Exponential Models 1

Simple Structures to Generate Exponential Growth and Decay

Exponential Growth

Exponential Decay

amount
of growth
per time unit

amount
of decrease
per time unit

growth rate

decay rate
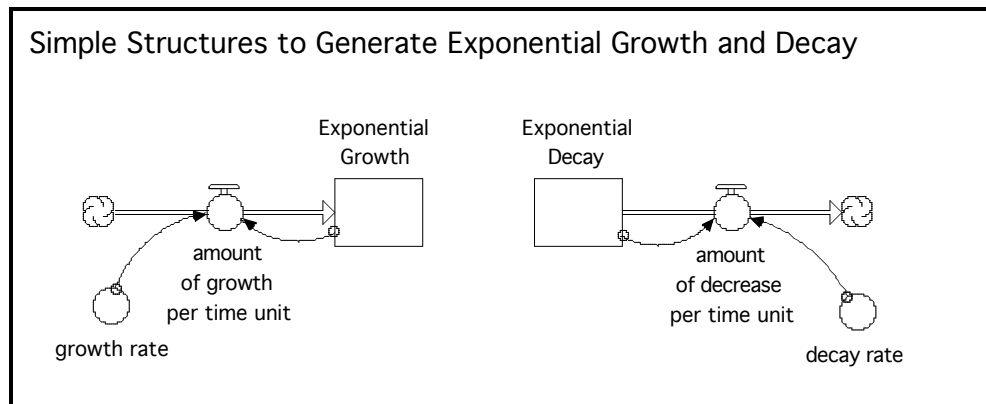
1. Bacteria

Suppose you have been doing an experiment in your biology lab with bacteria. You started the experiment on Wednesday and measured 1000 bacteria per square millimeter. You measure the number of bacteria each successive day for a week and determine that the exponential growth rate is 30% each day (that is, if you take yesterday's amount of bacteria and multiply it by 30% and add it to yesterday's amount of bacteria, you get today's amount of bacteria.)

In this problem there are three important components to consider: the amount of bacteria you have on any given day, the growth rate of the bacteria each day, and the amount of new bacteria that are added each day.

1(a) Determine which Stella diagram above to use for this problem. Draw the diagram, label each icon correctly using words that represent the ideas in the bacteria problem, and place the correct value or symbol in each icon so it will model the problem described above.

Notice that, in this diagram (which is very similar to a linear model) there is a connector from the stock (rectangle) back to the flow (growth amount per time unit). This connection didn't happen in the linear model. Why is this connection critical in an exponential model?

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1(b)   Construct the model you drew above, using the Stella software. When you double click on the flow icon, to define the value, you will notice that there will be two items listed in the "Required Inputs" box. You MUST use those items in your definition. You should multiply the two "Required Inputs" together for this model.   Just click on the name of the first required input, click on *, then click on the second name. Click the ☑ button in the bottom-right corner of the panel when done.   Click the ▷ button in the middle of the left side of the panel to close the panel, if desired.   Did you remember growth rate must be in decimal form? In the model's Properties panel set the **DT** to **1**, do not change anything else.   Then define a table in Stella. Include your stock name in the table. Run the simulation.

On which day did the bacteria triple?

| |
| |

Determine the number of bacteria per square millimeter one week (7 days) after the start of the study.

| |
| |

How many bacteria per square millimeter are there 12 days after the start of the study?

| |
| |

Write the math equation relating the amount of bacteria per square millimeter, B, to the time, t, since the first measurement was made.

| |
| |

1(c)   If the growth rate is doubled (from 30% to 60%) will there be twice as many bacteria per square millimeter after 12 days? First, make your prediction:

| |
| |

Now run the simulation and write down how many bacteria per square millimeter there were after 12 days:

| |
| |

1(d)   Let's see what influence the growth rate has. Fill in the table at the right using your simulation to get the answers.
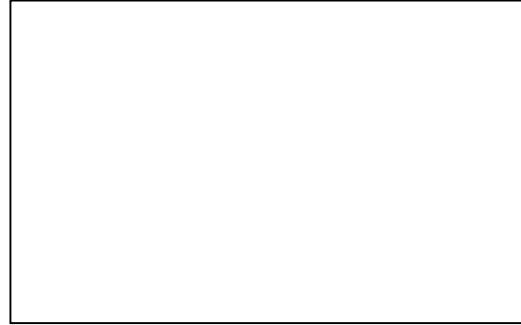
This type of growth is the reason people are either very happy (if it's money growing) or very concerned (if it's pollution growing) when they hear about exponential growth.

Growth Rate %	Amount of Bacteria on day 12
1	
10	
20	
30	
60	
90	

2. Medicine

Suppose you hurt your leg and are taking aspirin to help reduce the pain. Each tablet contains 325 mg of aspirin. You decide to take two tablets. Each hour, 20% of the amount of aspirin in an individual's bloodstream is removed from the system.

