

Functional Geometry Description of Escher's Fish

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Introduction

Square Limit

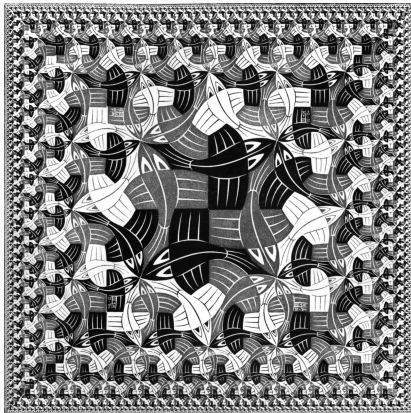


Figure 1: M.C. Escher's Square Limit

Source: <https://www.wikiart.org/en/m-c-escher/square-limit>

Functional Geometry

Functional Geometry is a paper by Peter Henderson[?, ?], which deconstructs the M.C. Escher woodcut “Square Limit”.

A picture is an example of a complex object that can be described in terms of its parts. Yet a picture needs to be rendered on a printer or a screen by a device that expects to be given a sequence of commands. Programming that sequence of commands directly is much harder than having an application generate the commands automatically from the simpler, denotational description.

Basic Operations

Note

The image it is located within a frame, but we do not consider the frame to be part of the picture.

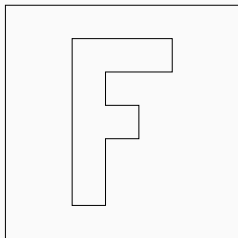


Figure 2: The value `f` denotes the picture of the letter F

Basic operations on pictures

- *rot*(*picture*) :: *picture*
- *flip*(*picture*) :: *picture*
- *rot45*(*picture*) :: *picture*
- *above*(*picture*, *picture*) :: *picture*
- *beside*(*picture*, *picture*) :: *picture*
- *over*(*picture*, *picture*) :: *picture*

Rotation

Rotates a picture 90° anticlockwise.

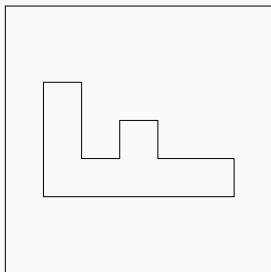


Figure 3: $\text{rot}(f)$

Flip

Flip a picture through its vertical centre axis.

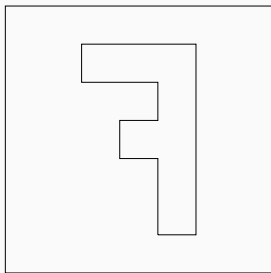


Figure 4: `flip(f)`

Rotation and Flip

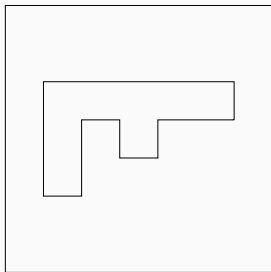


Figure 5: $\text{rot}(\text{flip}(f))$

Rotation 45°

Rotates a picture about its top left corner, through 45° anticlockwise.

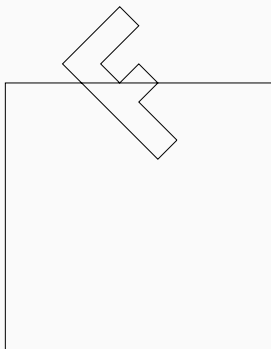


Figure 6: `rot45(f)`

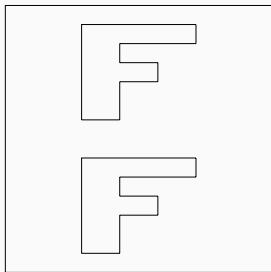


Figure 7: $\text{above}(f, f)$

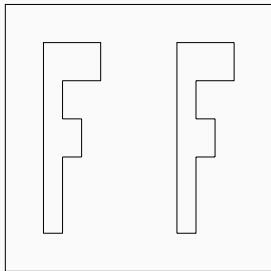


Figure 8: `beside(f, f)`

above/beside combination

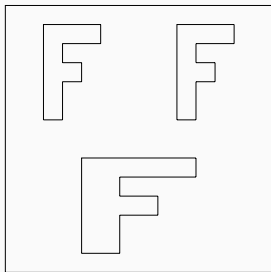


Figure 9: `above(beside(f, f) f)`

Superposition

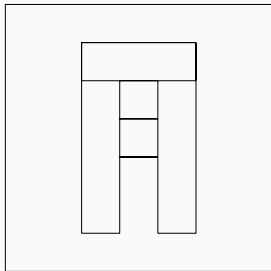


Figure 10: $\text{over}(f, \text{flip}(f))$

Laws

Laws (Unit Test)

