# From Exponential to Convergent to Logistic Models Using STELLA
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I. Exponential Growth/Decay Models   (Math Equation: $F(t) = Ab^t$)

"In biology, chemistry, and economics it is often necessary to study the behavior of a quantity that is increasing as time passes. **If, at every instant, the rate of increase of the quantity is proportional to the quantity at that instant, then ... the quantity is growing exponentially** or is exhibiting exponential growth. A simple example of exponential growth is exhibited by the growth of bacteria in a culture. Under ideal laboratory conditions a bacteria culture grows at a rate proportional to the number of bacteria present. It does so because the growth of the culture is accounted for by the division of the bacteria. The more bacteria there are at a given instant, the greater the possibilities for division and hence the more rapid is the rate of growth." [*]

DT = 1, Use Euler Integration Method
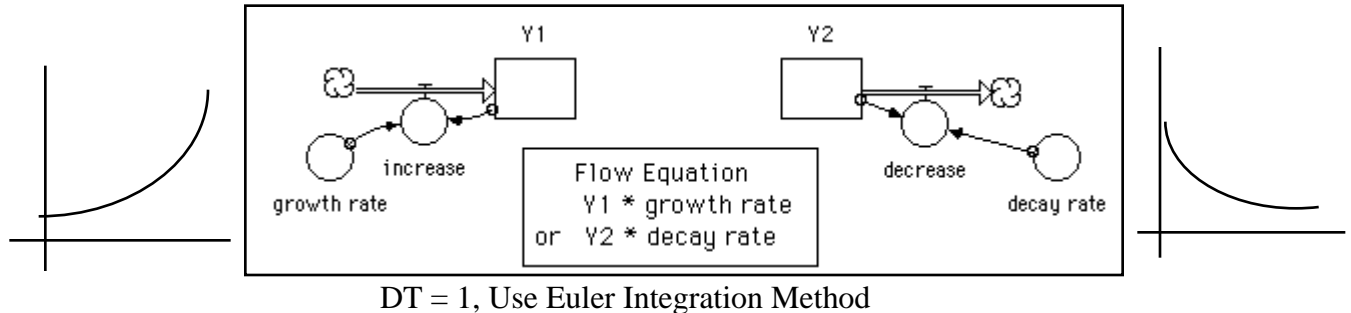
1. **(Bacteria)** Suppose you have been doing an experiment in your biology lab with bacteria. You started the experiment on Wednesday and measured 1000 bacteria per square millimeter. You measure the number of bacteria each successive day for a week and determine that the exponential growth rate is 50% each day.
    a.   Ask yourself:
        1.   What is increasing or decreasing with time? _____ Make that the stock. Be sure to label the stock and give it a beginning value.
        2.   What is causing the stock to increase or decrease? Use a flow directed into the stock if you want to increase. Use a flow out of the stock if you want it to decrease over time. What is the flow here?_____
        Is it causing the stock to increase or decrease?_____
        Set up the flow in the diagram. Be sure to label the flow icon and set its value.
        3.   Since this is exponential and not linear growth there is one small change in the diagram. You need to connect the stock back to the flow converter. Why is this connection important?_____
        _____
    b.   Set the DT to 1.00 (under Run/Time Specs). Then define a table in STELLA II. Include your stock name in the table. Run the model.
        On which day did the bacteria triple?_____
        Determine the number of bacteria per square millimeter one week (7days) after the start of the study. _____
    c.   Write the exponential equation for this situation._____

2. **(Poison)** You accidentally inhale some mildly poisonous fumes.   Twenty hours later you see a doctor. From a blood sample, she measures a poison concentration of 0.00372 mg per cubic cm. She asks you to wait at her office for one hour and takes another blood sample and measures the concentration to be 0.003482. The doctor expects the poison to decrease exponentially with the hours that pass since the first visit.
    a. Set up a STELLA II diagram for this situation.
    b. Define a table and run the simulation for 100 hours.

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[*] <u>Calculus and Its Applications,</u> Goldstein, Lay & Schneider, Prentice Hall, 1987, p242-243.

1. The doctor says you can resume normal activities when the poison concentration has dropped to 0.00010 mg/cu.cm. How long after the first office visit will you be able to resume normal activities?_____
2. What is the "biological half-life" of the poison?_____
3. Determine the mathematical equation for this problem._____

3. **(Generic)** The following tables represent exponential growth or decay data. Set up STELLA II diagrams for each. Run the simulation for 12 time units. Confirm that the STELLA table and the table below are the same. Determine the math equation that matches each function.

a.

