+ and λ^1_-, but assume that λ^3_- is shared across cameras.

Cameras are assumed to start in state 1, and we also allow the model to know the state of the camera at collection as this information was recorded.

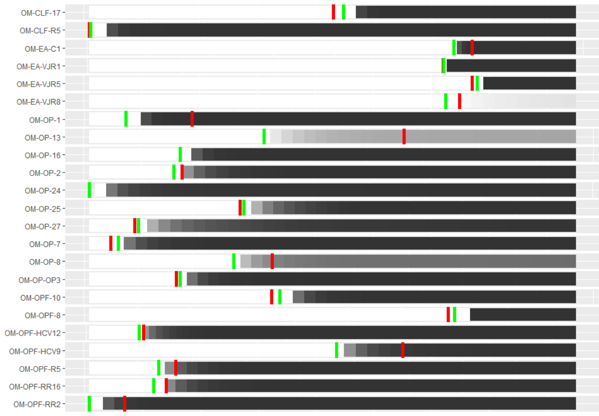


Fig. 3: HMM posterior for probability cameras are in state 2, shading is posterior mean. Red line shows estimated camera fail date, and green line shows last detection made by camera.

4 Discussion

Appendix A Proof for Theorem 1

Recall that $[\exp(C(x-y))]_{s,t} = \mathbb{P}(J_x = t, N_{y,x} = 0 | J_y = s)$. Then we can show that:

$$\begin{aligned}
&\mathbb{P}(J_{x_d} = J_{x_{d+1}} = 1, N_{x_d,x_{d+1}} = 0) \\
&\qquad = \mathbb{P}(J_{x_{d+1}} = 1, N_{x_d,x_{d+1}} = 0 | J_{x_d} = 1)\mathbb{P}(J_{x_d} = 1) \\
&\qquad = [\exp(C(x_{d+1} - x_d)]_{11}\mathbb{P}(J_{x_d} = 1).
\end{aligned}$$

Similarly we can show that:

$$\begin{aligned}
\mathbb{P}(J_x = s, &J_{x_{d+1}} = J_{x_d} = 1, N_{x_d,x_{d+1}} = 0) \\
&= \mathbb{P}(N_{x,x_{d+1}} = 0 | J_x = s, J_{x_{d+1}} = 1)\mathbb{P}(N_{x_d,x} = 0 | J_{x_d} = 1, J_x = s) \\
&\qquad \mathbb{P}(J_{x_{d+1}} = 1 | J_x = s)\mathbb{P}(J_x = s | J_{x_d} = 1)\mathbb{P}(J_{x_d} = 1) \\
&= \mathbb{P}(J_{x_{d+1}} = 1, N_{x,x_{d+1}} = 0 | J_x = s)\mathbb{P}(J_x = s, N_{x_d,x} = 0 | J_{x_d} = 1) \\
&\qquad \mathbb{P}(J_{x_d} = 1) \\
&= [\exp(C(x_{d+1} - x))]_{s1}[\exp(C(x - x_d))]_{1s}\mathbb{P}(J_{x_d} = 1)
\end{aligned}$$

Dividing the probabilities appropriately gives the result.

References