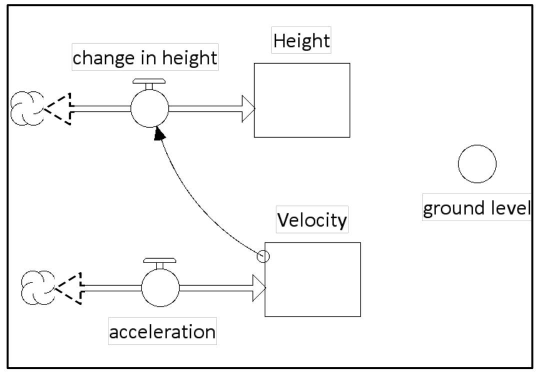
Quadratic Models 1[1]

Introduction and Background

The general Stella diagram for a quadratic function is shown at the right. The labels here are for an example that will model motion, such as throwing a ball straight up into the air. It would help us determine the height, each second, of the ball thrown upward at, say, an initial velocity of 30 m/sec.



In this model, the quantities that change over time are the *height* of the ball above the ground and the *velocity* of the ball. These are represented by stocks. The rate at which the height changes is called the ball's velocity. The rate at which velocity changes is called acceleration and that change results from the influence of gravity. Gravity pulls downward, toward the earth, causing the ball's velocity to slow down as the ball travels upward and then causing it to speed up as it falls back toward the earth. Since the height stock must increase and then decrease, the flow for this stock must be a *bi-flow*. In a bi-flow, positive numbers create an inflow into height, increasing the height value, and negative numbers create an outflow from the height, decreasing the height value.

We'll start out the system with velocity at an initial (positive) value of 30 m/sec. In this example, the acceleration will be the acceleration of gravity, which is always negative because it pulls in a direction downward toward the earth. Upward (away from the earth) is usually considered positive. Eventually, the acceleration resulting from gravity will drive the velocity stock value to a negative value. Why?

Since the velocity stock value is equal to the change in height flow, that flow value will also become negative. Once this happens, the height stock will begin to decline. The graph will show that the ball stops climbing and begins to fall.

[1] This lesson was written by Diana Fisher and Eileen Rogers.

The physics formula for this situation is:

$$\textbf{Height} = (\tfrac{1}{2})\mathbf{g}\mathbf{t}^2 + \mathbf{v_0}\mathbf{t} + \mathbf{h_0}$$

where **g** is the acceleration of gravity **(-9.8m/sec^2)**
$\mathbf{v_0}$ is the initial velocity **(m/sec)**
$\mathbf{h_0}$ is the initial height **(m)**
t is time **(sec)**

This equation is quadratic in time, **t**, and so the expected shape of the graph of this situation is a parabola. The Stella model shown on the previous page above allows you to create a parabolic graph using the information provided below.

Modeling Exercise

1. Build the quadratic model shown on the top of the first page of this handout. Be sure to click on the Map icon  and change it to the Model icon x^2. Check to see that the initial values and constants are set as follows:

Initial Height = 0 {m},
Initial Velocity = 30 {m/sec},
acceleration = -9.8 (m/sec^2),
Vel = Velocity {m/sec},
Ground Level = 0 {m}

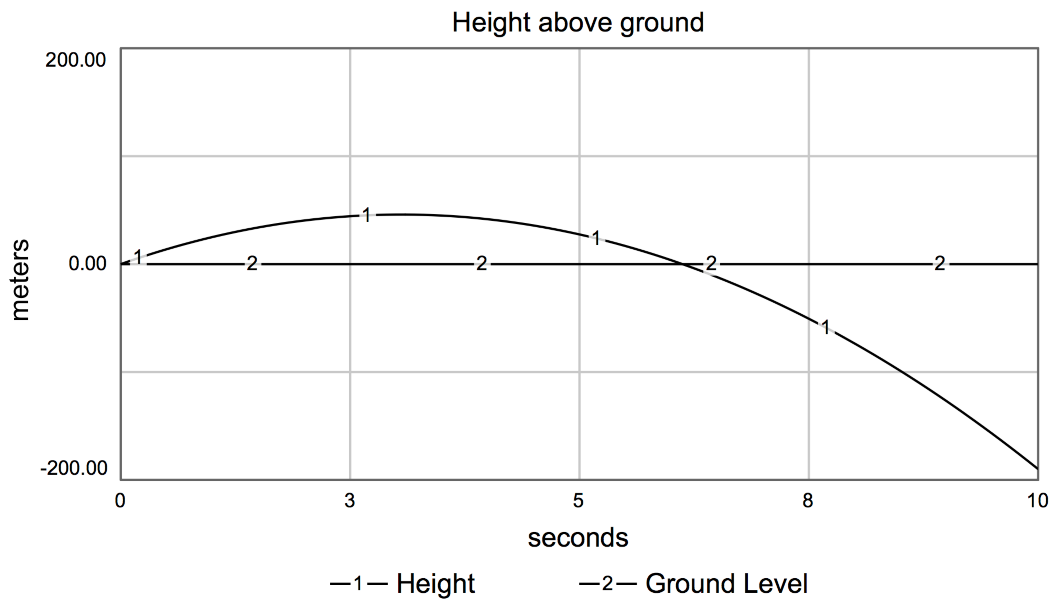
NOTE: Be sure to remove the check mark in each stock next to non-negative.

Double click in the modeling window to open the model Properties panel. Configure your simulation as follows:

Start time:	0
Stop time:	10
DT:	0.1
Time Units:	Seconds
Integration Method:	Runge-Kutta 4 (RK4)

Note: Make sure the Sim Speed is a value between 0 and 10.

Set up a graph that includes the Height and the Ground Level icon names. Set them both to use the same vertical scale—from -200 to 200. Now run the simulation. Your graph should look like the one at the top of the next page:



1. From the graph determine:

a. the approximate height of the ball at its highest point.

b. the approximate time the ball reached its highest point.

c. the approximate time the ball hit the ground.

Be sure to specify units for each of your answers.