X	Y
0	3
1	2.1
2	1.47
3	1.029

_____
equation

b.

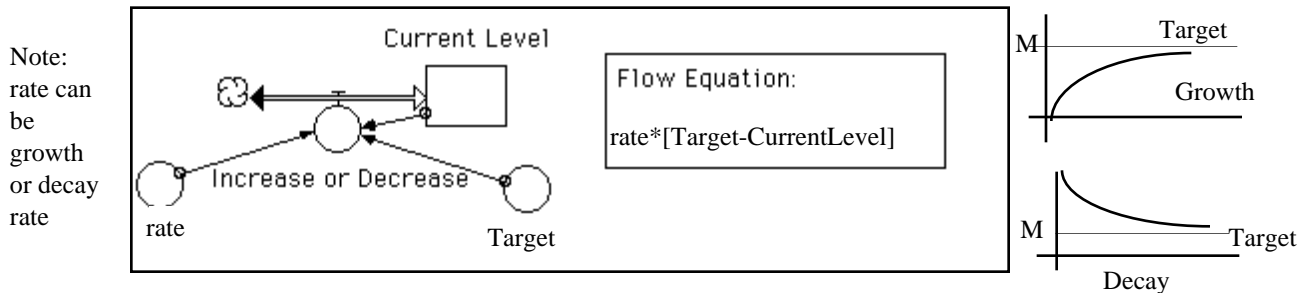| X	Y
0	1.21
1	1.331
2	1.4641
3	1.61051

_____
equation

Draw the STELLA diagram for each situation below. Label each icon and place the correct values in each icon.

II. Converging Exponential Models

Math Equation: $F(t) = [F(0) - M]e^{-kt} + M$

Converging exponential models have the characteristic that there is some value the growth or decay approaches, over time, instead of diverging to infinity. The system will change in such a way as to reduce the difference between the current level of a substance and it's target. **If, at every instant, the rate of increase of the quantity is proportional to the difference between the quantity at that instant and the target then ... the quantity is converging exponentially on the target.** A simple example of convergent behavior is a cup of coffee cooling to room temperature. (Notice that if the target, M, is zero, then this broader exponential case reduces to the simpler case of exponential decay described in part I.)

Note: rate can be growth or decay rate

Current Level

Increase or Decrease

rate

Target

Flow Equation:

rate*[Target-CurrentLevel]

M — Target
Growth

M — Target
Decay

Set BIFLOW, DT = 0.1, Use Runge-Kutta 4 Integration Method

4. **(Newton's Law of Cooling)** Newton's Law of Cooling states that the rate of change in the temperature of an object is proportional to the difference between the object's temperature and the temperature of the surrounding medium. Consider the following problem.

   You go to Starbucks and get a cup of coffee on a cold (35°) day. The coffee is at 95° when they pour it into a paper cup. Paper cups are not well insulated and loose heat at about 90% per minute. You like cold weather so you decide to sit outside to drink your coffee.
   Design a STELLA model for this situation. Define a table. Set the DT = 0.1 and use Runge-Kutta 4 as the integration method. Run the simulation for 12 minutes.
   a. What is the coffee temperature after one minute?_____
   b. Special Starbuck mugs only loose heat at about 20% per minute. How hot will the coffee be after minute if you use a Starbuck mug?_____
   c. Assume coffee is cool enough to drink when it is at 90° . What is the minimum time you should wait (after they pour your coffee in your mug) so you don't burn your mouth? _____
   d. (optional for SELLAites) If you put cream in your coffee after you get it (in a mug) the cream cools the coffee. Is there a difference in the coffee temperature if you put the cream in immediately when you det your coffee or if you wait five minutes and put it in just before you drink it? _____ Why do you think this happened? _____

