Oscillating Example for Algebra II, Using STELLA

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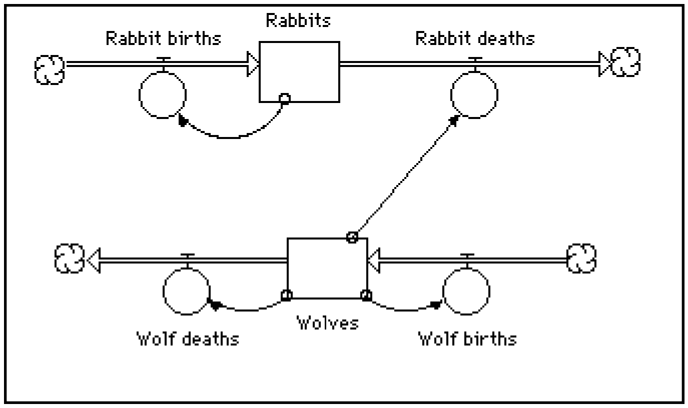
We will study an ecosystem involving rabbits and wolves in the usual predator/prey relationship. From our study we will see one example of oscillation.

Constructing the diagram: Create this diagram on your computer screen.



We will start our ecosystem with 50000 rabbits and 1250 wolves. Before you type in these values, be sure to click on the world in the left part of the screen to change it to X^2 . When you type in these starting values be sure to remove the X in the non-negative box for each stock of rabbits and wolves. To remove the X just click on it once.

New rabbit births depend upon the current rabbit population so that is why there is a connection from rabbits to rabbit births. The same is true for wolf births and wolf population.

The new rabbits come from the females (50% of the current rabbits are female). Each female has about 10 baby rabbits each year. About 1/4 of the baby rabbits survive to adulthood. Using this information we will define the rabbit births as follows: (Double-click on rabbit births and type: (remember don't type the word rabbits, click on rabbits in the required inputs box) then click OK)

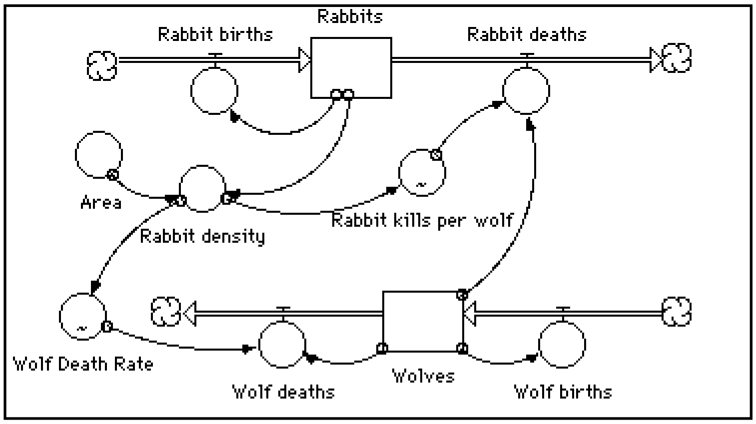
.5 * 10 * .25 * Rabbits

We will postpone defining the rabbit deaths for a few minutes.

Wolves births are a percentage of the current wolf population. That means the births should be defined as what kind of growth? (exponential/linear) The new wolves come from the females, which constitute 50% of the wolf population. Each female wolf has about 2 cubs every 2 years (so how many do they have each year?________). The survival rate of the cubs is 50%. Write the definition for wolf births:

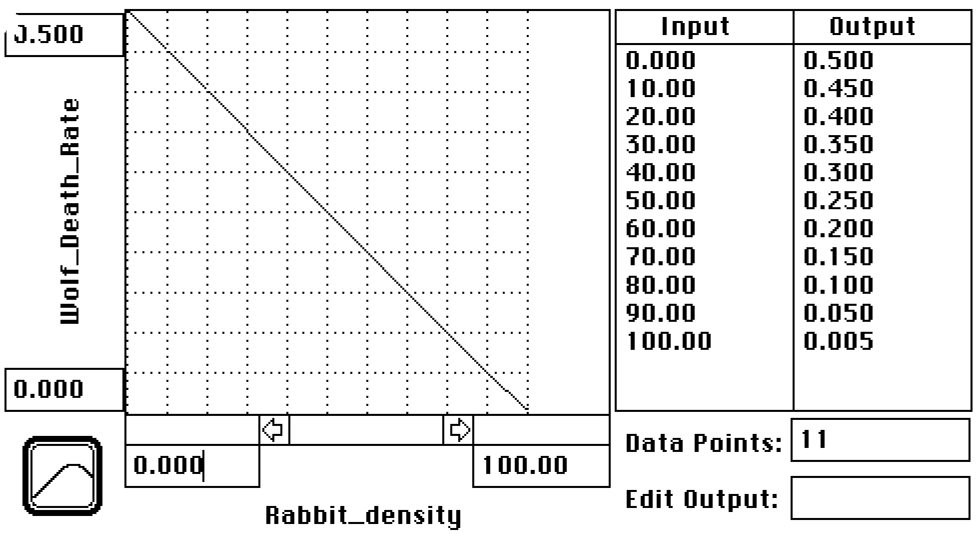
Wolf births = ____

Double click on wolf births and type in your definition.
In order to define the death rates we must add some new components. Add these components to the model: Rabbit density, area, Rabbits killed per wolf, and wolf death rate, so the model should now look like this.



Set the area to 1000. To figure out density think about the fact that density means how many of something you have in a given region. What should you define rabbit density to be? ______________________________ Double click on rabbit density and type in your definition, then click OK.

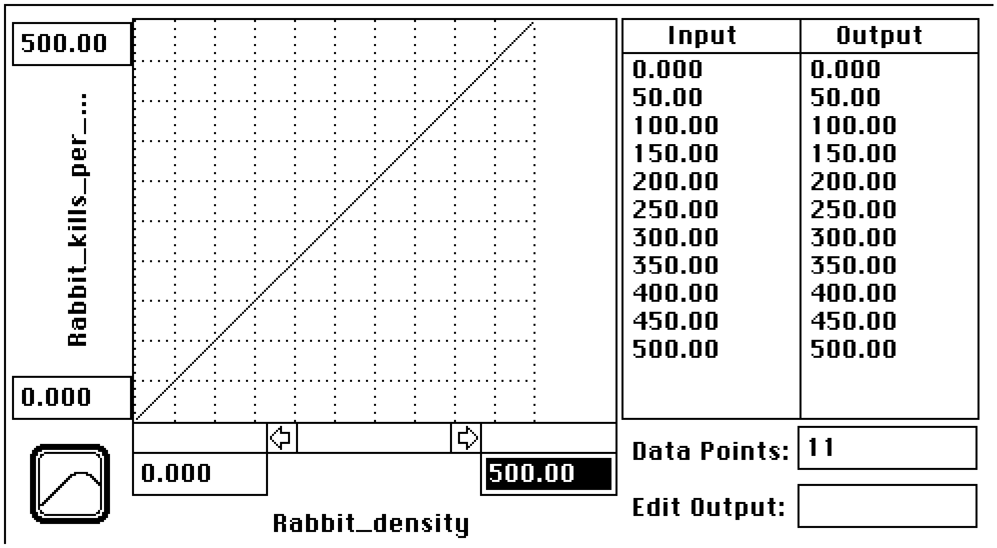
The parts that are left are a little complicated. You need to create definitions that are graphical. To do this for wolf death rate, double click on wolf death rate. Click on Rabbit density in the required inputs box, then click on become graph. Make the graph definition look like this.



What this means is that the wolf death rate depends upon the rabbit density. The fewer rabbits per given area the higher the death rate. The more rabbits per given area the lower the death rate. Click OK when you have finished this graphical definition.

Double-click on wolf deaths. Define the deaths to be wolf death rate * wolves.