$$\text{rot}(\text{rot}(\text{rot}(\text{rot}(p)))) = p$$

$$\text{rot}(\text{above}(p, q)) = \text{beside}(\text{rot}(p), \text{rot}(q))$$

$$\text{rot}(\text{beside}(p, q)) = \text{above}(\text{rot}(q), \text{rot}(p))$$

$$\text{flip}(\text{beside}(p, q)) = \text{beside}(\text{flip}(q), \text{flip}(p))$$

Square Limit

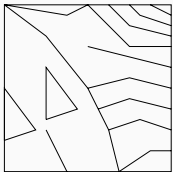


Figure 11: p

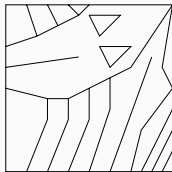


Figure 12: q

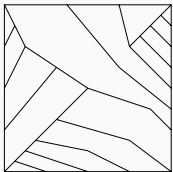


Figure 13: r

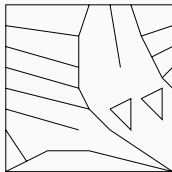


Figure 14: s

t

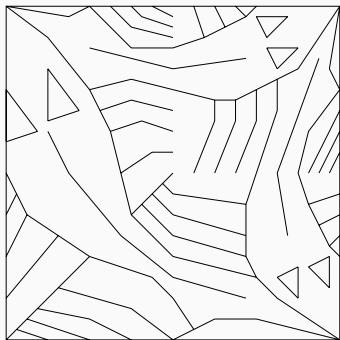


Figure 15: $t = \text{quartet}(p, q, r, s)$

u

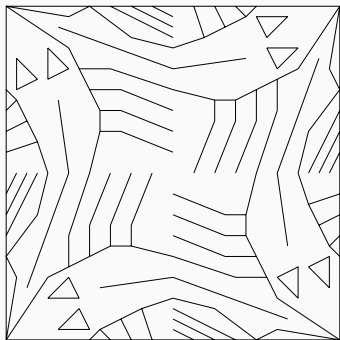


Figure 16: $u = \text{cycle}(\text{rot}(q))$

side1

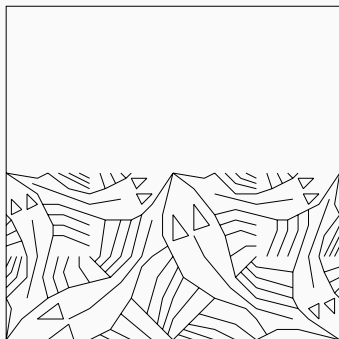


Figure 17: `side1 = quartet(blank, blank, rot(t), t)`

side2

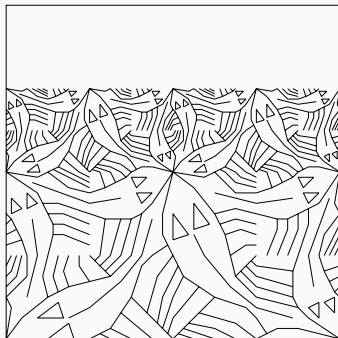


Figure 18: `side2 = quartet(side1, side1, rot(t), t)`

corner1

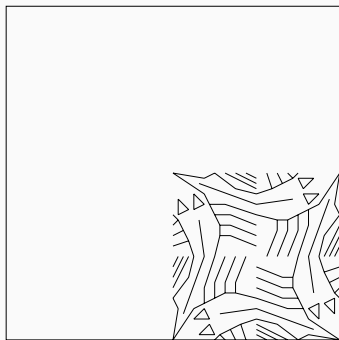


Figure 19: `corner1 = quartet(blank, blank, blank, u)`

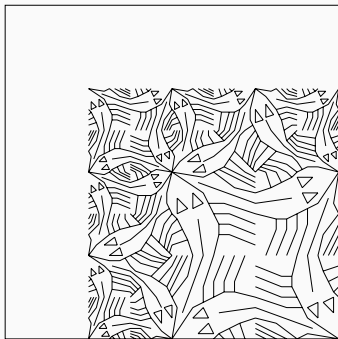


Figure 20: `corner2 = quartet(corner1, side1, rot(side1), u)`

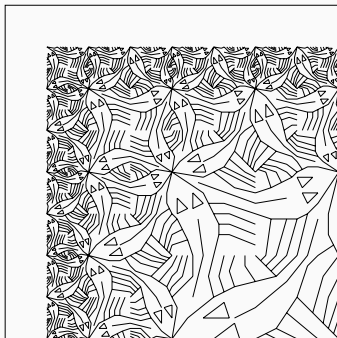


Figure 21: `pseudocorner = quartet(corner2, side2, rot(side2), rot(t))`

pseudolimit