   _____
   _____

III. Logistic (S-Shaped) Models:

Math Equation: $F(t) = \dfrac{M}{1 + \left[\dfrac{M}{F(0)} - 1\right] e^{-kt}}$

"The simple exponential growth model becomes inapplicable when the environment begins to inhibit the growth of the population. The logistic growth curve is an important type of ... model that takes into account some of the effects of the environment on a population. For small values of t, the curve has the same basic shape as an exponential growth curve. Then when the population begins to suffer from overcrowding or lack of food. the growth rate (the slope of the population curve) begins to slow down. Eventually, the growth rate tapers off to zero as the population reaches the maximum size that the environment will support. This latter part of the curve resembles"[**] ... the convergence model studied earlier. In this type of model, at every instant, **the rate of increase of the quantity is proportional to the product of the quantity itself, at that instant, times the difference between the quantity at that instant and the carrying capacity of the environment in which that quantity exists.**

Set BIFLOW, DT = 0.1, Use Runge-Kutta 4 Integration Method
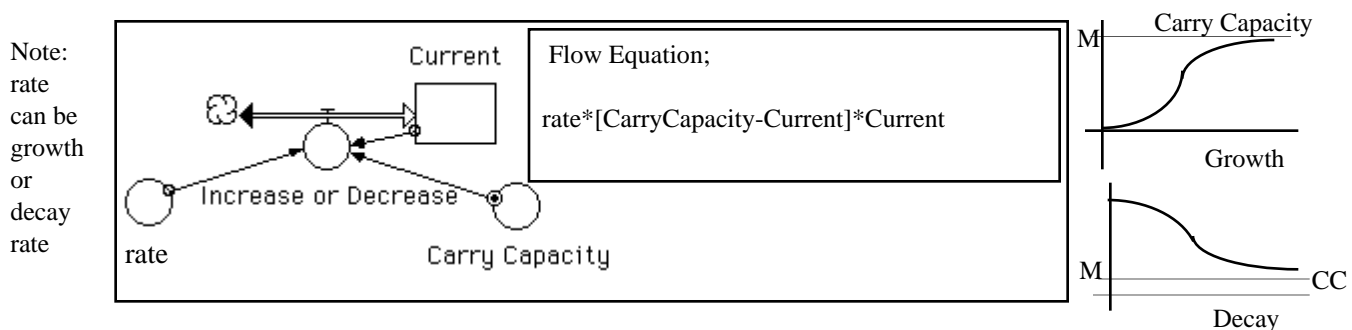
5. **(Fish Population)** Suppose a lake is stocked with 100 fish. After 3 months there are 250 fish. A study of the ecology of the lake predicts that the lake can support 1000 fish.
   a. Determine the value of k (the reproduction rate) for the fish. K = _____ Show how you determined the value of K below.

[**] Calculus and Its Applications, Goldstein, Lay, & Schneider, Prentice Hall, 1987, p275-276.

b. Now that you have the value of K (which should be .37) determine the rate = K/M. Then design a STELLA model to represent this situation.
   1. Define a graph so you can see the typical S-shaped curvature. To define a graph, choose the graph icon and place it in the diagram window. A blank grid will appear. Double-click on the grid. Choose both the Current Fish Population stock and the Carrying Capacity converter. Click on the >> to move both of these to the selected box. With these both in the selected box, click and slide over both of them to make them both selected(dark). Now click on the top double-headed arrow to the right of the Current Fish Population so that the arrow has an upper and lower bound (horizontal dash) placed around it. This will cause a MIN box and a MAX box to appear in the lower part of the window. Set the MIN to zero and the MAX to 1200. Click set. Click OK.
   Under Run/Time Specs set the simulation time to 25 months. Be sure the DT = 0.1 and set Runge-Kutta 4 integration method.
   Run the simulation.
c. Define a table. Find the number of fish present at time 10 months. _____
d. After how many months will the fish population be 600? _____
e. What would happen if the lake were stocked with 1200 mature fish? Find out by using the model. _____

6. **(Flu Epidemic)** On a college campus of 5000 students, one student returned from vacation with a contagious flu virus. The virus spread through the student body with a infection rate of 80% per day Determine the rate = K/M. Then   design a STELLA model for this situation.
Define a table and run the simulation for 20 days.
a. How many people were infected after 5 days?_____
b. The school will cancel classes when 40% of the students are ill. After how many days will the college cancel classes?_____
c. Just for fun find out what would happen if you put 6000 into the initial Students With Flu stock and ran the simulation. You should look at a graph of both the Students with Flu stock and the Carrying Capacity converter to get a good idea. Set up the graph like you did in the previous problem, except set the MIN to zero and the MAX to 7000.
d. Change the rate tp .0016. What happens in the graph? _____
_____