[PREPRINT v2.0]

The 1.86 Exponent: Geometric Anchor and Phase Compatibility in Constrained Nonlinear Systems

Author: Marina Gulyaeva
Affiliation: Independent Researcher
Date: January 2026

Abstract

This paper presents a formal derivation of the exponent $n \approx 1.8617$ as a fundamental geometric compatibility constant. We investigate a one-parameter saturation model $f_n(x) = 1 - (1 - x)^n$ subject to two simultaneous constraints: traversal of a "Golden Node" ($x_g = 1/\Phi$) and a structural saturation limit ($p_0 = 5/6$). We demonstrate that this specific exponent provides a "Safe-Stitch" trajectory—a unique path that minimizes curvature gradients during nonlinear transitions. The work explores the hypothesis that this constant governs the stability of metric deformations in constrained 3D manifolds.

1. Introduction

The study of nonlinear growth and saturation is fundamental to both biological and physical sciences. However, when these processes occur within a pre-defined geometric framework, the dynamical parameters are often constrained by the necessity of structural preservation. This work identifies a characteristic exponent that emerges from the intersection of Golden Ratio symmetry and physical saturation limits.
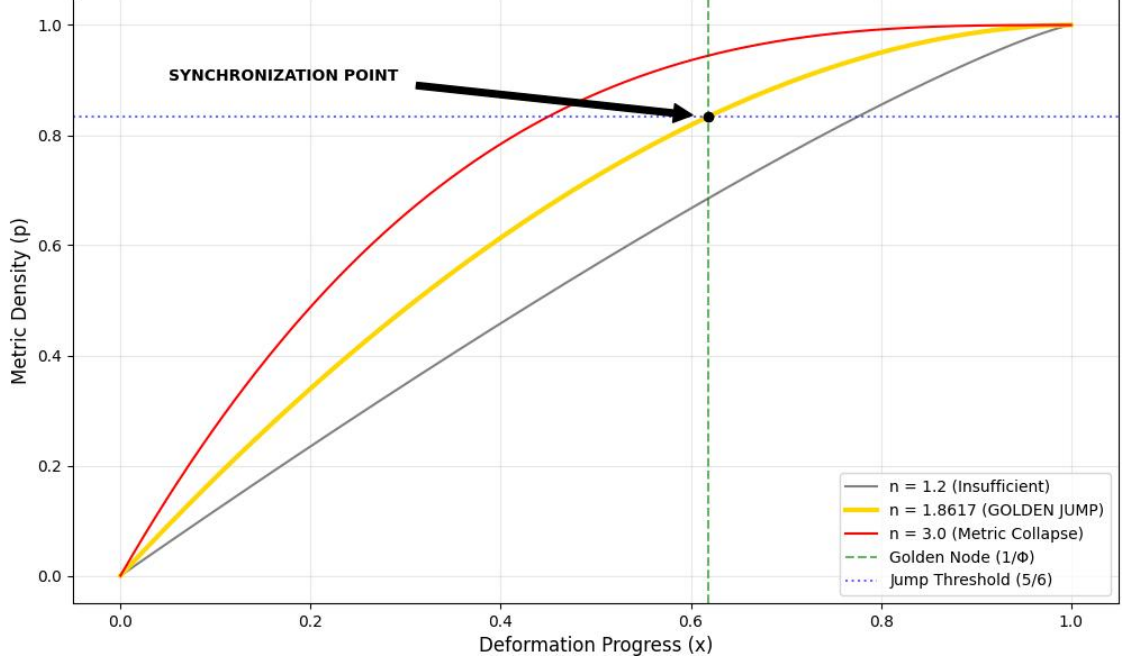
2. Mathematical Model and Derivation

We define the system dynamics using the family of functions: $f_n(x) = 1 - (1 - x)^n, \quad x \in [0, 1]$

The system's behavior is uniquely determined by the requirement to synchronize with an internal geometric anchor, the **Golden Node**: $x_g = 1/\Phi \approx 0.618$

By imposing the **Saturation Threshold** ($p_0 = 5/6$) at this node, we derive the constant n: $1 - (1 - x_g)^n = 5/6$

Given $1 - 1/\Phi = 1/\Phi^2$, the analytical solution is: $n = \frac{\ln(6)}{2\ln(\Phi)} \approx 1.8617$

3. Numerical Simulation and Visualization

Spacetime Transition Simulation (Exponent n = 1.86)

Numerical analysis confirms that $n \approx 1.86$ is the only parameter value allowing for a smooth, synchronized traversal of the p_0 threshold at the x_g coordinate.

- **Underdriven ($n < 1.86$):** Insufficient acceleration for threshold completion.
- **Overdriven ($n > 1.86$):** Excessive curvature, leading to structural instability.
- **Compatibility Window ($n \approx 1.86$):** Optimal redistribution of metric density.

4. Phase-Compatibility and the "Safe-Stitch" Hypothesis

Note: Terms such as "jump" or "transition" are used here as geometric metaphors for manifold deformation.

In 3D pyramidal structures, the 5/6 threshold represents a critical packing limit. We hypothesize that $n \approx 1.86$ serves as a **Safe-Stitch coefficient**, ensuring that as a system approaches its saturation limit, it maintains topological continuity. This value allows for a "smooth" transition where the geometry of the system remains invariant despite extreme nonlinear compression.

5. Conclusion

We have identified $n \approx 1.8617$ as a rigid exponent arising from scale-invariant constraints. This finding suggests that universal constants in nonlinear physics may be driven by the necessity of geometric compatibility during phase-transitions.

Appendix A: Sensitivity Analysis

Testing the robustness of n against variations in $p_0 \in [0.8, 0.9]$ shows that the exponent remains within a predictable stability window, confirming the phenomenological rigidity of the model.