To define rabbits killed per wolf, double-click on that converter. Click on Rabbit density in the required inputs box, then click on become graph. Make the graph look like this:



This means that as the number of rabbits increases, each wolf eats more rabbits in a year. Click OK when you have created this graphical definition.

Double-click on the rabbit deaths. Define the deaths to be rabbits killed per wolf * wolves.

Creating a graph of the results:

Select the graph icon in the upper part of the window by clicking on the icon once.

This is the graph icon

Place it near the diagram by clicking once. Double-click on the blank graph to get to the "define graph" window. Click on rabbits. Hold down the shift key and click on wolves. Click on the >> to copy them over into the Selected box. Click on rabbits in the selected box. Click on the double headed arrow to the <u>right</u> of rabbits so it looks like this:

Set the Min to 40000. Set the Max to 60000. Click on set.

Follow the same procedure to set wolves Min to 1000 and Max to 2000. Click on OK.
Now go to RUN/Time Specs in the top menu bar. Set the Simulation time from 0 to 100 with DT=1. Choose Runge-Kutta 4 as the calculation method.
Run the simulation. If you did everything right you should get 2 straight horizontal lines. Boring, huh! That means there are exactly the same number of rabbits dying and being born every year, and the same number of wolves dying and being born every year.

Now humans enter the picture. You are the manager of the "Fish and Wildlife Management Department" for the state of Oregon. The farmers and ranchers are angry because the wolves are eating some of their chickens and lambs. So they want open hunting season on the wolves. You sympathize with them but must also be aware of the environmentalists who don't want you to change anything. As a compromise you will allow a certain number of extra wolves to be killed starting in the 5th year of the simulation. You must decide on the number of wolves that can be killed and still have a balance in the ecosystem between the rabbits and the wolves.

To kill off extra wolves, double-click on wolf deaths. To what is already there **add**

wolf death rate * wolves + **pulse(___, 5, ___)**

In the 1st blank put the number of extra wolves to kill. In the 3rd blank put 1 if this number of wolves is to be killed every year, 2 if they are to be killed every other year, etc. Put in 999 if they are to be killed only once and no extra wolves are to be killed again for the next 200 years.

Try different values, running the simulation each time while looking at the graph. As long as the graph is an even (good), decreasing (best), or slowly increasing (ok) oscillation you have an acceptable balance.

Choose the best value for the number of wolves that can be hunted and killed each year (or whatever your time period is) and still maintain a balance in the ecosystem? Record your decision here ____.

In the past the government was not always known for its appropriate use of modeling to determine policy issues. They had come up with an arbitrary hunting policy for the wolves. They decided to set the number of wolves to kill at 10 every year. Use their numbers in this model. (You will have pulse(10,5,1)). Run the simulation. We will determine the equation for the old government's plan.

1. How many times in 50 years does the rabbit population reach a peak?__________ So the rabbits reach a peak every _________ years. (This is the period for the rabbit oscillation)
2. How many rabbits are there at the peak time near year 50. ___________ How many rabbits are there at the low point near year 50?_________ Subtract these two values and divide by 2. What did you get?___________ (This answer is the amplitude of the rabbit oscillation at that time.)
3. Choose the first high point after year 5 of the simulation. At what year of the simulation does this high point occur? _________ (This will be used as the starting point for our oscillation.)
4. What is the midpoint population for the rabbit oscillation? ___________ Calculate this point from the high and low points determined for question 2 above (near the 50 year mark). (Hint: a middle point is also considered an average or mean for the two numbers).
5. Write the equation for this oscillation as a cos function.
   Use the general form $Y = A \cos B(x-C) + D$ Where A = amplitude, $B = 2\pi/\text{period}$, C = starting value for the oscillation, D = midpoint of the oscillation. _________________________________

On another sheet of paper explain your policy and why you think your policy is better than the previous one. (Convince both the farmers/ranchers and the environmentalists). Determine the equation for your oscillation in the same way that the equation was determined for the old government's plan. Use each equation to explain the behavior of the ecosystem over time.