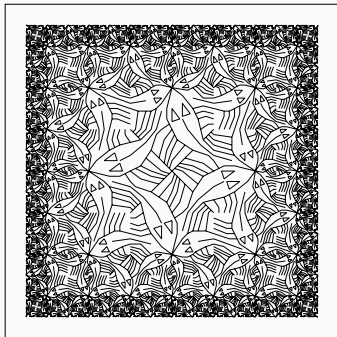


Figure 22: `pseudolimit = cycle(pseudocorner)`

corner

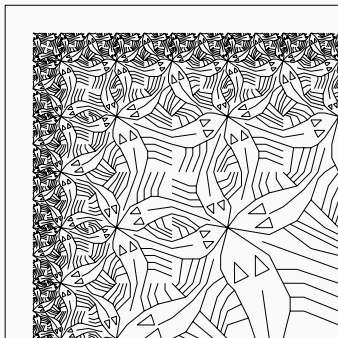


Figure 23: corner

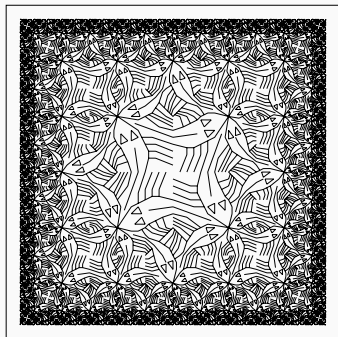


Figure 24: squarelimit

Implementation

Vectors

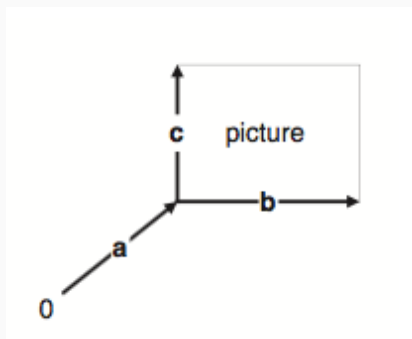


Figure 25: Basic vectors

Implementation

$$\text{over}(p, q)(a, b, c) = p(a, b, c) \cup q(a, b, c)$$

$$\text{blank}(a, b, c) = \{\}$$

$$\text{beside}(p, q)(a, b, c) = p(a, \frac{b}{2}, c) \cup q(a + \frac{b}{2}, \frac{b}{2}, c)$$

$$\text{above}(p, q)(a, b, c) = p(a, b, \frac{c}{2}) \cup q(a + \frac{c}{2}, b, \frac{c}{2})$$

Implementation

$$\text{rot}(p)(a, b, c) = p(a + b, c, -b)$$

$$\text{flip}(p)(a, b, c) = p(a + b, -b, c)$$

$$\text{rot45}(p)(a, b, c) = p(a + \frac{b+c}{2}, \frac{b+c}{2}, \frac{c-b}{2})$$

Demo

Future

- Add support for SVG Path.
- Build an API, in the front-end could exist something like Snap
- Escher's "Circuit Limit III" picture.

Circuit Limit III

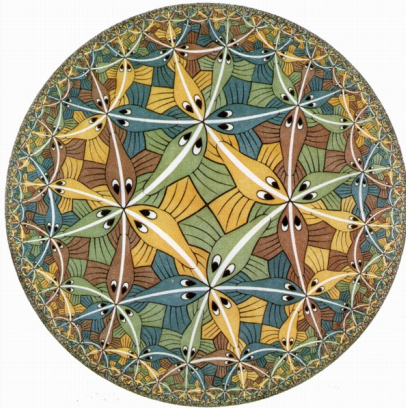


Figure 26: Circuit Limit III

Source: <http://mathstat.slu.edu/escher/upload/9/90/Circle-Limit-III.jpg>

Thanks!

https://github.com/milmazz/func_geo

References I



P. Henderson.

Functional geometry, 1982.



P. Henderson.

Functional geometry, 2002.