2(a) Determine which Stella diagram, on the top of the first page of this lesson, to use for this problem. Draw the diagram, label each icon correctly using words that represent the ideas in the aspirin problem, and place the correct value or symbol in each icon so it will model the problem described above.

2(b) You usually are allowed to take more aspirin every 4 hours. How much aspirin will there be in your system after the first 4 hours?
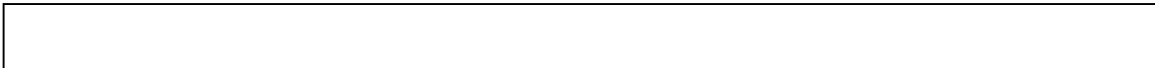
Assuming you take only one dose of aspirin, write the math equation that relates the amount of aspirin you have in your system, A, to the time, t.

2(c) Suppose you have a kidney infection in addition to the injured leg. Assume the kidney infection causes the aspirin to eliminate from your bloodstream at a rate of 10% instead of 20% per hour. Before you can take additional aspirin, you need to wait until the aspirin amount in your body drops to the same level that it would reach after 4 hours if your kidney was functioning normally.

Predict how long you should you wait before taking more aspirin, if you have the kidney problem?

Now run the simulation. How long does the simulation indicate that you should wait?
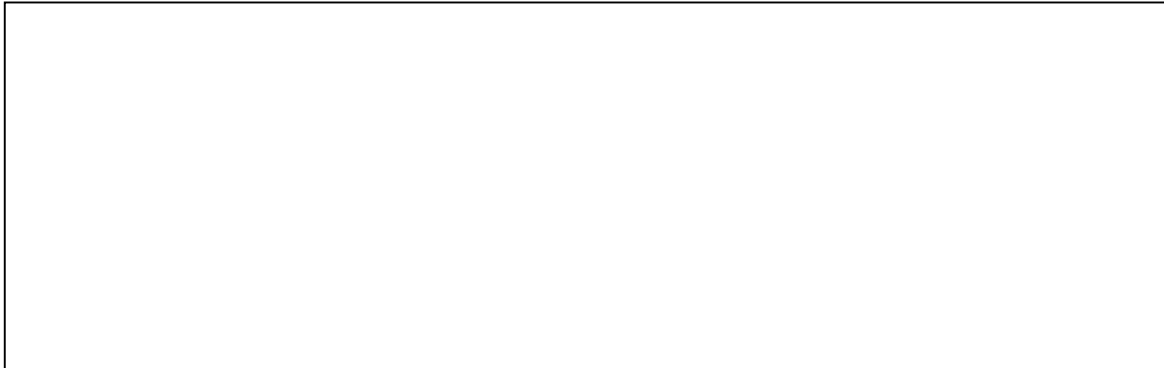
Explain any differences between your predicted answer and the simulation answer.

3. Extra for Experts: Social Issues

The population of Malawi in 1995 was 11,300,000. The population is reproducing at a rate of 5.45 % per year. 2.2 % of the population dies each year. We are interested in predicting the Malawi population in year 2095.

3(a) Construct a Stella diagram to use for this problem. Draw the diagram below, label each icon correctly using words that represent the ideas in the population problem, and place the correct value or symbol in each icon so it will model the problem described above.
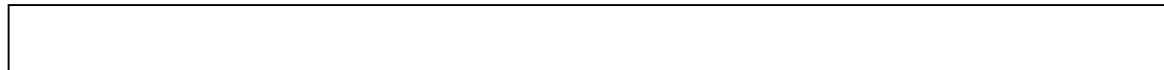
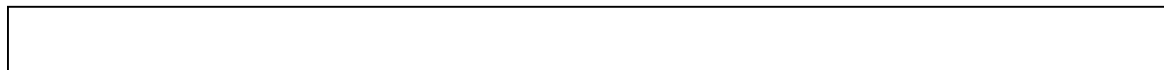Run the simulation. What is the Malawi population in 2095?

3(b) Assume a wonder drug is discovered, in 2020, that cuts the death rate in half. The United Nations makes this drug available to all third-world countries.

Predict the Malawi population in 2095, taking into consideration that they used this wonder drug as soon as it was available.

Now run the simulation [*Note*: you will have to use the STEP command in the death rate component to cause the change in rate to occur in your simulation at the correct time. See the Stella review sheet.] What did the simulation calculate as the Malawi population in 2095?

3(c)   (Remove the wonder drug scenario from your model.) You are a consultant to the president of Malawi people. The country is small and not very prosperous, so their economy will only be able to support 25 million people, even by 2095. What should you recommend to the president to help the country survive? You may not add any new components to the model (Be specific!   Explain **exactly** how you would alter the new simulation to show that the population does not grow beyond 25 million by 2095. Run the simulation with your suggestions to show that it actually does work. Your solution must be acceptable to the population.)

Write any new definitions you made for any components.

Use the table below to record the total population for the years given:

Year		Total Population {unit of measure = people}
1995	= year 0	
2020	= year 25	
2070	= year 75	
2095	= year 100	